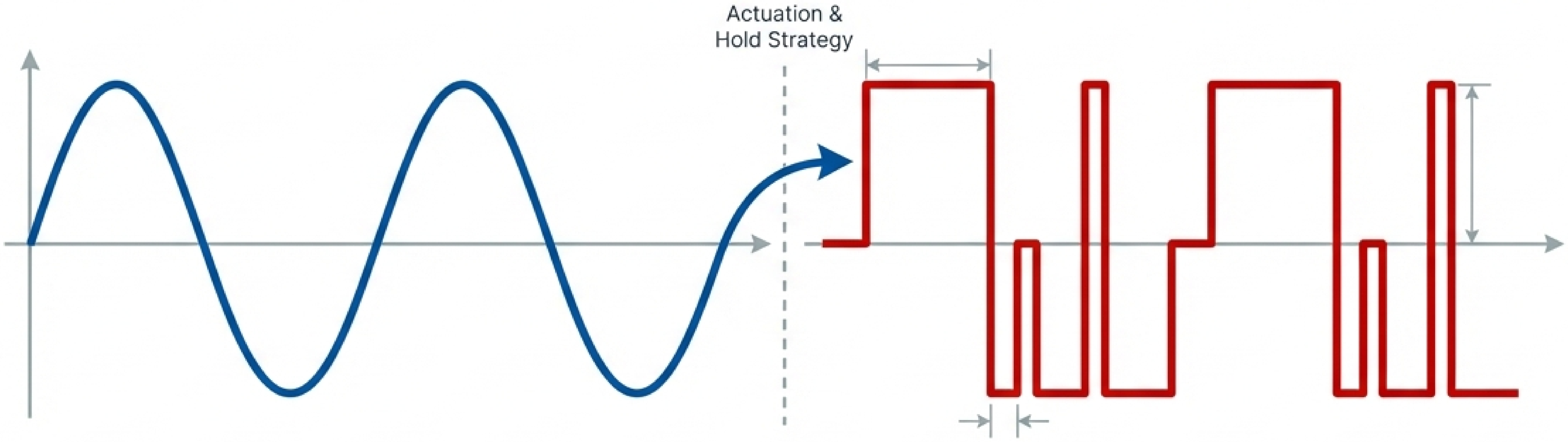
Digital Stabilization of Nonlinear Systems: PWM and Intermittent Hold Strategies

Synthesizing control laws for Strict-Feedback and Feedforward dynamics using finite-state actuation



Synthesized from Chang et al. (2023, 2024, 2025)

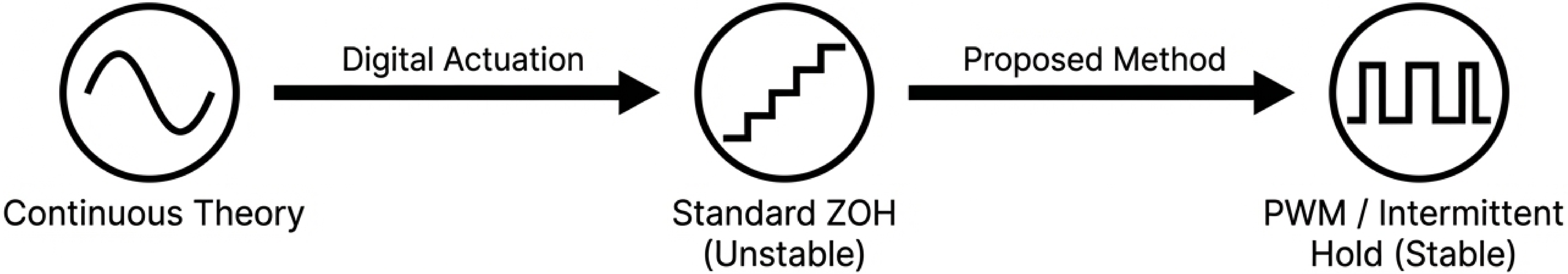
The Tension Between Continuous Theory and Digital Reality

The Challenge

- Physical systems (robotics, chemical reactors) are inherently nonlinear.
- Modern actuators are finite-state (ON/OFF) and networked.
- Standard quantization or Zero-Order Hold (ZOH) introduces “loss of resolution” that destabilizes nonlinear dynamics.

The Solution

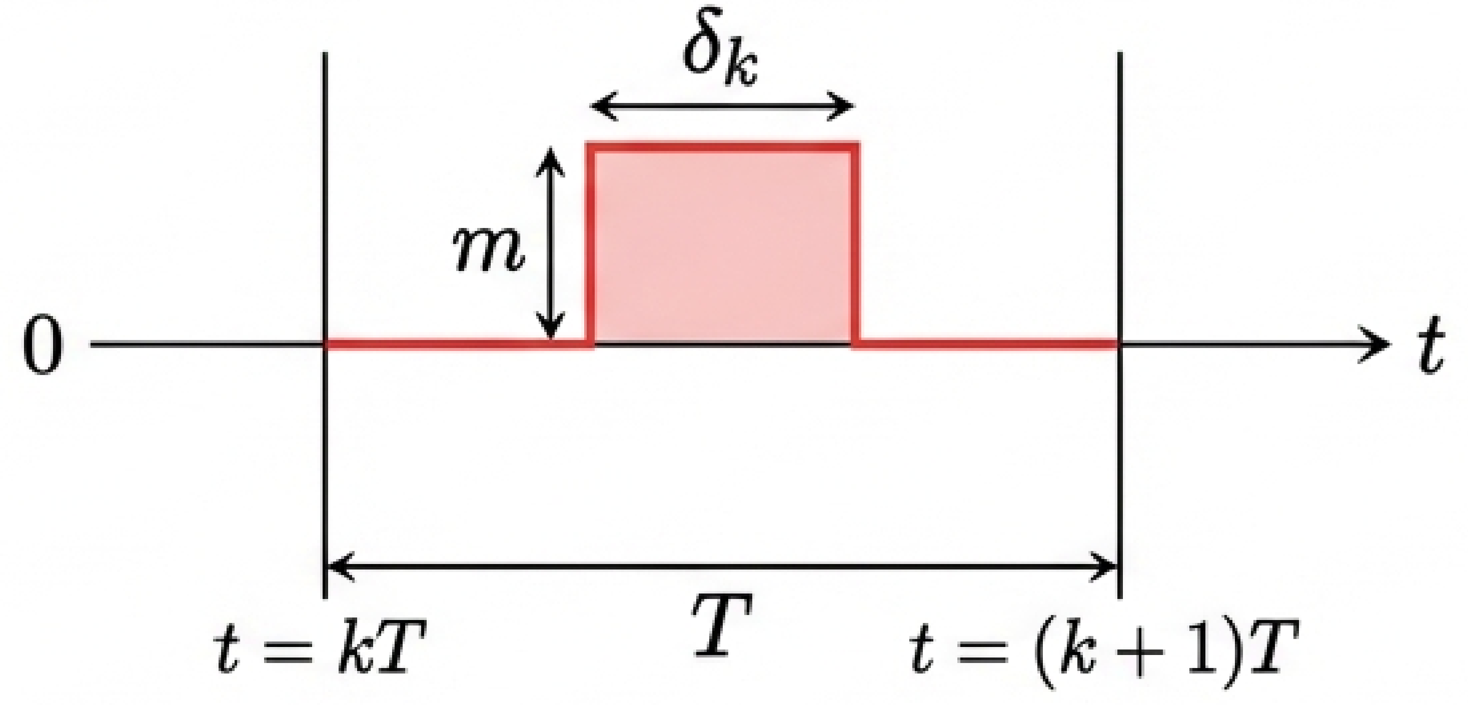
- **Control Variable:** Treat the *time duration* (Duty Cycle or Holding Time) as the continuous input.
- **Stability Mechanism:** Use *Dynamic High-Gain Scaling* to dominate nonlinear growth.



Defining the Actuation Models

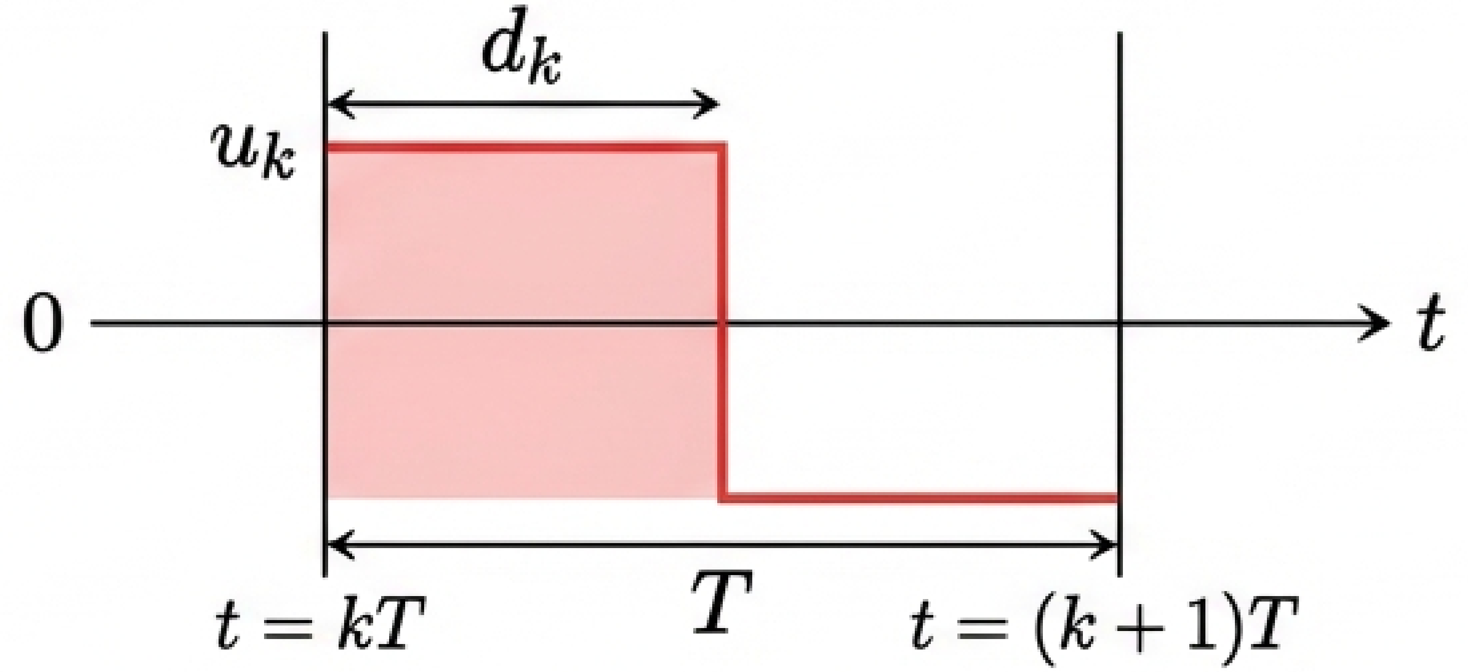
PWM Control (Chang et al., 2023, 2024)

- Actuator switches between discrete states $\{m, 0, -m\}$.
- Variable: Duty Cycle d_k (Pulse Width).
- Constraint: $0 \leq d_k \leq 1$.



Intermittent Hold (Chang et al., 2025)

- Control held constant, then zeroed.
- Metric: Activating Rate $\tau_k = d_k / T$.
- Innovation: Allows for "rest" periods to save energy.

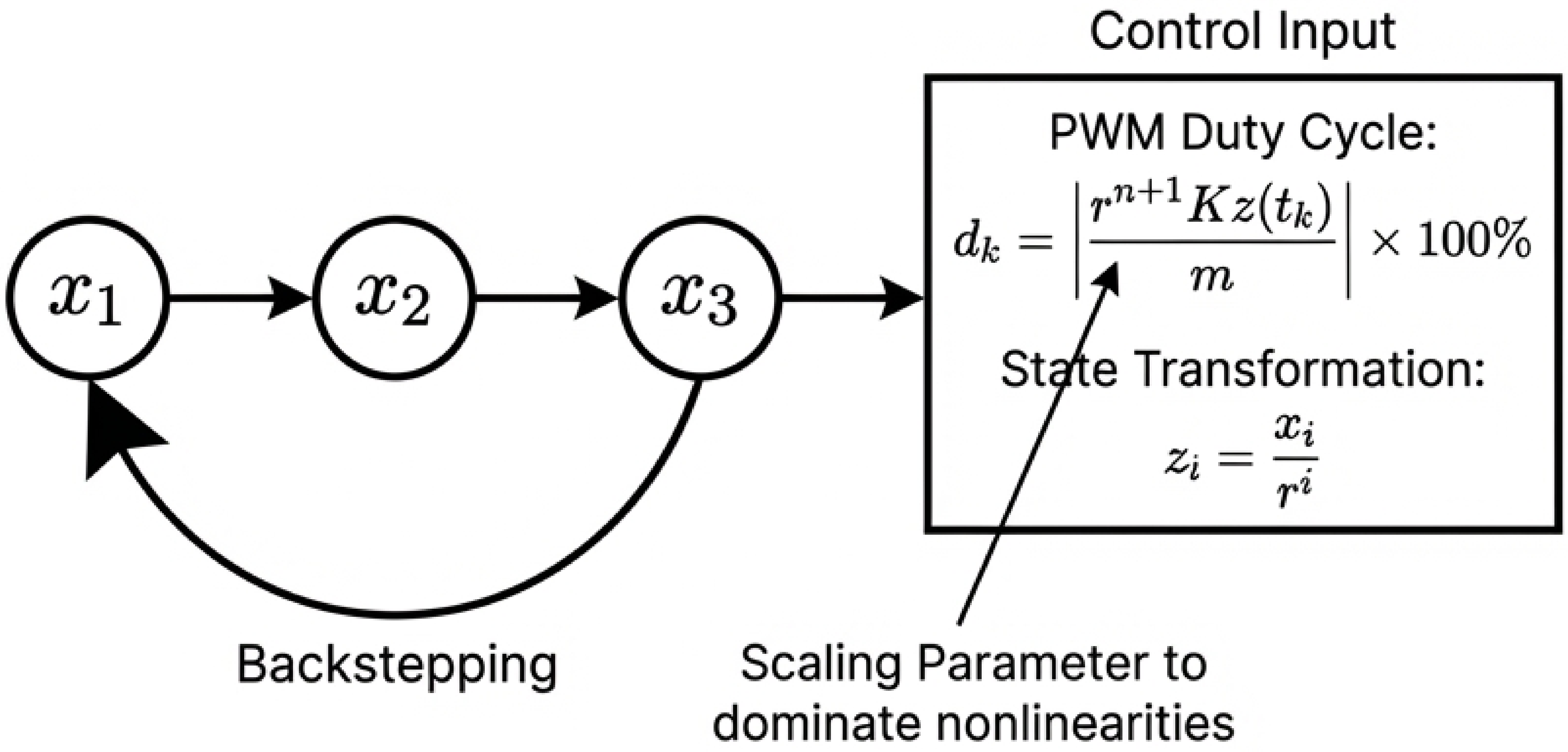


System Class 1: Strict-Feedback Systems

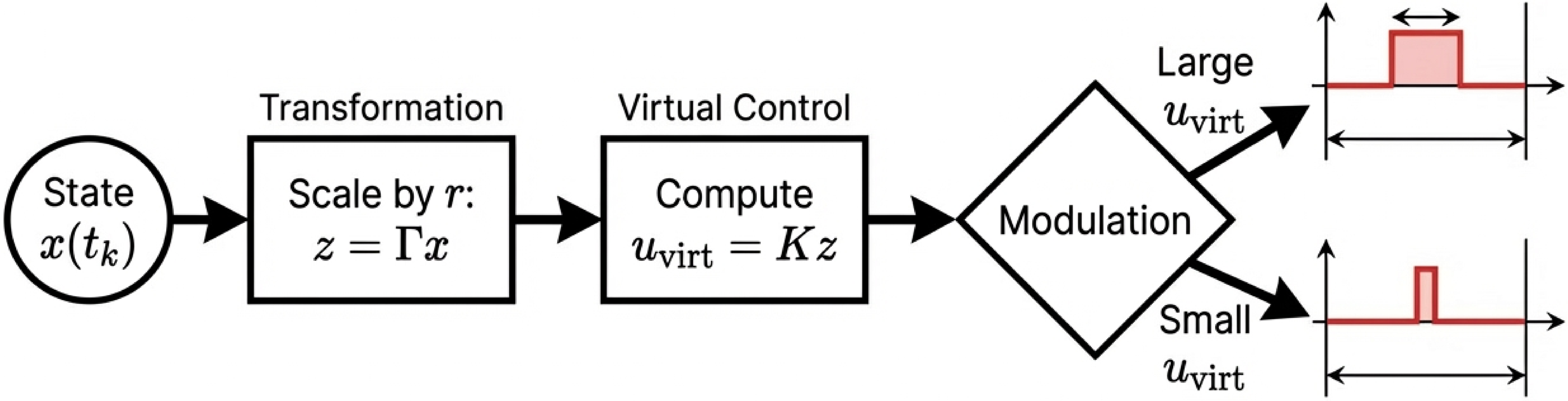
Definition:
Lower-triangular systems where state $\dot{x}_i$ depends on previous states.

Examples:
Mechanical linkages, single-link manipulators.

Solution:
3-Level State-Dependent PWM.



Stabilizing Strict-Feedback Dynamics via PWM



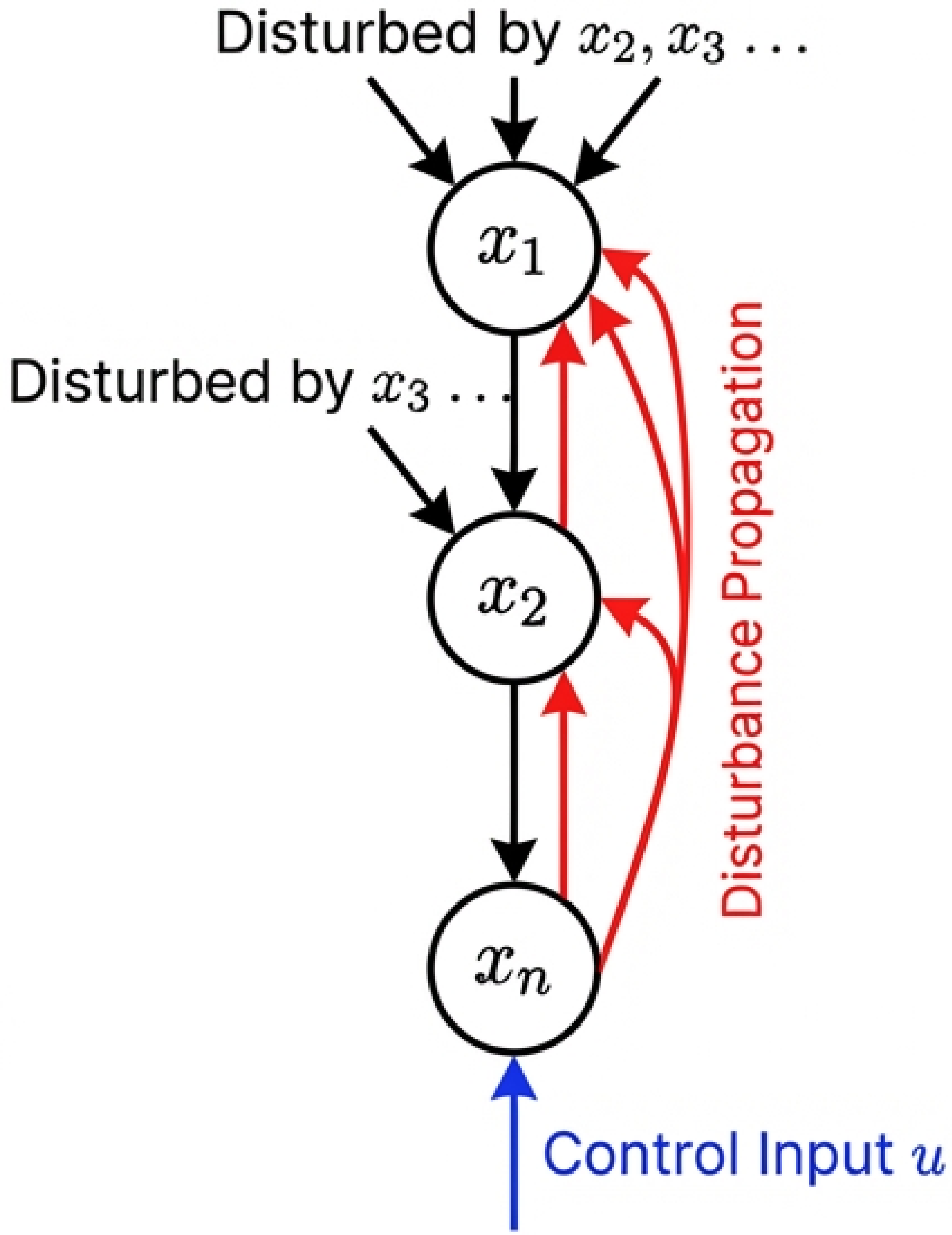
Stability Logic (Theorem 1):
If sampling time T is small and amplitude m is large ($m \geq r^n||K||$ ...), the discrete pulse effectively approximates the continuous stabilizer.

System Class 2: Feedforward Nonlinear Systems

Definition: Upper-triangular systems where $\dot{x}_i$ depends on future states.

Examples: Chemical reactors, underactuated vehicles (PVTOL).

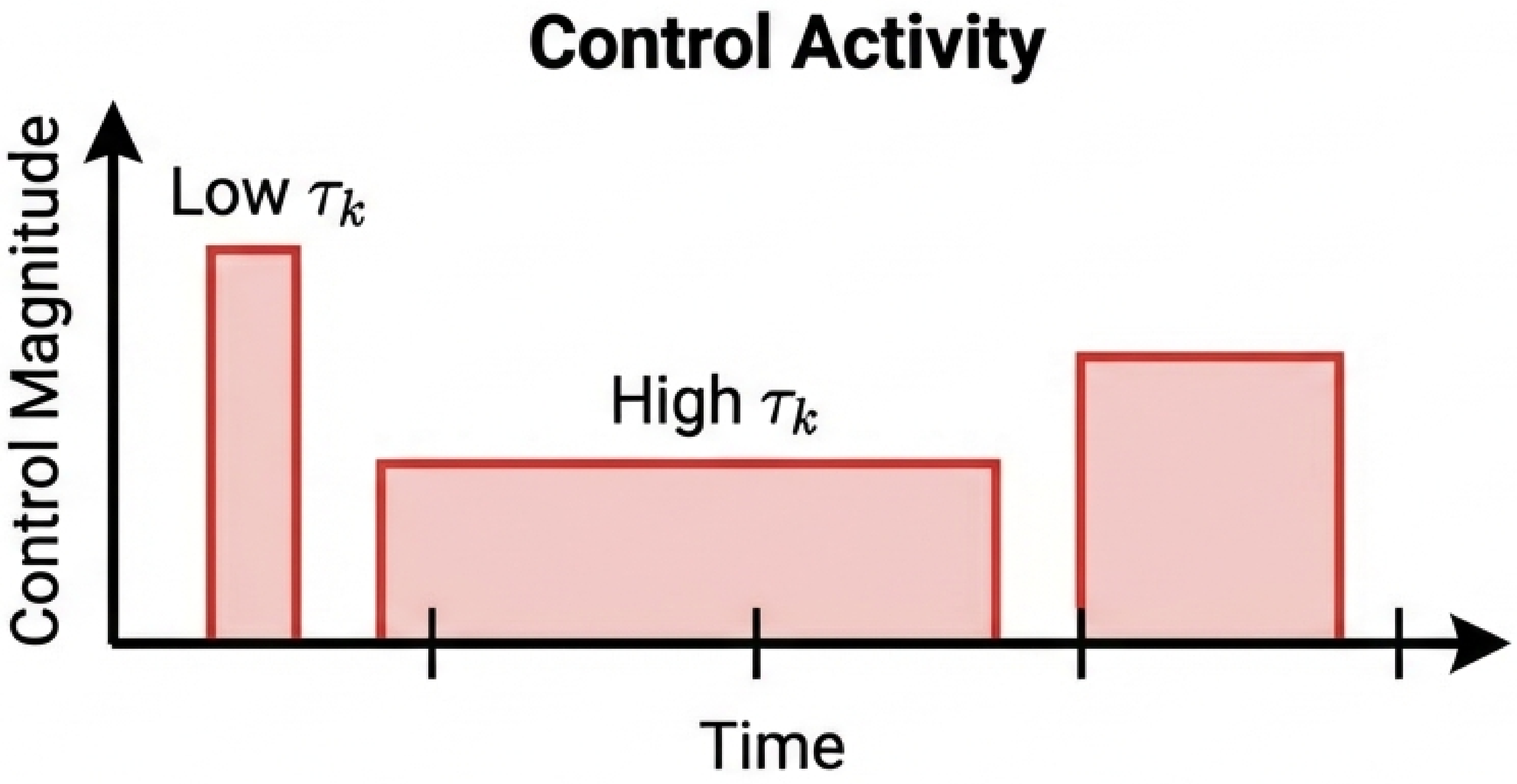
Challenge: Disturbances propagate forward from x_n to x_1.



1. Intermittent Hold (Focus on Activating Rate τ_k)

2. PWM with Hybrid Observer (Focus on Estimation)

Strategy A: Intermittent Hold for Feedforward Systems



Control Input:

$$u_k = \frac{1}{\tau_k} K\Gamma(\rho_k)\,\hat{x}_k$$

Key Feature: As duration τ_k shrinks, magnitude increases to compensate.

Robustness Result: Stability guaranteed for any activating rate $\tau \in [\tau_{\min}, 1]$. System can "rest" for significant portions of the cycle.

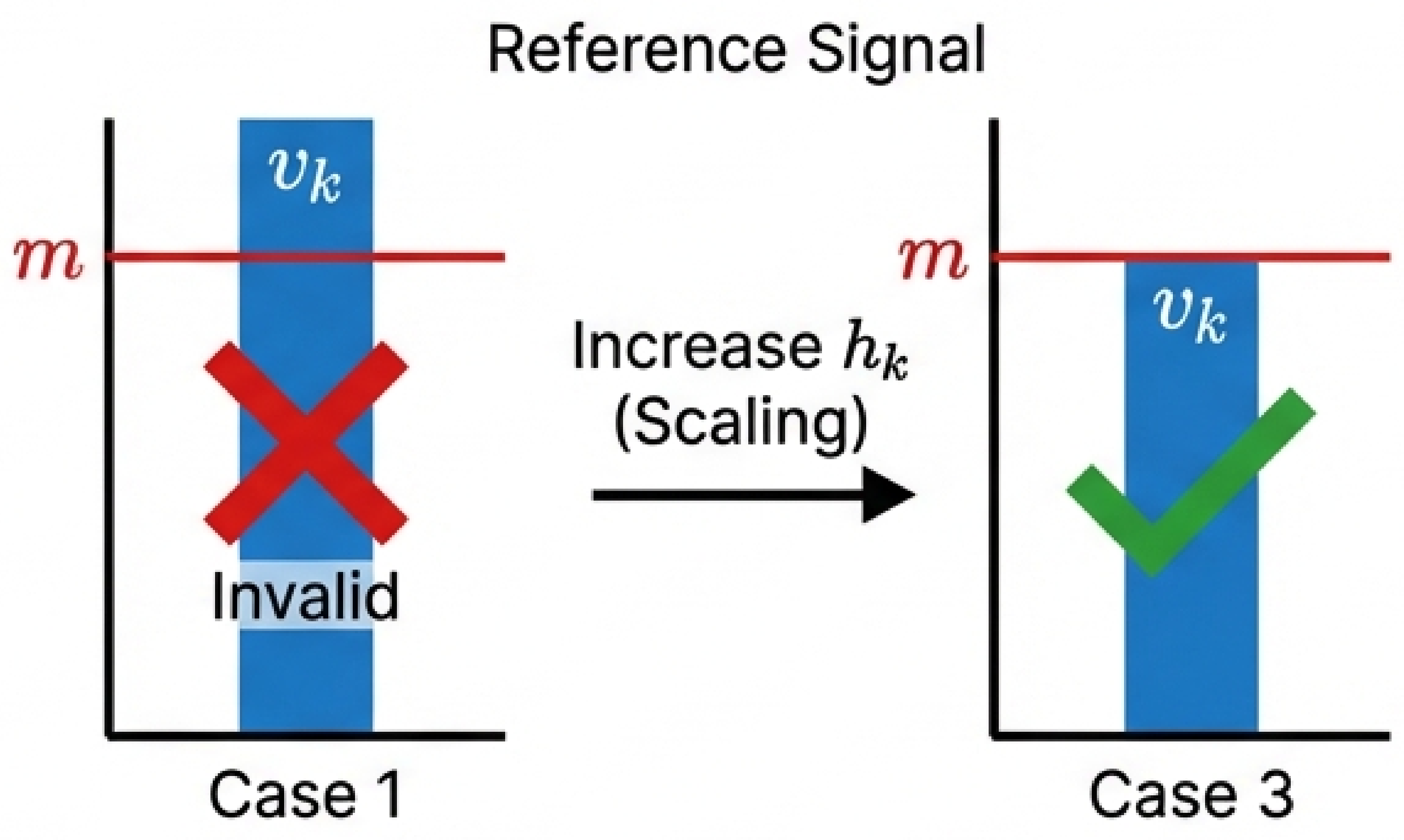
NotebookLM

Strategy B: PWM with Reference Signal

The Logic

1. Generate Reference: $v_k = KH_k\hat{x}(t_k)$
2. Convert to Duty Cycle: $d_k = \frac{|v_k|}{m}$
3. Constraint: $|v_k| \leq m$ (Must not saturate)

Dynamic Parameter h_k



The parameter h_k dynamically grows to scale the reference signal down, ensuring it always fits within the physical actuator limits.

The Common Enabler: Dynamic High-Gain Scaling

Intermittent Hold:
Parameter ρ_k
(Handles short duration)

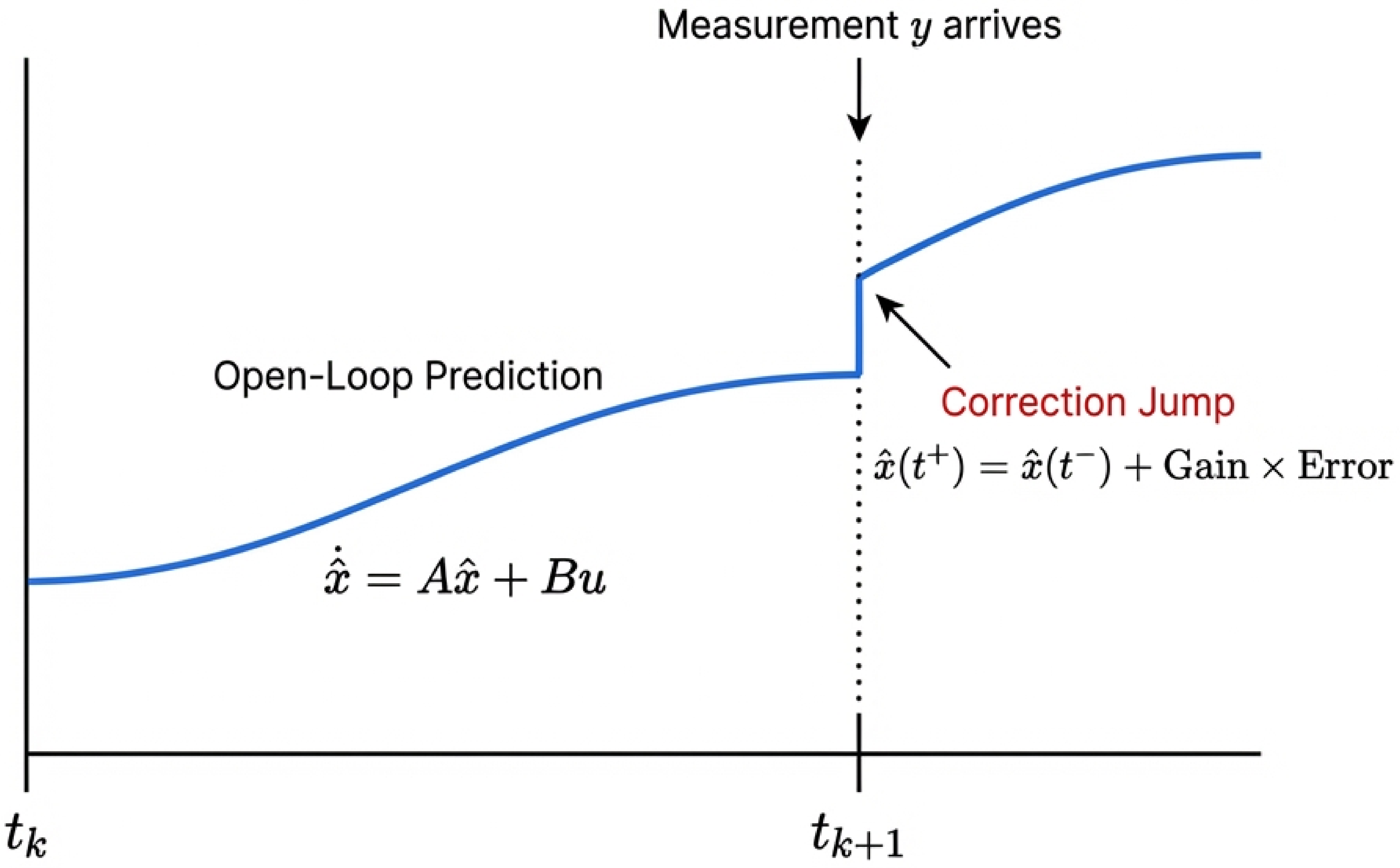
Strict-Feedback:
Parameter r
(Static or Dynamic)

$$\text{Scaling Matrix } \Gamma = \begin{bmatrix} \frac{1}{r} & & & \\ & \frac{1}{r^2} & & \\ & & \ddots & \\ & & & \frac{1}{r^n} \end{bmatrix}$$

PWM Feedforward:
Parameter h_k
(Handles saturation)

Mechanism: By scaling the states inversely to their power, the controller artificially amplifies the linear control authority to ‘overpower’ nonlinear growth.

Handling Missing States: The Hybrid Observer



The PWM controller uses this estimated state $\hat{x}$ to calculate pulse widths. Scaling parameter h_k ensures estimation error converges faster than system nonlinearities.

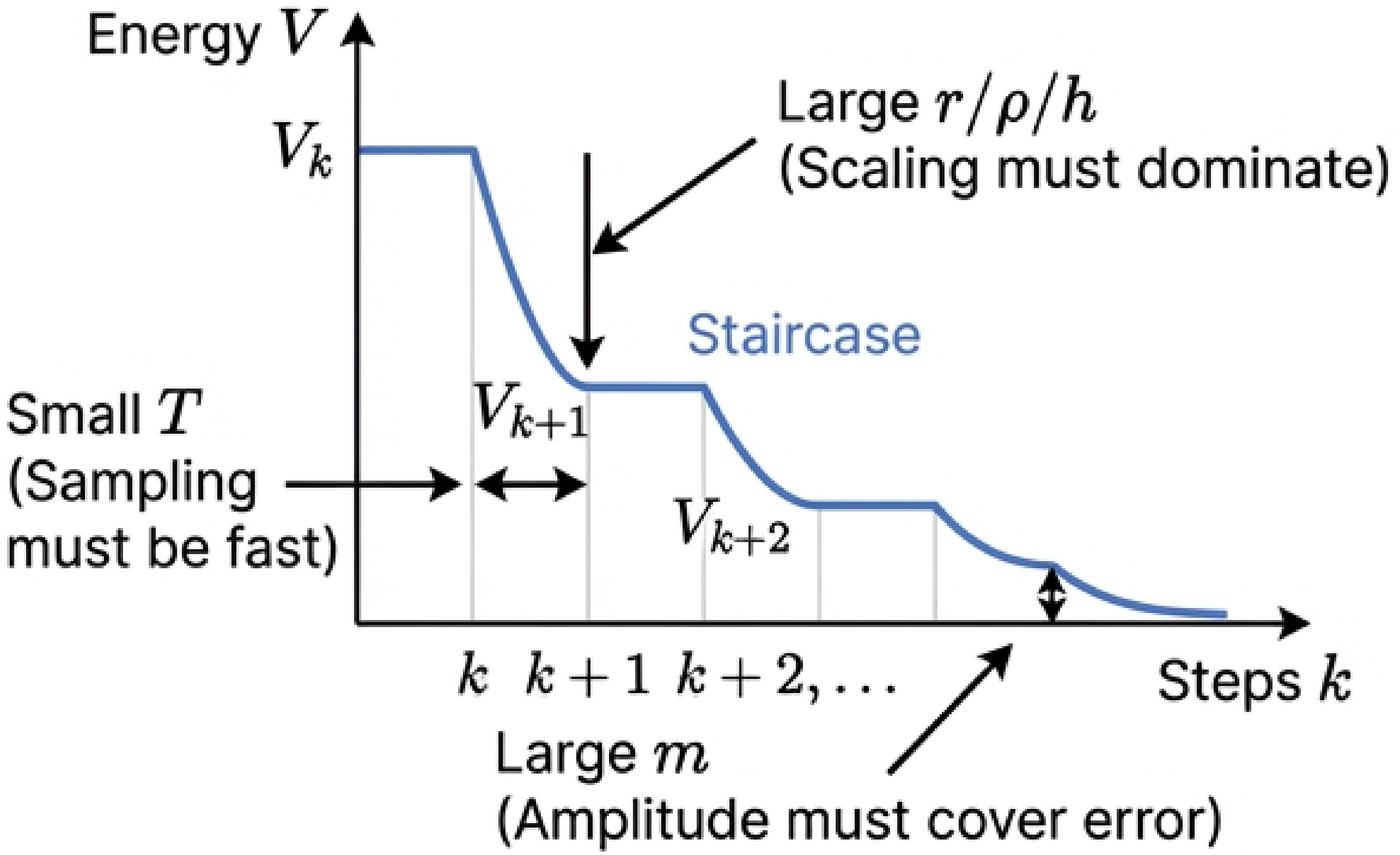
Stability Analysis: The Lyapunov Approach

The Math

- Energy Function: $V_k = z^T P z$
- Goal: Prove energy decrease $V_{k+1} - V_k < 0$

$$V_{k+1} - V_k < 0 \downarrow$$

Visual Proof Concept

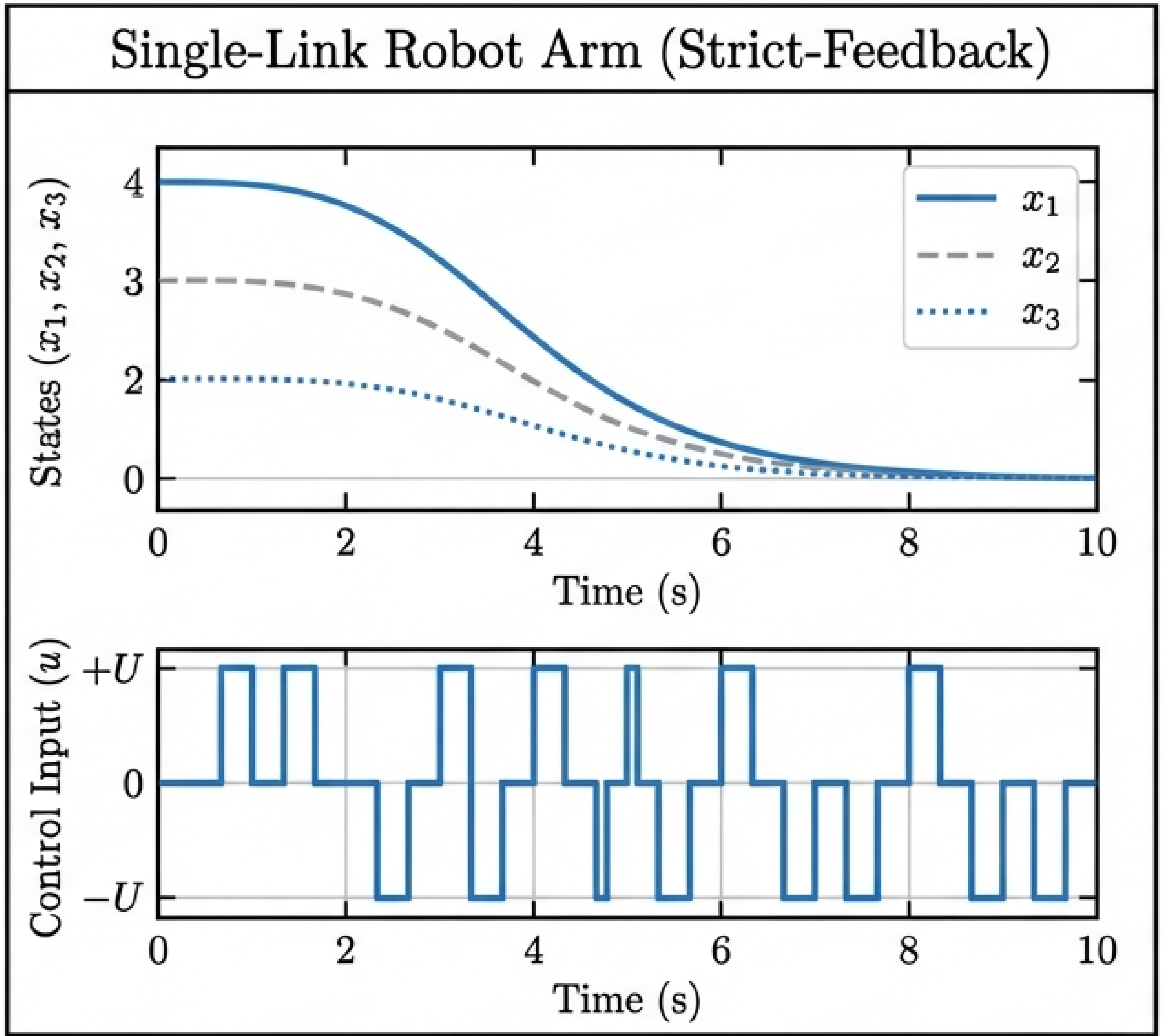


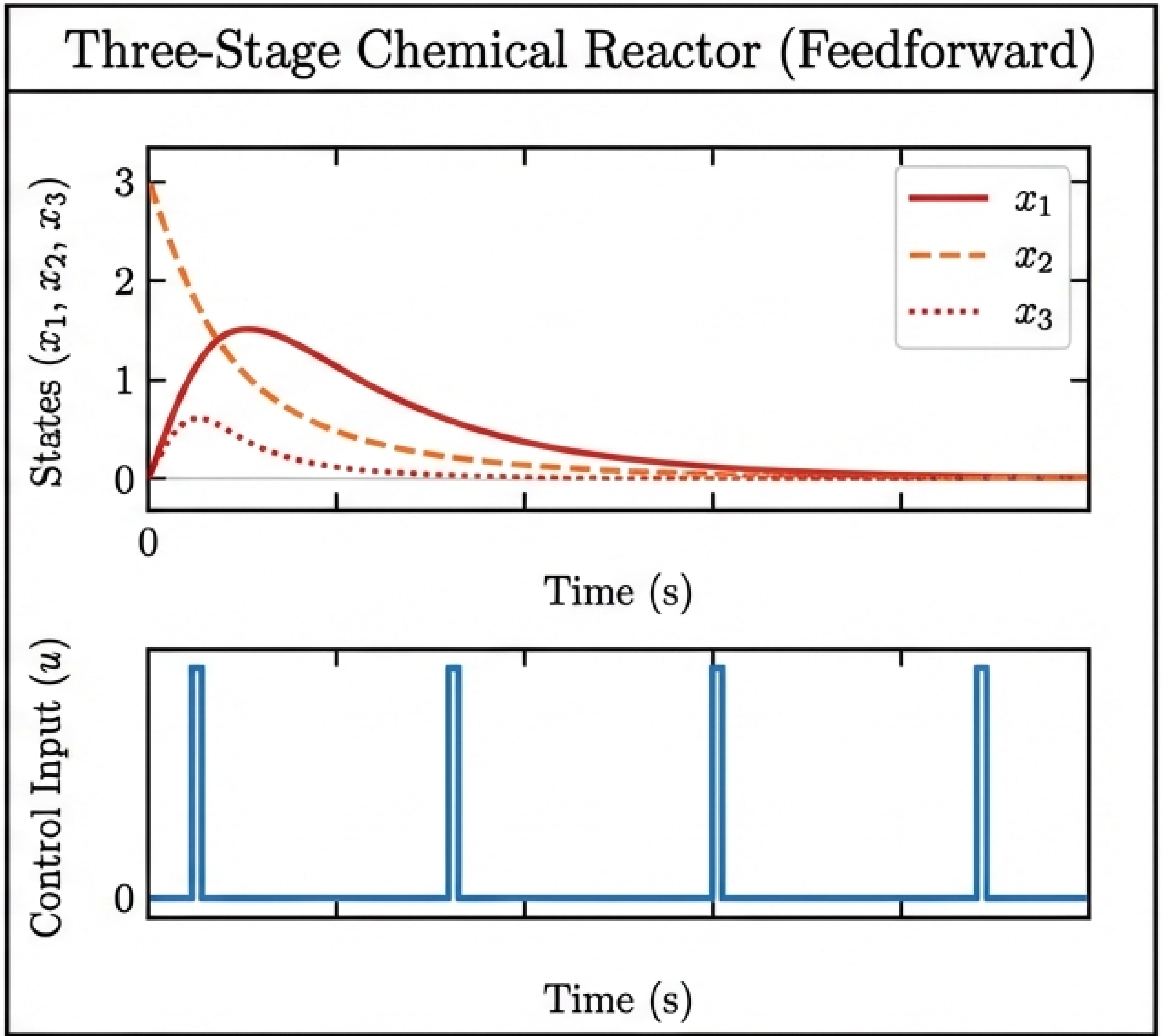
Conclusion: Despite the ‘choppy’ nature of PWM/Intermittent control, the scaled system energy **monotonically decreases** to zero.

Comparative Summary of Approaches

Strict-Feedback (Source 1)	Feedforward (Source 2)	Feedforward (Source 3)
Topology: Lower Triangular	Topology: Upper Triangular	Topology: Upper Triangular
Actuation: 3-Level PWM	Actuation: Intermittent Hold	Actuation: PWM w/ Reference
Parameter: r	Parameter: ρ_k	Parameter: h_k
Best For: Mechanical / Robotics	Best For: Energy Saving / Limited Bandwidth	Best For: Precise Output Feedback

Validation: Electromechanical & Chemical Systems

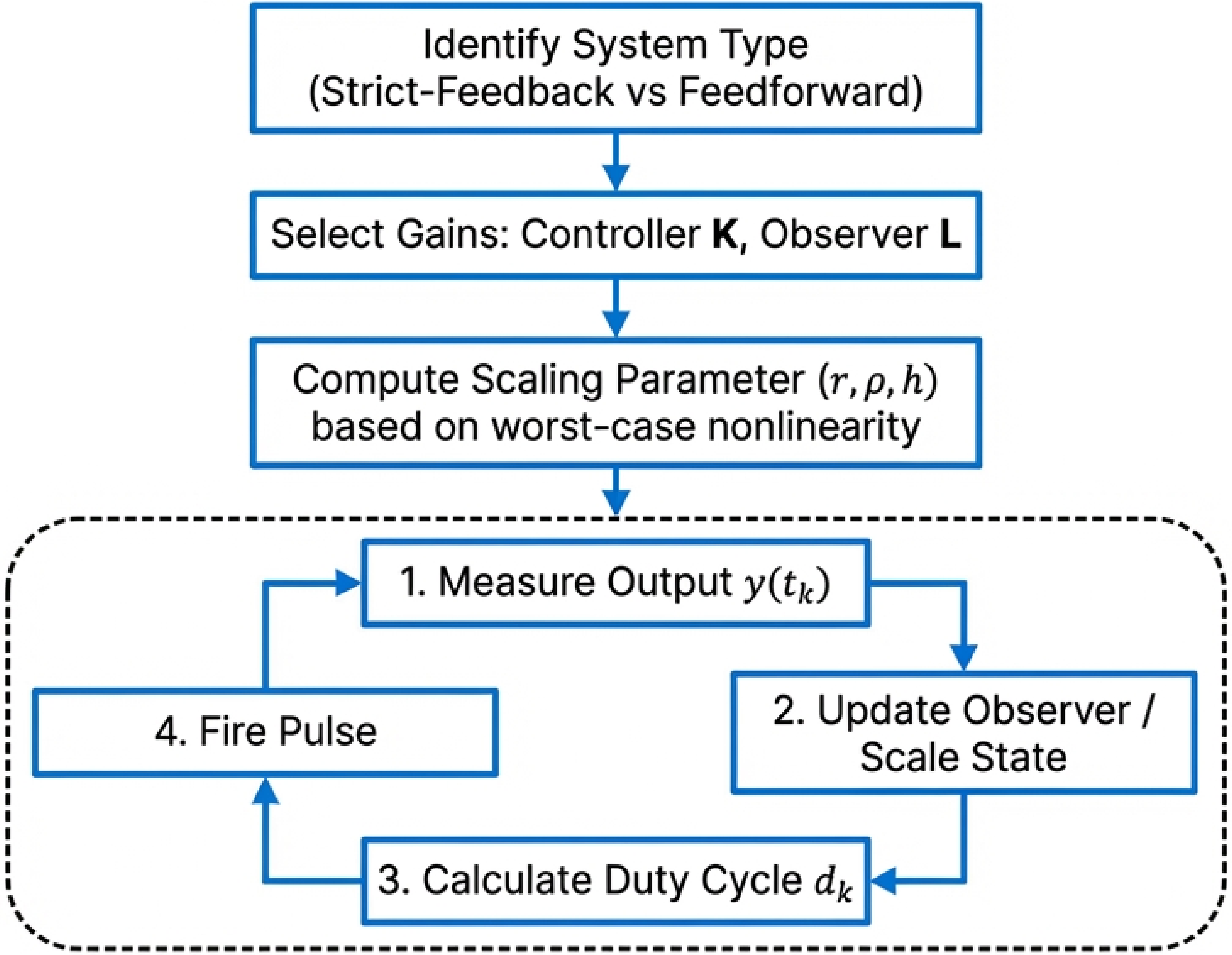




Inter	Convergence with coarse 3-level switching

Inter	Stable even with low Activating Rate

Practical Implementation Algorithm



Enabling the Digital Future of Nonlinear Control

Finite-state actuation is not a limitation to be mitigated, but a distinct control modality with rigorous stability guarantees.

Time as Control Variable

High-Gain Scaling

Digital-First Actuators

Impact: Enables cheaper, robust digital control for aerospace, robotics, and industrial automation.