Prescribed-Time Group Consensus for Heterogeneous Multi-Agent Systems

Resolving the Unknown Membership Dilemma via Deterministic Control Probing

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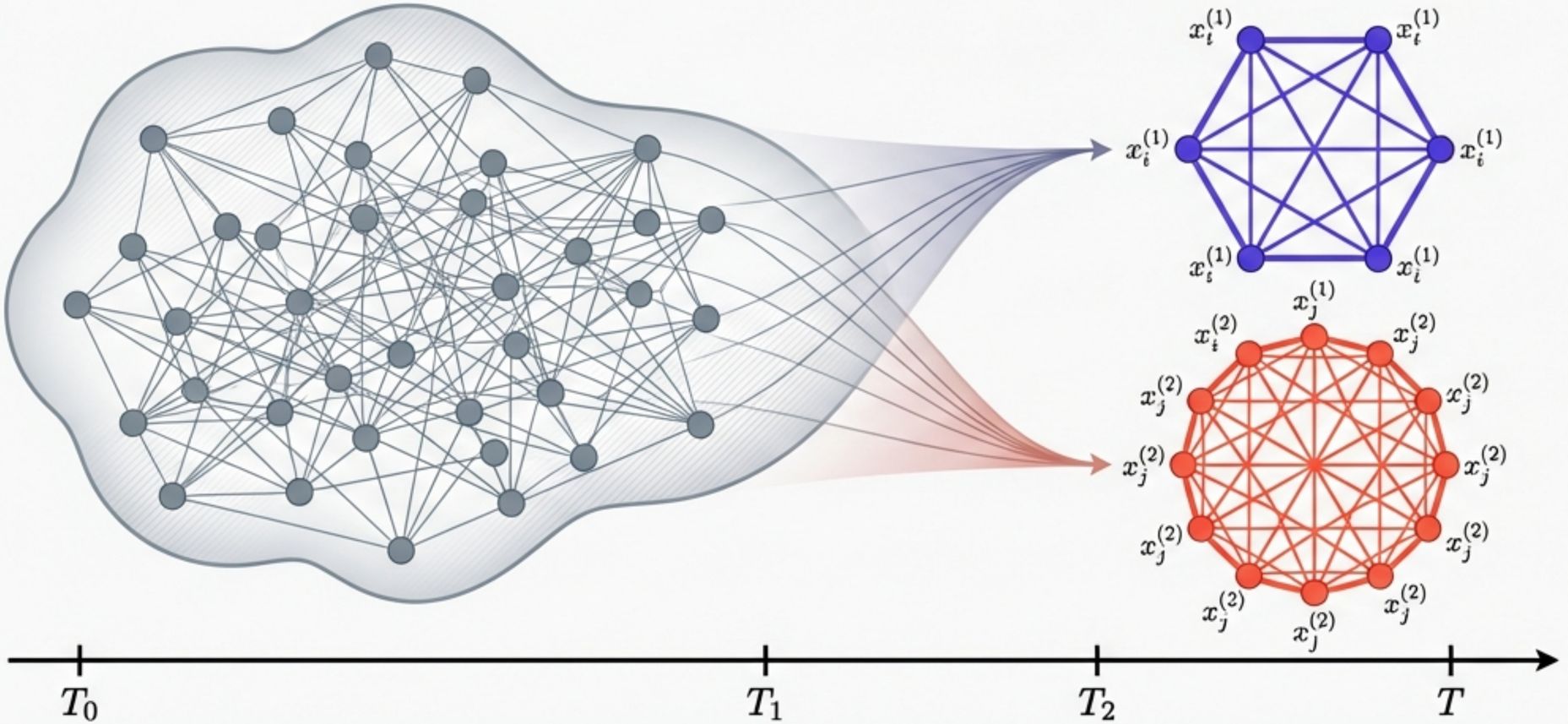

THE CHALLENGE:

Coordinating agents with unknown dynamics.

THE SOLUTION:

Time-constrained deterministic probing.

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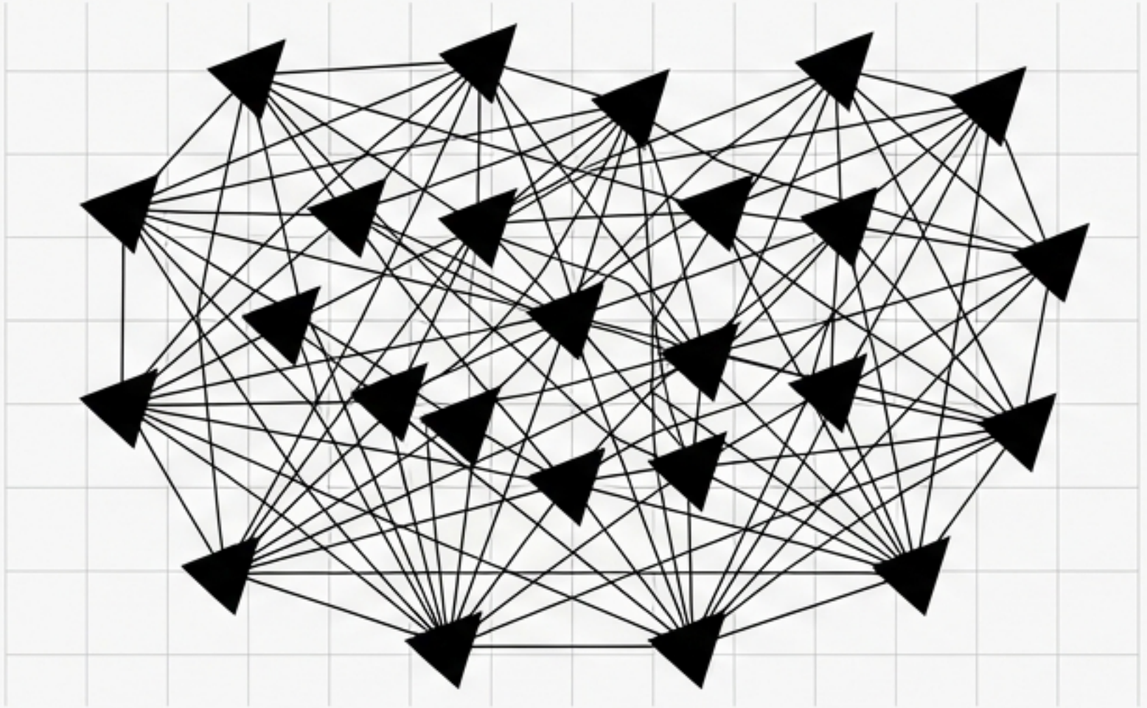
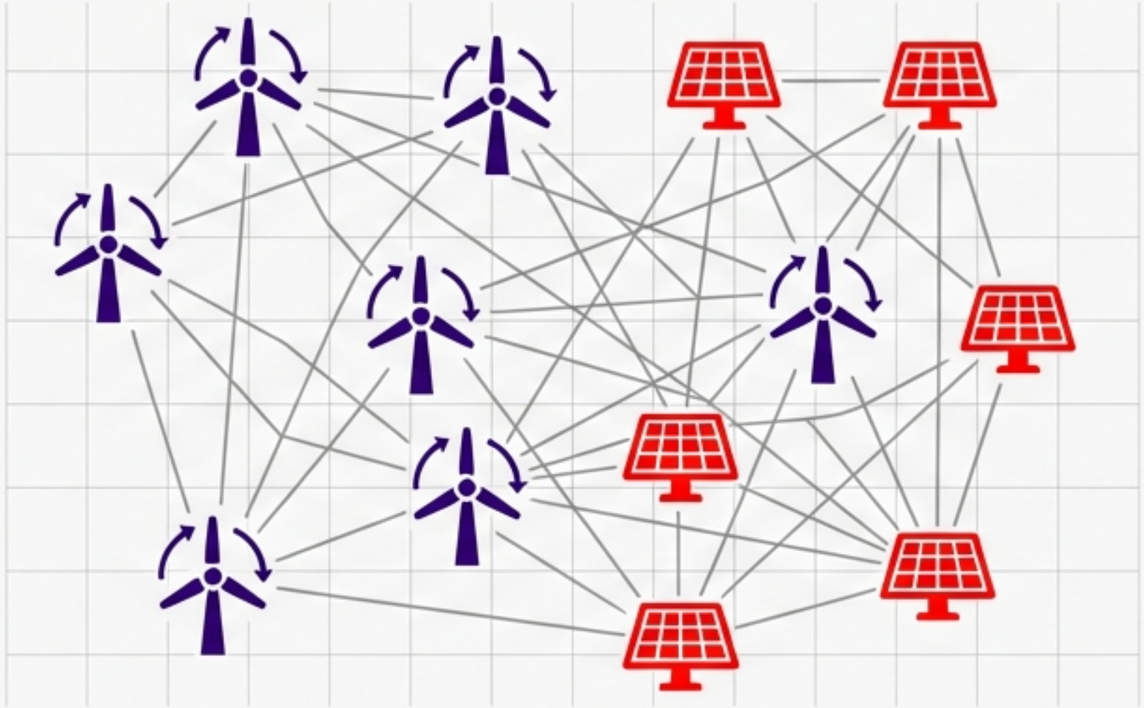
Beyond Global Synchronization: The Challenge of Heterogeneity

Global Consensus



Homogeneous Dynamics: Everyone does the same thing.

Group Consensus



Heterogeneous Dynamics: Agents must sync with their own kind, despite mixed connections.

CONSTRAINT: Agents communicate with neighbors but do not know if those neighbors share their dynamics (Group Ω_1 vs Ω_2).

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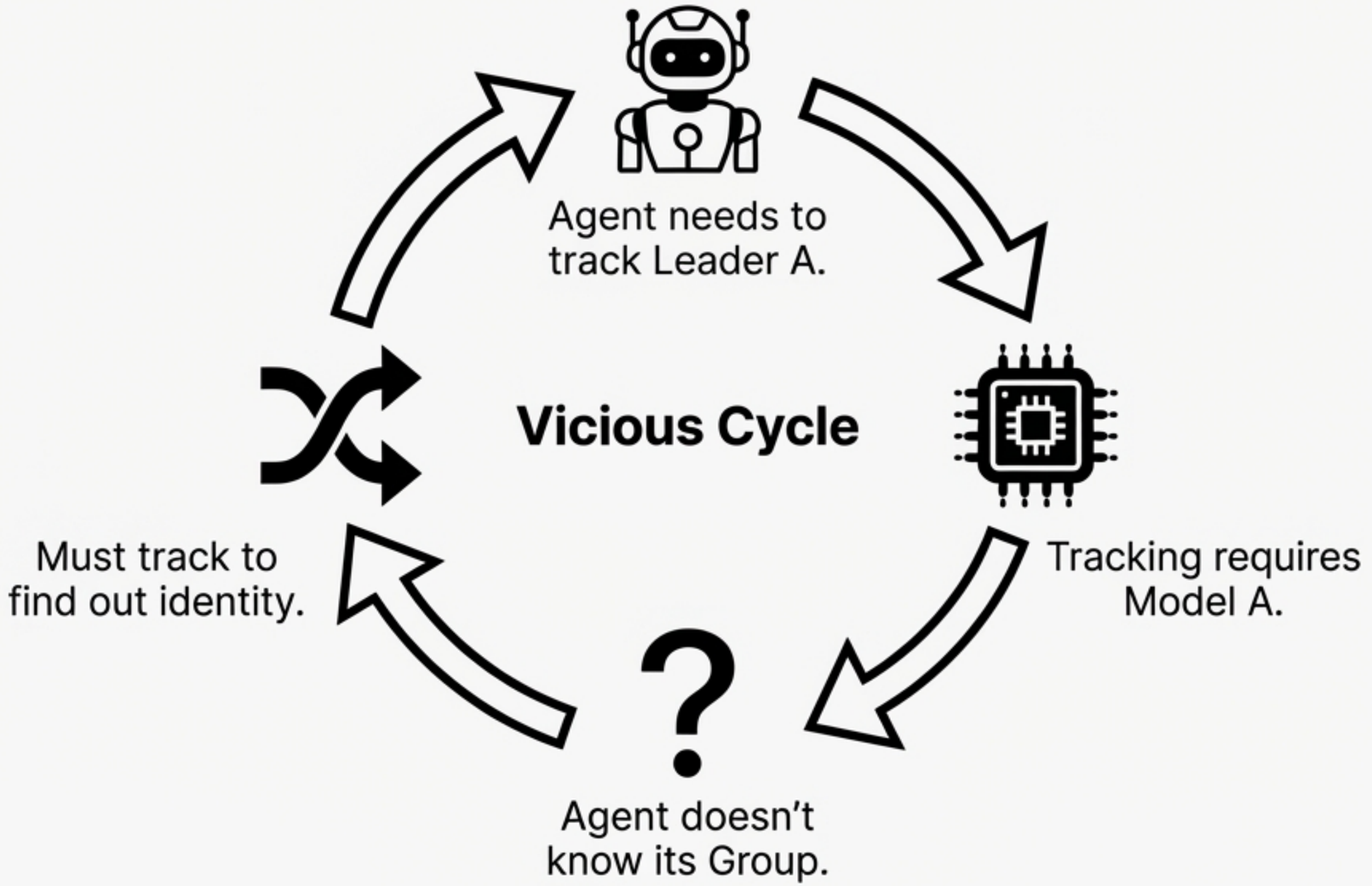
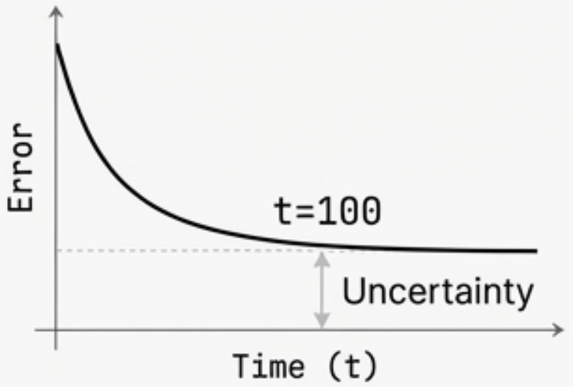
The Identification Paradox



Why Standard Control Fails



In finite time, is a non-zero error a mismatch or just lag?

Asymptotic convergence cannot tell us with 100% certainty.

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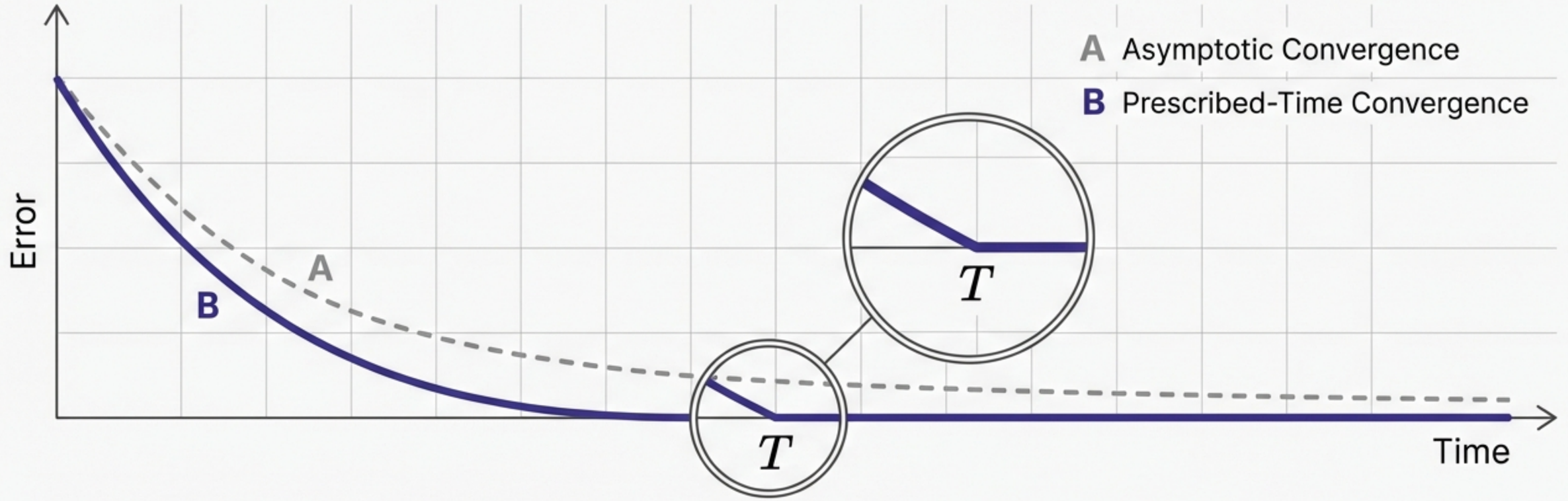
The Solution: Prescribed-Time Control

Using performance as a deterministic probe.



$$\lim_{t \to T} \|Error\| = 0$$

THE STRATEGY: If we force the system to converge by time T, any failure to hit zero is absolute proof of a model mismatch.

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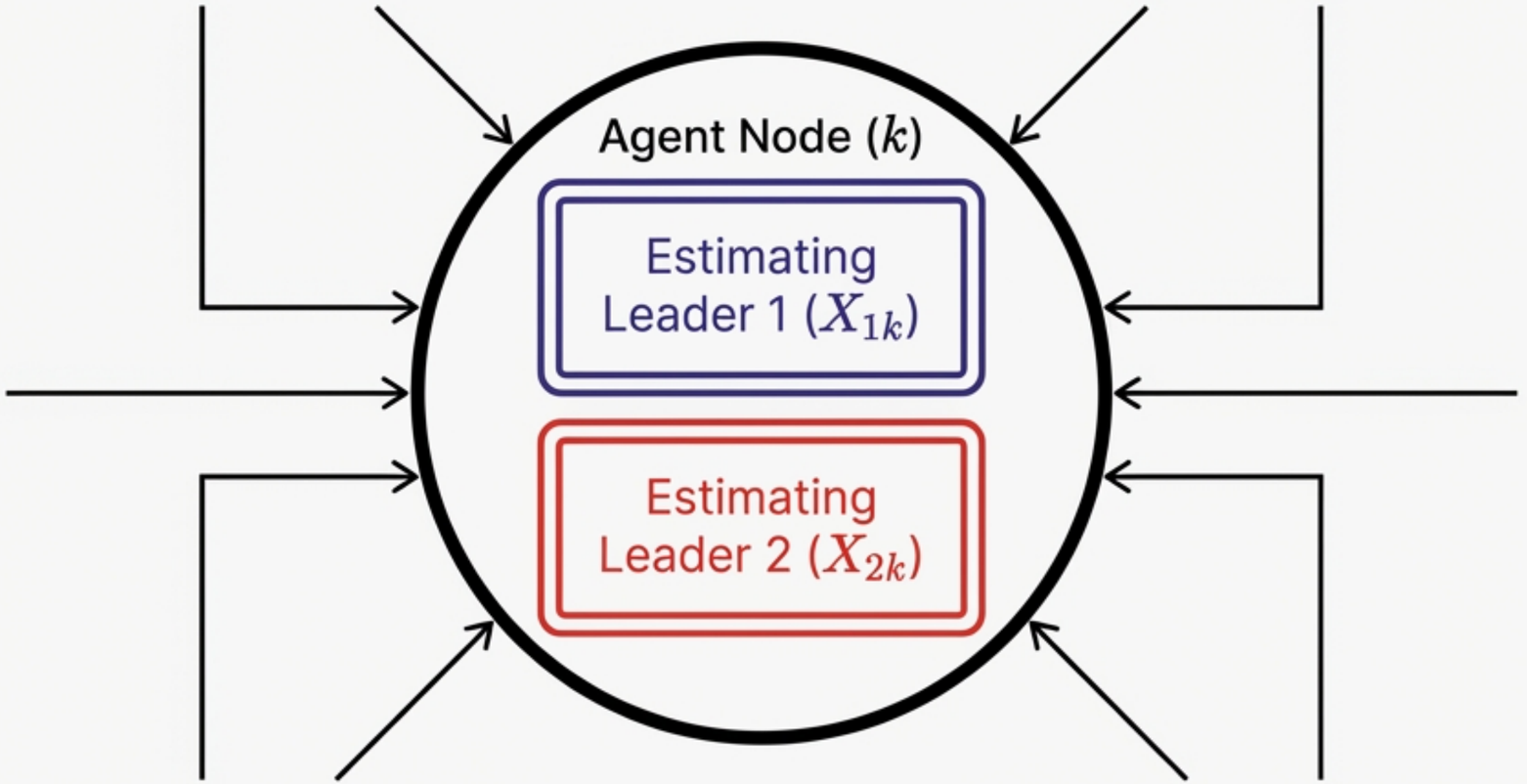
Step 1: The Two-Layer Distributed Observer



Objective: Before choosing a leader, the agent must see all options.

Mechanism: A distributed observer propagates leader states through the network topology.

Result: By time T_0, every follower knows exactly what Leader 1 and Leader 2 are doing (X_{01} & X_{02}).

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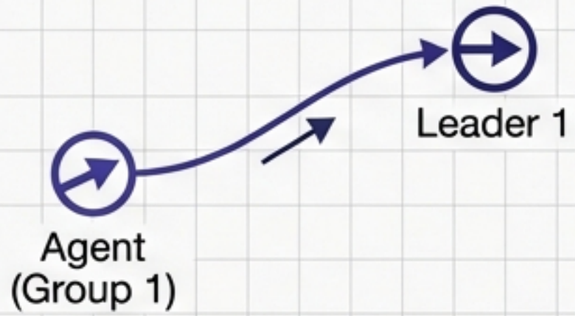
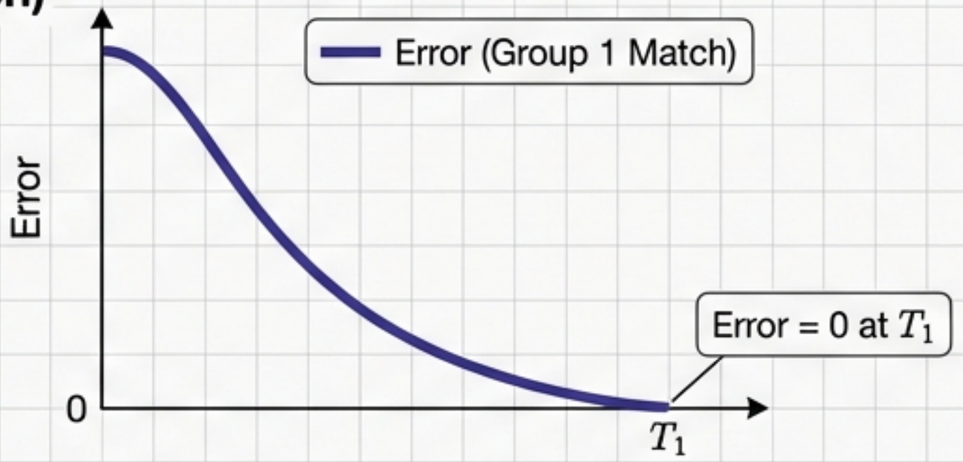
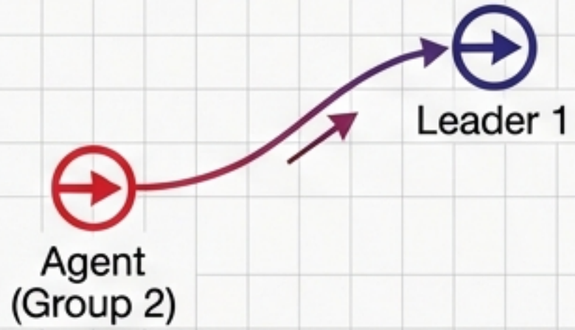
Step 2: The Trial (Testing Hypothesis 1)

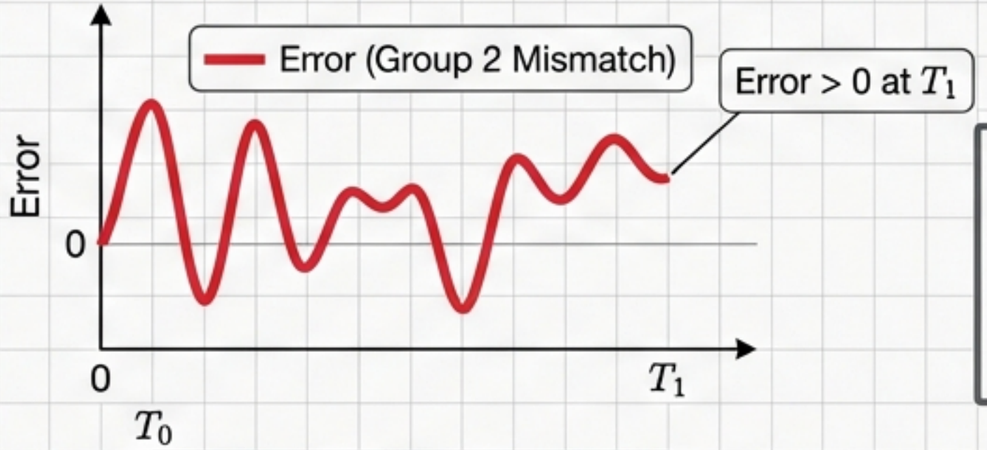
Scenario A: If Agent is Group 1 (Match)





Scenario B: If Agent is Group 2 (Mismatch)





Theorem 3 Guarantee:
Even with mismatched dynamics, the error remains bounded. No finite-time escape.

Trial Phase

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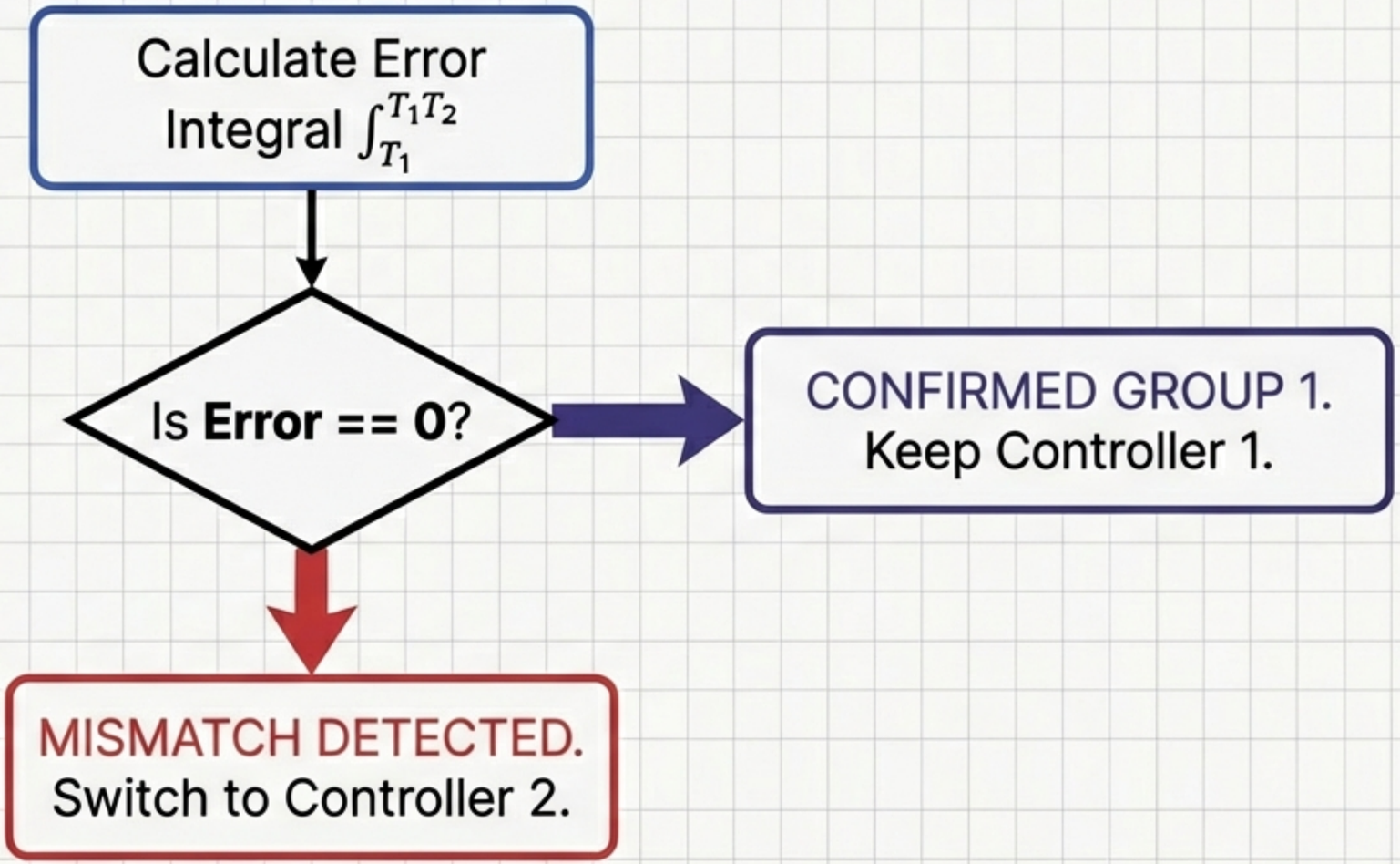
Step 3: The Logic Switch

Assumption 4: Output Distinguishability.

Different nonlinear systems cannot produce identical outputs from the same input.

Therefore, Non-Zero Error ⇒ Wrong Group.



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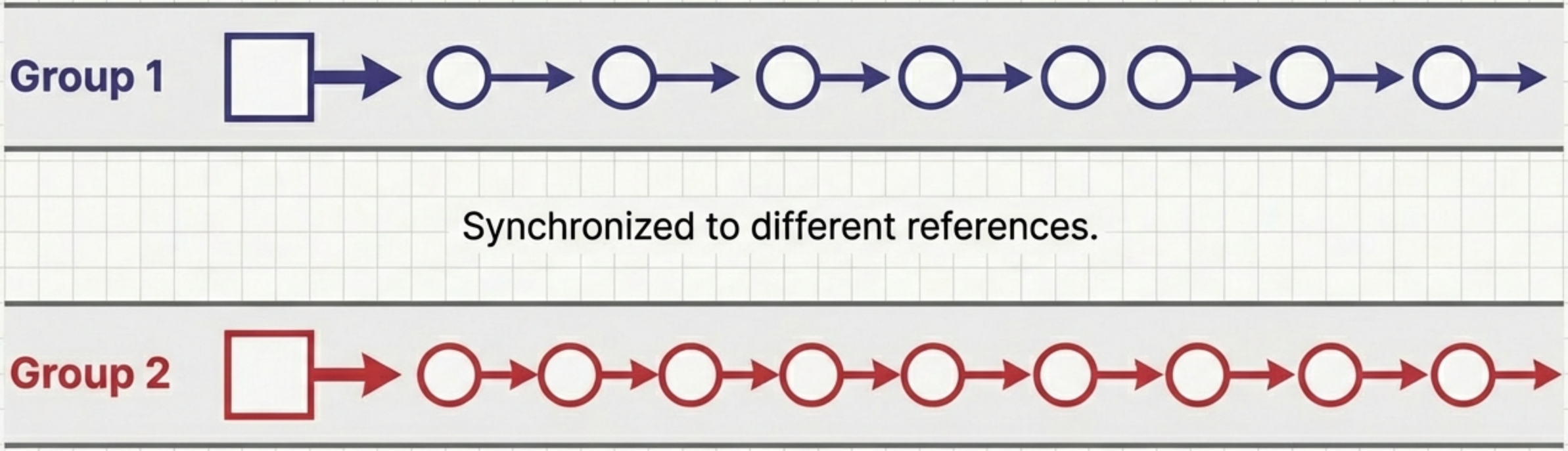
Step 4: Convergence



Synchronized to different references.

At time $t \geq T$: Tracking error is zero for all agents.

$$X_k(t) = X_{0\sigma(k)}(t)$$

0 T_0 T_1 T_2 T

Final Consensus

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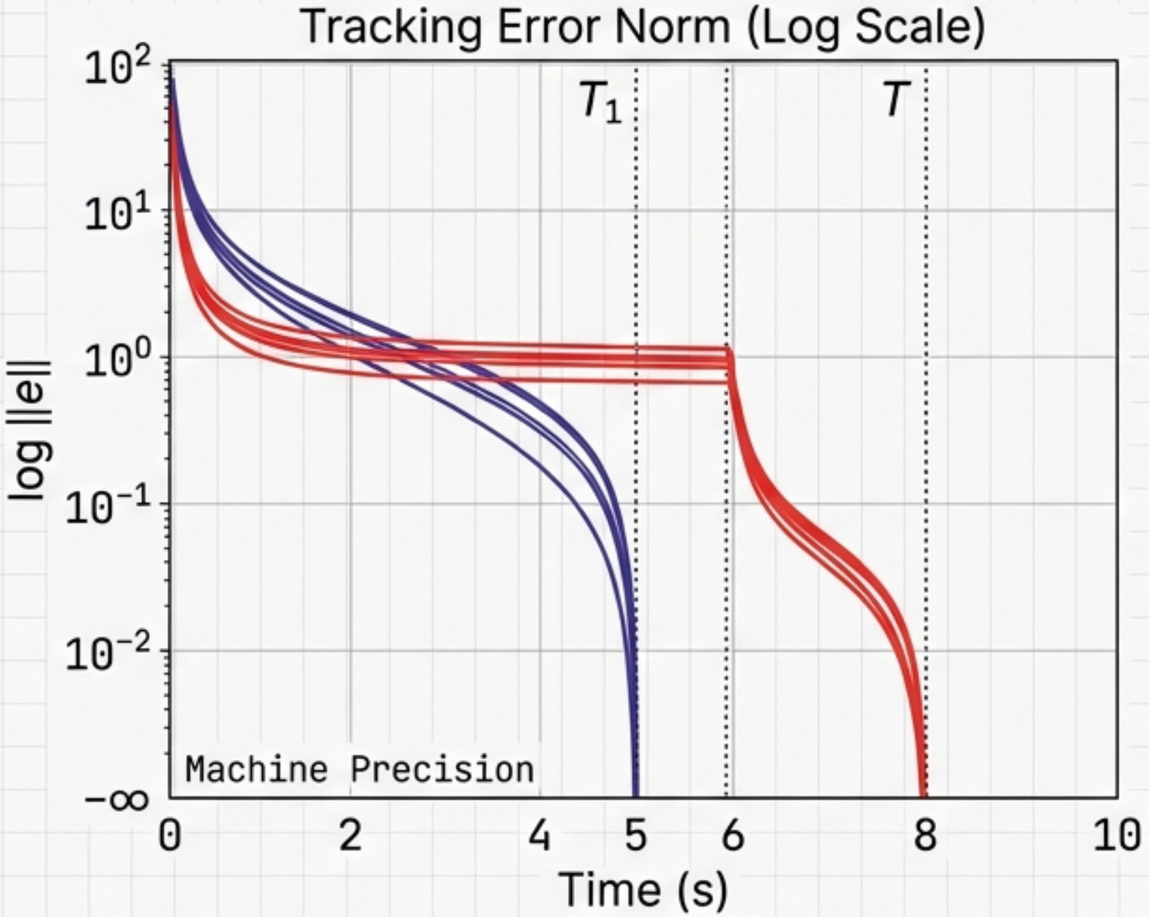
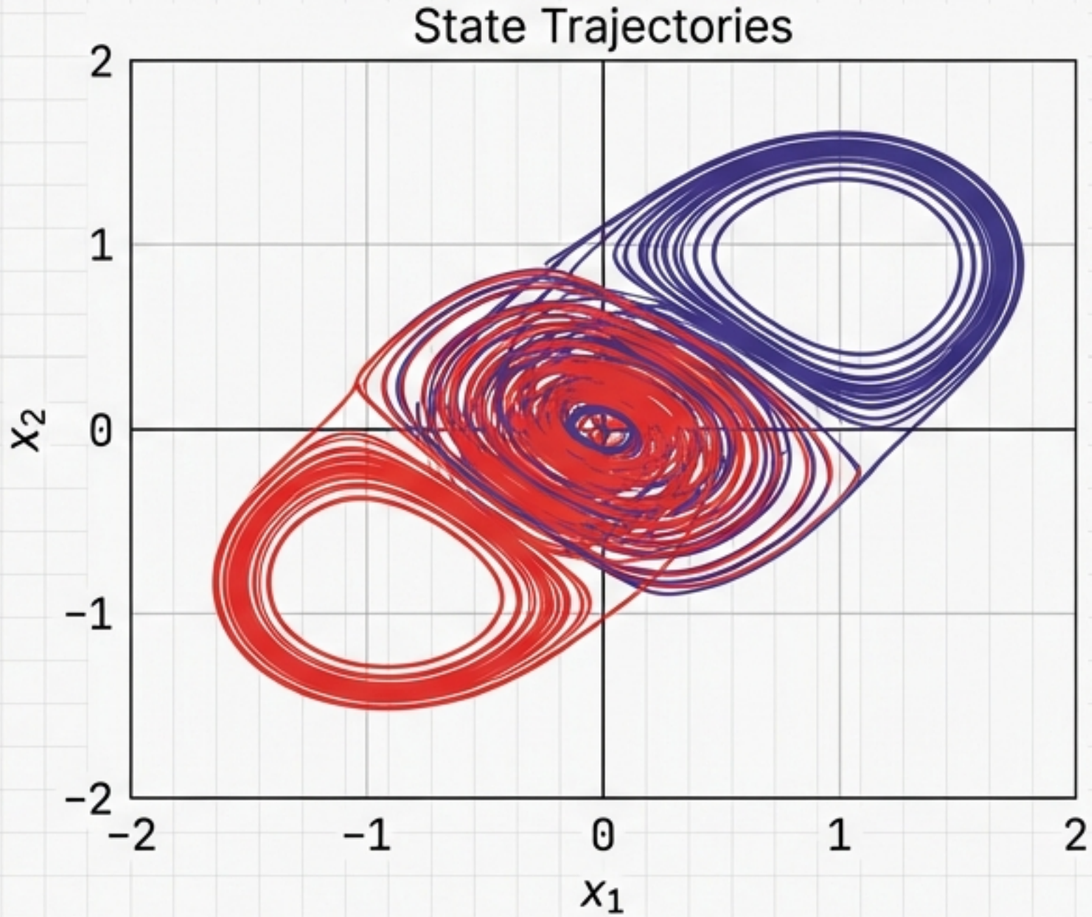
Simulation Validation
From Chaos to Precision.





Vertical drops in error (Left) confirm deterministic prescribed-time convergence.

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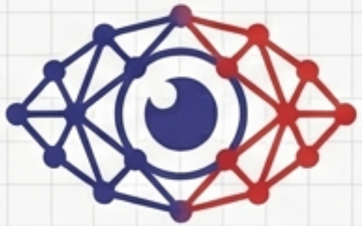
Summary of Contributions

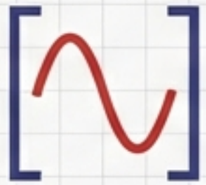


Two-Layer Observation

Followers estimate all leaders simultaneously, enabling informed decision-making.

Active Identification

Using control performance at time T as a sensor for group identity.
Solving the ‘Chicken-and-Egg’ paradox.



Bounded Safety

Rigorous proof that testing the ‘wrong’ controller does not cause instability.

Enabling autonomous organization in heterogeneous networks without pre-labeled identities.

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