Overview

Introduction

Modelling parallel systems

Linear Time Properties

Regular Properties

Linear Temporal Logic (LTL)

- syntax and semantics of LTL
- automata-based LTL model checking $\longleftarrow$
- complexity of LTL model checking

Computation-Tree Logic

Equivalences and Abstraction

LTL model checking problem

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given: finite transition system $\mathcal{T}$ over AP
(without terminal states)
LTL-formula φ over AP

question: does $\mathcal{T} \models \varphi$ hold ?

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$$\pi \not\models \varphi, \text{ i.e., } \pi \models \neg\varphi$$

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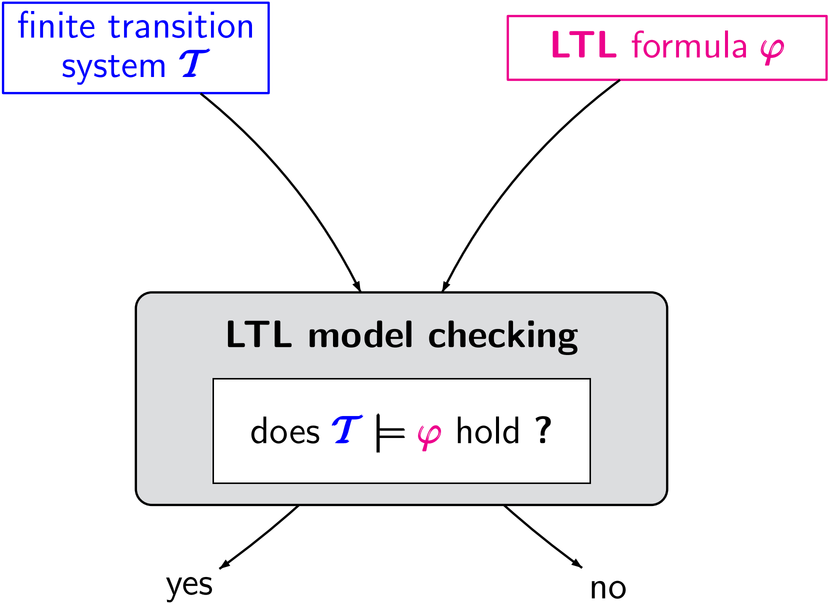
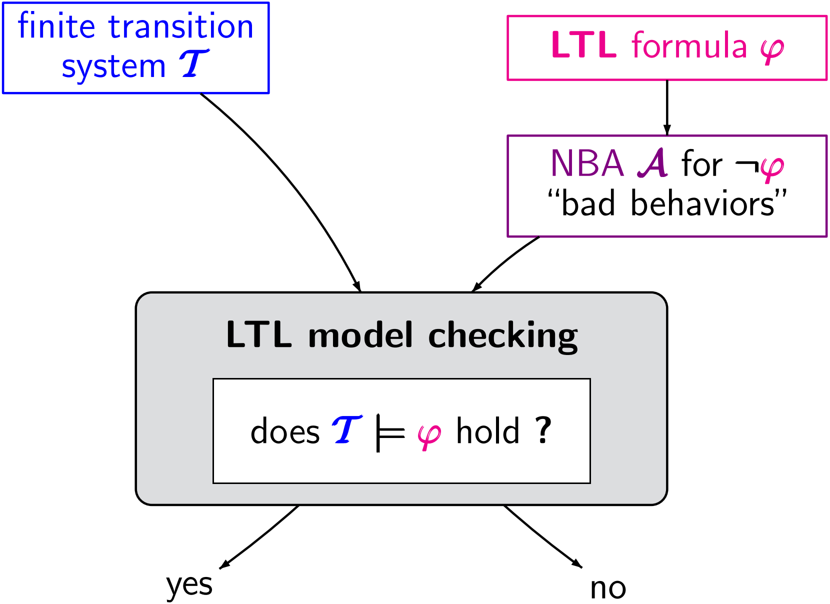
↑

construct the product-TS $\mathcal{T} \otimes \mathcal{A}$
search a path in the product that meets
the acceptance condition of $\mathcal{A}$


finite transition system $\mathcal{T}$
LTL formula φ
LTL model checking
does $\mathcal{T} \models \varphi$ hold **?**
yes
no

Automata-based LTL model checking



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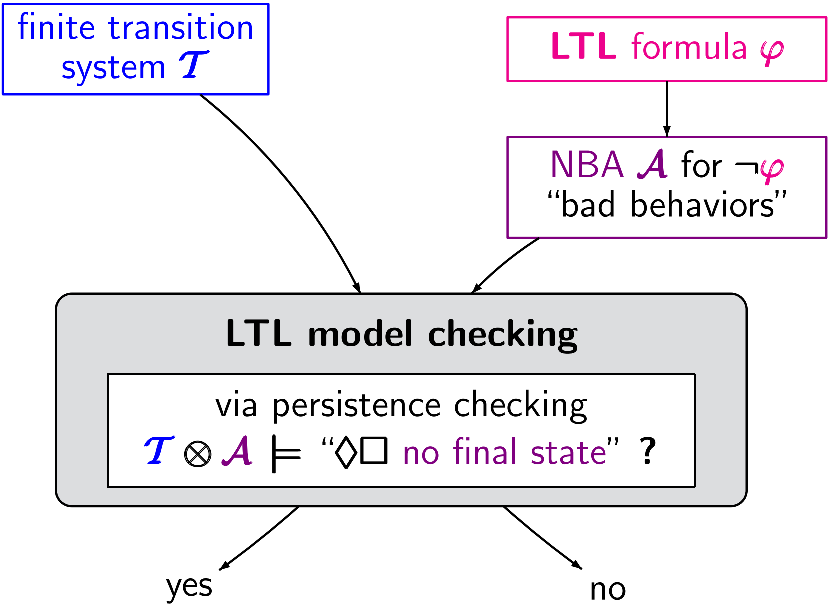

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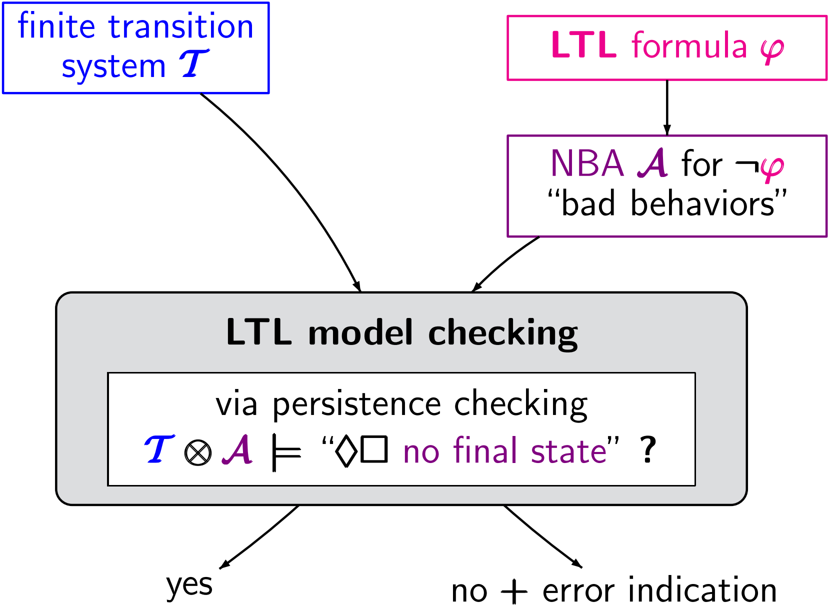

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$\mathcal{T} \models$ safety property E

iff $Traces_{fin}(\mathcal{T}) \cap \mathcal{L}(\mathcal{A}) = \varnothing$

where $\mathcal{A}$ is an NFA for the bad prefixes

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iff $\mathcal{T} \otimes \mathcal{A} \models \Diamond\Box \neg F$ $\longleftarrow$ persistence checking

Recall: nondeterministic Büchi automata

NBA $\mathcal{A} = (Q, \Sigma, \delta, Q_0, F)$

- Q finite set of states
- Σ alphabet
- $\delta : Q \times \Sigma \to 2^Q$ transition relation
- $Q_0 \subseteq Q$ set of initial states
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run for a word $A_0\, A_1\, A_2 \ldots \in \Sigma^\omega$:

state sequence $\pi = q_0\, q_1\, q_2 \ldots$ where $q_0 \in Q_0$
and $q_{i+1} \in \delta(q_i, A_i)$ for $i \geq 0$

run π is accepting if $\overset{\infty}{\exists}\, i \in \mathbb{N}.\ q_i \in F$

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From LTL to NBA

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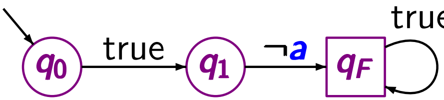
proof: ... later ...

NBA for LTL formulas



$\mathcal{L}_\omega(\mathcal{A}) =$ **?**

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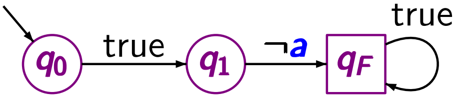
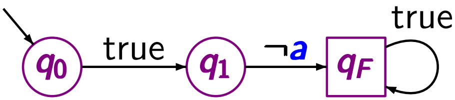


$$\mathcal{L}_\omega(\mathcal{A}) = \mathit{Words}(\bigcirc \neg a)$$



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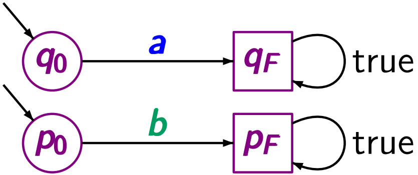

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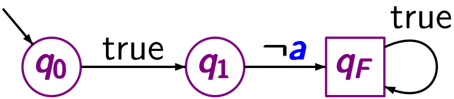

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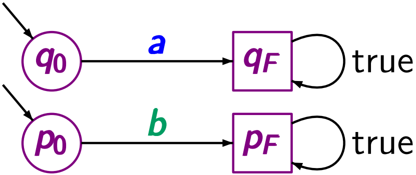

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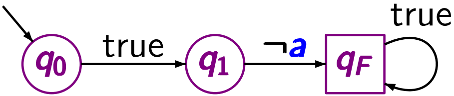

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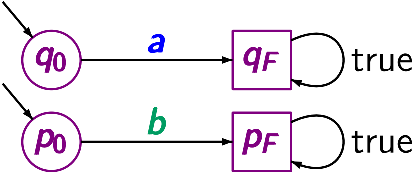

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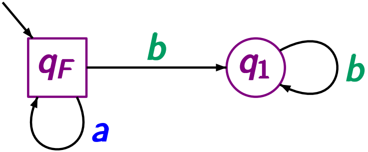

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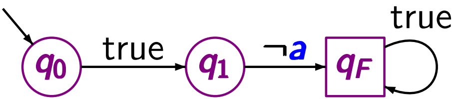

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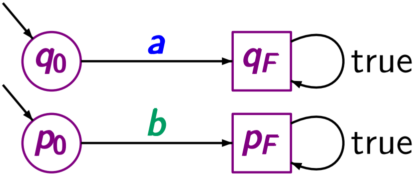

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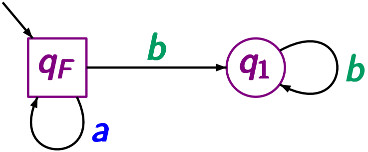

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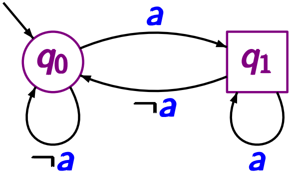

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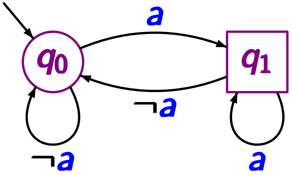

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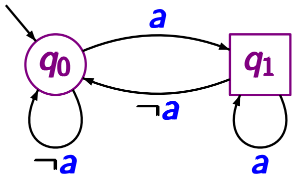

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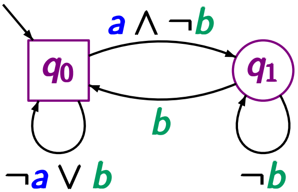

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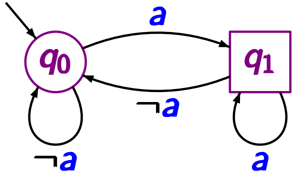

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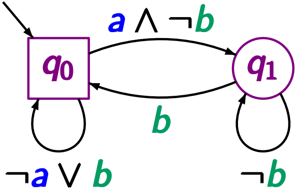

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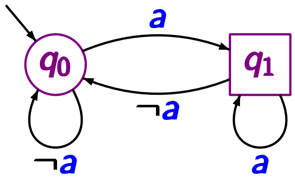
NBA for LTL formulas



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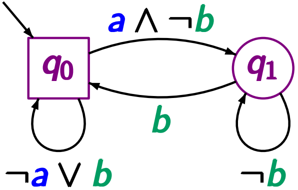

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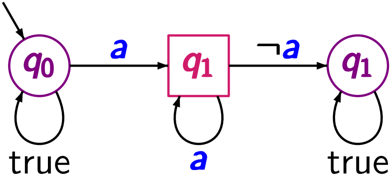

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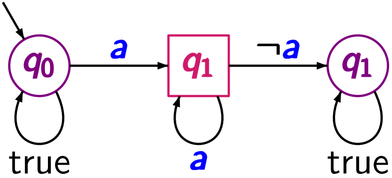

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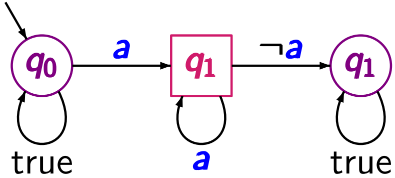

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possible runs for $\{a\}^\omega$

$q_0\ q_0\ q_0\ q_0\ q_0\ q_0\ \ldots$	not accepting
$q_0\ q_1\ q_1\ q_1\ q_1\ q_1\ \ldots$	accepting
$q_0\ q_0\ q_1\ q_1\ q_1\ q_1\ \ldots$	accepting
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$\vdots$	

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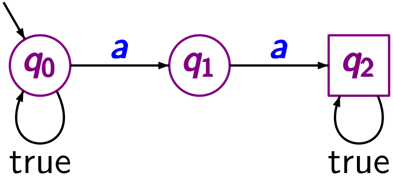
Example: $E \mathrel{\widehat{=}}$ "never a twice in a row"



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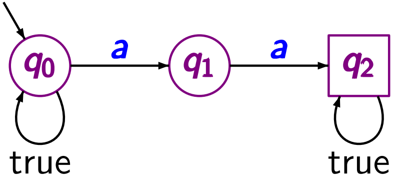
$$\mathcal{L}_\omega(\mathcal{A}) = \overline{E} = \left(2^{AP}\right)^\omega \setminus E = \mathit{Words}(\neg\varphi)$$

Example: $E \mathrel{\widehat{=}}$ "never a twice in a row"



$\varphi = \Box(a \rightarrow \bigcirc\neg a)$

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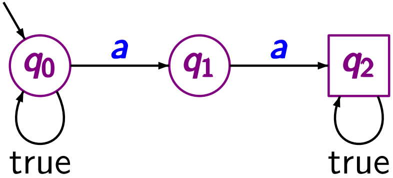

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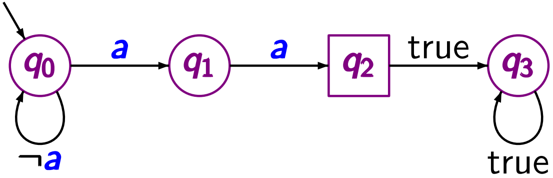

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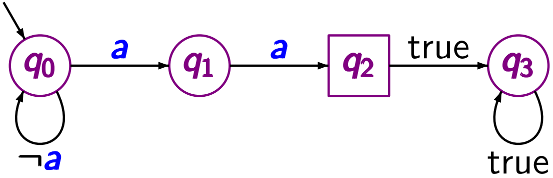

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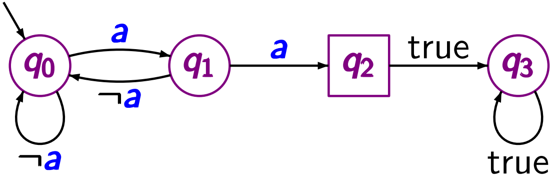

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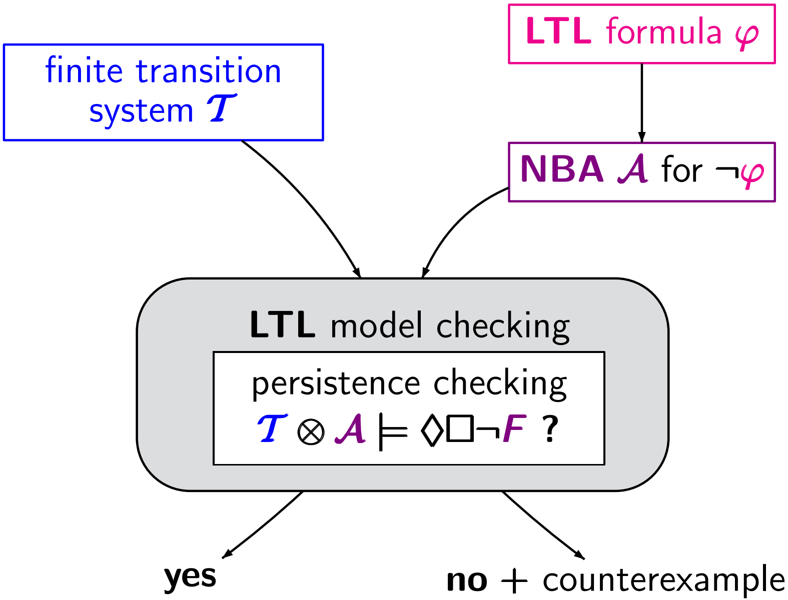
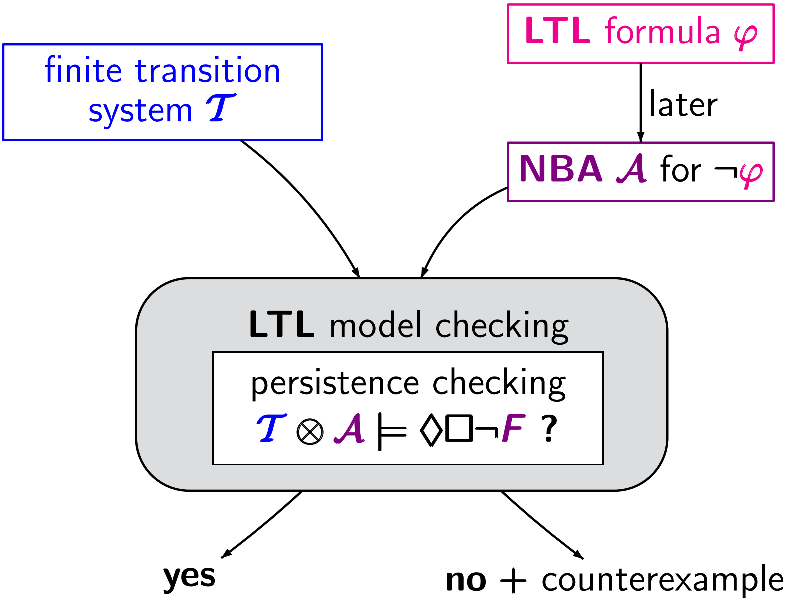


finite transition system $\mathcal{T}$
LTL formula φ
NBA $\mathcal{A}$ for $\neg\varphi$
LTL model checking
persistence checking
$\mathcal{T} \otimes \mathcal{A} \models \Diamond\Box\neg F$?
yes
no + counterexample

LTL model checking



Recall: product transition system

$\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$ TS without terminal states

$\mathcal{A} = (Q, 2^{AP}, \delta, Q_0, F)$ NBA or NFA

non-blocking, $Q_0 \cap F = \varnothing$

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product-TS $\mathcal{T} \otimes \mathcal{A} \overset{\text{def}}{=} (S \times Q, Act, \rightarrow', S_0', AP', L')$

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labeling: $AP' = Q$, $L'(\langle s, q\rangle) = \{q\}$

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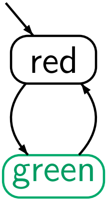
transition relation:

$$\frac{s \xrightarrow{\alpha} s' \ \wedge \ q' \in \delta(q, L(s'))}{\langle s, q \rangle \xrightarrow{\alpha}{}' \langle s', q' \rangle}$$

Example: LTL model checking

TS $\mathcal{T}$



LTL formula $\varphi = \Box\Diamond green$

Example: LTL model checking

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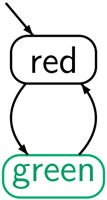

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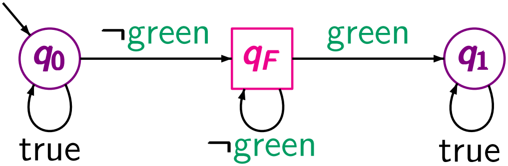
NBA $\mathcal{A}$ for the complement

$\neg\varphi \equiv \Diamond\Box\neg\text{green}$



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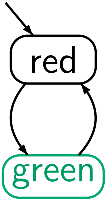

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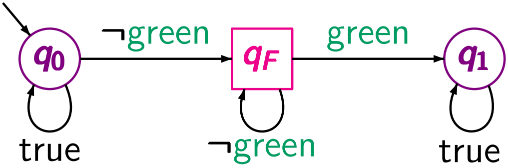
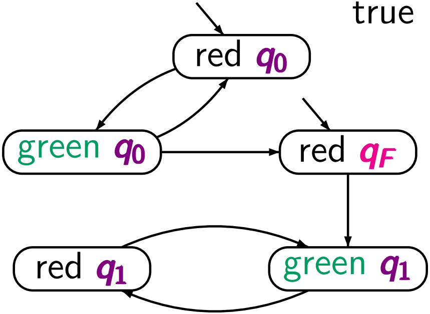
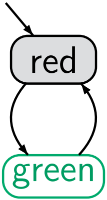




reachable fragment of the product TS $\mathcal{T} \otimes \mathcal{A}$

Example: LTL model checking

TS $\mathcal{T}$



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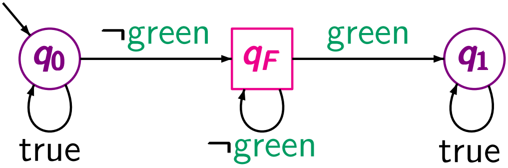
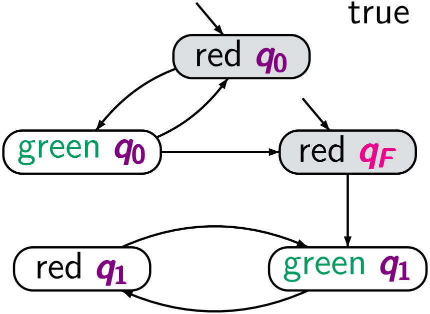




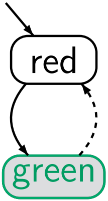
initial states:

$\langle \text{red}, q \rangle$ where

$$q \in \delta(q_0, L(\text{red}))$$
$$= \delta(q_0, \varnothing)$$
$$= \{q_0, q_F\}$$

Example: LTL model checking

TS $\mathcal{T}$



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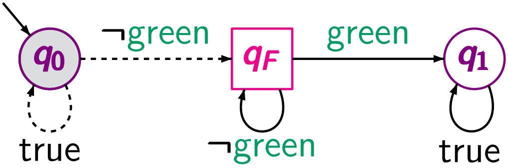
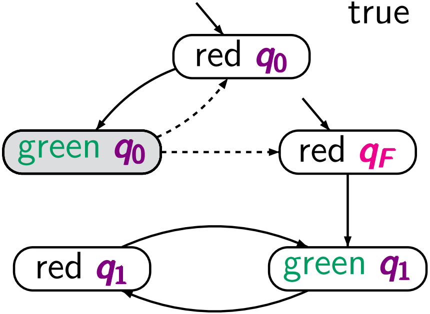




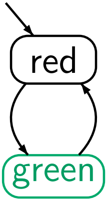
transition

$$\langle \text{green}, q_0 \rangle \rightarrow \langle \text{red}, q \rangle$$

$$\begin{aligned} q &\in \delta(q_0, L(\text{red})) \\ &= \delta(q_0, \varnothing) \\ &= \{q_0, q_F\} \end{aligned}$$

Example: LTL model checking

TS $\mathcal{T}$



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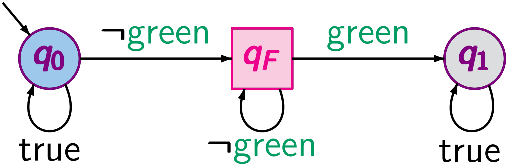
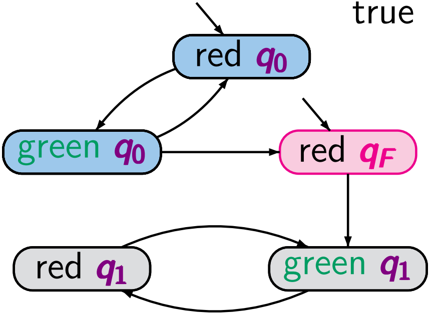


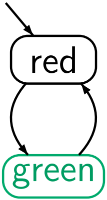


atomic propositions

$AP' = \{q_0, q_F, q_1\}$

obvious labeling function

TS $\mathcal{T}$



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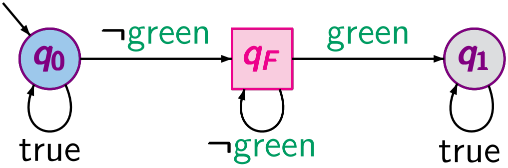
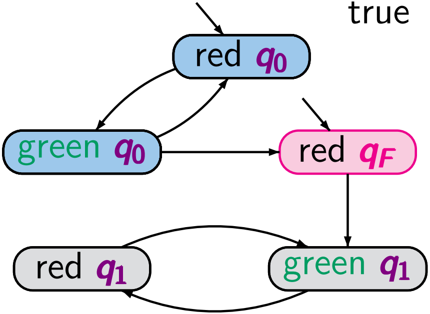
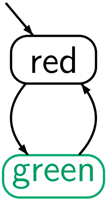




$\mathcal{T} \otimes \mathcal{A} \models \Diamond\Box\neg F$

Example: LTL model checking

TS $\mathcal{T}$



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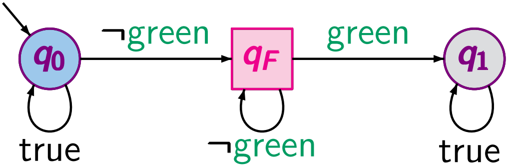
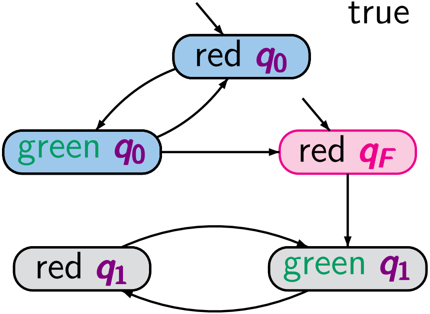


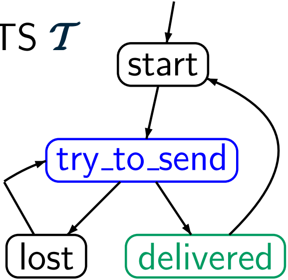


$\mathcal{T} \otimes \mathcal{A} \models \Diamond\Box\neg F$

hence: $\mathcal{T} \not\models \varphi$

Example: LTL model checking

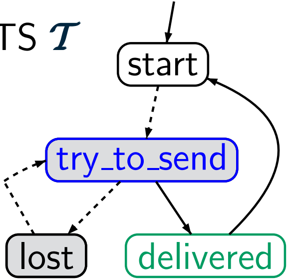
TS $\mathcal{T}$



LTL formula $\varphi = \Box(\boldsymbol{try} \to \Diamond \boldsymbol{del})$

"each (repeatedly) sent message will eventually be delivered"

TS $\mathcal{T}$



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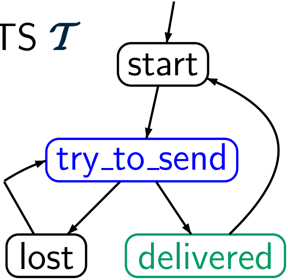
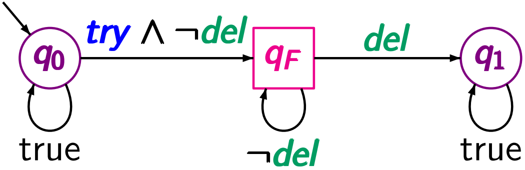


NBA $\mathcal{A}$ for $\neg\varphi \equiv \Diamond(try \wedge \Box\neg del)$



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Example: LTL model checking

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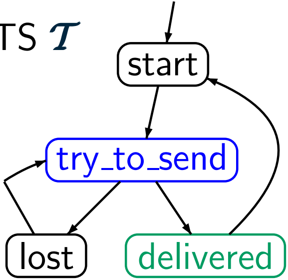

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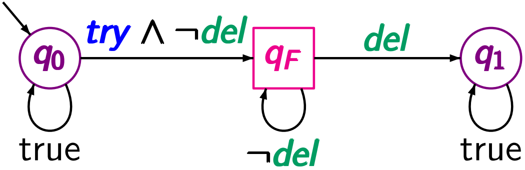
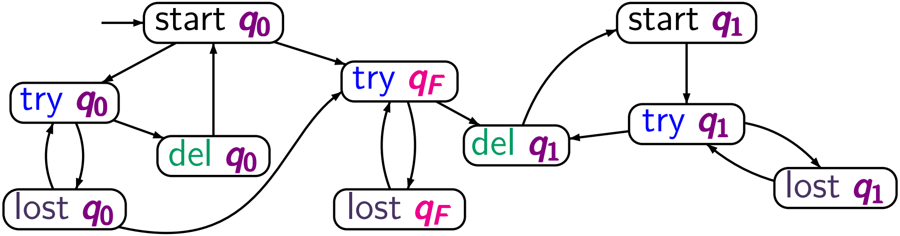
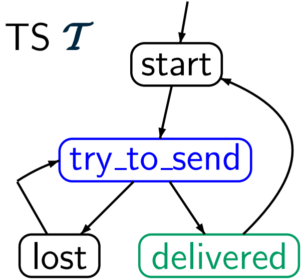




reachable fragment of the product-TS

Example: LTL model checking



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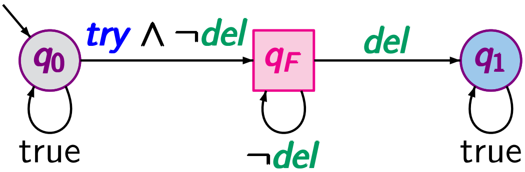
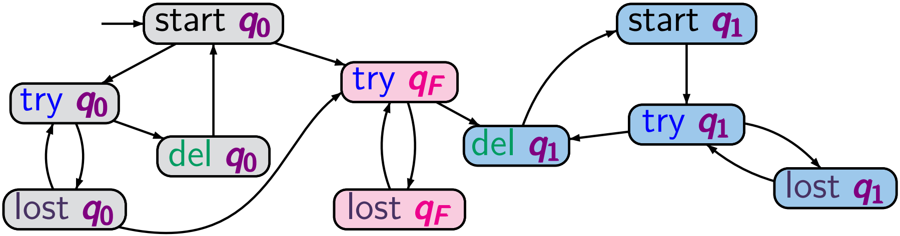
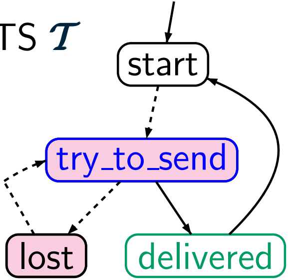




set of atomic propositions $AP' = \{q_0, q_1, q_F\}$

Example: LTL model checking

TS $\mathcal{T}$



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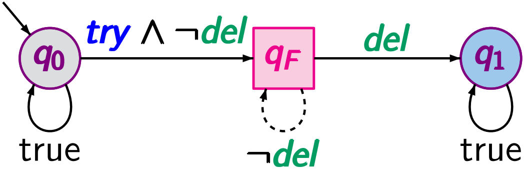
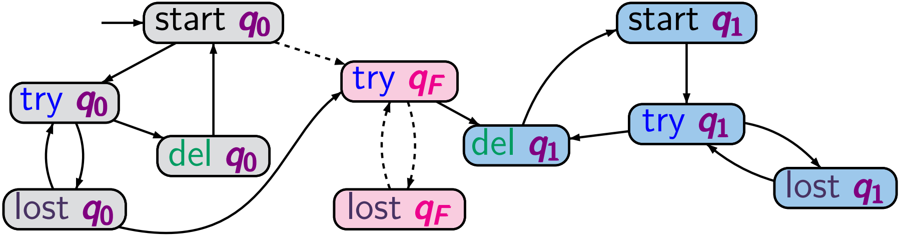


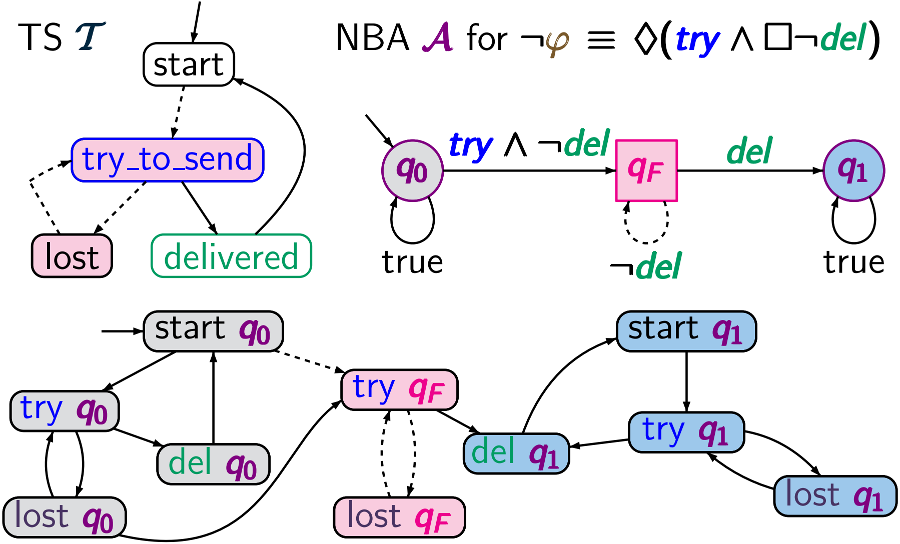


$$\mathcal{T} \otimes \mathcal{A} \not\models \Diamond\Box\neg F$$

Example: LTL model checking

TS $\mathcal{T}$

NBA $\mathcal{A}$ for $\neg\varphi \equiv \Diamond(\mathit{try} \wedge \Box\neg \mathit{del})$



$\mathcal{T} \otimes \mathcal{A} \not\models \Diamond\Box\neg F$ hence: $\mathcal{T} \not\models \varphi$

LTL model checking

given: finite TS $\mathcal{T}$, LTL-formula φ

question: does $\mathcal{T} \models \varphi$ hold **?**

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question: does $\mathcal{T} \models \varphi$ hold **?**

construct an NBA $\mathcal{A}$ for $\neg\varphi$ and the product $\mathcal{T} \otimes \mathcal{A}$

check whether $\mathcal{T} \otimes \mathcal{A} \models \Diamond\Box\neg F$

given: finite TS $\mathcal{T}$, LTL-formula φ

question: does $\mathcal{T} \models \varphi$ hold **?**

construct an NBA $\mathcal{A}$ for $\neg\varphi$ and the product $\mathcal{T} \otimes \mathcal{A}$

check whether $\mathcal{T} \otimes \mathcal{A} \models \Diamond\Box\neg F$ $\longleftarrow$ persistence checking nested **DFS**

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construct an NBA $\mathcal{A}$ for $\neg\varphi$ and the product $\mathcal{T} \otimes \mathcal{A}$

check whether $\mathcal{T} \otimes \mathcal{A} \models \Diamond\Box\neg F$ $\longleftarrow$ persistence checking nested **DFS**

IF $\mathcal{T} \otimes \mathcal{A} \models \Diamond\Box\neg F$

THEN return "**yes**"

ELSE compute a counterexample

$$\langle s_0, p_0\rangle \ldots \langle s_n, p_n\rangle \ldots \langle s_n, p_n\rangle$$

for $\mathcal{T} \otimes \mathcal{A}$ and $\Diamond\Box\neg F$

return "**no**" and $s_0 \ldots s_n \ldots s_n$

Complexity of LTL model checking

given: finite TS $\mathcal{T}$, LTL-formula φ

question: does $\mathcal{T} \models \varphi$ hold ?

~~construct an NBA~~ $\mathcal{A}$ for $\neg\varphi$ and the product $\mathcal{T} \otimes \mathcal{A}$

check whether $\mathcal{T} \otimes \mathcal{A} \models \Diamond\Box\neg F$ ⟵ persistence checking nested **DFS**

IF $\mathcal{T} \otimes \mathcal{A} \models \Diamond\Box\neg F$

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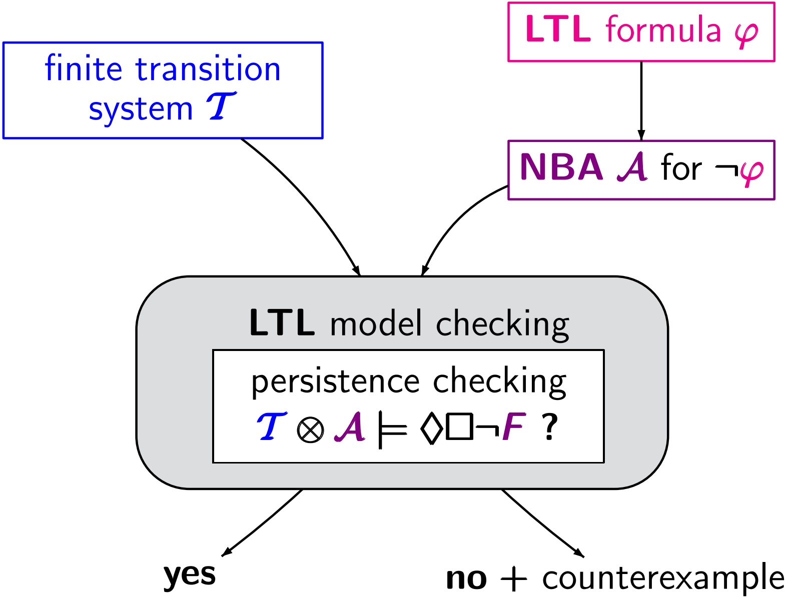
$\langle s_0, p_0 \rangle \ldots \langle s_n, p_n \rangle \ldots \langle s_n, p_n \rangle$

for $\mathcal{T} \otimes \mathcal{A}$ and $\Diamond\Box\neg F$

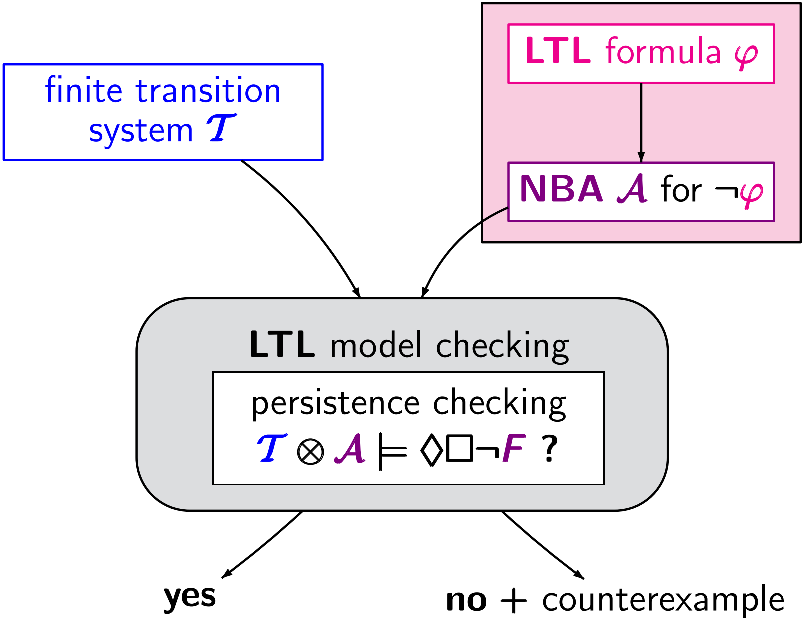
return "**no**" and $s_0 \ldots s_n \ldots s_n$

time complexity: $\mathcal{O}(\mathit{size}(\mathcal{T}) \cdot \mathit{size}(\mathcal{A}))$

LTL model checking



LTL model checking



From LTL to NBA

From LTL to NBA

For each **LTL** formula φ there is an **NBA** $\mathcal{A}$ s.t.

$$\mathcal{L}_\omega(\mathcal{A}) \;=\; \mathit{Words}(\varphi)$$

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For each **LTL** formula φ there is an **NBA** $\mathcal{A}$ s.t.

$$\mathcal{L}_\omega(\mathcal{A}) = \mathit{Words}(\varphi)$$

$$\boxed{\textbf{LTL formula } \varphi} \longrightarrow \boxed{\begin{array}{c}\textbf{NBA } \mathcal{A} \text{ s.t.}\\ \mathcal{L}_\omega(\mathcal{A}) = \mathit{Words}(\varphi)\end{array}}$$

nondeterministic
Büchi automaton

From LTL to NBA

For each **LTL** formula φ there is an **NBA** $\mathcal{A}$ s.t.

$$\mathcal{L}_\omega(\mathcal{A}) = \mathit{Words}(\varphi)$$

LTL formula φ

$\downarrow$

GNBA $\mathcal{G}$ s.t. $\mathcal{L}_\omega(\mathcal{G}) = \mathit{Words}(\varphi)$ — generalized NBA, several acceptance sets

$\downarrow$

NBA $\mathcal{A}$ s.t. $\mathcal{L}_\omega(\mathcal{A}) = \mathcal{L}_\omega(\mathcal{G})$ — nondeterministic Büchi automaton, **1** acceptance set

From LTL to NBA

For each **LTL** formula φ there is an **NBA** $\mathcal{A}$ s.t.

$$\mathcal{L}_\omega(\mathcal{A}) \;=\; Words(\varphi)$$

LTL formula φ

↓

GNBA $\mathcal{G}$ s.t.
$\mathcal{L}_\omega(\mathcal{G}) = Words(\varphi)$

generalized NBA
k acceptance sets

↓ k copies of $\mathcal{G}$

NBA $\mathcal{A}$ s.t.
$\mathcal{L}_\omega(\mathcal{A}) = \mathcal{L}_\omega(\mathcal{G})$

nondeterministic
Büchi automaton
1 acceptance set

Encoding of LTL semantics in a GNBA

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idea: encode the semantics of the operators appearing in φ by appropriate components of the GNBA $\mathcal{G}$

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semantics of ...	encoding
propositional logic ***true***, $\neg$, $\wedge$	
next $\bigcirc$	
until $\mathbf{U}$	

Encoding of LTL semantics in a GNBA

idea: encode the semantics of the operators appearing in φ by appropriate components of the GNBA $\mathcal{G}$

semantics of ...	encoding
propositional logic *true*, $\neg$, $\wedge$	in the states
next $\bigcirc$	
until U	

Encoding of LTL semantics in a GNBA

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semantics of ...	encoding
propositional logic ***true***, $\neg$, $\wedge$	in the states
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until **U**	

Encoding of LTL semantics in a GNBA

idea: encode the semantics of the operators appearing in φ by appropriate components of the GNBA $\mathcal{G}$

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Encoding of LTL semantics in a GNBA

idea: encode the semantics of the operators appearing in φ by appropriate components of the GNBA $\mathcal{G}$

semantics of ...	encoding
propositional logic ***true***, $\neg$, $\wedge$	in the states
next $\bigcirc$	in the transition relation
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$$\psi_1 \,\mathsf{U}\, \psi_2 \;\equiv\; \psi_2 \vee \big(\psi_1 \wedge \bigcirc(\psi_1 \,\mathsf{U}\, \psi_2)\big)$$

Encoding of LTL semantics in a GNBA

idea: encode the semantics of the operators appearing in φ by appropriate components of the GNBA $\mathcal{G}$

semantics of ...	encoding
propositional logic *true*, $\neg$, $\wedge$	in the states
next $\bigcirc$	in the transition relation
until U	via expansion law

$$\psi_1 \,\mathsf{U}\, \psi_2 \;\equiv\; \psi_2 \vee (\psi_1 \wedge \bigcirc(\psi_1 \,\mathsf{U}\, \psi_2))$$

encoded in the states

Encoding of LTL semantics in a GNBA

idea: encode the semantics of the operators appearing in φ by appropriate components of the GNBA $\mathcal{G}$

semantics of ...	encoding
propositional logic *true*, $\neg$, $\wedge$	in the states
next $\bigcirc$	in the transition relation
until U	via expansion law

$$\psi_1 \,\mathsf{U}\, \psi_2 \;\equiv\; \psi_2 \vee \big(\psi_1 \wedge \bigcirc(\psi_1 \,\mathsf{U}\, \psi_2)\big)$$

encoded in the states

encoded in the transition relation

Encoding of LTL semantics in a GNBA

idea: encode the semantics of the operators appearing in φ by appropriate components of the GNBA $\mathcal{G}$

semantics of ...	encoding
propositional logic *true*, $\neg$, $\wedge$	in the states
next $\bigcirc$	in the transition relation
until U	expansion law, **least fixed point**

$$\psi_1 \,\mathsf{U}\, \psi_2 \;\equiv\; \psi_2 \vee (\psi_1 \wedge \bigcirc(\psi_1 \,\mathsf{U}\, \psi_2))$$

encoded in the states

encoded in the transition relation

acceptance condition

LTL $\rightsquigarrow$ GNBA

LTL formula $\varphi \rightsquigarrow$ GNBA $\mathcal{G}$ for *Words*(φ)

LTL formula $\varphi \rightsquigarrow$ GNBA $\mathcal{G}$ for $\mathit{Words}(\varphi)$

states of $\mathcal{G} \quad \widehat{=} \quad$ (certain) sets of subformulas of φ

LTL formula $\varphi \rightsquigarrow$ GNBA $\mathcal{G}$ for $\mathit{Words}(\varphi)$

states of $\mathcal{G}$ $\widehat{=}$ (certain) sets of subformulas of φ s.t. each word $\sigma = A_0 A_1 A_2 \ldots \in \mathit{Words}(\varphi)$ can be extended to an accepting run $B_0 B_1 B_2 \ldots$ in $\mathcal{G}$

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states of $\mathcal{G}$ $\widehat{=}$ (certain) sets of subformulas of φ s.t. each word $\sigma = A_0\, A_1\, A_2 \ldots \in Words(\varphi)$ can be extended to an accepting run $B_0\, B_1\, B_2 \ldots$ in $\mathcal{G}$

$A_0 \quad A_1 \quad A_2 \quad A_3 \quad \ldots \quad \in Words(\varphi)$

LTL formula $\varphi \rightsquigarrow$ GNBA $\mathcal{G}$ for $Words(\varphi)$

states of $\mathcal{G}$ $\hat{=}$ (certain) sets of subformulas of φ s.t. each word $\sigma = A_0 A_1 A_2 \ldots \in Words(\varphi)$ can be extended to an accepting run $B_0 B_1 B_2 \ldots$ in $\mathcal{G}$

$$\begin{array}{cccccl} A_0 & A_1 & A_2 & A_3 & \ldots & \in Words(\varphi) \\ \downarrow & \downarrow & \downarrow & \downarrow & & \\ B_0 & B_1 & B_2 & B_3 & \ldots & \text{accepting run} \end{array}$$

where $B_i = \{\psi \in cl(\varphi) : A_i A_{i+1} A_{i+2} \ldots \models \psi\}$

LTL formula $\varphi \rightsquigarrow$ GNBA $\mathcal{G}$ for $Words(\varphi)$

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$$\begin{array}{ccccccl} A_0 & A_1 & A_2 & A_3 & \ldots & \in & Words(\varphi) \\ \downarrow & \downarrow & \downarrow & \downarrow & & & \\ B_0 & B_1 & B_2 & B_3 & \ldots & & \text{accepting run} \end{array}$$

where $B_i = \{\psi \in cl(\varphi) : A_i A_{i+1} A_{i+2} \ldots \models \psi\}$

set of subformulas of φ and their negations

LTL $\rightsquigarrow$ GNBA

LTL formula $\varphi \rightsquigarrow$ GNBA $\mathcal{G}$ for $Words(\varphi)$

states of $\mathcal{G} \;\; \widehat{=} \;\;$ (certain) sets of subformulas of φ
s.t. each word $\sigma = A_0 A_1 A_2 \ldots \in Words(\varphi)$ can be extended to an accepting run $B_0 B_1 B_2 \ldots$ in $\mathcal{G}$

Example: $\varphi \;=\; a \, \mathsf{U} (\neg a \wedge b)$

LTL $\rightsquigarrow$ GNBA

LTL formula $\varphi \rightsquigarrow$ GNBA $\mathcal{G}$ for $\mathit{Words}(\varphi)$

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Example: $\varphi = a\, \mathsf{U} (\neg a \wedge b)$

$$\{a\} \quad \{a\} \quad \{a, b\} \quad \{b\} \quad \varnothing \quad \varnothing \quad \ldots \models \varphi$$

LTL formula $\varphi \rightsquigarrow$ GNBA $\mathcal{G}$ for $Words(\varphi)$

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Example: $\varphi \;=\; a\, \mathsf{U}(\neg a \wedge b)$

$$\begin{array}{ccccccc} \{a\} & \{a\} & \{a,b\} & \{b\} & \varnothing & \varnothing & \ldots \models \varphi \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \\ B_0 & B_1 & B_2 & B_3 & B_4 & B_5 & \end{array}$$

LTL formula $\varphi \rightsquigarrow$ GNBA $\mathcal{G}$ for $Words(\varphi)$

states of $\mathcal{G}$ $\widehat{=}$ (certain) sets of subformulas of φ s.t. each word $\sigma = A_0\, A_1\, A_2 \ldots \in Words(\varphi)$ can be extended to an accepting run $B_0\, B_1\, B_2 \ldots$ in $\mathcal{G}$

Example: $\varphi = a\, \mathsf{U}(\neg a \wedge b)$ $\qquad \psi = \neg a \wedge b$

$$\begin{array}{cccccccc} \{a\} & \{a\} & \{a,b\} & \{b\} & \varnothing & \varnothing & \ldots \models \varphi \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \\ B_0 & B_1 & B_2 & B_3 & B_4 & B_5 & \end{array}$$

where the B_i's are subsets of $\{a, \neg a, b, \neg b, \psi, \neg\psi, \varphi, \neg\varphi\}$

LTL formula $\varphi \rightsquigarrow$ GNBA $\mathcal{G}$ for $Words(\varphi)$

states of $\mathcal{G}$ $\widehat{=}$ (certain) sets of subformulas of φ s.t. each word $\sigma = A_0 A_1 A_2 \ldots \in Words(\varphi)$ can be extended to an accepting run $B_0 B_1 B_2 \ldots$ in $\mathcal{G}$

Example: $\varphi = a \, \mathsf{U} (\neg a \wedge b)$ $\qquad \psi = \neg a \wedge b$

$$\{a\} \quad \{a\} \quad \{a, b\} \quad \{b\} \quad \varnothing \quad \varnothing \quad \ldots \models \varphi$$

$$\downarrow$$

$$\boxed{\begin{array}{r} a \\ \neg b \\ \neg \psi \\ \varphi \end{array}}$$

just for better readability:
tuple rather than set notation

LTL $\rightsquigarrow$ GNBA

LTL formula $\varphi \rightsquigarrow$ GNBA $\mathcal{G}$ for $Words(\varphi)$

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Example: $\varphi = a\,\mathsf{U}(\neg a \wedge b)$ $\qquad \psi = \neg a \wedge b$

$$\begin{array}{ccccccccc}
\{a\} & \{a\} & \{a,b\} & \{b\} & \varnothing & \varnothing & \ldots & \models \varphi \\
\downarrow & \downarrow & & & & & & \\
\boxed{\begin{array}{r} a \\ \neg b \\ \neg\psi \\ \varphi \end{array}} & \boxed{\begin{array}{r} a \\ \neg b \\ \neg\psi \\ \varphi \end{array}} & & & & & &
\end{array}$$

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LTL formula $\varphi \rightsquigarrow$ GNBA $\mathcal{G}$ for $\mathit{Words}(\varphi)$

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Example: $\varphi = a\,\mathsf{U}(\neg a \wedge b)$ $\qquad \psi = \neg a \wedge b$

$$
\begin{array}{cccccccl}
\{a\} & \{a\} & \{a, b\} & \{b\} & \varnothing & \varnothing & \ldots & \models \varphi \\
\downarrow & \downarrow & \downarrow & & & & & \\
\boxed{\begin{array}{r} a \\ \neg b \\ \neg\psi \\ \varphi \end{array}} & \boxed{\begin{array}{r} a \\ \neg b \\ \neg\psi \\ \varphi \end{array}} & \boxed{\begin{array}{r} a \\ b \\ \neg\psi \\ \varphi \end{array}} & & & & &
\end{array}
$$

LTL formula $\varphi \rightsquigarrow$ GNBA $\mathcal{G}$ for $\mathit{Words}(\varphi)$

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$$
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\{a\} & \{a\} & \{a,b\} & \{b\} & \varnothing & \varnothing & \ldots & \models \varphi \\
\downarrow & \downarrow & \downarrow & \downarrow & & & & \\
\boxed{\begin{array}{r} a \\ \neg b \\ \neg\psi \\ \varphi \end{array}} & \boxed{\begin{array}{r} a \\ \neg b \\ \neg\psi \\ \varphi \end{array}} & \boxed{\begin{array}{r} a \\ b \\ \neg\psi \\ \varphi \end{array}} & \boxed{\begin{array}{r} \neg a \\ b \\ \psi \\ \varphi \end{array}} & & & &
\end{array}
$$

LTL formula $\varphi \rightsquigarrow$ GNBA $\mathcal{G}$ for $Words(\varphi)$

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$$
\begin{array}{ccccccccc}
\{a\} & \{a\} & \{a,b\} & \{b\} & \varnothing & \varnothing & \ldots & \models \varphi \\
\downarrow & \downarrow & \downarrow & \downarrow & \downarrow & & & \\
\boxed{\begin{array}{r} a \\ \neg b \\ \neg\psi \\ \varphi \end{array}} & \boxed{\begin{array}{r} a \\ \neg b \\ \neg\psi \\ \varphi \end{array}} & \boxed{\begin{array}{r} a \\ b \\ \neg\psi \\ \varphi \end{array}} & \boxed{\begin{array}{r} \neg a \\ b \\ \psi \\ \varphi \end{array}} & \boxed{\begin{array}{r} \neg a \\ \neg b \\ \neg\psi \\ \neg\varphi \end{array}} & & &
\end{array}
$$

LTL $\rightsquigarrow$ GNBA

LTL formula $\varphi \rightsquigarrow$ GNBA $\mathcal{G}$ for $\mathit{Words}(\varphi)$

states of $\mathcal{G}$ $\widehat{=}$ (certain) sets of subformulas of φ s.t. each word $\sigma = A_0\, A_1\, A_2 \ldots \in \mathit{Words}(\varphi)$ can be extended to an accepting run $B_0\, B_1\, B_2 \ldots$ in $\mathcal{G}$

Example: $\varphi = a\, \mathsf{U} (\neg a \wedge b)$ $\qquad \psi = \neg a \wedge b$

$\{a\}$	$\{a\}$	$\{a, b\}$	$\{b\}$	$\varnothing$	$\varnothing$	$\ldots$	$\models \varphi$
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$		
a $\neg b$ $\neg\psi$ φ	a $\neg b$ $\neg\psi$ φ	a b $\neg\psi$ φ	$\neg a$ b ψ φ	$\neg a$ $\neg b$ $\neg\psi$ $\neg\varphi$	$\neg a$ $\neg b$ $\neg\psi$ $\neg\varphi$	$\ldots$	

Closure of LTL formulas

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Let φ be an LTL formula. Then:

$$\mathit{subf}(\varphi) \overset{\text{def}}{=} \text{set of all subformulas of } \varphi$$

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where ψ and $\neg\neg\psi$ are identified

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Example: if $\varphi = a\,\mathsf{U}(\neg a \wedge b)$ then

$$cl(\varphi) = \{a, b, \neg a \wedge b, \varphi\} \cup \{\neg a, \neg b, \neg(\neg a \wedge b), \neg\varphi\}$$

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Example: if $\varphi' = \Box a$

Closure of LTL formulas

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where ψ and $\neg\neg\psi$ are identified

Example: if $\varphi = a \,\mathsf{U}\, (\neg a \wedge b)$ then

$$\mathit{cl}(\varphi) = \{a, b, \neg a \wedge b, \varphi\} \cup \{\neg a, \neg b, \neg(\neg a \wedge b), \neg\varphi\}$$

Example: if $\varphi' = \Box a = \neg\Diamond\neg a = \neg(\mathit{true}\,\mathsf{U}\,\neg a)$

Let φ be an LTL formula. Then:

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Example: if $\varphi = a \,\mathsf{U}(\neg a \wedge b)$ then

$$cl(\varphi) = \{a, b, \neg a \wedge b, \varphi\} \cup \{\neg a, \neg b, \neg(\neg a \wedge b), \neg\varphi\}$$

Example: if $\varphi' = \Box a = \neg\Diamond\neg a = \neg(\mathit{true}\,\mathsf{U}\,\neg a)$ then

$$cl(\varphi') = \{a, \neg a, \mathit{true}, \neg\mathit{true}, \Box a, \neg\Box a\}$$

Elementary formula-sets

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Let $B \subseteq cl(\varphi)$. B is called elementary if:

Elementary formula-sets

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(1) B is consistent w.r.t. propositional logic

(2) B is maximal consistent

(3) B is locally consistent with respect to until $\mathbf{U}$:

Let $B \subseteq cl(\varphi)$. B is called elementary if:

(1) B is consistent w.r.t. propositional logic

if $\psi \in B$ then $\neg\psi \notin B$

(2) B is maximal consistent

(3) B is locally consistent with respect to until $\mathbf{U}$:

Let $B \subseteq cl(\varphi)$. B is called elementary if:

(1) B is consistent w.r.t. propositional logic

if $\psi \in B$ then $\neg\psi \notin B$

if $\psi_1 \wedge \psi_2 \in B$ then $\neg\psi_1 \notin B$ and $\neg\psi_2 \notin B$

(2) B is maximal consistent

(3) B is locally consistent with respect to until $\mathbf{U}$:

Elementary formula-sets

Let $B \subseteq cl(\varphi)$. B is called elementary if:

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if $\psi_1 \wedge \psi_2 \in B$ then $\neg\psi_1 \notin B$ and $\neg\psi_2 \notin B$
if $\psi_1 \in B$ and $\psi_2 \in B$ then $\neg(\psi_1 \wedge \psi_2) \notin B$

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Elementary formula-sets

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if $\psi_1 \wedge \psi_2 \in B$ then $\neg\psi_1 \notin B$ and $\neg\psi_2 \notin B$
if $\psi_1 \in B$ and $\psi_2 \in B$ then $\neg(\psi_1 \wedge \psi_2) \notin B$
if $\mathit{false} \in cl(\varphi)$ then $\mathit{false} \notin B$

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Elementary formula-sets

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if $\psi \in cl(\varphi) \setminus B$ then $\neg\psi \in B$

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Let $B \subseteq cl(\varphi)$. B is called elementary if:

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if $\psi \in B$ then $\neg\psi \notin B$
if $\psi_1 \wedge \psi_2 \in B$ then $\neg\psi_1 \notin B$ and $\neg\psi_2 \notin B$
if $\psi_1 \in B$ and $\psi_2 \in B$ then $\neg(\psi_1 \wedge \psi_2) \notin B$
if $\mathit{false} \in cl(\varphi)$ then $\mathit{false} \notin B$

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if $\psi \in cl(\varphi) \setminus B$ then $\neg\psi \in B$

(3) B is locally consistent with respect to until $\mathbf{U}$:

if $\psi_1 \,\mathbf{U}\, \psi_2 \in B$ and $\neg\psi_2 \in B$ then $\neg\psi_1 \notin B$

Elementary formula-sets

Let $B \subseteq cl(\varphi)$. B is called elementary if:

(1) B is consistent w.r.t. propositional logic

if $\psi \in B$ then $\neg\psi \notin B$
if $\psi_1 \wedge \psi_2 \in B$ then $\neg\psi_1 \notin B$ and $\neg\psi_2 \notin B$
if $\psi_1 \in B$ and $\psi_2 \in B$ then $\neg(\psi_1 \wedge \psi_2) \notin B$
if $\mathit{false} \in cl(\varphi)$ then $\mathit{false} \notin B$

(2) B is maximal consistent

if $\psi \in cl(\varphi) \setminus B$ then $\neg\psi \in B$

(3) B is locally consistent with respect to until U:

if $\psi_1 \mathsf{U} \psi_2 \in B$ and $\neg\psi_2 \in B$ then $\neg\psi_1 \notin B$

if $\psi_2 \in B$ and $\psi_1 \mathsf{U} \psi_2 \in cl(\varphi)$ then $\neg(\psi_1 \mathsf{U} \psi_2) \notin B$

Elementary formula-sets

$B \subseteq cl(\varphi)$ is elementary iff:

(i) B is maximal consistent w.r.t. prop. logic, i.e., if ψ, $\psi_1 \wedge \psi_2 \in cl(\varphi)$ then:

$$\boxed{\begin{array}{rcl} \psi \notin B & \text{iff} & \neg\psi \in B \\ \psi_1 \wedge \psi_2 \in B & \text{iff} & \psi_1 \in B \text{ and } \psi_2 \in B \\ \mathit{true} \in cl(\varphi) & \text{implies} & \mathit{true} \in B \end{array}}$$

(ii) B is locally consistent with respect to until $\mathbf{U}$, i.e., if $\psi_1 \mathbf{U} \psi_2 \in cl(\varphi)$ then:

$$\boxed{\begin{array}{l} \text{if } \psi_1 \mathbf{U} \psi_2 \in B \text{ and } \psi_2 \notin B \text{ then } \psi_1 \in B \\ \text{if } \psi_2 \in B \text{ then } \psi_1 \mathbf{U} \psi_2 \in B \end{array}}$$

Elementary or not?

Let $\varphi = a \, \mathsf{U} (\neg a \wedge b)$.

$B_1 = \{a, b, \neg a \wedge b, \varphi\}$

Elementary or not?

Let $\varphi = a \, \mathsf{U} (\neg a \wedge b)$.

$B_1 = \{a, b, \neg a \wedge b, \varphi\}$ not elementary

propositional inconsistent

Elementary or not?

Let $\varphi = a \, \mathsf{U}(\neg a \wedge b)$.

$B_1 = \{a, b, \neg a \wedge b, \varphi\}$ not elementary propositional inconsistent

$B_2 = \{\neg a, b, \varphi\}$

Elementary or not?

Let $\varphi = a \, \mathsf{U}(\neg a \wedge b)$.

$B_1 = \{a, b, \neg a \wedge b, \varphi\}$ not elementary
propositional inconsistent

$B_2 = \{\neg a, b, \varphi\}$ not elementary, not maximal
as $\neg a \wedge b \notin B_2$
$\neg(\neg a \wedge b) \notin B_2$

Elementary or not?

Let $\varphi = a \, \mathsf{U}(\neg a \wedge b)$.

$B_1 = \{a, b, \neg a \wedge b, \varphi\}$ — not elementary, propositional inconsistent

$B_2 = \{\neg a, b, \varphi\}$ — not elementary, not maximal

as $\neg a \wedge b \notin B_2$

$\neg(\neg a \wedge b) \notin B_2$

$B_3 = \{\neg a, b, \neg a \wedge b, \neg \varphi\}$

Elementary or not?

Let $\varphi = a\,\mathsf{U}(\neg a \wedge b)$.

$B_1 = \{a, b, \neg a \wedge b, \varphi\}$ — not elementary

propositional inconsistent

$B_2 = \{\neg a, b, \varphi\}$ — not elementary, not maximal

as $\neg a \wedge b \notin B_2$

$\neg(\neg a \wedge b) \notin B_2$

$B_3 = \{\neg a, b, \neg a \wedge b, \neg\varphi\}$ — not elementary

not locally consistent for U

Elementary or not?

Let $\varphi = a\,\mathsf{U}(\neg a \wedge b)$.

$B_1 = \{a, b, \neg a \wedge b, \varphi\}$ — not elementary
propositional inconsistent

$B_2 = \{\neg a, b, \varphi\}$ — not elementary, not maximal
as $\neg a \wedge b \notin B_2$
$\neg(\neg a \wedge b) \notin B_2$

$B_3 = \{\neg a, b, \neg a \wedge b, \neg\varphi\}$ — not elementary
not locally consistent for U

$B_4 = \{\neg a, \neg b, \neg(\neg a \wedge b), \neg\varphi\}$

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$B_4 = \{\neg a, \neg b, \neg(\neg a \wedge b), \neg\varphi\}$ elementary

Example: elementary formula-sets

closure $cl(\varphi)$:

- set of all subformulas of φ and their negations
- ψ and $\neg\neg\psi$ are identified

elementary formula-sets: subsets B of $cl(\varphi)$

- maximal consistent w.r.t. propositional logic
- locally consistent w.r.t. **U**

For $\varphi = a\,\mathsf{U}(\neg a \wedge b)$, the elementary sets are:

$$
\begin{array}{ll}
\{\ a,\ \ b, \neg(\neg a \wedge b), \varphi\} & \{\ a,\ \ b, \neg(\neg a \wedge b), \neg\varphi\} \\
\{\ a, \neg b, \neg(\neg a \wedge b), \varphi\} & \{\ a, \neg b, \neg(\neg a \wedge b), \neg\varphi\} \\
\{\neg a,\ \ b,\ \ \neg a \wedge b\ , \varphi\} & \{\neg a, \neg b, \neg(\neg a \wedge b), \neg\varphi\}
\end{array}
$$

Encoding of LTL semantics in a GNBA

idea: encode the semantics of the operators appearing in φ by appropriate components of the GNBA $\mathcal{G}$:

semantics of ...	encoding
propositional logic *true*, $\neg$, $\wedge$	in the states
next $\bigcirc$	in the transition relation
until **U**	expansion law, least fixed point

$$\psi_1 \, \mathsf{U} \, \psi_2 \;\equiv\; \psi_2 \vee (\psi_1 \wedge \bigcirc(\psi_1 \, \mathsf{U} \, \psi_2))$$

encoded in the states

encoded in the transition relation

acceptance condition

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encoded in the transition relation

acceptance condition

GNBA for LTL-formula φ

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$$\mathcal{G} = (Q, 2^{AP}, \delta, Q_0, \mathcal{F})$$

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state space: $Q = \{ B \subseteq cl(\varphi) : B \text{ is elementary} \}$

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if $A \neq B \cap AP$ then $\delta(B, A) = \varnothing$

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if $A = B \cap AP$ then $\delta(B, A) =$ set of all $B' \in Q$ s.t.

$$\begin{array}{rcl} \bigcirc\psi \in B & \text{iff} & \psi \in B' \\ \psi_1 \,\mathsf{U}\, \psi_2 \in B & \text{iff} & (\psi_2 \in B) \vee (\psi_1 \in B \wedge \psi_1 \,\mathsf{U}\, \psi_2 \in B') \end{array}$$

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acceptance set $\mathcal{F} = \{ F_{\psi_1 \mathsf{U} \psi_2} : \psi_1 \,\mathsf{U}\, \psi_2 \in cl(\varphi) \}$

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where $F_{\psi_1 \mathsf{U} \psi_2} = \{B \in Q : \psi_1 \,\mathsf{U}\, \psi_2 \notin B \vee \psi_2 \in B\}$

Example: GNBA for $\varphi = \bigcirc a$

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$a, \bigcirc a$

$a, \neg \bigcirc a$

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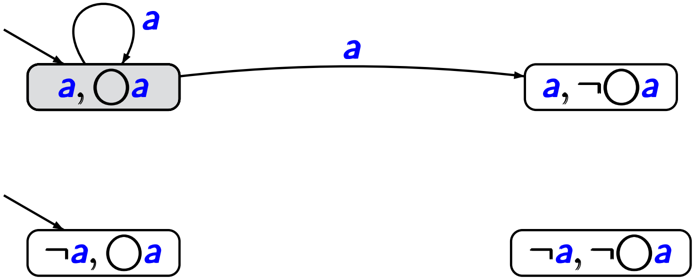

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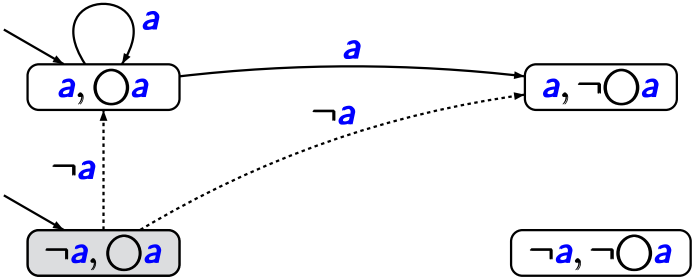

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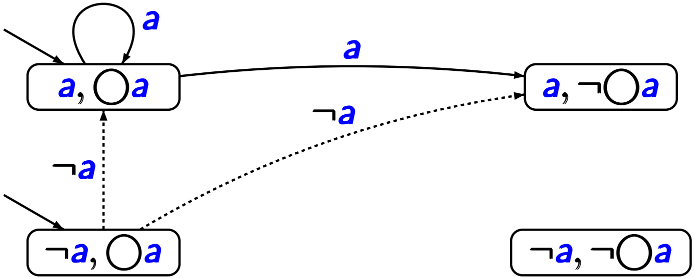

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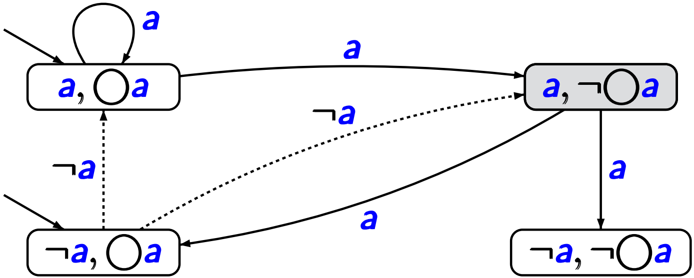

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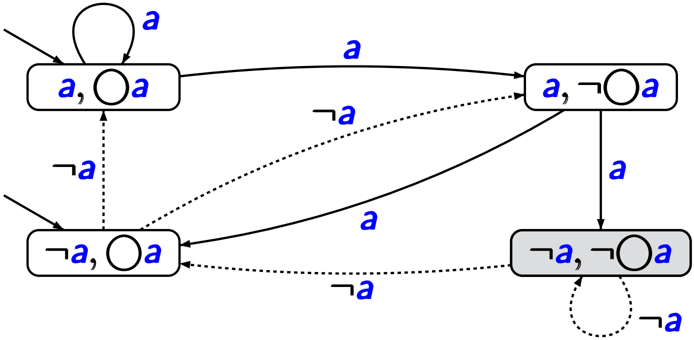

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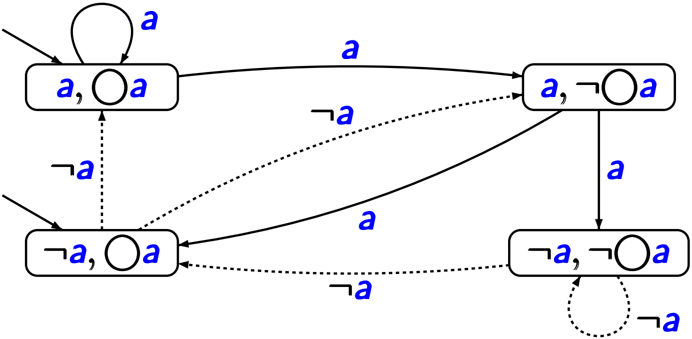
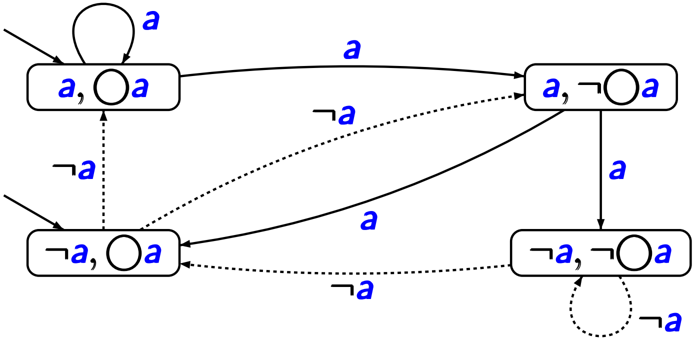


set of acceptance sets:

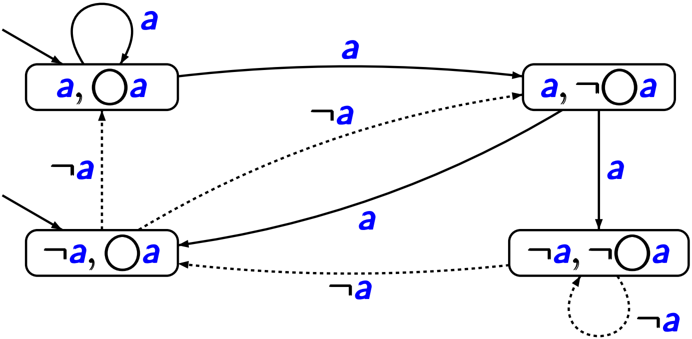
Example: GNBA for $\varphi = \bigcirc a$



set of acceptance sets: $\mathcal{F} = \varnothing$

hence: all words having an infinite run are accepted

Example: GNBA for $\varphi = \bigcirc a$



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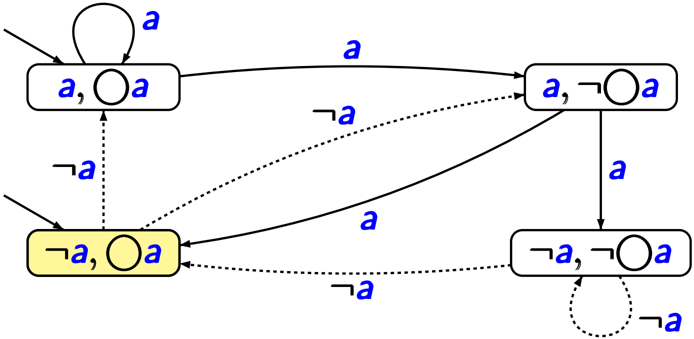

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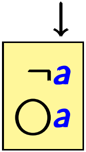

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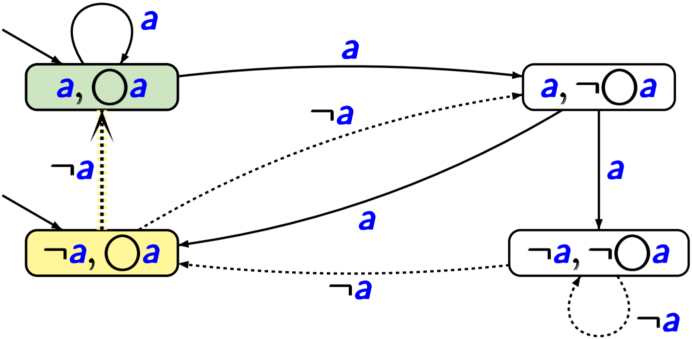

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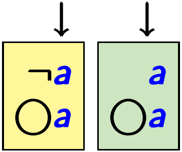

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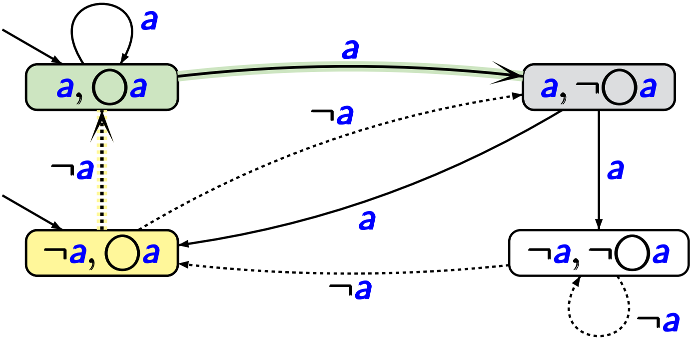

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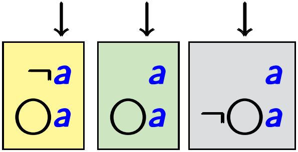

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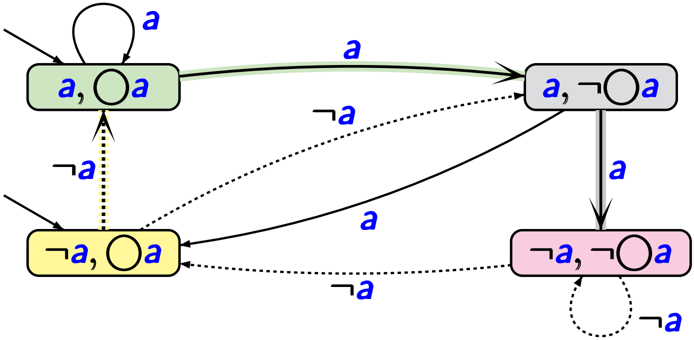

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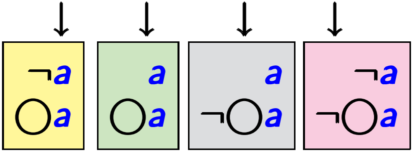

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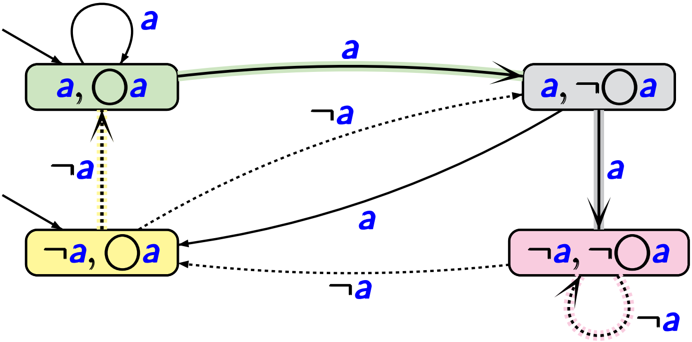

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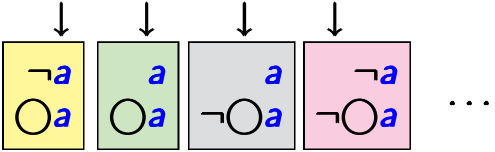
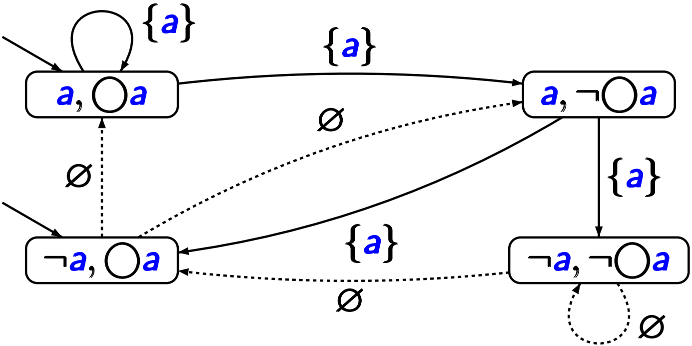


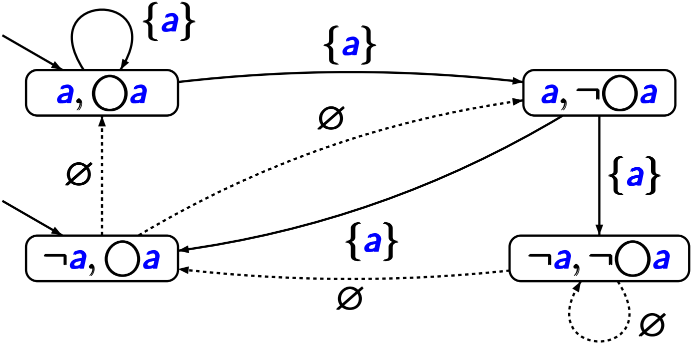
accepting run

Soundness of the GNBA for $\varphi = \bigcirc a$



for all words $\sigma = A_0\, A_1\, A_2\, A_3 \ldots \in \mathcal{L}_\omega(\mathcal{G})$: $\quad A_1 = \{a\}$

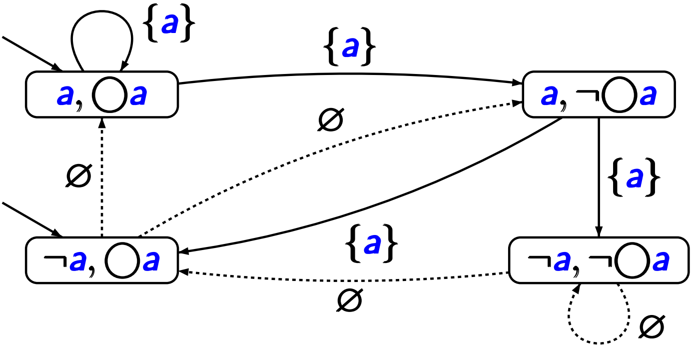
Soundness of the GNBA for $\varphi = \bigcirc a$



for all words $\sigma = A_0\, A_1\, A_2\, A_3 \ldots \in \mathcal{L}_\omega(\mathcal{G})$: $\quad A_1 = \{a\}$

proof:

Soundness of the GNBA for $\varphi = \bigcirc a$



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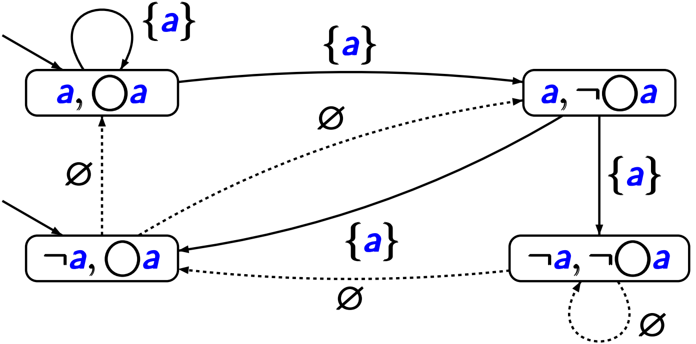

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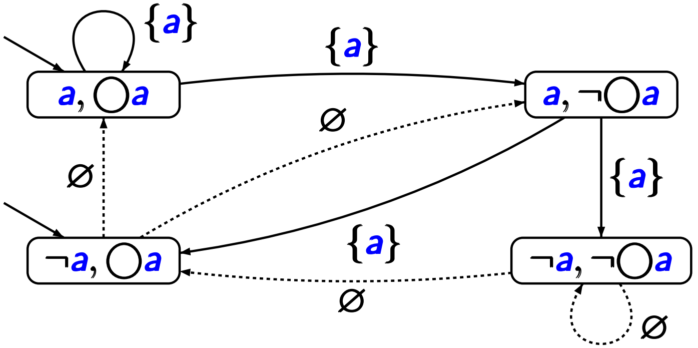
$\Longrightarrow \quad \bigcirc a \in B_0$

Soundness of the GNBA for $\varphi = \bigcirc a$



for all words $\sigma = A_0\, A_1\, A_2\, A_3 \ldots \in \mathcal{L}_\omega(\mathcal{G})$: $\quad A_1 = \{a\}$

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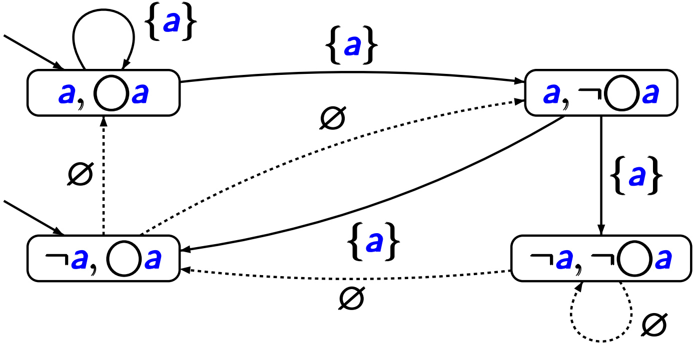
$\Longrightarrow \quad \bigcirc a \in B_0$ and therefore $a \in B_1$

Soundness of the GNBA for $\varphi = \bigcirc a$



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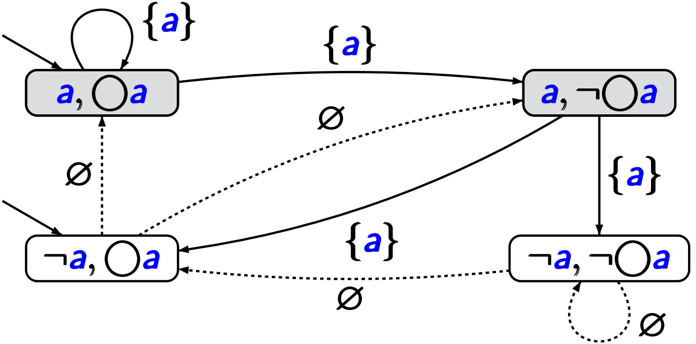
$\Longrightarrow$ $\bigcirc a \in B_0$ and therefore $a \in B_1$

$\Longrightarrow$ the outgoing edges of B_1 have label $\{a\}$

Soundness of the GNBA for $\varphi = \bigcirc a$



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$\Longrightarrow \quad \{a\} = B_1 \cap AP = A_1$

Example: GNBA for $\varphi = a \, \mathsf{U} \, b$

Example: GNBA for $\varphi = a \,\mathsf{U}\, b$

$a, b, a \,\mathsf{U}\, b$

$\neg a, \neg b, \neg(a \,\mathsf{U}\, b)$

$a, \neg b, a \,\mathsf{U}\, b$

$a, \neg b, \neg(a \,\mathsf{U}\, b)$

$\neg a, b, a \,\mathsf{U}\, b$

locally inconsistent: $\{a, b, \neg(a \,\mathsf{U}\, b)\}$

$\{\neg a, b, \neg(a \,\mathsf{U}\, b)\}$

$\{\neg a, \neg b, a \,\mathsf{U}\, b\}$

Example: GNBA for $\varphi = a \,\mathsf{U}\, b$

$a, b, a \,\mathsf{U}\, b$

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$a, \neg b, a \,\mathsf{U}\, b$

$a, \neg b, \neg(a \,\mathsf{U}\, b)$

$\neg a, b, a \,\mathsf{U}\, b$

initial states: B with $\varphi = a \,\mathsf{U}\, b \in B$

Example: GNBA for $\varphi = a \,\mathsf{U}\, b$

$\longrightarrow$ $a, b, a \,\mathsf{U}\, b$

$\neg a, \neg b, \neg(a \,\mathsf{U}\, b)$

$\longrightarrow$ $a, \neg b, a \,\mathsf{U}\, b$

$a, \neg b, \neg(a \,\mathsf{U}\, b)$

$\longrightarrow$ $\neg a, b, a \,\mathsf{U}\, b$

initial states: B with $\varphi = a \,\mathsf{U}\, b \in B$

Example: GNBA for $\varphi = a \,\mathsf{U}\, b$

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$\longrightarrow$ $a, \neg b, a \,\mathsf{U}\, b$

$a, \neg b, \neg(a \,\mathsf{U}\, b)$

$\longrightarrow$ $\neg a, b, a \,\mathsf{U}\, b$

initial states: B with $\varphi = a \,\mathsf{U}\, b \in B$

acceptance condition: just one set of accept states

$F =$ set of all B with $\varphi \notin B$ or $b \in B$

$\longrightarrow$ $a, b, a \, \mathsf{U} \, b$ $\qquad$ $\neg a, \neg b, \neg(a \, \mathsf{U} \, b)$

$\longrightarrow$ $a, \neg b, a \, \mathsf{U} \, b$ $\qquad$ $a, \neg b, \neg(a \, \mathsf{U} \, b)$

$\longrightarrow$ $\neg a, b, a \, \mathsf{U} \, b$

initial states: B with $\varphi = a \, \mathsf{U} \, b \in B$

acceptance condition: $\boxed{\text{just one set of accept states}}$

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Example: (G)NBA for $\varphi = a \mathbin{\mathsf{U}} b$

$\rightarrow$ $a, b, a \mathbin{\mathsf{U}} b$

$\neg a, \neg b, \neg(a \mathbin{\mathsf{U}} b)$

$\rightarrow$ $a, \neg b, a \mathbin{\mathsf{U}} b$

$a, \neg b, \neg(a \mathbin{\mathsf{U}} b)$

$\rightarrow$ $\neg a, b, a \mathbin{\mathsf{U}} b$

initial states: B with $\varphi = a \mathbin{\mathsf{U}} b \in B$

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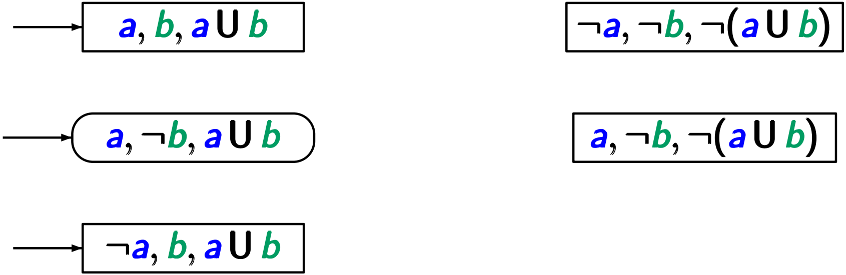
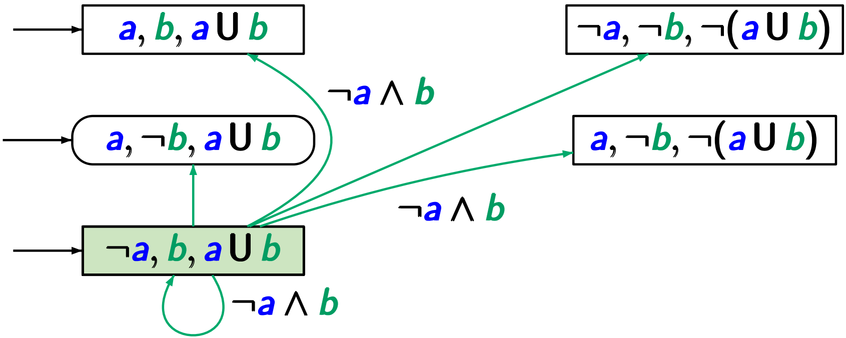
Example: (G)NBA for $\varphi = a \,\mathsf{U}\, b$



transition relation: $B' \in \delta(B, B \cap AP)$ iff

$$a \,\mathsf{U}\, b \in B \iff \big(b \in B \ \vee \ (a \in B \ \wedge \ a \,\mathsf{U}\, b \in B')\big)$$

Example: (G)NBA for $\varphi = a \, \mathsf{U} \, b$



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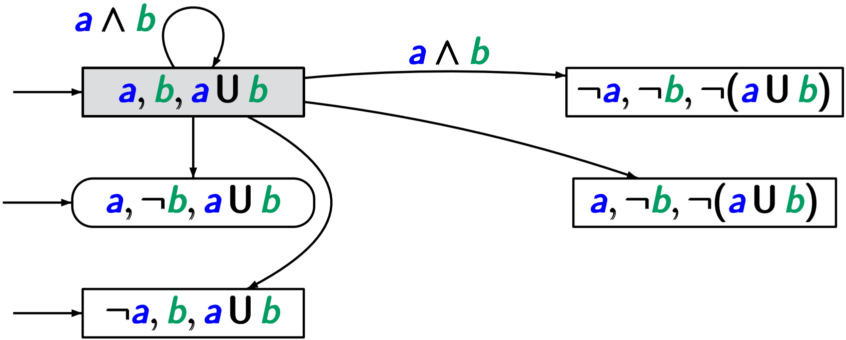
Example: (G)NBA for $\varphi = a \,\mathsf{U}\, b$



transition relation: $B' \in \delta(B, B \cap AP)$ iff

$$a \,\mathsf{U}\, b \in B \iff \big(b \in B \;\vee\; (\,a \in B \;\wedge\; a \,\mathsf{U}\, b \in B'\,)\big)$$

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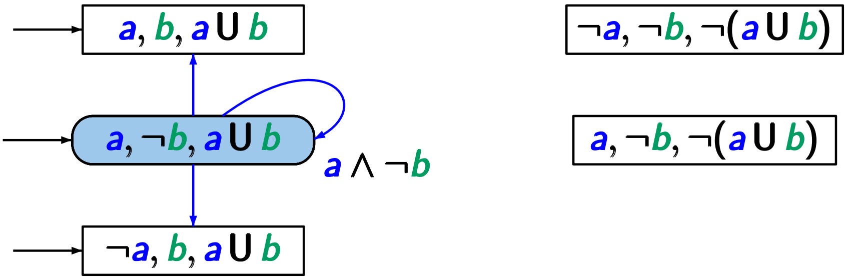

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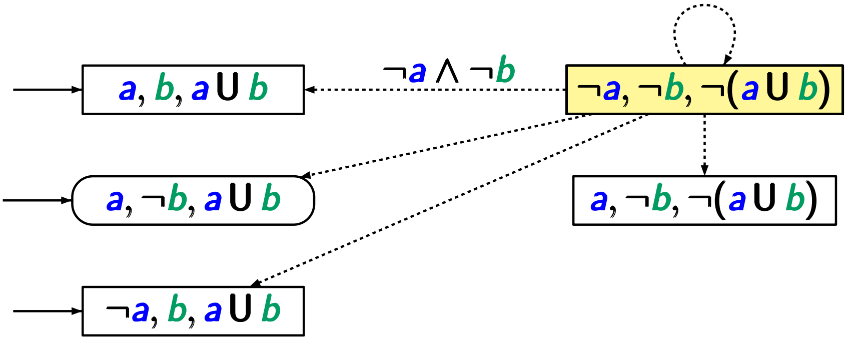

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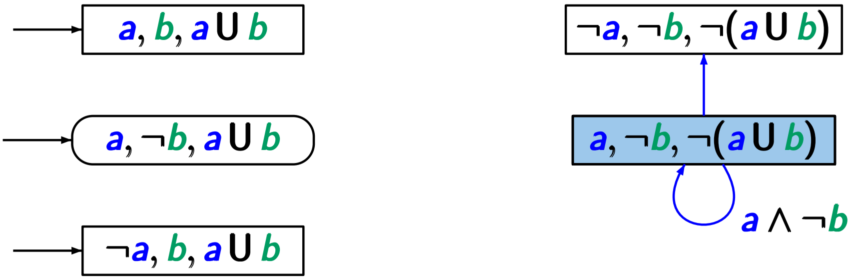

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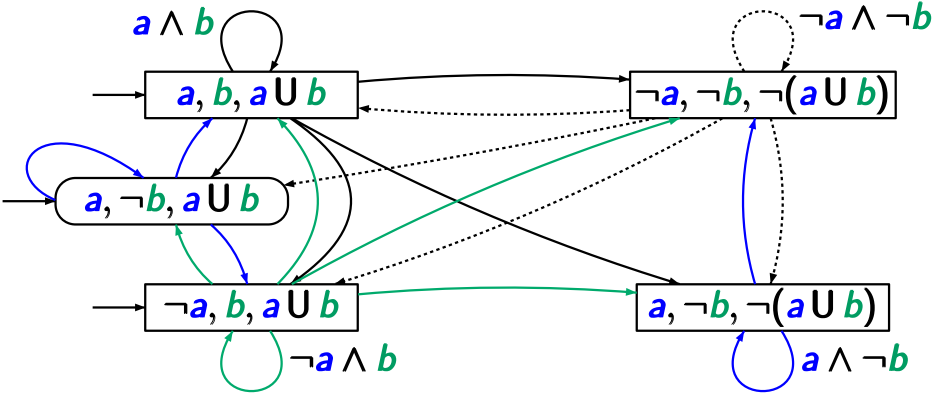
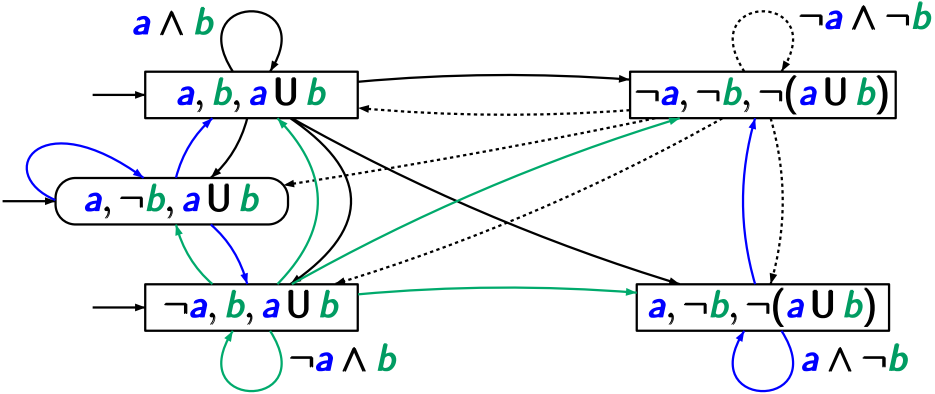


Example: (G)NBA for $\varphi = a \,\mathsf{U}\, b$



$$\{a\} \quad \{a\} \quad \{a, b\} \quad \varnothing \quad \varnothing \quad \varnothing \quad \ldots \models a \,\mathsf{U}\, b$$

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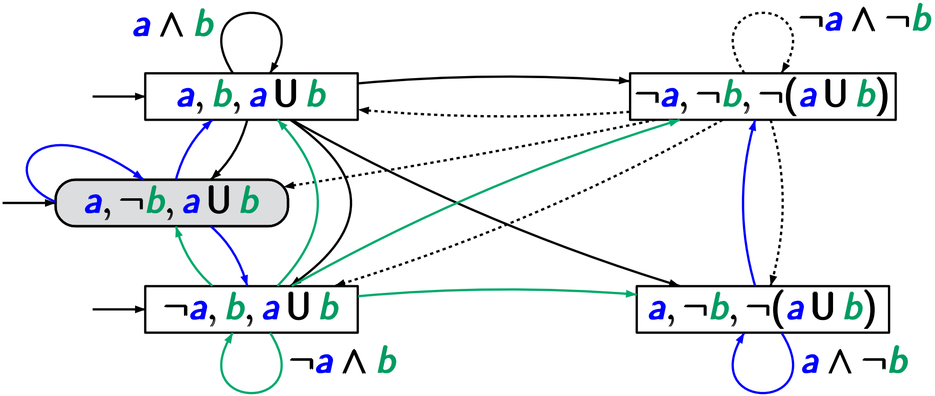

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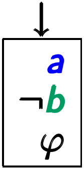
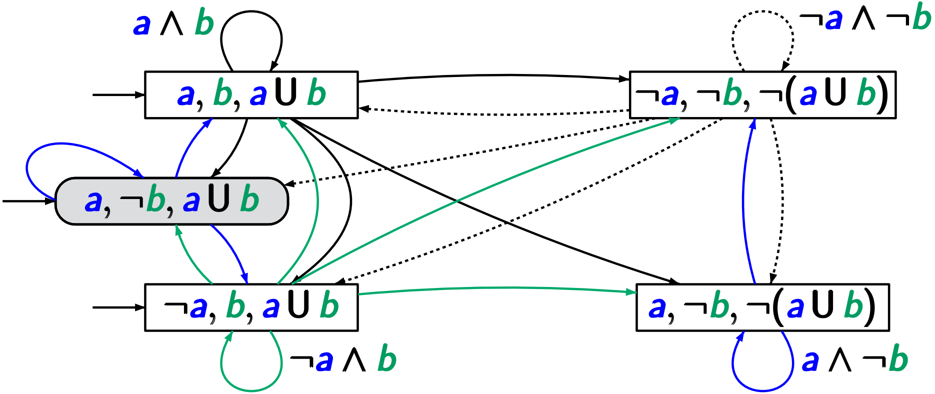


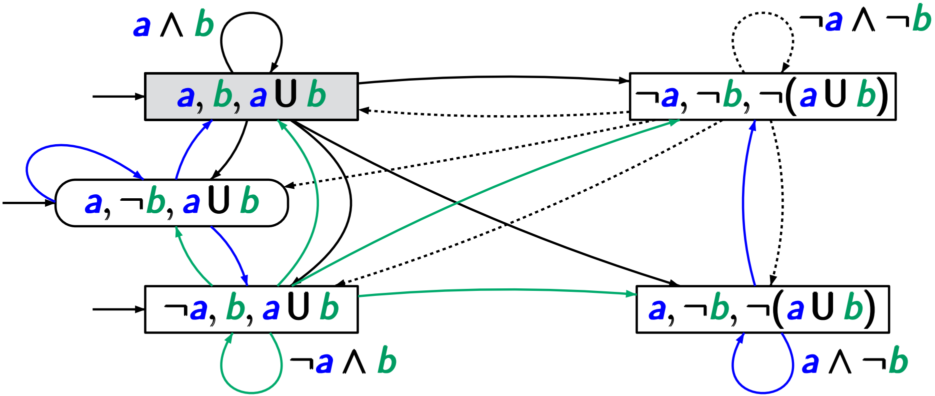
Example: (G)NBA for $\varphi = a \mathsf{U} b$



$\{a\} \quad \{a\} \quad \{a, b\} \quad \varnothing \quad \varnothing \quad \varnothing \quad \ldots \models a \mathsf{U} b$

$\{a\}$	$\{a\}$
a	a
$\neg b$	$\neg b$
φ	φ

Example: (G)NBA for $\varphi = a \,\mathsf{U}\, b$



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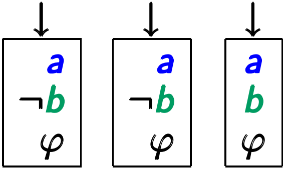

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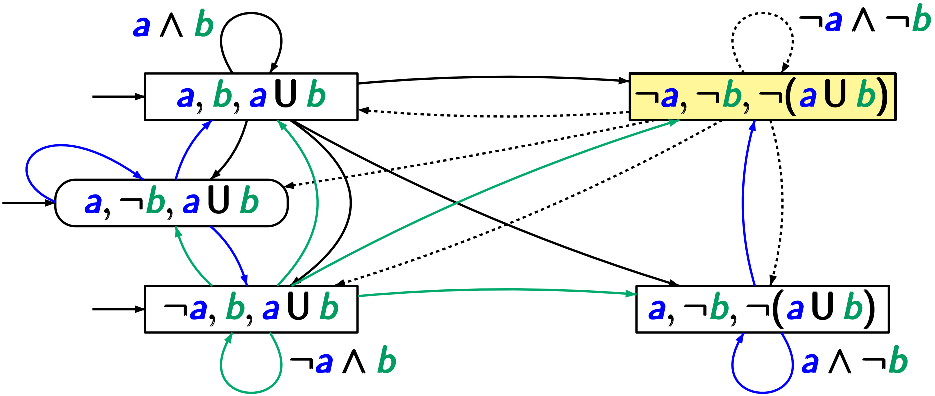

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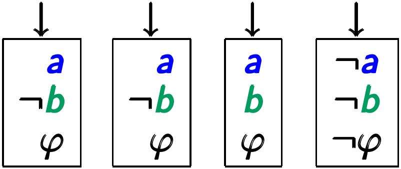

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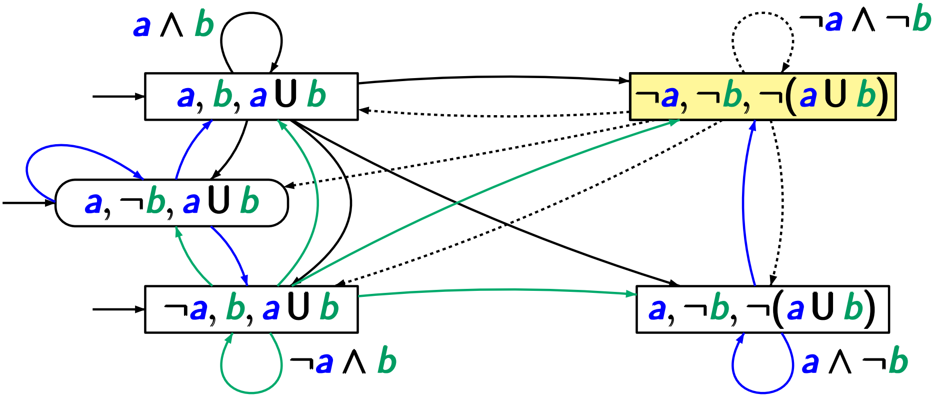

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$\{a\}$	$\{a\}$	$\{a,b\}$	$\varnothing$	$\varnothing$
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
a	a	a	$\neg a$	$\neg a$
$\neg b$	$\neg b$	b	$\neg b$	$\neg b$
φ	φ	φ	$\neg\varphi$	$\neg\varphi$

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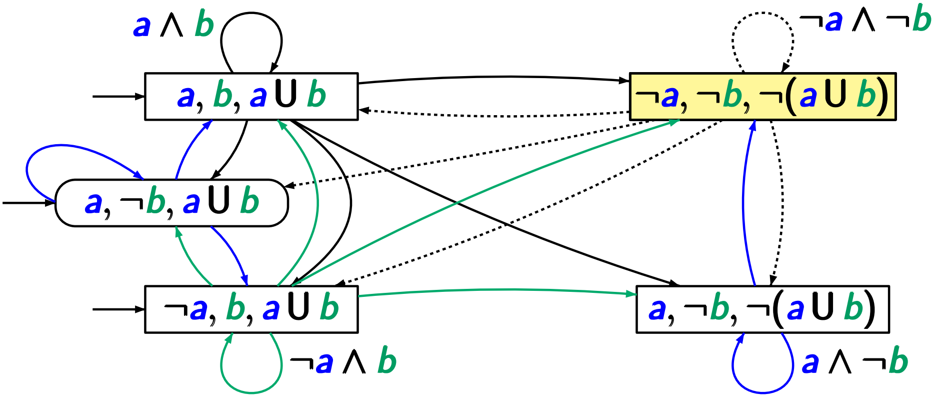

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φ	φ	φ	$\neg\varphi$	$\neg\varphi$	$\neg\varphi$

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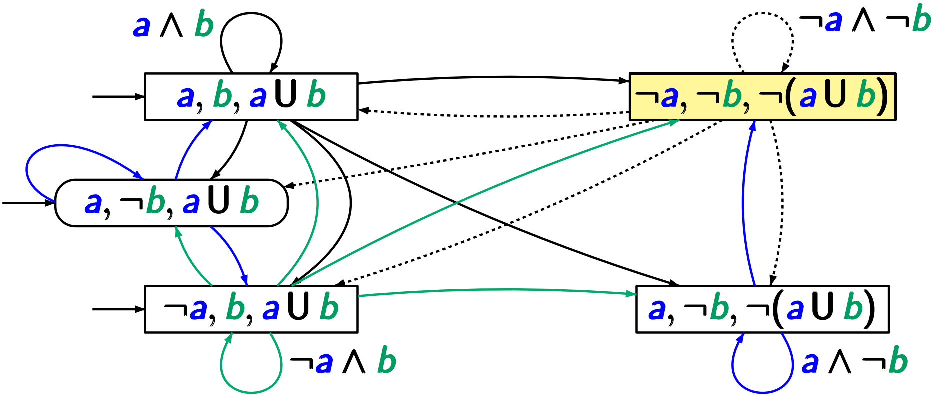

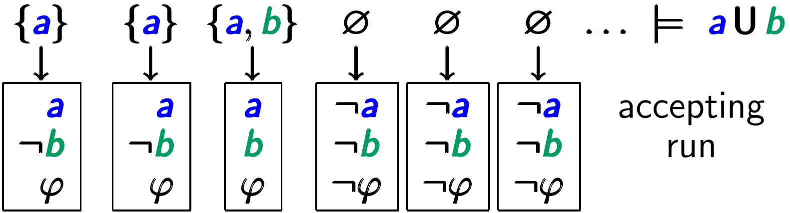

accepting run

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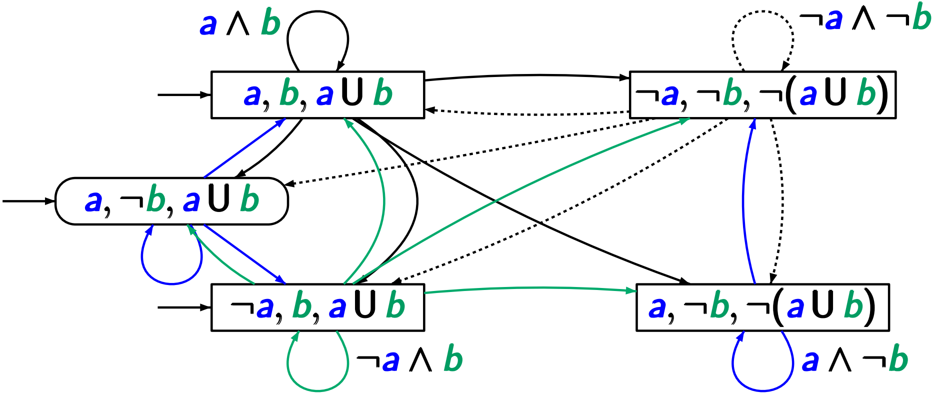

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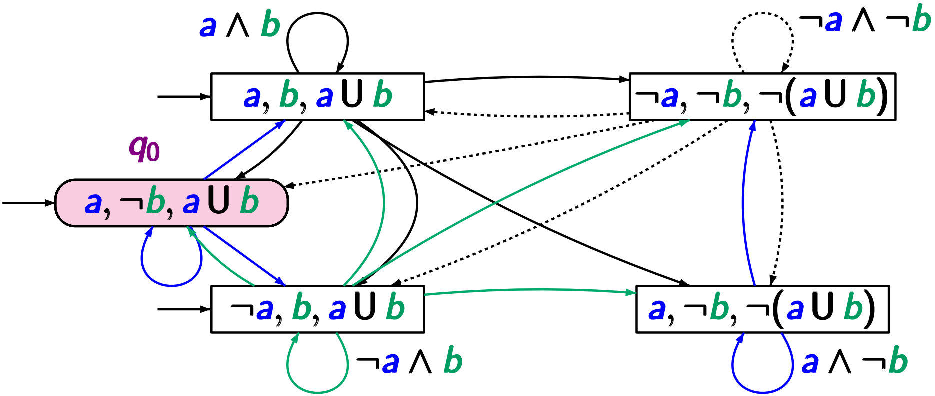

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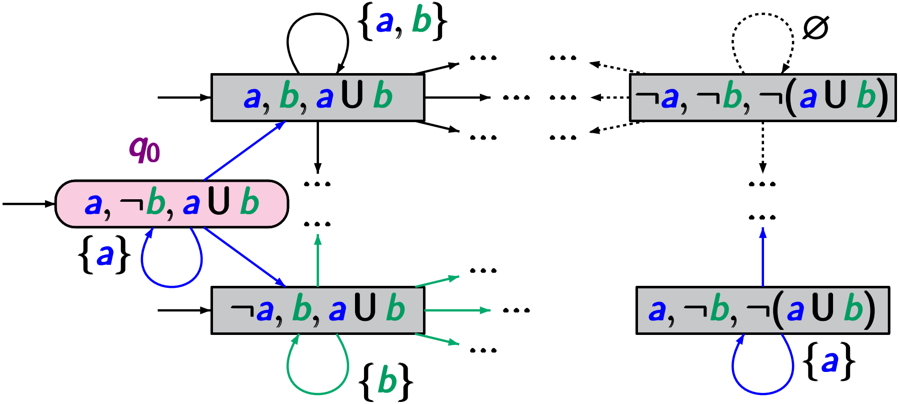

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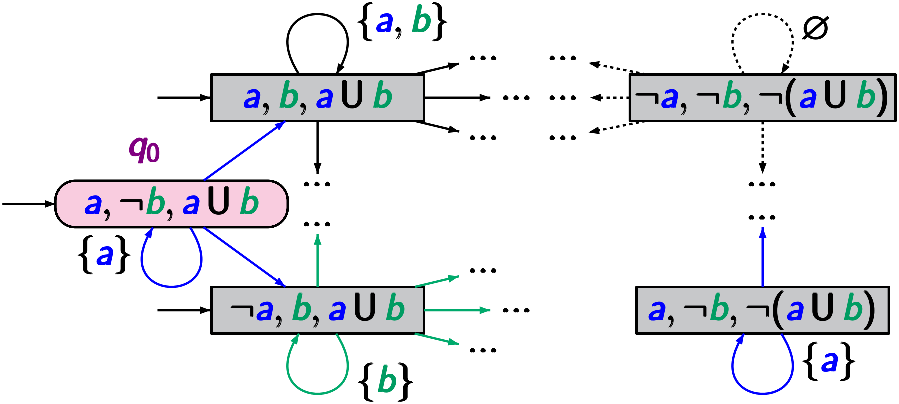

$\{a\}\,\{a\}\,\{a\}\,\{a\} \ldots \not\models \varphi$

only 1 infinite run: $q_0\, q_0\, q_0 \ldots$ not accepting

$$\mathcal{G} = (Q, 2^{AP}, \delta, Q_0, \mathcal{F})$$

state space: $Q = \{B \subseteq cl(\varphi) : B \text{ is elementary}\}$

initial states: $Q_0 = \{B \in Q : \varphi \in B\}$

transition relation: for $B \in Q$ and $A \in 2^{AP}$:

if $A \neq B \cap AP$ then $\delta(B, A) = \varnothing$

if $A = B \cap AP$ then $\delta(B, A) =$ set of all $B' \in Q$ s.t.

$$\bigcirc\psi \in B \quad \text{iff} \quad \psi \in B'$$

$$\psi_1 \mathsf{U} \psi_2 \in B \quad \text{iff} \quad (\psi_2 \in B) \vee (\psi_1 \in B \wedge \psi_1 \mathsf{U} \psi_2 \in B')$$

acceptance set $\mathcal{F} = \{F_{\psi_1 \mathsf{U} \psi_2} : \psi_1 \mathsf{U} \psi_2 \in cl(\varphi)\}$

where $F_{\psi_1 \mathsf{U} \psi_2} = \{B \in Q : \psi_1 \mathsf{U} \psi_2 \notin B \vee \psi_2 \in B\}$

Complexity: LTL $\rightsquigarrow$ NBA

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For each **LTL** formula φ, there is an **NBA** $\mathcal{A}$ s.t.

$$\mathcal{L}_\omega(\mathcal{A}) \quad = \quad \mathit{Words}(\varphi)$$

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$$\begin{tikzcd}
\boxed{\textbf{LTL} \text{ formula } \varphi} \arrow[d] \\
\boxed{\textbf{GNBA } \mathcal{G}} \arrow[d] \\
\boxed{\textbf{NBA } \mathcal{A}}
\end{tikzcd}$$

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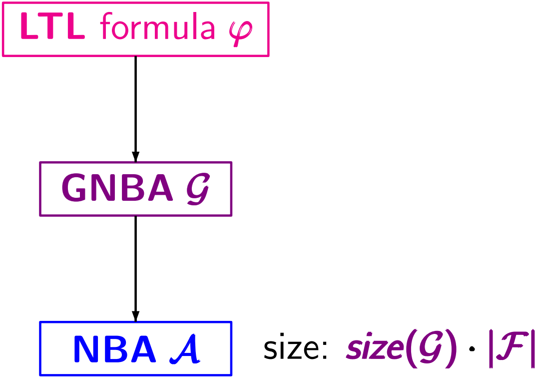

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LTL formula φ

$\downarrow$

GNBA $\mathcal{G}$

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NBA $\mathcal{A}$ size: $\mathit{size}(\mathcal{G}) \cdot |\mathcal{F}|$

$|\mathcal{F}|$ = number of acceptance sets in $\mathcal{G}$

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Complexity: LTL $\rightsquigarrow$ NBA

For each **LTL** formula φ, there is an **NBA** $\mathcal{A}$ s.t.

$$\begin{aligned} \mathcal{L}_\omega(\mathcal{A}) &= \mathit{Words}(\varphi) \text{ and} \\ \mathit{size}(\mathcal{A}) &\leq 2^{|cl(\varphi)|} \cdot |\varphi| \end{aligned}$$

$$\begin{array}{ll} \boxed{\textbf{LTL}\text{ formula } \varphi} & \\ \quad\downarrow & \\ \boxed{\textbf{GNBA } \mathcal{G}} & \text{size: } 2^{|cl(\varphi)|} \\ \quad\downarrow & \\ \boxed{\textbf{NBA } \mathcal{A}} & \text{size: } \mathit{size}(\mathcal{G}) \cdot |\mathcal{F}| \end{array}$$

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↓

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↓

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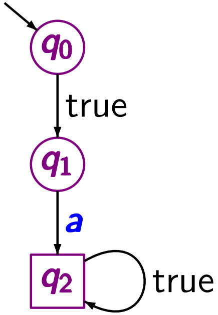
NBA for $\bigcirc a$



constructed GNBA has **4** states and **8** edges

Size of NBA for LTL formulas

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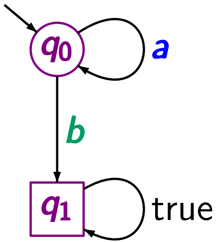
NBA for $a\, \mathbf{U}\, b$



constructed (G)NBA has
5 states and **20** edges

For the proposed transformation **LTL** $\rightsquigarrow$ **NBA**:

> The constructed NBA for LTL formulas are often unnecessarily complicated

... but there exists LTL formulas φ_n such that

- $|\varphi_n| = \mathcal{O}(poly(n))$
- each NBA for φ_n has at least 2^n states

LT-properties that have no "small" NBA

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consider the following family of LT-properties $(E_n)_{n\geq 1}$:

$$E_n = \left\{ \begin{array}{l} \text{set of all infinite words over } 2^{AP} \text{ of the form} \\ A_1\, A_2\, A_3 \ldots A_n\, A_1\, A_2\, A_3 \ldots A_n\, B_1\, B_2\, B_3\, B_4 \ldots \end{array} \right.$$

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$$\varphi_n = \bigwedge_{a\in AP} \bigwedge_{0\leq i<n} (\bigcirc^i a \leftrightarrow \bigcirc^{i+n} a)$$

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$$\varphi_n = \bigwedge_{a \in AP} \bigwedge_{0 \leq i < n} (\bigcirc^i a \leftrightarrow \bigcirc^{i+n} a) \longleftarrow \boxed{\begin{array}{c}\text{length} \\ \mathcal{O}(poly(n))\end{array}}$$

LT-property E_n for n=1

$$E_1 = \left\{ \begin{array}{l} \text{set of all infinite words over } 2^{AP} \text{ of the form} \\ \quad A\,A\,B_1\,B_2\,B_3\,B_4 \ldots \text{ where } A, B_j \subseteq AP \text{ for } j \geq 0 \end{array} \right.$$

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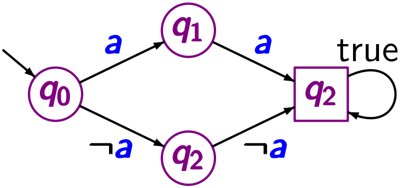
NBA for E_1 if $AP = \{a\}$:



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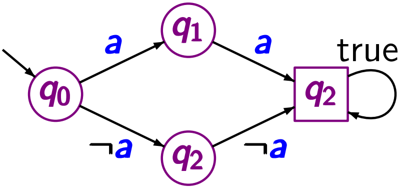

LTL-formula:
$a \leftrightarrow \bigcirc a$

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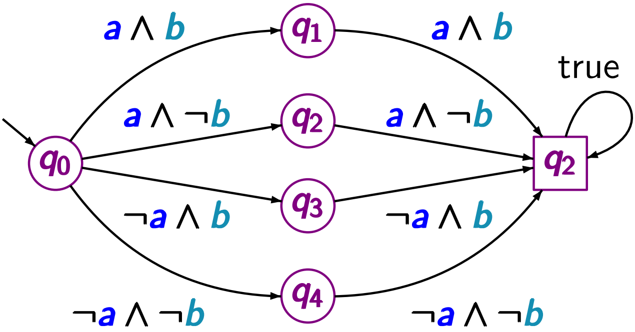

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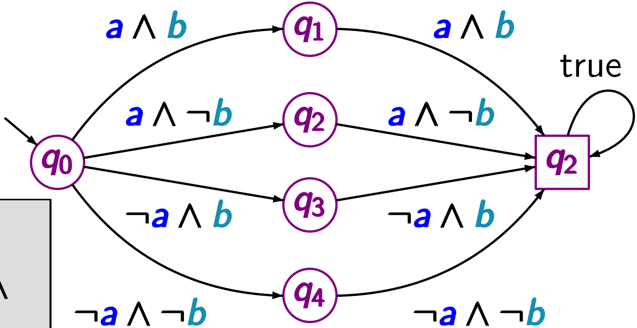
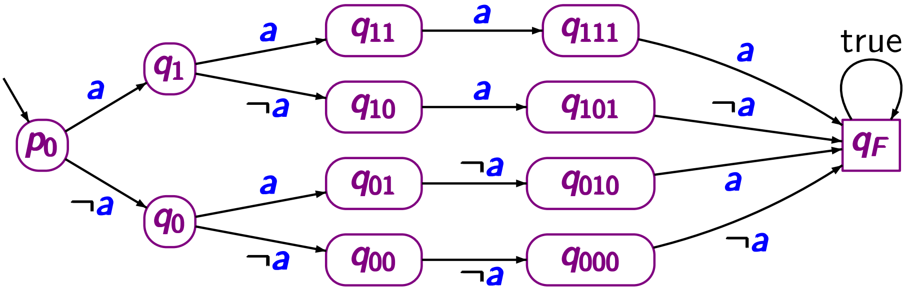


LTL-formula:

$(a \leftrightarrow \bigcirc a) \wedge (b \leftrightarrow \bigcirc b)$

LT property E_n for $n=2$ and $AP = \{a\}$



$$E_2 = \{ A_1 A_2 A_1 A_2 \sigma : \; A_1, A_2 \subseteq AP, \sigma \in (2^{AP})^\omega \}$$

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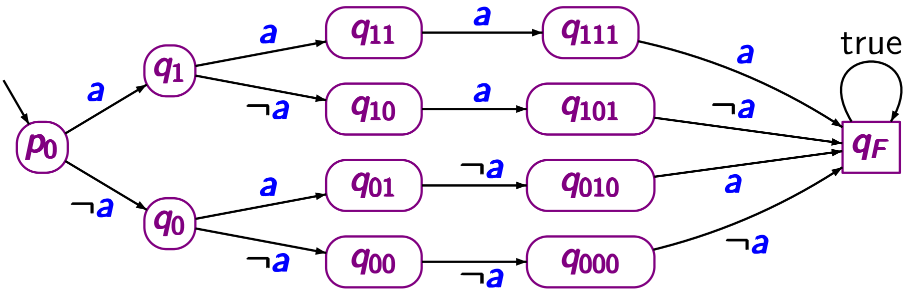

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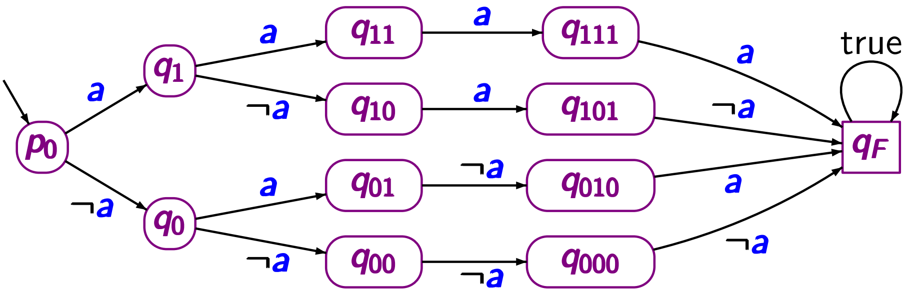
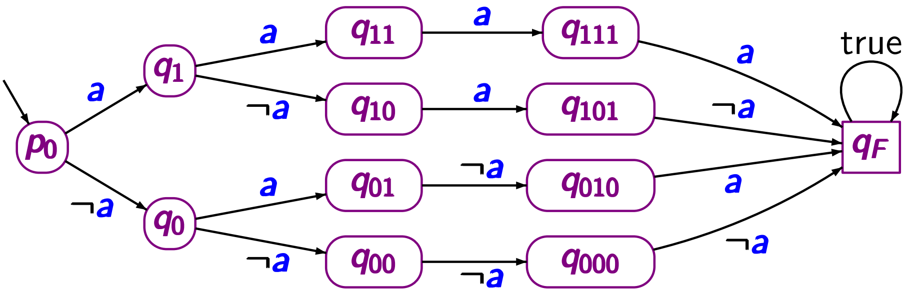
LTL-formula: $(a \leftrightarrow \bigcirc\bigcirc a) \wedge (\bigcirc a \leftrightarrow \bigcirc\bigcirc\bigcirc a)$

LT property E_n for n=2 and $AP = \{a\}$



general case: each **NBA** for E_n has $\geq 2^n$ states

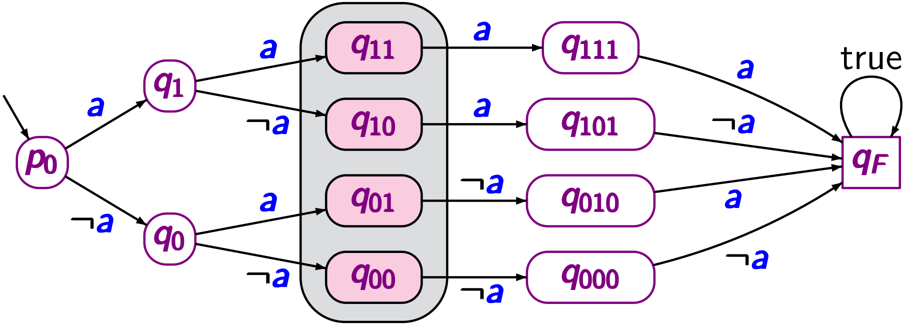
LT property E_n for $n=2$ and $AP = \{a\}$



> *general case:* each **NBA** for E_n has $\geq 2^n$ states

$$E_n = Words(\varphi_n) \text{ where } \varphi_n = \bigwedge_{a \in AP} \bigwedge_{0 \leq i < n} (\bigcirc^i a \leftrightarrow \bigcirc^{n+i} a)$$

LT property E_n for n=2 and $AP = \{a\}$



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