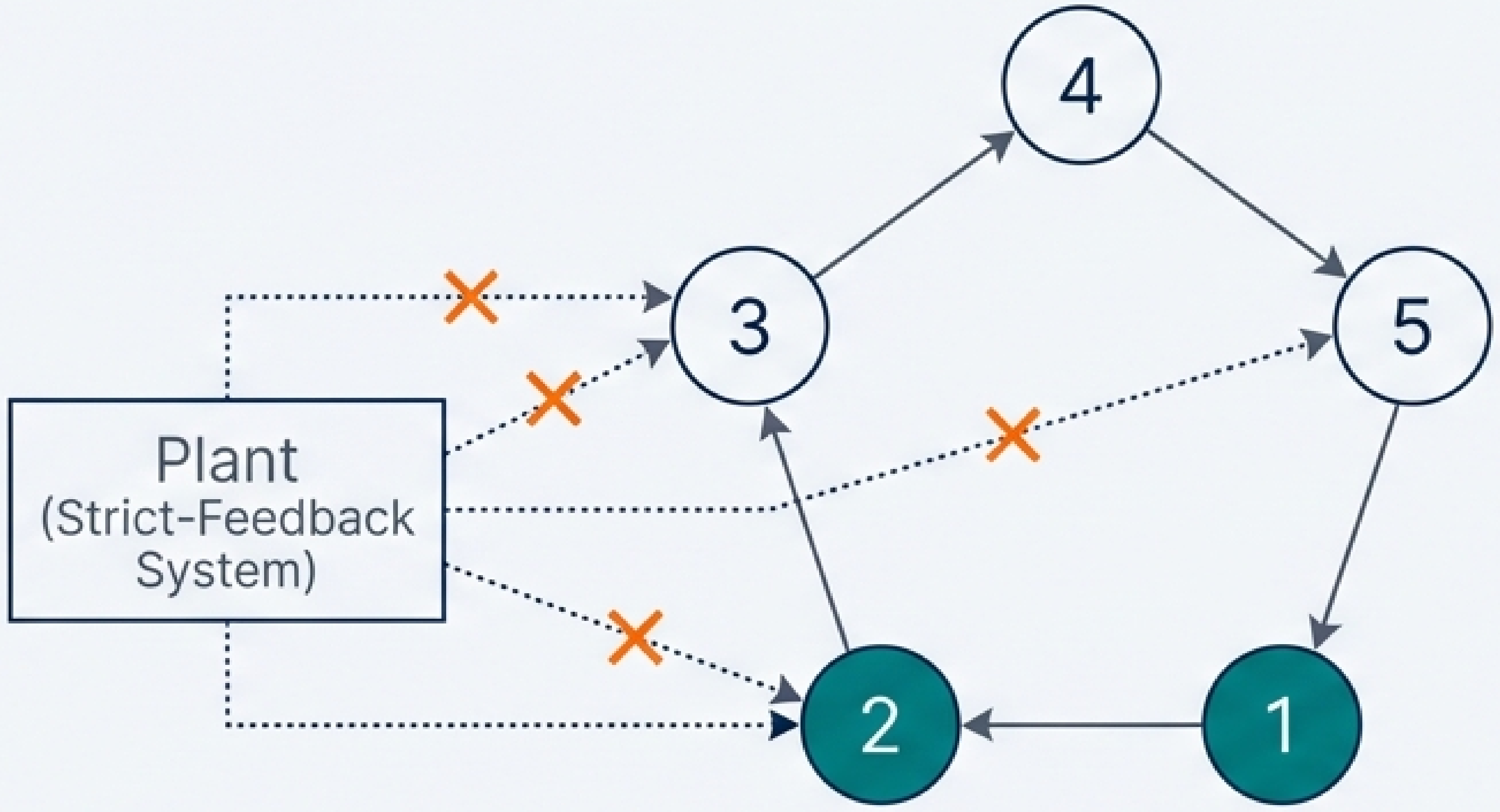
Designing Distributed Prescribed Finite-Time Observers

For Strict-Feedback Nonlinear Systems with Partial Observability



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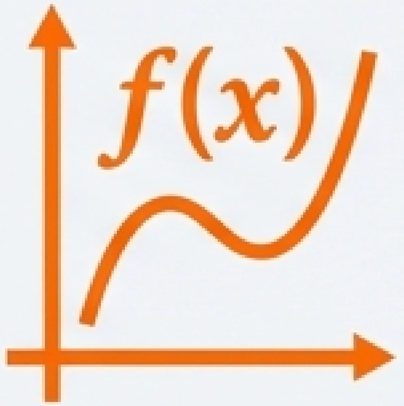
Executive Summary: Core Contributions

The “Blind” Node Problem

Solves state reconstruction for distributed networks where only a subset of nodes ($m < N$) have access to system measurements. Achieves full state estimation via consensus, even for nodes with zero direct visibility.

Prescribed Finite-Time Convergence

Shifts from asymptotic convergence (infinite time) to a hard deadline T^*. The settling time T^* is user-defined and completely independent of initial system conditions or control parameters.



Handling Strict-Feedback Nonlinearity

accommodates strict-feedback systems satisfying a Lipschitz condition on the finite interval $[0, T)$. This relaxes global Lipschitz requirements, allowing for more aggressive nonlinear dynamics.

Robustness via Boundedness

Introduces “Prescribed Finite-Time Boundedness” to handle external disturbances. Ensures estimation error remains trapped within a specific, user-controllable bound after time T^*.

System Class: Strict-Feedback Nonlinear Dynamics

The structural constraints of the plant model.

The Math

X_dot = AX + F(t, X) + Bd(t)

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}$$

Teal highlights indicate strict-feedback coupling.

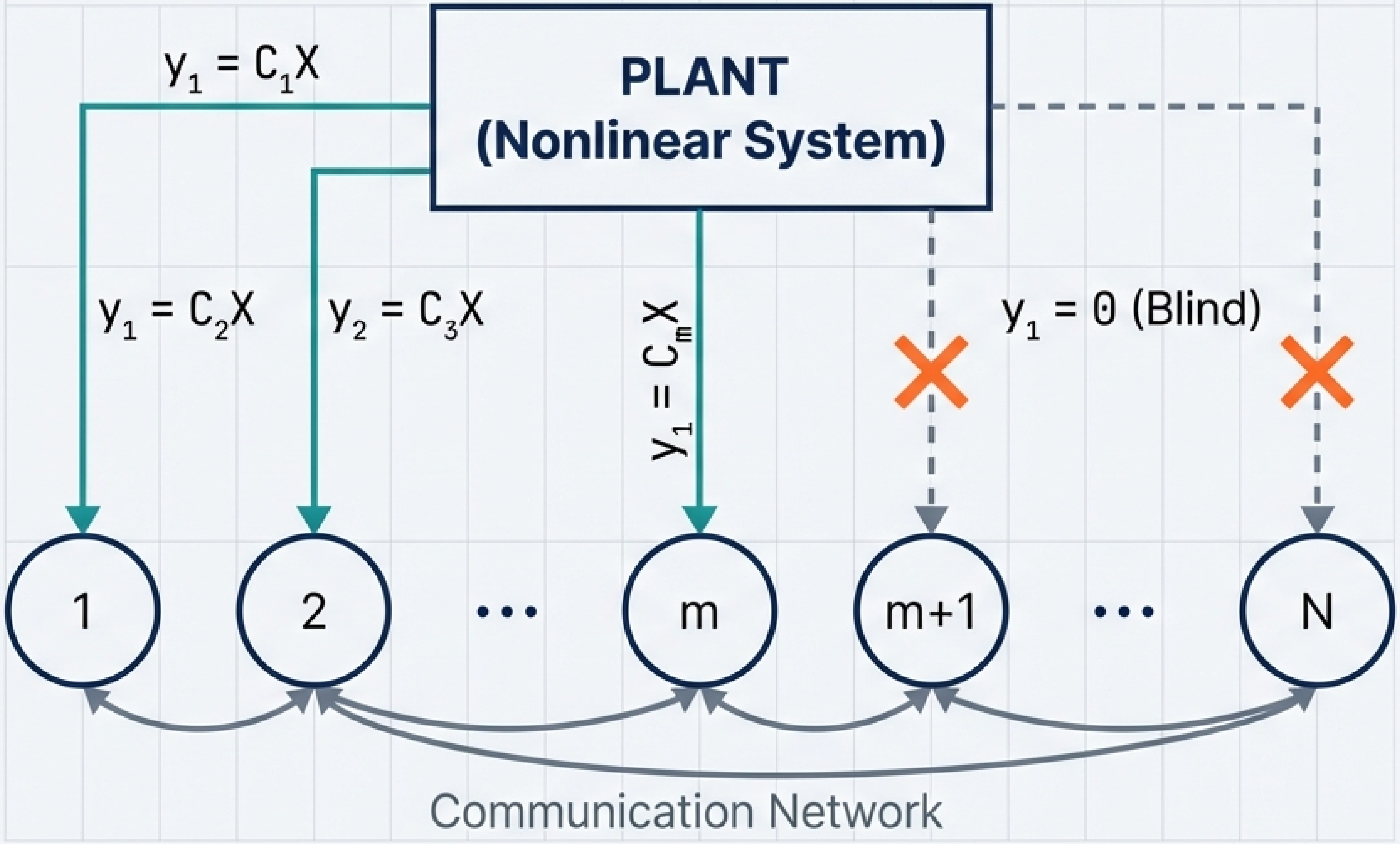
System Characteristics

- **Strict-Feedback Form:** The i-th state derivative depends only on states x_1 through x_i+1.
- **Triangular Coupling:** Allows for sequential design (backstepping).
- **Applications:** Mechanical linkages, hydraulic actuators, electric servomotors.
- **Constraint:** Standard linear observers fail due to nonlinear couplings F(t,X).

Assumption: Nonlinear term F(t,X) satisfies a local Lipschitz condition on the interval [0, T].

The Challenge: Distributed Partial Observability

Fig 1. Topology & Sensing



Sensing Constraints

1. **Limited Access:** Only m sensors see the plant. The remaining N-m nodes are blind.
2. **Low Dimension:** Even the seeing sensors (1...m) may only measure 1 dimension of the n-dimensional state.
3. **The Objective:** Every node i must estimate the full state vector $X_i(t)$ such that $X_i \to X$.

The Gap: Why Standard Finite-Time Methods Fail

A comparison of convergence properties and design limitations.

Method	Convergence Time	Dependency	Verdict
Asymptotic Convergence	$t \to$ Infinity	System Eigenvalues	Too Slow (Impractical)
Standard Finite-Time	T (Variable)	Initial Conditions $X(0)$	Unpredictable T
Fixed-Time	$T < T_{\max}$	Design Parameters	Conservative Bound
Prescribed Finite-Time (Proposed)	Exactly T^*	User Defined Only	**Deterministic & Optimal**

Key Insight: In the proposed method, the settling time T^* is a **design parameter**, not a result of system dynamics.

The Solution: Distributed Prescribed Finite-Time Observer

$$\dot{\mathbf{X}}_i = \underbrace{\mathbf{A}\mathbf{X}_i + \mathbf{F}(t, \mathbf{X}_i)}_{\substack{\text{Model Replica} \\ \text{(Internal Dynamics)}}} - \underbrace{\mathbf{L}_i(\mathbf{y}_i - \mathbf{C}_i\mathbf{X}_i)}_{\substack{\text{Measurement Injection} \\ \text{(Own Sensor)}}} - \underbrace{\mathbf{K}_i \sum a_{ik}(\mathbf{X}_i - \mathbf{X}_k)}_{\substack{\text{Consensus Term} \\ \text{(Neighbor Correction)}}}$$

Time-Varying Gains

Gain Scaling Function: $\dfrac{\mu}{T^* - t}$

Mathematical Constraints: Nonlinearity

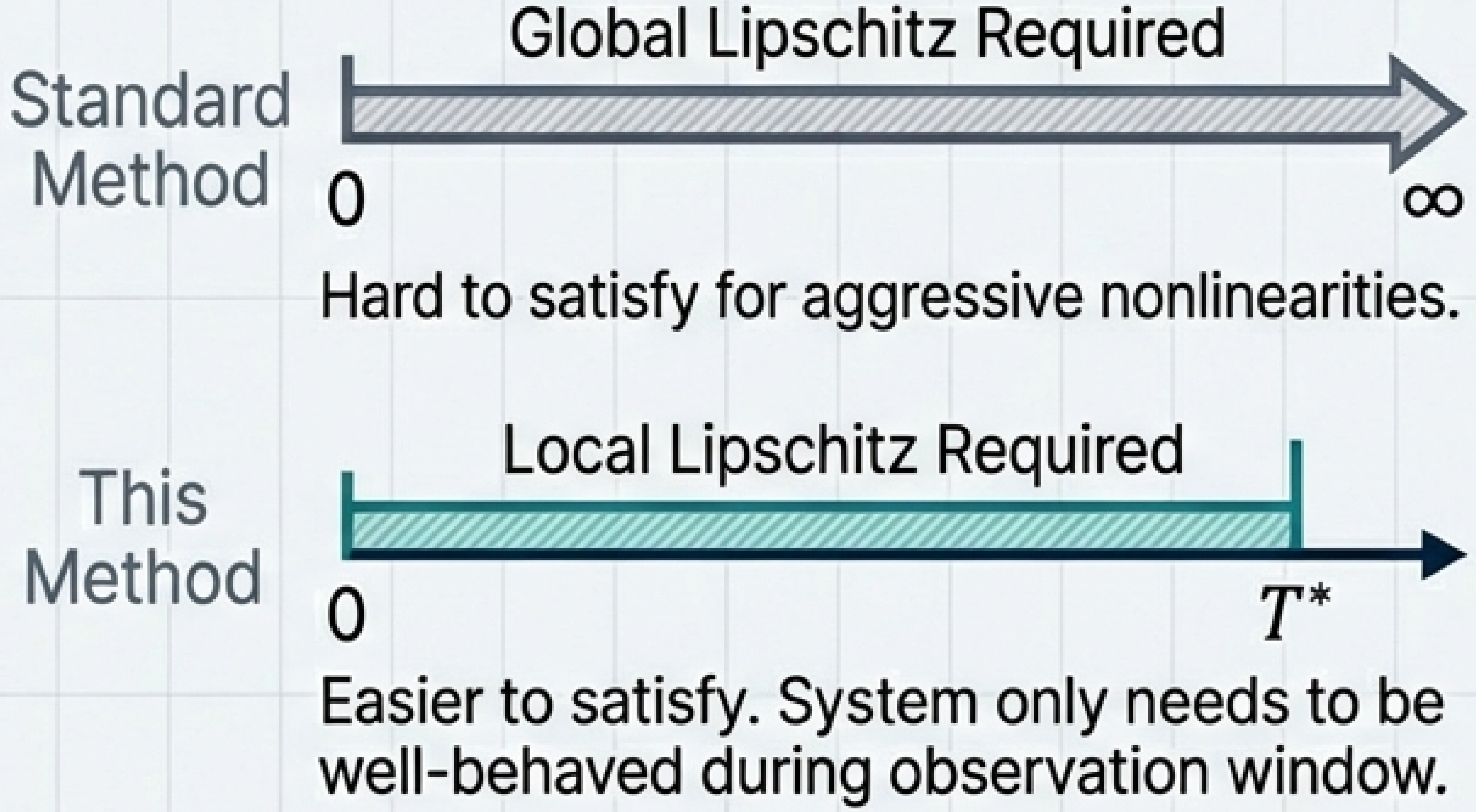
Assumption 1: Local Lipschitz Condition

The nonlinear function $f_j(t, x)$ must satisfy the inequality:

$$|f_j(t,x) - f_j(t,z)| \leq \theta \sum |x_k - z_k|$$

Crucially, this condition only needs to hold on the finite interval $[0, T^*)$.

Visual Comparison of Time Horizons



Significance: Allows application to systems that might be unstable or chaotic over long time scales, as long as they are observable for the short window T^*.

Design Procedure: Algorithm 1

Topology Analysis

- Calculate Graph Laplacian L
- Find eigenvalues λ_2
- Check connectivity vector ξ

Parameter Tuning

- Select ε
- Solve Linear Matrix Inequalities (LMIs)
- Find matrices P_i and $\bar{L}_i$

Gain Selection

- Compute coupling parameter γ
- Set scaling factor μ
- Define T^* (Prescribed Time)

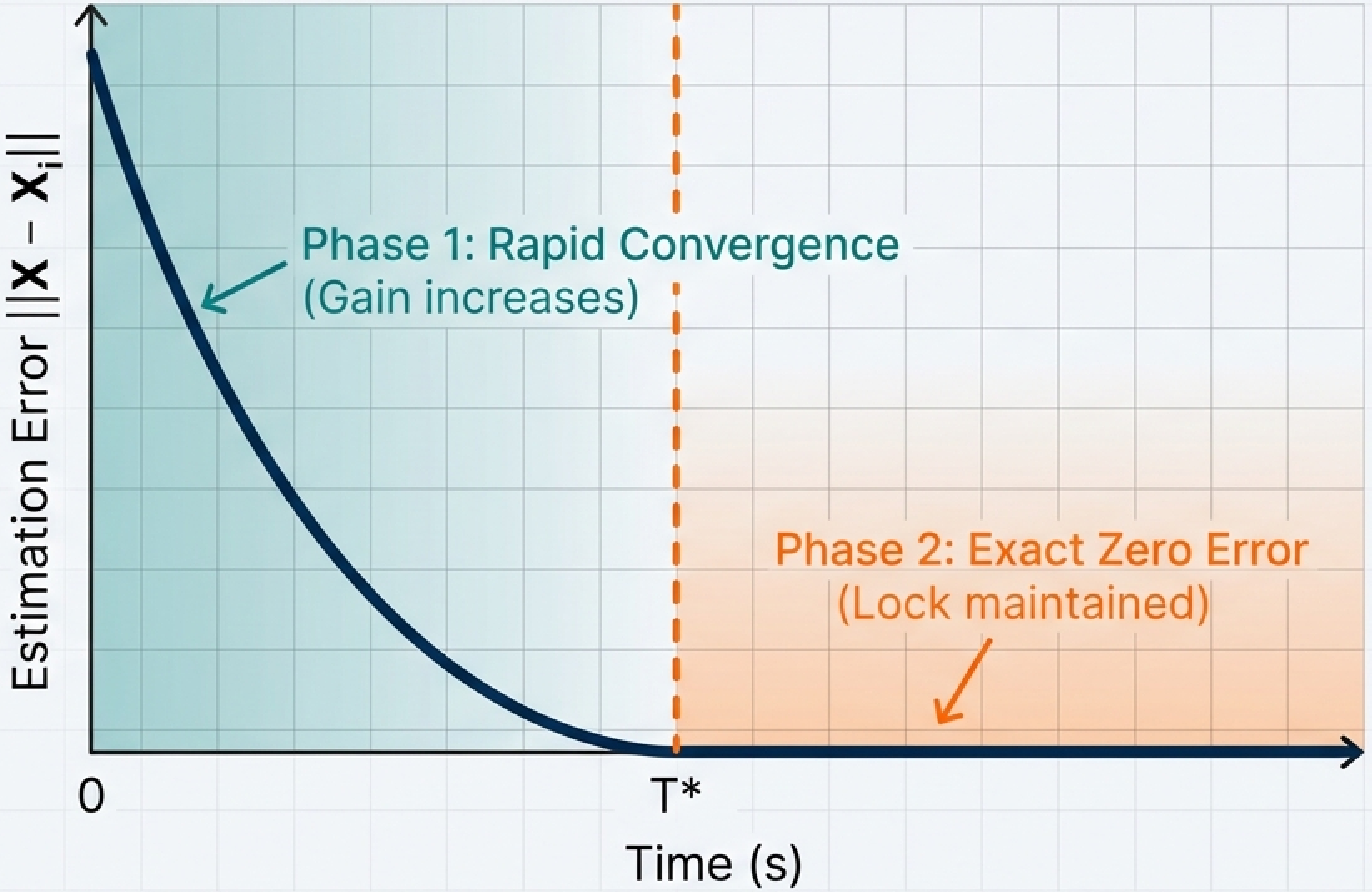
Execution

- Apply Time-Varying Gains
- $L_i(t)$ and $K_i(t)$
- Using scaling function Φ

Note: The calculation of L_i does not require (A, C_i) to be observable for every node, only for the network jointly.

Scenario A: The Ideal Case (No Disturbance)

Theorem 1: Distributed Prescribed Finite-Time Convergence



Outcome: The observer state $\mathbf{X}_i$ becomes mathematically becomes mathematically identical to system state $\mathbf{X}$ at exactly T*, and remains so indefinitely.

NotebookLM

Scenario B: The Realistic Case (With Disturbance)

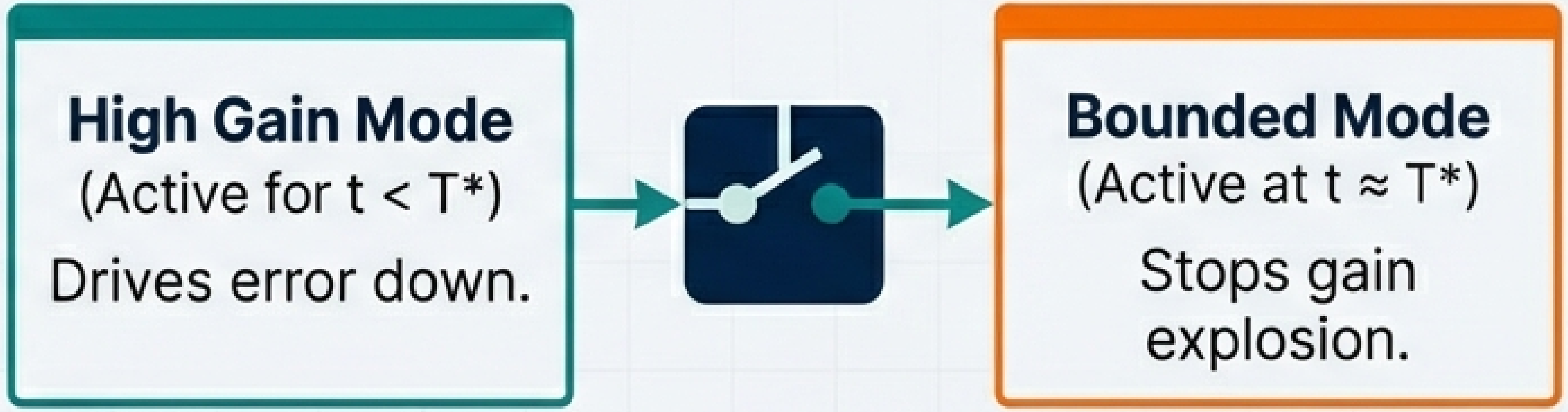
The Complication

In the presence of external disturbance $d(t) \neq 0$:

1. As $t \to T^*$, the gain $1/(T^*-t) \to$ **Infinity**.
2. This infinite gain amplifies the disturbance $d(t)$.
3. Result: Potential **instability** or **singularity** at T^*.

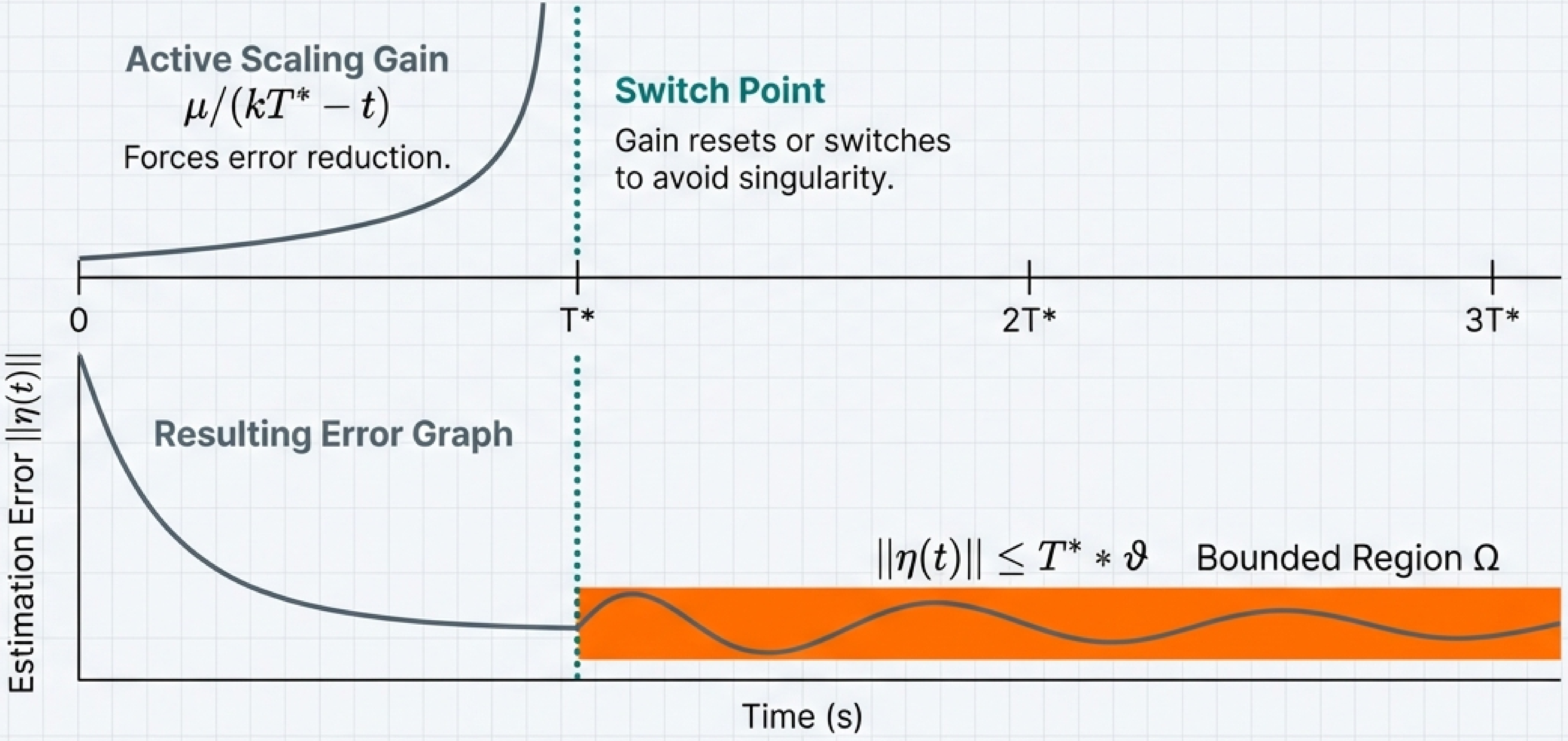
The Solution: Prescribed Finite-Time Boundedness

Theorem 2 introduces a switching strategy.



Result:
Error does not go to 0, but is trapped in a small set **Ω**. **||Error|| <= Bound**.

Robustness Strategy: The Switching Logic



Key Takeaway: The error bound is proportional to T*. A smaller prescribed time T* yields a tighter error bound.

Numerical Validation: 4th-Order Nonlinear System

Setup Details

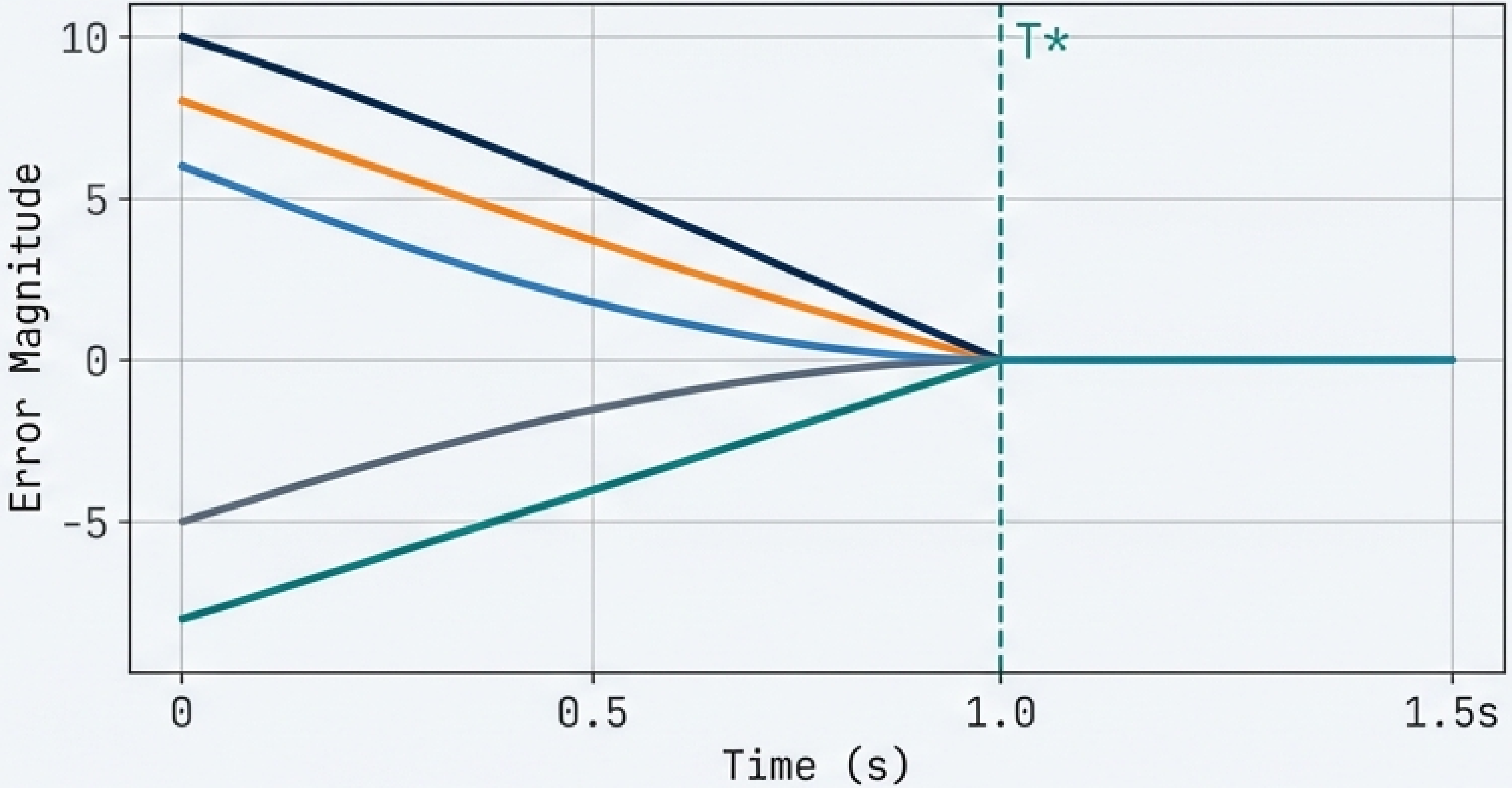
System Order: $n = 4$

Observers: $N = 5$

Sensors: Only Nodes 1 & 4 have sensors. Nodes 2, 3, 5 are blind.

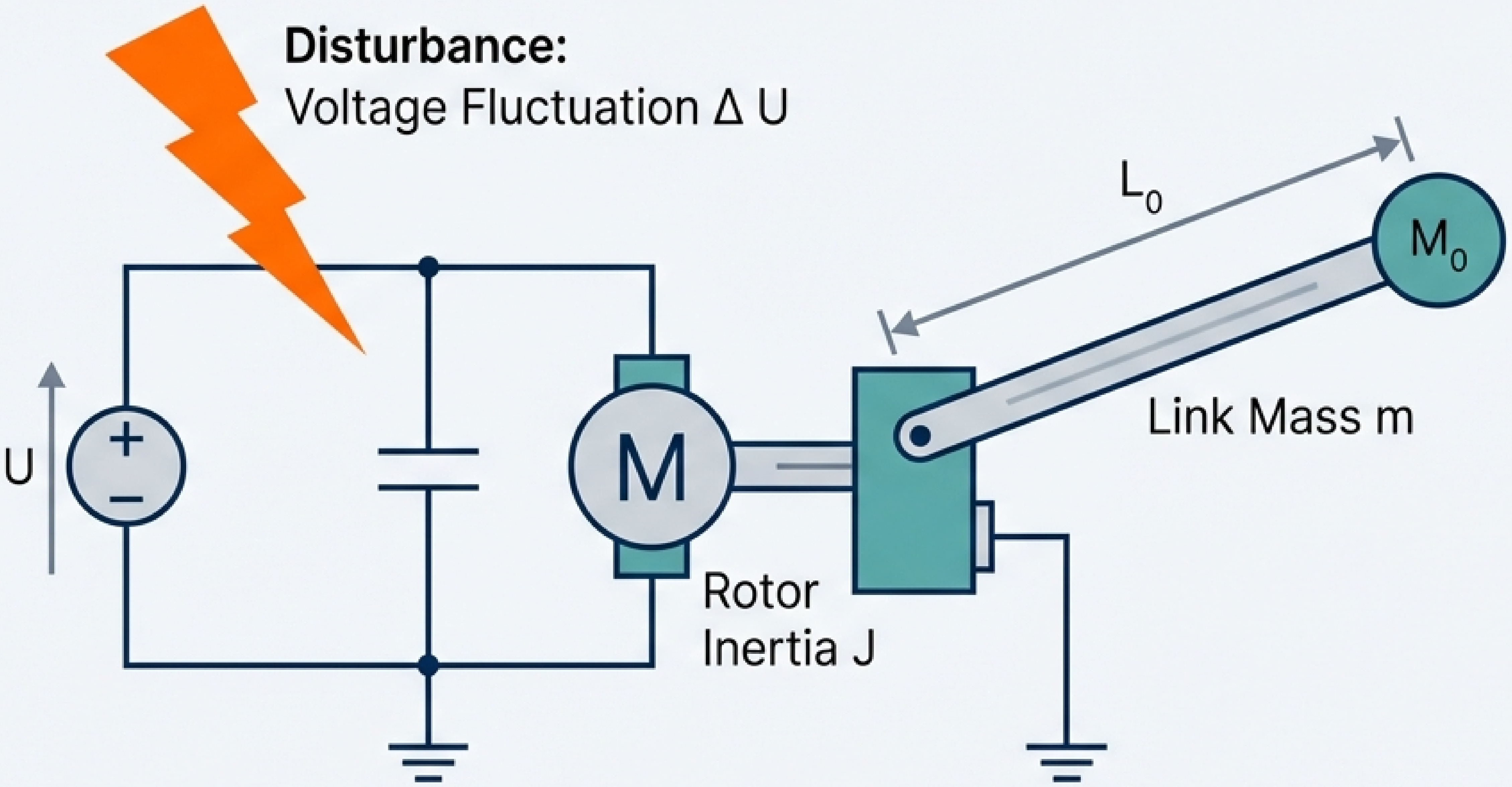
Prescribed Time: $T^* = 1.0$ s

Observer Estimation Errors



Complete synchronization achieved for all nodes, including blind ones.

Application: Electromechanical Single-Link Manipulator



Estimation Goal

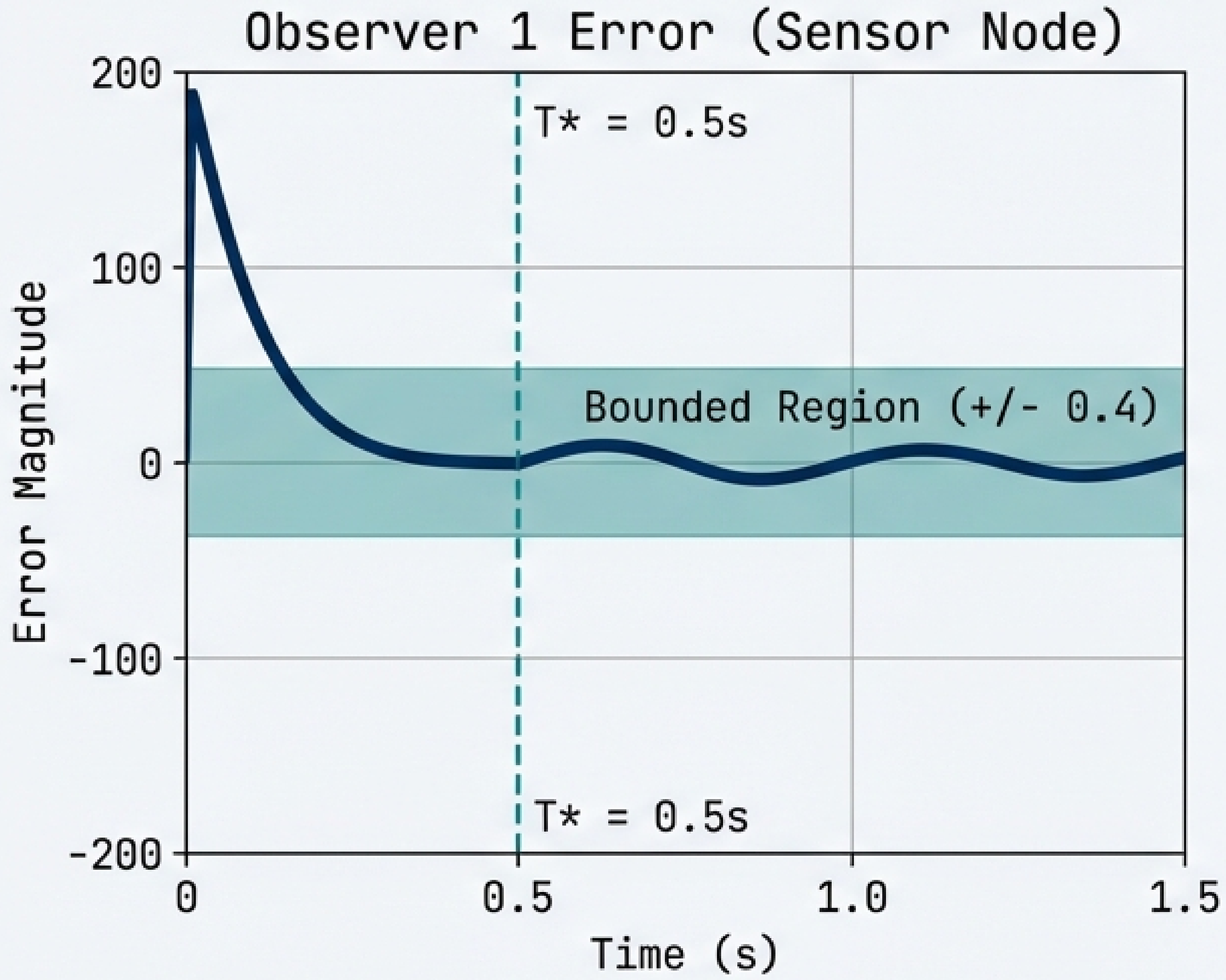
Target: Estimate Angular Position (**r**) and Current (**I**).

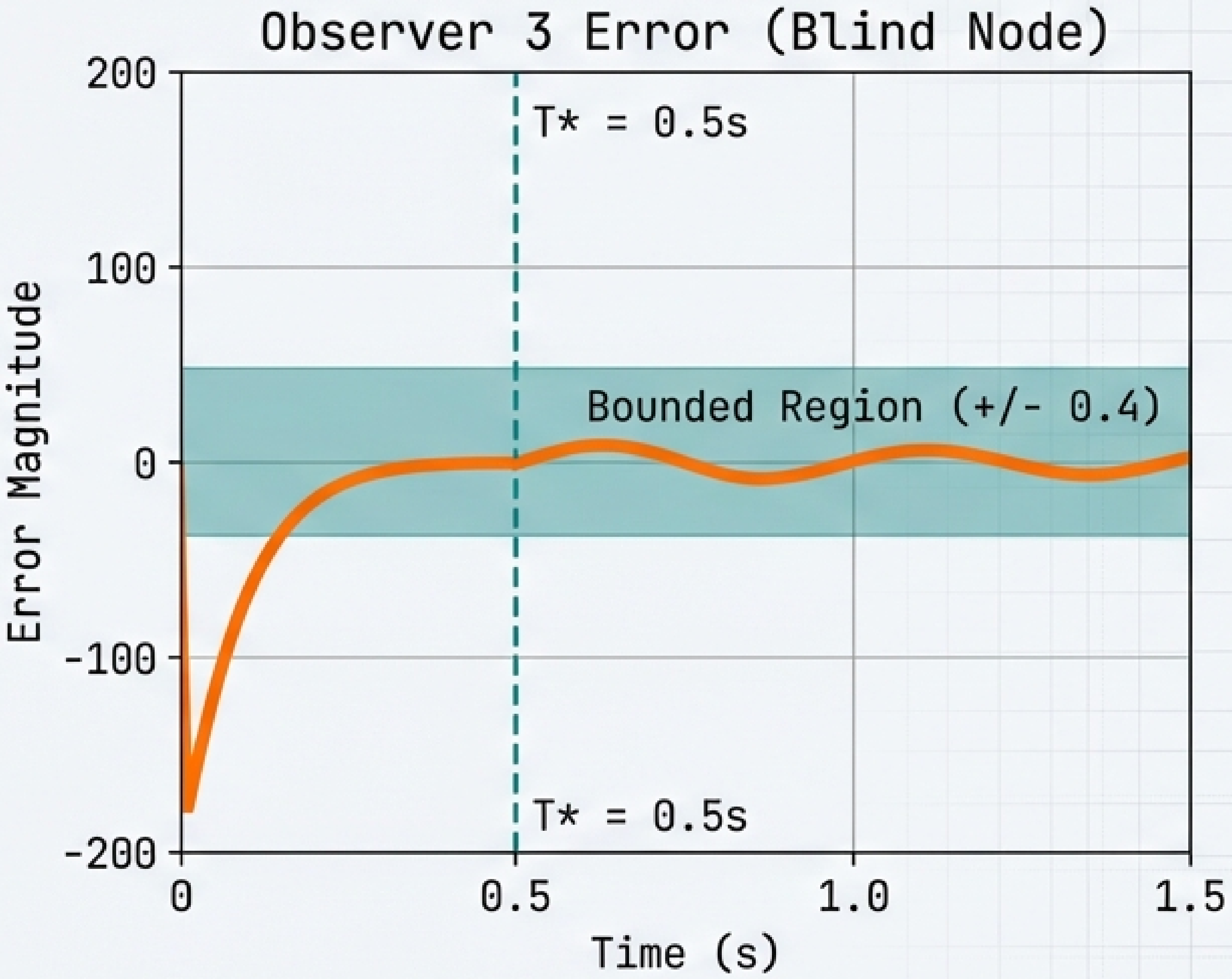
Network: 3 Observers.

Constraint: Only Nodes 1 & 2 have sensors. Node 3 is blind.

Parameter: Prescribed Time T* = 0.5s.

Results: Robustness Under Disturbance





Conclusion: Despite voltage noise $d(t)$ bounded by 10, the switching gain strategy keeps estimation error within +/- 0.4 after $T*$.

Summary & Impact

01.

Distributed Vision

Solved the "blind node" problem. Successfully reconstructs full states in strict-feedback nonlinear systems using only partial, local measurements.

02.

Prescribed Timing

Decoupled convergence time from initial conditions.
Engineers can now set an exact deadline T^* for system convergence.

03.

Robustness

Guaranteed bounded error in the presence of disturbances via a novel switching gain strategy.
Prevents instability while maintaining precision.

Enabling precise, time-critical monitoring for large-scale industrial sensor networks.