

Lecture 6.2: Advanced Policy Gradients

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July 1st, 2022

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- Solve the constrained optimization problem
- Natural gradient

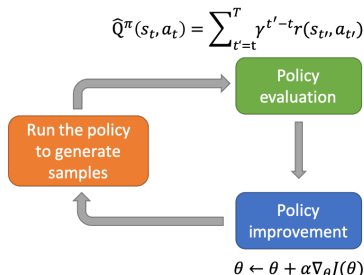
2 Trust region policy optimization (TRPO)

3 Proximal policy optimization (PPO)

Review: Vanilla policy gradient (REINFORCE)

REINFORCE algorithm: Loop:

1. sample $\{\tau^i\}$ from $\pi_\theta(a_t|s_t)$ (run the policy)
2. $\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_{i=1}^N \sum_{t=0}^T \nabla_\theta \log \pi_\theta(a_t^i|s_t^i) \sum_{t'=t}^T \gamma^{t'-t} r(s_t^i, a_t^i)$
3. $\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$



Problems of vanilla policy gradient (REINFORCE)

$$\nabla_{\theta} J(\theta) \approx \frac{1}{N} \sum_{i=1}^N \sum_{t=1}^T \nabla_{\theta} \log \pi_{\theta}(a_t^i | s_t^i) Q^{\pi}(s_t^i, a_t^i)$$
$$\theta \leftarrow \theta + \alpha \nabla_{\theta} J(\theta)$$

- Hard to select the step size α
 - Too big step: Bad policy \rightarrow data collected under bad policy \rightarrow we cannot recover (in Supervised Learning, data does not depend on neural network weights)
 - Too small step: Not efficient use of experience (in Supervised Learning, data can be trivially re-used)

Problems of vanilla policy gradient (REINFORCE)



- Small changes in the policy parameters can unexpectedly lead to big **changes** in the policy

Gradient descent in parameter space

- The step size in gradient descent results from solving the following optimization problem, e.g., using line search

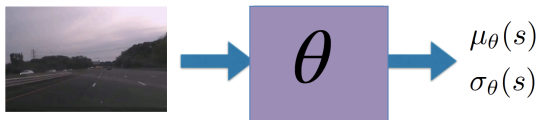
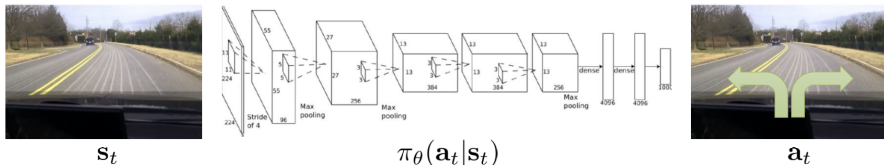
$$d^* = \arg \max_{||d|| \leq \epsilon} J(\theta + d)$$

- Euclidean distance in parameter space
- Stochastic gradient descent (SGD)

$$\theta \leftarrow \theta + d^*$$

Hard to pick the threshold ϵ

- It is hard to predict the result on the parameterized distribution
 - Especially for nonlinear function approximators, e.g., neural networks



Gradient descent in distribution space

- Gradient descent in parameter space

$$d^* = \arg \max_{||d|| \leq \epsilon} J(\theta + d)$$

- **Natural gradient descent:** the step size in parameter space is determined by considering the KL divergence in the distributions before and after the update

$$d^* = \arg \max_d J(\theta + d), \quad s.t. D_{\text{KL}}(\pi_\theta || \pi_{\theta+d}) \leq \epsilon$$

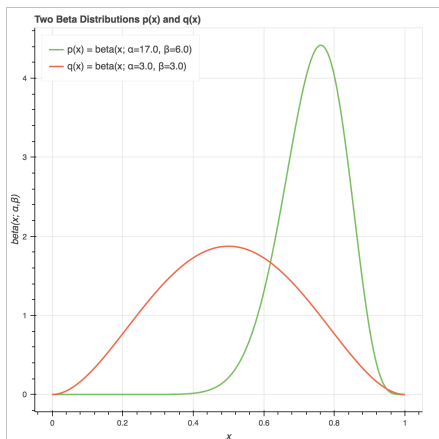
- **KL divergence** in distribution space
- Easier to pick the distance threshold!!!

Distance for probability distributions

- How to calculate the distance between two points in a 2D coordinate?

$$\text{distance} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

- Euclidean distance



- How to calculate the distance between two **probability distributions**, $p(x)$ and $q(x)$?

Kullback-Leibler (KL) divergence

- A measure of how one probability distribution, $p(x)$, is different from a second, reference probability distribution, $q(x)$

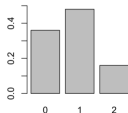
$$D_{\text{KL}}(p(x)||q(x)) = \sum_i p(x_i) \log \frac{p(x_i)}{q(x_i)}$$

$$D_{\text{KL}}(p(x)||q(x)) = \int_x p(x) \log \frac{p(x)}{q(x)} dx$$

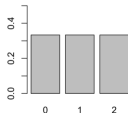
- A KL divergence of 0 indicates that the two distributions are identical

KL divergence: An example

Distribution P
Binomial with $p = 0.4$, $N = 2$



Distribution Q
Uniform with $p = 1/3$



$$\begin{aligned} D_{\text{KL}}(P \parallel Q) &= \sum_{x \in \mathcal{X}} P(x) \ln \left(\frac{P(x)}{Q(x)} \right) \\ &= 0.36 \ln \left(\frac{0.36}{0.333} \right) + 0.48 \ln \left(\frac{0.48}{0.333} \right) + 0.16 \ln \left(\frac{0.16}{0.333} \right) \\ &= 0.0852996 \end{aligned}$$

$$\begin{aligned} D_{\text{KL}}(Q \parallel P) &= \sum_{x \in \mathcal{X}} Q(x) \ln \left(\frac{Q(x)}{P(x)} \right) \\ &= 0.333 \ln \left(\frac{0.333}{0.36} \right) + 0.333 \ln \left(\frac{0.333}{0.48} \right) + 0.333 \ln \left(\frac{0.333}{0.16} \right) \\ &= 0.097455 \end{aligned}$$

x	0	1	2
Distribution P(x)	0.36	0.48	0.16
Distribution Q(x)	0.333	0.333	0.333

KL divergence: A test

- Suppose two Gaussian distributions:

$$p(x) \sim \mathcal{N}(\mu_1, \sigma_1^2), \quad q(x) \sim \mathcal{N}(\mu_2, \sigma_2^2)$$

- What is $D_{\text{KL}}(p(x)||q(x))$?

$$\log \frac{\sigma_2}{\sigma_1} + \frac{\sigma_1^2 + (\mu_1 - \mu_2)^2}{2\sigma_2^2} - \frac{1}{2}$$

KL divergence between two Gaussians

$$p(x) \sim \mathcal{N}(\mu_1, \sigma_1^2), \quad \mathbb{E}_{p(x)}[x] = \mu_1, \quad \text{var}_{p(x)}[x] = \mathbb{E}[(x - \mu_1)^2] = \sigma_1^2$$

$$\begin{aligned} D_{\text{KL}}(p(x)||q(x)) &= \mathbb{E}_{p(x)}[\log p(x) - \log q(x)] \\ &= \mathbb{E}_{p(x)} \left[-\log(\sqrt{2\pi}\sigma_1) - \frac{(x - \mu_1)^2}{2\sigma_1^2} + \log(\sqrt{2\pi}\sigma_2) + \frac{(x - \mu_2)^2}{2\sigma_2^2} \right] \\ &= \log \frac{\sigma_2}{\sigma_1} - \frac{\mathbb{E}_{p(x)}[(x - \mu_1)^2]}{2\sigma_1^2} + \frac{\mathbb{E}_{p(x)}[(x - \mu_1 + \mu_1 - \mu_2)^2]}{2\sigma_2^2} \\ &= \log \frac{\sigma_2}{\sigma_1} - \frac{\sigma_1^2}{2\sigma_1^2} + \frac{\mathbb{E}_{p(x)}[(x - \mu_1)^2 + 2(x - \mu_1)(\mu_1 - \mu_2) + (\mu_1 - \mu_2)^2]}{2\sigma_2^2} \\ &= \log \frac{\sigma_2}{\sigma_1} + \frac{\sigma_1^2 + (\mu_1 - \mu_2)^2}{2\sigma_2^2} - \frac{1}{2} \end{aligned}$$

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Back to natural gradient descent

- How to solve this **constrained optimization problem**?

$$d^* = \arg \max_d J(\theta + d), \quad s.t. D_{\text{KL}}(\pi_\theta || \pi_{\theta+d}) \leq \epsilon$$

- What tool to use?
 - Turn the constrained optimization problem to an unconstrained one?

- How to solve this constrained optimization problem?

$$d^* = \arg \max_d J(\theta + d), \quad s.t. D_{\text{KL}}(\pi_\theta || \pi_{\theta+d}) \leq \epsilon$$

- Use the **Lagrangian multiplier** λ , turn to the **unconstrained penalized objective**

$$d^* = \arg \max_d J(\theta + d) - \lambda(D_{\text{KL}}(\pi_\theta || \pi_{\theta+d}) - \epsilon)$$

Taylor expansion for the unconstrained penalized objective

$$d^* = \arg \max_d J(\theta + d) - \lambda(\text{D}_{\text{KL}}(\pi_\theta || \pi_{\theta+d}) - \epsilon)$$

- First-order Taylor expansion for the loss

$$J(\theta + d) \approx J(\theta) + \nabla_{\theta'} J(\theta')|_{\theta'=\theta} \cdot d$$

- Second-order Taylor expansion for the KL

$$\text{D}_{\text{KL}}(\pi_\theta || \pi_{\theta+d}) \approx \frac{1}{2} d^T \cdot \nabla_{\theta'}^2 \text{D}_{\text{KL}}(\pi_\theta || \pi_{\theta'})|_{\theta'=\theta} \cdot d$$

Taylor series/expansion

- A representation of a function as an infinite sum of terms that are calculated from the values of the function's derivatives at a single point

$$\begin{aligned}f(x) &= \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x-a)^n \\&= f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2 + \dots\end{aligned}$$

- Examples

$$e^x = ?$$

$$\frac{1}{1-x} = ?$$

Taylor series/expansion

- A representation of a function as an infinite sum of terms that are calculated from the values of the function's derivatives at a single point

$$\begin{aligned} f(x) &= \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x-a)^n \\ &= f(a) + f'(a)(x-a) + \frac{f''(a)}{2} (x-a)^2 + \dots \end{aligned}$$

- Examples

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \dots$$

Taylor expansion of $J(\theta + d)$

- Let $\theta' = \theta + d$ is the independent variable
- That is, $x = \theta'$, $a = \theta$, $x - a = d$
- **What is the Taylor expansion of $J(\theta + d)$?**

$$\begin{aligned} f(x) &= \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x - a)^n \\ &= f(a) + f'(a)(x - a) + \frac{f''(a)}{2}(x - a)^2 + \dots \end{aligned}$$

Taylor expansion of $J(\theta + d)$

- Let $\theta' = \theta + d$ is the independent variable
- That is, $x = \theta'$, $a = \theta$, $x - a = d$
- **What is the Taylor expansion of $J(\theta + d)$?**
- First-order Taylor expansion for the loss:

$$J(\theta + d) \approx J(\theta) + \nabla_{\theta'} J(\theta')|_{\theta'=\theta} \cdot d$$

Taylor expansion of KL

- Let $\theta' = \theta + d$ is the independent variable
- That is, $x = \theta'$, $a = \theta$, $x - a = d$
- **What is the Taylor expansion of $D_{\text{KL}}(\pi_{\theta} || \pi_{\theta+d})$?**

$$\begin{aligned} f(x) &= \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x - a)^n \\ &= f(a) + f'(a)(x - a) + \frac{f''(a)}{2}(x - a)^2 + \dots \end{aligned}$$

Taylor expansion of KL

- Let $\theta' = \theta + d$ is the independent variable
- That is, $x = \theta'$, $a = \theta$, $x - a = d$
- **What is the Taylor expansion of $D_{\text{KL}}(\pi_\theta || \pi_{\theta+d})$?**
- Second-order Taylor expansion for $D_{\text{KL}}(\pi_\theta || \pi_{\theta'})$:

$$D_{\text{KL}}(\pi_\theta || \pi_{\theta'}) \approx D_{\text{KL}}(\pi_\theta || \pi_\theta) + d^T \nabla_{\theta'} D_{\text{KL}}(\pi_\theta || \pi_{\theta'})|_{\theta'=\theta} \\ + \frac{1}{2} d^T \nabla_{\theta'}^2 D_{\text{KL}}(\pi_\theta || \pi_{\theta'})|_{\theta'=\theta} d$$

Taylor expansion of KL

$$D_{\text{KL}}(\pi_\theta || \pi_{\theta'}) \approx D_{\text{KL}}(\pi_\theta || \pi_\theta) + d^T \nabla_{\theta'} D_{\text{KL}}(\pi_\theta || \pi_{\theta'})|_{\theta'=\theta} + \frac{1}{2} d^T \nabla_{\theta'}^2 D_{\text{KL}}(\pi_\theta || \pi_{\theta'})|_{\theta'=\theta} d$$

$$D_{\text{KL}}(\pi_\theta || \pi_{\theta'}) = \int \pi_\theta(x) \log \frac{\pi_\theta(x)}{\pi_{\theta'}(x)} dx = \underbrace{\int \pi_\theta(x) \log \pi_\theta(x) dx}_{\text{independent of } \theta'} - \int \pi_\theta(x) \log \pi_{\theta'}(x) dx$$

$$\begin{aligned} \nabla_{\theta'} D_{\text{KL}}(\pi_\theta || \pi_{\theta'})|_{\theta'=\theta} &= -\nabla_{\theta'} \int \pi_\theta(x) \log \pi_{\theta'}(x) dx|_{\theta'=\theta} \\ &= -\int \pi_\theta(x) \nabla_{\theta'} \log \pi_{\theta'}(x) dx|_{\theta'=\theta} \\ &= -\int \frac{\pi_\theta(x)}{\pi_{\theta'}(x)} \nabla_{\theta'} \pi_{\theta'}(x) dx|_{\theta'=\theta} \\ &= -\nabla_{\theta'} \int \pi_{\theta'}(x) dx|_{\theta'=\theta} \\ &= 0 \end{aligned}$$

Taylor expansion of KL

$$D_{\text{KL}}(\pi_\theta || \pi_{\theta'}) \approx D_{\text{KL}}(\pi_\theta || \pi_\theta) + d^T \nabla_{\theta'} D_{\text{KL}}(\pi_\theta || \pi_{\theta'})|_{\theta'=\theta} + \frac{1}{2} d^T \nabla_{\theta'}^2 D_{\text{KL}}(\pi_\theta || \pi_{\theta'})|_{\theta'=\theta} d$$

$$\begin{aligned} \nabla_{\theta'}^2 D_{\text{KL}}(\pi_\theta || \pi_{\theta'})|_{\theta'=\theta} &= - \int \pi_\theta(x) \nabla_{\theta'}^2 \log \pi_{\theta'}(x) dx|_{\theta'=\theta} \\ &= - \int \pi_\theta(x) \frac{\pi_{\theta'}(x) \nabla_{\theta'}^2 \pi_{\theta'}(x) - \nabla_{\theta'} \pi_{\theta'}(x) \nabla_{\theta'} \pi_{\theta'}(x)^T}{\pi_{\theta'}(x)^2} dx|_{\theta'=\theta} \\ &= \underbrace{- \nabla_{\theta'}^2 \int \pi_{\theta'}(x) dx|_{\theta'=\theta}}_0 + \int \pi_\theta(x) \nabla_{\theta'} \log \pi_{\theta'}(x) \nabla_{\theta'} \log \pi_{\theta'}(x)^T dx|_{\theta'=\theta} \\ &= \mathbb{E}_{x \sim \pi_\theta} [\nabla_{\theta'} \log \pi_{\theta'}(x) \nabla_{\theta'} \log \pi_{\theta'}(x)^T |_{\theta'=\theta}] \end{aligned}$$

Hessian of KL = Fisher information matrix (FIM)

- **Hessian:** A square matrix of second-order partial derivatives of a scalar-valued function, which describes the local curvature of a function of many variables

$$H = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} & \cdots & \frac{\partial^2 f}{\partial x_1 \partial x_n} \\ \frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} & \cdots & \frac{\partial^2 f}{\partial x_2 \partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial^2 f}{\partial x_n \partial x_1} & \frac{\partial^2 f}{\partial x_n \partial x_2} & \cdots & \frac{\partial^2 f}{\partial x_n^2} \end{bmatrix}$$

- **Fisher information:** a way of measuring the amount of information that an observable random variable X carries about an unknown parameter θ upon which the probability of X depends

$$F(\theta) = \mathbb{E}_{x \sim \pi_\theta} [\nabla_\theta \log \pi_\theta(x) \nabla_\theta \log \pi_\theta(x)^T]$$

Hessian of KL = Fisher information matrix (FIM)

- The FIM is exactly the **Hessian matrix** of KL divergence

$$\underbrace{\nabla_{\theta'}^2 D_{\text{KL}}(\pi_{\theta} || \pi_{\theta'})|_{\theta'=\theta}}_{\text{Hessian of KL}} = \underbrace{\mathbb{E}_{x \sim \pi_{\theta}} [\nabla_{\theta'} \log \pi_{\theta'}(x) \nabla_{\theta'} \log \pi_{\theta'}(x)^T |_{\theta'=\theta}]}_{\text{FIM}}$$

$$\begin{aligned} D_{\text{KL}}(\pi_{\theta} || \pi_{\theta'}) &\approx \underbrace{D_{\text{KL}}(\pi_{\theta} || \pi_{\theta})}_0 + d^T \underbrace{\nabla_{\theta'} D_{\text{KL}}(\pi_{\theta} || \pi_{\theta'})|_{\theta'=\theta}}_0 + \frac{1}{2} d^T \underbrace{\nabla_{\theta'}^2 D_{\text{KL}}(\pi_{\theta} || \pi_{\theta'})|_{\theta'=\theta}}_{\mathbf{F}(\theta)} d \\ &= \frac{1}{2} d^T \mathbf{F}(\theta) d \\ &= \frac{1}{2} (\theta' - \theta)^T \mathbf{F}(\theta) (\theta' - \theta) \end{aligned}$$

Back to Taylor expansion of KL

$$D_{\text{KL}}(\pi_{\theta} || \pi_{\theta+d}) \approx \frac{1}{2} d^T \mathbf{F}(\theta) d$$

- KL divergence is roughly analogous to a distance measure between distributions
- Fisher information serves as a local distance metric between distributions: how much you change the distribution if you move the parameters a little bit in a given direction

Back to solving the KL constrained problem

$$\begin{aligned}d^* &= \arg \max_d J(\theta + d) - \lambda(\mathbf{D}_{\text{KL}}(\pi_\theta || \pi_{\theta+d}) - \epsilon) \\&\approx \arg \max_d J(\theta) + \nabla_{\theta'} J(\theta')|_{\theta'=\theta} \cdot d - \lambda\left(\frac{1}{2}d^T \nabla_{\theta'}^2 \mathbf{D}_{\text{KL}}(\pi_\theta || \pi_{\theta'})|_{\theta'=\theta} d - \epsilon\right) \\&= \arg \max_d \nabla_{\theta'} J(\theta')|_{\theta'=\theta} \cdot d - \frac{1}{2}\lambda d^T \mathbf{F}(\theta) d\end{aligned}$$

- Set the gradient to 0:

$$\begin{aligned}0 &= \frac{\partial}{\partial d} \left(\nabla_{\theta'} J(\theta')|_{\theta'=\theta} \cdot d - \frac{1}{2}\lambda d^T \mathbf{F}(\theta) d \right) \\&= \nabla_{\theta'} J(\theta')|_{\theta'=\theta} - \lambda \mathbf{F}(\theta) d\end{aligned}$$

$$d^* = \frac{1}{\lambda} \mathbf{F}^{-1}(\theta) \nabla_{\theta'} J(\theta')|_{\theta'=\theta} = \frac{1}{\lambda} \mathbf{F}^{-1}(\theta) \nabla_{\theta} J(\theta)$$

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Natural gradient descent

- The natural gradient:

$$\tilde{\nabla}_{\theta} J(\theta) = \mathbf{F}^{-1}(\theta) \underbrace{\nabla_{\theta} J(\theta)}_{\hat{g}}$$

- Natural gradient ascent:

$$\theta' = \theta + \alpha \cdot \mathbf{F}^{-1}(\theta) \hat{g}$$

- How to determine the learning rate α :

$$D_{\text{KL}}(\pi_{\theta} || \pi_{\theta} + d) \approx \frac{1}{2} (\theta' - \theta)^T \mathbf{F}(\theta) (\theta' - \theta) \leq \epsilon$$

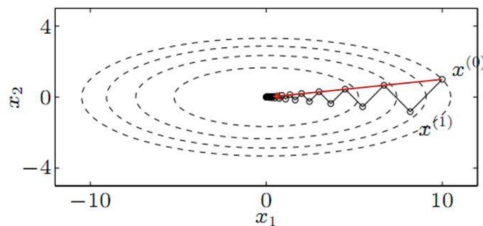
$$\frac{1}{2} (\alpha \hat{g})^T \mathbf{F}(\alpha \hat{g}) = \epsilon$$

$$\boxed{\alpha = \sqrt{\frac{2\epsilon}{\hat{g}^T \mathbf{F} \hat{g}}}}$$

Geometric interpretation of natural policy gradient

- Find **the steepest direction** for parameter updating

Essentially the same problem as this:



Natural gradient descent \rightarrow Natural policy gradient (NPG)

Algorithm 1 Natural Policy Gradient

Input: initial policy parameters θ_0

for $k = 0, 1, 2, \dots$ **do**

Collect set of trajectories \mathcal{D}_k on policy $\pi_k = \pi(\theta_k)$

Estimate advantages $\hat{A}_t^{\pi_k}$ using any advantage estimation algorithm

Form sample estimates for

- policy gradient \hat{g}_k (using advantage estimates)
- and KL-divergence Hessian / Fisher Information Matrix \hat{H}_k

Compute Natural Policy Gradient update:

$$\theta_{k+1} = \theta_k + \sqrt{\frac{2 \epsilon}{\hat{g}_k^T \hat{H}_k \hat{g}_k}} \hat{H}_k^{-1} \hat{g}_k$$

end for

- Originated from **natural gradient descent** in supervised learning
- Very **expensive** to compute the **inverse of Hessian matrix** for a large number of parameters

Review of natural policy gradient

- The gradient
 - Constrain parameter update in parameter space (using Euclidean distance)
- The natural gradient
 - Constrain parameter update in distribution space (using KL divergence)
 - The meaning of “natural”: the distance metric is **invariant** to function parameterization
- Fisher information matrix (FIM)
 - Second-order information: a local distance metric between distributions
 - The FIM is exactly the Hessian matrix of KL divergence
 - Expensive to compute for a large number of parameters

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Trust region policy optimization (TRPO)

- John Schulman, Sergey Levine, Philipp Moritz, Michael Jordan, and Pieter Abbeel, **Trust Region Policy Optimization**, ICML, 2015.
- The family of **statistical learning**
 - John Schulman → Pieter Abbeel → Andrew Ng → Michael Jordan

John Schulman's Homepage

I'm a research scientist at [OpenAI](#). I co-lead the reinforcement learning (RL) team, where we work on (1) designing better RL algorithms that enable agents to learn much faster in novel situations; (2) designing better training environments that teach agents transferrable skills. We mostly use [games](#) as a [testbed](#).

Previously, I received my [PhD](#) in Computer Science from UC Berkeley, where I had the good fortune of being advised by [Pieter Abbeel](#). Prior to my recent work in RL, I spent some time working on robotics, enabling robots to [tie knots](#) and [stitches](#) and plan movement using [trajectory optimization](#).

- [Publications](#)
- [Presentations](#)
- [Code](#)
- [Awards](#)

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Trust region policy optimization (TRPO)



Michael I. Jordan

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2003

[On spectral clustering: Analysis and an algorithm](#)

AY Ng, MI Jordan, Y Weiss

Advances in neural information processing systems, 849-856

7927

2002

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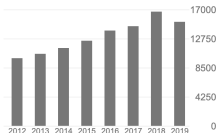
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TRPO - The KL constrained problem

- The objective function:

$$\underset{\theta}{\text{maximize}} \quad \hat{\mathbb{E}}_t \left[\frac{\pi_{\theta}(a_t|s_t)}{\pi_{\theta_{old}}(a_t|s_t)} \hat{A}_t \right]$$

$$\text{subject to} \quad \hat{\mathbb{E}}_t [\text{D}_{\text{KL}}[\pi_{\theta_{old}}(\cdot|s_t), \pi_{\theta}(\cdot|s_t)]] \leq \delta$$

- Also worth considering using a penalty instead of a constraint:

$$\underset{\theta}{\text{maximize}} \quad \hat{\mathbb{E}}_t \left[\frac{\pi_{\theta}(a_t|s_t)}{\pi_{\theta_{old}}(a_t|s_t)} \hat{A}_t \right] - \beta \hat{\mathbb{E}}_t [\text{D}_{\text{KL}}[\pi_{\theta_{old}}(\cdot|s_t), \pi_{\theta}(\cdot|s_t)]]$$

- Again the KL penalized problem!

Algorithm 3 Trust Region Policy Optimization

Input: initial policy parameters θ_0

for $k = 0, 1, 2, \dots$ **do**

Collect set of trajectories \mathcal{D}_k on policy $\pi_k = \pi(\theta_k)$

Estimate advantages $\hat{A}_t^{\pi_k}$ using any advantage estimation algorithm

Form sample estimates for

- policy gradient \hat{g}_k (using advantage estimates)
- and KL-divergence Hessian-vector product function $f(v) = \hat{H}_k v$

Use CG with n_{cg} iterations to obtain $x_k \approx \hat{H}_k^{-1} \hat{g}_k$

Estimate proposed step $\Delta_k \approx \sqrt{\frac{2\delta}{x_k^T \hat{H}_k x_k}} x_k$

Perform backtracking line search with exponential decay to obtain final update

$$\theta_{k+1} = \theta_k + \alpha^j \Delta_k$$

end for

Line search with monotonic policy improvement

Algorithm 2 Line Search for TRPO

Compute proposed policy step $\Delta_k = \sqrt{\frac{2\delta}{\hat{g}_k^T \hat{H}_k^{-1} \hat{g}_k}} \hat{H}_k^{-1} \hat{g}_k$

for $j = 0, 1, 2, \dots, L$ **do**

 Compute proposed update $\theta = \theta_k + \alpha^j \Delta_k$

if $\mathcal{L}_{\theta_k}(\theta) \geq 0$ and $\bar{D}_{KL}(\theta || \theta_k) \leq \delta$ **then**

 accept the update and set $\theta_{k+1} = \theta_k + \alpha^j \Delta_k$

 break

end if

end for

- Still very **expensive** to compute the **inverse of Hessian matrix** for a large number of parameters

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Proximal policy optimization (PPO): Clipped objective

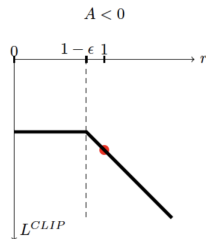
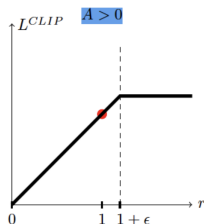
- The surrogate objective function:

$$\mathcal{L}^{\text{IS}}(\theta) = \hat{\mathbb{E}}_t \left[\frac{\pi_{\theta}(a_t|s_t)}{\pi_{\theta_{\text{old}}}(a_t|s_t)} \hat{A}_t \right] = \hat{\mathbb{E}}_t [r_t(\theta) \hat{A}_t]$$

- Form a lower bound via clipped importance ratios

$$\mathcal{L}^{\text{CLIP}}(\theta) = \hat{\mathbb{E}}_t \left[\min \left(r_t(\theta) \hat{A}_t, \text{clip}(r_t(\theta), 1 - \epsilon, 1 + \epsilon) \hat{A}_t \right) \right]$$

- Prevent large changes of policies, constrain the policy update
- Achieve similar performance to TRPO without second-order information (no Fisher matrix!)**



Proximal policy optimization (PPO): Adaptive KL penalty

Input: initial policy parameters θ_0 , initial KL penalty β_0 , target KL-divergence δ

for $k = 0, 1, 2, \dots$ **do**

Collect set of partial trajectories \mathcal{D}_k on policy $\pi_k = \pi(\theta_k)$

Estimate advantages $\hat{A}_t^{\pi_k}$ using any advantage estimation algorithm

Compute policy update

$$\theta_{k+1} = \arg \max_{\theta} \mathcal{L}_{\theta_k}(\theta) - \beta_k \bar{D}_{KL}(\theta || \theta_k)$$

by taking K steps of minibatch SGD (via Adam)

if $\bar{D}_{KL}(\theta_{k+1} || \theta_k) \geq 1.5\delta$ **then**

$$\beta_{k+1} = 2\beta_k$$

else if $\bar{D}_{KL}(\theta_{k+1} || \theta_k) \leq \delta/1.5$ **then**

$$\beta_{k+1} = \beta_k/2$$

end if

end for

Don't use second order approximation for KL which is expensive, use standard gradient descent

- Penalty coefficient β changes between iterations to approximately enforce KL-divergence constraint
- Achieve similar performance to TRPO without second-order information (no Fisher matrix!)

- TRPO: again the KL penalty problem
 - Natural policy gradient + Monotonic policy improvement + Line search
 - Still need to compute the natural gradient with Hessian matrix
- PPO
 - Achieve TRPO-like performance without second-order computation
 - Clipped objective, adaptive KL penalty

$$\underset{\theta}{\text{maximize}} \quad \hat{\mathbb{E}}_t \left[\frac{\pi_{\theta}(a_t|s_t)}{\pi_{\theta_{old}}(a_t|s_t)} \hat{A}_t \right]$$

$$\text{subject to} \quad \hat{\mathbb{E}}_t [\text{D}_{\text{KL}}[\pi_{\theta_{old}}(\cdot|s_t), \pi_{\theta}(\cdot|s_t)]] \leq \delta$$

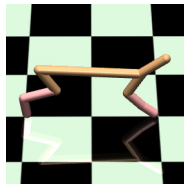
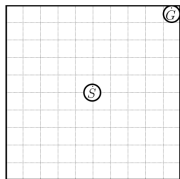
Learning objectives of this lecture

- You should be able to...
 - Know how to derive the natural policy gradient
 - Be aware of several advanced algorithms, e.g., TRPO, PPO
 - Enhance your mathematical skills

- Lecture 9 of CS285 at UC Berkeley, *Deep Reinforcement Learning, Decision Making, and Control*
 - <http://rail.eecs.berkeley.edu/deeprlcourse/static/slides/lec-9.pdf>
- Classic papers
 - Peters & Schaal (2008). **Reinforcement learning of motor skills with policy gradients**: very accessible overview of optimal baselines and natural gradient.
- DRL policy gradient papers
 - Schulman, L., Moritz, Jordan, Abbeel (2015). **Trust region policy optimization**: deep RL with natural policy gradient and adaptive step size.
 - Schulman, Wolski, Dhariwal, Radford, Klimov (2017). **Proximal policy optimization algorithms**: deep RL with importance sampled policy gradient.
 - Y. Duan, et al., **Benchmarking Deep Reinforcement Learning for Continuous Control**, *ICML*, 2016.

Volunteer Homework 2

- Study the policy gradient algorithm in detail
- Implement the series of policy gradient algorithms on problems 1 & 2
 - Problem 1: the point maze navigation, continuous state-action space ($s, a \in \mathbb{R}^2$, $s \in [-0.5, 0.5]^2$, $a \in [-0.1, 0.1]^2$)
 - Problem 2: the MuJoCo HalfCheetah, make the robot run forward
 - Must use vanilla policy gradient and natural policy gradient, encourage to use TRPO and PPO
- Write a report introducing the algorithms and your experimentation
 - Explanations, steps, evaluation results, visualizations...



THE END