PlotDatesCurves.Rmd

Code

This code plots the yield curves for the Brazilian Bonds using the public data available from the Tesouro Direto site. https://www.tesourotransparente.gov.br/ (https://www.tesourotransparente.gov.br/). The input worksheets can be downloaded, together with this code and R and Python function libraries, from the author's GitHub.

A specific given date is used to optimize the curves for parametric models using both the Differential Evolution and Steepest Descent methods, as well as a regression for a non parametric curve using the Kernel-Ridge model.

This code can be used as an example to plot the curves and calculate errors. Our work includes additional codes to plot conclusions considering the whole historic data.

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```
# Identification ------------------------------------------------------------
#
# FGV - Fundacao Getúlio Vargas
# EESP - Escola de Economia São Paulo
# MPEF - Mestrado Profissional em Economia e Finanças
#
# Projeto de dissertação
#
# YIELD CURVES COMPARISON
# Autor: Luis Giovanni Faria
#
# Date: 23/01/2023
#

# Initialization ------------------------------------------------------------

# Remove working set
rm(list=ls())
graphics.off()

# Working Directory
sysInfo <- Sys.info()
if (sysInfo[[4]] == "GIO-YOGA") {
  workingDirectory <- "C:/Users/giovanni/OneDrive/$FGV/Yield Curves/R/dev"
} else if (sysInfo[[4]] == "DESKTOP-G3EH8EA") {
  workingDirectory <- "C:/Users/lcgfa/OneDrive/$FGV/Yield Curves/R/dev"
} else {
  workingDirectory <- getwd()
}
setwd(workingDirectory)
cat("Working directory =>",getwd())
```

```
Working directory => C:/Users/lcgfa/OneDrive/$FGV/Yield Curves/R/dev
```

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```
inputDirectory <- "../database/"
outputDirectory <- "../output/"

# Load R and Python libraries
library(readxl)
#library(xlsx)
library(reticulate)
#library(Matrix)
library(ggplot2)
#library(grid)
#library(gridExtra)

source("../lib/yc_lib.r")
source("../lib/yc_loaddata.r")
source_python("../lib/kr_model.py")
source_python("../lib/kr_utils.py")
source_python("../lib/yc_utils.py")

nsimDE <- 10 # number of simulations differential evolution
nsimSD <- 10 # number of simulations steepest descent
faceValue <- 1000
daysYear <- 365
#datePlots <- list() # list of date reference curve plots

# Parametric models to be run with optimization methods DE and SD
# NS = Nelson-Siegel
# BL = Bliss
# NSS= Svensson
# DP = DePooter
# BC = Bjork-Christensen
models <- c("NS","BL","NSS","DP","BC") # Models to optimize
```

Here we define the reference date to be used to calculate and plot the curves.

Hide

```
################################################################################
## Load data

# run data from tesouro diretoto
bondData <- ReadTesouroDireto(inputDirectory,coupon=0.1)
```

```
Reading file:  ../database/LTN_2004.xls
File=LTN_2004 Sheet=1 Maturity=2004-01-07 Records=252
File=LTN_2004 Sheet=2 Maturity=2004-04-01 Records=252
File=LTN_2004 Sheet=3 Maturity=2004-07-01 Records=252
File=LTN_2004 Sheet=4 Maturity=2004-10-01 Records=252
File=LTN_2004 Sheet=5 Maturity=2005-01-04 Records=252
File=LTN_2004 Sheet=6 Maturity=2005-04-01 Records=252
File=LTN_2004 Sheet=7 Maturity=2005-07-01 Records=252
File=LTN_2004 Sheet=8 Maturity=2005-10-01 Records=252
File=LTN_2004 Sheet=9 Maturity=2006-01-01 Records=252
File=LTN_2004 Sheet=10 Maturity=2006-07-01 Records=252
Reading file:  ../database/LTN_2005.xls
File=LTN_2005 Sheet=1 Maturity=2005-01-04 Records=251
File=LTN_2005 Sheet=2 Maturity=2005-04-01 Records=251
File=LTN_2005 Sheet=3 Maturity=2005-07-01 Records=251
File=LTN_2005 Sheet=4 Maturity=2005-10-01 Records=251
File=LTN_2005 Sheet=5 Maturity=2006-01-01 Records=251
File=LTN_2005 Sheet=6 Maturity=2006-04-01 Records=251
File=LTN_2005 Sheet=7 Maturity=2006-07-01 Records=251
File=LTN_2005 Sheet=8 Maturity=2006-10-01 Records=251
File=LTN_2005 Sheet=9 Maturity=2007-01-01 Records=251
File=LTN_2005 Sheet=10 Maturity=2007-04-01 Records=251
File=LTN_2005 Sheet=11 Maturity=2007-07-01 Records=251
File=LTN_2005 Sheet=12 Maturity=2008-01-01 Records=251
File=LTN_2005 Sheet=13 Maturity=2008-07-01 Records=251
Reading file:  ../database/LTN_2006.xls
File=LTN_2006 Sheet=1 Maturity=2006-04-01 Records=248
File=LTN_2006 Sheet=2 Maturity=2006-07-01 Records=248
File=LTN_2006 Sheet=3 Maturity=2006-10-01 Records=248
File=LTN_2006 Sheet=4 Maturity=2007-01-01 Records=248
File=LTN_2006 Sheet=5 Maturity=2007-04-01 Records=248
File=LTN_2006 Sheet=6 Maturity=2007-07-01 Records=248
File=LTN_2006 Sheet=7 Maturity=2007-10-01 Records=248
File=LTN_2006 Sheet=8 Maturity=2008-01-01 Records=248
File=LTN_2006 Sheet=9 Maturity=2008-04-01 Records=248
File=LTN_2006 Sheet=10 Maturity=2008-07-01 Records=248
File=LTN_2006 Sheet=11 Maturity=2009-01-01 Records=248
Reading file:  ../database/LTN_2007.xls
File=LTN_2007 Sheet=1 Maturity=2007-04-01 Records=250
File=LTN_2007 Sheet=2 Maturity=2007-07-01 Records=250
File=LTN_2007 Sheet=3 Maturity=2007-10-01 Records=250
File=LTN_2007 Sheet=4 Maturity=2008-01-01 Records=250
File=LTN_2007 Sheet=5 Maturity=2008-04-01 Records=250
File=LTN_2007 Sheet=6 Maturity=2008-07-01 Records=250
File=LTN_2007 Sheet=7 Maturity=2008-10-01 Records=250
File=LTN_2007 Sheet=8 Maturity=2009-01-01 Records=250
File=LTN_2007 Sheet=9 Maturity=2009-07-01 Records=250
File=LTN_2007 Sheet=10 Maturity=2009-10-01 Records=250
File=LTN_2007 Sheet=11 Maturity=2010-01-01 Records=250
Reading file:  ../database/LTN_2008.xls
```

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Warning: Coercing numeric to date in A254 / R254C1Warning: Coercing numeric to date in A255 /
R255C1Warning: Coercing numeric to date in A256 / R256C1
```

```
File=LTN_2008 Sheet=1 Maturity=2008-04-01 Records=254
```

```
Warning: Coercing numeric to date in A247 / R247C1Warning: Coercing numeric to date in A248 / R248C1Warning: Coercing numeric to date in A249 / R249C1
```

```
File=LTN_2008 Sheet=2 Maturity=2008-07-01 Records=247
```

```
Warning: Coercing numeric to date in A247 / R247C1Warning: Coercing numeric to date in A248 / R248C1Warning: Coercing numeric to date in A249 / R249C1
```

```
File=LTN_2008 Sheet=3 Maturity=2008-10-01 Records=247
File=LTN_2008 Sheet=4 Maturity=2009-01-01 Records=247
File=LTN_2008 Sheet=5 Maturity=2009-04-01 Records=247
File=LTN_2008 Sheet=6 Maturity=2009-07-01 Records=247
File=LTN_2008 Sheet=7 Maturity=2009-10-01 Records=247
File=LTN_2008 Sheet=8 Maturity=2010-01-01 Records=247
File=LTN_2008 Sheet=9 Maturity=2010-07-01 Records=247
File=LTN_2008 Sheet=10 Maturity=2011-01-01 Records=247
Reading file:  ../database/LTN_2009.xls
File=LTN_2009 Sheet=1 Maturity=2009-04-01 Records=250
File=LTN_2009 Sheet=2 Maturity=2009-07-01 Records=250
File=LTN_2009 Sheet=3 Maturity=2009-10-01 Records=250
File=LTN_2009 Sheet=4 Maturity=2010-01-01 Records=250
File=LTN_2009 Sheet=5 Maturity=2010-07-01 Records=250
File=LTN_2009 Sheet=6 Maturity=2011-01-01 Records=250
File=LTN_2009 Sheet=7 Maturity=2012-01-01 Records=250
Reading file:  ../database/LTN_2010.xls
File=LTN_2010 Sheet=1 Maturity=2010-07-01 Records=251
File=LTN_2010 Sheet=2 Maturity=2011-01-01 Records=251
File=LTN_2010 Sheet=3 Maturity=2011-07-01 Records=251
File=LTN_2010 Sheet=4 Maturity=2012-01-01 Records=251
File=LTN_2010 Sheet=5 Maturity=2013-01-01 Records=251
Reading file:  ../database/LTN_2011.xls
File=LTN_2011 Sheet=1 Maturity=2011-07-01 Records=251
File=LTN_2011 Sheet=2 Maturity=2012-01-01 Records=251
File=LTN_2011 Sheet=3 Maturity=2013-01-01 Records=251
File=LTN_2011 Sheet=4 Maturity=2014-01-01 Records=251
File=LTN_2011 Sheet=5 Maturity=2015-01-01 Records=251
Reading file:  ../database/LTN_2012.xls
File=LTN_2012 Sheet=1 Maturity=2013-01-01 Records=249
File=LTN_2012 Sheet=2 Maturity=2014-01-01 Records=249
File=LTN_2012 Sheet=3 Maturity=2015-01-01 Records=249
File=LTN_2012 Sheet=4 Maturity=2016-01-01 Records=227
Reading file:  ../database/LTN_2013.xls
File=LTN_2013 Sheet=1 Maturity=2014-01-01 Records=251
File=LTN_2013 Sheet=2 Maturity=2015-01-01 Records=251
File=LTN_2013 Sheet=3 Maturity=2016-01-01 Records=251
File=LTN_2013 Sheet=4 Maturity=2017-01-01 Records=243
Reading file:  ../database/LTN_2014.xls
File=LTN_2014 Sheet=1 Maturity=2015-01-01 Records=253
File=LTN_2014 Sheet=2 Maturity=2016-01-01 Records=253
File=LTN_2014 Sheet=3 Maturity=2017-01-01 Records=253
File=LTN_2014 Sheet=4 Maturity=2018-01-01 Records=236
Reading file:  ../database/LTN_2015.xls
File=LTN_2015 Sheet=1 Maturity=2016-01-01 Records=250
File=LTN_2015 Sheet=2 Maturity=2017-01-01 Records=250
File=LTN_2015 Sheet=3 Maturity=2018-01-01 Records=250
File=LTN_2015 Sheet=4 Maturity=2021-01-01 Records=205
Reading file:  ../database/LTN_2016.xls
File=LTN_2016 Sheet=1 Maturity=2017-01-01 Records=250
File=LTN_2016 Sheet=2 Maturity=2018-01-01 Records=250
File=LTN_2016 Sheet=3 Maturity=2019-01-01 Records=234
File=LTN_2016 Sheet=4 Maturity=2021-01-01 Records=250
File=LTN_2016 Sheet=5 Maturity=2023-01-01 Records=234
Reading file:  ../database/LTN_2017.xls
```

```
File=LTN_2017 Sheet=1 Maturity=2018-01-01 Records=245
File=LTN_2017 Sheet=2 Maturity=2019-01-01 Records=247
File=LTN_2017 Sheet=3 Maturity=2020-01-01 Records=221
File=LTN_2017 Sheet=4 Maturity=2021-01-01 Records=247
File=LTN_2017 Sheet=5 Maturity=2023-01-01 Records=247
Reading file:  ../database/LTN_2018.xls
File=LTN_2018 Sheet=1 Maturity=2019-01-01 Records=246
File=LTN_2018 Sheet=2 Maturity=2020-01-01 Records=246
File=LTN_2018 Sheet=3 Maturity=2021-01-01 Records=246
File=LTN_2018 Sheet=4 Maturity=2023-01-01 Records=246
File=LTN_2018 Sheet=5 Maturity=2025-01-01 Records=222
Reading file:  ../database/LTN_2019.xls
File=LTN_2019 Sheet=1 Maturity=2020-01-01 Records=250
File=LTN_2019 Sheet=2 Maturity=2021-01-01 Records=250
File=LTN_2019 Sheet=3 Maturity=2022-01-01 Records=228
File=LTN_2019 Sheet=4 Maturity=2023-01-01 Records=250
File=LTN_2019 Sheet=5 Maturity=2025-01-01 Records=250
Reading file:  ../database/LTN_2020.xls
File=LTN_2020 Sheet=1 Maturity=2021-01-01 Records=249
File=LTN_2020 Sheet=2 Maturity=2022-01-01 Records=249
File=LTN_2020 Sheet=3 Maturity=2023-01-01 Records=249
File=LTN_2020 Sheet=4 Maturity=2025-01-01 Records=249
File=LTN_2020 Sheet=5 Maturity=2026-01-01 Records=222
Reading file:  ../database/LTN_2021.xls
File=LTN_2021 Sheet=1 Maturity=2022-01-01 Records=247
File=LTN_2021 Sheet=2 Maturity=2023-01-01 Records=247
File=LTN_2021 Sheet=3 Maturity=2024-07-01 Records=223
File=LTN_2021 Sheet=4 Maturity=2025-01-01 Records=247
File=LTN_2021 Sheet=5 Maturity=2026-01-01 Records=247
Reading file:  ../database/LTN_2022.xls
File=LTN_2022 Sheet=1 Maturity=2023-01-01 Records=189
File=LTN_2022 Sheet=2 Maturity=2024-07-01 Records=189
File=LTN_2022 Sheet=3 Maturity=2025-01-01 Records=189
File=LTN_2022 Sheet=4 Maturity=2026-01-01 Records=189
File=LTN_2022 Sheet=5 Maturity=2029-01-01 Records=154
Reading file:  ../database/NTN-F_2004.xls
File=NTN-F_2004 Sheet=1 Maturity=2008-01-01 Records=252
Reading file:  ../database/NTN-F_2005.xls
File=NTN-F_2005 Sheet=1 Maturity=2008-01-01 Records=251
File=NTN-F_2005 Sheet=2 Maturity=2010-01-01 Records=251
Reading file:  ../database/NTN-F_2006.xls
File=NTN-F_2006 Sheet=1 Maturity=2008-01-01 Records=248
File=NTN-F_2006 Sheet=2 Maturity=2010-01-01 Records=248
File=NTN-F_2006 Sheet=3 Maturity=2012-01-01 Records=248
File=NTN-F_2006 Sheet=4 Maturity=2014-01-01 Records=248
Reading file:  ../database/NTN-F_2007.xls
File=NTN-F_2007 Sheet=1 Maturity=2008-01-01 Records=250
File=NTN-F_2007 Sheet=2 Maturity=2010-01-01 Records=250
File=NTN-F_2007 Sheet=3 Maturity=2010-07-01 Records=250
File=NTN-F_2007 Sheet=4 Maturity=2011-01-01 Records=250
File=NTN-F_2007 Sheet=5 Maturity=2012-01-01 Records=250
File=NTN-F_2007 Sheet=6 Maturity=2013-01-01 Records=250
File=NTN-F_2007 Sheet=7 Maturity=2014-01-01 Records=250
File=NTN-F_2007 Sheet=8 Maturity=2017-01-01 Records=250
Reading file:  ../database/NTN-F_2008.xls
File=NTN-F_2008 Sheet=1 Maturity=2010-01-01 Records=247
```

```
File=NTN-F_2008 Sheet=2 Maturity=2010-07-01 Records=247
File=NTN-F_2008 Sheet=3 Maturity=2011-01-01 Records=247
File=NTN-F_2008 Sheet=4 Maturity=2012-01-01 Records=247
File=NTN-F_2008 Sheet=5 Maturity=2013-01-01 Records=247
File=NTN-F_2008 Sheet=6 Maturity=2014-01-01 Records=247
File=NTN-F_2008 Sheet=7 Maturity=2017-01-01 Records=247
Reading file:  ../database/NTN-F_2009.xls
File=NTN-F_2009 Sheet=1 Maturity=2010-01-01 Records=250
File=NTN-F_2009 Sheet=2 Maturity=2010-07-01 Records=250
File=NTN-F_2009 Sheet=3 Maturity=2011-01-01 Records=250
File=NTN-F_2009 Sheet=4 Maturity=2012-01-01 Records=250
File=NTN-F_2009 Sheet=5 Maturity=2013-01-01 Records=250
File=NTN-F_2009 Sheet=6 Maturity=2014-01-01 Records=250
File=NTN-F_2009 Sheet=7 Maturity=2017-01-01 Records=250
Reading file:  ../database/NTN-F_2010.xls
File=NTN-F_2010 Sheet=1 Maturity=2010-07-01 Records=251
File=NTN-F_2010 Sheet=2 Maturity=2011-01-01 Records=251
File=NTN-F_2010 Sheet=3 Maturity=2012-01-01 Records=251
File=NTN-F_2010 Sheet=4 Maturity=2013-01-01 Records=251
File=NTN-F_2010 Sheet=5 Maturity=2014-01-01 Records=251
File=NTN-F_2010 Sheet=6 Maturity=2017-01-01 Records=251
File=NTN-F_2010 Sheet=7 Maturity=2021-01-01 Records=251
Reading file:  ../database/NTN-F_2011.xls
File=NTN-F_2011 Sheet=1 Maturity=2012-01-01 Records=251
File=NTN-F_2011 Sheet=2 Maturity=2013-01-01 Records=251
File=NTN-F_2011 Sheet=3 Maturity=2014-01-01 Records=251
File=NTN-F_2011 Sheet=4 Maturity=2017-01-01 Records=251
File=NTN-F_2011 Sheet=5 Maturity=2021-01-01 Records=251
Reading file:  ../database/NTN-F_2012.xls
File=NTN-F_2012 Sheet=1 Maturity=2013-01-01 Records=249
File=NTN-F_2012 Sheet=2 Maturity=2014-01-01 Records=249
File=NTN-F_2012 Sheet=3 Maturity=2017-01-01 Records=249
File=NTN-F_2012 Sheet=4 Maturity=2021-01-01 Records=249
File=NTN-F_2012 Sheet=5 Maturity=2023-01-01 Records=144
Reading file:  ../database/NTN-F_2013.xls
File=NTN-F_2013 Sheet=1 Maturity=2014-01-01 Records=251
File=NTN-F_2013 Sheet=2 Maturity=2017-01-01 Records=251
File=NTN-F_2013 Sheet=3 Maturity=2021-01-01 Records=251
File=NTN-F_2013 Sheet=4 Maturity=2023-01-01 Records=251
Reading file:  ../database/NTN-F_2014.xls
File=NTN-F_2014 Sheet=1 Maturity=2017-01-01 Records=253
File=NTN-F_2014 Sheet=2 Maturity=2021-01-01 Records=253
File=NTN-F_2014 Sheet=3 Maturity=2023-01-01 Records=253
File=NTN-F_2014 Sheet=4 Maturity=2025-01-01 Records=236
Reading file:  ../database/NTN-F_2015.xls
File=NTN-F_2015 Sheet=1 Maturity=2017-01-01 Records=250
File=NTN-F_2015 Sheet=2 Maturity=2021-01-01 Records=250
File=NTN-F_2015 Sheet=3 Maturity=2023-01-01 Records=250
File=NTN-F_2015 Sheet=4 Maturity=2025-01-01 Records=250
Reading file:  ../database/NTN-F_2016.xls
File=NTN-F_2016 Sheet=1 Maturity=2017-01-01 Records=250
File=NTN-F_2016 Sheet=2 Maturity=2021-01-01 Records=250
File=NTN-F_2016 Sheet=3 Maturity=2023-01-01 Records=250
File=NTN-F_2016 Sheet=4 Maturity=2025-01-01 Records=250
File=NTN-F_2016 Sheet=5 Maturity=2027-01-01 Records=234
Reading file:  ../database/NTN-F_2017.xls
```

```
File=NTN-F_2017 Sheet=1 Maturity=2021-01-01 Records=247
File=NTN-F_2017 Sheet=2 Maturity=2023-01-01 Records=247
File=NTN-F_2017 Sheet=3 Maturity=2025-01-01 Records=247
File=NTN-F_2017 Sheet=4 Maturity=2027-01-01 Records=247
Reading file:  ../database/NTN-F_2018.xls
File=NTN-F_2018 Sheet=1 Maturity=2021-01-01 Records=246
File=NTN-F_2018 Sheet=2 Maturity=2023-01-01 Records=246
File=NTN-F_2018 Sheet=3 Maturity=2025-01-01 Records=246
File=NTN-F_2018 Sheet=4 Maturity=2027-01-01 Records=246
File=NTN-F_2018 Sheet=5 Maturity=2029-01-01 Records=222
Reading file:  ../database/NTN-F_2019.xls
File=NTN-F_2019 Sheet=1 Maturity=2021-01-01 Records=250
File=NTN-F_2019 Sheet=2 Maturity=2023-01-01 Records=250
File=NTN-F_2019 Sheet=3 Maturity=2025-01-01 Records=250
File=NTN-F_2019 Sheet=4 Maturity=2027-01-01 Records=250
File=NTN-F_2019 Sheet=5 Maturity=2029-01-01 Records=250
Reading file:  ../database/NTN-F_2020.xls
File=NTN-F_2020 Sheet=1 Maturity=2021-01-01 Records=249
File=NTN-F_2020 Sheet=2 Maturity=2023-01-01 Records=249
File=NTN-F_2020 Sheet=3 Maturity=2025-01-01 Records=249
File=NTN-F_2020 Sheet=4 Maturity=2027-01-01 Records=249
File=NTN-F_2020 Sheet=5 Maturity=2029-01-01 Records=249
File=NTN-F_2020 Sheet=6 Maturity=2031-01-01 Records=222
Reading file:  ../database/NTN-F_2021.xls
File=NTN-F_2021 Sheet=1 Maturity=2023-01-01 Records=247
File=NTN-F_2021 Sheet=2 Maturity=2025-01-01 Records=247
File=NTN-F_2021 Sheet=3 Maturity=2027-01-01 Records=247
File=NTN-F_2021 Sheet=4 Maturity=2029-01-01 Records=247
File=NTN-F_2021 Sheet=5 Maturity=2031-01-01 Records=247
Reading file:  ../database/NTN-F_2022.xls
File=NTN-F_2022 Sheet=1 Maturity=2023-01-01 Records=189
File=NTN-F_2022 Sheet=2 Maturity=2025-01-01 Records=189
File=NTN-F_2022 Sheet=3 Maturity=2027-01-01 Records=189
File=NTN-F_2022 Sheet=4 Maturity=2029-01-01 Records=189
File=NTN-F_2022 Sheet=5 Maturity=2031-01-01 Records=189
File=NTN-F_2022 Sheet=6 Maturity=2033-01-01 Records=154
```

Hide

```
dateReference <- as.Date("2021-09-30") # DATE REFERENCE

cat("=> Working with reference date",format(dateReference,"%Y-%m-%d"),"\n")
```

```
=> Working with reference date 2021-09-30
```

Hide

```
# Set to generate price vector and cashfow matrix (subset of ondData
bondSet <- bondData[c(1,2,3,8,9)]
bondSet <- subset(bondSet, DateReference==dateReference)

################################################################################
# Compute YTM for bonds

## Calculate yield to maturity
bondSet$Yield <- c(0)

for (i in 1:nrow(bondSet)) {
  bondSet$Yield[i] <- YieldToMaturity(price=bondSet$PricePU[i],x0=0.00,
                                      days=as.numeric(rev(seq(from=bondSet$DateMaturity[i],to
=dateReference, by="-6 month"))-dateReference),
                                      faceValue=faceValue,
                                      coupon=bondSet$Coupon[i]/2,
                                      daysYear=daysYear,epson=1e-6,nmax=100)
} # end-for

# view working set
print(bondSet)
```

	DateReference <date>	BondType <chr>	DateMaturity <date>	Coupon <dbl>	PricePU <dbl>	Yield <dbl>
28181	2021-09-30	LTN	2022-01-01	0.0	982.49	0.06933082
28428	2021-09-30	LTN	2023-01-01	0.0	895.20	0.08822809
28651	2021-09-30	LTN	2024-07-01	0.0	765.93	0.09684830
28898	2021-09-30	LTN	2025-01-01	0.0	726.37	0.09814041
29145	2021-09-30	LTN	2026-01-01	0.0	653.19	0.10003143
50823	2021-09-30	NTN-F	2023-01-01	0.1	1032.31	0.09094994
51070	2021-09-30	NTN-F	2025-01-01	0.1	1017.81	0.09976925
51317	2021-09-30	NTN-F	2027-01-01	0.1	999.93	0.10338872
51564	2021-09-30	NTN-F	2029-01-01	0.1	982.28	0.10560073
51811	2021-09-30	NTN-F	2031-01-01	0.1	962.08	0.10789482

1-10 of 10 rows

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```
NA
```

Hide

```
################################################################################
## Build Constrained Cubic Spline
##

# Initialize yield vectors and CCS parameters
bondSet$CCS <- c(0)
# sort by maturity
bondSort <- bondSet[order(bondSet$DateMaturity,decreasing=FALSE),]

x <- as.numeric(bondSort$DateMaturity-bondSort$DateMaturity[1])/nrow(bondSet)
y <- bondSort$Yield

# Compute fitted CCS yields for maturity dates
for (i in 1:nrow(bondSet)) {
  xstar <- (x[length(x)]-x[1])*as.numeric(bondSet$DateMaturity[i]-bondSet$DateMaturity[1])/
    as.numeric(bondSet$DateMaturity[nrow(bondSet)]-bondSet$DateMaturity[1])
  bondSet$CCS[i] <- CCSpline(x,y,xstar)
}

# Generate CCS curve
numberOfPoints <- 100
maxMaturity <- max(bondSet$DateMaturity)
CCScurve <- data.frame(matrix(ncol=3, nrow=numberOfPoints))
colnames(CCScurve) <- c("X", "DateMaturity", "CCS")
class(CCScurve$DateMaturity) <- "Date"

for (i in 1:numberOfPoints) {
  xstar <- x[1]+(x[length(x)]-x[1])/numberOfPoints*(i-1)
  CCScurve[i,1] <- xstar
  CCScurve[i,2] <- as.Date(bondSet$DateMaturity[1]+
                             as.numeric(bondSet$DateMaturity[nrow(bondSet)]-bondSet$DateMatur
ity[1])/numberOfPoints*(i-1))
  CCScurve[i,3] <- CCSpline(x,y,xstar)
} # end-for

print(bondSet)
```

	DateReference <date>	BondTy… <chr>	DateMaturity <date>	Cou… <dbl>	PricePU <dbl>	Yield <dbl>	CCS <dbl>
28181	2021-09-30	LTN	2022-01-01	0.0	982.49	0.06933082	0.06933082
28428	2021-09-30	LTN	2023-01-01	0.0	895.20	0.08822809	0.09094994
28651	2021-09-30	LTN	2024-07-01	0.0	765.93	0.09684830	0.09684830
28898	2021-09-30	LTN	2025-01-01	0.0	726.37	0.09814041	0.09976925
29145	2021-09-30	LTN	2026-01-01	0.0	653.19	0.10003143	0.10003143
50823	2021-09-30	NTN-F	2023-01-01	0.1	1032.31	0.09094994	0.09094994
51070	2021-09-30	NTN-F	2025-01-01	0.1	1017.81	0.09976925	0.09976925
51317	2021-09-30	NTN-F	2027-01-01	0.1	999.93	0.10338872	0.10338872
51564	2021-09-30	NTN-F	2029-01-01	0.1	982.28	0.10560073	0.10560073

	DateReference <date>	BondTy… <chr>	DateMaturity <date>	Cou… <dbl>	PricePU <dbl>	Yield <dbl>	CCS <dbl>
51811	2021-09-30	NTN-F	2031-01-01	0.1	962.08	0.10789482	0.10789482

1-10 of 10 rows

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```
NA
```

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```
################################################################################
## Build parametric curves
##

## Define model functions as a list
ModelFunction = list();

ModelFunction[["NS"]] = function(params,t) {
  ## Description: Compute Nelson-Siegel (1987) function
  return(params["Beta1"]+
           params["Beta2"]*((1-exp(-t/params["Lambda1"]))/(t/params["Lambda1"]))+
           params["Beta3"]*((1-exp(-t/params["Lambda1"]))/(t/params["Lambda1"])-exp(-t/params
["Lambda1"])))
} # end-function

ModelFunction[["DPR"]] = function(params,t) {
  ## Description: Compute Diebold-Piazzesi-Rudesbusch (2005) function
  return(params["Beta1"]+
           params["Beta2"]*((1-exp(-t/params["Lambda1"]))/(t/params["Lambda1"])))
} # end-function

ModelFunction[["BL"]] = function(params,t) {
  ## Description: Compute Bliss (1997) function
  return(params["Beta1"]+
           params["Beta2"]*((1-exp(-t/params["Lambda1"]))/(t/params["Lambda1"]))+
           params["Beta3"]*((1-exp(-t/params["Lambda2"]))/(t/params["Lambda2"])-exp(-t/params
["Lambda2"])))
} # end-function

ModelFunction[["NSS"]] = function(params,t) {
  ## Description: Compute Nelson-Siegel-Svensson (1994) function
  return(params["Beta1"]+
           params["Beta2"]*((1-exp(-t/params["Lambda1"]))/(t/params["Lambda1"]))+
           params["Beta3"]*((1-exp(-t/params["Lambda1"]))/(t/params["Lambda1"])-exp(-t/params
["Lambda1"]))+
           params["Beta4"]*((1-exp(-t/params["Lambda2"]))/(t/params["Lambda2"])-exp(-t/params
["Lambda2"])))
} # end-function

ModelFunction[["DP"]] = function(params,t) {
  ## Description: Compute De Pooter (2007) function
  return(params["Beta1"]+
           params["Beta2"]*((1-exp(-t/params["Lambda1"]))/(t/params["Lambda1"]))+
           params["Beta3"]*((1-exp(-t/params["Lambda1"]))/(t/params["Lambda1"])-exp(-t/params
["Lambda1"]))+
           params["Beta4"]*((1-exp(-t/params["Lambda2"]))/(t/params["Lambda2"])-exp(-2*t/para
ms["Lambda2"])))
} # end-function

ModelFunction[["BC"]] = function(params,t) {
  ## Description: Compute Björk-Christensen (1999) function
  return(params["Beta1"]+
           params["Beta2"]*t/(2*params["Lambda1"])+
```

```
           params["Beta3"]*((1-exp(-t/params["Lambda1"]))/(t/params["Lambda1"]))+
           params["Beta4"]*((1-exp(-t/params["Lambda1"]))/(t/params["Lambda1"])-exp(-t/params
["Lambda1"]))+
           params["Beta5"]*((1-exp(-2*t/params["Lambda1"]))/(2*t/params["Lambda1"])))
} # end-function


################################################################################
## Run Differential Evolution for  parametric curves
##

NP <- 100 # population size
nmaxG = 500 # number of generations
set.seed(0)
lower = c(Beta1=0,Beta2=-0.05,Beta3=-0.5,Beta4=-0.5,Beta5=-0.5,Lambda1=0,Lambda2=0);
upper = c(Beta1=1,Beta2=0.05,Beta3=0.5,Beta4=0.5,Beta5=-0.5,Lambda1=1.25,Lambda2=1.25);

### Restrictionsestrições
gx = list();
gx[[1]] = function(params){return(-params["Beta1"]-params["Beta2"])}

nsim <- nsimDE # number of simulations

for (m in models) {
  cat("Reference date ",format(dateReference,"%Y-%m-%d")," Calculating Model - Differential E
volution = ",m,"\n")
  # data structure to store results
  paramsOpt = matrix(NA,nrow=7,ncol=nsim)
  errors <- c()
  rownames(paramsOpt) = c("Beta1","Beta2","Beta3","Beta4","Beta5","Lambda1","Lambda2")

  for (i in 1:nsim) {
    cat(sprintf("Simulation %d/%d\n",i,nsim));
    # Initial population
    paramsG0 <- t(as.matrix(data.frame(Beta1 = runif(NP,lower["Beta1"],upper["Beta1"]),
                                       Beta2 = runif(NP,lower["Beta2"],upper["Beta2"]),
                                       Beta3 = runif(NP,lower["Beta3"],upper["Beta3"]),
                                       Beta4 = runif(NP,lower["Beta4"],upper["Beta4"]),
                                       Beta5 = runif(NP,lower["Beta5"],upper["Beta5"]),
                                       Lambda1 = runif(NP,lower["Lambda1"],upper["Lambda1"]),
                                       Lambda2 = runif(NP,lower["Lambda2"],upper["Lambda
2"]))))
    # get optimum parameters
    paramsOpt[,i] <- DifferentialEvolution(RootMeanSquareError,ModelFunction[[m]],paramsG0,
                                           t=as.numeric(as.Date(bondSet$DateMaturity)-dateRef
erence)/daysYear,
                                           yield=bondSet$Yield,
                                           F = 1,
                                           CR = 0.5,
                                           lower=lower,
                                           upper=upper,
                                           gx = gx,
                                           nmaxG=500)
    # get errors
    errors[i] <- RootMeanSquareError(ModelFunction[[m]],paramsOpt[,i],
                                     t=as.numeric(as.Date(bondSet$DateMaturity)-dateReferenc
```

```
e)/daysYear,
                                    yield=bondSet$Yield)
  } # end-simulations
  # get fitted models

  bondSet[,ncol(bondSet)+1] <- ModelFunction[[m]](params=paramsOpt[,which.min(errors)],
                                                  t=as.numeric(as.Date(bondSet$DateMaturity)-
dateReference)/daysYear)
  colnames(bondSet)[ncol(bondSet)] <- m
} # end-for
```

```
Reference date  2021-09-30  Calculating Model - Differential Evolution =  NS
Simulation 1/10
Simulation 2/10
Simulation 3/10
Simulation 4/10
Simulation 5/10
Simulation 6/10
Simulation 7/10
Simulation 8/10
Simulation 9/10
Simulation 10/10
Reference date  2021-09-30  Calculating Model - Differential Evolution =  BL
Simulation 1/10
Simulation 2/10
Simulation 3/10
Simulation 4/10
Simulation 5/10
Simulation 6/10
Simulation 7/10
Simulation 8/10
Simulation 9/10
Simulation 10/10
Reference date  2021-09-30  Calculating Model - Differential Evolution =  NSS
Simulation 1/10
Simulation 2/10
Simulation 3/10
Simulation 4/10
Simulation 5/10
Simulation 6/10
Simulation 7/10
Simulation 8/10
Simulation 9/10
Simulation 10/10
Reference date  2021-09-30  Calculating Model - Differential Evolution =  DP
Simulation 1/10
Simulation 2/10
Simulation 3/10
Simulation 4/10
Simulation 5/10
Simulation 6/10
Simulation 7/10
Simulation 8/10
Simulation 9/10
Simulation 10/10
Reference date  2021-09-30  Calculating Model - Differential Evolution =  BC
Simulation 1/10
Simulation 2/10
Simulation 3/10
Simulation 4/10
Simulation 5/10
Simulation 6/10
Simulation 7/10
Simulation 8/10
Simulation 9/10
Simulation 10/10
```

Hide

```
print(bondSet)
```

	DateReference <date>	BondT… <chr>	DateMaturity <date>	Cou… <dbl>	Price… <dbl>	Yield <dbl>	CCS <dbl>	
28181	2021-09-30	LTN	2022-01-01	0.0	982.49	0.06933082	0.06933082	0.06
28428	2021-09-30	LTN	2023-01-01	0.0	895.20	0.08822809	0.09094994	0.08
28651	2021-09-30	LTN	2024-07-01	0.0	765.93	0.09684830	0.09684830	0.09
28898	2021-09-30	LTN	2025-01-01	0.0	726.37	0.09814041	0.09976925	0.09
29145	2021-09-30	LTN	2026-01-01	0.0	653.19	0.10003143	0.10003143	0.10
50823	2021-09-30	NTN-F	2023-01-01	0.1	1032.31	0.09094994	0.09094994	0.08
51070	2021-09-30	NTN-F	2025-01-01	0.1	1017.81	0.09976925	0.09976925	0.09
51317	2021-09-30	NTN-F	2027-01-01	0.1	999.93	0.10338872	0.10338872	0.10
51564	2021-09-30	NTN-F	2029-01-01	0.1	982.28	0.10560073	0.10560073	0.10
51811	2021-09-30	NTN-F	2031-01-01	0.1	962.08	0.10789482	0.10789482	0.10

1-10 of 10 rows | 1-9 of 12 columns

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```
NA
```

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```
##############################################################################
## Run Gradient method for  parametric curves
##

## Number of simulations
nsim <- nsimSD

## Set seed
set.seed(0)

for (m in models) {
  cat("Reference date ",format(dateReference,"%Y-%m-%d")," Calculating Model - Steepest Desce
nt = ",m,"\n")
  ## Intervals
  params0 <- as.matrix(data.frame(Beta1 = runif(nsim,0,1),
                                  Beta2 = runif(nsim,-1,1),
                                  Beta3 = runif(nsim,-1,1),
                                  Beta4 = runif(nsim,-1,1),
                                  Beta5 = runif(nsim,-1,1),
                                  Lambda1 = runif(nsim,1,5),
                                  Lambda2 = runif(nsim,1,5)));
  ## data structure with parameters in each interaction
  params <- matrix(NA,nrow(params0),ncol(params0));
  colnames(params) = colnames(params0);
  ## data structure with errors
  errors <- c();
  ## data structure with fitted curves
  curves <- matrix(NA,nrow(bondSet),nsim);
  for (i in 1:nsim){
    cat(sprintf("Simulation %d/%d\n",i,nsim));
    #RmseFunc=paste("RootMeanSquareError",m,sep="");
    params[i,] <- OptimizeSteepestDescent(RootMeanSquareError,ModelFunction[[m]],
                                          x0=params0[i,],
                                          t=as.numeric(as.Date(bondSet$DateMaturity)-dateRefe
rence)/daysYear,
                                          yield=bondSet$Yield,
                                          lower = c(0,-Inf,-Inf,-Inf,1,1),
                                          upper = c(Inf, Inf, Inf, Inf, Inf, Inf))
    errors[i] <- RootMeanSquareError(ModelFunction[[m]],
                                     params=params[i,],
                                     t=as.numeric(as.Date(bondSet$DateMaturity)-dateReferenc
e)/daysYear,
                                     yield=bondSet$Yield)
    curves[,i] <- ModelFunction[[m]](params=params[i,],
                                     t=as.numeric(as.Date(bondSet$DateMaturity)-dateReferenc
e)/daysYear)
  } # end-for

  # get fitted yiels for minimum error
  bondSet[,ncol(bondSet)+1] <- curves[,which.min(errors)]
  colnames(bondSet)[ncol(bondSet)] <- paste(m,"_G",sep="")
} # end for
```

```
Reference date  2021-09-30  Calculating Model - Steepest Descent =  NS
Simulation 1/10
Simulation 2/10
Simulation 3/10
Simulation 4/10
Simulation 5/10
Simulation 6/10
Simulation 7/10
Simulation 8/10
Simulation 9/10
Simulation 10/10
Reference date  2021-09-30  Calculating Model - Steepest Descent =  BL
Simulation 1/10
Simulation 2/10
Simulation 3/10
Simulation 4/10
Simulation 5/10
Simulation 6/10
Simulation 7/10
Simulation 8/10
Simulation 9/10
Simulation 10/10
Reference date  2021-09-30  Calculating Model - Steepest Descent =  NSS
Simulation 1/10
Simulation 2/10
Simulation 3/10
Simulation 4/10
Simulation 5/10
Simulation 6/10
Simulation 7/10
Simulation 8/10
Simulation 9/10
Simulation 10/10
Reference date  2021-09-30  Calculating Model - Steepest Descent =  DP
Simulation 1/10
Simulation 2/10
Simulation 3/10
Simulation 4/10
Simulation 5/10
Simulation 6/10
Simulation 7/10
Simulation 8/10
Simulation 9/10
Simulation 10/10
Reference date  2021-09-30  Calculating Model - Steepest Descent =  BC
Simulation 1/10
Simulation 2/10
Simulation 3/10
Simulation 4/10
Simulation 5/10
Simulation 6/10
Simulation 7/10
Simulation 8/10
Simulation 9/10
Simulation 10/10
```

Hide

```
print(bondSet)
```

	DateReference <date>	BondT… <chr>	DateMaturity <date>	Cou… <dbl>	Price… <dbl>	Yield <dbl>	CCS <dbl>	
28181	2021-09-30	LTN	2022-01-01	0.0	982.49	0.06933082	0.06933082	0.06
28428	2021-09-30	LTN	2023-01-01	0.0	895.20	0.08822809	0.09094994	0.08
28651	2021-09-30	LTN	2024-07-01	0.0	765.93	0.09684830	0.09684830	0.09
28898	2021-09-30	LTN	2025-01-01	0.0	726.37	0.09814041	0.09976925	0.09
29145	2021-09-30	LTN	2026-01-01	0.0	653.19	0.10003143	0.10003143	0.10
50823	2021-09-30	NTN-F	2023-01-01	0.1	1032.31	0.09094994	0.09094994	0.08
51070	2021-09-30	NTN-F	2025-01-01	0.1	1017.81	0.09976925	0.09976925	0.09
51317	2021-09-30	NTN-F	2027-01-01	0.1	999.93	0.10338872	0.10338872	0.10
51564	2021-09-30	NTN-F	2029-01-01	0.1	982.28	0.10560073	0.10560073	0.10
51811	2021-09-30	NTN-F	2031-01-01	0.1	962.08	0.10789482	0.10789482	0.10

1-10 of 10 rows | 1-9 of 17 columns

Hide

```
NA
```

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```
################################################################################
## KERNEL RIDGE MODEL

# Generate price vector

cat("Reference date ",format(dateReference,"%Y-%m-%d")," Calculating Model - Kernel Ridge\n")
```

```
Reference date  2021-09-30  Calculating Model - Kernel Ridge
```

Hide

```
# Generate price vector
priceVector <- as.matrix(bondSet$PricePU)

# Generate cashflow matrix
maxYearsToMaturity <- as.integer(trunc(as.numeric(max(bondSet$DateMaturity)-dateReference)/da
ysYear+1))
cashflowMatrix <- matrix(0, nrow=nrow(bondSet),ncol=maxYearsToMaturity*daysYear)

## Load bonds cashflow
for (i in 1:nrow(bondSet)) {
  dateCoupon <- rev(seq(from=bondSet$DateMaturity[i],to=dateReference, by="-6 month"))
  timeToCoupon <- as.numeric(dateCoupon-dateReference)
  for (j in 1:length(dateCoupon)) {
    cashflowMatrix[i,timeToCoupon[j]] <- (trunc(j/length(dateCoupon))+bondSet$Coupon[i]/2)*fa
ceValue
  } #end-for
} # end-for

# View reduced cashflow matrix
print(as.vector(priceVector))
```

```
 [1]  982.49  895.20  765.93  726.37  653.19 1032.31 1017.81  999.93  982.28
[10]  962.08
```

Hide

```
print(ReduceSparseMatrix(cashflowMatrix))
```

93 <dbl>	274 <dbl>	458 <dbl>	639 <dbl>	823 <dbl>	1005 <dbl>	1189 <dbl>	1370 <dbl>	1554 <dbl>	1735 <dbl>
1000	0	0	0	0	0	0	0	0	0
0	0	1000	0	0	0	0	0	0	0
0	0	0	0	0	1000	0	0	0	0
0	0	0	0	0	0	1000	0	0	0
0	0	0	0	0	0	0	0	1000	0
50	50	1050	0	0	0	0	0	0	0
50	50	50	50	50	50	1050	0	0	0
50	50	50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50	50	50

1-10 of 10 rows | 1-10 of 19 columns

Hide

```
# converts objects from R to Python
priceVector <- r_to_py(priceVector)
cashflowMatrix <- r_to_py(cashflowMatrix)

# Compute fitted KR curve in python
g_hat <- KRcurve(dateReference,priceVector,cashflowMatrix,maxYearsToMaturity)[[1]]
bondSet$KR <- FitKR(cashflowMatrix,g_hat,dateReference)[[2]]

################################################################################

print(bondSet)
```

	DateReference <date>	BondT… <chr>	DateMaturity <date>	Cou… <dbl>	Price… <dbl>	Yield <dbl>	CCS <dbl>	
28181	2021-09-30	LTN	2022-01-01	0.0	982.49	0.06933082	0.06933082	0.06
28428	2021-09-30	LTN	2023-01-01	0.0	895.20	0.08822809	0.09094994	0.08
28651	2021-09-30	LTN	2024-07-01	0.0	765.93	0.09684830	0.09684830	0.09
28898	2021-09-30	LTN	2025-01-01	0.0	726.37	0.09814041	0.09976925	0.09
29145	2021-09-30	LTN	2026-01-01	0.0	653.19	0.10003143	0.10003143	0.10
50823	2021-09-30	NTN-F	2023-01-01	0.1	1032.31	0.09094994	0.09094994	0.08
51070	2021-09-30	NTN-F	2025-01-01	0.1	1017.81	0.09976925	0.09976925	0.09
51317	2021-09-30	NTN-F	2027-01-01	0.1	999.93	0.10338872	0.10338872	0.10
51564	2021-09-30	NTN-F	2029-01-01	0.1	982.28	0.10560073	0.10560073	0.10
51811	2021-09-30	NTN-F	2031-01-01	0.1	962.08	0.10789482	0.10789482	0.10

1-10 of 10 rows | 1-9 of 18 columns

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```
NA
```

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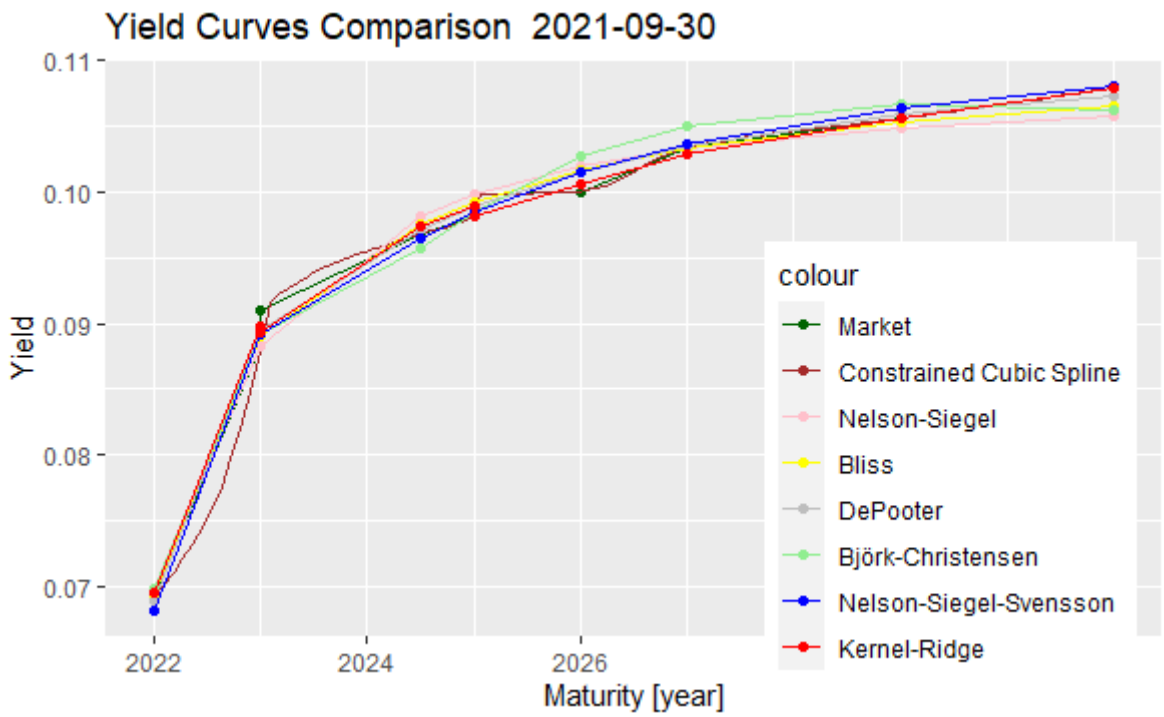
```
# Plot model curves curve
ggplot(data=bondSet,aes(x=DateMaturity))+
  geom_point(data=bondSet,aes(y=Yield,colour="Market")) +  geom_line(data=bondSet,aes(y=Yiel
d,colour="Market"))+
  geom_line(data=CCScurve,aes(y=CCS,colour="Constrained Cubic Spline"))+
  geom_point(data=bondSet,aes(y=NS,colour="Nelson-Siegel"))+geom_line(aes(y=NS,colour="Nelson
-Siegel"))+
  geom_point(data=bondSet,aes(y=BL,colour="Bliss"))+geom_line(aes(y=BL,colour="Bliss"))+
  geom_point(data=bondSet,aes(y=DP,colour="DePooter"))+geom_line(aes(y=DP,colour="DePooter"))
+
  geom_point(data=bondSet,aes(y=BC,colour="Björk-Christensen"))+geom_line(aes(y=BC,colour="Bj
örk-Christensen"))+
  geom_point(data=bondSet,aes(y=NSS,colour="Nelson-Siegel-Svensson"))+geom_line(aes(y=NSS,col
our="Nelson-Siegel-Svensson"))+
  geom_point(data=bondSet,aes(y=KR,colour="Kernel-Ridge"))+geom_line(aes(y=KR,colour="Kernel-
Ridge"))+
  theme(legend.position = c(0.8, 0.3),
        legend.direction = "vertical") +
  scale_colour_manual(
    breaks = c("Market","Constrained Cubic Spline","Nelson-Siegel","Diebold-Piazzesi-Rudebus
h",
               "Bliss","DePooter","Björk-Christensen","Nelson-Siegel-Svensson","Kernel-Ridg
e"),
    values = c("darkgreen","brown","pink","orange","yellow","grey","lightgreen","blue","re
d"))+
  labs(title = paste("Yield Curves Comparison ",format(dateReference,"%Y-%m-%d")),
       y="Yield",
       x="Maturity [year]")
```



Hide

```
##########################################################################################################
#######
```