Stochastic Processes

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Probability Space: A measure space(Ω,A,P)is a probability space if P(Ω)=1. In this case, P is called a probability measure.

Expected Value of Random Variable X on Probability Space Ω where X is any Measurable Function:

$$\mathsf{E}[X] = \int_{\Omega} X(\omega)\mathsf{P}(d\omega)$$

Integral is for $\omega \in \Omega$

Discrete Time Stochastic Process : A discrete time stochastic process is a sequence of random variables: $\{X_n \,:\, n \in Z\}$ where the index n is conventionally interpreted as time

Discrete Time Markov Chain : A discrete time stochastic process is a Markov Chain iff P($X_{\text{n+1}}$ = j | X_n = i, $X_{\text{n-1}} = i-1$, ... $X_{\text{n-d}} = i-d$) = P($X_{\text{n+1}}$ = j | X_n = i) for all n $\in Z, d >= 0$

Continuous Time Stochastic Process : A continuous time stochastic process is a family of random variables: $\{X(t) \,:\, t \in R\}$ where the index t is conventionally interpreted as time. An alternate viewpoint is that the entire function $(t \mapsto X(t))$ is a random function of time. This is called the **sample path**.

Poisson Process : A Poisson point process is a counting process that has a parameter of λ, often called the rate or intensity.
Properties
1) N(0)=0;
2) It has independent increments; and the number of events (or points) in any interval of length t is a Poisson random variable with parameter (or mean)
3) It has probability density function $\Pr\{N(t) = n\} = \frac{(\lambda t)^n}{n!}e^{-\lambda t}$