27.1 The Electron Current

As you connect 2 capacitor plates with a metal wire, the plates become neutral. The capacitor has been <u>discharged!</u>

- the net charge of each plate decreases

Current: the motion of charges

Charge Carriers: charges that move in a conductor

Recall, when atoms come together to form a solid, the outer electrons become detached from their parent nuclei to form a fluid-like <u>sea of electrons</u> that can move through a solid

- Electrons are the charge carriers in metals

- <u>No</u> net motion!

Can create net motion by pushing this sea of electrons with an electric field

- this net motion is called the <u>drift speed</u> v_d

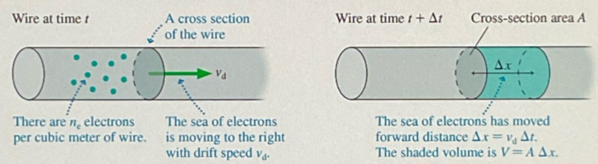
Electron Current (i_e): <u>the number of electrons per second</u> that pass through a cross section of a wire or another conductor

- units: s^{-1}

$N_e = i_e \Delta t$ → number N_e of electrons that pass through the cross section during time interval Δt

- the electron current depends on the electrons' drift speed

FIGURE 27.4 The sea of electrons moves to the right with drift speed v_d.



$$N_e = \underbrace{n_e}_{\text{number density of conduction electrons}} \underbrace{V}_{\text{volume}} = n_e \underbrace{A}_{\text{area of cross section}} \underbrace{\Delta x}_{\text{distance}} = n_e A \underbrace{v_d}_{\text{drift speed}} \underbrace{\Delta t}_{\text{time interval}}$$

Electron Current in the Wire: $i_e = n_e A v_d$

To increase the current, make them move faster, increase the amount per cubic meter, or increase the size of the pipe they're flowing through

Conduction-electron density in metals:

Metal	Electron density (m^{-3})
Aluminum	18×10^{28}
Iron	17×10^{28}
Copper	8.5×10^{28}
Gold	5.9×10^{28}
Silver	5.8×10^{28}

Example 27.1: The size of the electron current

What is the electron current in a 2.0 mm diameter copper wire if the electron speed is $1.0 \cdot 10^{-4}$ m/s?

$$i_e = n_e A v_d = (8.5 \cdot 10^{28}\,m^{-3})(\pi(1.0 \cdot 10^{-3}\,m)^2)(1.0 \cdot 10^{-4}\,m/s) = 2.7 \cdot 10^{19}\,s^{-1}$$

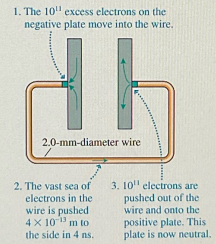
How long does it take to discharge a capacitor?

A fairly typical drift speed of the electron current through a wire is 10^{-4} m/s

But, the discharge of a capacitor is instantaneous!

Wire that connects the capacitor already has electrons.

FIGURE 27.6 The sea of electrons needs only a minuscule rearrangement.



So as soon as excess electrons move from the (−) plate to the wire, an equal number of electrons at the other end of the wire transfer to the (+) plate, thus neutralizing it.

27.2 Creating a Current

As the sea of electrons move through the conductor, collisions between the electrons and the atoms of the metal transform the electrons' kinetic energy into the thermal energy of the metal, making the metal warmer.

Recall from Ch 24,

$\vec{E} = \vec{0}$ inside a conductor in electrostatic equilibrium

a conductor with electrons moving through it is not in electrostatic equilibrium

An electron current is a net motion of charges sustained by an internal electric field

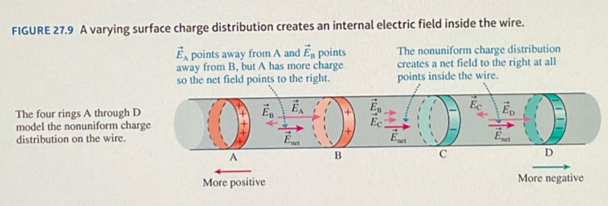


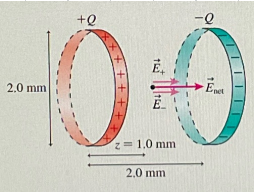
FIGURE 27.9 A varying surface charge distribution creates an internal electric field inside the wire.

A nonuniform distribution of surface charges along a wire creates a net electric field inside the wire that points from the more positive end of the wire toward the more negative end of the wire. This is the internal electric field $\vec{E}$ that pushes the electron current through the wire.

Example 27.2: The surface charge on a current-carrying wire

Two 2.0 mm diameter rings are 2.0 mm apart. They are charged to $\pm Q$. What value of Q causes the electric field at the midpoint to be 0.010 V/m?

★ Typical electric field strength in a current-carrying wire: 0.01 N/C = 0.01 V/m



Looking at the model, both contribute equally to the field strength,

electric field strength of the positive ring is $E_+ = 0.0050$ V/m

the distance $z = 1.0$ mm is half the ring spacing

Recall from Ch 23,

$$E_+ = \frac{1}{4\pi\varepsilon_0}\frac{zQ}{(z^2+R^2)^{3/2}}$$

So charge required is...

$$Q = \frac{4\pi\varepsilon_0 (z^2+R^2)^{3/2}}{z} E_+$$

$$= \frac{((0.0010\,m)^2+(0.0010\,m)^2)^{3/2}}{(9.0\cdot10^9\,Nm^2/C^2)(0.0010\,m)}(0.0050\,V/m)$$

$$Q = 1.6\cdot10^{-18}\,C$$

average electron speed at room temperature: 10^5 m/s

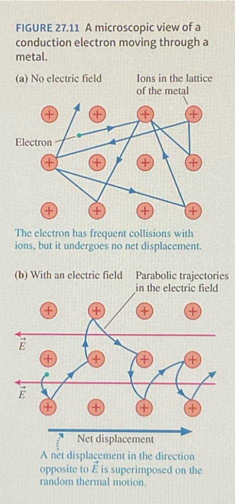


FIGURE 27.11 A microscopic view of a conduction electron moving through a metal.

a steady electric force causes the electrons to move along parabolic trajectories between collisions

these trajectories cause the (-) electrons to drift slowly in the direction of the electric field

Acceleration of the electron between collisions: $a_x = \frac{F}{m} = \frac{eE}{m}$

X component of electron's velocity: $v_x = v_{ox} + a_x \Delta t = v_{ox} + \frac{eE}{m}\Delta t$

The electron speeds up, with increasing kinetic energy, until its next collision with an ion.

The collision transfers much of the electron's kinetic energy of the ion and thus to the thermal energy of the metal.

This energy transfer is the "friction" that raises the temperature of the wire.

The magnitude of the electron's average velocity, due to the electric field, is the drift speed v_d of the electron

Avg Velocity: $v_d = \overline{v_x} = \overline{v_{ox}} + \frac{eE}{m}\overline{\Delta t}$

Avg value of Δt is the mean time between collisions, can be considered as a constant (τ)

Avg Speed in which the electrons are pushed along by the electric field is: $v_d = \frac{e\tau}{m}E$

Electron Current (Updated): $i_e = n_e A v_d = \frac{n_e e \tau A}{m} E$

The electron current is directly proportional to the electric field strength

Example 27.3: Collisions of a Copper Wire

In a 2.0 mm diameter wire with a drift speed of $1.0 \cdot 10^{-4}$ m/s, the electron current = $2.7 \cdot 10^{19}\ s^{-1}$. If an internal electric field of 0.020 V/m is needed to sustain this current, a typical value, how many collisions/sec, on average, do the electrons in copper undergo?

$$\tau = \frac{m v_d}{eE} = \frac{9.11 \cdot 10^{-31}\ kg\ (1 \cdot 10^{-4}\ m/s)}{1.6 \cdot 10^{-19}\ C\ (0.020\ V/m)} = 2.8 \cdot 10^{-14}\ s$$

The avg number of collisions/sec is the inverse.

Collision Rate: $\frac{1}{\tau} = 3.5 \cdot 10^{13}\ s^{-1}$

27.3 Current and Current Density

Current (I) in the wire = rate of charge flow

$$I = \frac{dQ}{dt}$$

For a steady current, $Q = I\Delta t$

SI unit for current = $\frac{\text{Coulomb}}{\text{second}}$ = ampere (A)

named after French scientist André Marie Ampère

"Amp" is the informal abbreviation of ampere

Typically measured in milliamps ($1\,mA = 10^{-3}A$) or microamps ($1\,\mu A = 10^{-6}A$)

$$I = \frac{Q}{\Delta t} = \frac{eN_e}{\Delta t} = \frac{e(i_e\Delta t)}{\Delta t} = ei_e$$

Electron Current (i_e), the rate at which electrons move through a wire, is more fundamental b/c it looks directly at the charge carriers

Current (I), the rate at which the charge of the electrons moves through the wire, is more practical b/c we can measure charge more easily than we can count electrons

The direction of current:

is the direction in which positive charges seem to move

(in a metal) is opposite the direction of motion of the electrons

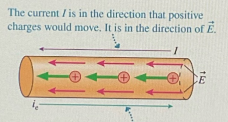


FIGURE 27.13 The current I is opposite the direction of motion of the electrons in a metal.

$$I = ei_e = n_e e v_d A \qquad (J \cdot A)$$

Current Density (J) in a wire: $\frac{I}{A} = n_e e v_d$

Units: A/m^2

Example 27.4: Finding the electron drift speed

A 1.0 A current passes through a 1.0-mm-diameter copper wire. What are the current density and the drift speed of the electrons in the wire?

$$J = \frac{I}{A} = \frac{I}{\pi r^2} = \frac{1.0\text{ A}}{\pi (0.0005\text{ m})^2} = 1.3 \cdot 10^6 \text{ A/m}^2$$

$$J = n_e e v_d$$

$\downarrow$

$$v_d = \frac{J}{n_e e} = 9.6 \cdot 10^{-5} \text{ m/s} = 0.096 \text{ mm/s}$$

Only 2 ways to decrease I:

either decrease the amount of charge

OR

decrease the charge's drift speed through the wire

The rate of electrons leaving a lightbulb (or any other device) is exactly the same as the rate of electrons entering the lightbulb. Current doesn't change.

* Due to conservation of charge, the current must be the same at all points in an individual current-carrying wire

Junction: a point where a wire branches

Kirchoff's Junction Law: $\sum I_{in} = \sum I_{out}$

$\boxed{27.4}$ Conductivity and Resistivity

Current Density: $J = n_e e v_d = n_e e \left(\frac{e\tau E}{m}\right) = \frac{n_e e^2 \tau}{m} E$

Conductivity (σ) of a material: $\sigma = \frac{n_e e^2 \tau}{m}$ Units: $\Omega^{-1} m^{-1}$

Thus, $J = \sigma E$

Resistivity (ρ) $= \frac{1}{\sigma} = \frac{m}{n_e e^2 \tau}$ Units: Ωm

(Inverse of Conductivity!)

TABLE 27.2 Resistivity and conductivity of conducting materials

Material	Resistivity (Ω m)	Conductivity (Ω^{-1}m^{-1})
Aluminum	2.8×10^{-8}	3.5×10^{7}
Copper	1.7×10^{-8}	6.0×10^{7}
Gold	2.4×10^{-8}	4.1×10^{7}
Iron	9.7×10^{-8}	1.0×10^{7}
Silver	1.6×10^{-8}	6.2×10^{7}
Tungsten	5.6×10^{-8}	1.8×10^{7}
Nichrome*	1.5×10^{-6}	6.7×10^{5}
Carbon	3.5×10^{-5}	2.9×10^{4}

*Nickel-chromium alloy used for heating wires.

$J = \sigma E$ tells us ....

1) Current is caused by an electric field exerting forces on the charge carriers

2) The current density, and hence the current $I = JA$, depends linearly on the strength of the electric field. To double the current, you must double the strength of the electric field that pushes the charges along.

3) The current density also depends on the conductivity of the material. Different conducting materials have different conductivities b/c they have different values of the electron density and, especially, different values of the mean time between electron collisions w/ the lattice of atoms.

Example 27.5: The electric field in a wire

A 2.0-mm-diameter aluminum wire carries a current of 800 mA. What is the electric field strength inside the wire?

$$E = \frac{J}{\sigma} = \frac{I}{\sigma \pi r^2} = \frac{0.80\ A}{(3.5 \cdot 10^7\ \Omega^{-1} m^{-1})\pi(0.0010\ m)^2} = 0.0073\ V/m$$

Superconductivity: complete loss of resistance at low temperatures

resistivity of a superconducting metal is truly zero

27.5 Resistance and Ohm's Law

Inside a constant diameter conductor, $E = \frac{\Delta V}{\Delta s} = \frac{\Delta V}{L}$

$$I = JA = A\sigma E = \frac{A}{\rho} E = \frac{A}{\rho L} \Delta V$$

$$I = \frac{A}{\rho L} \Delta V$$

The current is directly proportional to the potential difference between the ends of a conductor

Resistance (of a conductor): $R = \frac{\rho L}{A}$

SI unit of resistance: ohm, $1\ \text{ohm} = 1\ \Omega = 1\ V/A$

Ohm's Law: $I = \frac{\Delta V}{R}$

Example 27.6: The resistivity of a leaf

Resistivity measurements on the leaves of corn plants are a good way to assess stress and the plant's overall health. To determine resistivity, the current is measured when a voltage is applied between two electrodes placed 20 cm apart on a leaf that is 2.5 cm wide and 0.20 mm thick. The following data are obtained by using several different voltages:

Voltage (V)	Current (μA)
5.0	2.3
10.0	5.1
15.0	7.5
20.0	10.3
25.0	12.2

What is the resistivity of the leaf tissue?

$I = \frac{1}{R} \Delta V$

Current vs. ΔV table, so its slope $= \frac{1}{R}$

$$R = \frac{1}{0.5\ \mu A/V} = 2.0 \cdot 10^6 \frac{V}{A} = 2.0 \cdot 10^6\ \Omega$$

$$\rho = \frac{AR}{L} = \frac{(5.0 \cdot 10^{-6} m^2)(2.0 \cdot 10^6\ \Omega)}{0.20\ m} = 50\ \Omega m$$

1) A battery is a source of potential difference ΔV_{bat}. An ideal battery has $\Delta V_{bat} = \mathcal{E}$

2) The battery creates a potential difference $\Delta V_{wire} = \Delta V_{bat}$ between the ends of a wire

3) The potential difference ΔV_{wire} causes an electric field $E = \Delta V_{wire}/L$ in the wire

4) The electric field establishes a current $I = JA = \sigma A E$ in the wire

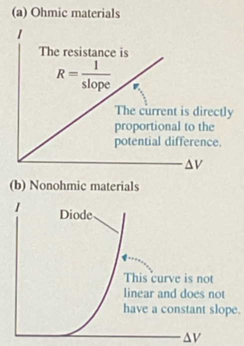
5) The magnitude of the current is determined jointly by the battery and the wire's resistance R to be $I = \Delta V_{wire}/R$

Ohm's Law is limited to ohmic materials, materials whose resistance (R) remains constant during use (metals)

Nonohmic materials: current isn't directly proportional to ΔV

Resistors: ohmic devices that limit the current in a circuit, whose resistance is larger than the metal wires

FIGURE 27.20 Current-versus-potential-difference graphs for ohmic and nonohmic materials.



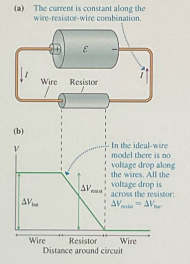
Three Classes of Ohmic Materials:

1) Wires are metals with small resistivities ρ and thus very small resistances ($R \ll 1\Omega$). An <u>Ideal wire</u> has $R = 0\Omega$; hence the potential difference between the ends of an ideal wire is $\Delta V = 0V$ even if there is a current in it.

2) Resistors are poor conductors with resistances usually in the range 10^1 to $10^6\Omega$. They are used to control a circuit. Most resistors in a circuit have a specified value of R, such as $500\,\Omega$.

3) Insulators are materials such as glass, plastic, or air. An ideal insulator has $R = \infty\Omega$; hence there is no current in an insulator even if there is a potential difference across it ($I = \Delta V/R = 0A$). This is why insulators can be used to hold apart two conductors at different potentials. All practical insulators have $R \gg 10^9\Omega$ and can be treated as ideal.

FIGURE 27.21 The potential along a wire-resistor-wire combination.



Apply Ohm's Law only to resistors

$R_{wire} \ll R_{resist}$, $\Delta V_{wire} = IR_{wire}$ between ends of each wire $\ll \Delta V_{resist} = IR_{resist}$ across the resistor

In an ideal-wire model, wires are equipotentials, and the segments of the voltage graph corresponding to the wires are horizontal

<u>Example 27.7</u>: A battery and a resistor

What resistor would have a 15 mA current if connected across the terminals of a 9.0V battery?

$\Delta V_{bat} = 9.0V = \Delta V_{resist}$

$$I = \frac{\Delta V}{R}$$

$$R = \frac{\Delta V_{resist}}{I} = \frac{9.0V}{0.015A} = 600\,\Omega$$