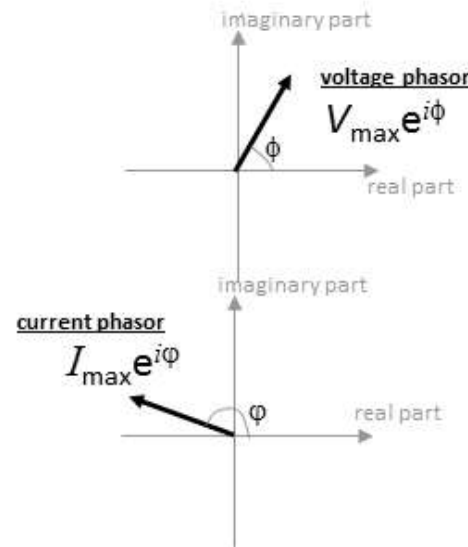
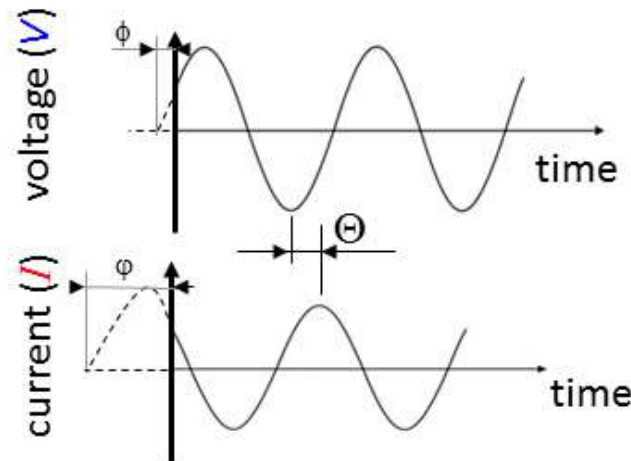
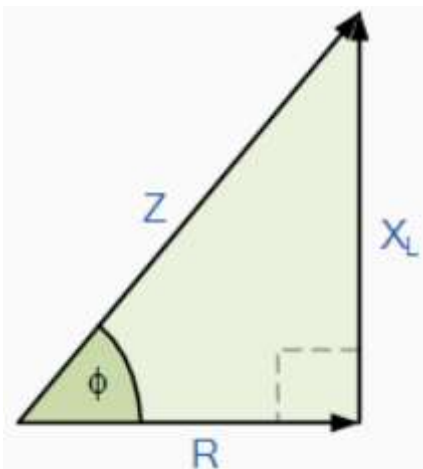
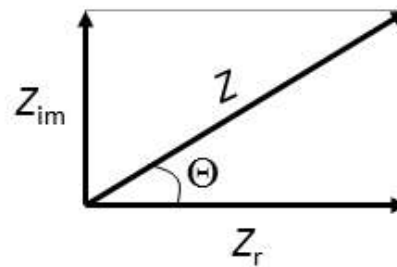


(3 ϕ) Three Phase Alternating Voltage and Current

Monophase

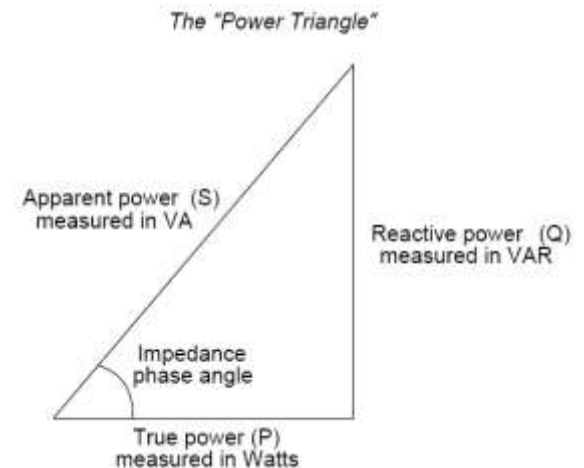


$$Z = \frac{V_{\max} e^{i\phi}}{I_{\max} e^{i\phi}} = |Z| e^{i\Theta} = Z_r + i \cdot Z_{\text{im}}$$



$$Z^2 = R^2 + X_L^2$$

$$\cos \phi = \frac{R}{Z}$$



$$P = \text{true power} \quad P = I^2 R \quad P = \frac{E^2}{R}$$

Measured in units of **Watts**

$$Q = \text{reactive power} \quad Q = I^2 X \quad Q = \frac{E^2}{X}$$

Measured in units of **Volt-Amps-Reactive (VAR)**

$$S = \text{apparent power} \quad S = I^2 Z \quad S = \frac{E^2}{Z} \quad S = IE$$

Measured in units of **Volt-Amps (VA)**

Three phase

Three-phase electric power is a common method of alternating-current electric power generation, transmission, and distribution.

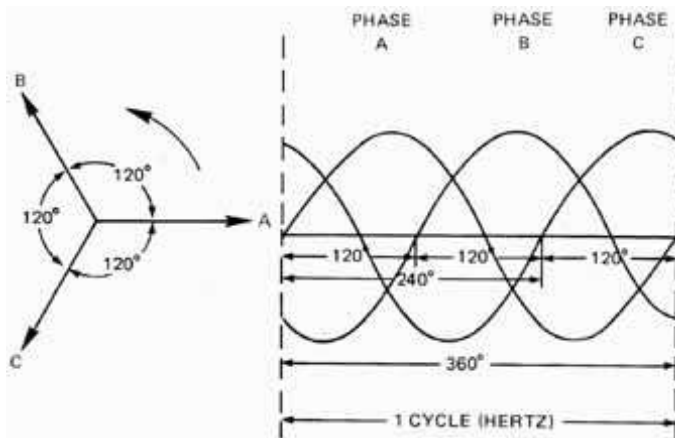
It is a type of polyphase system and is the most common method used by electrical grids worldwide to transfer power. It is also used to power large motors and other heavy loads.

A three-phase system is usually more economical than an equivalent single-phase or two-phase system at the same voltage because it uses less conductor material to transmit electrical power

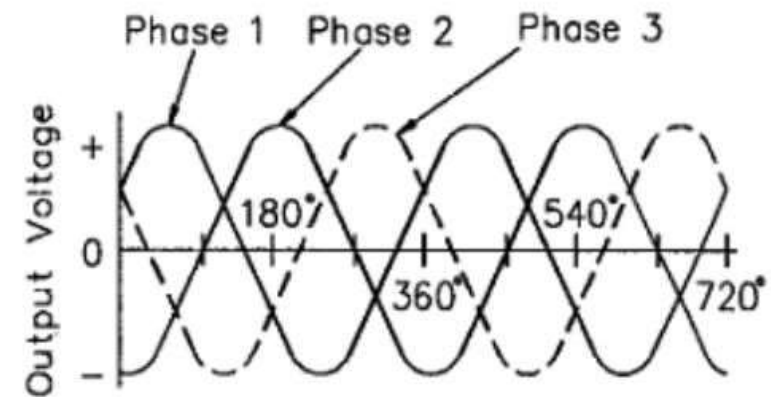
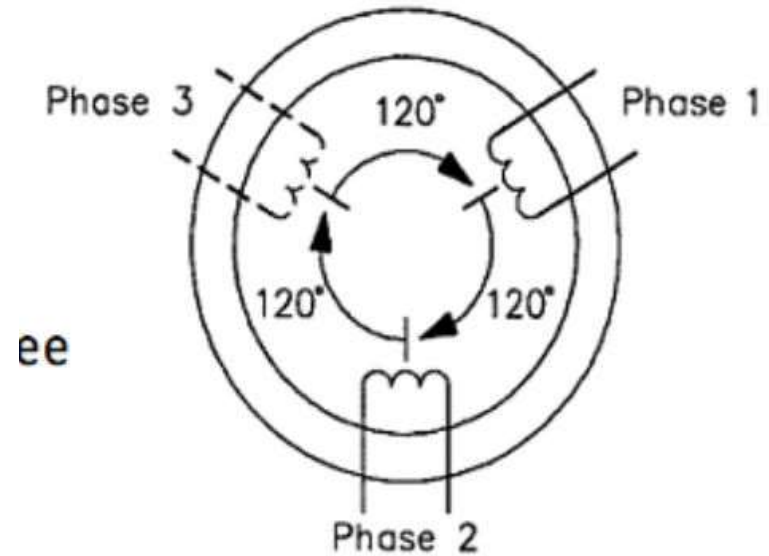
Definition

A three-phase (3 ϕ) system is a combination of three single-phase systems. In a 3 ϕ balanced system, power comes from a 3 ϕ AC generator that produces three separate and equal voltages, each of which is 120° out of phase with the other voltages

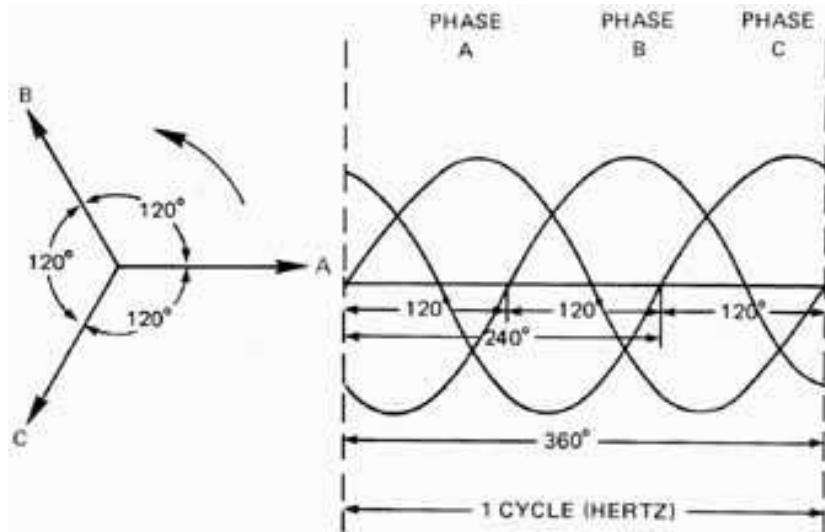
A three-phase AC system consists of three-phase generators, transmission lines, and loads.



Three Phase Generation



Three Phase Waveform



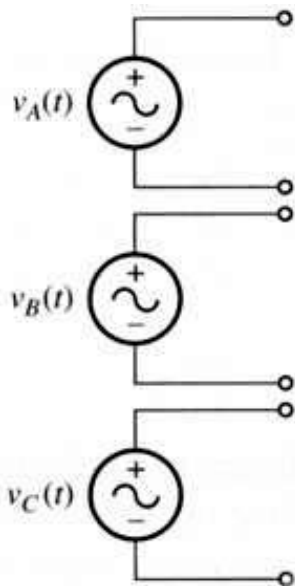
$$V_{\text{rms}} = 220[\text{V}]$$

$$f = 50[\text{Hz}]$$

$$V_A(t) = 220\sqrt{2} \sin(2\pi 50t)$$

$$V_A(t) = 220\sqrt{2} \sin(2\pi 50t - 120^\circ)$$

$$V_A(t) = 220\sqrt{2} \sin(2\pi 50t - 240^\circ)$$



$$v_A(t) = \sqrt{2} V \sin \omega t \text{ V}$$

$$\mathbf{V}_A = V \angle 0^\circ \text{ V}$$

$$v_B(t) = \sqrt{2} V \sin (\omega t - 120^\circ) \text{ V}$$

$$\mathbf{V}_B = V \angle -120^\circ \text{ V}$$

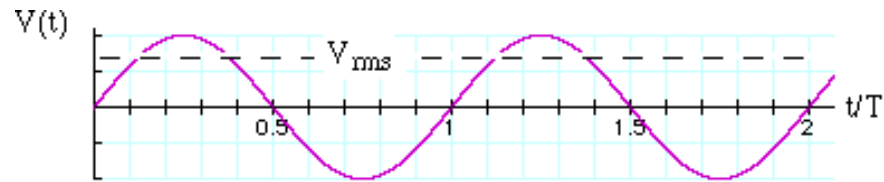
$$v_C(t) = \sqrt{2} V \sin (\omega t - 240^\circ) \text{ V}$$

$$\mathbf{V}_C = V \angle -240^\circ \text{ V}$$

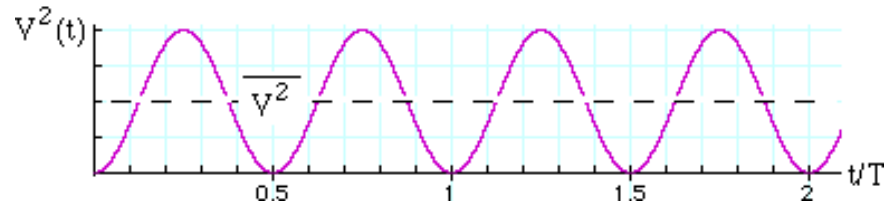
One Critical Advatage of Three Phase Waveform

Power delivered to a three-phase load is constant at all time, instead of pulsing as it does in a single-phase system.

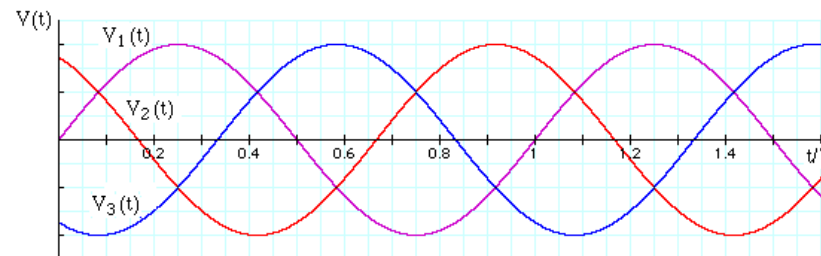
Concerning that the power is proportional to square of the voltage signal



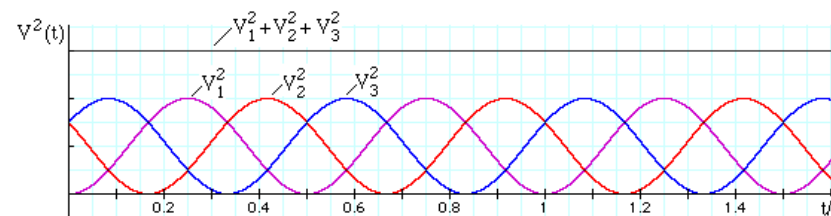
Monophase Power:



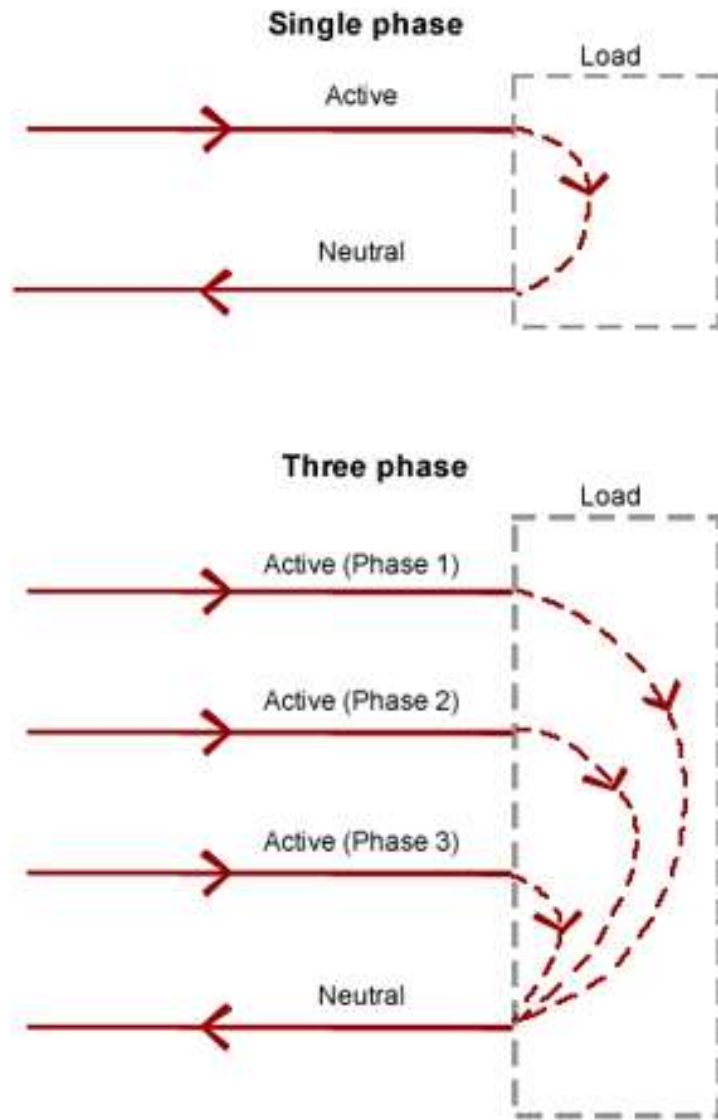
Three Phase Power:



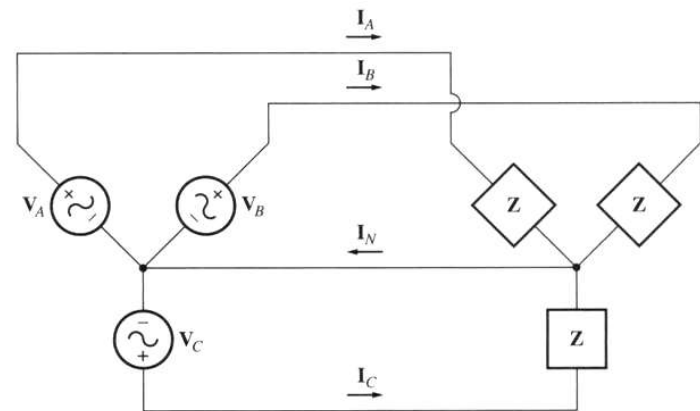
Power transfer into a linear balanced load is constant, which helps to reduce generator and motor vibrations.



Three Phase Distribution Lines

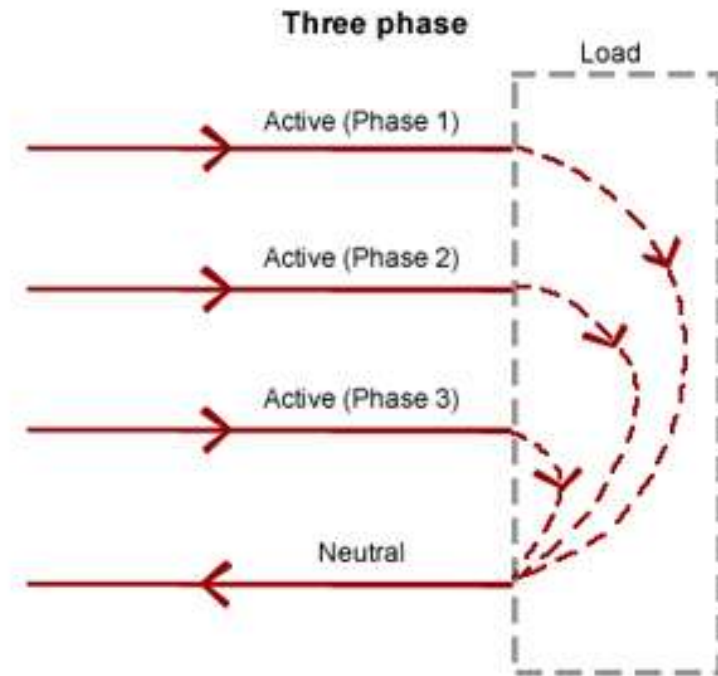
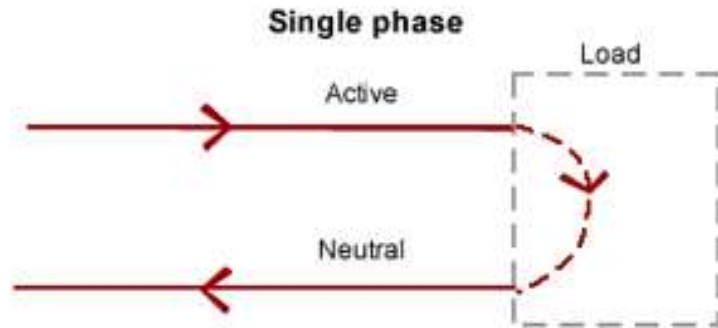


We can connect the negative (ground) ends of the three single-phase generators and loads together, so they share the common return line (neutral).

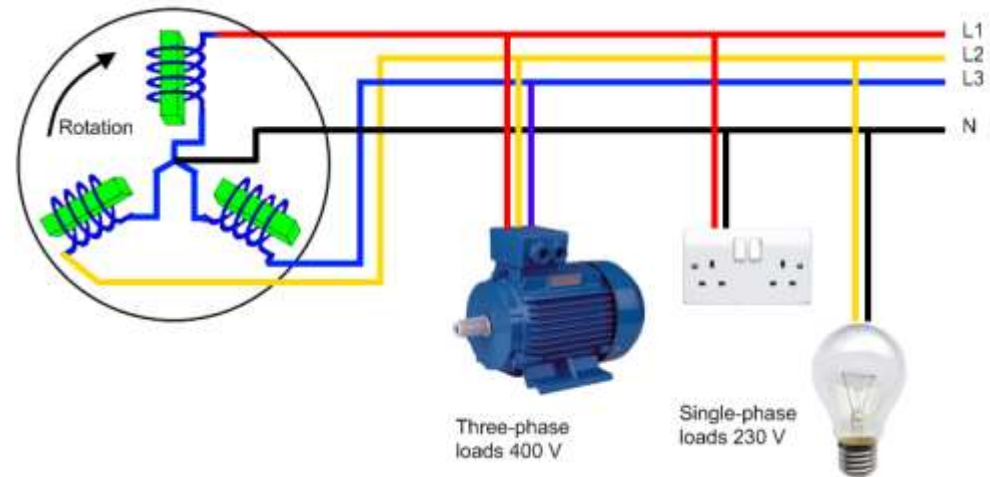


Then the three phase system reduces to necessary number of cable to transfer power from 6 to 4.

Three Phase Distribution Lines

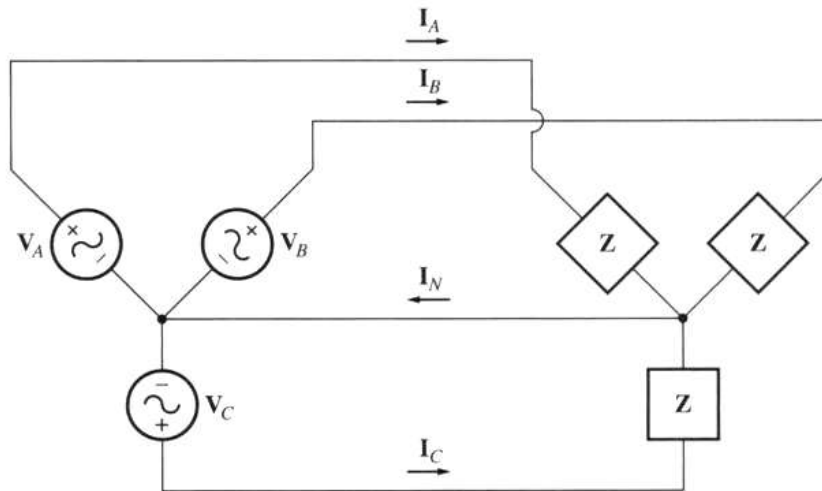


Three-phase systems may have a neutral wire. A neutral wire allows the three-phase system to use a higher voltage while still supporting lower-voltage single-phase loads. In high-voltage distribution situations, it is common not to have a neutral wire as the loads can simply be connected between phases (phase-phase connection).



Three Phase Currents

If each of the phases transfers power to loads that have same impedance, such three-phase power systems (equal magnitude, phase differences of 120° , identical loads) are called **balanced**.



$$I_A = \frac{V \angle 0^\circ}{Z \angle \theta} = I \angle -\theta$$

$$I_B = \frac{V \angle -120^\circ}{Z \angle \theta} = I \angle -120^\circ - \theta$$

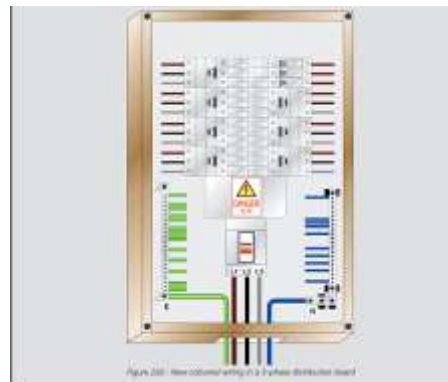
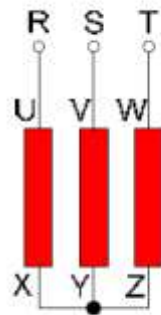
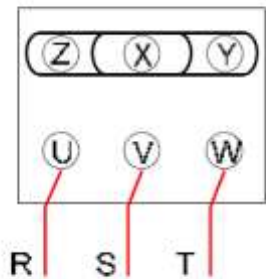
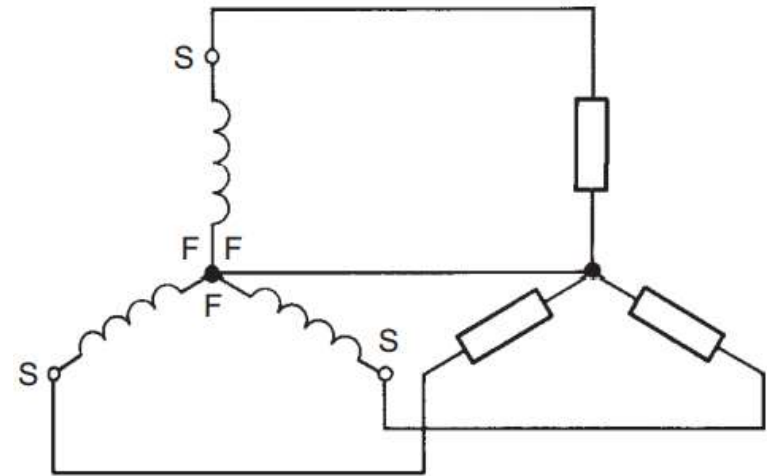
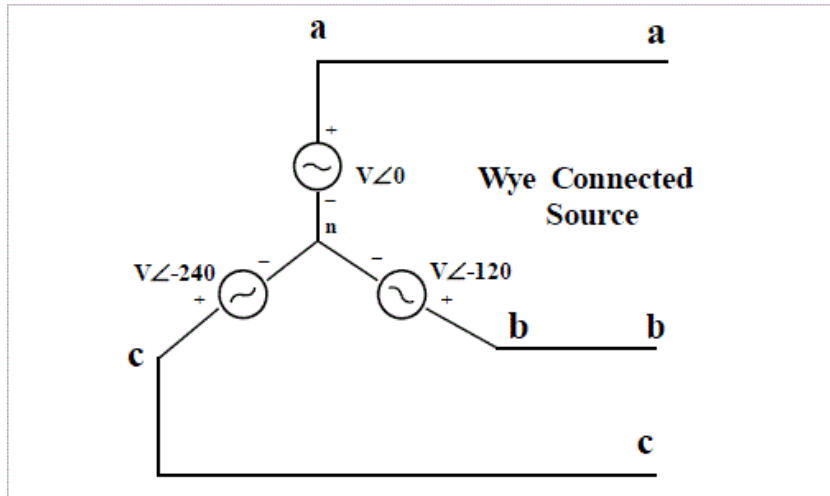
$$I_C = \frac{V \angle -240^\circ}{Z \angle \theta} = I \angle -240^\circ - \theta$$

The phase currents tend to cancel out one another, summing to zero in the case of a linear balanced load. This makes it possible to reduce the size of the neutral conductor because it carries little to no current; all the phase conductors carry the same current and so can be the same size, for a balanced load.

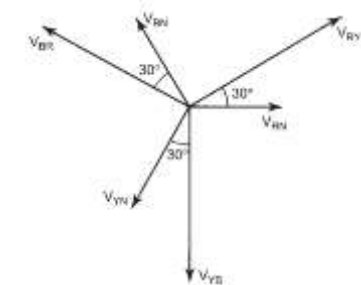
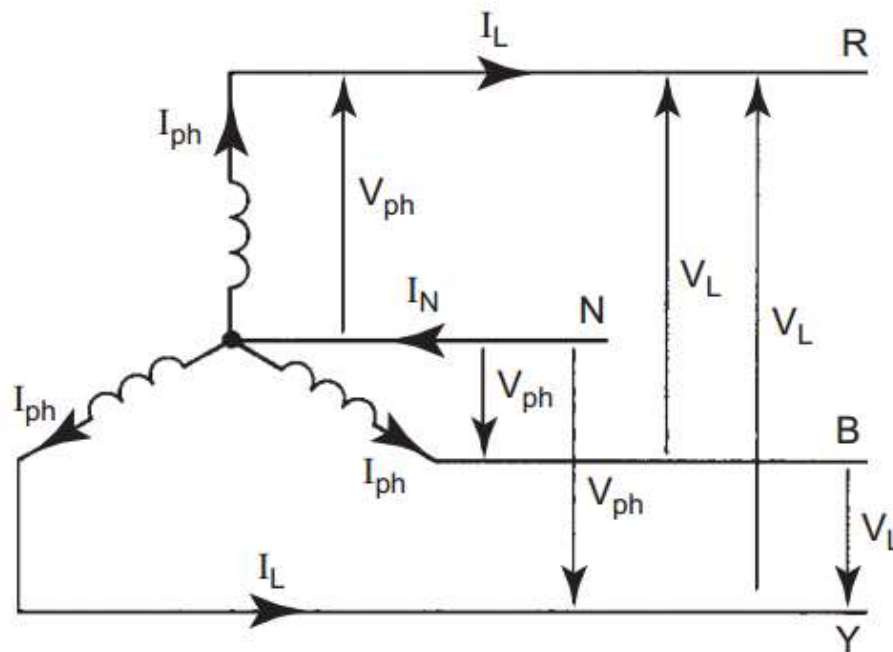
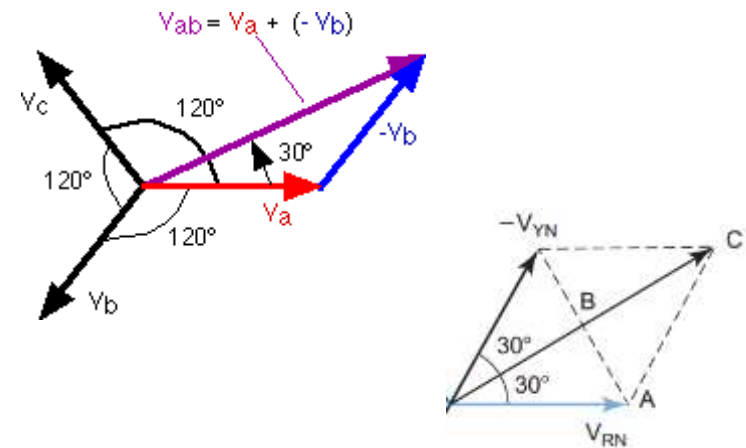
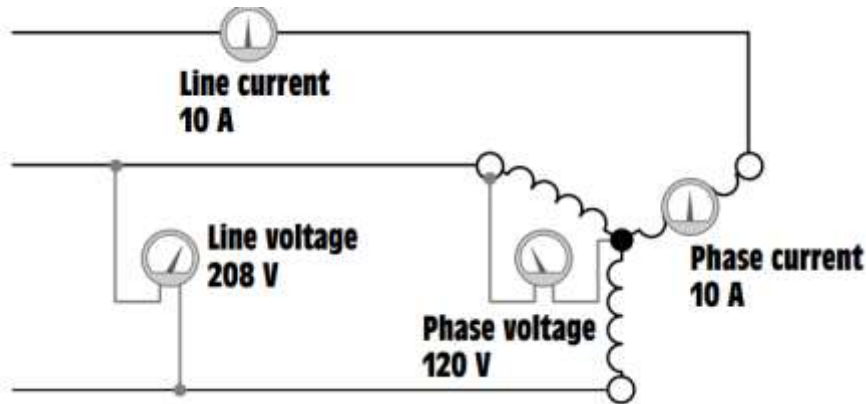
$$\begin{aligned} I_N &= I \left[\cos(-\theta) - \frac{1}{2} \cos(-\theta) + \frac{\sqrt{3}}{2} \sin(-\theta) - \frac{1}{2} \cos(-\theta) - \frac{\sqrt{3}}{2} \sin(-\theta) \right] \\ &\quad + jI \left[\sin(-\theta) - \frac{1}{2} \sin(-\theta) + \frac{\sqrt{3}}{2} \cos(-\theta) - \frac{1}{2} \sin(-\theta) - \frac{\sqrt{3}}{2} \cos(-\theta) \right] \\ &= 0 \end{aligned}$$

Star (Wye or Y) Connected Three Phase Systems

It is not necessary to have six wires from the three phase windings to the three loads. Each winding will have a 'start' (S) and a 'finish' (F) end. The star or wye (Y) connection mentioned is achieved by connecting the corresponding ends of the three phases together.



Star (Wye or Y) Connected Three Phase Systems



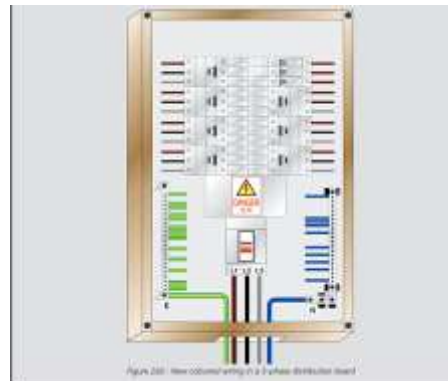
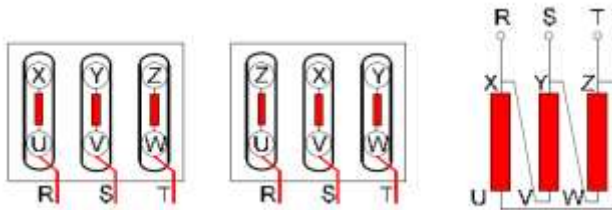
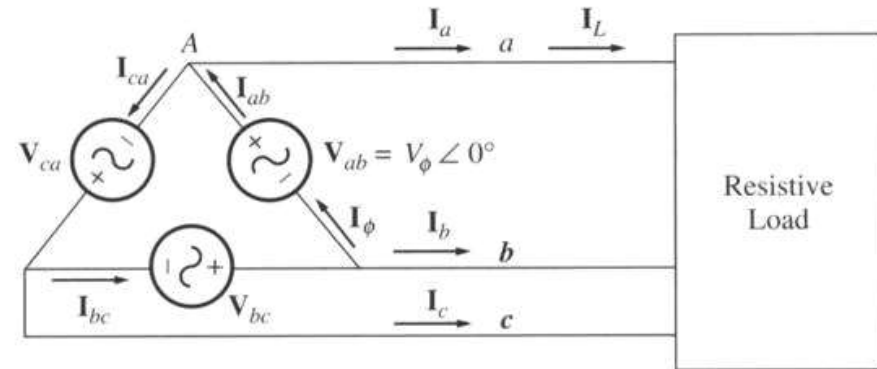
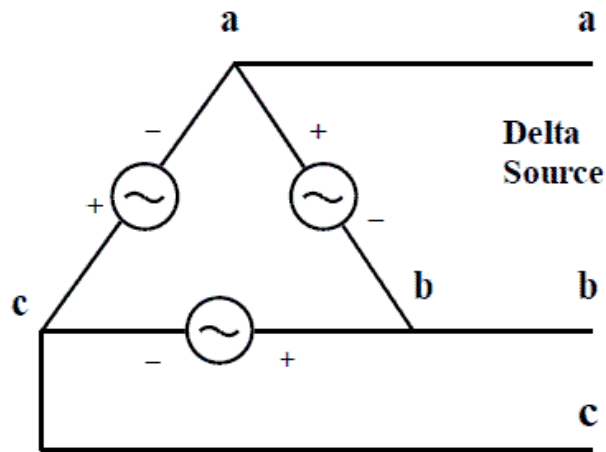
in star configuration, $V_L = \sqrt{3}V_{ph}$

in star configuration, $I_L = I_{ph}$

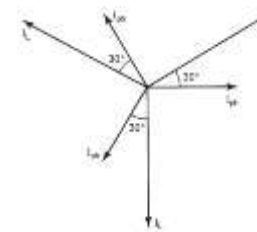
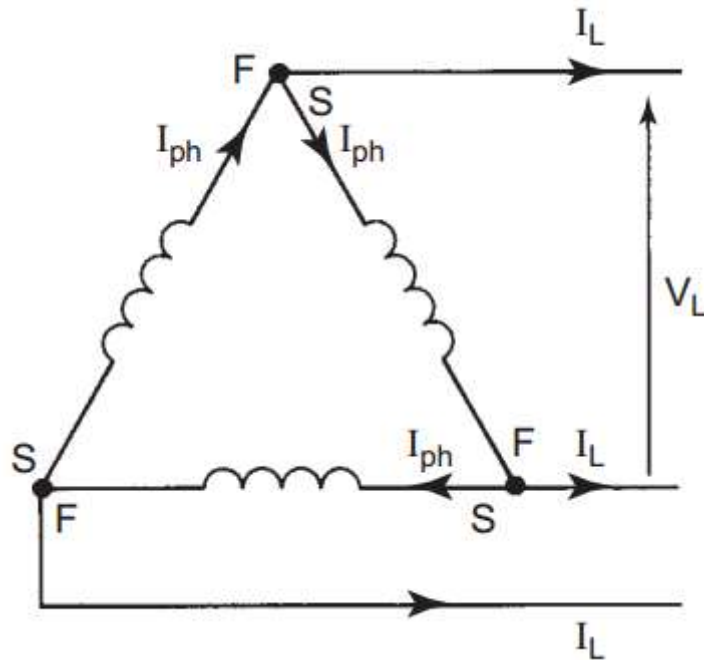
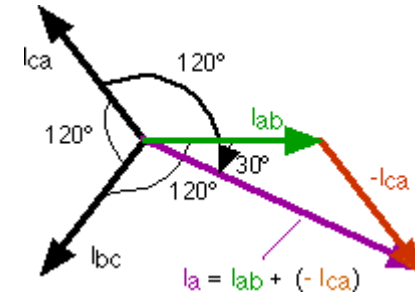
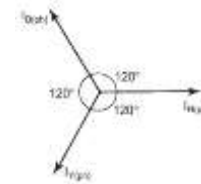
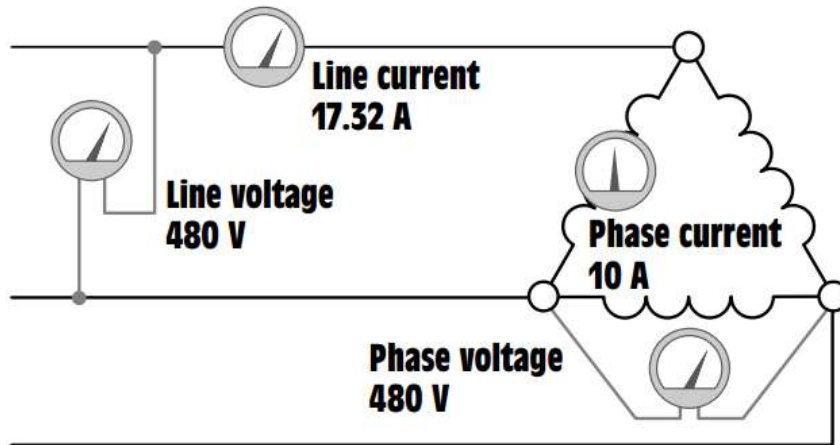
V_{ph} = Phase Voltage, V_L = Line Voltage, I_{ph} = Phase Current, I_L = Line Current

Delta (Mesh or Δ) Connected Three Phase Systems

It is not necessary to have six wires from the three phase windings to the three loads. Each winding will have a 'start' (S) and a 'finish' (F) end. The star or wye (Y) connection mentioned is achieved by connecting the corresponding ends of the three phases together.



Delta (Mesh or Δ) Connected Three Phase Systems



$$V_L = V_{ph}$$

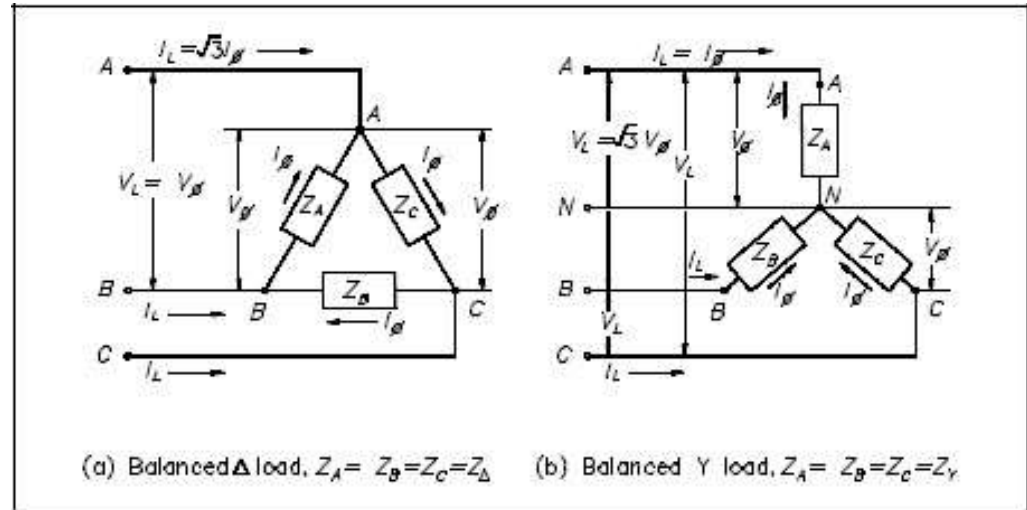
$$I_L = \sqrt{3}I_{ph}$$

V_{ph} = Phase Voltage, V_L = Line Voltage, I_{ph} = Phase Current, I_L = Line Current

Three Phase Power

Total power (P_T) is equal to three times the single-phase power. Then the mathematical representation for total power in a balanced delta or wye load is

$$P_{3\phi} = 3P_{\phi} = 3V_{\phi}I_{\phi}$$



For Star or Y connected systems

$$V_L = \sqrt{3}V_{\phi}, I_L = I_{\phi}$$

$$P_{3\phi} = 3V_{\phi}I_{\phi} = 3 \frac{V_L}{\sqrt{3}} I_L$$

$$P_{3\phi} = \sqrt{3}V_L I_L$$

V_{ϕ} = Phase Voltage, V_L = Line Voltage, I_{ϕ} = Phase Current, I_L = Line Current

For Delta or Δ connected systems

$$V_L = V_{\phi}, I_L = \sqrt{3}I_{\phi}$$

$$P_{3\phi} = 3V_{\phi}I_{\phi} = 3V_L \frac{I_L}{\sqrt{3}}$$

$$P_{3\phi} = \sqrt{3}V_L I_L$$

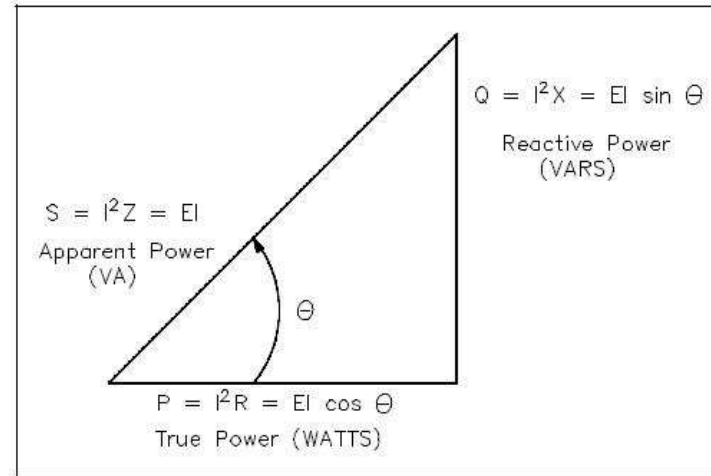
$$P_{3\phi} = \sqrt{3}V_L I_L$$

Three Phase Power Triangle

If the 3 ϕ system is connected to a balanced load with impedance:

$$Z_{\phi} = |Z_{\phi}| \angle \varphi^{\circ}$$

Then the power triangle and the power values of the 3 ϕ system is similar to monophas systems:



$$S_{3\phi} = \sqrt{3} V_L I_L \quad [\text{VA}]$$

$$Q_{3\phi} = \sqrt{3} V_L I_L \sin \varphi \quad [\text{VAR}]$$

$$P_{3\phi} = \sqrt{3} V_L I_L \cos \varphi \quad [\text{W}]$$

$$S_{3\phi} = 3 V_{\phi} I_{\phi} \quad [\text{VA}]$$

$$Q_{3\phi} = 3 V_{\phi} I_{\phi} \sin \varphi \quad [\text{VAR}]$$

$$P_{3\phi} = 3 V_{\phi} I_{\phi} \cos \varphi \quad [\text{W}]$$

V_{ϕ} = Phase Voltage, V_L = Line Voltage, I_{ϕ} = Phase Current, I_L = Line Current

Three Phase Analysis

Problem: An asynchronous electric that runs using three phase grid has a power factor of 0.85 and measured that it gets 7 [A] from grid. What are the powers it gets from R-S-T phases if the phases balanced?

Solution: P : Real Power[Watt] ; Q : Apparent Power [VAR]; S : ReactivePower [VA]

U : 380 [V] → The line voltage is 380 [V]

I : It is the current taken from one phase [A]

$$P = \sqrt{3} \times U \times I \times \cos \phi = \sqrt{3} \times 380 \times 7 \times 0,85 = 3916 \text{ [W]}$$

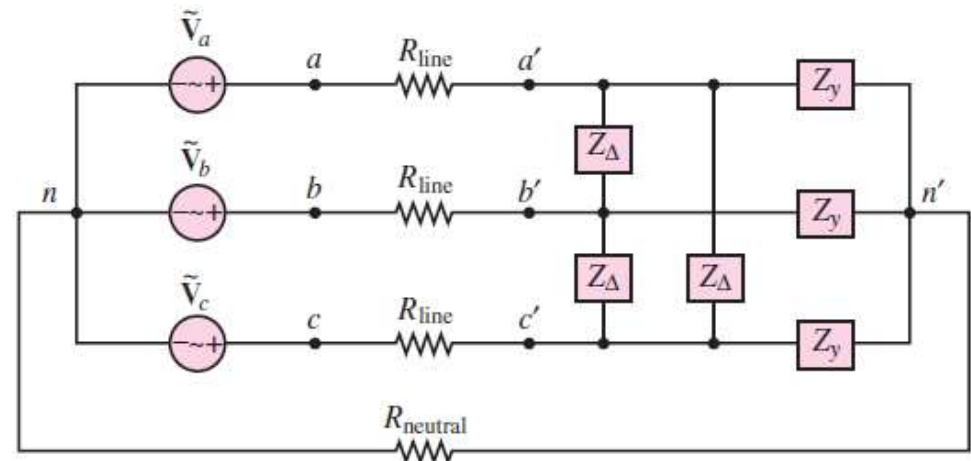
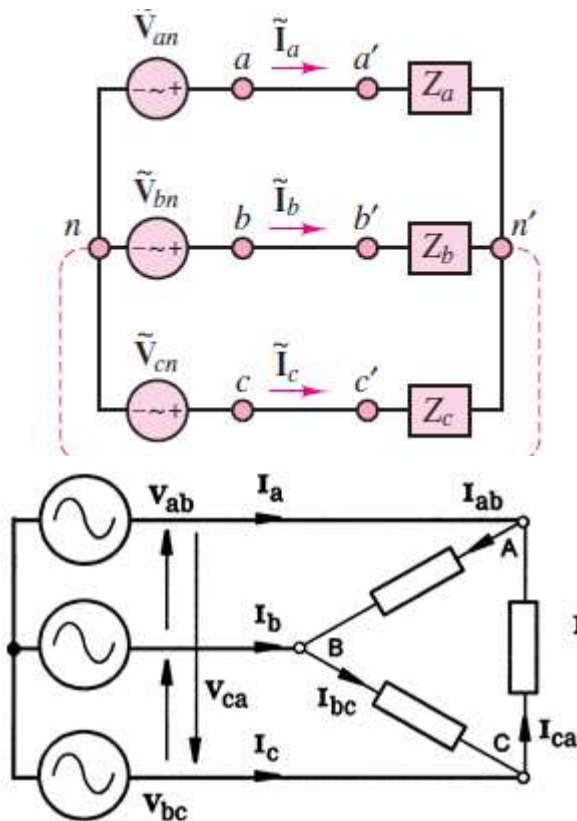
$$Q = \sqrt{3} \times U \times I \times \sin \phi = \sqrt{3} \times 380 \times 7 \times 0,5268 = 2427 \text{ [VAR]}$$

$$S = \sqrt{3} \times U \times I = \sqrt{3} \times 380 \times 7 = 4607 \text{ [VA]}$$

Three Phase Analysis

To do per phase analysis

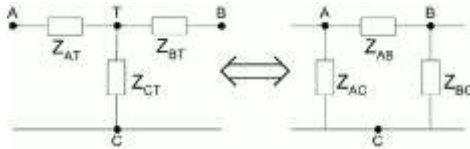
1. Convert all Δ load/sources to equivalent Y's
2. Solve phase "a" independent of the other phases
3. Total system power $S = 3 V_a I_a^*$
4. If desired, phase "b" and "c" values can be determined by inspection (i.e., $\pm 120^\circ$ degree phase shifts)
5. If necessary, go back to original circuit to determine line-line values or internal Δ values



Three Phase Analysis

Delta to wye conversion

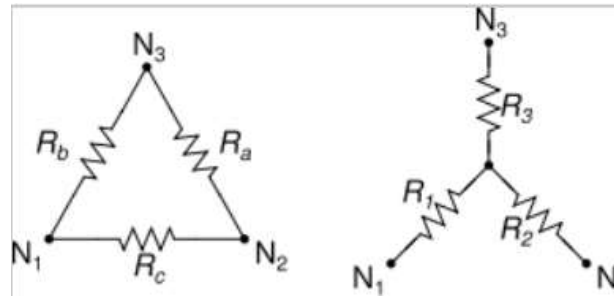
The Y-Δ transform is known by a variety of other names, mostly based upon the two shapes involved, listed in either order. The Y, spelled out as wye, can also be called T or star; the Δ, spelled out as delta, can also be called triangle, Π (spelled out as pi), or mesh. Thus, common names for the transformation include wye-delta or delta-wye, star-delta, star-mesh, or T-Π.



$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}$$

$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$

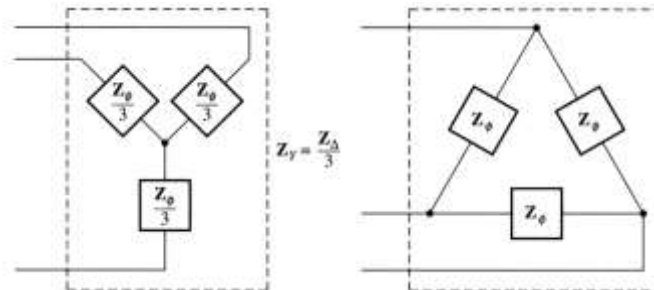
$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}$$



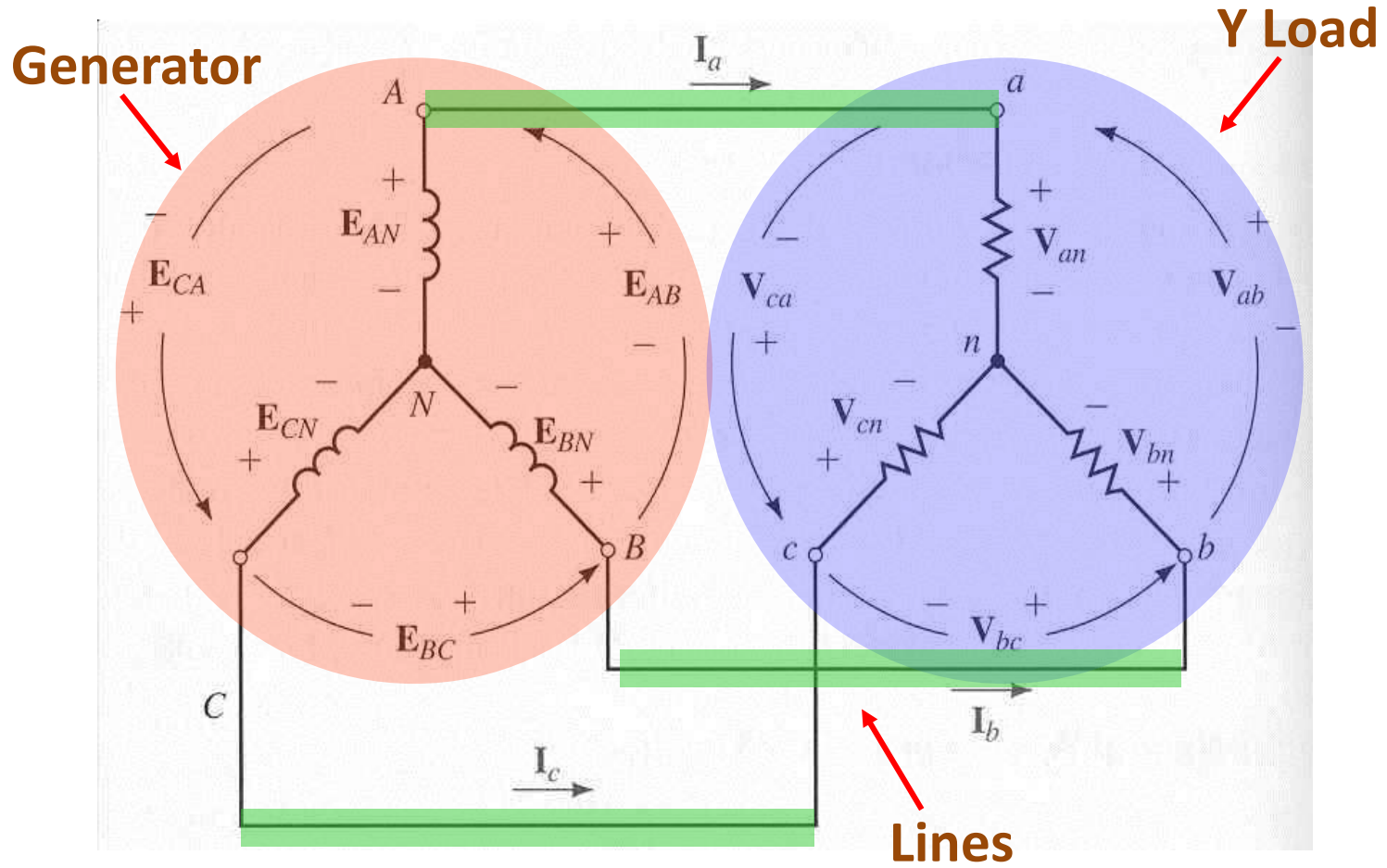
$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_a R_c}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$



Y-Y System



Line/Phase Voltages for a Balanced Y Circuit

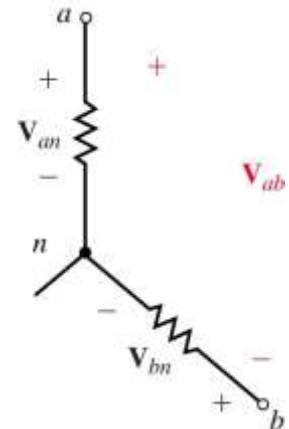
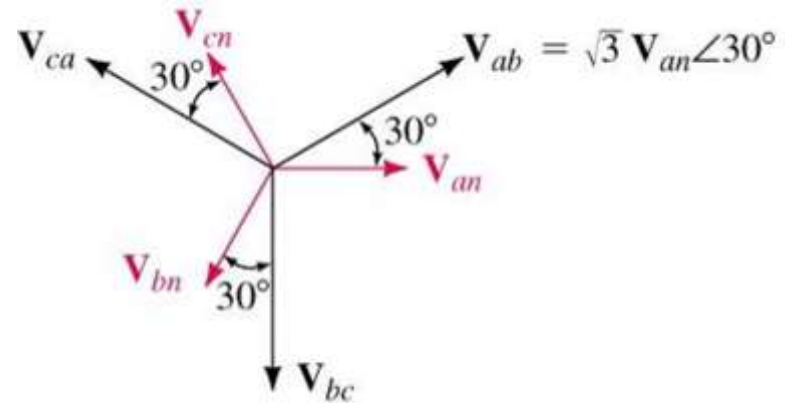
Using the following relationship we can use the line voltage V_{ab} to find the phase voltage V_{an} :

$$V_{ab} = \sqrt{3} V_{an} \angle 30^\circ$$

we can also convert back...

$$V_{an} = \frac{V_{ab}}{\sqrt{3} \angle 30^\circ} \quad E_{AN} = \frac{V_{AB}}{\sqrt{3} \angle 30^\circ}$$




Therefore, given any voltage at a point in a balanced, 3-phase Y system, *you can determine the remaining 5 voltages by inspection* – remember, there are 3 line voltages and 3 phase voltages.



Nominal Voltages

Short Cut

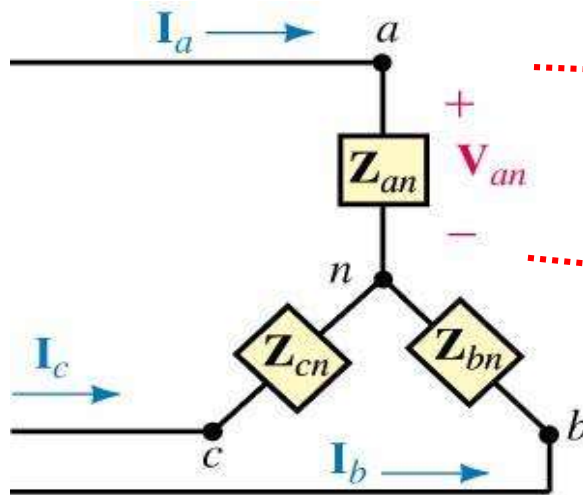
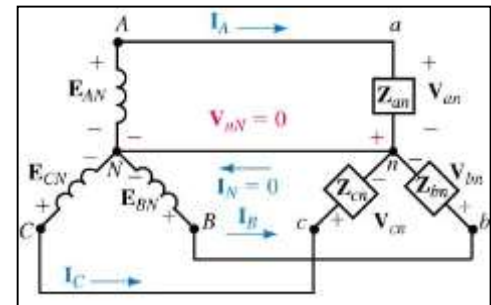
- If phase voltage $V_{an} = 120 \text{ V}$ then line voltages $V_{ab} = 208 \text{ V} \Rightarrow V_{an}\sqrt{3}$.
- This is referred to as **120/208-V** system.
- Two other common nominal voltages are **220/380-V** and **347/600-V**.

<u>Phase Voltages</u>		<u>Line Voltages</u>
$\vec{E}_{AN} = E \angle (\theta + 0^\circ)$		$\vec{E}_{AB} = \sqrt{3}E \angle (30^\circ + \theta)$
$\vec{E}_{BN} = E \angle (\theta - 120^\circ)$		$\vec{E}_{BC} = \sqrt{3}E \angle (-90^\circ + \theta)$
$\vec{E}_{CN} = E \angle (\theta + 120^\circ)$		$\vec{E}_{CA} = \sqrt{3}E \angle (150^\circ + \theta)$

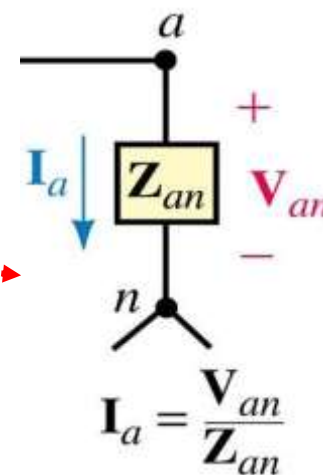
Currents for a Y Circuit

- Similar to finding line and phase currents.
- Recall that for Y loads, line current and phase current are the same.

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_{an}}$$



Line current



Phase current

Law of Sines

- The ratio of the length of a side to the sine of its corresponding opposite angle is constant:

$$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \chi}$$

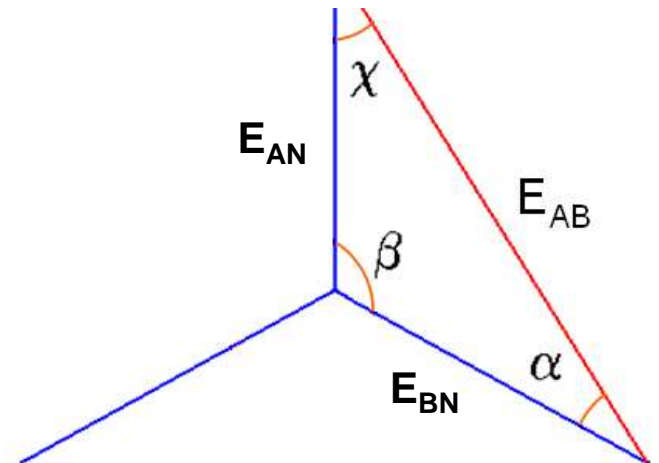
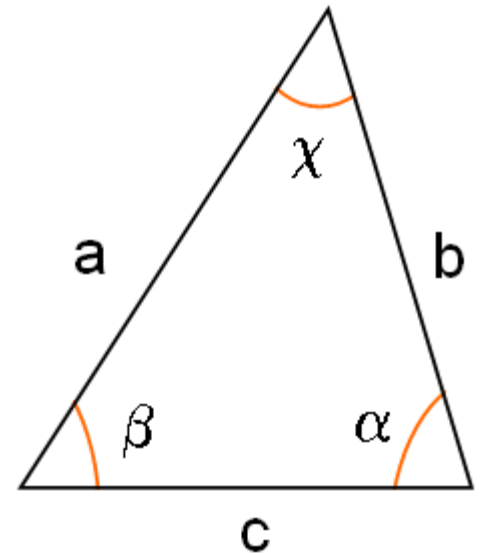
- Applying the law of sines to the vector E_{AB} :

$$\beta = 120^\circ$$

$$\alpha = \chi = \frac{180 - 120}{2} = 30^\circ$$

$$\frac{E_{AN}}{\sin 30^\circ} = \frac{E_{AB}}{\sin 120^\circ}$$

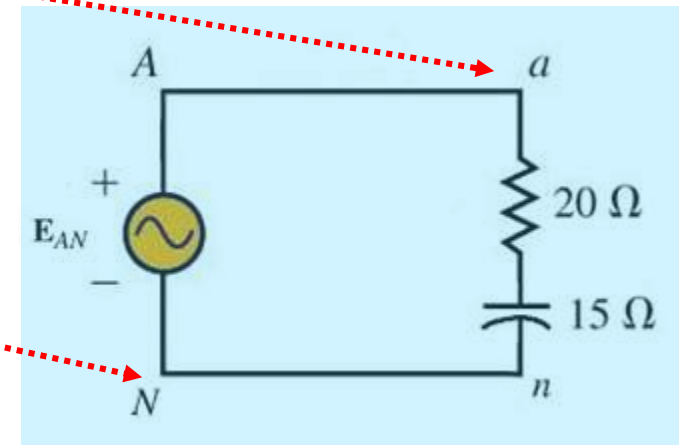
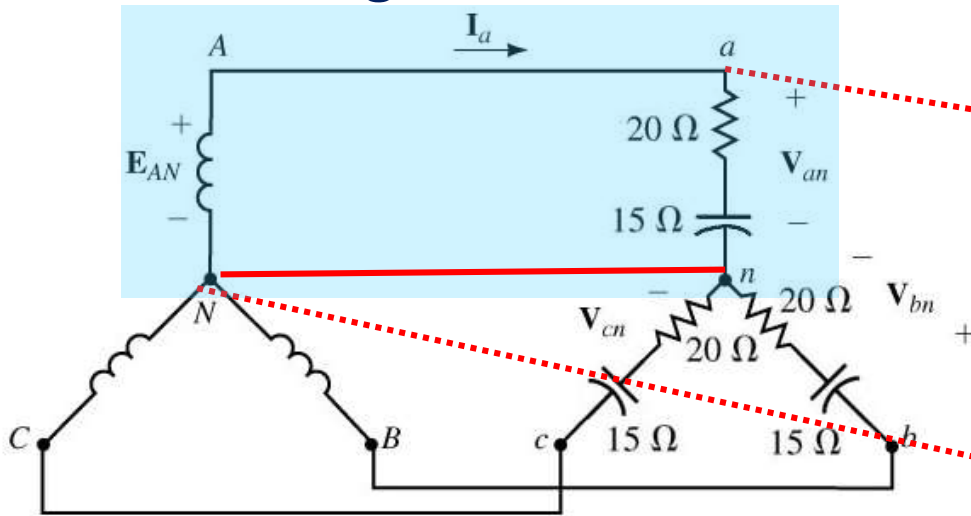
$$E_{AB} = E_{AN} \frac{\sin 120^\circ}{\sin 30^\circ} = E_{AN} \sqrt{3}$$



Y-Y Single Phase Equivalent

Imagining a wire connecting the N and n in the figure below allows for a more simple analysis of the circuit called the Single-Phase Equivalent:

Original Circuit



This works because the loads are balanced and $E_{an} = V_{an}$, $I_a = I_b = I_c$

Single-Phase Equivalent

Example Problem 1

$$\mathbf{E}_{AB} = 208 \angle 0^\circ \text{ V}.$$

Find the phase voltages and line currents.

Line Voltages:

$$\bar{\mathbf{E}}_{AB} = 208 \angle 0^\circ \text{ V}$$

$$\bar{\mathbf{E}}_{BC} = 208 \angle -120^\circ \text{ V}$$

$$\bar{\mathbf{E}}_{CA} = 208 \angle 120^\circ \text{ V}$$

Phase Voltages:

$$\mathbf{E}_{AN} = \frac{\mathbf{E}_{AB}}{\sqrt{3} \angle 30^\circ} = \frac{208 \angle 0^\circ \text{ V}}{\sqrt{3} \angle 30^\circ} = 120 \angle -30^\circ \text{ V}$$

$$\mathbf{E}_{BN} = \mathbf{E}_{AN} \angle (\theta - 120^\circ) = 120 \angle (-30^\circ - 120^\circ) = 120 \angle -150^\circ \text{ V}$$

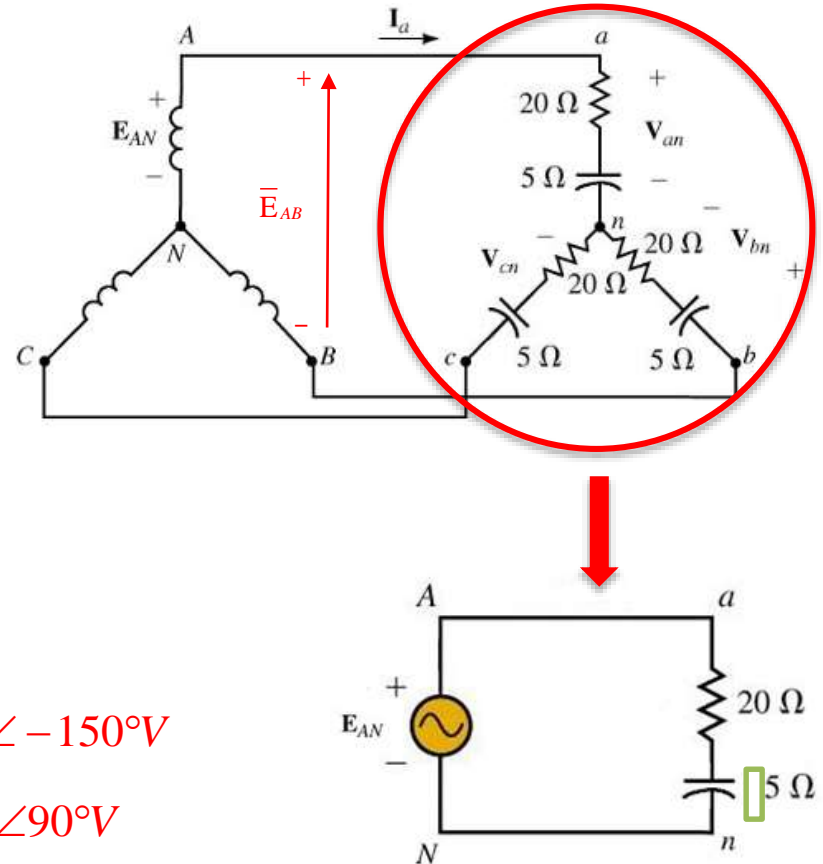
$$\mathbf{E}_{CN} = \mathbf{E}_{AN} \angle (\theta + 120^\circ) = 120 \angle (-30^\circ + 120^\circ) = 120 \angle 90^\circ \text{ V}$$

Phase/Line Currents:

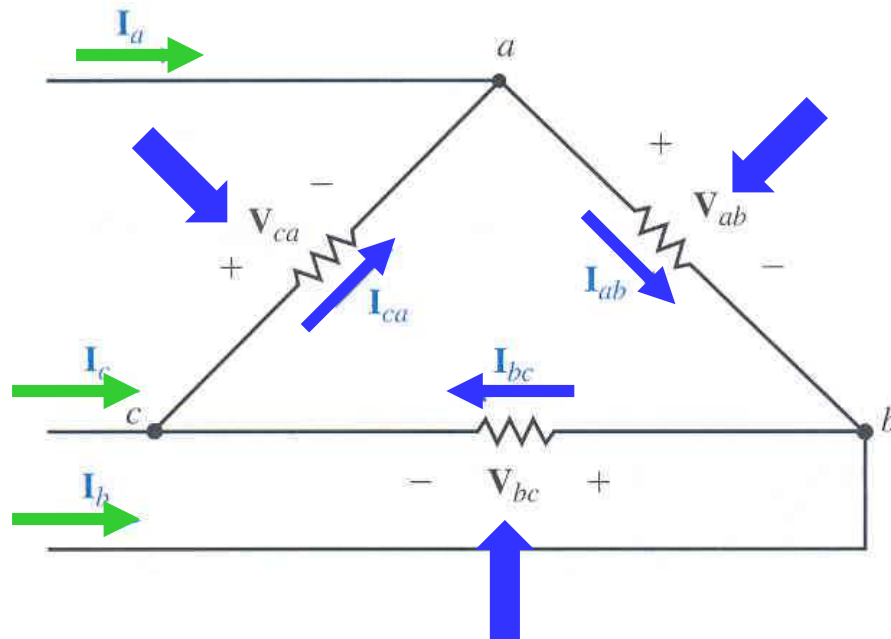
$$\mathbf{I}_a = \frac{\mathbf{E}_{an}}{\mathbf{Z}_{an}} = \frac{208 \angle 0^\circ}{20 - j5} = 5.8 \angle -16^\circ \text{ A}$$

$$\mathbf{I}_b = \mathbf{I}_a \angle (\theta - 120^\circ) = 5.8 \angle (-16^\circ - 120^\circ) = 5.8 \angle -136^\circ \text{ A}$$

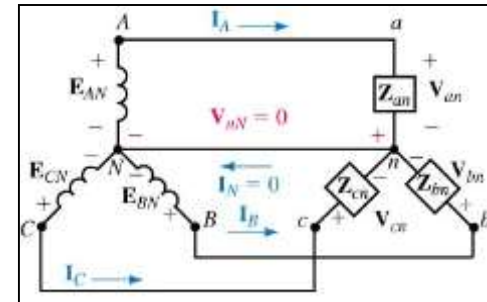
$$\mathbf{I}_c = \mathbf{I}_a \angle (\theta + 120^\circ) = 5.8 \angle (-16^\circ + 120^\circ) = 5.8 \angle 104^\circ \text{ A}$$



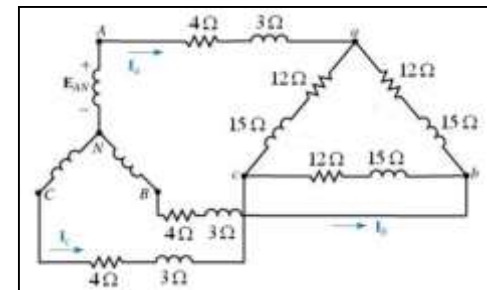
Δ Load Definitions




Y-Y



Y- Δ



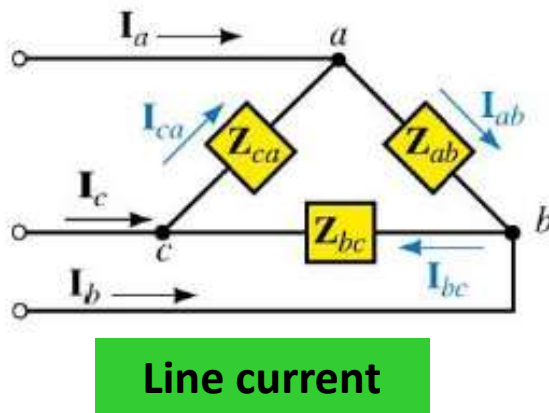
 For Δ loads, **phase voltage** and **line voltage** are the same thing.

 **Line currents** are the currents in the line conductors.

 **Phase currents** are the currents through phases .

Line/Phase Currents for a Δ Circuit

- Relationship between line and phase currents

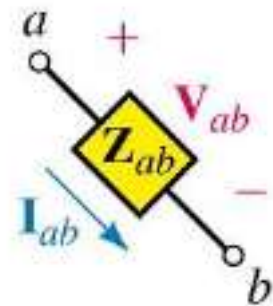


$$I_{ab} = \frac{V_{ab}}{Z_{ab}}$$

$$I_a = I_{ab} - I_{ca}$$

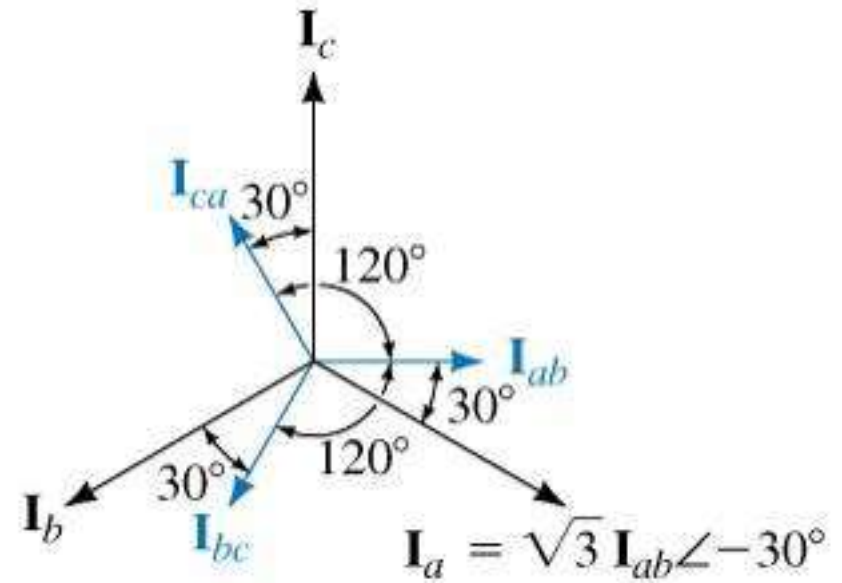
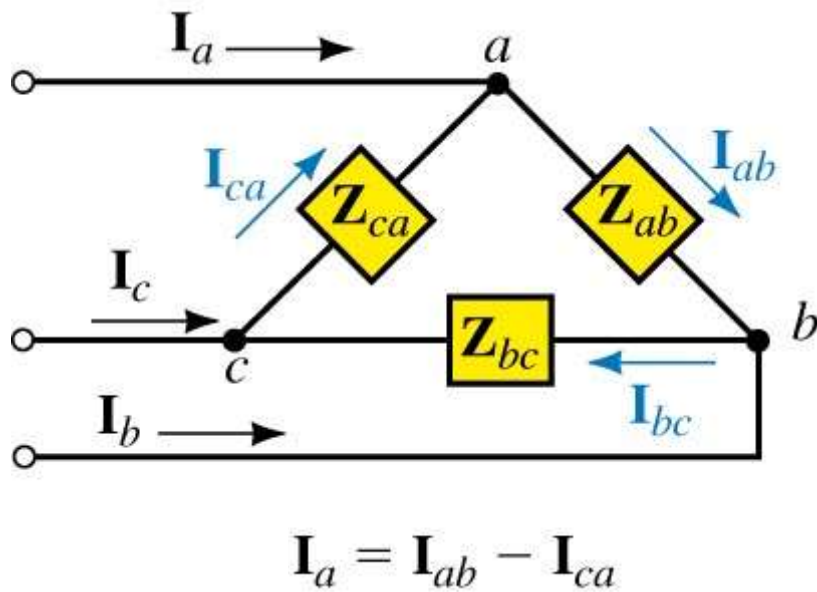
$$I_a = \sqrt{3} I_{ab} \angle -30^\circ$$

Note the -30
here



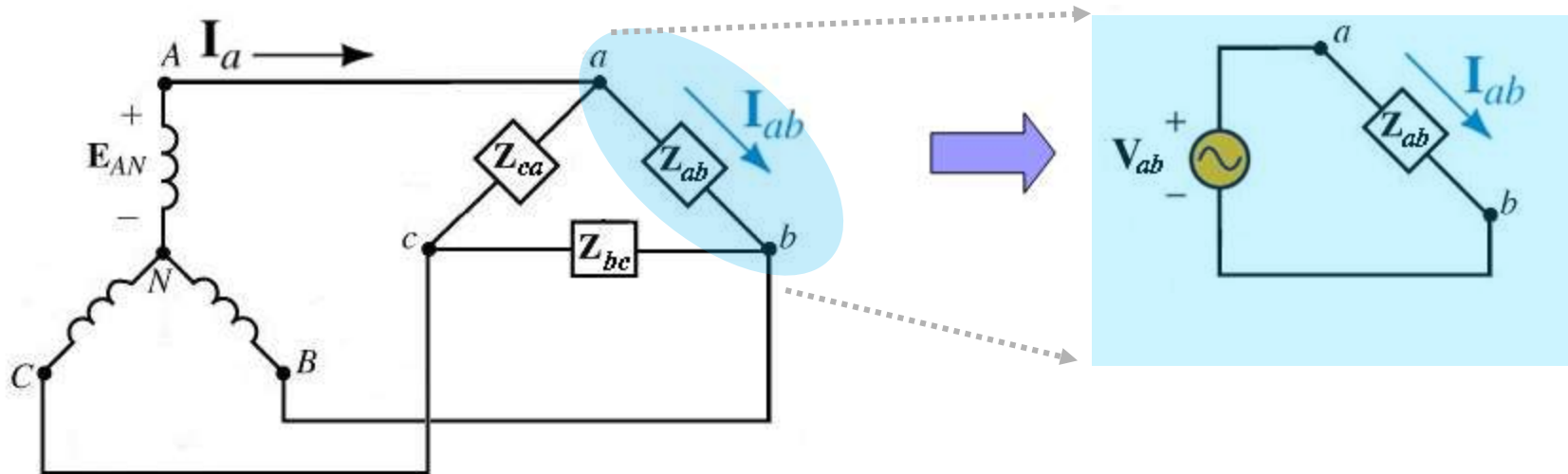
- For a balanced Δ system, the magnitude of line current is **1.732** ($\sqrt{3}$) **times** the magnitude of the phase current and line current **lags** phase current by **30°**.
- Therefore, given any current at a point in a balanced, 3-phase Δ system, you can determine the remaining 5 currents by inspection.

Line/Phase Currents for a Δ Circuit



Phasor diagram

Single Phase Equivalent Circuit to Solve Y- Δ Problem



- This is just an equivalent circuit **of the load side!**
- What is missing from the equivalent circuit?
 - I_a, E_{ab}, E_{an}
- You can only use the equivalent circuit to solve for load side voltages and currents:
 - V_{ab}, I_{ab}
- Notice that this equivalent circuit uses **LINE VOLTAGE**.
 - #1 mistake with DELTA circuits is using the wrong kind of voltage!

Example Problem 2

$$I_a = 41.6 \angle 6.9^\circ \text{ A}.$$

Determine the phase currents in the load.

Determine the supply line voltage E_{AB} .

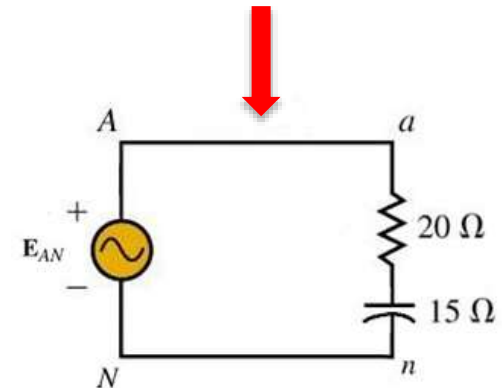
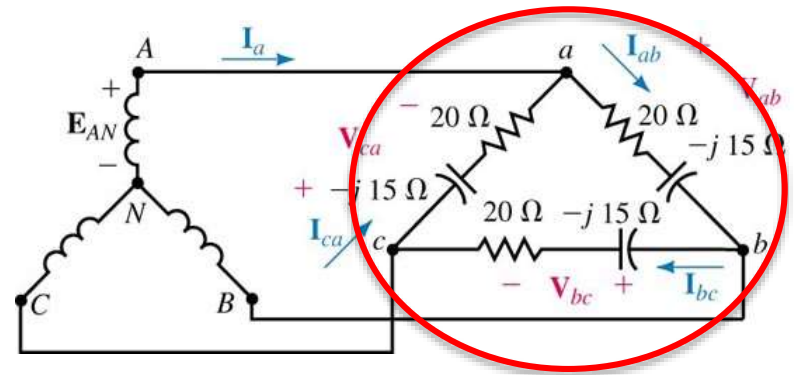
$$I_a = \sqrt{3} I_{ab} \angle -30^\circ$$

$$I_{ab} = \frac{I_a}{\sqrt{3} \angle -30^\circ} = \frac{41.6 \text{ A} \angle 6.9^\circ}{\sqrt{3} \angle -30^\circ} = \boxed{23.7 \text{ A} \angle 36.9^\circ}$$

$$I_{bc} = I_{ab} \angle \theta^\circ - 120^\circ = 23.7 \text{ A} \angle (36.9^\circ - 120^\circ) = \boxed{23.7 \text{ A} \angle -83.1^\circ}$$

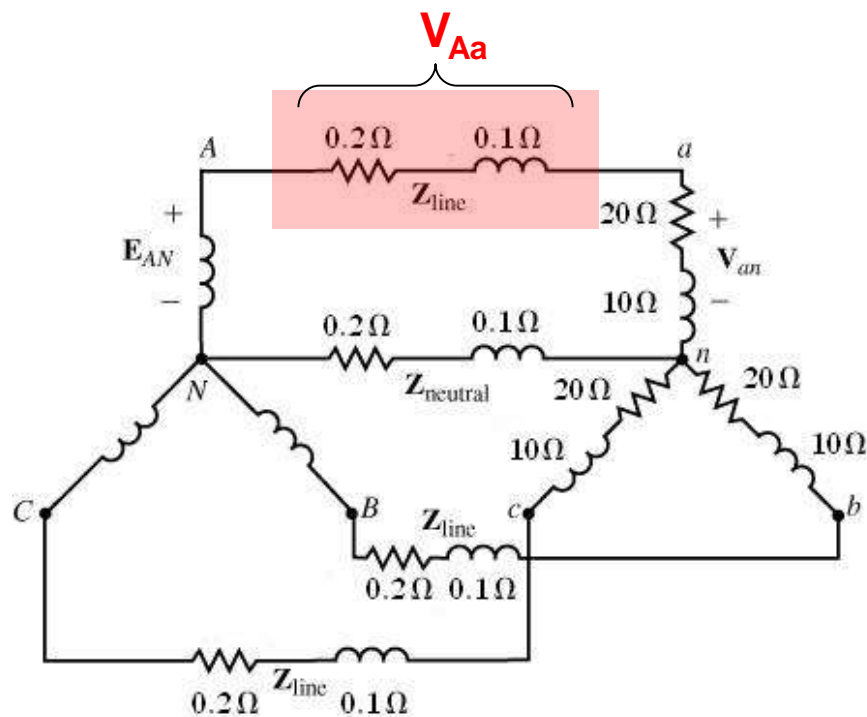
$$I_{ca} = I_{ab} \angle \theta^\circ + 120^\circ = 23.7 \text{ A} \angle (36.9^\circ + 120^\circ) = \boxed{23.7 \text{ A} \angle 156.9^\circ}$$

$$I_a = \frac{V_{an}}{Z_{an}} \Rightarrow V_{ab} = E_{ab} = I_{ab} Z_{ab} = (23.7 \text{ A} \angle 36.9^\circ)(20 - j15) = \boxed{591.8 \text{ V} \angle 0^\circ}$$



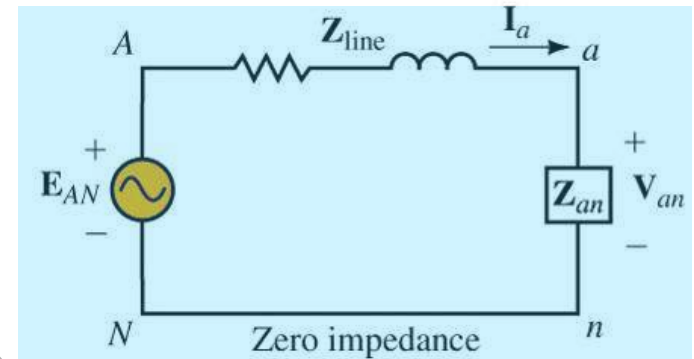
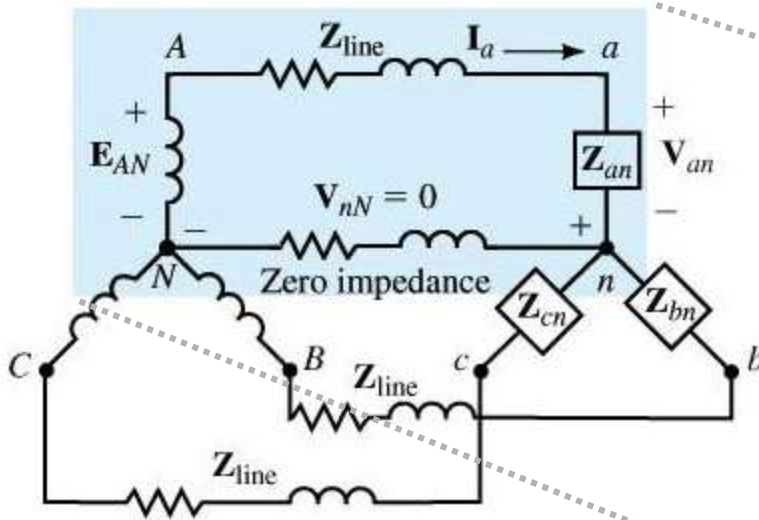
Line Impedances in 3 Phase AC

- Impedances in transmission lines complicate three phase analysis.
- Voltage drop in the transmission line must be accounted for:
 E_{AN} will not be the same as V_{an} .



Y-Y Single Phase Equivalent

Original circuit

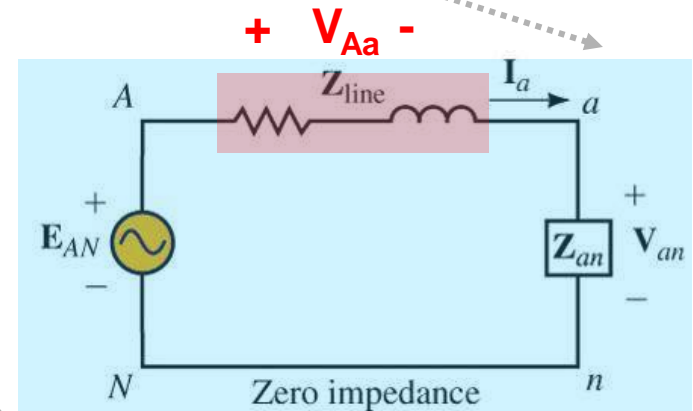
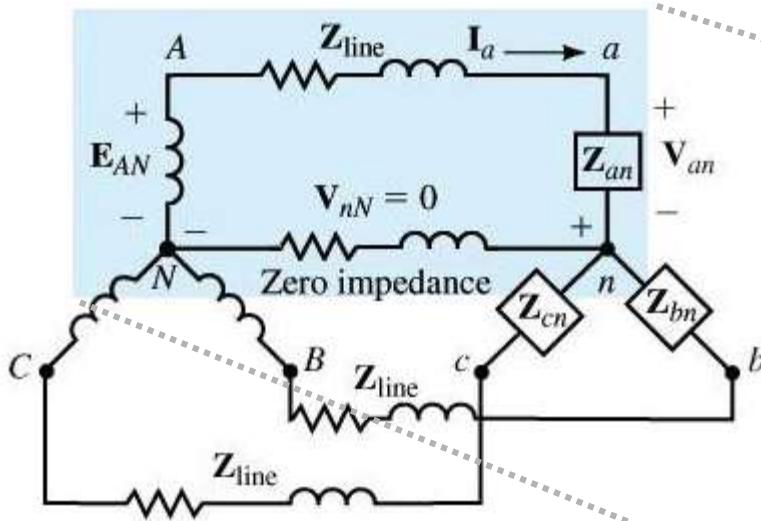


Single-phase equivalent

- Notice that the neutral line impedance drop is ignored on the single phase equivalent.
- No voltage drop occurs across this neutral line since in reality, no current flows through it.

Y-Y Single Phase Equivalent

Original circuit



Single-phase equivalent

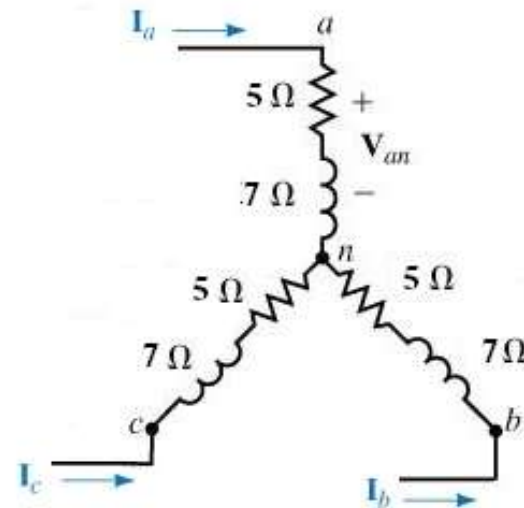
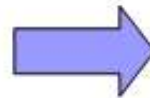
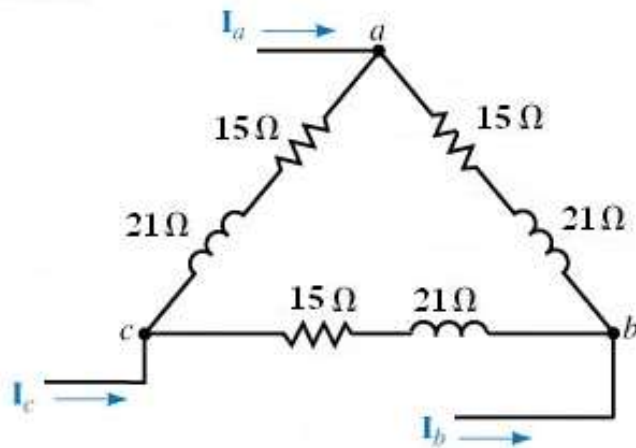
- Use KVL on the single phase equivalent to find unknowns:

$$E_{AN} - V_{Aa} - V_{an} = 0$$

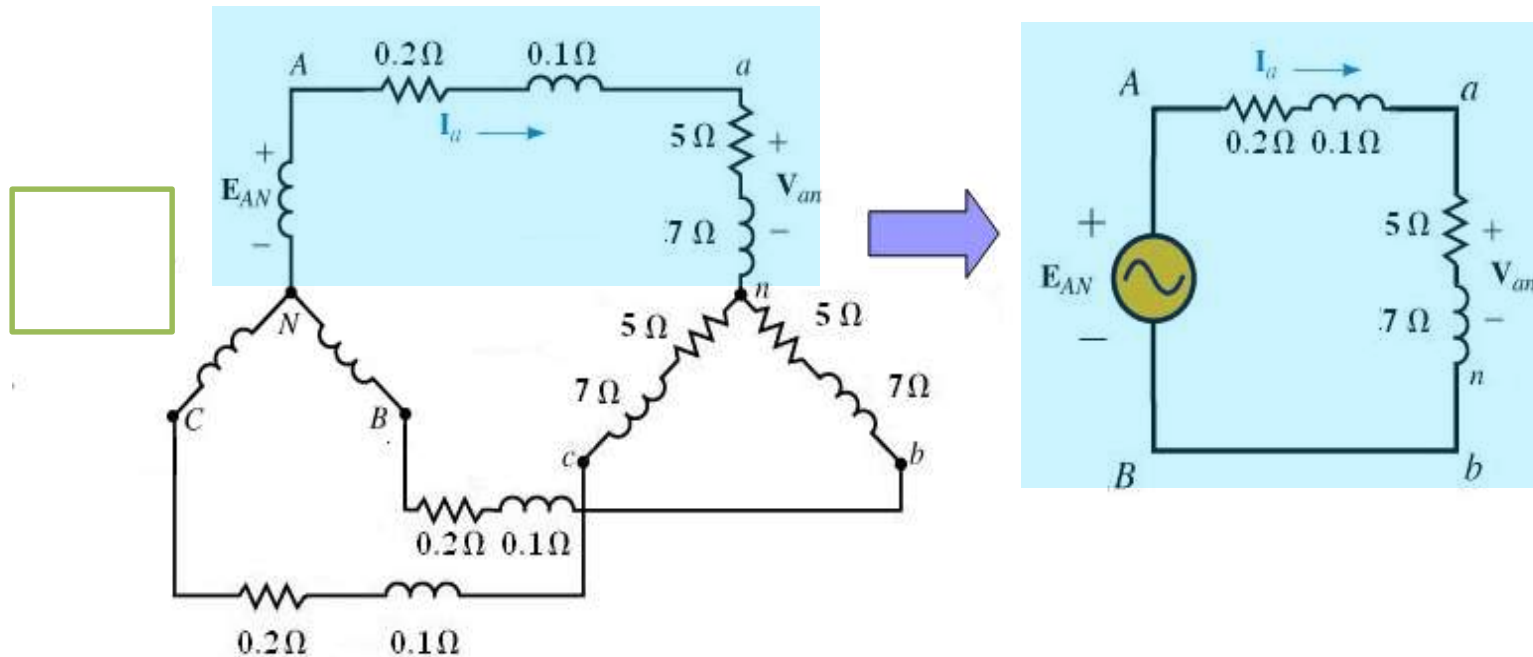
Δ to Y Conversion

- A Delta circuit with line impedances **cannot** be solved without first performing a $\Delta \rightarrow Y$ conversion.
- The Delta load can be converted to an equivalent Wye load as shown:

$$Z_Y = \frac{Z_{\Delta}}{3}$$



Equivalent Delta Circuit With Line Impedances



Y to Δ CONVERSION

$$Z_Y = \frac{Z_\Delta}{3} = \frac{15 + 21j}{3} = 5 + 7j$$

Example Problem 3

$E_{AB} = 208 \angle 0^\circ$ V. Determine:

- Find load phase voltage V_{ab} .
- Find the load phase current I_{ab} .

$$Z_Y = \frac{Z_\Delta}{3} \Rightarrow Z_Y = \frac{12 + j15}{3} = 4 + j5$$

$$E_{an} = \frac{E_{ab}}{\sqrt{3} \angle 30^\circ} = \frac{208V \angle 0^\circ}{\sqrt{3} \angle 30^\circ} = 120V \angle -30^\circ$$

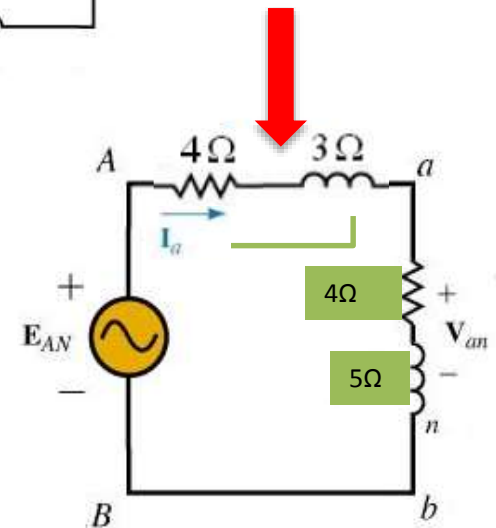
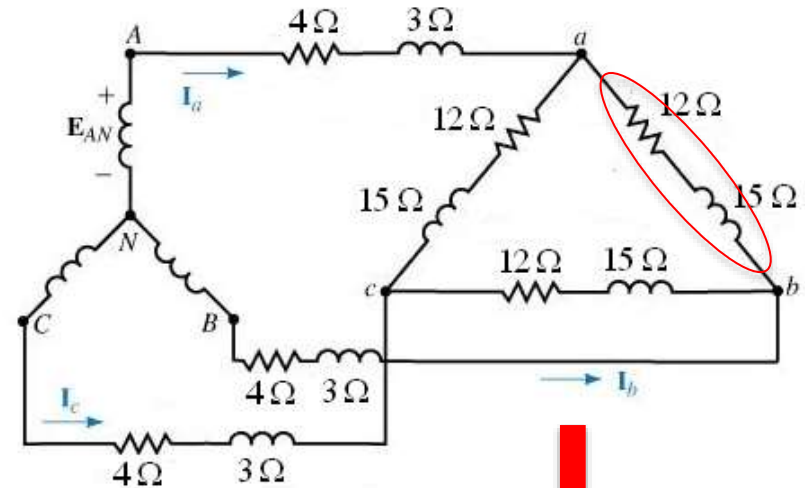
Take note that E_{an} does not equal V_{an} in this case due to the line impedances.

$$I_a = \frac{E_{an}}{Z_{Tan}} = \frac{120V \angle 30^\circ}{(4 + j3) + (4 + j5)} = 10.6A \angle -75^\circ$$

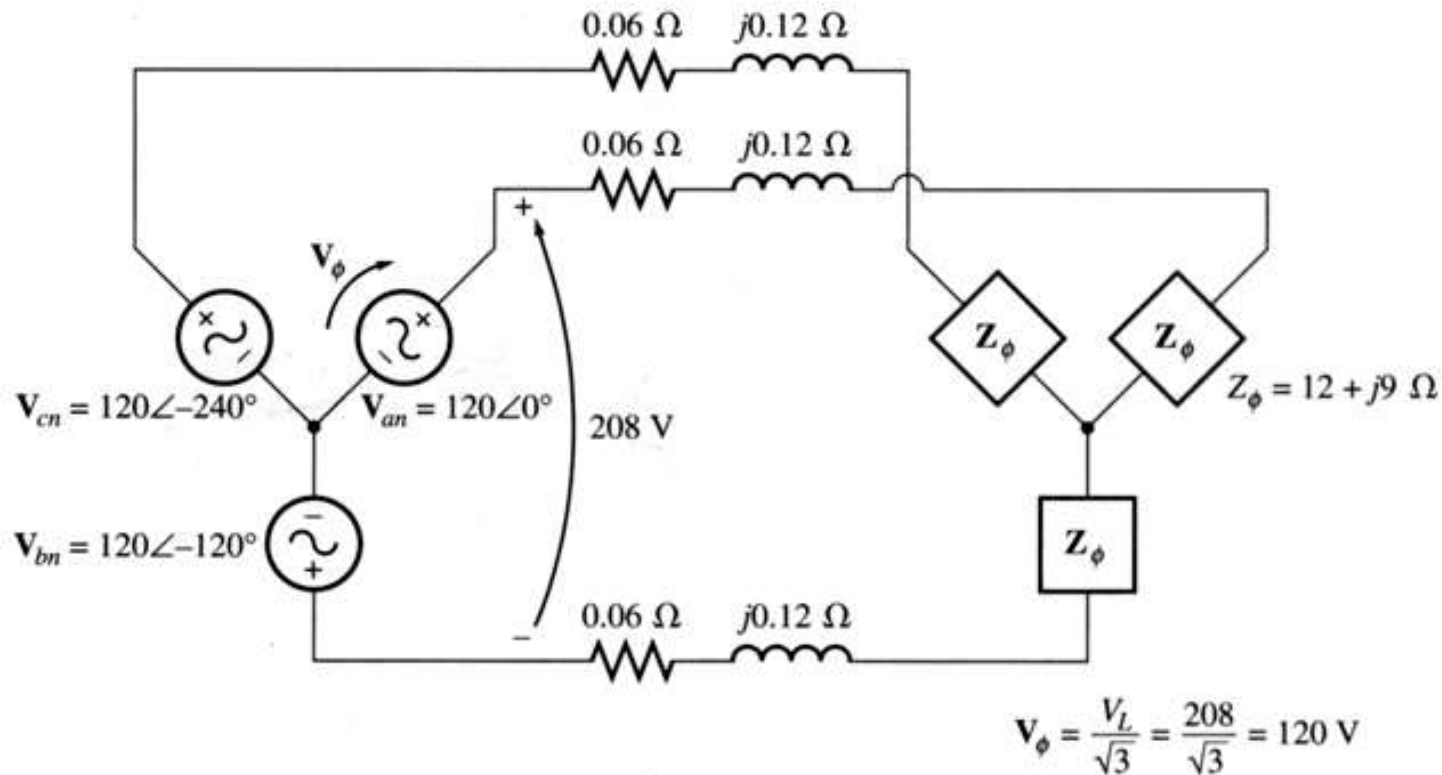
$$I_{ab} = \frac{I_a}{\sqrt{3} \angle -30^\circ} = \frac{10.6A \angle -75^\circ}{\sqrt{3} \angle -30^\circ} = \boxed{6.12A \angle -45^\circ}$$

$$V_{ab} = I_{ab} Z_{ab} = (6.12A \angle -45^\circ)(12 + j15) = \boxed{118V \angle 6^\circ}$$

NOTE: When calculating V_{ab} use the original Δ -circuit line impedances.



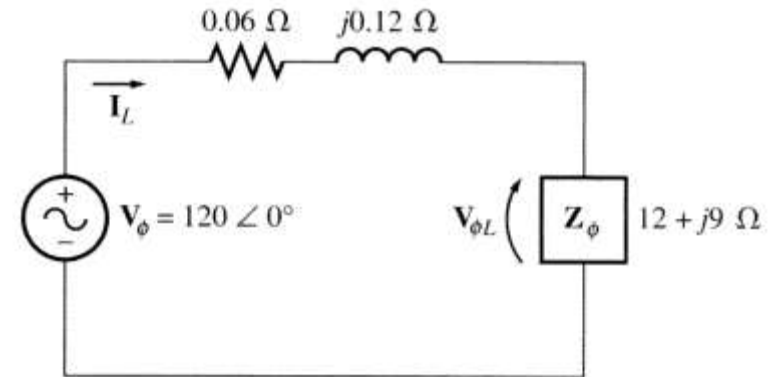
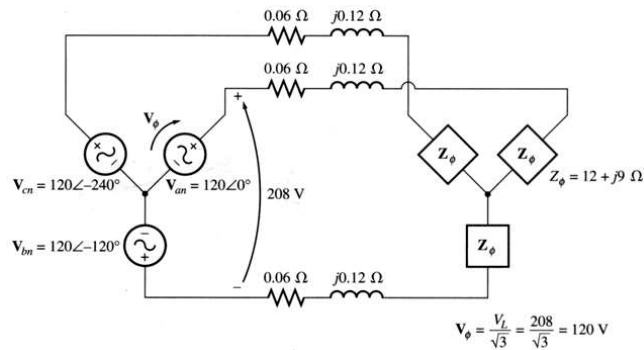
Three Phase Analysis



For a 208-V three-phase ideally balanced system, find:

- the magnitude of the line current I_L ;
- The magnitude of the load's line and phase voltages V_{LL} and $V_{\phi L}$;
- The real, reactive, and the apparent powers consumed by the load;
- The power factor of the load.

Three Phase Analysis



a) The line current:

Is the load inductive or capacitive??

$$I_L = \frac{V}{Z_L + Z_{load}} = \frac{120 \angle 0^\circ}{(0.06 + j0.12) + (12 + j9)} = \frac{120 \angle 0^\circ}{12.06 + j9.12} = \frac{120 \angle 0^\circ}{15.12 \angle 37.1^\circ} = 7.94 \angle -37.1^\circ \text{ A}$$

b) The phase voltage on the load:

$$V_{\phi L} = I_{\phi L} Z_{\phi L} = (7.94 \angle -37.1^\circ)(12 + j9) = (7.94 \angle -37.1^\circ)(15 \angle 36.9^\circ) = 119.1 \angle -0.2^\circ \text{ V}$$

The magnitude of the line voltage on the load:

$$V_{LL} = \sqrt{3} V_{\phi L} = 206.3 \text{ V}$$

Three Phase Analysis

c) The real power consumed by the load:

$$P_{load} = 3V_{\phi}I_{\phi}\cos\theta = 3 \cdot 119.1 \cdot 7.94 \cos 36.9^{\circ} = 2270 \text{ W}$$

The reactive power consumed by the load:

$$Q_{load} = 3V_{\phi}I_{\phi}\sin\theta = 3 \cdot 119.1 \cdot 7.94 \sin 36.9^{\circ} = 1702 \text{ var}$$

The apparent power consumed by the load:

$$S_{load} = 3V_{\phi}I_{\phi} = 3 \cdot 119.1 \cdot 7.94 = 2839 \text{ VA}$$

d) The load power factor:

$$PF_{load} = \cos\theta = \cos 36.9^{\circ} = 0.8 \text{ --lagging}$$

Per phase circuit :

