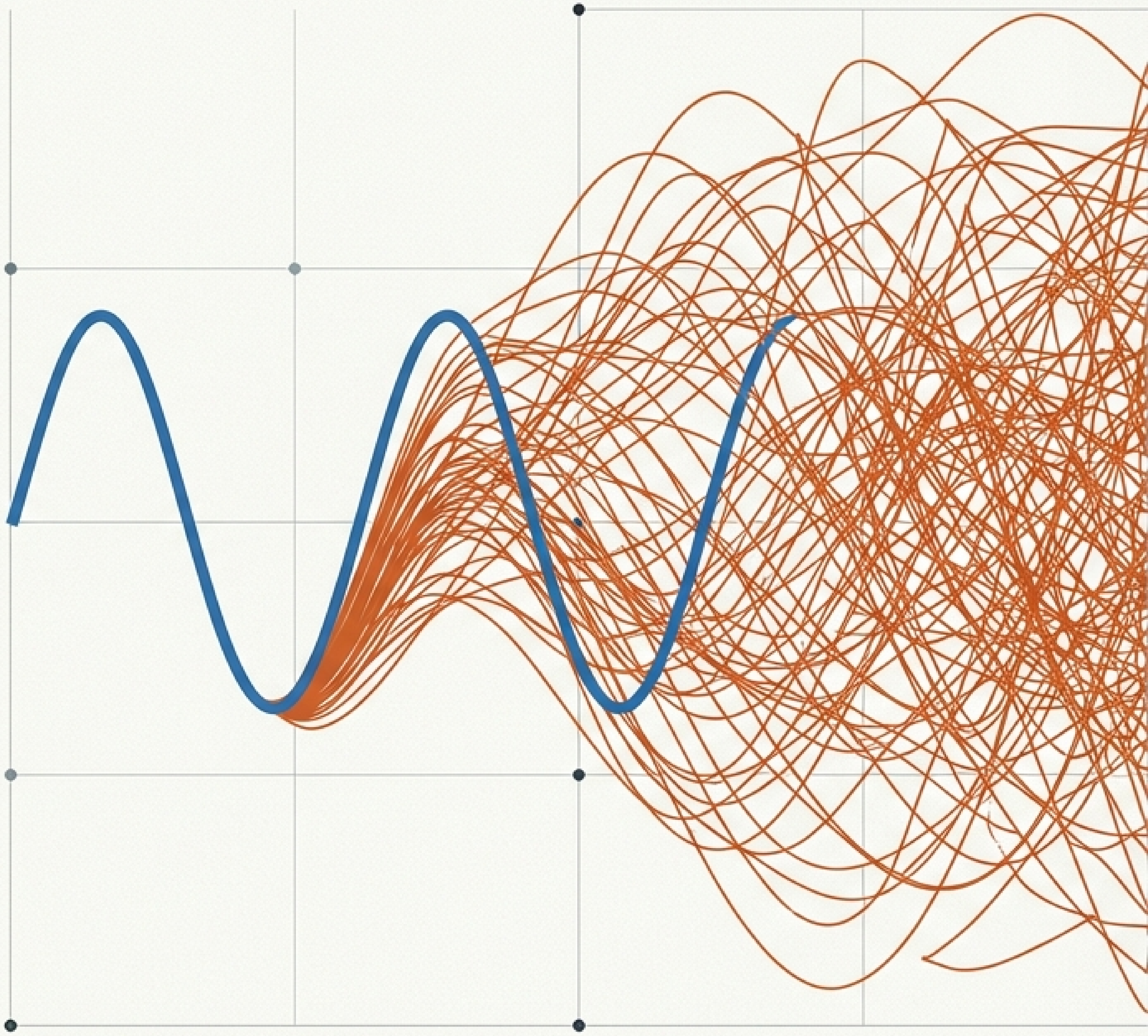
Orchestrating Nonlinearity

A Review of Decentralized Regulation & Consensus Protocols

Exploring the evolution of control theory in the works of Le Chang, Chenghui Zhang, and Xianfu Zhang.

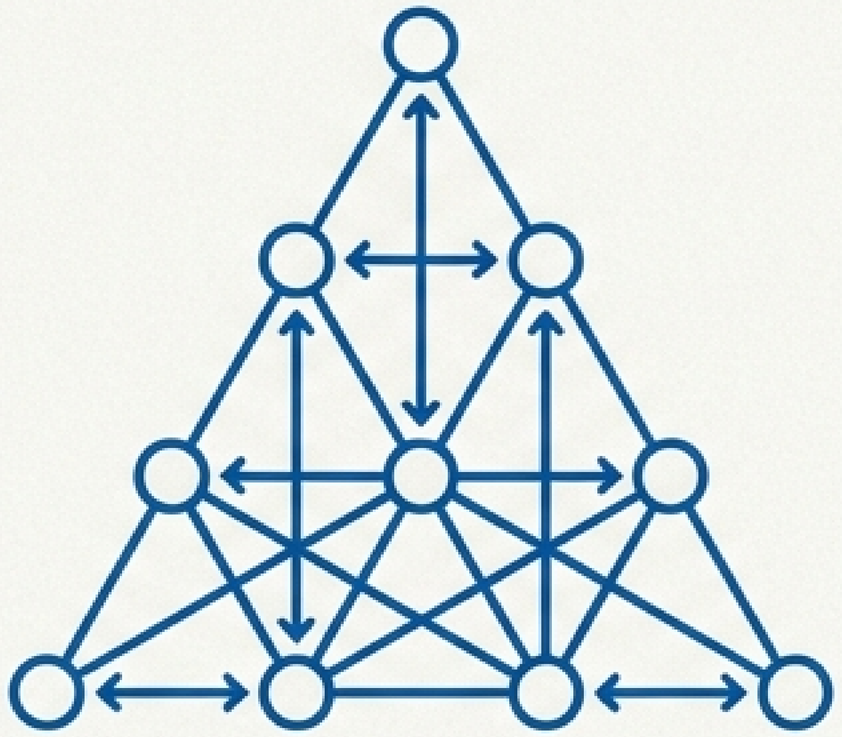
2014 & 2017 • Shandong University



The Quest for Autonomous Coordination

The Core Challenge

How do we force a group of autonomous agents—robots, UAVs, or microgrids—to follow a leader when their internal behaviors are chaotic and communication is limited?



The Protagonist

Le Chang and the research team at the School of Control Science and Engineering, Shandong University.

2014: The Foundation

Mastering complex internal dynamics (Upper-triangular) within simple, undirected networks.

2017: The Evolution

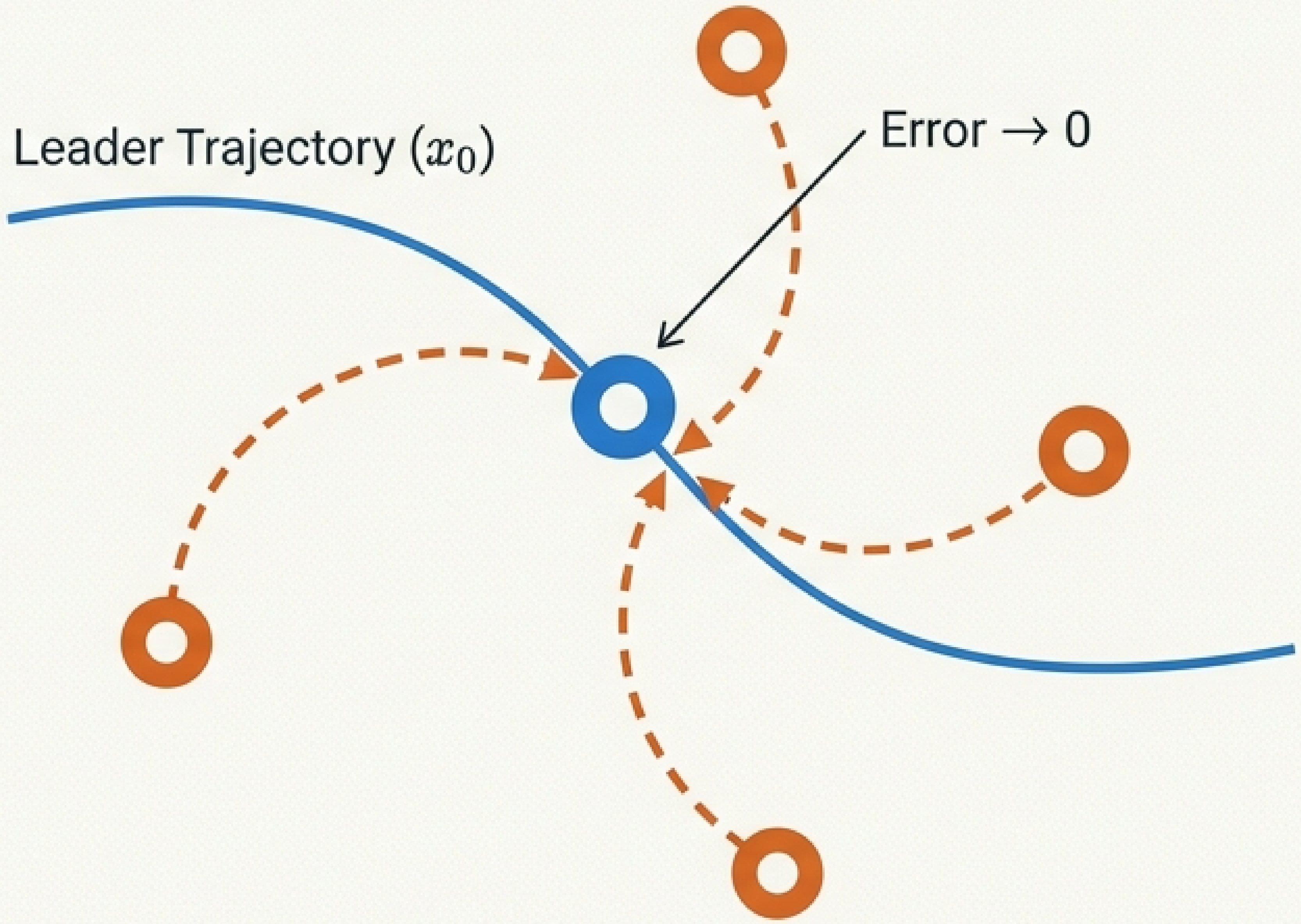
Mastering uncertain dynamics (Feedforward) within complex, directed networks.

Defining Leader-Follower Consensus

The Objective

To design a protocol u_k using local information such that all follower states converge to the leader's state.

$$\lim_{t \to \infty} \|x_k(t) - x_1(t)\| = 0$$

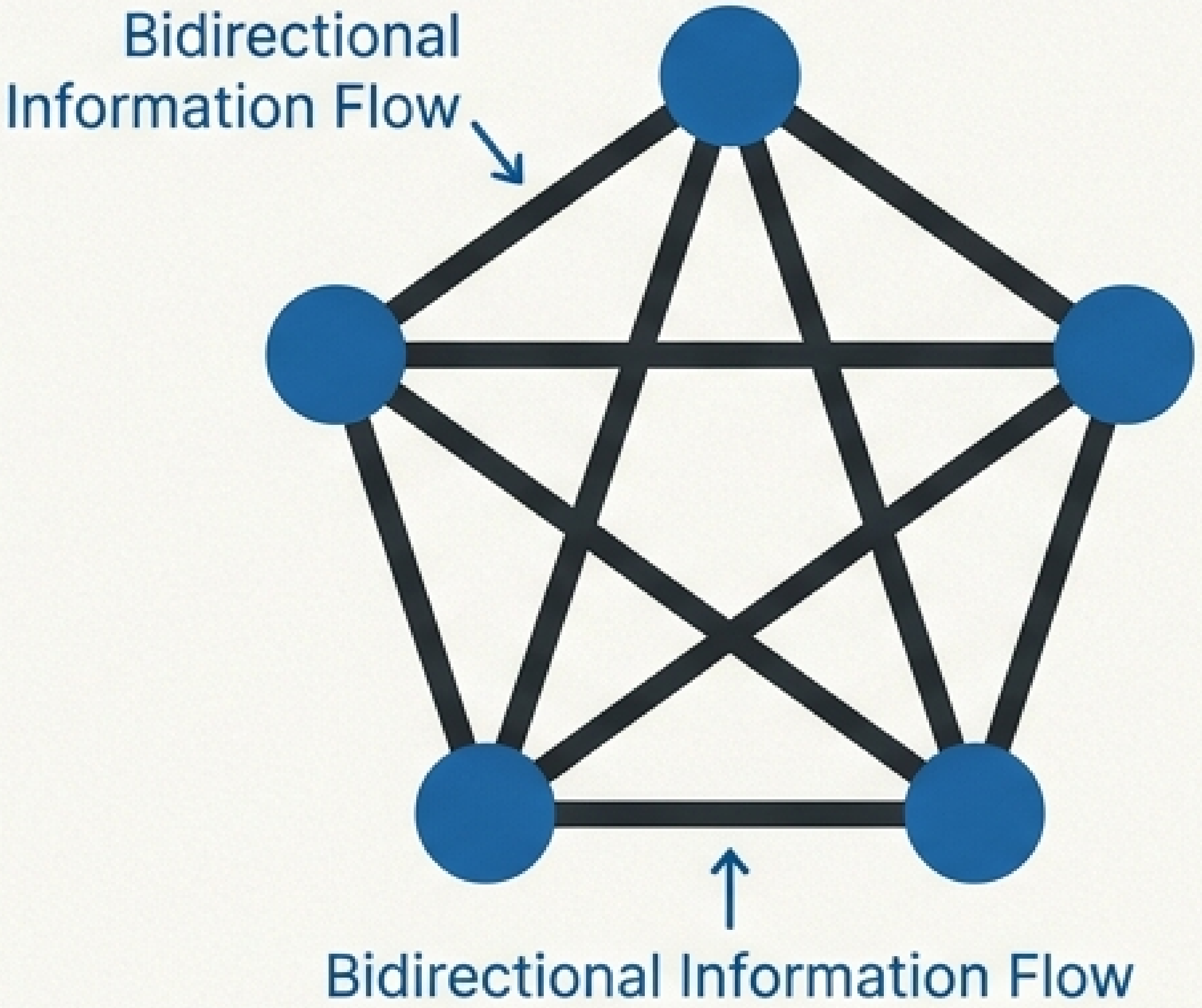


The Constraints

- **Nonlinear Dynamics:** Agents react unpredictably via function $f(\cdot)$.
- **Local Information:** Agents only see immediate neighbors, not the global view.

2014: The Upper-Triangular Foundation

Leader-follower Consensus of Upper-triangular Nonlinear Multi-agent Systems



System Type

Upper-Triangular Nonlinear Systems. The control input u_k is buried inside every nonlinear term, making standard cancellation techniques impossible.

Topology

Undirected Graph. Symmetry in communication ($A \leftrightarrow B$). The Laplacian matrix is symmetric.

The Challenge: The Lipschitz Barrier

$$\dot{x}_{k,i} = x_{k,i+1} + f_i(x_{k,i+1}, \ldots, u_k)$$

The Critical Flaw. The control input is entangled within the nonlinearity.

The Assumption: The nonlinear functions satisfy the Lipschitz condition, but because the input is nested, we need a method to **decouple this complexity without destabilizing the system.**

Unlocking Consensus via Rescaling

The Key: Rescaling Transformation

A scaling gain (L) introduces a design freedom ($E_k = H\, e_k$). If L is large enough, the controller dominates the nonlinear terms.

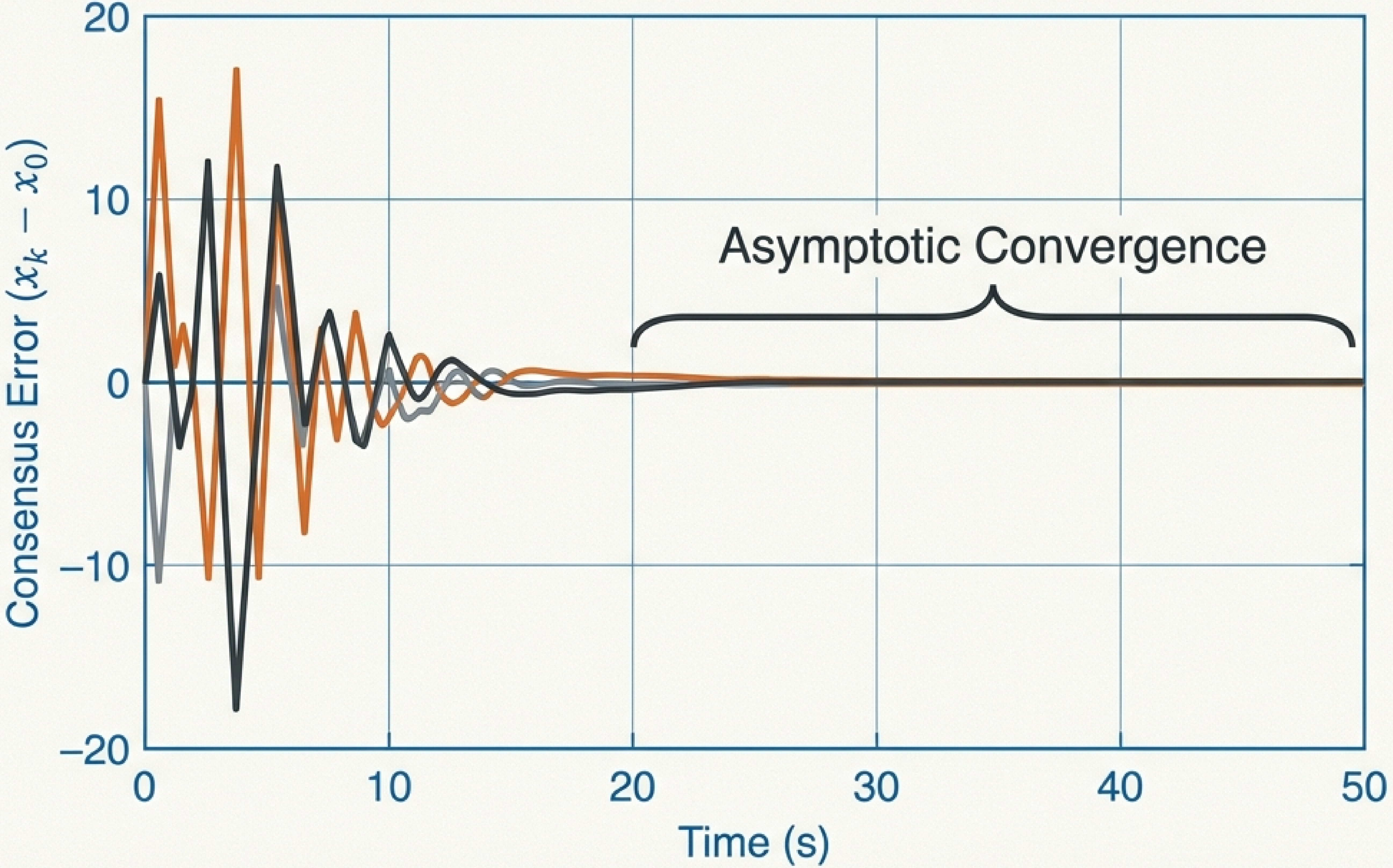
The Mechanism: Observer-Based Design

Since full states are not measurable, an observer estimates states ($\hat{x}$) based on output (y_k).

$$u_k = \frac{1}{L^n} K H \sum a_{kj} (\hat{x}_k - \hat{x}_j)$$

Scaling Gain Connection

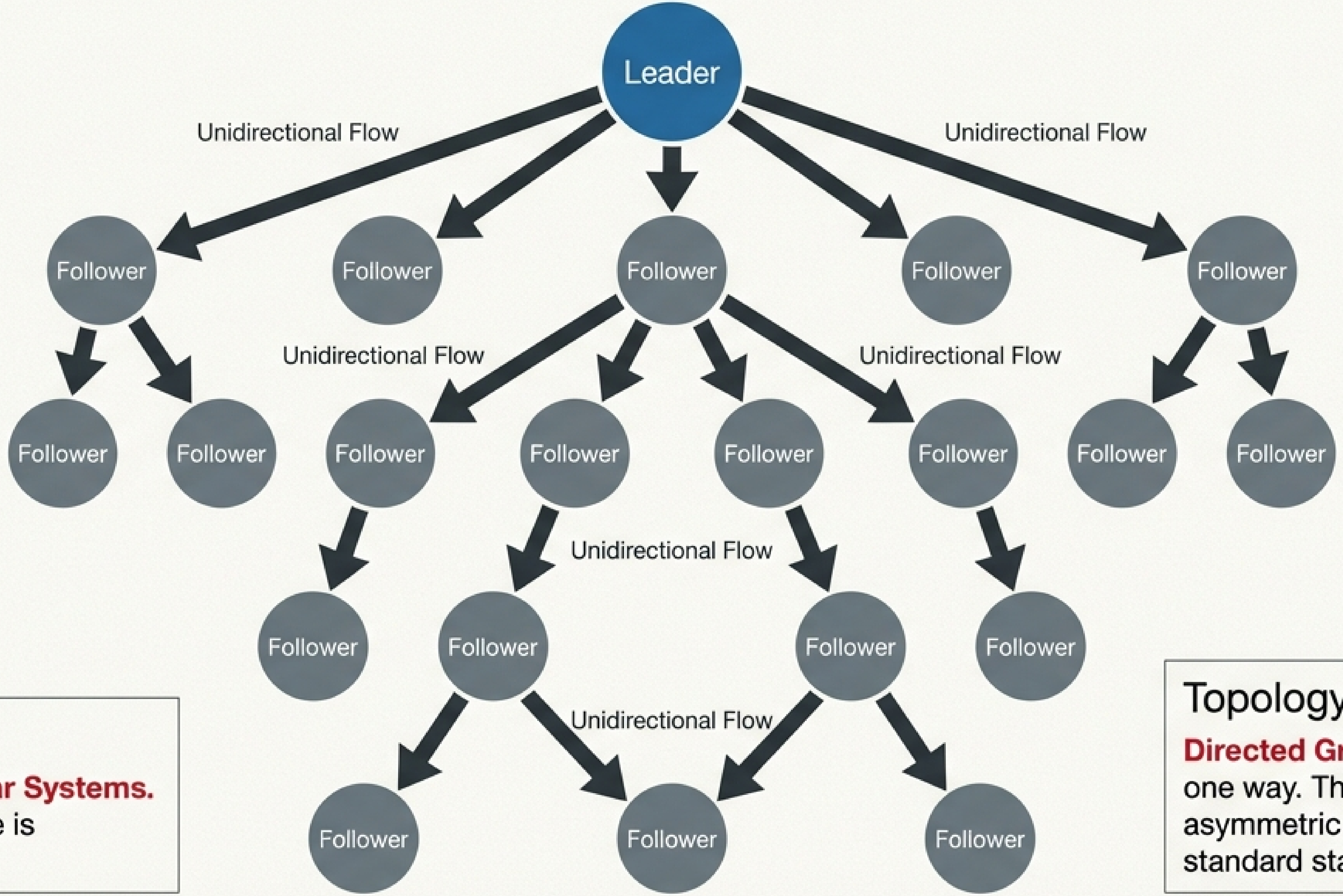
Evidence of Stabilization (2014)



- **Setup:** 4 Agents (1 Leader, 3 Followers).
- **Dynamics:** Included $\sin^2(u_k)$ terms.
- **Result:** Global stabilization achieved despite entangled input nonlinearities.

2017: The Directed Feedforward Evolution

Decentralised regulation of nonlinear multi-agent systems with directed network topologies



System Type

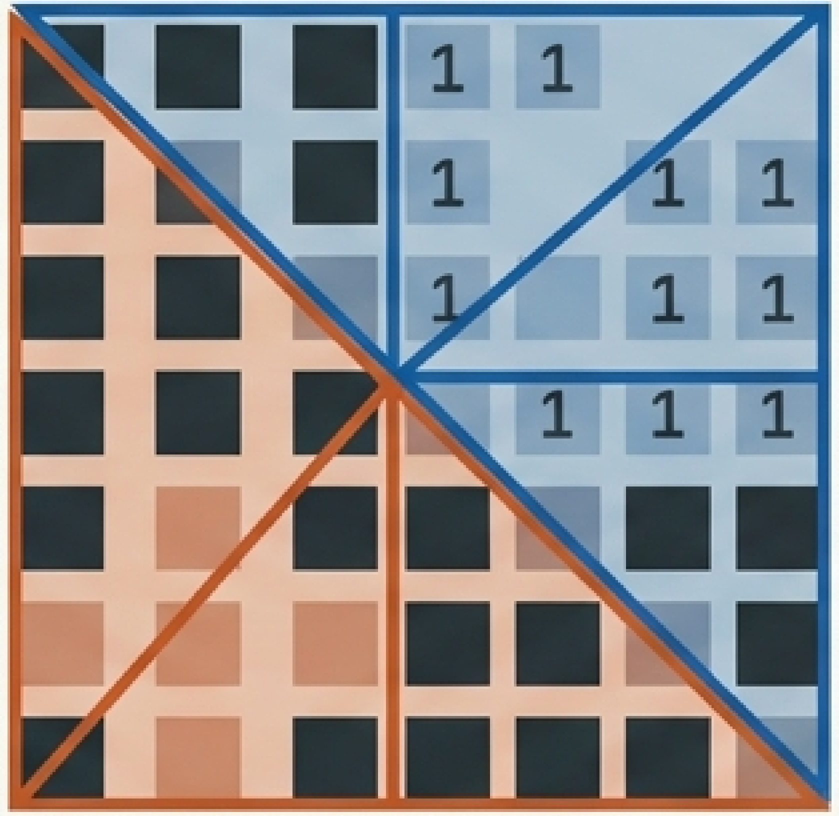
Feedforward Nonlinear Systems. The system growth rate is **unknown** a *priori.*

Topology

Directed Graph. Information flows one way. The Laplacian matrix is asymmetric ($\mathcal{L} \neq \mathcal{L}^T$), breaking standard stability proofs.

Navigating Uncertainty and Asymmetry

The Asymmetric Laplacian



In directed graphs, symmetry is lost. We must utilize properties of **M-matrices** to guarantee stability.

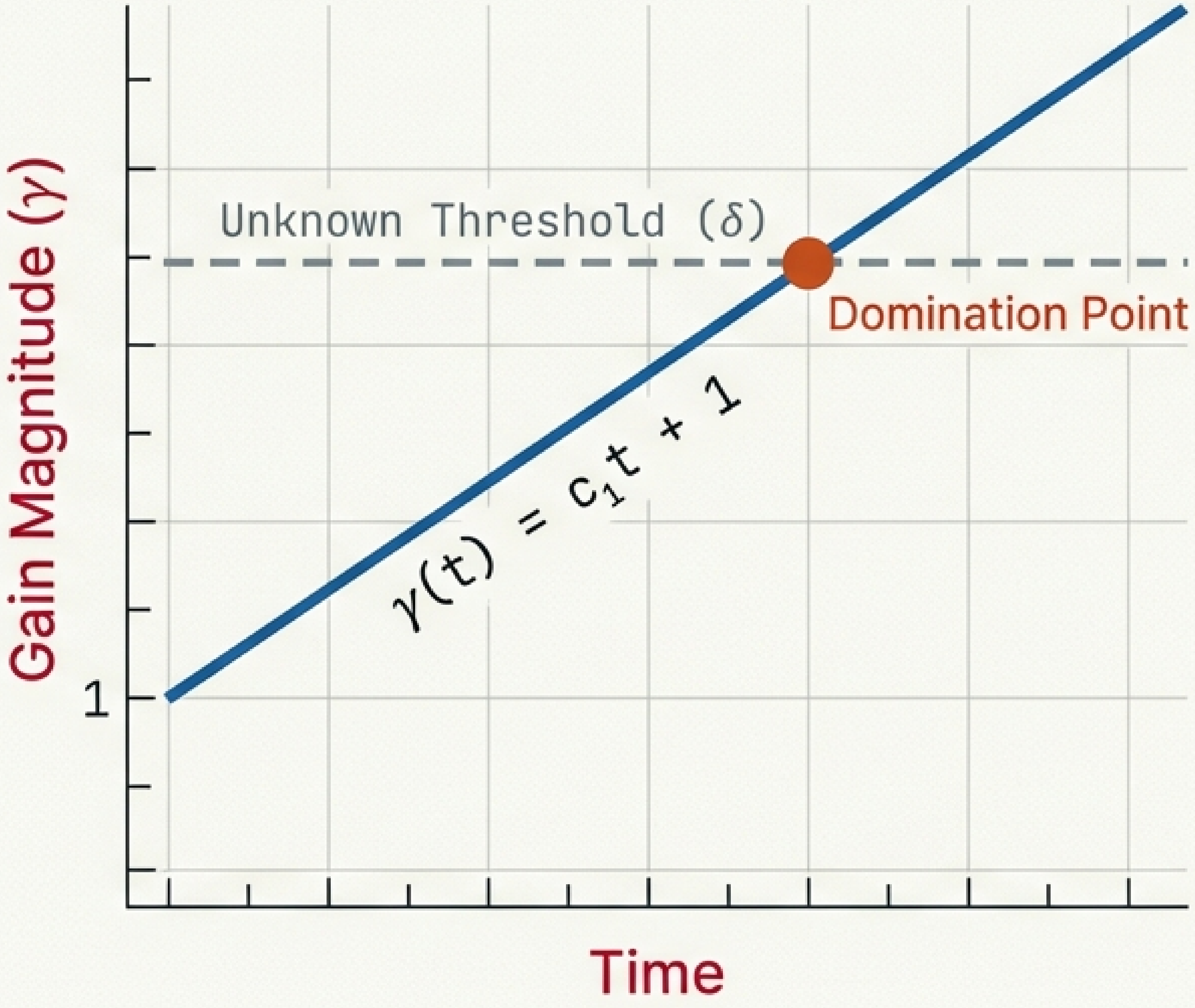
The Unknown Growth Rate

$$f(x) \leq \delta ||x||$$

The nonlinearity is bounded by an unknown constant δ.

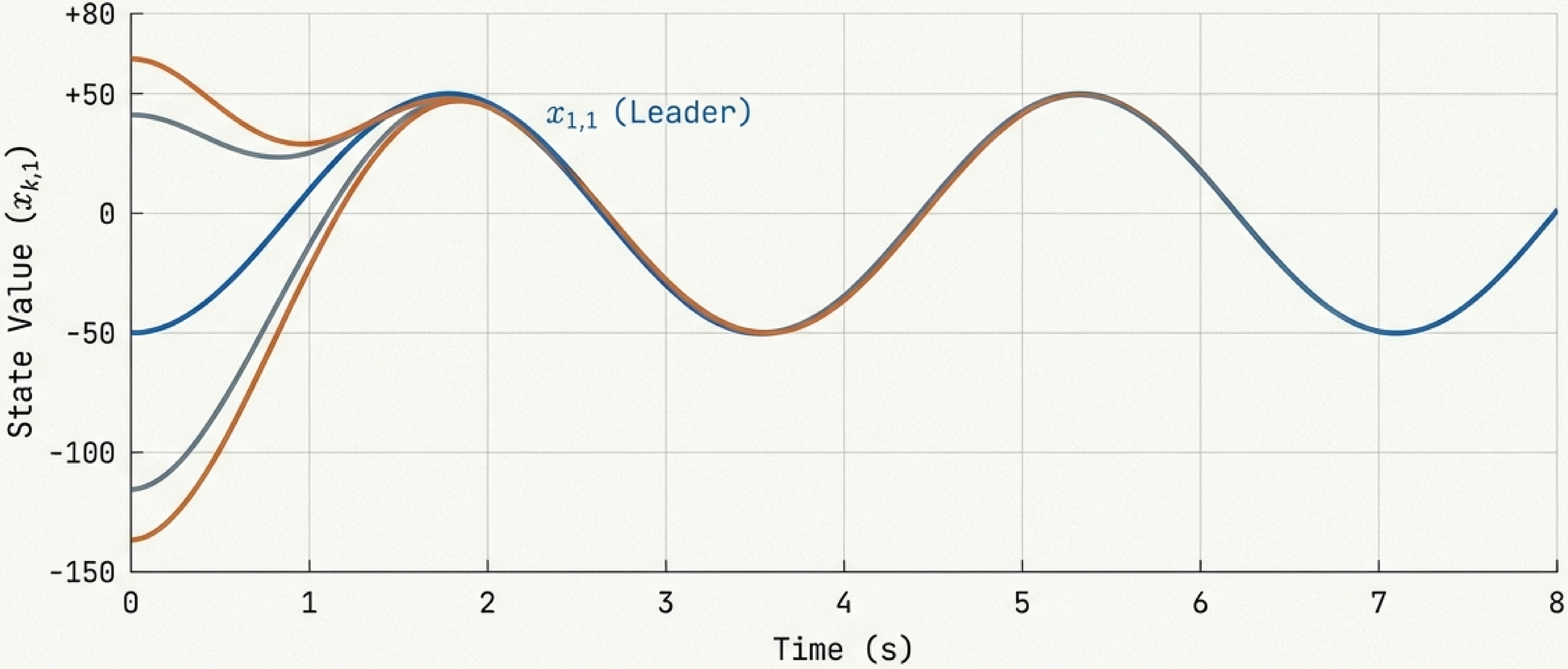
> Standard high-gain methods fail because we don't know how high the gain needs to be.

The Solution: Dynamic Low-Gain Feedback



- **Start Low:** Begin with small gain to avoid amplifying noise.
- **Grow Dynamically:** The gain adapts linearly over time.
- **Guarantee:** Eventually, $\gamma(t)$ surpasses the unknown growth rate δ, ensuring convergence.

Global Convergence in Directed Networks (2017)

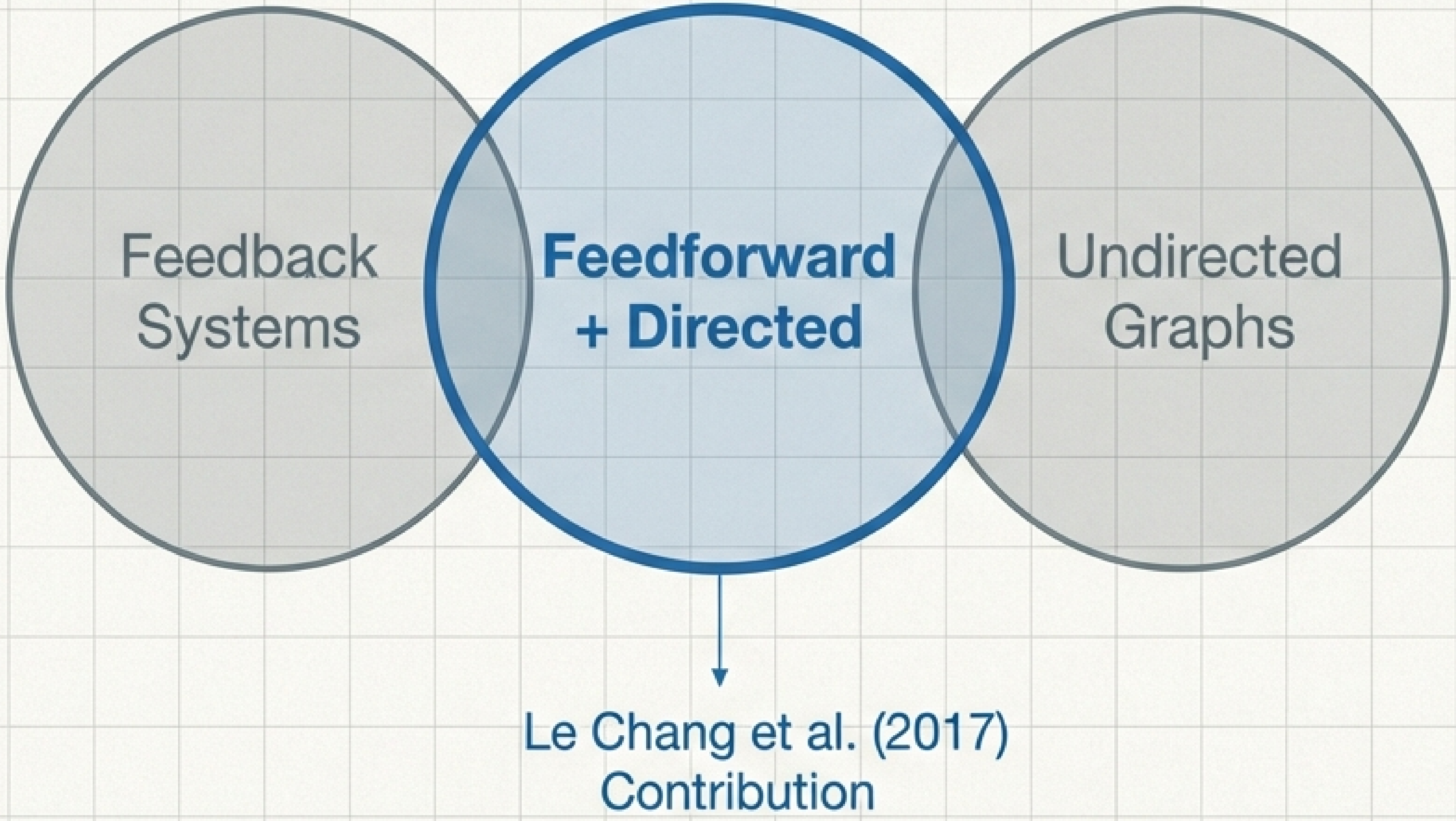


5 Agents in a Directed Spanning Tree topology. Perfect synchronization achieved despite dead-zones and unknown growth rates.

A Tale of Two Systems

Feature	2014 Approach	2017 Approach
System Form	Upper-Triangular (Input entangled)	Feedforward (Input drives chain)
Topology	Undirected (Bidirectional)	Directed (Unidirectional)
Uncertainty	Lipschitz (Known Constraints)	Growth Rate (Unknown δ)
Key Tool	Rescaling Transformation (L)	Dynamic Low-Gain ($\gamma(t)$)

Filling the Gap in Control Theory



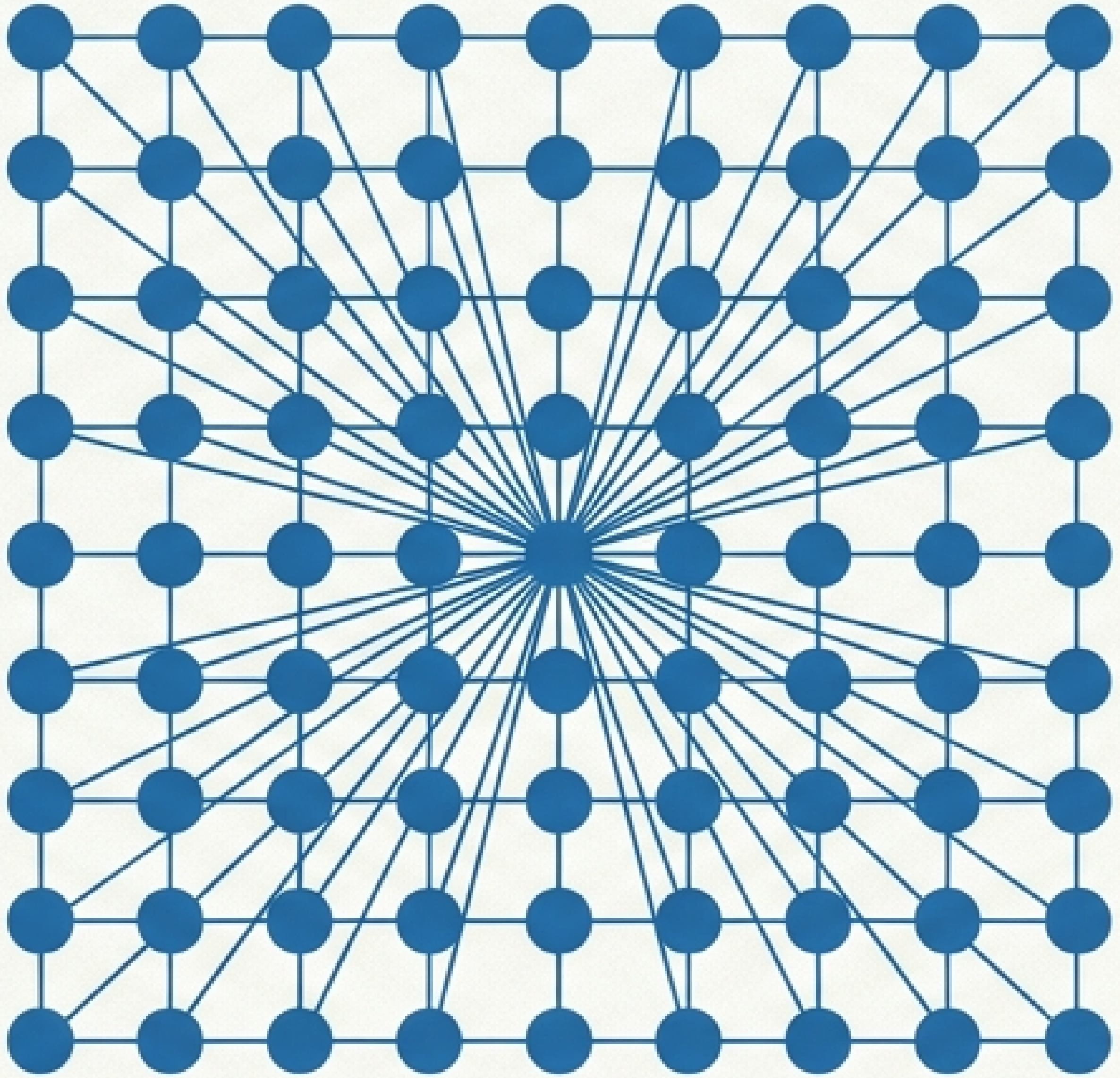
Theoretical Rigor: Proved stability using M-matrix properties and nonlinear estimation.

Practicality: Addressed one-way communication common in sensor networks.

Versatility: Solutions provided for both full-state feedback and observer-based output feedback.

From Chaos to Consensus

Le Chang's work demonstrates that global consensus is mathematically guaranteed, even when agents behave chaotically, communication is restricted, and system parameters are unknown.



References

Le Chang, Chenghui Zhang, Xianfu Zhang & Xiandong Chen (2017). Decentralised regulation of nonlinear multi-agent systems with directed network topologies. *International Journal of Control*, 90:11, 2338-2348.

Chenghui Zhang, Le Chang, Xianfu Zhang (2014). Leader-follower Consensus of Upper-triangular Nonlinear Multi-agent Systems. *IEEE/CAA Journal of Automatica Sinica*, 1(2), 210-217.