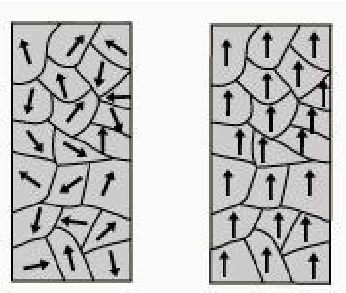
MEE210 Electrical Machines

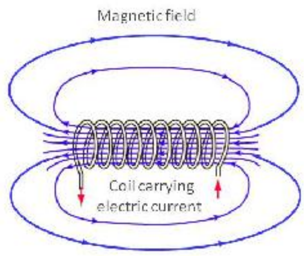
Electromagnetism

Electromagnetism

Magnetism is associated with charges in motion (currents):

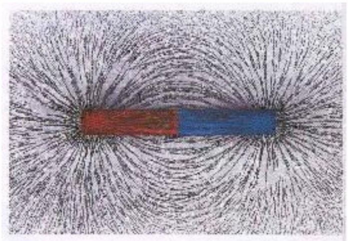
- *microscopic* currents in the atoms of magnetic materials.
- *macroscopic* currents in the windings of an *electromagnet*.



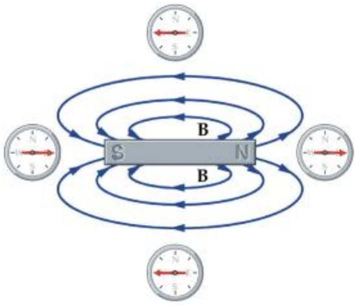


Electromagnetism

The region around a moving charge is disturbed by the charge's motion.



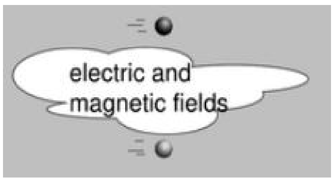
The Magnetic field "B" defines the effect of a magnet at a point in vector form.

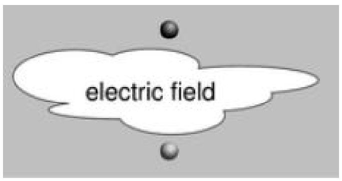


The magnetic field lines show the path, an object would take if it were attracted to one pole of magnet (and, of course, repelled by the other).

Electromagnetism

If an isolated charge is moving, the space contains both an electric field AND a magnetic field. If the charge is stationary, only an electric field is present.





Similar to electric force in strength and direction, magnetic objects are said to have `poles' (north and south, instead of positive and negative charge).

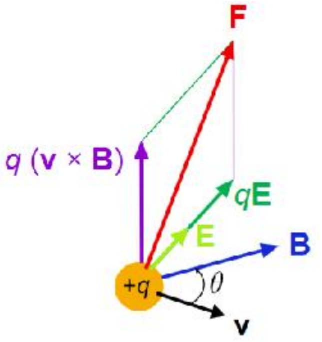
However, magnetic objects are always found in pairs, there exists no isolated magnetic poles in Nature.

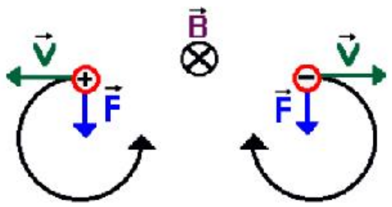
Electromagnetic Force

The magnetic field B is defined due to the Lorentz Force Law, and specifically from the magnetic force on a moving charge:

Magnetic Force

$$F_{Lorentz} = q\vec{E} + \boxed{q\vec{v} \times \vec{B}}$$





Electromagnetic Force

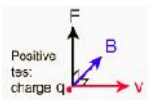
1. The force is perpendicular to both the velocity v of the charge q and the magnetic field B.
2. The magnitude of the force is F = qvB sinθ where θ is the angle < 180 degrees between the velocity and the magnetic field.

This implies that the magnetic force on a stationary charge or a charge moving parallel to the magnetic field is zero.

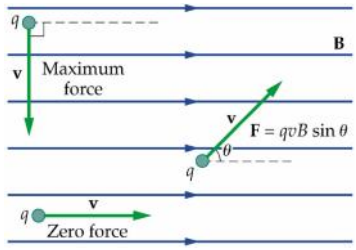
3. The direction of the force is given by the right hand rule.

Magnetic Force

$$F_{Lorentz} = q\vec{E} + \boxed{q\vec{v} \times \vec{B}}$$



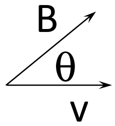
$$\left|F_{magnetic}\right| = q\left|\vec{v}\right|\left|\vec{B}\right|\sin^{-}$$

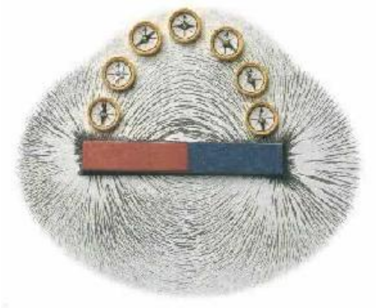


Magnetic Fields

The magnitude of the magnetic field by measuring the force on a moving charge:

$$|B| = \frac{\left|\vec{F}_{mag}\right|}{q|\vec{v}|\sin}$$





The magnetic field's units are N·s/C·m, which are usually abbreviated as Teslas,

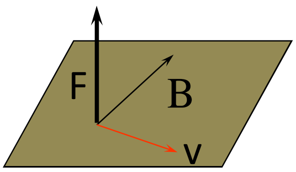
1 Tesla=1 N·s/C·m.

A smaller magnetic field unit is the Gauss

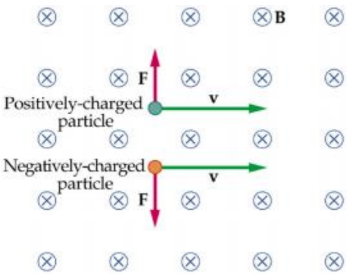
1 Tesla = 10,000 Gauss

Direction of Magnetic Forces

The direction of the magnetic force on a moving charge is perpendicular to the plane formed by B and v.



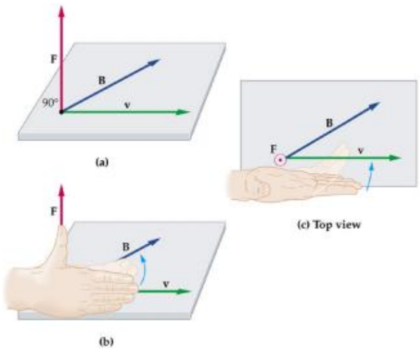
To determine the direction, the Right Hand Rule (RHR) must be applied



To depict a vector oriented perpendicular to the page we use crosses and dots.

- A cross indicates a vector going into the page (think of the tail feathers of an arrow disappearing into the page).
- A dot indicates a vector coming out of the page (think of the tip of an arrow coming at you, out of the page).

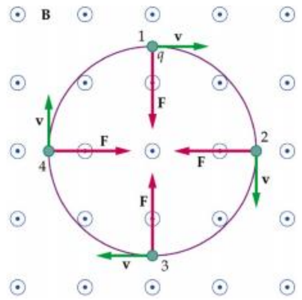
Direction of Magnetic Forces (Right Hand Rule)



- Draw vectors **v** and **B** with their tails at the location of the charge q.
- Point fingers of right hand along velocity vector **v**.
- Curl fingers towards Magnetic field vector **B.**
- Thumb points in direction of magnetic force **F** on q, perpendicular to both **v** and **B**.

Motion of Charges in B Fields

If a charged particle is moving perpendicular to a uniform magnetic field, its trajectory will be a circle because the force F=qvB is always perpendicular to the motion, and therefore centripetal.



$$F_a - F_{mag} = 0$$

$$F_a = ma = \frac{mv^2}{r}$$

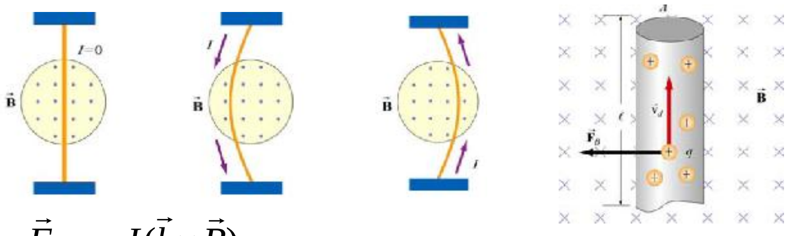
$$F_{mag} = qvB$$

From which we find the radius of the circular trajectory is:

$$r = \frac{mv}{qB}$$

Magnetic Force on a Current-Carrying Wire

Since electric current consists of a collection of charged particles in motion, when placed in a magnetic field, a current-carrying wire will also experience a magnetic force.

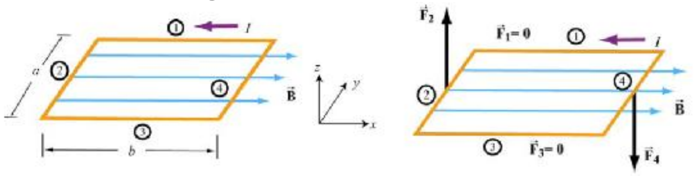


$$\vec{F}_{mag} = I(\vec{l} \times \vec{B})$$

$\vec{l}$: *length vector with a magnitude l and directed along the* direction of the electric current.

Torque on a Current Loop

The magnetic forces acting on sides 1 and 3 vanish because their length vectors are parallel to B and their cross products vanish. On the other hand, the magnetic forces acting on segments 2 and 4 are non-vanishing:



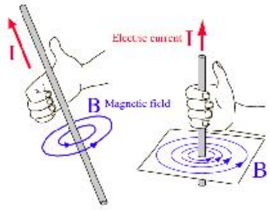
Even though the net force on the loop vanishes, the forces F_2 and F_4 will produce a torque which causes the loop to rotate

$$\vec{\tau}_{mag} = I(\vec{A} \times \vec{B})$$

$\vec{A}$: It is convenient to introduce the area vector *is defined with a magnitude of area of the loop and the direction normal to the plane of the loop. The direction of the positive sense is set by the conventional right-hand rule.*

Magnetic Field of Current

The magnetic field lines around a long wire which carries an electric current form concentric circles around the wire.



$$B = \frac{\mu_0 I}{2\pi r}$$

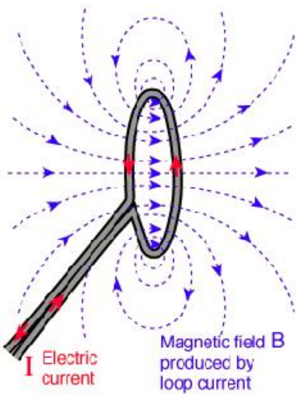
The constant μ_0 is the permeability of free space.

The direction of the magnetic field is perpendicular to the wire

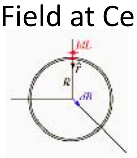
and is in the direction the fingers of your right hand would curl if you wrapped them around the wire with your thumb in the direction of the current.

Magnetic Field of Current Loop

Electric current in a circular loop creates a magnetic field which is more concentrated in the center of the loop than outside the loop.



Field at Center of Current Loop



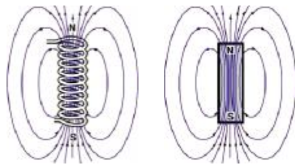
$$B = \frac{\mu_0 I}{2R}$$

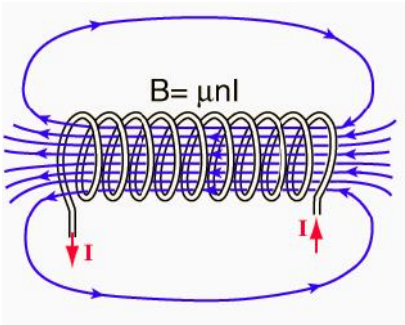
Field on Axis of Current Loop

$$B = \frac{\mu_0}{4\pi} \frac{2\pi I R^2}{(z^2 + R^2)^{2/3}}$$

Solenoid

A long straight coil of wire can be used to generate a nearly uniform magnetic field similar to that of a bar magnet. Such coils, called solenoids, have an enormous number of practical applications.

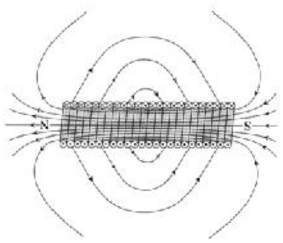


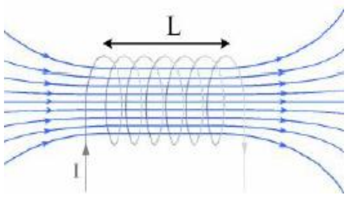


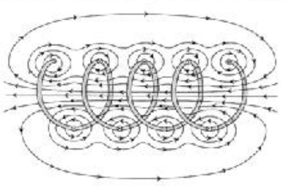
The magnetic field is concentrated into a nearly uniform field in the center of a long solenoid. The field outside is weak and divergent.

Solenoid

Each loop of the coil is assumed to be seperate sources for magnetic fiels then the resultant magnetic field is simply superposition of the vector sum of each loop





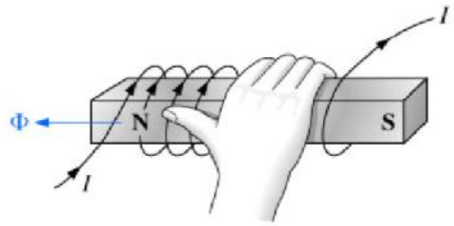


$$B = \mu_0 I \frac{N}{L} = \mu_0 I n$$

N is the # of windings and the L is the Length of coil.

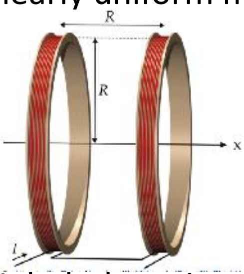
Solenoid

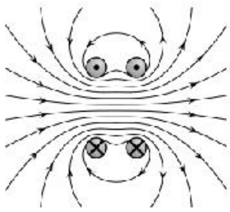
The direction of the magnetic flux lines can be found by placing the thumb of the *right* hand in the direction of *conventional current* flow and noting the direction of the fingers



Helmholtz Coil

A Helmholtz coil is a device for producing a region of nearly uniform magnetic field.





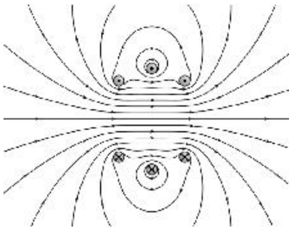
$$B = \left(\frac{4}{5}\right)^{3/2} \frac{\mu_0 In}{R}$$

A Helmholtz pair consists of two identical circular magnetic coils that are placed symmetrically one on each side of a common axis, and separated by a distance equal to the radius of the coil.

Each coil carries an equal electrical current flowing in the same direction.

Maxwell Coil (Uniform)

A Maxwell coil is a device for producing a large volume of almost constant or constant-gradient magnetic field.

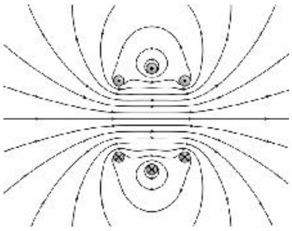


A constant-field Maxwell coil set consists of three coils oriented on the surface of a virtual sphere.

Each of the outer coils should be of radius $\sqrt{4/7}$ R and distance from the plane of $\sqrt{3/7}$ R the central coil of radius R . The number of ampere-turns of each of the smaller coils should equal exactly 49/64 of the middle coil.

Maxwell Coil (Gradient)

A Maxwell coil is a device for producing a large volume of almost constant or constant-gradient magnetic field.



A constant-field Maxwell coil set consists of three coils oriented on the surface of a virtual sphere.

Each of the outer coils should be of radius $\sqrt{4/7}$ R and distance from the plane of $\sqrt{3/7}$ R the central coil of radius R . The number of ampere-turns of each of the smaller coils should equal exactly 49/64 of the middle coil.

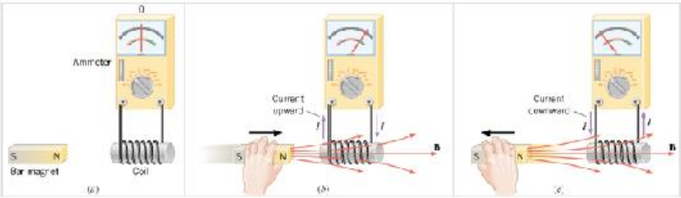
Electromagnetic Induction

Imposing an electric field on a conductor gives rise to a current which in turn generates a magnetic field.

One could then inquire whether or not an electric field could be produced by a magnetic field.

In 1831, Michael Faraday discovered that, by varying magnetic field with time, an electric field could be generated.

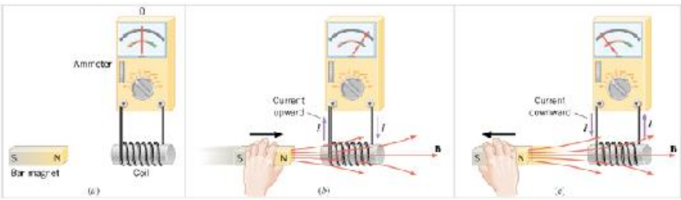
The phenomenon is known as electromagnetic induction.



Faraday's Law of Induction

Faraday showed that no current is registered in the galvanometer when bar magnet is stationary with respect to the loop.

However, a current is induced in the loop when a relative motion exists between the bar magnet and the loop.



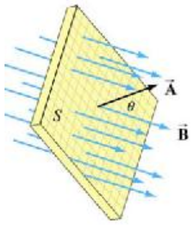
In particular, the ampermeter deflects in one direction as the magnet approaches the loop, and the opposite direction as it moves away.

Magnetic Flux

Faraday's experiment demonstrates that an electric current is induced in the loop by changing the magnetic field. The coil behaves as if it were connected to an voltage source.

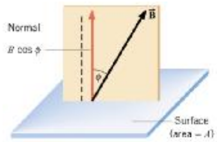
Experimentally it is found that the induced voltage depends on the rate of change of magnetic flux through the coil.

The magnetic flux through a surface is given by:



$$\Phi = B \cdot A$$

$$\Phi = BA \cos ¯$$



The SI unit of magnetic flux is the weber (Wb):

1 Wb =1 T m ^{2}

Faraday's Law of Induction

The induced emf ε (voltage) in a coil is proportional to the negative of the rate of change of magnetic flux:

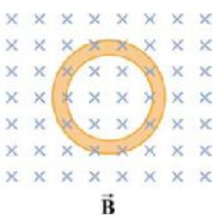
$$£ = -\frac{d¤}{dt}$$

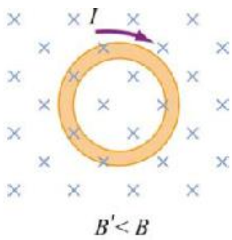
For a coil of N windings:

$$£ = -N\frac{d¤}{dt}$$

An e.m.f. may be induced in the following ways:

by varying the magnitude of B with time





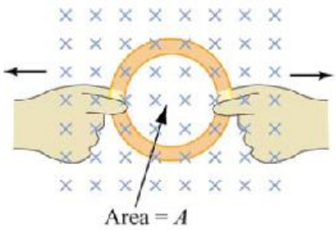
Faraday's Law of Induction

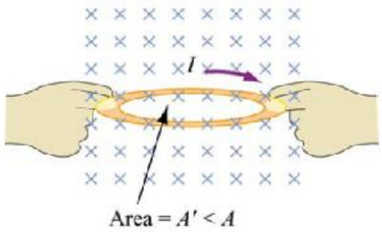
The induced emf ε (voltage) in a coil is proportional to the negative of the rate of change of magnetic flux:

$£ = -\frac{d¤}{dt}$ For a coil of N windings: $£ = -N\frac{d¤}{dt}$

An e.m.f. may be induced in the following ways:

by varying the magnitude of A, i.e., the area enclosed by the loop with time





$\Phi = B \cdot A$ $\Phi = BA \cos ¯$

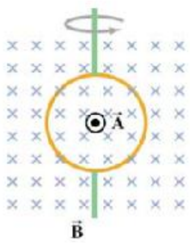
Faraday's Law of Induction

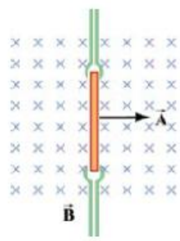
The induced emf ε (voltage) in a coil is proportional to the negative of the rate of change of magnetic flux:

$£ = -\frac{d¤}{dt}$ For a coil of N windings: $£ = -N\frac{d¤}{dt}$

An e.m.f. may be induced in the following ways:

varying the angle between B and the area vector A with time

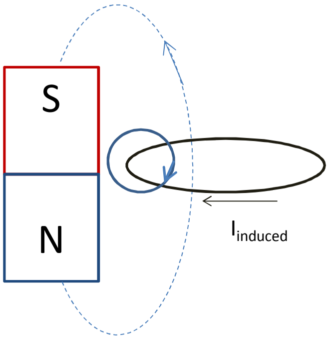




$\Phi = B \cdot A$ $\Phi = BA \cos ¯$

Lenz's Law Direction of Induction Current

An induced current always flows in a direction that opposes the change that caused it.

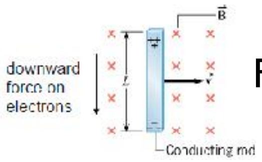


In this example the magnetic field in the upward direction through the loop is increasing.

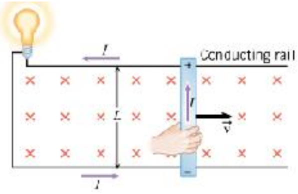
So a current is generated in the loop which produces an downward magnetic field inside the loop to oppose the change.

Motional E.M.F.

Each charge within the conductor is moving and experiences a magnetic force



$$F_{mag} = qvB$$



The separated charges on the ends of the conductor give rise to an induced emf, called a **motional emf.**

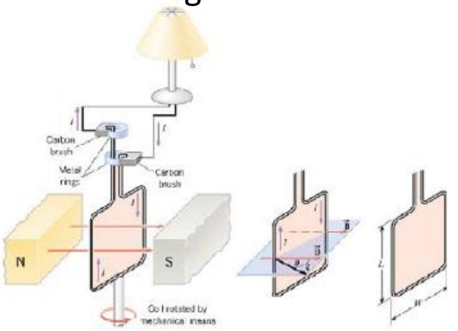
In equilibrium, the electric force of repulsion from charge buildup at the ends, *qE (where E is the electric field), balances* F_{mag}: $F_{mag} = qvB = qE$

$$E = Bv \longrightarrow E = \frac{\Delta V}{\Delta s} = \frac{£}{L} = Bv \Rightarrow £ = BLv$$

Motional Emf

The Electric Generator

A generator converts mechanical energy to electrical energy. Consider a current loop which rotates in a constant magnetic field



$$£ = 2BLv \sin ¯$$

$$v = μ \frac{W}{2}$$

$$£ = 2BLμ \frac{W}{2} \sin ¯$$

$$£ = BLWμ \sin ¯$$

For a coil of N loops and the rotation speed of w, The resultant motional e.m.f. İs alternating voltage $£(t) = NBLWμ \sin wt$

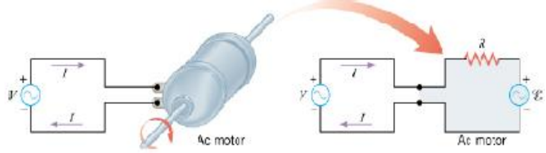
The Back E.M.F. Of a Motor

When a motor is operating, two sources of emf are present:

(1) the applied emf *V that provides current to drive the motor,*

(2) the emf induced by the generator-like action of the rotating coil.

Consistent with Lenz's law, the induced e.m.f. acts to oppose the applied e.m.f. and is called back e.m.f.:



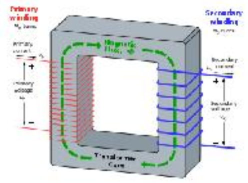
$$v - £ = IR$$

Transformers

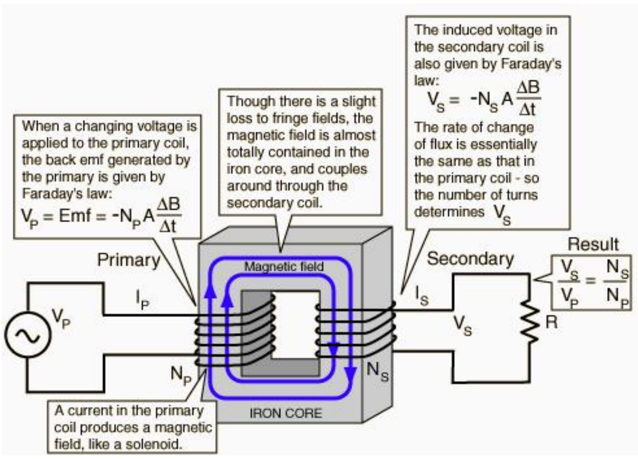
A transformer is a power converter that transfers electrical energy from one circuit to another through inductively coupled conductors—the transformer's coils.

A varying current in the first or primary winding creates a varying magnetic flux in the transformer's core and thus a varying magnetic field through the secondary winding.

This varying magnetic field induces a varying electromotive force (EMF), or "voltage", in the secondary winding. This effect is called inductive coupling.



Transformer and Faraday's Law



Transformer calculations

$$V_P = V_m \sin(wt)$$

$$V_P(t) = -N_P \frac{d\text{¤}_P(t)}{dt}, \quad \int d\text{¤}_P(t) = \frac{1}{N_P}\int -V_P(t)dt$$

$$\text{¤}_P(t) = -\frac{1}{N_P}\int V_P(t)dt$$

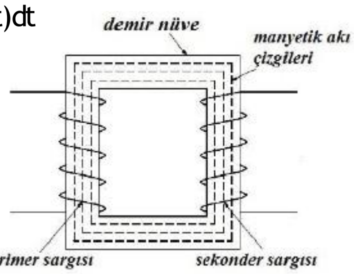
$$\text{¤}_P(t) = -\frac{V_m}{N_P}\left(\frac{-\cos(wt)}{w}\right) = \frac{V_m}{N_P}\left(\frac{\cos(wt)}{w}\right)$$

$$\text{¤}_P(t) = \text{¤}_S(t)$$

$$V_S(t) = -N_S \frac{d\text{¤}_S(t)}{dt}$$

$$V_S(t) = -N_S\left(\frac{V_m}{N_P}\left(\frac{-w\sin(wt)}{w}\right)\right) = \frac{N_S}{N_P} V_m \sin(wt) = \frac{N_S}{N_P} V_P(t)$$

$$\frac{V_P(t)}{V_S(t)} = \frac{N_P}{N_S} = K_{trafo}$$



Transformer calculations

Voltage

$$\frac{V_P(t)}{V_S(t)} = \frac{N_P}{N_S} = K_{trafo}$$

Current

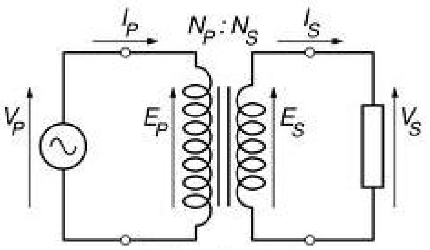
$$P_P = P_S$$

$$V_P I_P = V_S I_S$$

$$V_P = K_{trafo} V_S$$

$$V_S I_S = K_{trafo} V_S I_P$$

$$\frac{I_P}{I_S} = \frac{1}{K_{trafo}}$$



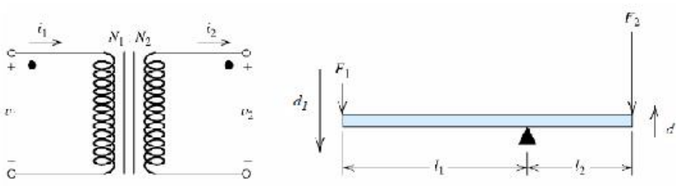
Impedance

$$Z_L = \frac{V_L}{I_L} = \frac{V_S}{I_S}$$

$$Z_L = \frac{V_S}{I_S} = \frac{\frac{1}{K_{trafo}} V_P}{K_{trafo} I_P} = \frac{1}{K_{trafo}^2}\frac{V_P}{I_P} = \frac{1}{K_{trafo}^2}\,{}^{P}Z_L$$

$$\frac{{}^{P}Z_L}{Z_L} = K_{trafo}^2$$

Transformer – Mechanical analogy



$$v_2(t) - \frac{N_2}{N_1} v_1(t)$$

$$i_2(t) = \frac{N_1}{N_2} i_1(t)$$

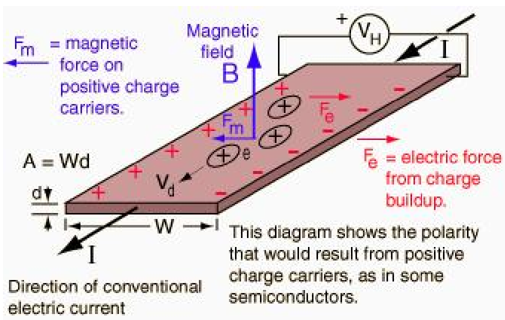
$$d_2 - \frac{l_2}{l_1} d_1$$

$$F_2 = \frac{l_1}{l_2} F_1$$

The Hall effect

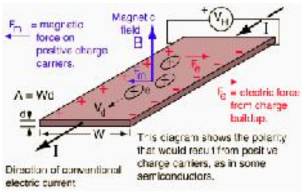
If an electric current flows through a conductor in a magnetic field, the magnetic field exerts a transverse force on the moving charge carriers which tends to push them to one side of the conductor.

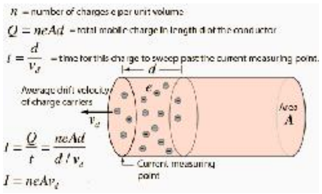
This becomes evident in a thin flat conductor.



The Hall effect

A buildup of charge at the sides of the conductors will balance this magnetic influence, producing a measurable voltage between the two sides of the conductor





$$F_M = ev_d B$$ V_d is the drift velocity of the charge.

$$F_E = eE = F_M = ev_d B$$

$$E = v_d B$$

$$I = neAv_d$$ n is the density of charge carriers.

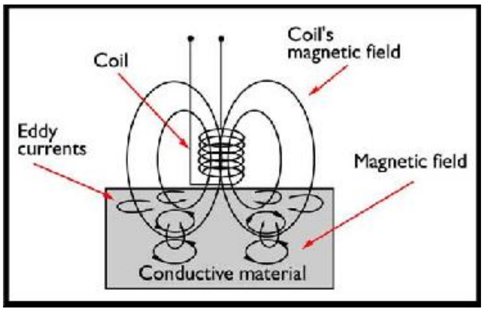
$$v_d = \frac{I}{neA}$$

$$E = \frac{I}{neA} B \qquad \Delta V = \frac{Fw}{q} = \frac{qEw}{q} = Ew$$

$$\frac{\Delta V}{w} = \frac{I}{nedw} B \qquad V_H = \frac{IB}{nedw}$$

The Eddy Currents

In a bulk material effected by a electromaget,eddy currents occurs in following mannner:



An alternating current creates a magnetic field.The magnetic field causes a resulting eddy current in a part, which creates an induced magnetic field. The magnetic field from the coil is opposed to the induced magnetic field from the eddy current.