## First-Order Logic with Domain Conditions

Logic and Computation Seminar, 23 July 2004

Luís Cruz-Filipe
(joint work with Herman Geuvers & Freek Wiedijk)

University of Nijmegen, Netherlands
Center for Logic and Computation, Portugal





### 1. Introduction



- 1. Introduction
- 2. Systems FOL, T and D

- 1. Introduction
- 2. Systems FOL, T and D
- 3. Equivalences

- 1. Introduction
- 2. Systems FOL, T and D
- 3. Equivalences
- 4. Completeness Results

- 1. Introduction
- 2. Systems FOL, T and D
- 3. Equivalences
- 4. Completeness Results
- 5. Conclusions



no adequate treatment of partiality in logic

- o no adequate treatment of partiality in logic
- paper by Wiedijk and Zwanenburg (TPHOLs 2003)

- o no adequate treatment of partiality in logic
- o paper by Wiedijk and Zwanenburg (TPHOLs 2003)
  - syntactic system, equivalent to FOL

- o no adequate treatment of partiality in logic
- paper by Wiedijk and Zwanenburg (TPHOLs 2003)
  - syntactic system, equivalent to FOL
  - no semantics

- no adequate treatment of partiality in logic
- paper by Wiedijk and Zwanenburg (TPHOLs 2003)
  - syntactic system, equivalent to FOL
  - no semantics
  - overpermissive presentation of FOL

Three equivalences:

Three equivalences:

6 D ↔ P: easy, uninteresting

#### Three equivalences:

- 6 D ↔ P: easy, uninteresting
- 6 D ↔ T: syntactic level on the paper

$$\mathcal{DC}(\Gamma), \mathcal{DC}_{\Gamma}(\varphi), \Gamma \vdash^{\mathsf{T}} \varphi \text{ iff } \Gamma \vdash^{\mathsf{D}} \varphi$$

#### Three equivalences:

- 6 D ↔ P: easy, uninteresting
- 6 D ↔ T: syntactic level on the paper

$$\mathcal{DC}(\Gamma), \mathcal{DC}_{\Gamma}(\varphi), \Gamma \vdash^{\mathsf{T}} \varphi \text{ iff } \Gamma \vdash^{\mathsf{D}} \varphi$$

semantics new

#### Three equivalences:

- 6 D ↔ P: easy, uninteresting
- 6 D ↔ T: syntactic level on the paper

$$\mathcal{DC}(\Gamma), \mathcal{DC}_{\Gamma}(\varphi), \Gamma \vdash^{\mathsf{T}} \varphi \text{ iff } \Gamma \vdash^{\mathsf{D}} \varphi$$

#### semantics new

6 T  $\leftrightarrow$  FOL: not trivial, not done on the paper

#### Derivation Rules for FOL

$$(\textit{assum}) \frac{\Gamma \vdash^{\mathsf{FOL}} \varphi}{\Gamma \vdash^{\mathsf{FOL}} \varphi} \varphi \in \Gamma \quad (\neg \neg \cdot E) \frac{\Gamma \vdash^{\mathsf{FOL}} \neg \neg \varphi}{\Gamma \vdash^{\mathsf{FOL}} \varphi}$$
 
$$(\rightarrow \cdot I) \frac{\Gamma, \varphi \vdash^{\mathsf{FOL}} \psi}{\Gamma \vdash^{\mathsf{FOL}} (\varphi \rightarrow \psi)} \quad (\rightarrow \cdot E) \frac{\Gamma \vdash^{\mathsf{FOL}} (\varphi \rightarrow \psi) \quad \Gamma \vdash^{\mathsf{FOL}} \varphi}{\Gamma \vdash^{\mathsf{FOL}} (\psi x_i \cdot \varphi)}$$
 
$$(\forall \cdot I) \frac{\Gamma \vdash^{\mathsf{FOL}} \varphi}{\Gamma \vdash^{\mathsf{FOL}} (\forall x_i \cdot \varphi)} x_i \not\in FV(\Gamma) \quad (\forall \cdot E) \frac{\Gamma \vdash^{\mathsf{FOL}} (\forall x_i \cdot \varphi)}{\Gamma \vdash^{\mathsf{FOL}} (\psi x_i \cdot \varphi)} *$$
 
$$(refl) \frac{\Gamma \vdash^{\mathsf{FOL}} (\psi x_i \cdot \varphi)}{\Gamma \vdash^{\mathsf{FOL}} (\psi x_i \cdot \varphi)} x_i \not\in FV(\Gamma) \quad (\forall \cdot E) \frac{\Gamma \vdash^{\mathsf{FOL}} (\psi x_i \cdot \varphi)}{\Gamma \vdash^{\mathsf{FOL}} (\psi x_i \cdot \varphi)} *$$
 
$$(refl) \frac{\Gamma \vdash^{\mathsf{FOL}} (\psi x_i \cdot \varphi)}{\Gamma \vdash^{\mathsf{FOL}} (\psi x_i \cdot \varphi)} x_i \not\in FV(\Gamma) \quad (\forall \cdot E) \frac{\Gamma \vdash^{\mathsf{FOL}} (\psi x_i \cdot \varphi)}{\Gamma \vdash^{\mathsf{FOL}} (\psi x_i \cdot \varphi)} *$$
 
$$(refl) \frac{\Gamma \vdash^{\mathsf{FOL}} (\psi x_i \cdot \varphi)}{\Gamma \vdash^{\mathsf{FOL}} (\psi x_i \cdot \varphi)} x_i \not\in FV(\Gamma) \quad (\forall \cdot E) \frac{\Gamma \vdash^{\mathsf{FOL}} (\psi x_i \cdot \varphi)}{\Gamma \vdash^{\mathsf{FOL}} (\psi x_i \cdot \varphi)} *$$
 
$$(refl) \frac{\Gamma \vdash^{\mathsf{FOL}} (\psi x_i \cdot \varphi)}{\Gamma \vdash^{\mathsf{FOL}} (\psi x_i \cdot \varphi)} x_i \not\in FV(\Gamma) \quad (\forall \cdot E) \frac{\Gamma \vdash^{\mathsf{FOL}} (\psi x_i \cdot \varphi)}{\Gamma \vdash^{\mathsf{FOL}} (\psi x_i \cdot \varphi)} *$$
 
$$(refl) \frac{\Gamma \vdash^{\mathsf{FOL}} (\psi x_i \cdot \varphi)}{\Gamma \vdash^{\mathsf{FOL}} (\psi x_i \cdot \varphi)} x_i \not\in FV(\Gamma) \quad (\forall \cdot E) \frac{\Gamma \vdash^{\mathsf{FOL}} (\psi x_i \cdot \varphi)}{\Gamma \vdash^{\mathsf{FOL}} (\psi x_i \cdot \varphi)} *$$
 
$$(refl) \frac{\Gamma \vdash^{\mathsf{FOL}} (\psi x_i \cdot \varphi)}{\Gamma \vdash^{\mathsf{FOL}} (\psi x_i \cdot \varphi)} x_i \not\in FV(\Gamma) \quad (\forall \cdot E) \frac{\Gamma \vdash^{\mathsf{FOL}} (\psi x_i \cdot \varphi)}{\Gamma \vdash^{\mathsf{FOL}} (\psi x_i \cdot \varphi)} *$$
 
$$(refl) \frac{\Gamma \vdash^{\mathsf{FOL}} (\psi x_i \cdot \varphi)}{\Gamma \vdash^{\mathsf{FOL}} (\psi x_i \cdot \varphi)} x_i \not\in FV(\Gamma) \quad (\forall \cdot E) \frac{\Gamma \vdash^{\mathsf{FOL}} (\psi x_i \cdot \varphi)}{\Gamma \vdash^{\mathsf{FOL}} (\psi x_i \cdot \varphi)} *$$
 
$$(refl) \frac{\Gamma \vdash^{\mathsf{FOL}} (\psi x_i \cdot \varphi)}{\Gamma \vdash^{\mathsf{FOL}} (\psi x_i \cdot \varphi)} x_i \not\in FV(\Gamma) \quad (\forall \cdot E) \frac{\Gamma \vdash^{\mathsf{FOL}} (\psi x_i \cdot \varphi)}{\Gamma \vdash^{\mathsf{FOL}} (\psi x_i \cdot \varphi)} *$$
 
$$(refl) \frac{\Gamma \vdash^{\mathsf{FOL}} (\psi x_i \cdot \varphi)}{\Gamma \vdash^{\mathsf{FOL}} (\psi x_i \cdot \varphi)} x_i \not\in FV(\Gamma) \quad (\forall \cdot E) \frac{\Gamma \vdash^{\mathsf{FOL}} (\psi x_i \cdot \varphi)}{\Gamma \vdash^{\mathsf{FOL}} (\psi x_i \cdot \varphi)} *$$
 
$$(refl) \frac{\Gamma \vdash^{\mathsf{FOL}} (\psi x_i \cdot \varphi)}{\Gamma \vdash^{\mathsf{FOL}} (\psi x_i \cdot \varphi)} x_i \not\in FV(\Gamma) \quad (\forall \cdot E) \frac{\Gamma \vdash^{\mathsf{FOL}} (\psi x_i \cdot \varphi)}{\Gamma \vdash^{\mathsf{FOL}} (\psi x_i \cdot \varphi)} *$$
 
$$(refl) \frac{\Gamma \vdash^{\mathsf{FOL}} (\psi x_i \cdot \varphi)}{\Gamma \vdash^{\mathsf{FOL}} (\psi x_i \cdot \varphi)} x_i \not\in FV(\Gamma) \quad (\forall \cdot E) \frac{\Gamma \vdash^{\mathsf{FOL}} (\psi x_i \cdot \varphi)}{\Gamma \vdash^{\mathsf{FOL}} (\psi x_i \cdot \varphi)} *$$
 
$$(refl) \frac{\Gamma \vdash^{\mathsf{FOL}} (\psi x_i \cdot \varphi)}{\Gamma \vdash^{\mathsf{FOL}} (\psi x_i \cdot \varphi)} x_i \not\in FV(\Gamma) \quad (\forall \cdot E) \frac{\Gamma \vdash^{\mathsf{FOL}} (\psi x_i \cdot \varphi)}{\Gamma \vdash^$$

#### Derivation Rules for T – I

$$(\epsilon\text{-wf}) \frac{\Gamma \vdash^{\mathsf{T}} wf}{\epsilon \vdash^{\mathsf{T}} wf} \quad (\text{decl-wf}) \frac{\Gamma \vdash^{\mathsf{T}} wf}{\Gamma, x_i \vdash^{\mathsf{T}} wf} \quad (\text{assum-wf}) \frac{\Gamma \vdash^{\mathsf{T}} \varphi wf}{\Gamma, \varphi \vdash^{\mathsf{T}} wf}$$
 
$$(var\text{-wf}) \frac{\Gamma \vdash^{\mathsf{T}} wf}{\Gamma \vdash^{\mathsf{T}} x_i wf} x_i \in \Gamma \quad (\text{const-wf}) \frac{\Gamma \vdash^{\mathsf{T}} wf}{\Gamma \vdash^{\mathsf{T}} c_i wf}$$
 
$$(\text{fun-wf}) \frac{\Gamma \vdash^{\mathsf{T}} t_1 wf \cdots \Gamma \vdash^{\mathsf{T}} t_{a_i} wf \Gamma \vdash^{\mathsf{T}} wf}{\Gamma \vdash^{\mathsf{T}} t_1 (t_1, \dots, t_{a_i}) wf}$$
 
$$(\text{if-wf}) \frac{\Gamma \vdash^{\mathsf{T}} \vartheta wf \Gamma \vdash^{\mathsf{T}} t_1 wf \Gamma \vdash^{\mathsf{T}} t_2 wf}{\Gamma \vdash^{\mathsf{T}} (\text{if } \vartheta \text{ then } t_1 \text{ else } t_2) wf}$$
 
$$(\bot\text{-wf}) \frac{\Gamma \vdash^{\mathsf{T}} wf}{\Gamma \vdash^{\mathsf{T}} \bot wf} \quad (\to\text{-wf}) \frac{\Gamma \vdash^{\mathsf{T}} \varphi wf \Gamma \vdash^{\mathsf{T}} \psi wf}{\Gamma \vdash^{\mathsf{T}} (\varphi \to \psi) wf} \quad (\forall\text{-wf}) \frac{\Gamma, x_i \vdash^{\mathsf{T}} \varphi wf}{\Gamma \vdash^{\mathsf{T}} (\forall x_i, \varphi) wf}$$
 
$$(=\text{-wf}) \frac{\Gamma \vdash^{\mathsf{T}} t_1 wf \Gamma \vdash^{\mathsf{T}} t_2 wf}{\Gamma \vdash^{\mathsf{T}} t_1 = t_2 wf} \quad (\text{pred-wf}) \frac{\Gamma \vdash^{\mathsf{T}} t_1 wf \cdots \Gamma \vdash^{\mathsf{T}} t_{r_i} wf \Gamma \vdash^{\mathsf{T}} wf}{\Gamma \vdash^{\mathsf{T}} t_1 (t_1, \dots, t_{r_i}) wf}$$

#### Derivation Rules for T – II

$$(assum) \ \frac{\Gamma \vdash^{\mathsf{T}} \textit{wf}}{\Gamma \vdash^{\mathsf{T}} \varphi} \ \varphi \in \Gamma \quad (\rightarrow \text{-}l) \ \frac{\Gamma, \varphi \vdash^{\mathsf{T}} \psi}{\Gamma \vdash^{\mathsf{T}} (\varphi \to \psi)} \quad (\rightarrow \text{-}E) \ \frac{\Gamma \vdash^{\mathsf{T}} (\varphi \to \psi) \quad \Gamma \vdash^{\mathsf{T}} \varphi}{\Gamma \vdash^{\mathsf{T}} \psi}$$
 
$$(\neg \neg \cdot E) \ \frac{\Gamma \vdash^{\mathsf{T}} \neg \neg \varphi}{\Gamma \vdash^{\mathsf{T}} \varphi} \quad (\forall \neg \cdot l) \ \frac{\Gamma, x_i \vdash^{\mathsf{T}} \varphi}{\Gamma \vdash^{\mathsf{T}} (\forall x_i, \varphi)} \quad (\forall \neg \cdot E) \ \frac{\Gamma \vdash^{\mathsf{T}} (\forall x_i, \varphi) \quad \Gamma \vdash^{\mathsf{T}} t \textit{wf}}{\Gamma \vdash^{\mathsf{T}} \varphi [x_i := t]}$$
 
$$(\textit{refl}) \ \frac{\Gamma \vdash^{\mathsf{T}} t \textit{wf}}{\Gamma \vdash^{\mathsf{T}} t = t} \quad (\textit{sym}) \ \frac{\Gamma \vdash^{\mathsf{T}} t_1 = t_2}{\Gamma \vdash^{\mathsf{T}} t_2 = t_1} \quad (\textit{trans}) \ \frac{\Gamma \vdash^{\mathsf{T}} t_1 = t_2 \quad \Gamma \vdash^{\mathsf{T}} t_2 = t_3}{\Gamma \vdash^{\mathsf{T}} t_1 = t_3}$$
 
$$(=-\textit{fun}) \ \frac{\Gamma \vdash^{\mathsf{T}} t_1 = t_1' \quad \cdots \quad \Gamma \vdash^{\mathsf{T}} t_{a_i} = t_{a_i}' \quad \Gamma \vdash^{\mathsf{T}} \textit{wf}}{\Gamma \vdash^{\mathsf{T}} t_1 = t_1' \quad \cdots \quad \Gamma \vdash^{\mathsf{T}} t_{a_i} = t_{a_i}' \quad \Gamma \vdash^{\mathsf{T}} \textit{wf}}$$
 
$$(=-\textit{pred}) \ \frac{\Gamma \vdash^{\mathsf{T}} t_1 = t_1' \quad \cdots \quad \Gamma \vdash^{\mathsf{T}} t_{a_i} = t_{a_i}' \quad \Gamma \vdash^{\mathsf{T}} \textit{wf}}{\Gamma \vdash^{\mathsf{T}} t_1 = t_1' \quad \cdots \quad \Gamma \vdash^{\mathsf{T}} t_{a_i} = t_{a_i}' \quad \Gamma \vdash^{\mathsf{T}} \textit{wf}}$$
 
$$(=-\textit{if-true}) \ \frac{\Gamma \vdash^{\mathsf{T}} t_1 = t_1' \quad \cdots \quad \Gamma \vdash^{\mathsf{T}} t_1 = t_1' \quad \cdots \quad t_{a_i}' = t_1' = t_1' \quad \cdots \quad t_{a_i}' = t_1' =$$

#### Derivation Rules for D - I

$$(\epsilon\text{-wt}) \frac{\Gamma \vdash^{\mathsf{D}} \mathsf{wf}}{\epsilon \vdash^{\mathsf{D}} \mathsf{wf}} \quad (\mathsf{decl\text{-wt}}) \frac{\Gamma \vdash^{\mathsf{D}} \mathsf{wf}}{\Gamma, x_i \vdash^{\mathsf{D}} \mathsf{wf}} \quad (\mathsf{assum\text{-wt}}) \frac{\Gamma \vdash^{\mathsf{D}} \varphi \; \mathsf{wf}}{\Gamma, \varphi \vdash^{\mathsf{D}} \mathsf{wf}}$$
 
$$(\mathsf{var\text{-wt}}) \frac{\Gamma \vdash^{\mathsf{D}} \mathsf{wf}}{\Gamma \vdash^{\mathsf{D}} x_i \; \mathsf{wf}} x_i \in \Gamma \quad (\mathsf{const\text{-wt}}) \frac{\Gamma \vdash^{\mathsf{D}} \mathsf{wf}}{\Gamma \vdash^{\mathsf{D}} c_i \; \mathsf{wf}}$$
 
$$(\mathsf{fun\text{-wt}}) \frac{\Gamma \vdash^{\mathsf{D}} D_{f_i}(t_1, \dots, t_{a_i})}{\Gamma \vdash^{\mathsf{D}} f_i(t_1, \dots, t_{a_i}) \; \mathsf{wf}}$$
 
$$(\mathsf{if\text{-wt}}) \frac{\Gamma, \vartheta \vdash^{\mathsf{D}} t_1 \; \mathsf{wf} \quad \Gamma, \neg \vartheta \vdash^{\mathsf{D}} t_2 \; \mathsf{wf}}{\Gamma \vdash^{\mathsf{D}} (\mathsf{if} \; \vartheta \; \mathsf{then} \; t_1 \; \mathsf{else} \; t_2) \; \mathsf{wf}}$$
 
$$(\bot - \mathsf{wf}) \frac{\Gamma \vdash^{\mathsf{D}} \mathsf{wf}}{\Gamma \vdash^{\mathsf{D}} \bot \; \mathsf{wf}} \quad (\to - \mathsf{wf}) \frac{\Gamma, \varphi \vdash^{\mathsf{D}} \psi \; \mathsf{wf}}{\Gamma \vdash^{\mathsf{D}} (\varphi \to \psi) \; \mathsf{wf}} \quad (\forall - \mathsf{wf}) \frac{\Gamma, x_i \vdash^{\mathsf{D}} \varphi \; \mathsf{wf}}{\Gamma \vdash^{\mathsf{D}} (\forall x_i, \varphi) \; \mathsf{wf}}$$
 
$$(=-\mathsf{wf}) \frac{\Gamma \vdash^{\mathsf{D}} t_1 \; \mathsf{wf} \quad \Gamma \vdash^{\mathsf{D}} t_2 \; \mathsf{wf}}{\Gamma \vdash^{\mathsf{D}} t_1 = t_2 \; \mathsf{wf}} \quad (\mathsf{pred\text{-wf}}) \frac{\Gamma \vdash^{\mathsf{D}} t_1 \; \mathsf{wf} \quad \cdots \quad \Gamma \vdash^{\mathsf{D}} t_{r_i} \; \mathsf{wf} \quad \Gamma \vdash^{\mathsf{D}} \mathsf{wf}}{\Gamma \vdash^{\mathsf{D}} t_1, \dots, t_{r_i}) \; \mathsf{wf}}$$

#### Derivation Rules for D - II

$$\begin{array}{l} \textit{(assum)} \ \frac{\Gamma \vdash^{\mathsf{D}} \textit{Wf}}{\Gamma \vdash^{\mathsf{D}} \varphi} \ \varphi \in \Gamma \quad (\rightarrow \text{-}l) \ \frac{\Gamma, \varphi \vdash^{\mathsf{D}} \psi}{\Gamma \vdash^{\mathsf{D}} (\varphi \to \psi)} \quad (\rightarrow \text{-}E) \ \frac{\Gamma \vdash^{\mathsf{D}} (\varphi \to \psi) \quad \Gamma \vdash^{\mathsf{D}} \varphi}{\Gamma \vdash^{\mathsf{D}} \psi} \\ \\ (\neg \neg \text{-}E) \ \frac{\Gamma \vdash^{\mathsf{D}} \neg \neg \varphi}{\Gamma \vdash^{\mathsf{D}} \varphi} \quad (\forall \text{-}l) \ \frac{\Gamma, x_i \vdash^{\mathsf{D}} \varphi}{\Gamma \vdash^{\mathsf{D}} (\forall x_i. \varphi)} \quad (\forall \text{-}E) \ \frac{\Gamma \vdash^{\mathsf{D}} (\forall x_i. \varphi) \quad \Gamma \vdash^{\mathsf{D}} t \ \textit{Wf}}{\Gamma \vdash^{\mathsf{D}} \varphi [x_i := t]} \\ \textit{(refl)} \ \frac{\Gamma \vdash^{\mathsf{D}} t \ \textit{Wf}}{\Gamma \vdash^{\mathsf{D}} t = t} \quad \textit{(sym)} \ \frac{\Gamma \vdash^{\mathsf{D}} t_1 = t_2}{\Gamma \vdash^{\mathsf{D}} t_2 = t_1} \quad \textit{(trans)} \ \frac{\Gamma \vdash^{\mathsf{D}} t_1 = t_2 \quad \Gamma \vdash^{\mathsf{D}} t_2 = t_3}{\Gamma \vdash^{\mathsf{D}} t_1 = t_3} \\ \textit{(=-fun)} \ \frac{\Gamma \vdash^{\mathsf{D}} t_1 = t_1' \cdots \Gamma \vdash^{\mathsf{D}} t_{a_i} = t_{a_i}' \quad \Gamma \vdash^{\mathsf{D}} D_{f_i}(t_1, \dots, t_{a_i}) \quad \Gamma \vdash^{\mathsf{D}} D_{f_i}(t_1', \dots, t_{a_i}')}{\Gamma \vdash^{\mathsf{D}} f_i(t_1, \dots, t_{a_i}) = f_i(t_1', \dots, t_{a_i}')} \\ \textit{(=-fun)} \ \frac{\Gamma \vdash^{\mathsf{D}} t_1 = t_1' \quad \cdots \quad \Gamma \vdash^{\mathsf{D}} t_1 = t_1' \quad \cdots \quad \Gamma \vdash^{\mathsf{D}} t_n = t_{r_i}' \quad \Gamma \vdash^{\mathsf{D}} \textit{Wf}}{\Gamma \vdash^{\mathsf{D}} P_i(t_1, \dots, t_{r_i}) \rightarrow P_i(t_1', \dots, t_{r_i}')} \\ \textit{(=-if-true)} \ \frac{\Gamma \vdash^{\mathsf{D}} \vartheta \quad \Gamma, \vartheta \vdash^{\mathsf{D}} t_1 \ \textit{Wf} \quad \Gamma, \neg \vartheta \vdash^{\mathsf{D}} t_2 \ \textit{Wf}}{\Gamma \vdash^{\mathsf{D}} (\text{if} \vartheta \text{ then } t_1 \text{ else } t_2) = t_1} \\ \textit{(=-if-false)} \ \frac{\Gamma \vdash^{\mathsf{D}} \neg \vartheta \quad \Gamma, \vartheta \vdash^{\mathsf{D}} t_1 \ \textit{Wf} \quad \Gamma, \neg \vartheta \vdash^{\mathsf{D}} t_2 \ \textit{Wf}}{\Gamma \vdash^{\mathsf{D}} (\text{if} \vartheta \text{ then } t_1 \text{ else } t_2) = t_2} \\ \end{array}$$

6 T-models are FOL-models

- T-models are FOL-models
- 6 a T-substitution for  $\mathfrak{M}$  is a partial function that assigns values in A to some variables  $x_i$

- T-models are FOL-models
- 6 a T-substitution for  $\mathfrak M$  is a partial function that assigns values in A to some variables  $x_i$

- T-models are FOL-models
- 6 a T-substitution for  $\mathfrak{M}$  is a partial function that assigns values in A to some variables  $x_i$

$$\models^{\mathsf{T}}_{\mathfrak{M},\rho} \perp \textit{wf}$$

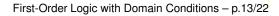
$$\models^{\mathsf{T}}_{\mathfrak{M},\rho} P_i(t_1,\ldots,t_{r_i}) \textit{ wf } \textit{ iff } \models^{\mathsf{T}}_{\mathfrak{M},\rho} t_1 \textit{ wf},\ldots,\models^{\mathsf{T}}_{\mathfrak{M},\rho} t_{r_i} \textit{ wf}$$

$$\models^{\mathsf{T}}_{\mathfrak{M},\rho} t_1 = t_2 \textit{ wf } \textit{ iff } \models^{\mathsf{T}}_{\mathfrak{M},\rho} t_1 \textit{ wf } \textit{and } \models^{\mathsf{T}}_{\mathfrak{M},\rho} t_2 \textit{ wf}$$

$$\models^{\mathsf{T}}_{\mathfrak{M},\rho} (\varphi \to \psi) \textit{ wf } \textit{ iff } \models^{\mathsf{T}}_{\mathfrak{M},\rho} \varphi \textit{ wf } \textit{and } \models^{\mathsf{T}}_{\mathfrak{M},\rho} \psi \textit{ wf}$$

$$\models^{\mathsf{T}}_{\mathfrak{M},\rho} (\forall x_i.\varphi) \textit{ wf } \textit{ iff } \models^{\mathsf{T}}_{\mathfrak{M},\rho[x_i:=a]} \varphi \textit{ wf } \textit{for } \textit{all } a \in A$$

$$\begin{split} & \biguplus_{\mathfrak{M},\rho}^{\mathsf{T}} \bot \\ & \models_{\mathfrak{M},\rho}^{\mathsf{T}} P_i(t_1,\ldots,t_{r_i}) \quad \text{iff} \quad \left(\llbracket t_1 \rrbracket_{\mathfrak{M},\rho}^{\mathsf{T}},\ldots,\llbracket t_{r_i} \rrbracket_{\mathfrak{M},\rho}^{\mathsf{T}}\right) \in \llbracket P_i \rrbracket_{\mathfrak{M},\rho}^{\mathsf{T}} \\ & \models_{\mathfrak{M},\rho}^{\mathsf{T}} t_1 = t_2 \quad \text{iff} \quad \llbracket t_1 \rrbracket_{\mathfrak{M},\rho}^{\mathsf{T}} = \llbracket t_2 \rrbracket_{\mathfrak{M},\rho}^{\mathsf{T}} \\ & \models_{\mathfrak{M},\rho}^{\mathsf{T}} \varphi \to \psi \quad \text{iff} \quad \models_{\mathfrak{M},\rho}^{\mathsf{T}} (\varphi \to \psi) \text{ wf and} \\ & \qquad \qquad \qquad \not\models_{\mathfrak{M},\rho}^{\mathsf{T}} \varphi \text{ or } \models_{\mathfrak{M},\rho}^{\mathsf{T}} \psi \\ & \models_{\mathfrak{M},\rho}^{\mathsf{T}} \forall x_i. \varphi \quad \text{iff} \quad \models_{\mathfrak{M},\rho[x_i:=a]}^{\mathsf{T}} \varphi \text{ for all } a \in A \end{split}$$



#### Well-formation of contexts:

- 1.  $\epsilon \models_{\mathfrak{M},\rho}^{\mathsf{T}} \mathsf{wf}$
- 2.  $\varphi, \Gamma \models_{\mathfrak{M}, \rho}^{\mathsf{T}} \mathsf{wf} \text{ iff } \models_{\mathfrak{M}, \rho}^{\mathsf{T}} \varphi \mathsf{wf} \text{ and } \Gamma \models_{\mathfrak{M}, \rho}^{\mathsf{T}} \mathsf{wf}$
- 3.  $x_i, \Gamma \models_{\mathfrak{M}, \rho}^{\mathsf{T}} \mathsf{wf} \mathsf{iff} \Gamma \models_{\mathfrak{M}, \rho[x_i:=a]}^{\mathsf{T}} \mathsf{wf} \mathsf{for all } a \in A$

#### Well-formation of contexts:

- 1.  $\epsilon \models_{\mathfrak{M},\rho}^{\mathsf{T}} \mathsf{wf}$
- 2.  $\varphi, \Gamma \models_{\mathfrak{M}, \rho}^{\mathsf{T}} \mathsf{wf} \mathsf{iff} \models_{\mathfrak{M}, \rho}^{\mathsf{T}} \varphi \mathsf{wf} \mathsf{and} \Gamma \models_{\mathfrak{M}, \rho}^{\mathsf{T}} \mathsf{wf}$
- 3.  $x_i, \Gamma \models_{\mathfrak{M}, \rho}^{\mathsf{T}} \mathsf{wf} \mathsf{iff} \Gamma \models_{\mathfrak{M}, \rho[x_i:=a]}^{\mathsf{T}} \mathsf{wf} \mathsf{for all } a \in A$

 $\Gamma \models_{\mathfrak{M},\rho}^{\mathsf{T}} t \text{ wf defined in a similar way with}$ 

1'. 
$$\epsilon \models_{\mathfrak{M},\rho}^{\mathsf{T}} t \ \textit{wf} \ \text{iff} \ \models_{\mathfrak{M},\rho}^{\mathsf{T}} t \ \textit{wf}$$

#### Well-formation of contexts:

- 1.  $\epsilon \models_{\mathfrak{M},\rho}^{\mathsf{T}} \mathsf{wf}$
- 2.  $\varphi, \Gamma \models_{\mathfrak{M}, \rho}^{\mathsf{T}} \mathsf{wf} \mathsf{iff} \models_{\mathfrak{M}, \rho}^{\mathsf{T}} \varphi \mathsf{wf} \mathsf{and} \Gamma \models_{\mathfrak{M}, \rho}^{\mathsf{T}} \mathsf{wf}$
- 3.  $x_i, \Gamma \models_{\mathfrak{M}, \rho}^{\mathsf{T}} \mathsf{wf} \mathsf{iff} \Gamma \models_{\mathfrak{M}, \rho[x_i:=a]}^{\mathsf{T}} \mathsf{wf} \mathsf{for all } a \in A$

 $\Gamma \models_{\mathfrak{M},\rho}^{\mathsf{T}} t \text{ wf defined in a similar way with}$ 

1'. 
$$\epsilon \models_{\mathfrak{M},\rho}^{\mathsf{T}} t \ \textit{wf} \ \text{iff} \ \models_{\mathfrak{M},\rho}^{\mathsf{T}} t \ \textit{wf}$$

 $\Gamma \models_{\mathfrak{M},\rho}^{\mathsf{T}} \varphi \text{ wf defined by } 1', 2 \text{ and } 3$ 

## Semantics of T – V

#### Consequence:

1'. 
$$\epsilon \models_{\mathfrak{M},\rho}^{\mathsf{T}} \psi \text{ iff } \models_{\mathfrak{M},\rho}^{\mathsf{T}} \psi$$

$$2'$$
.  $\varphi, \Gamma \models_{\mathfrak{M},\rho}^{\mathsf{T}} \psi$  iff

$$(a) \models_{\mathfrak{M},\rho}^{\mathsf{T}} \neg \varphi \text{ and } \Gamma \models_{\mathfrak{M},\rho}^{\mathsf{T}} \psi \text{ wf}$$

(b) 
$$\models_{\mathfrak{M},\rho}^{\mathsf{T}} \varphi \text{ and } \Gamma \models_{\mathfrak{M},\rho}^{\mathsf{T}} \psi$$

3. 
$$x_i, \Gamma \models_{\mathfrak{M}, \psi}^{\mathsf{T}} \text{ iff } \Gamma \models_{\mathfrak{M}, \rho[x_i:=a]}^{\mathsf{T}} \psi \text{ for all } a \in A$$

#### Semantics of T - V

#### Consequence:

1'. 
$$\epsilon \models_{\mathfrak{M},\rho}^{\mathsf{T}} \psi \text{ iff } \models_{\mathfrak{M},\rho}^{\mathsf{T}} \psi$$

$$2'$$
.  $\varphi, \Gamma \models_{\mathfrak{M},\rho}^{\mathsf{T}} \psi$  iff

$$(a) \models_{\mathfrak{M},\rho}^{\mathsf{T}} \neg \varphi \text{ and } \Gamma \models_{\mathfrak{M},\rho}^{\mathsf{T}} \psi \text{ wf}$$

(b) 
$$\models_{\mathfrak{M},\rho}^{\mathsf{T}} \varphi \text{ and } \Gamma \models_{\mathfrak{M},\rho}^{\mathsf{T}} \psi$$

3. 
$$x_i, \Gamma \models_{\mathfrak{M}, \psi}^{\mathsf{T}} \text{ iff } \Gamma \models_{\mathfrak{M}, \rho[x_i:=a]}^{\mathsf{T}} \psi \text{ for all } a \in A$$

$$\Gamma \models^\mathsf{T}_\mathfrak{M} \mathcal{X} \text{ iff } \Gamma \models^\mathsf{T}_{\mathfrak{M},\emptyset} \mathcal{X}$$

#### Semantics of T - V

#### Consequence:

1'. 
$$\epsilon \models_{\mathfrak{M},\rho}^{\mathsf{T}} \psi \text{ iff } \models_{\mathfrak{M},\rho}^{\mathsf{T}} \psi$$

$$2'$$
.  $\varphi, \Gamma \models_{\mathfrak{M},\rho}^{\mathsf{T}} \psi$  iff

$$(a) \models_{\mathfrak{M},\rho}^{\mathsf{T}} \neg \varphi \text{ and } \Gamma \models_{\mathfrak{M},\rho}^{\mathsf{T}} \psi \text{ wf}$$

(b) 
$$\models_{\mathfrak{M},\rho}^{\mathsf{T}} \varphi \text{ and } \Gamma \models_{\mathfrak{M},\rho}^{\mathsf{T}} \psi$$

3. 
$$x_i, \Gamma \models_{\mathfrak{M}, \psi}^{\mathsf{T}} \text{ iff } \Gamma \models_{\mathfrak{M}, \rho[x_i:=a]}^{\mathsf{T}} \psi \text{ for all } a \in A$$

$$\Gamma \models^\mathsf{T}_\mathfrak{M} \mathcal{X} \text{ iff } \Gamma \models^\mathsf{T}_{\mathfrak{M},\emptyset} \mathcal{X}$$

$$\Gamma \models^{\mathsf{T}} \mathcal{X} \text{ iff } \Gamma \models^{\mathsf{T}}_{\mathfrak{M}} \mathcal{X} \text{ for all T-models } \mathfrak{M}.$$



6 Similar to T, but functions now may be partial:

$$[\![f]\!]^{\mathsf{D}}_{\mathfrak{M}}:A^n \not\to A$$



$$[\![f]\!]_{\mathfrak{M}}^{\mathsf{D}}:A^n \not\to A$$

 $D_f s$  become 'relevant':

$$\llbracket f \rrbracket_{\mathfrak{M}}^{\mathsf{D}}(a_1, \dots, a_n) \downarrow \mathsf{iff}(a_1, \dots, a_n) \in \llbracket D_f \rrbracket_{\mathfrak{M}}^{\mathsf{D}}$$

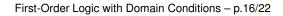
Similar to T, but functions now may be partial:

$$[\![f]\!]_{\mathfrak{M}}^{\mathsf{D}}:A^n \not\to A$$

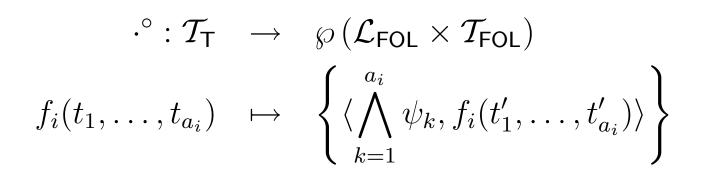
 $D_f s$  become 'relevant':

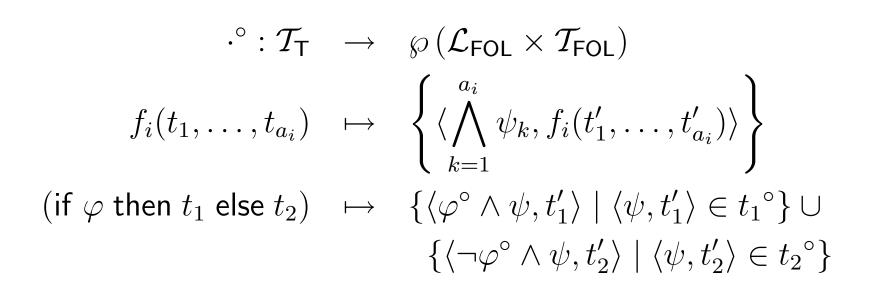
$$\llbracket f \rrbracket_{\mathfrak{M}}^{\mathsf{D}}(a_1, \dots, a_n) \downarrow \mathsf{iff}(a_1, \dots, a_n) \in \llbracket D_f \rrbracket_{\mathfrak{M}}^{\mathsf{D}}$$

(modulo some technical problems...)



$$\cdot^{\circ}: \mathcal{T}_{\mathsf{T}} \ o \ \wp\left(\mathcal{L}_{\mathsf{FOL}} imes \mathcal{T}_{\mathsf{FOL}}
ight)$$





 $\cdot^{\circ}:\mathcal{L}_{\mathsf{T}} \ o \ \mathcal{L}_{\mathsf{FOL}}$ 



Let  $\Gamma$  be a context in T and  $\varphi \in \mathcal{L}_T$  such that  $\Gamma \vdash^T \varphi$  wf. Then  $\Gamma \vdash^T \varphi$  iff  $\Gamma \models^T \varphi$ .

Let  $\Gamma$  be a context in T and  $\varphi \in \mathcal{L}_T$  such that  $\Gamma \vdash^T \varphi$  wf. Then  $\Gamma \vdash^T \varphi$  iff  $\Gamma \models^T \varphi$ .

$$\Gamma \vdash^{\mathsf{T}} \varphi \text{ iff (1) } \Gamma^{\circ} \vdash^{\mathsf{FOL}} \varphi^{\circ} \text{ and (2) } \Gamma \vdash^{\mathsf{T}} \varphi \text{ } \textit{wf.}$$

Let  $\Gamma$  be a context in T and  $\varphi \in \mathcal{L}_T$  such that  $\Gamma \vdash^T \varphi$  wf. Then  $\Gamma \vdash^T \varphi$  iff  $\Gamma \models^T \varphi$ .

$$\Gamma \vdash^{\mathsf{T}} \varphi \text{ iff (1) } \Gamma^{\circ} \vdash^{\mathsf{FOL}} \varphi^{\circ} \text{ and (2) } \Gamma \vdash^{\mathsf{T}} \varphi \text{ } \textit{wf.}$$

6 (1) is equivalent to  $\Gamma^{\circ} \models^{\mathsf{FOL}} \varphi^{\circ}$ .

Let  $\Gamma$  be a context in T and  $\varphi \in \mathcal{L}_T$  such that  $\Gamma \vdash^T \varphi$  wf. Then  $\Gamma \vdash^T \varphi$  iff  $\Gamma \models^T \varphi$ .

$$\Gamma \vdash^{\mathsf{T}} \varphi \text{ iff (1) } \Gamma^{\circ} \vdash^{\mathsf{FOL}} \varphi^{\circ} \text{ and (2) } \Gamma \vdash^{\mathsf{T}} \varphi \text{ } \textit{wf.}$$

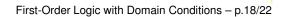
- 6 (1) is equivalent to  $\Gamma^{\circ} \models^{\mathsf{FOL}} \varphi^{\circ}$ .
- 6 (2) is equivalent to  $\Gamma \models^{\mathsf{T}} \varphi$  wf.

Let  $\Gamma$  be a context in T and  $\varphi \in \mathcal{L}_T$  such that  $\Gamma \vdash^T \varphi$  wf. Then  $\Gamma \vdash^T \varphi$  iff  $\Gamma \models^T \varphi$ .

$$\Gamma \vdash^{\mathsf{T}} \varphi \text{ iff (1) } \Gamma^{\circ} \vdash^{\mathsf{FOL}} \varphi^{\circ} \text{ and (2) } \Gamma \vdash^{\mathsf{T}} \varphi \text{ } \textit{wf.}$$

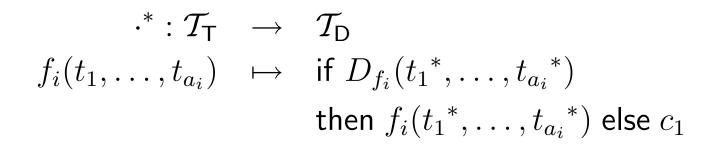
- 6 (1) is equivalent to  $\Gamma^{\circ} \models^{\mathsf{FOL}} \varphi^{\circ}$ .
- 6 (2) is equivalent to  $\Gamma \models^{\mathsf{T}} \varphi$  wf.

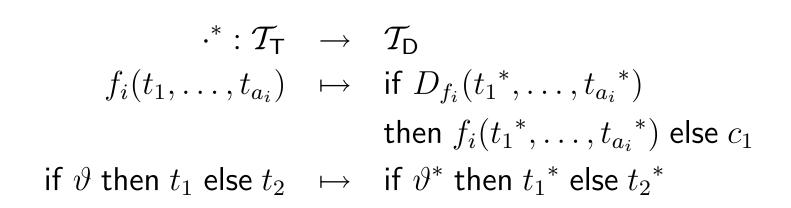
The conjunction of these two holds iff  $\Gamma \models^{\mathsf{T}} \varphi$ .

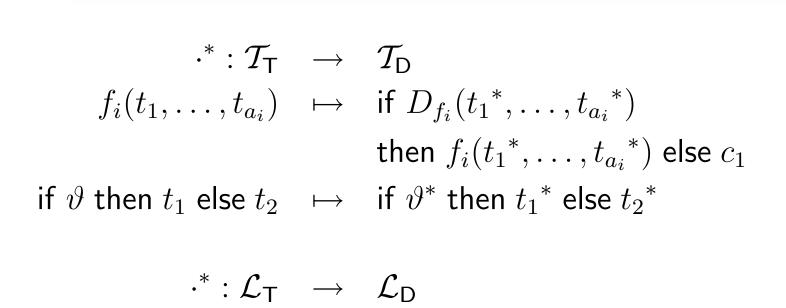


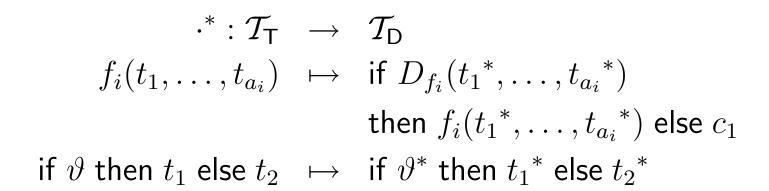


$$\cdot^*:\mathcal{T}_\mathsf{T} \ o \ \mathcal{T}_\mathsf{D}$$









### The domain conditions



#### The domain conditions

$$\begin{array}{rcl} \mathcal{DC}_{\Gamma}^{\vdash}:\mathcal{T}_{\mathsf{T}} & \to & \wp(\mathcal{J}_{\mathsf{T}}) \\ \mathcal{DC}_{\Gamma}^{\vdash}(f_{i}(t_{1},\ldots,t_{a_{i}})) & = & \mathcal{DC}_{\Gamma}^{\vdash}(t_{1}) \cup \ldots \cup \mathcal{DC}_{\Gamma}^{\vdash}(t_{a_{i}}) \cup \\ & & \cup \left\{\Gamma \vdash^{\mathsf{T}} D_{f_{i}}(t_{1},\ldots,t_{a_{i}})\right\} \\ \mathcal{DC}_{\Gamma}^{\vdash}(\text{if }\vartheta \text{ then } t_{1} \text{ else } t_{2}) & = & \mathcal{DC}_{\Gamma}^{\vdash}(\vartheta) \cup \mathcal{DC}_{\Gamma,\vartheta}^{\vdash}(t_{1}) \cup \mathcal{DC}_{\Gamma,\neg\vartheta}^{\vdash}(t_{2}) \end{array}$$

#### The domain conditions

$$\begin{array}{rcl} \mathcal{DC}_{\Gamma}^{\vdash}:\mathcal{T}_{\mathsf{T}} & \to & \wp(\mathcal{J}_{\mathsf{T}}) \\ \mathcal{DC}_{\Gamma}^{\vdash}(f_{i}(t_{1},\ldots,t_{a_{i}})) & = & \mathcal{DC}_{\Gamma}^{\vdash}(t_{1}) \cup \ldots \cup \mathcal{DC}_{\Gamma}^{\vdash}(t_{a_{i}}) \cup \\ & & \cup \left\{\Gamma \vdash^{\mathsf{T}} D_{f_{i}}(t_{1},\ldots,t_{a_{i}})\right\} \\ \mathcal{DC}_{\Gamma}^{\vdash}(\text{if }\vartheta \text{ then } t_{1} \text{ else } t_{2}) & = & \mathcal{DC}_{\Gamma}^{\vdash}(\vartheta) \cup \mathcal{DC}_{\Gamma,\vartheta}^{\vdash}(t_{1}) \cup \mathcal{DC}_{\Gamma,\neg\vartheta}^{\vdash}(t_{2}) \end{array}$$

$$\mathcal{DC}_{\Gamma}^{\vdash} : \mathcal{L}_{\mathsf{T}} \to \wp(\mathcal{J}_{\mathsf{T}})$$

$$\mathcal{DC}_{\Gamma}^{\vdash}(P_{i}(t_{1}, \dots, t_{r_{i}})) = \mathcal{DC}_{\Gamma}^{\vdash}(t_{1}) \cup \dots \cup \mathcal{DC}_{\Gamma}^{\vdash}(t_{r_{i}})$$

$$\mathcal{DC}_{\Gamma}^{\vdash}(\forall x_{i}. \varphi) = \mathcal{DC}_{\Gamma, x_{i}}^{\vdash}(\varphi)$$

# The domain conditions (cont.)



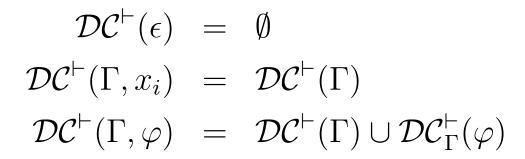
### The domain conditions (cont.)

$$\mathcal{DC}^{\vdash}(\epsilon) = \emptyset$$

$$\mathcal{DC}^{\vdash}(\Gamma, x_i) = \mathcal{DC}^{\vdash}(\Gamma)$$

$$\mathcal{DC}^{\vdash}(\Gamma, \varphi) = \mathcal{DC}^{\vdash}(\Gamma) \cup \mathcal{DC}^{\vdash}_{\Gamma}(\varphi)$$

### The domain conditions (cont.)



Theorem [Wiedijk & Zwanenburg 2003]:

 $\Gamma \vdash^{\mathsf{D}} \varphi \text{ iff } \mathcal{DC}(\Gamma) \text{ and } \mathcal{DC}_{\Gamma}(\varphi) \text{ hold and } \Gamma \vdash^{\mathsf{T}} \varphi.$ 



Let  $\mathfrak{M}=\langle A,F,P,C\rangle$  be a D-model. Then  $\mathfrak{M}_*$  is the T-model defined by  $\mathfrak{M}_*=\langle A,F_*,P,C\rangle$ , where

Let  $\mathfrak{M}=\langle A,F,P,C\rangle$  be a D-model. Then  $\mathfrak{M}_*$  is the T-model defined by  $\mathfrak{M}_*=\langle A,F_*,P,C\rangle$ , where

$$F_* = \{ \llbracket f_1 \rrbracket_{\mathfrak{M}_*}^{\mathsf{T}}, \dots, \llbracket f_n \rrbracket_{\mathfrak{M}_*}^{\mathsf{D}} \}$$

Let  $\mathfrak{M}=\langle A,F,P,C\rangle$  be a D-model. Then  $\mathfrak{M}_*$  is the T-model defined by  $\mathfrak{M}_*=\langle A,F_*,P,C\rangle$ , where

$$F_* = \{ [\![f_1]\!]_{\mathfrak{M}_*}^{\mathsf{T}}, \dots, [\![f_n]\!]_{\mathfrak{M}_*}^{\mathsf{D}} \}$$

$$\llbracket f_i \rrbracket_{\mathfrak{M}_*}^{\mathsf{T}}(e_1, \dots, e_{a_i}) = \left\{ \begin{array}{ll} \llbracket f_i \rrbracket_{\mathfrak{M}}^{\mathsf{D}}(e_1, \dots, e_{a_i}) & \llbracket f_i \rrbracket_{\mathfrak{M}}^{\mathsf{D}}(e_1, \dots, e_{a_i}) \downarrow \\ \llbracket c_1 \rrbracket_{\mathfrak{M}}^{\mathsf{D}} & \text{otherwise} \end{array} \right.$$

Let  $\mathfrak{M}=\langle A,F,P,C\rangle$  be a D-model. Then  $\mathfrak{M}_*$  is the T-model defined by  $\mathfrak{M}_*=\langle A,F_*,P,C\rangle$ , where

$$F_* = \{ [\![f_1]\!]_{\mathfrak{M}_*}^{\mathsf{T}}, \dots, [\![f_n]\!]_{\mathfrak{M}_*}^{\mathsf{D}} \}$$

$$\llbracket f_i \rrbracket_{\mathfrak{M}_*}^{\mathsf{T}}(e_1, \dots, e_{a_i}) = \left\{ \begin{array}{ll} \llbracket f_i \rrbracket_{\mathfrak{M}}^{\mathsf{D}}(e_1, \dots, e_{a_i}) & \llbracket f_i \rrbracket_{\mathfrak{M}}^{\mathsf{D}}(e_1, \dots, e_{a_i}) \downarrow \\ \llbracket c_1 \rrbracket_{\mathfrak{M}}^{\mathsf{D}} & \text{otherwise} \end{array} \right.$$

Let  $\mathfrak{M}=\langle A,F,P,C\rangle$  be a D-model. Then  $\mathfrak{M}_*$  is the T-model defined by  $\mathfrak{M}_*=\langle A,F_*,P,C\rangle$ , where

$$F_* = \{ [\![f_1]\!]_{\mathfrak{M}_*}^{\mathsf{T}}, \dots, [\![f_n]\!]_{\mathfrak{M}_*}^{\mathsf{D}} \}$$

$$\llbracket f_i \rrbracket_{\mathfrak{M}_*}^{\mathsf{T}}(e_1, \dots, e_{a_i}) = \left\{ \begin{array}{ll} \llbracket f_i \rrbracket_{\mathfrak{M}}^{\mathsf{D}}(e_1, \dots, e_{a_i}) & \llbracket f_i \rrbracket_{\mathfrak{M}}^{\mathsf{D}}(e_1, \dots, e_{a_i}) \downarrow \\ \llbracket c_1 \rrbracket_{\mathfrak{M}}^{\mathsf{D}} & \text{otherwise} \end{array} \right.$$

Let  $\mathfrak{M}=\langle A,F,P,C\rangle$  be a D-model. Then  $\mathfrak{M}_*$  is the T-model defined by  $\mathfrak{M}_*=\langle A,F_*,P,C\rangle$ , where

$$F_* = \{ \llbracket f_1 \rrbracket_{\mathfrak{M}_*}^{\mathsf{T}}, \dots, \llbracket f_n \rrbracket_{\mathfrak{M}_*}^{\mathsf{D}} \}$$

$$\llbracket f_i \rrbracket_{\mathfrak{M}_*}^{\mathsf{T}}(e_1, \dots, e_{a_i}) = \left\{ \begin{array}{ll} \llbracket f_i \rrbracket_{\mathfrak{M}}^{\mathsf{D}}(e_1, \dots, e_{a_i}) & \llbracket f_i \rrbracket_{\mathfrak{M}}^{\mathsf{D}}(e_1, \dots, e_{a_i}) \downarrow \\ \llbracket c_1 \rrbracket_{\mathfrak{M}}^{\mathsf{D}} & \text{otherwise} \end{array} \right.$$

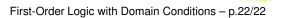
$$\models_{\mathfrak{M},\rho}^{\mathsf{D}} \varphi^* \mathsf{\textit{wf}} \mathsf{\textit{iff}} \models_{\mathfrak{M}_*,\rho}^{\mathsf{T}} \varphi \mathsf{\textit{\textit{wf}}}$$

Let  $\mathfrak{M}=\langle A,F,P,C\rangle$  be a D-model. Then  $\mathfrak{M}_*$  is the T-model defined by  $\mathfrak{M}_*=\langle A,F_*,P,C\rangle$ , where

$$F_* = \{ \llbracket f_1 \rrbracket_{\mathfrak{M}_*}^{\mathsf{T}}, \dots, \llbracket f_n \rrbracket_{\mathfrak{M}_*}^{\mathsf{D}} \}$$

$$\llbracket f_i \rrbracket_{\mathfrak{M}_*}^{\mathsf{T}}(e_1, \dots, e_{a_i}) = \left\{ \begin{array}{ll} \llbracket f_i \rrbracket_{\mathfrak{M}}^{\mathsf{D}}(e_1, \dots, e_{a_i}) & \llbracket f_i \rrbracket_{\mathfrak{M}}^{\mathsf{D}}(e_1, \dots, e_{a_i}) \downarrow \\ \llbracket c_1 \rrbracket_{\mathfrak{M}}^{\mathsf{D}} & \text{otherwise} \end{array} \right.$$

- $\models_{\mathfrak{M},\rho}^{\mathsf{D}} \varphi^* \mathsf{\textit{wf}} \mathsf{\textit{iff}} \models_{\mathfrak{M}_*,\rho}^{\mathsf{T}} \varphi \mathsf{\textit{\textit{wf}}}$
- $\sqsubseteq_{\mathfrak{M},\rho}^{\mathsf{D}} \varphi^* \text{ iff } \models_{\mathfrak{M}_*,\rho}^{\mathsf{T}} \varphi$





6 find 'right' definition of  $\Gamma \models^{\mathsf{D}}_{\mathfrak{M},\rho} \varphi$ 



- 6 find 'right' definition of  $\Gamma \models_{\mathfrak{M},\rho}^{\mathsf{D}} \varphi$
- 6 define semantic domain conditions



- 6 find 'right' definition of  $\Gamma \models^{\mathsf{D}}_{\mathfrak{M},\rho} \varphi$
- 6 define semantic domain conditions
- prove equivalence between T and D semantically



- 6 find 'right' definition of  $\Gamma \models^{\mathsf{D}}_{\mathfrak{M},\rho} \varphi$
- 6 define semantic domain conditions
- prove equivalence between T and D semantically
- 6 completeness of D