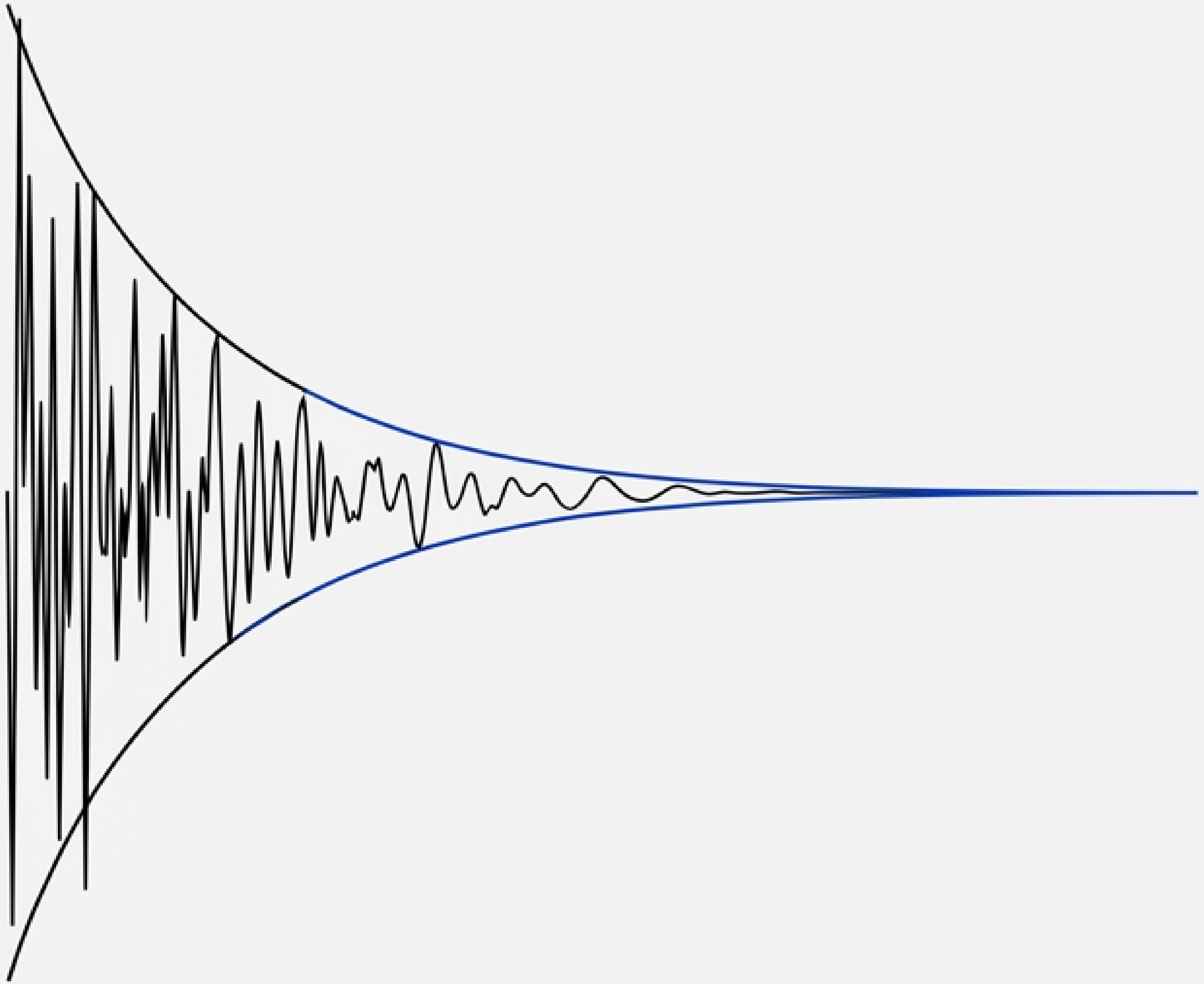
# Advanced Stabilization & Regulation in Nonlinear Systems

*A technical portfolio of recent contributions by Le Chang*

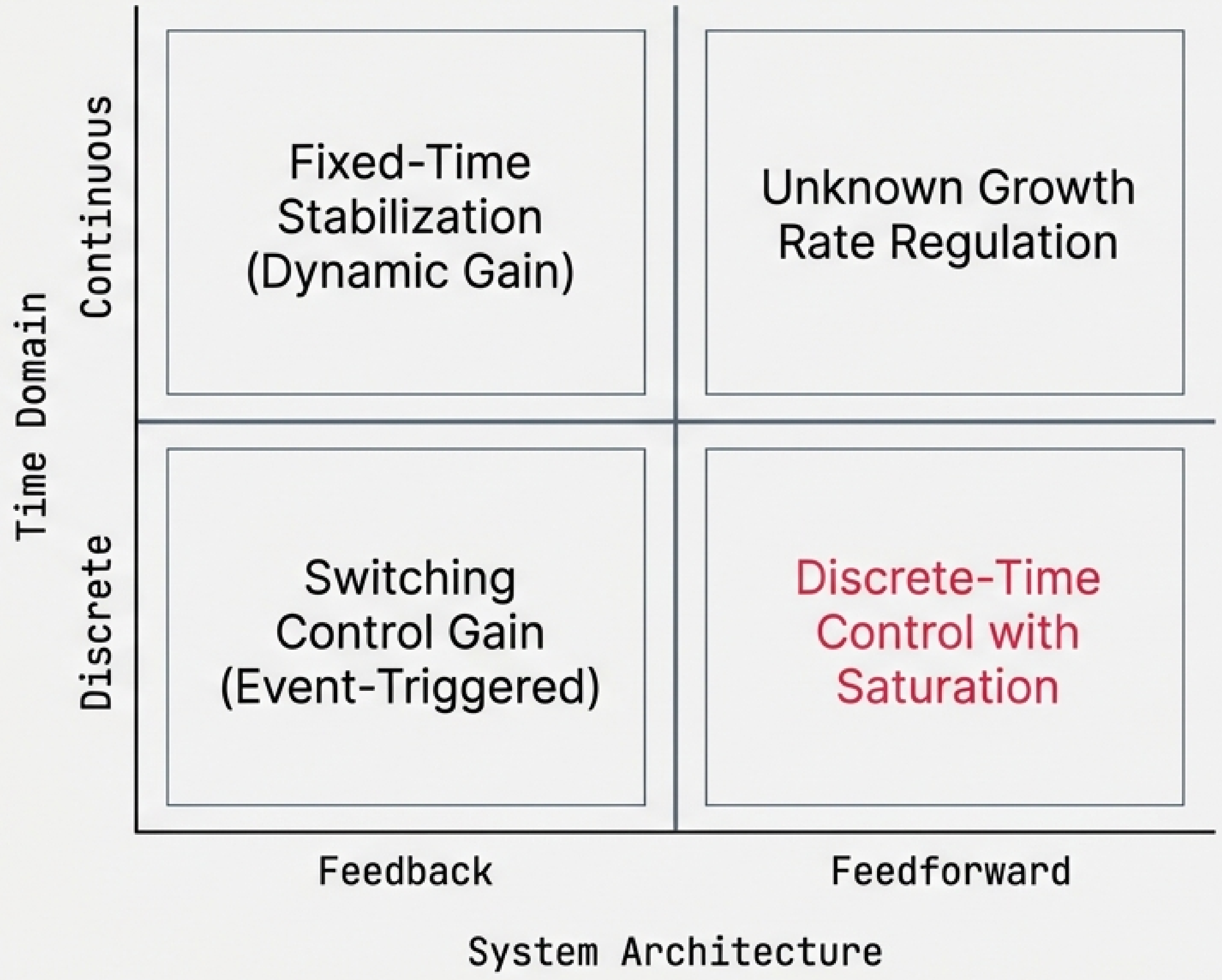


Synthesized from IEEE/CAA Journal of Automatica Sinica
and International Journal of Robust and Nonlinear Control.

# Orchestrating Stability Across System Classes

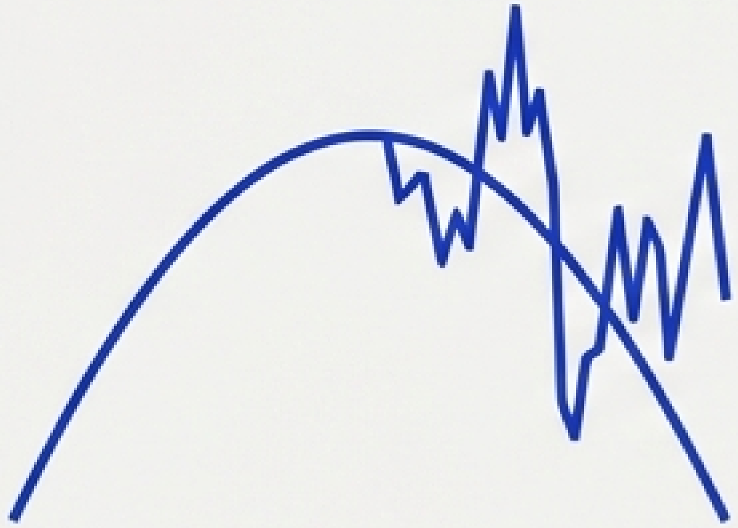
Le Chang, currently with the School of Aeronautics and Astronautics at Shanghai Jiao Tong University, focuses on distributed estimation and control of nonlinear systems. His background includes research at Shandong University and Swinburne University of Technology." Technology.

**“The Unifying Mission: Robustness under Uncertainty.”**



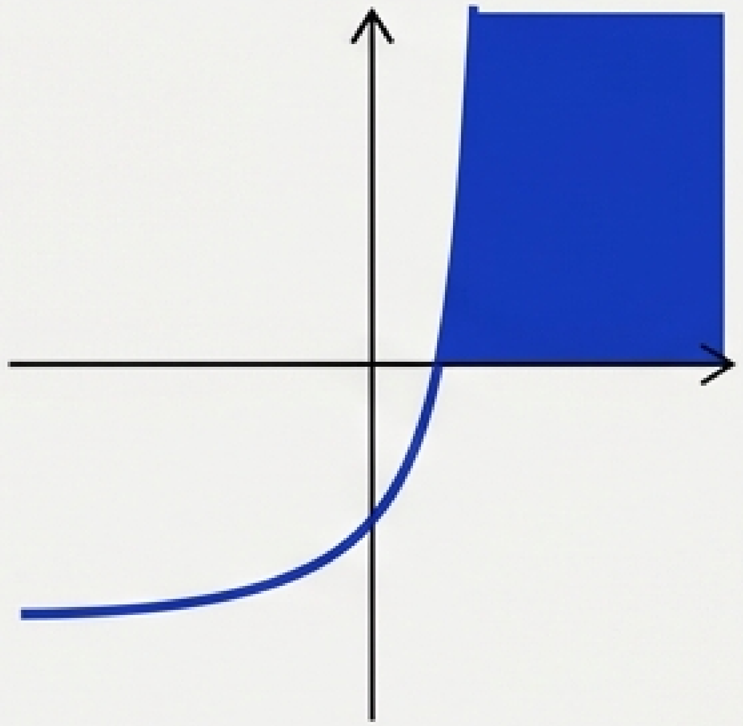
# The Landscape of Control Challenges

Le Chang's work specifically targets four "enemies" of stability in nonlinear systems.



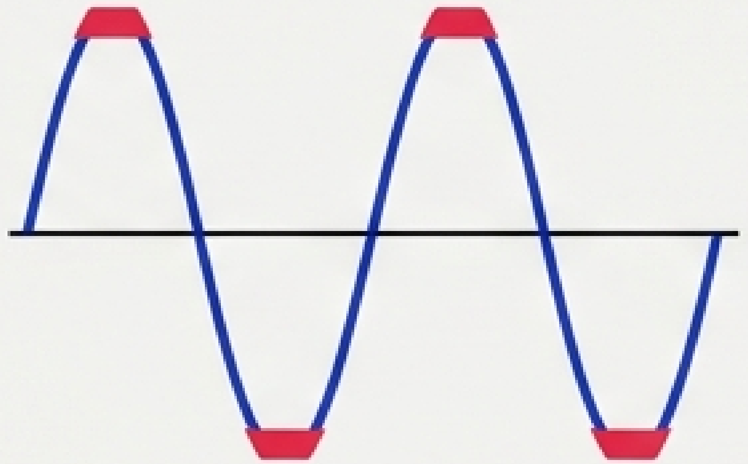
## Unmodeled Dynamics

Mathematical models fail to capture full real-world complexity, creating uncertainty in function $f(\cdot)$.



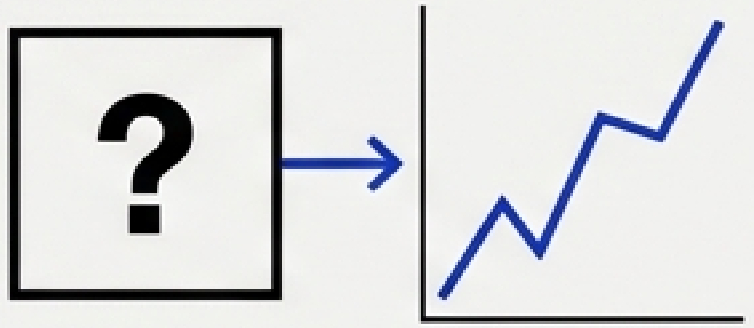
## Explosion of Complexity

In backstepping control, derivative calculations become exponentially complex as system order increases.



## Unknown Input Saturation

Physical actuators have limits. Demanding infinite energy causes system failure.

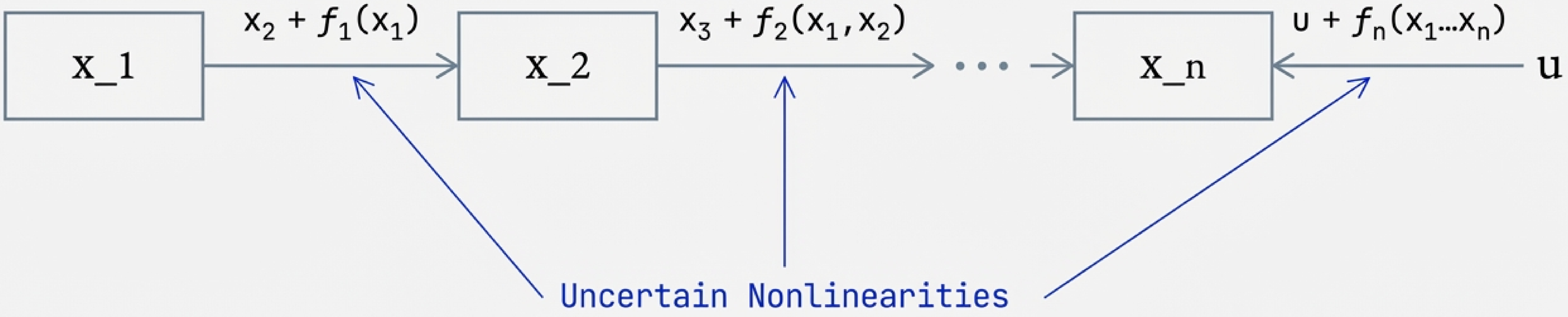



## Unknown Growth Rates

Nonlinearities that scale unpredictably, making it difficult to design a standard gain parameter.

# The Cascade of Strict-Feedback Systems

Strict-feedback systems are characterized by a triangular structure where state x_i is driven by x_i+1. This structure is common in mechanical and electromechanical systems.



# Fixed-Time Stabilization via Dynamic Gain

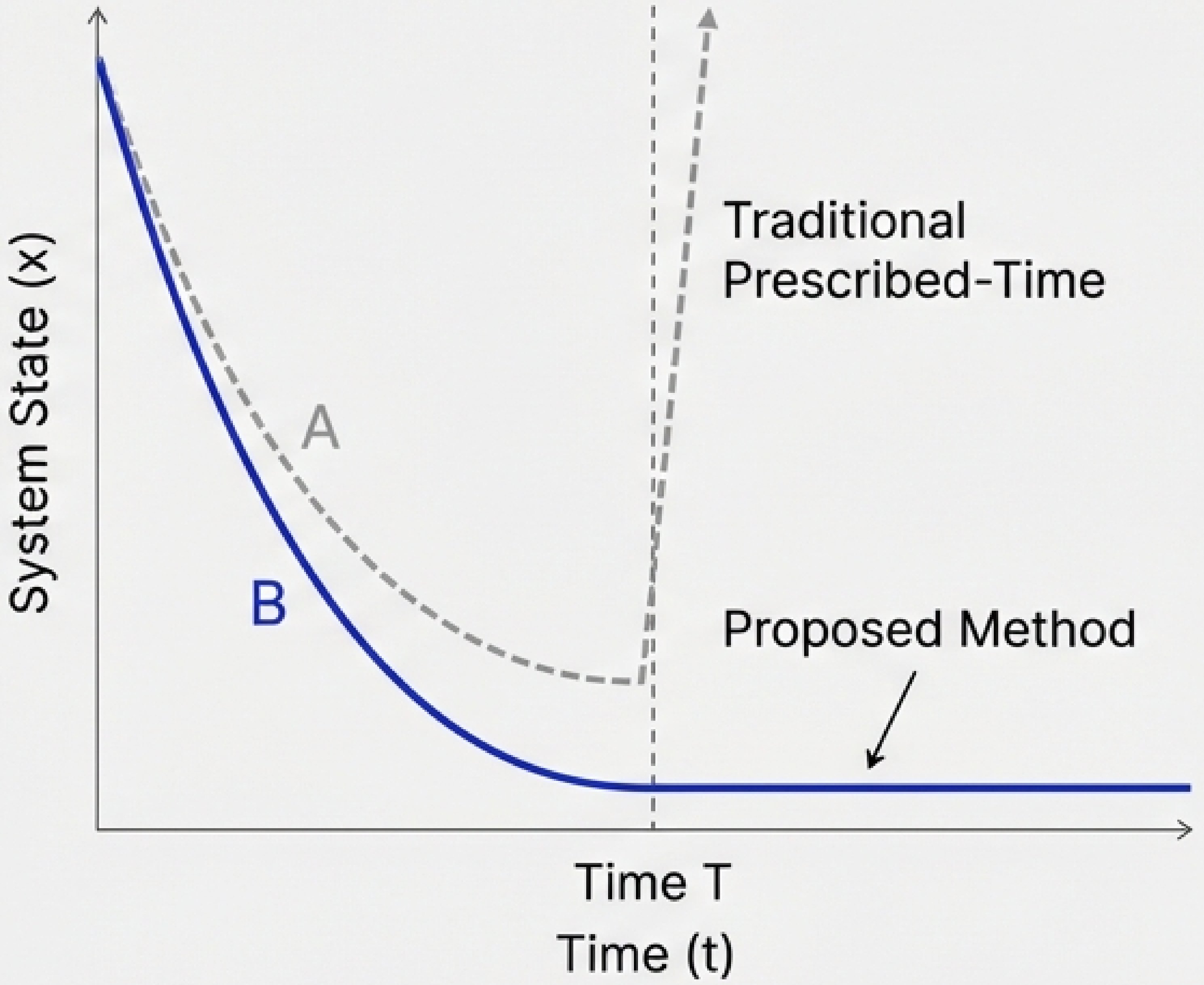
Source: IEEE/CAA **Journal of Automatica Sinica, Vol. 10, No. 2, 2023**

**The Challenge:** Standard "Prescribed-Time Control" uses time-varying functions that often blow up to infinity or stop working after time T.

**The Solution:** A State-Dependent Dynamic Gain method.

- Introduces two dynamic parameters (r_1, r_2) that depend on system states, not just time.
- Controller Form (Quasi-linear): $u = -k\_1 * (r_2{}^{n} / r_1{}^{n(1-\tau)}) * x\_1 - \ldots$

**Key Innovation:** Eliminates "explosion of complexity" and remains active beyond convergence time T.



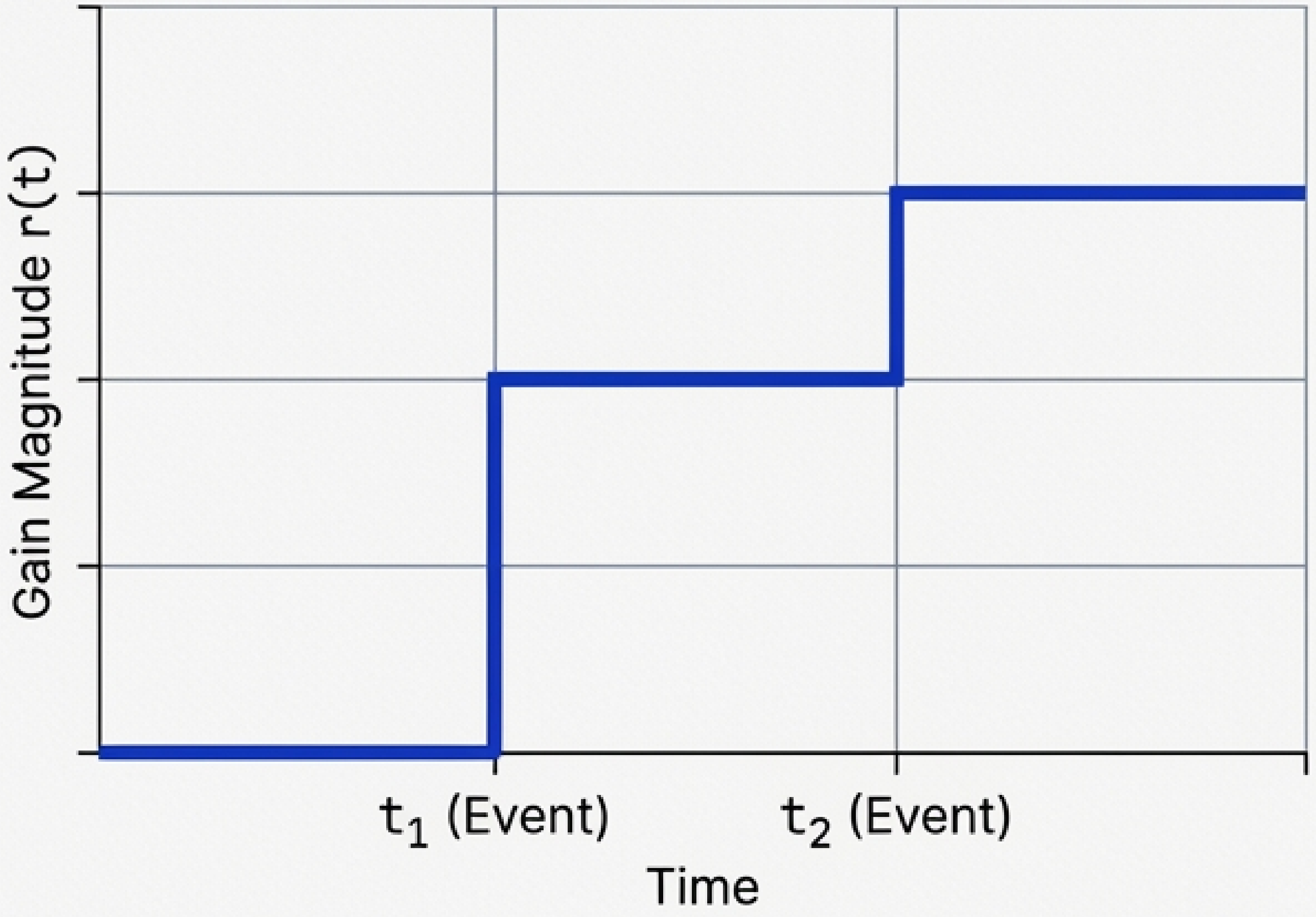
# Handling Non-Polynomial Growth with Switching Gains

Source: International Journal of Control, 2025

The Challenge: When nonlinear growth rates aren't simple polynomials, fixed gains fail and continuous high-gains are wasteful.

The Solution: An Event-Triggered Switching Mechanism.

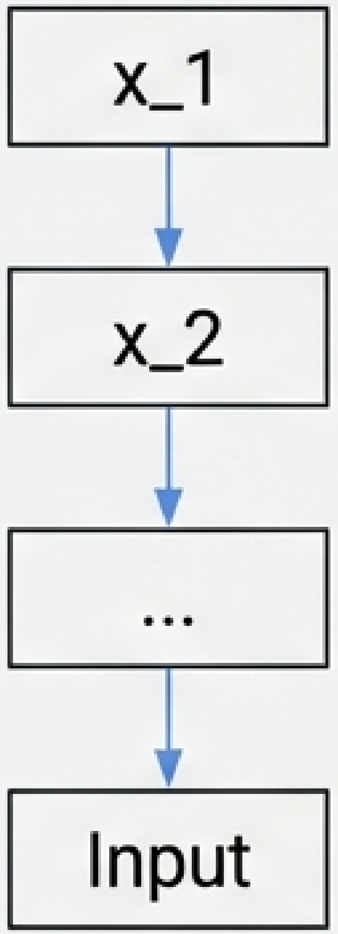
- Uses a high-gain observer with a switched parameter $r(t)$.
- Trigger: Gain updates only when error exceeds threshold defined by $\lambda_{1,k}$.
- Update Law: $r(t) = \max\{\lambda_2 * \phi_k, \lambda_{1,k} * r(t_{k-1})\}$.




Gain remains constant between events to save computation, settling at the necessary level for stability.

# Shifting Architectures: The Feedforward Challenge

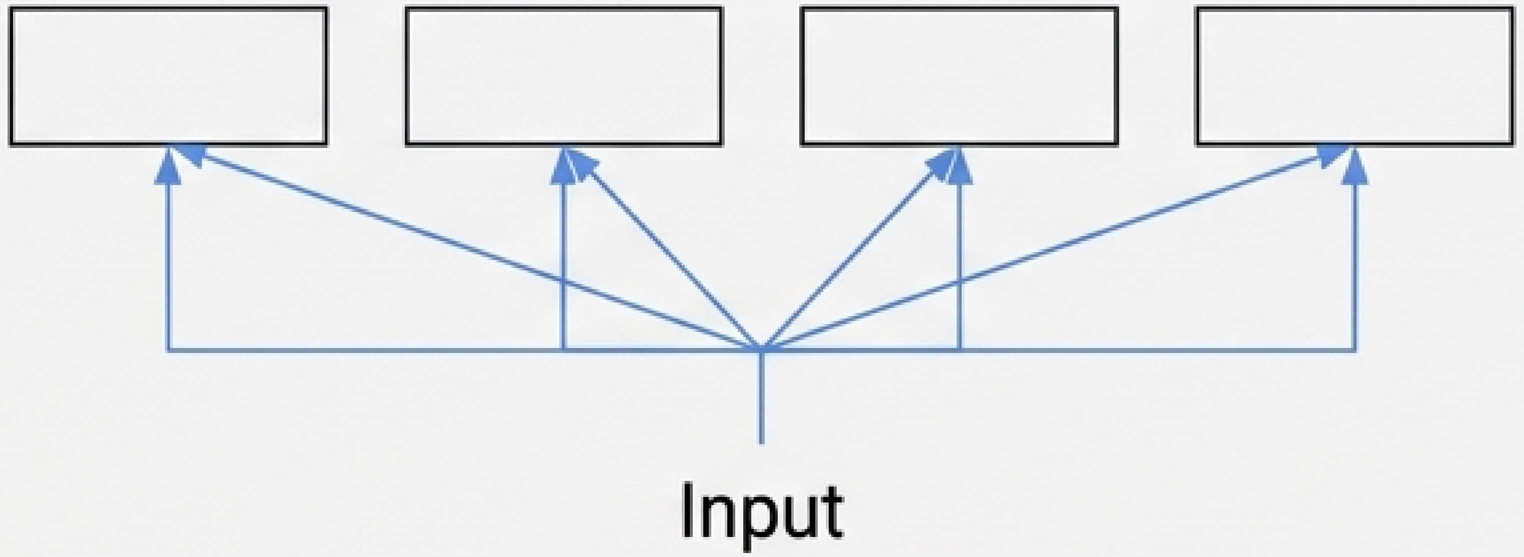
## **Feedback Architecture** (Papers 1 & 2)




**Cascading States:**
x_1 -> x_2 -> ... -> Input

Generally feedback linearizable.

## **Feedforward Architecture** (Papers 3 & 4)




**Broadcast Input:** Input affects all dynamics directly.

Harder to control. Lack of feedback linearization.

New Constraints for this section: **Discrete-Time Sampling** & **Unknown Input Saturation**.

# Discrete-Time Control with Unknown Saturation

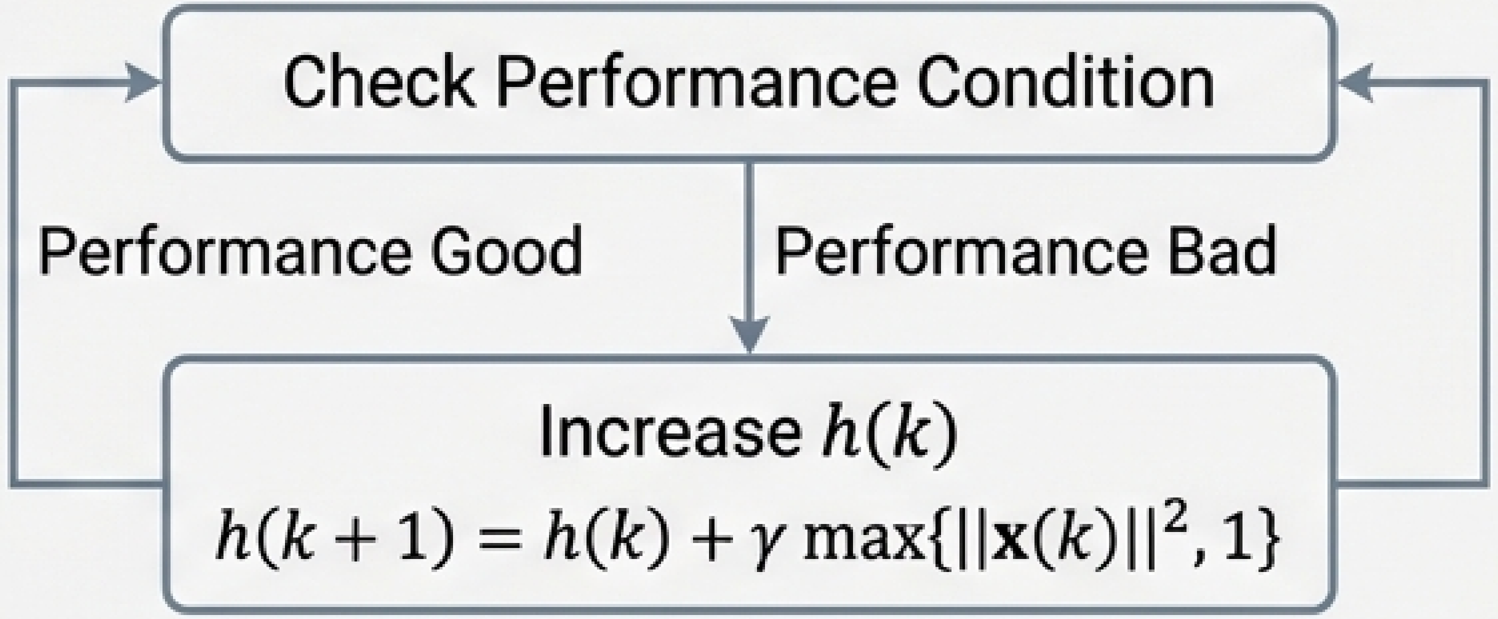
Source: Int. J. Robust Nonlinear Control, 2022

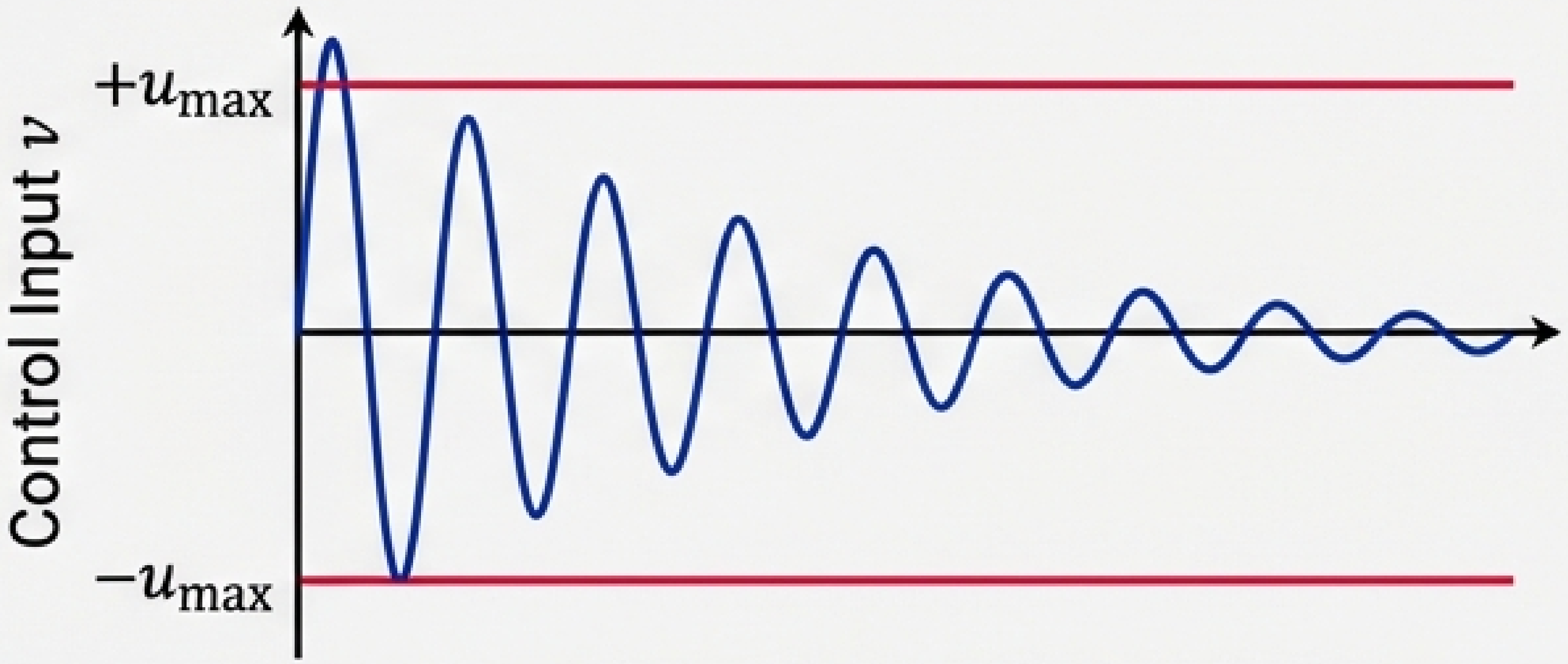
The Challenge: Controlling discrete (digital) systems where the physical actuator limit ($u_{\max}$) is unknown.

The Solution: Low Gain Feedback with Adaptive Parameter $h(k)$.

$$v(k) = -k_1 \frac{x_1(k)}{h^n(k)} - \cdots - k_n \frac{x_n(k)}{h(k)}$$

Parameter $h(k)$ grows to suppress the control signal magnitude until it fits inside the unknown saturation limit.




NotebookLM

# Regulating the 'Black Box': Unknown Input-Dependent Rates

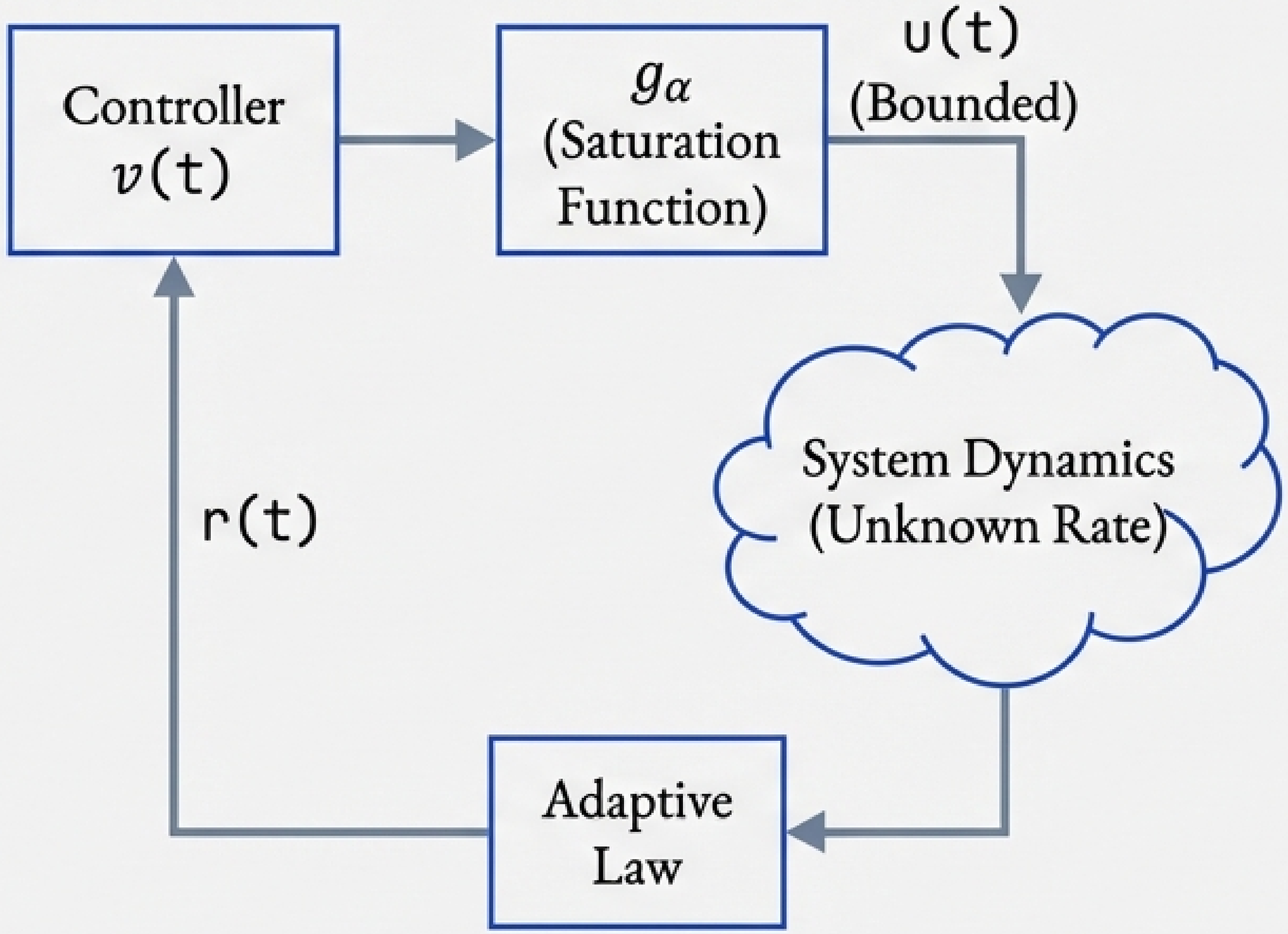
Source: Int. J. Robust Nonlinear Control, 2025

**The Challenge:**

The nonlinear growth rate involves $\phi(u)$, an entirely unknown function of the input. Standard adaptive methods fail.

**The Solution:** Bounded Input Transformation.

1. Enforce bound on input using $u(t) = g_\alpha(v(t))$.
2. Transform "unknown input rate" problem into a simpler "unknown constant" problem.
3. Use time-varying parameter $r(t)$ to dominate the unknown constant.



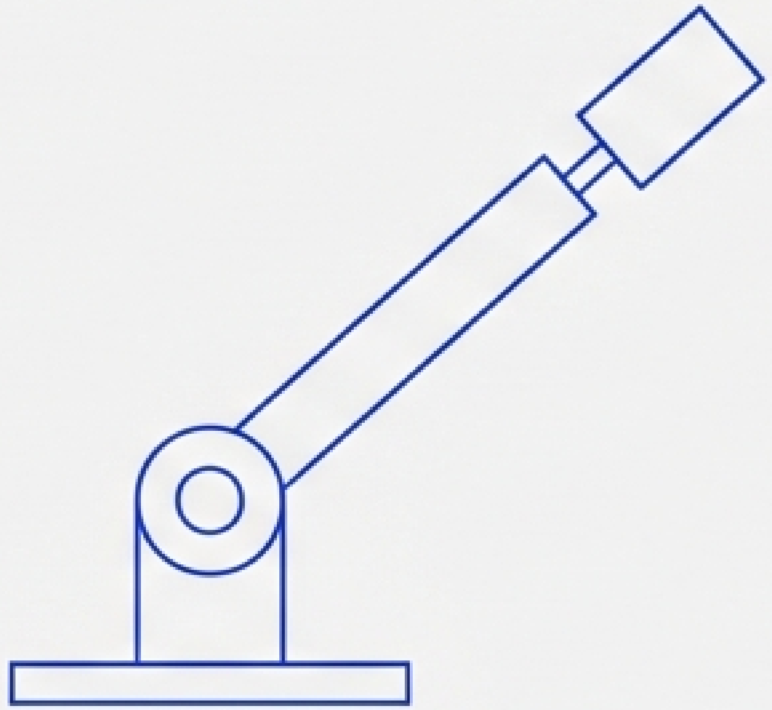
# The Gain Toolkit: Matching Strategy to Uncertainty

| Strategy | System Type | Best For... |
|---|---|---|
| Dynamic Gain (State-Dependent) | Strict-Feedback | Fixed-Time convergence without complexity explosion. |
| Switching Gain (Event-Triggered) | Output Feedback | Reducing computation when updates are expensive. |
| Low Gain (Parameter $h$) | Discrete Feedforward | Handling unknown saturation limits. |
| Adaptive/Time-Varying Gain | Uncertain Feedforward | Managing entirely unknown growth structures. |

**There is no universal controller; the gain dynamics must mirror the structure of the uncertainty.**

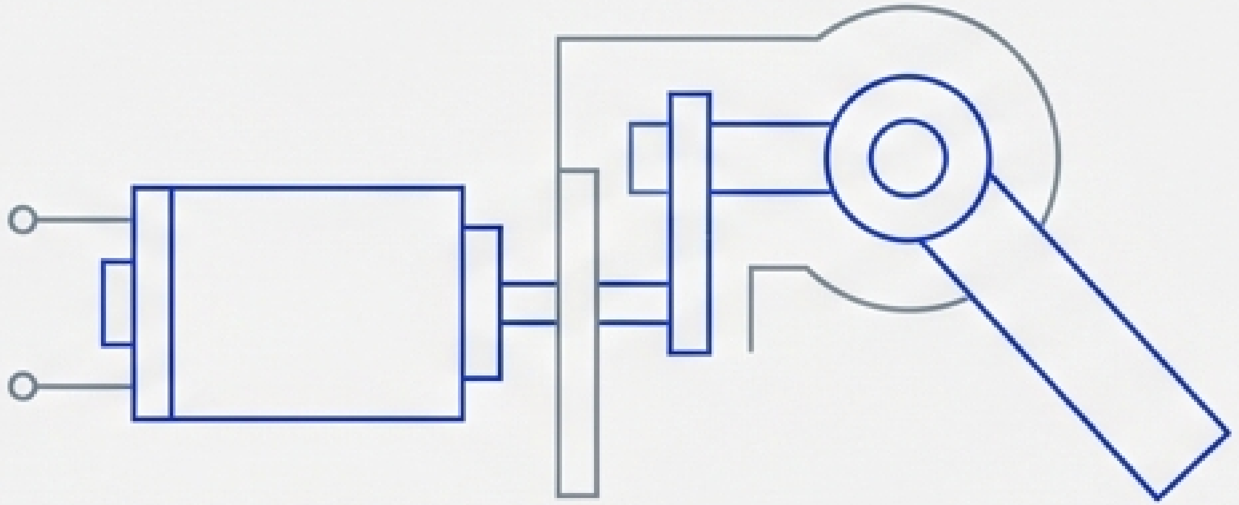
# Verification: Strict-Feedback Applications

## One-Link Manipulator (Paper 1)

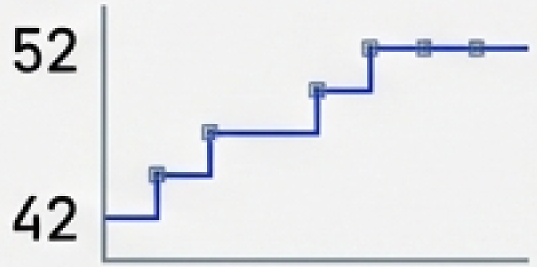


- States (x_1, x_2, x_3) converge to zero before T = 4s.
- Dynamic gain r_1 remains bounded [0, 1].

## Robotic Manipulator with DC Motor (Paper 2)



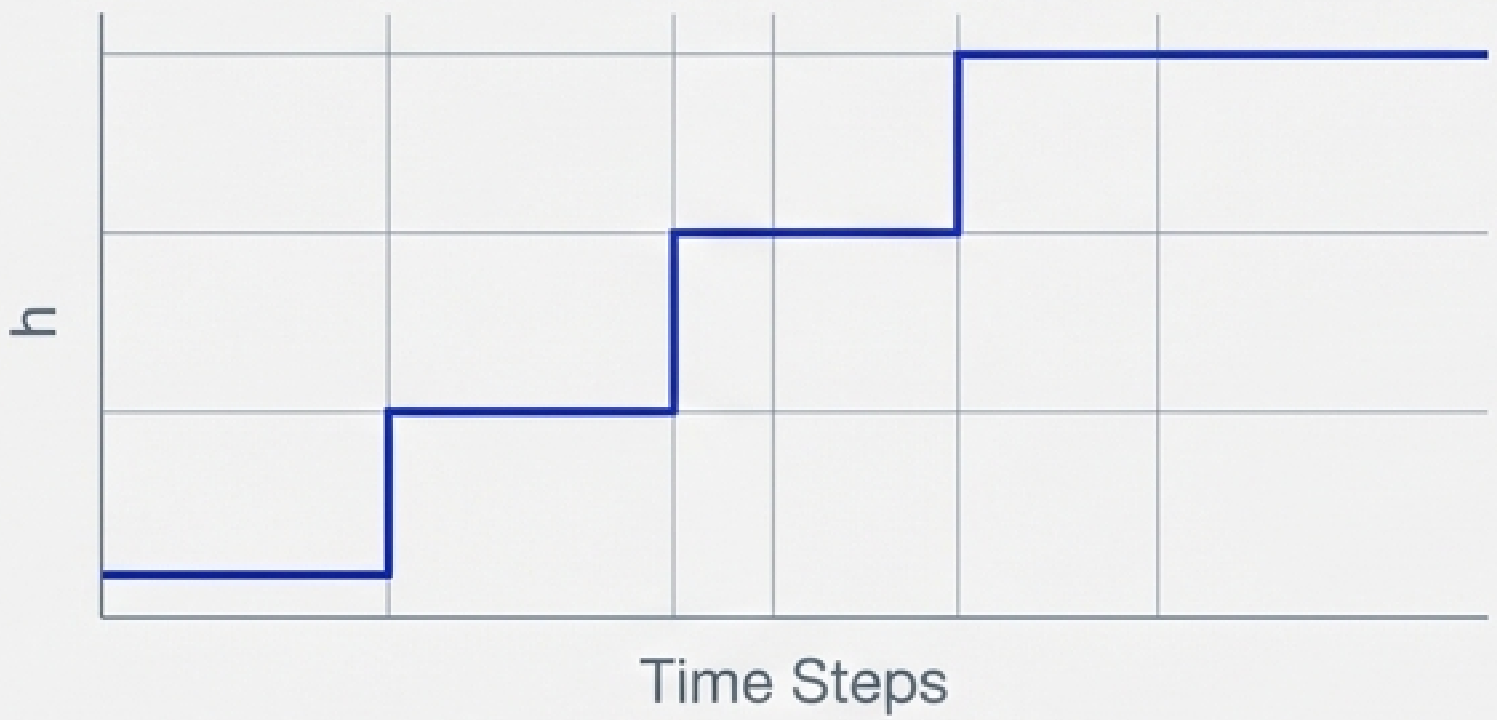
- Comparison vs. Existing Methods (Zhao & Jiang).
- Le Chang Method: Faster convergence, smaller control amplitude.
- Gain Update: Discrete steps from ~42 to ~52.



# Verification: Feedforward Applications

## Numerical Discrete System (Paper 3)

4th-order system with unknown saturation.

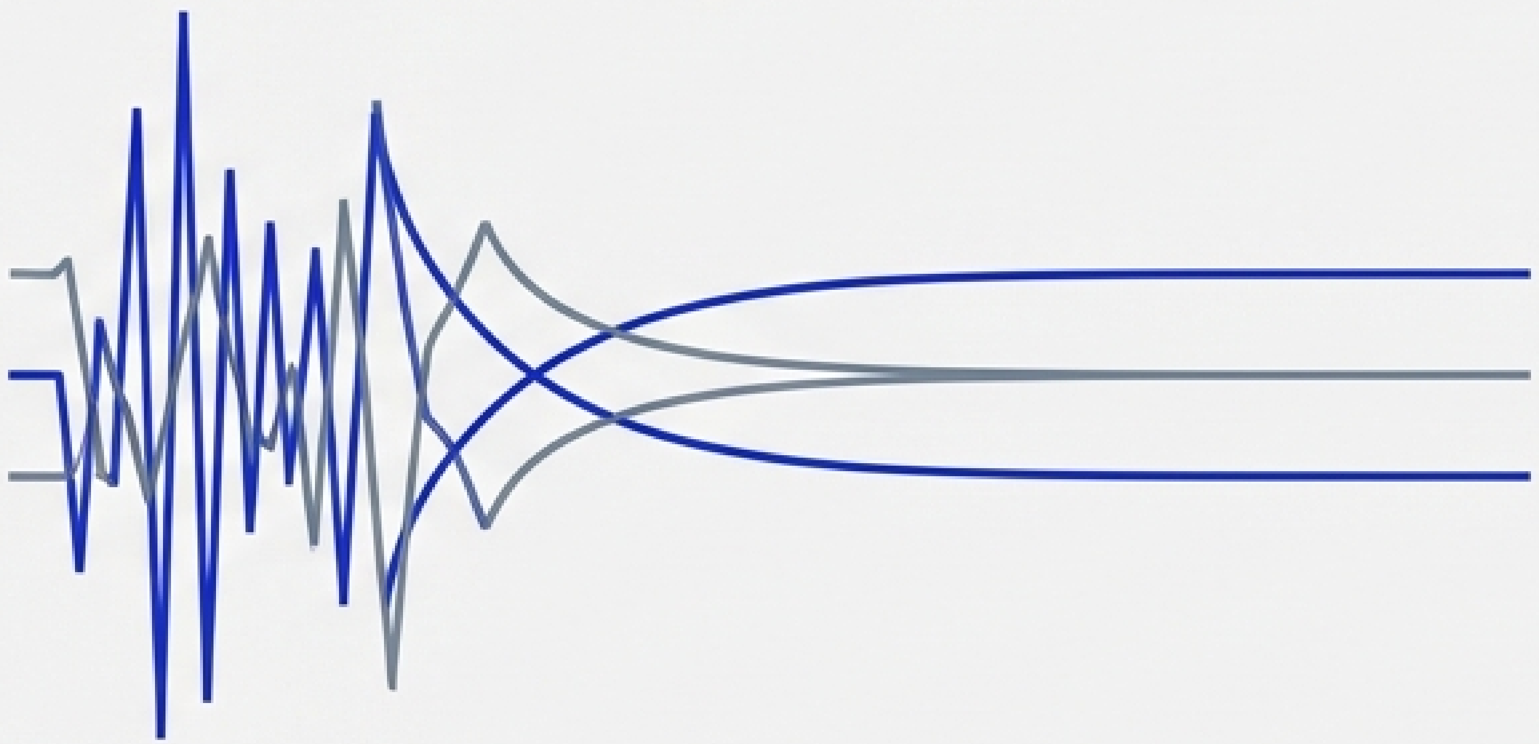



Input kept bounded (<0.3) while states drive to zero.

## Liquid Level Resonant Circuit (Paper 4)

LC circuit with unknown resistance dynamics:

$$R_a = \ln(c_1 t + c_2)\phi(u)$$



Time-varying gain $r(t)$ adjusts to unknown resistance changes.

# Redefining Control Robustness

1. **Removing Singularities**
   Proved fixed-time stabilization is possible without control signal blow-up near time T.
2. **Unified Digital Control**
   Bridged the gap between low-gain feedback theory and discrete-time implementation.
3. **The 'Black Box' Solution**
   First rigorous method for handling entirely unknown input-dependent growth rates.
4. **Computational Efficiency**
   Reduced computational load via switching and event-triggered designs.

# Stability in an Uncertain World

> "The progression from Fixed-Time Feedback to Unknown-Rate Feedforward systems demonstrates a singular philosophy: We do not need to model every uncertainty to control it."

By designing intelligent, dynamic gains that react to system states—whether through switching, adapting, or time-varying parameters—we can ensure stability for complex mechanical and electrical systems, even when the underlying dynamics remain partially unknown.

# Source Material


Zhang, C. H., Chang, L., et al. "Fixed-Time Stabilization of a Class of Strict-Feedback Nonlinear Systems via Dynamic Gain Feedback Control." *IEEE/CAA J. Autom. Sinica*, 2023.

Chang, L., Fu, C., & Zhang, H. "Global output feedback stabilisation for nonlinear systems via a switching control gain approach." *International Journal of Control*, 2025.

Chang, L., & Fu, C. "Designing a stabilizing controller for discrete-time nonlinear feedforward systems with unknown input saturation." *Int. J. Robust Nonlinear Control*, 2022.

Chang, L., & Zhang, X. "Global Output Regulation for Uncertain Feedforward Nonlinear Systems With Unknown Nonlinear Growth Rate." *Int. J. Robust Nonlinear Control*, 2025.