symbolic ai and mathematics

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what is ai?

the goal

build computer systems that behave in an “intelligent” way

- how do we measure success?

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two main directions

- data-driven ai
- symbolic ai

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- data-driven ai – learning
- symbolic ai – reasoning

symbolic ai

the central question

how do we represent knowledge?

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- implicitly – using procedures
- explicitly – using logic

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decidability ...but you need to make decisions...

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expressivity knowledge is structured and complex. . .

decidability . . . but you need to make decisions. . .

complexity . . . preferably before the end of the universe

human logic vs mathematical logic

some examples of (non-)problematic reasoning:

human logic vs mathematical logic

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belief revision we continuously adjust what (we think) we know

logic for mathematics

a simpler world?

mathematics is consistent[a] and monotonic

[a] modulo gödel

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still many options

- propositional logic – simple but not structure
- first-order logic – everyone knows it, but not enough
- higher-order logic with inductive types – ouch

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$\rightsquigarrow$ and let's not discuss intuitionism

it all comes up to what the goal is

automated theorem proving

sat-solving is a success story!

- encode your problem as a VERY large propositional formula and push the button

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large number of new results in combinatorics in the last years

- can deal with millions of variables, millions of clauses
- proofs can be "efficiently" verified
- encodings can be made formal

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$\rightsquigarrow$ more recent success stories in first-order logic (but gödel. . .)

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interactive theorem proving

use expressive (undecidable) logics, and help the computer with the proof

- also success stories – e.g. four-color theorem, kepler conjecture
- starting to be usable in active research
- potential for automation, learning, etc

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still mostly a tool for experts

- not very intuitive
- requires understanding both mathematics, proof theory, and a bit of programming

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computational algebra systems

the operational approach

- no explicit semantics
- no formal guarantees of correctness
- can give very strange results (but you have to try hard)

the future

what do we want?

computers doing mathematics

when do we want it?

as soon as possible

how do we get there?

probably: combining a bit of everything

thank you!