

Bài tập tuần 5 (11/12/2023 – 16/12/2023)

Câu 1. Thực hiện phát hiện biên bằng bộ lọc Sobel.

Câu 2. Thực hiện phát hiện biên bằng bộ lọc Prewitt

Câu 3. Thực hiện phát hiện biên bằng bộ lọc Kirsch

Câu 4. Thực hiện phát hiện biên bằng bộ lọc Laplacian:

0	-1	0
-1	4	-1
0	-1	0

Yêu cầu: Thay công thức và tính toán từng bước cụ thể. Không được làm tắt và ghi mỗi đáp số.

Ma trận 30:

I =	4	6	7	2	2
	6	4	7	2	2
	3	5	3	5	4
	4	6	6	4	4
	1	2	3	6	6

Ta có ma trận cửa sổ 3x3 tại các vị trí như sau:

COL ROW	0	1	2	3	4
0	0 0 0 0 4 6 0 6 4	0 0 0 4 6 7 6 4 7	0 0 0 6 7 2 4 7 2	0 0 0 7 2 2 7 2 2	0 0 0 2 2 0 2 2 0
1	0 4 6 0 6 4 0 3 5	4 6 7 6 4 7 3 5 3	6 7 2 4 7 2 5 3 5	7 2 2 7 2 2 3 5 4	2 2 0 2 2 0 5 4 0
2	0 6 4 0 3 5 0 4 6	6 4 7 3 5 3 4 6 6	4 7 2 5 3 5 6 6 4	7 2 2 3 5 4 6 4 4	2 2 0 5 4 0 4 4 0
3	0 3 5 0 4 6 0 1 2	3 5 3 4 6 6 1 2 3	5 3 5 6 6 4 2 3 6	3 5 4 6 4 4 3 6 6	5 4 0 4 4 0 6 6 0
4	0 4 6	4 6 6	6 6 4	6 4 4	4 4 0

	0 1 2	1 2 3	2 3 6	3 6 6	6 6 0
	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0

Câu 1. Thực hiện phát hiện biên bằng bộ lọc Sobel.

I =

4	6	7	2	2
6	4	7	2	2
3	5	3	5	4
4	6	6	4	4
1	2	3	6	6

K1 =

1	0	-1
2	0	-2
1	0	-1

$$G(0, 0) = 0*1 + 0*0 + 0*-1 + 0*2 + 4*0 + 6*-2 + 0*1 + 6*0 + 4*-1 = -16$$

$$G(0, 1) = 0*1 + 0*0 + 0*-1 + 4*2 + 6*0 + 7*-2 + 6*1 + 4*0 + 7*-1 = -7$$

$$G(0, 2) = 0*1 + 0*0 + 0*-1 + 6*2 + 7*0 + 2*-2 + 4*1 + 7*0 + 2*-1 = 10$$

$$G(0, 3) = 0*1 + 0*0 + 0*-1 + 7*2 + 2*0 + 2*-2 + 7*1 + 2*0 + 2*-1 = 15$$

$$G(0, 4) = 0*1 + 0*0 + 0*-1 + 2*2 + 2*0 + 0*-2 + 2*1 + 2*0 + 0*-1 = 6$$

$$G(1, 0) = 0*1 + 4*0 + 6*-1 + 0*2 + 6*0 + 4*-2 + 0*1 + 3*0 + 5*-1 = -19$$

$$G(1, 1) = 4*1 + 6*0 + 7*-1 + 6*2 + 4*0 + 7*-2 + 3*1 + 5*0 + 3*-1 = -5$$

$$G(1, 2) = 6*1 + 7*0 + 2*-1 + 4*2 + 7*0 + 2*-2 + 5*1 + 3*0 + 5*-1 = 8$$

$$G(1, 3) = 7*1 + 2*0 + 2*-1 + 7*2 + 2*0 + 2*-2 + 3*1 + 5*0 + 4*-1 = 14$$

$$G(1, 4) = 2*1 + 2*0 + 0*-1 + 2*2 + 2*0 + 0*-2 + 5*1 + 4*0 + 0*-1 = 11$$

$$G(2, 0) = 0*1 + 6*0 + 4*-1 + 0*2 + 3*0 + 5*-2 + 0*1 + 4*0 + 6*-1 = -20$$

$$G(2, 1) = 6*1 + 4*0 + 7*-1 + 3*2 + 5*0 + 3*-2 + 4*1 + 6*0 + 6*-1 = -3$$

$$G(2, 2) = 4*1 + 7*0 + 2*-1 + 5*2 + 3*0 + 5*-2 + 6*1 + 6*0 + 4*-1 = 4$$

$$\begin{aligned}
G(2, 3) &= 7*1 + 2*0 + 2*-1 + 3*2 + 5*0 + 4*-2 + 6*1 + 4*0 + 4*-1 = 5 \\
G(2, 4) &= 2*1 + 2*0 + 0*-1 + 5*2 + 4*0 + 0*-2 + 4*1 + 4*0 + 0*-1 = 16 \\
G(3, 0) &= 0*1 + 3*0 + 5*-1 + 0*2 + 4*0 + 6*-2 + 0*1 + 1*0 + 2*-1 = -19 \\
G(3, 1) &= 3*1 + 5*0 + 3*-1 + 4*2 + 6*0 + 6*-2 + 1*1 + 2*0 + 3*-1 = -6 \\
G(3, 2) &= 5*1 + 3*0 + 5*-1 + 6*2 + 6*0 + 4*-2 + 2*1 + 3*0 + 6*-1 = 0 \\
G(3, 3) &= 3*1 + 5*0 + 4*-1 + 6*2 + 4*0 + 4*-2 + 3*1 + 6*0 + 6*-1 = 0 \\
G(3, 4) &= 5*1 + 4*0 + 0*-1 + 4*2 + 4*0 + 0*-2 + 6*1 + 6*0 + 0*-1 = 19 \\
G(4, 0) &= 0*1 + 4*0 + 6*-1 + 0*2 + 1*0 + 2*-2 + 0*1 + 0*0 + 0*-1 = -10 \\
G(4, 1) &= 4*1 + 6*0 + 6*-1 + 1*2 + 2*0 + 3*-2 + 0*1 + 0*0 + 0*-1 = -6 \\
G(4, 2) &= 6*1 + 6*0 + 4*-1 + 2*2 + 3*0 + 6*-2 + 0*1 + 0*0 + 0*-1 = -6 \\
G(4, 3) &= 6*1 + 4*0 + 4*-1 + 3*2 + 6*0 + 6*-2 + 0*1 + 0*0 + 0*-1 = -4 \\
G(4, 4) &= 4*1 + 4*0 + 0*-1 + 6*2 + 6*0 + 0*-2 + 0*1 + 0*0 + 0*-1 = 16
\end{aligned}$$

Kết quả:

Gx =

-16	-7	10	15	6
-19	-5	8	14	11
-20	-3	4	5	16
-19	-6	0	0	19
-10	-6	-6	-4	16

K2 =

1	0	-1
2	0	-2
1	0	-1

$$\begin{aligned}
G(0, 0) &= 0*1 + 0*2 + 0*1 + 0*0 + 4*0 + 6*0 + 0*-1 + 6*-2 + 4*-1 = -16 \\
G(0, 1) &= 0*1 + 0*2 + 0*1 + 4*0 + 6*0 + 7*0 + 6*-1 + 4*-2 + 7*-1 = -21 \\
G(0, 2) &= 0*1 + 0*2 + 0*1 + 6*0 + 7*0 + 2*0 + 4*-1 + 7*-2 + 2*-1 = -20
\end{aligned}$$

$$G(0, 3) = 0*1 + 0*2 + 0*1 + 7*0 + 2*0 + 2*0 + 7*-1 + 2*-2 + 2*-1 = -13$$

$$G(0, 4) = 0*1 + 0*2 + 0*1 + 2*0 + 2*0 + 0*0 + 2*-1 + 2*-2 + 0*-1 = -6$$

$$G(1, 0) = 0*1 + 4*2 + 6*1 + 0*0 + 6*0 + 4*0 + 0*-1 + 3*-2 + 5*-1 = 3$$

$$G(1, 1) = 4*1 + 6*2 + 7*1 + 6*0 + 4*0 + 7*0 + 3*-1 + 5*-2 + 3*-1 = 7$$

$$G(1, 2) = 6*1 + 7*2 + 2*1 + 4*0 + 7*0 + 2*0 + 5*-1 + 3*-2 + 5*-1 = 6$$

$$G(1, 3) = 7*1 + 2*2 + 2*1 + 7*0 + 2*0 + 2*0 + 3*-1 + 5*-2 + 4*-1 = -4$$

$$G(1, 4) = 2*1 + 2*2 + 0*1 + 2*0 + 2*0 + 0*0 + 5*-1 + 4*-2 + 0*-1 = -7$$

$$G(2, 0) = 0*1 + 6*2 + 4*1 + 0*0 + 3*0 + 5*0 + 0*-1 + 4*-2 + 6*-1 = 2$$

$$G(2, 1) = 6*1 + 4*2 + 7*1 + 3*0 + 5*0 + 3*0 + 4*-1 + 6*-2 + 6*-1 = -1$$

$$G(2, 2) = 4*1 + 7*2 + 2*1 + 5*0 + 3*0 + 5*0 + 6*-1 + 6*-2 + 4*-1 = -2$$

$$G(2, 3) = 7*1 + 2*2 + 2*1 + 3*0 + 5*0 + 4*0 + 6*-1 + 4*-2 + 4*-1 = -5$$

$$G(2, 4) = 2*1 + 2*2 + 0*1 + 5*0 + 4*0 + 0*0 + 4*-1 + 4*-2 + 0*-1 = -6$$

$$G(3, 0) = 0*1 + 3*2 + 5*1 + 0*0 + 4*0 + 6*0 + 0*-1 + 1*-2 + 2*-1 = 7$$

$$G(3, 1) = 3*1 + 5*2 + 3*1 + 4*0 + 6*0 + 6*0 + 1*-1 + 2*-2 + 3*-1 = 8$$

$$G(3, 2) = 5*1 + 3*2 + 5*1 + 6*0 + 6*0 + 4*0 + 2*-1 + 3*-2 + 6*-1 = 2$$

$$G(3, 3) = 3*1 + 5*2 + 4*1 + 6*0 + 4*0 + 4*0 + 3*-1 + 6*-2 + 6*-1 = -4$$

$$G(3, 4) = 5*1 + 4*2 + 0*1 + 4*0 + 4*0 + 0*0 + 6*-1 + 6*-2 + 0*-1 = -5$$

$$G(4, 0) = 0*1 + 4*2 + 6*1 + 0*0 + 1*0 + 2*0 + 0*-1 + 0*-2 + 0*-1 = 14$$

$$G(4, 1) = 4*1 + 6*2 + 6*1 + 1*0 + 2*0 + 3*0 + 0*-1 + 0*-2 + 0*-1 = 22$$

$$G(4, 2) = 6*1 + 6*2 + 4*1 + 2*0 + 3*0 + 6*0 + 0*-1 + 0*-2 + 0*-1 = 22$$

$$G(4, 3) = 6*1 + 4*2 + 4*1 + 3*0 + 6*0 + 6*0 + 0*-1 + 0*-2 + 0*-1 = 18$$

$$G(4, 4) = 4*1 + 4*2 + 0*1 + 6*0 + 6*0 + 0*0 + 0*-1 + 0*-2 + 0*-1 = 12$$

Kết quả:

Gy =

-16	-21	-20	-13	-6
3	7	6	-4	-7

2	-1	-2	-5	-6
7	8	2	-4	-5
14	22	22	18	12

$$G = |G_x| + |G_y|$$

32	28	30	28	12
22	12	14	18	18
22	4	6	10	22
26	14	2	4	24
24	28	28	22	28

Bài 2: Thực hiện phát hiện biên bằng bộ lọc Prewitt

I =

4	6	7	2	2
6	4	7	2	2
3	5	3	5	4
4	6	6	4	4
1	2	3	6	6

K1 =

1	0	-1
1	0	-1
1	0	-1

$$G(0, 0) = 0*1 + 0*0 + 0*-1 + 0*1 + 4*0 + 6*-1 + 0*1 + 6*0 + 4*-1 = -10$$

$$G(0, 1) = 0*1 + 0*0 + 0*-1 + 4*1 + 6*0 + 7*-1 + 6*1 + 4*0 + 7*-1 = -4$$

$$G(0, 2) = 0*1 + 0*0 + 0*-1 + 6*1 + 7*0 + 2*-1 + 4*1 + 7*0 + 2*-1 = 6$$

$$G(0, 3) = 0*1 + 0*0 + 0*-1 + 7*1 + 2*0 + 2*-1 + 7*1 + 2*0 + 2*-1 = 10$$

$$G(0, 4) = 0*1 + 0*0 + 0*-1 + 2*1 + 2*0 + 0*-1 + 2*1 + 2*0 + 0*-1 = 4$$

$$G(1, 0) = 0*1 + 4*0 + 6*-1 + 0*1 + 6*0 + 4*-1 + 0*1 + 3*0 + 5*-1 = -15$$

$$G(1, 1) = 4*1 + 6*0 + 7*-1 + 6*1 + 4*0 + 7*-1 + 3*1 + 5*0 + 3*-1 = -4$$

$$G(1, 2) = 6*1 + 7*0 + 2*-1 + 4*1 + 7*0 + 2*-1 + 5*1 + 3*0 + 5*-1 = 6$$

$$\begin{aligned}
G(1, 3) &= 7*1 + 2*0 + 2*-1 + 7*1 + 2*0 + 2*-1 + 3*1 + 5*0 + 4*-1 = 9 \\
G(1, 4) &= 2*1 + 2*0 + 0*-1 + 2*1 + 2*0 + 0*-1 + 5*1 + 4*0 + 0*-1 = 9 \\
G(2, 0) &= 0*1 + 6*0 + 4*-1 + 0*1 + 3*0 + 5*-1 + 0*1 + 4*0 + 6*-1 = -15 \\
G(2, 1) &= 6*1 + 4*0 + 7*-1 + 3*1 + 5*0 + 3*-1 + 4*1 + 6*0 + 6*-1 = -3 \\
G(2, 2) &= 4*1 + 7*0 + 2*-1 + 5*1 + 3*0 + 5*-1 + 6*1 + 6*0 + 4*-1 = 4 \\
G(2, 3) &= 7*1 + 2*0 + 2*-1 + 3*1 + 5*0 + 4*-1 + 6*1 + 4*0 + 4*-1 = 6 \\
G(2, 4) &= 2*1 + 2*0 + 0*-1 + 5*1 + 4*0 + 0*-1 + 4*1 + 4*0 + 0*-1 = 11 \\
G(3, 0) &= 0*1 + 3*0 + 5*-1 + 0*1 + 4*0 + 6*-1 + 0*1 + 1*0 + 2*-1 = -13 \\
G(3, 1) &= 3*1 + 5*0 + 3*-1 + 4*1 + 6*0 + 6*-1 + 1*1 + 2*0 + 3*-1 = -4 \\
G(3, 2) &= 5*1 + 3*0 + 5*-1 + 6*1 + 6*0 + 4*-1 + 2*1 + 3*0 + 6*-1 = -2 \\
G(3, 3) &= 3*1 + 5*0 + 4*-1 + 6*1 + 4*0 + 4*-1 + 3*1 + 6*0 + 6*-1 = -2 \\
G(3, 4) &= 5*1 + 4*0 + 0*-1 + 4*1 + 4*0 + 0*-1 + 6*1 + 6*0 + 0*-1 = 15 \\
G(4, 0) &= 0*1 + 4*0 + 6*-1 + 0*1 + 1*0 + 2*-1 + 0*1 + 0*0 + 0*-1 = -8 \\
G(4, 1) &= 4*1 + 6*0 + 6*-1 + 1*1 + 2*0 + 3*-1 + 0*1 + 0*0 + 0*-1 = -4 \\
G(4, 2) &= 6*1 + 6*0 + 4*-1 + 2*1 + 3*0 + 6*-1 + 0*1 + 0*0 + 0*-1 = -2 \\
G(4, 3) &= 6*1 + 4*0 + 4*-1 + 3*1 + 6*0 + 6*-1 + 0*1 + 0*0 + 0*-1 = -1 \\
G(4, 4) &= 4*1 + 4*0 + 0*-1 + 6*1 + 6*0 + 0*-1 + 0*1 + 0*0 + 0*-1 = 10
\end{aligned}$$

Kết quả:

-10	-4	6	10	4
-15	-4	6	9	9
-15	-3	4	6	11
-13	-4	-2	-2	15
-8	-4	-2	-1	10

K2 =

1	0	-1
1	0	-1
1	0	-1

$$G(0, 0) = 0*1 + 0*1 + 0*1 + 0*0 + 4*0 + 6*0 + 0*-1 + 6*-1 + 4*-1 = -10$$

$$G(0, 1) = 0*1 + 0*1 + 0*1 + 4*0 + 6*0 + 7*0 + 6*-1 + 4*-1 + 7*-1 = -17$$

$$G(0, 2) = 0*1 + 0*1 + 0*1 + 6*0 + 7*0 + 2*0 + 4*-1 + 7*-1 + 2*-1 = -13$$

$$G(0, 3) = 0*1 + 0*1 + 0*1 + 7*0 + 2*0 + 2*0 + 7*-1 + 2*-1 + 2*-1 = -11$$

$$G(0, 4) = 0*1 + 0*1 + 0*1 + 2*0 + 2*0 + 0*0 + 2*-1 + 2*-1 + 0*-1 = -4$$

$$G(1, 0) = 0*1 + 4*1 + 6*1 + 0*0 + 6*0 + 4*0 + 0*-1 + 3*-1 + 5*-1 = 2$$

$$G(1, 1) = 4*1 + 6*1 + 7*1 + 6*0 + 4*0 + 7*0 + 3*-1 + 5*-1 + 3*-1 = 6$$

$$G(1, 2) = 6*1 + 7*1 + 2*1 + 4*0 + 7*0 + 2*0 + 5*-1 + 3*-1 + 5*-1 = 2$$

$$G(1, 3) = 7*1 + 2*1 + 2*1 + 7*0 + 2*0 + 2*0 + 3*-1 + 5*-1 + 4*-1 = -1$$

$$G(1, 4) = 2*1 + 2*1 + 0*1 + 2*0 + 2*0 + 0*0 + 5*-1 + 4*-1 + 0*-1 = -5$$

$$G(2, 0) = 0*1 + 6*1 + 4*1 + 0*0 + 3*0 + 5*0 + 0*-1 + 4*-1 + 6*-1 = 0$$

$$G(2, 1) = 6*1 + 4*1 + 7*1 + 3*0 + 5*0 + 3*0 + 4*-1 + 6*-1 + 6*-1 = 1$$

$$G(2, 2) = 4*1 + 7*1 + 2*1 + 5*0 + 3*0 + 5*0 + 6*-1 + 6*-1 + 4*-1 = -3$$

$$G(2, 3) = 7*1 + 2*1 + 2*1 + 3*0 + 5*0 + 4*0 + 6*-1 + 4*-1 + 4*-1 = -3$$

$$G(2, 4) = 2*1 + 2*1 + 0*1 + 5*0 + 4*0 + 0*0 + 4*-1 + 4*-1 + 0*-1 = -4$$

$$G(3, 0) = 0*1 + 3*1 + 5*1 + 0*0 + 4*0 + 6*0 + 0*-1 + 1*-1 + 2*-1 = 5$$

$$G(3, 1) = 3*1 + 5*1 + 3*1 + 4*0 + 6*0 + 6*0 + 1*-1 + 2*-1 + 3*-1 = 5$$

$$G(3, 2) = 5*1 + 3*1 + 5*1 + 6*0 + 6*0 + 4*0 + 2*-1 + 3*-1 + 6*-1 = 2$$

$$G(3, 3) = 3*1 + 5*1 + 4*1 + 6*0 + 4*0 + 4*0 + 3*-1 + 6*-1 + 6*-1 = -3$$

$$G(3, 4) = 5*1 + 4*1 + 0*1 + 4*0 + 4*0 + 0*0 + 6*-1 + 6*-1 + 0*-1 = -3$$

$$G(4, 0) = 0*1 + 4*1 + 6*1 + 0*0 + 1*0 + 2*0 + 0*-1 + 0*-1 + 0*-1 = 10$$

$$G(4, 1) = 4*1 + 6*1 + 6*1 + 1*0 + 2*0 + 3*0 + 0*-1 + 0*-1 + 0*-1 = 16$$

$$G(4, 2) = 6*1 + 6*1 + 4*1 + 2*0 + 3*0 + 6*0 + 0*-1 + 0*-1 + 0*-1 = 16$$

$$G(4, 3) = 6*1 + 4*1 + 4*1 + 3*0 + 6*0 + 6*0 + 0*-1 + 0*-1 + 0*-1 = 14$$

$$G(4, 4) = 4*1 + 4*1 + 0*1 + 6*0 + 6*0 + 0*0 + 0*-1 + 0*-1 + 0*-1 = 8$$

Kết quả:

-10	-17	-13	-11	-4
2	6	2	-1	-5
0	1	-3	-3	-4
5	5	2	-3	-3
10	16	16	14	8

$$G = |Gx| + |Gy|$$

20	21	19	21	8
17	10	8	10	14
15	4	7	9	15
18	9	4	5	18
18	20	18	15	18

Câu 3. Thực hiện phát hiện biên bằng bộ lọc Kirsch

$$K_1 = \begin{bmatrix} -3 & -3 & 5 \\ -3 & 0 & 5 \\ -3 & -3 & 5 \end{bmatrix}$$

$$K_5 = \begin{bmatrix} 5 & -3 & -3 \\ 5 & 0 & -3 \\ 5 & -3 & -3 \end{bmatrix}$$

$$K_2 = \begin{bmatrix} -3 & 5 & 5 \\ -3 & 0 & 5 \\ -3 & -3 & -3 \end{bmatrix}$$

$$K_6 = \begin{bmatrix} -3 & -3 & -3 \\ 5 & 0 & -3 \\ 5 & 5 & -3 \end{bmatrix}$$

$$K_3 = \begin{bmatrix} 5 & 5 & 5 \\ -3 & 0 & -3 \\ -3 & -3 & -3 \end{bmatrix}$$

$$K_7 = \begin{bmatrix} -3 & -3 & -3 \\ -3 & 0 & -3 \\ 5 & 5 & 5 \end{bmatrix}$$

$$K_4 = \begin{bmatrix} 5 & 5 & -3 \\ 5 & 0 & -3 \\ -3 & -3 & -3 \end{bmatrix}$$

$$K_8 = \begin{bmatrix} -3 & -3 & -3 \\ -3 & 0 & 5 \\ -3 & 5 & 5 \end{bmatrix}$$

I =

4	6	7	2	2
6	4	7	2	2
3	5	3	5	4
4	6	6	4	4
1	2	3	6	6

K1 =

-3	-3	5
-3	0	5
-3	-3	5

$$G1(0, 0) = 0*-3 + 0*-3 + 0*5 + 0*-3 + 4*0 + 6*5 + 0*-3 + 6*-3 + 4*5 = 32$$

$$G1(0, 1) = 0*-3 + 0*-3 + 0*5 + 4*-3 + 6*0 + 7*5 + 6*-3 + 4*-3 + 7*5 = 28$$

$$G1(0, 2) = 0*-3 + 0*-3 + 0*5 + 6*-3 + 7*0 + 2*5 + 4*-3 + 7*-3 + 2*5 = -31$$

$$G1(0, 3) = 0*-3 + 0*-3 + 0*5 + 7*-3 + 2*0 + 2*5 + 7*-3 + 2*-3 + 2*5 = -28$$

$$G1(0, 4) = 0*-3 + 0*-3 + 0*5 + 2*-3 + 2*0 + 0*5 + 2*-3 + 2*-3 + 0*5 = -18$$

$$G1(1, 0) = 0*-3 + 4*-3 + 6*5 + 0*-3 + 6*0 + 4*5 + 0*-3 + 3*-3 + 5*5 = 54$$

$$G1(1, 1) = 4*-3 + 6*-3 + 7*5 + 6*-3 + 4*0 + 7*5 + 3*-3 + 5*-3 + 3*5 = 13$$

$$G1(1, 2) = 6*-3 + 7*-3 + 2*5 + 4*-3 + 7*0 + 2*5 + 5*-3 + 3*-3 + 5*5 = -30$$

$$G1(1, 3) = 7*-3 + 2*-3 + 2*5 + 7*-3 + 2*0 + 2*5 + 3*-3 + 5*-3 + 4*5 = -32$$

$$G1(1, 4) = 2*-3 + 2*-3 + 0*5 + 2*-3 + 2*0 + 0*5 + 5*-3 + 4*-3 + 0*5 = -45$$

$$G1(2, 0) = 0*-3 + 6*-3 + 4*5 + 0*-3 + 3*0 + 5*5 + 0*-3 + 4*-3 + 6*5 = 45$$

$$G1(2, 1) = 6*-3 + 4*-3 + 7*5 + 3*-3 + 5*0 + 3*5 + 4*-3 + 6*-3 + 6*5 = 11$$

$$G1(2, 2) = 4*-3 + 7*-3 + 2*5 + 5*-3 + 3*0 + 5*5 + 6*-3 + 6*-3 + 4*5 = -29$$

$$G1(2, 3) = 7*-3 + 2*-3 + 2*5 + 3*-3 + 5*0 + 4*5 + 6*-3 + 4*-3 + 4*5 = -16$$

$$G1(2, 4) = 2*-3 + 2*-3 + 0*5 + 5*-3 + 4*0 + 0*5 + 4*-3 + 4*-3 + 0*5 = -51$$

$$G1(3, 0) = 0*-3 + 3*-3 + 5*5 + 0*-3 + 4*0 + 6*5 + 0*-3 + 1*-3 + 2*5 = 53$$

$$G1(3, 1) = 3*-3 + 5*-3 + 3*5 + 4*-3 + 6*0 + 6*5 + 1*-3 + 2*-3 + 3*5 = 15$$

$$G1(3, 2) = 5*-3 + 3*-3 + 5*5 + 6*-3 + 6*0 + 4*5 + 2*-3 + 3*-3 + 6*5 = 18$$

$$G1(3, 3) = 3*-3 + 5*-3 + 4*5 + 6*-3 + 4*0 + 4*5 + 3*-3 + 6*-3 + 6*5 = 1$$

$$G1(3, 4) = 5*-3 + 4*-3 + 0*5 + 4*-3 + 4*0 + 0*5 + 6*-3 + 6*-3 + 0*5 = -75$$

$$G1(4, 0) = 0*-3 + 4*-3 + 6*5 + 0*-3 + 1*0 + 2*5 + 0*-3 + 0*-3 + 0*5 = 28$$

$$G1(4, 1) = 4*-3 + 6*-3 + 6*5 + 1*-3 + 2*0 + 3*5 + 0*-3 + 0*-3 + 0*5 = 12$$

$$G1(4, 2) = 6*-3 + 6*-3 + 4*5 + 2*-3 + 3*0 + 6*5 + 0*-3 + 0*-3 + 0*5 = 8$$

$$G1(4, 3) = 6*-3 + 4*-3 + 4*5 + 3*-3 + 6*0 + 6*5 + 0*-3 + 0*-3 + 0*5 = 11$$

$$G1(4, 4) = 4*-3 + 4*-3 + 0*5 + 6*-3 + 6*0 + 0*5 + 0*-3 + 0*-3 + 0*5 = -42$$

G1 =

32	28	-31	-28	-18
54	13	-30	-32	-45
45	11	-29	-16	-51
53	15	18	1	-75
28	12	8	11	-42

|G1| =

32	28	31	28	18
54	13	30	32	45
45	11	29	16	51
53	15	18	1	75
28	12	8	11	42

K2 =

-3	5	5
-3	0	5
-3	-3	-3

$$G2(0, 0) = 0*-3 + 0*5 + 0*5 + 0*-3 + 4*0 + 6*5 + 0*-3 + 6*-3 + 4*-3 = 0$$

$$\begin{aligned}
G2(0, 1) &= 0*-3 + 0*5 + 0*5 + 4*-3 + 6*0 + 7*5 + 6*-3 + 4*-3 + 7*-3 = -28 \\
G2(0, 2) &= 0*-3 + 0*5 + 0*5 + 6*-3 + 7*0 + 2*5 + 4*-3 + 7*-3 + 2*-3 = -47 \\
G2(0, 3) &= 0*-3 + 0*5 + 0*5 + 7*-3 + 2*0 + 2*5 + 7*-3 + 2*-3 + 2*-3 = -44 \\
G2(0, 4) &= 0*-3 + 0*5 + 0*5 + 2*-3 + 2*0 + 0*5 + 2*-3 + 2*-3 + 0*-3 = -18 \\
G2(1, 0) &= 0*-3 + 4*5 + 6*5 + 0*-3 + 6*0 + 4*5 + 0*-3 + 3*-3 + 5*-3 = 46 \\
G2(1, 1) &= 4*-3 + 6*5 + 7*5 + 6*-3 + 4*0 + 7*5 + 3*-3 + 5*-3 + 3*-3 = 37 \\
G2(1, 2) &= 6*-3 + 7*5 + 2*5 + 4*-3 + 7*0 + 2*5 + 5*-3 + 3*-3 + 5*-3 = -14 \\
G2(1, 3) &= 7*-3 + 2*5 + 2*5 + 7*-3 + 2*0 + 2*5 + 3*-3 + 5*-3 + 4*-3 = -48 \\
G2(1, 4) &= 2*-3 + 2*5 + 0*5 + 2*-3 + 2*0 + 0*5 + 5*-3 + 4*-3 + 0*-3 = -29 \\
G2(2, 0) &= 0*-3 + 6*5 + 4*5 + 0*-3 + 3*0 + 5*5 + 0*-3 + 4*-3 + 6*-3 = 45 \\
G2(2, 1) &= 6*-3 + 4*5 + 7*5 + 3*-3 + 5*0 + 3*5 + 4*-3 + 6*-3 + 6*-3 = -5 \\
G2(2, 2) &= 4*-3 + 7*5 + 2*5 + 5*-3 + 3*0 + 5*5 + 6*-3 + 6*-3 + 4*-3 = -5 \\
G2(2, 3) &= 7*-3 + 2*5 + 2*5 + 3*-3 + 5*0 + 4*5 + 6*-3 + 4*-3 + 4*-3 = -32 \\
G2(2, 4) &= 2*-3 + 2*5 + 0*5 + 5*-3 + 4*0 + 0*5 + 4*-3 + 4*-3 + 0*-3 = -35 \\
G2(3, 0) &= 0*-3 + 3*5 + 5*5 + 0*-3 + 4*0 + 6*5 + 0*-3 + 1*-3 + 2*-3 = 61 \\
G2(3, 1) &= 3*-3 + 5*5 + 3*5 + 4*-3 + 6*0 + 6*5 + 1*-3 + 2*-3 + 3*-3 = 31 \\
G2(3, 2) &= 5*-3 + 3*5 + 5*5 + 6*-3 + 6*0 + 4*5 + 2*-3 + 3*-3 + 6*-3 = -6 \\
G2(3, 3) &= 3*-3 + 5*5 + 4*5 + 6*-3 + 4*0 + 4*5 + 3*-3 + 6*-3 + 6*-3 = -7 \\
G2(3, 4) &= 5*-3 + 4*5 + 0*5 + 4*-3 + 4*0 + 0*5 + 6*-3 + 6*-3 + 0*-3 = -43 \\
G2(4, 0) &= 0*-3 + 4*5 + 6*5 + 0*-3 + 1*0 + 2*5 + 0*-3 + 0*-3 + 0*-3 = 60 \\
G2(4, 1) &= 4*-3 + 6*5 + 6*5 + 1*-3 + 2*0 + 3*5 + 0*-3 + 0*-3 + 0*-3 = 60 \\
G2(4, 2) &= 6*-3 + 6*5 + 4*5 + 2*-3 + 3*0 + 6*5 + 0*-3 + 0*-3 + 0*-3 = 56 \\
G2(4, 3) &= 6*-3 + 4*5 + 4*5 + 3*-3 + 6*0 + 6*5 + 0*-3 + 0*-3 + 0*-3 = 43 \\
G2(4, 4) &= 4*-3 + 4*5 + 0*5 + 6*-3 + 6*0 + 0*5 + 0*-3 + 0*-3 + 0*-3 = -10
\end{aligned}$$

G2 =

0	-28	-47	-44	-18
46	37	-14	-48	-29
45	-5	-5	-32	-35
61	31	-6	-7	-43
60	60	56	43	-10

|G2| =

0	28	47	44	18
46	37	14	48	29
45	5	5	32	35
61	31	6	7	43
60	60	56	43	10

K3 =

5	5	5
-3	0	-3
-3	-3	-3

$$G3(0, 0) = 0*5 + 0*5 + 0*5 + 0*-3 + 4*0 + 6*-3 + 0*-3 + 6*-3 + 4*-3 = -48$$

$$G3(0, 1) = 0*5 + 0*5 + 0*5 + 4*-3 + 6*0 + 7*-3 + 6*-3 + 4*-3 + 7*-3 = -84$$

$$G3(0, 2) = 0*5 + 0*5 + 0*5 + 6*-3 + 7*0 + 2*-3 + 4*-3 + 7*-3 + 2*-3 = -63$$

$$G3(0, 3) = 0*5 + 0*5 + 0*5 + 7*-3 + 2*0 + 2*-3 + 7*-3 + 2*-3 + 2*-3 = -60$$

$$G3(0, 4) = 0*5 + 0*5 + 0*5 + 2*-3 + 2*0 + 0*-3 + 2*-3 + 2*-3 + 0*-3 = -18$$

$$G3(1, 0) = 0*5 + 4*5 + 6*5 + 0*-3 + 6*0 + 4*-3 + 0*-3 + 3*-3 + 5*-3 = 14$$

$$G3(1, 1) = 4*5 + 6*5 + 7*5 + 6*-3 + 4*0 + 7*-3 + 3*-3 + 5*-3 + 3*-3 = 13$$

$$G3(1, 2) = 6*5 + 7*5 + 2*5 + 4*-3 + 7*0 + 2*-3 + 5*-3 + 3*-3 + 5*-3 = 18$$

$$G3(1, 3) = 7*5 + 2*5 + 2*5 + 7*-3 + 2*0 + 2*-3 + 3*-3 + 5*-3 + 4*-3 = -8$$

$$G3(1, 4) = 2*5 + 2*5 + 0*5 + 2*-3 + 2*0 + 0*-3 + 5*-3 + 4*-3 + 0*-3 = -13$$

$$\begin{aligned}
G_3(2, 0) &= 0*5 + 6*5 + 4*5 + 0*-3 + 3*0 + 5*-3 + 0*-3 + 4*-3 + 6*-3 = 5 \\
G_3(2, 1) &= 6*5 + 4*5 + 7*5 + 3*-3 + 5*0 + 3*-3 + 4*-3 + 6*-3 + 6*-3 = 19 \\
G_3(2, 2) &= 4*5 + 7*5 + 2*5 + 5*-3 + 3*0 + 5*-3 + 6*-3 + 6*-3 + 4*-3 = -13 \\
G_3(2, 3) &= 7*5 + 2*5 + 2*5 + 3*-3 + 5*0 + 4*-3 + 6*-3 + 4*-3 + 4*-3 = -8 \\
G_3(2, 4) &= 2*5 + 2*5 + 0*5 + 5*-3 + 4*0 + 0*-3 + 4*-3 + 4*-3 + 0*-3 = -19 \\
G_3(3, 0) &= 0*5 + 3*5 + 5*5 + 0*-3 + 4*0 + 6*-3 + 0*-3 + 1*-3 + 2*-3 = 13 \\
G_3(3, 1) &= 3*5 + 5*5 + 3*5 + 4*-3 + 6*0 + 6*-3 + 1*-3 + 2*-3 + 3*-3 = 7 \\
G_3(3, 2) &= 5*5 + 3*5 + 5*5 + 6*-3 + 6*0 + 4*-3 + 2*-3 + 3*-3 + 6*-3 = 2 \\
G_3(3, 3) &= 3*5 + 5*5 + 4*5 + 6*-3 + 4*0 + 4*-3 + 3*-3 + 6*-3 + 6*-3 = -15 \\
G_3(3, 4) &= 5*5 + 4*5 + 0*5 + 4*-3 + 4*0 + 0*-3 + 6*-3 + 6*-3 + 0*-3 = -3 \\
G_3(4, 0) &= 0*5 + 4*5 + 6*5 + 0*-3 + 1*0 + 2*-3 + 0*-3 + 0*-3 + 0*-3 = 44 \\
G_3(4, 1) &= 4*5 + 6*5 + 6*5 + 1*-3 + 2*0 + 3*-3 + 0*-3 + 0*-3 + 0*-3 = 68 \\
G_3(4, 2) &= 6*5 + 6*5 + 4*5 + 2*-3 + 3*0 + 6*-3 + 0*-3 + 0*-3 + 0*-3 = 56 \\
G_3(4, 3) &= 6*5 + 4*5 + 4*5 + 3*-3 + 6*0 + 6*-3 + 0*-3 + 0*-3 + 0*-3 = 43 \\
G_3(4, 4) &= 4*5 + 4*5 + 0*5 + 6*-3 + 6*0 + 0*-3 + 0*-3 + 0*-3 + 0*-3 = 22
\end{aligned}$$

$G_3 =$

-48	-84	-63	-60	-18
14	13	18	-8	-13
5	19	-13	-8	-19
13	7	2	-15	-3
44	68	56	43	22

$|G_3| =$

48	84	63	60	18
14	13	18	8	13
5	19	13	8	19
13	7	2	15	3
44	68	56	43	22

K4 =

5	5	-3
5	0	-3
-3	-3	-3

$$G4(0, 0) = 0*5 + 0*5 + 0*-3 + 0*5 + 4*0 + 6*-3 + 0*-3 + 6*-3 + 4*-3 = -48$$

$$G4(0, 1) = 0*5 + 0*5 + 0*-3 + 4*5 + 6*0 + 7*-3 + 6*-3 + 4*-3 + 7*-3 = -52$$

$$G4(0, 2) = 0*5 + 0*5 + 0*-3 + 6*5 + 7*0 + 2*-3 + 4*-3 + 7*-3 + 2*-3 = -15$$

$$G4(0, 3) = 0*5 + 0*5 + 0*-3 + 7*5 + 2*0 + 2*-3 + 7*-3 + 2*-3 + 2*-3 = -4$$

$$G4(0, 4) = 0*5 + 0*5 + 0*-3 + 2*5 + 2*0 + 0*-3 + 2*-3 + 2*-3 + 0*-3 = -2$$

$$G4(1, 0) = 0*5 + 4*5 + 6*-3 + 0*5 + 6*0 + 4*-3 + 0*-3 + 3*-3 + 5*-3 = -34$$

$$G4(1, 1) = 4*5 + 6*5 + 7*-3 + 6*5 + 4*0 + 7*-3 + 3*-3 + 5*-3 + 3*-3 = 5$$

$$G4(1, 2) = 6*5 + 7*5 + 2*-3 + 4*5 + 7*0 + 2*-3 + 5*-3 + 3*-3 + 5*-3 = 34$$

$$G4(1, 3) = 7*5 + 2*5 + 2*-3 + 7*5 + 2*0 + 2*-3 + 3*-3 + 5*-3 + 4*-3 = 32$$

$$G4(1, 4) = 2*5 + 2*5 + 0*-3 + 2*5 + 2*0 + 0*-3 + 5*-3 + 4*-3 + 0*-3 = 3$$

$$G4(2, 0) = 0*5 + 6*5 + 4*-3 + 0*5 + 3*0 + 5*-3 + 0*-3 + 4*-3 + 6*-3 = -27$$

$$G4(2, 1) = 6*5 + 4*5 + 7*-3 + 3*5 + 5*0 + 3*-3 + 4*-3 + 6*-3 + 6*-3 = -13$$

$$G4(2, 2) = 4*5 + 7*5 + 2*-3 + 5*5 + 3*0 + 5*-3 + 6*-3 + 6*-3 + 4*-3 = 11$$

$$G4(2, 3) = 7*5 + 2*5 + 2*-3 + 3*5 + 5*0 + 4*-3 + 6*-3 + 4*-3 + 4*-3 = 0$$

$$G4(2, 4) = 2*5 + 2*5 + 0*-3 + 5*5 + 4*0 + 0*-3 + 4*-3 + 4*-3 + 0*-3 = 21$$

$$G4(3, 0) = 0*5 + 3*5 + 5*-3 + 0*5 + 4*0 + 6*-3 + 0*-3 + 1*-3 + 2*-3 = -27$$

$$G4(3, 1) = 3*5 + 5*5 + 3*-3 + 4*5 + 6*0 + 6*-3 + 1*-3 + 2*-3 + 3*-3 = 15$$

$$G4(3, 2) = 5*5 + 3*5 + 5*-3 + 6*5 + 6*0 + 4*-3 + 2*-3 + 3*-3 + 6*-3 = 10$$

$$G4(3, 3) = 3*5 + 5*5 + 4*-3 + 6*5 + 4*0 + 4*-3 + 3*-3 + 6*-3 + 6*-3 = 1$$

$$G4(3, 4) = 5*5 + 4*5 + 0*-3 + 4*5 + 4*0 + 0*-3 + 6*-3 + 6*-3 + 0*-3 = 29$$

$$G4(4, 0) = 0*5 + 4*5 + 6*-3 + 0*5 + 1*0 + 2*-3 + 0*-3 + 0*-3 + 0*-3 = -4$$

$$G4(4, 1) = 4*5 + 6*5 + 6*-3 + 1*5 + 2*0 + 3*-3 + 0*-3 + 0*-3 + 0*-3 = 28$$

$$G4(4, 2) = 6*5 + 6*5 + 4*-3 + 2*5 + 3*0 + 6*-3 + 0*-3 + 0*-3 + 0*-3 = 40$$

$$G4(4, 3) = 6*5 + 4*5 + 4*-3 + 3*5 + 6*0 + 6*-3 + 0*-3 + 0*-3 + 0*-3 = 35$$

$$G4(4, 4) = 4*5 + 4*5 + 0*-3 + 6*5 + 6*0 + 0*-3 + 0*-3 + 0*-3 + 0*-3 = 70$$

G4 =

-48	-52	-15	-4	-2
-34	5	34	32	3
-27	-13	11	0	21
-27	15	10	1	29
-4	28	40	35	70

|G4| =

48	52	15	4	2
34	5	34	32	3
27	13	11	0	21
27	15	10	1	29
4	28	40	35	70

K5 =

5	-3	-3
5	0	-3
5	-3	-3

$$G5(0, 0) = 0*5 + 0*-3 + 0*-3 + 0*5 + 4*0 + 6*-3 + 0*5 + 6*-3 + 4*-3 = -48$$

$$G5(0, 1) = 0*5 + 0*-3 + 0*-3 + 4*5 + 6*0 + 7*-3 + 6*5 + 4*-3 + 7*-3 = -4$$

$$G5(0, 2) = 0*5 + 0*-3 + 0*-3 + 6*5 + 7*0 + 2*-3 + 4*5 + 7*-3 + 2*-3 = 17$$

$$G5(0, 3) = 0*5 + 0*-3 + 0*-3 + 7*5 + 2*0 + 2*-3 + 7*5 + 2*-3 + 2*-3 = 52$$

$$G5(0, 4) = 0*5 + 0*-3 + 0*-3 + 2*5 + 2*0 + 0*-3 + 2*5 + 2*-3 + 0*-3 = 14$$

$$G5(1, 0) = 0*5 + 4*-3 + 6*-3 + 0*5 + 6*0 + 4*-3 + 0*5 + 3*-3 + 5*-3 = -66$$

$$G5(1, 1) = 4*5 + 6*-3 + 7*-3 + 6*5 + 4*0 + 7*-3 + 3*5 + 5*-3 + 3*-3 = -19$$

$$G5(1, 2) = 6*5 + 7*-3 + 2*-3 + 4*5 + 7*0 + 2*-3 + 5*5 + 3*-3 + 5*-3 = 18$$

$$G5(1, 3) = 7*5 + 2*-3 + 2*-3 + 7*5 + 2*0 + 2*-3 + 3*5 + 5*-3 + 4*-3 = 40$$

$$G5(1, 4) = 2*5 + 2*-3 + 0*-3 + 2*5 + 2*0 + 0*-3 + 5*5 + 4*-3 + 0*-3 = 27$$

$$G5(2, 0) = 0*5 + 6*-3 + 4*-3 + 0*5 + 3*0 + 5*-3 + 0*5 + 4*-3 + 6*-3 = -75$$

$$G5(2, 1) = 6*5 + 4*-3 + 7*-3 + 3*5 + 5*0 + 3*-3 + 4*5 + 6*-3 + 6*-3 = -13$$

$$G5(2, 2) = 4*5 + 7*-3 + 2*-3 + 5*5 + 3*0 + 5*-3 + 6*5 + 6*-3 + 4*-3 = 3$$

$$G5(2, 3) = 7*5 + 2*-3 + 2*-3 + 3*5 + 5*0 + 4*-3 + 6*5 + 4*-3 + 4*-3 = 32$$

$$G5(2, 4) = 2*5 + 2*-3 + 0*-3 + 5*5 + 4*0 + 0*-3 + 4*5 + 4*-3 + 0*-3 = 37$$

$$G5(3, 0) = 0*5 + 3*-3 + 5*-3 + 0*5 + 4*0 + 6*-3 + 0*5 + 1*-3 + 2*-3 = -51$$

$$G5(3, 1) = 3*5 + 5*-3 + 3*-3 + 4*5 + 6*0 + 6*-3 + 1*5 + 2*-3 + 3*-3 = -17$$

$$G5(3, 2) = 5*5 + 3*-3 + 5*-3 + 6*5 + 6*0 + 4*-3 + 2*5 + 3*-3 + 6*-3 = 2$$

$$G5(3, 3) = 3*5 + 5*-3 + 4*-3 + 6*5 + 4*0 + 4*-3 + 3*5 + 6*-3 + 6*-3 = -15$$

$$G5(3, 4) = 5*5 + 4*-3 + 0*-3 + 4*5 + 4*0 + 0*-3 + 6*5 + 6*-3 + 0*-3 = 45$$

$$G5(4, 0) = 0*5 + 4*-3 + 6*-3 + 0*5 + 1*0 + 2*-3 + 0*5 + 0*-3 + 0*-3 = -36$$

$$G5(4, 1) = 4*5 + 6*-3 + 6*-3 + 1*5 + 2*0 + 3*-3 + 0*5 + 0*-3 + 0*-3 = -20$$

$$G5(4, 2) = 6*5 + 6*-3 + 4*-3 + 2*5 + 3*0 + 6*-3 + 0*5 + 0*-3 + 0*-3 = -8$$

$$G5(4, 3) = 6*5 + 4*-3 + 4*-3 + 3*5 + 6*0 + 6*-3 + 0*5 + 0*-3 + 0*-3 = 3$$

$$G5(4, 4) = 4*5 + 4*-3 + 0*-3 + 6*5 + 6*0 + 0*-3 + 0*5 + 0*-3 + 0*-3 = 38$$

G5 =

-48	-4	17	52	14
-66	-19	18	40	27
-75	-13	3	32	37
-51	-17	2	-15	45
-36	-20	-8	3	38

|G5| =

48	4	17	52	14
66	19	18	40	27
75	13	3	32	37
51	17	2	15	45
36	20	8	3	38

K6 =

-3	-3	-3
5	0	-3
5	5	-3

$$G6(0, 0) = 0^{*-3} + 0^{*-3} + 0^{*-3} + 0^{*5} + 4^{*0} + 6^{*-3} + 0^{*5} + 6^{*5} + 4^{*-3} = 0$$

$$G6(0, 1) = 0^{*-3} + 0^{*-3} + 0^{*-3} + 4^{*5} + 6^{*0} + 7^{*-3} + 6^{*5} + 4^{*5} + 7^{*-3} = 28$$

$$G6(0, 2) = 0^{*-3} + 0^{*-3} + 0^{*-3} + 6^{*5} + 7^{*0} + 2^{*-3} + 4^{*5} + 7^{*5} + 2^{*-3} = 73$$

$$G6(0, 3) = 0^{*-3} + 0^{*-3} + 0^{*-3} + 7^{*5} + 2^{*0} + 2^{*-3} + 7^{*5} + 2^{*5} + 2^{*-3} = 68$$

$$G6(0, 4) = 0^{*-3} + 0^{*-3} + 0^{*-3} + 2^{*5} + 2^{*0} + 0^{*-3} + 2^{*5} + 2^{*5} + 0^{*-3} = 30$$

$$G6(1, 0) = 0^{*-3} + 4^{*-3} + 6^{*-3} + 0^{*5} + 6^{*0} + 4^{*-3} + 0^{*5} + 3^{*5} + 5^{*-3} = -42$$

$$G6(1, 1) = 4^{*-3} + 6^{*-3} + 7^{*-3} + 6^{*5} + 4^{*0} + 7^{*-3} + 3^{*5} + 5^{*5} + 3^{*-3} = -11$$

$$G6(1, 2) = 6^{*-3} + 7^{*-3} + 2^{*-3} + 4^{*5} + 7^{*0} + 2^{*-3} + 5^{*5} + 3^{*5} + 5^{*-3} = -6$$

$$G6(1, 3) = 7^{*-3} + 2^{*-3} + 2^{*-3} + 7^{*5} + 2^{*0} + 2^{*-3} + 3^{*5} + 5^{*5} + 4^{*-3} = 24$$

$$G6(1, 4) = 2^{*-3} + 2^{*-3} + 0^{*-3} + 2^{*5} + 2^{*0} + 0^{*-3} + 5^{*5} + 4^{*5} + 0^{*-3} = 43$$

$$G6(2, 0) = 0^{*-3} + 6^{*-3} + 4^{*-3} + 0^{*5} + 3^{*0} + 5^{*-3} + 0^{*5} + 4^{*5} + 6^{*-3} = -43$$

$$G6(2, 1) = 6^{*-3} + 4^{*-3} + 7^{*-3} + 3^{*5} + 5^{*0} + 3^{*-3} + 4^{*5} + 6^{*5} + 6^{*-3} = -13$$

$$G6(2, 2) = 4^{*-3} + 7^{*-3} + 2^{*-3} + 5^{*5} + 3^{*0} + 5^{*-3} + 6^{*5} + 6^{*5} + 4^{*-3} = 19$$

$$G6(2, 3) = 7^{*-3} + 2^{*-3} + 2^{*-3} + 3^{*5} + 5^{*0} + 4^{*-3} + 6^{*5} + 4^{*5} + 4^{*-3} = 8$$

$$G6(2, 4) = 2^{*-3} + 2^{*-3} + 0^{*-3} + 5^{*5} + 4^{*0} + 0^{*-3} + 4^{*5} + 4^{*5} + 0^{*-3} = 53$$

$$G6(3, 0) = 0^{*-3} + 3^{*-3} + 5^{*-3} + 0^{*5} + 4^{*0} + 6^{*-3} + 0^{*5} + 1^{*5} + 2^{*-3} = -43$$

$$G6(3, 1) = 3^{*-3} + 5^{*-3} + 3^{*-3} + 4^{*5} + 6^{*0} + 6^{*-3} + 1^{*5} + 2^{*5} + 3^{*-3} = -25$$

$$G6(3, 2) = 5*-3 + 3*-3 + 5*-3 + 6*5 + 6*0 + 4*-3 + 2*5 + 3*5 + 6*-3 = -14$$

$$G6(3, 3) = 3*-3 + 5*-3 + 4*-3 + 6*5 + 4*0 + 4*-3 + 3*5 + 6*5 + 6*-3 = 9$$

$$G6(3, 4) = 5*-3 + 4*-3 + 0*-3 + 4*5 + 4*0 + 0*-3 + 6*5 + 6*5 + 0*-3 = 53$$

$$G6(4, 0) = 0*-3 + 4*-3 + 6*-3 + 0*5 + 1*0 + 2*-3 + 0*5 + 0*5 + 0*-3 = -36$$

$$G6(4, 1) = 4*-3 + 6*-3 + 6*-3 + 1*5 + 2*0 + 3*-3 + 0*5 + 0*5 + 0*-3 = -52$$

$$G6(4, 2) = 6*-3 + 6*-3 + 4*-3 + 2*5 + 3*0 + 6*-3 + 0*5 + 0*5 + 0*-3 = -56$$

$$G6(4, 3) = 6*-3 + 4*-3 + 4*-3 + 3*5 + 6*0 + 6*-3 + 0*5 + 0*5 + 0*-3 = -45$$

$$G6(4, 4) = 4*-3 + 4*-3 + 0*-3 + 6*5 + 6*0 + 0*-3 + 0*5 + 0*5 + 0*-3 = 6$$

G6 =

0	28	73	68	30
-42	-11	-6	24	43
-43	-13	19	8	53
-43	-25	-14	9	53
-36	-52	-56	-45	6

|G6| =

0	28	73	68	30
42	11	6	24	43
43	13	19	8	53
43	25	14	9	53
36	52	56	45	6

K7 =

-3	-3	-3
-3	0	-3
5	5	5

$$G7(0, 0) = 0*-3 + 0*-3 + 0*-3 + 0*-3 + 4*0 + 6*-3 + 0*5 + 6*5 + 4*5 = 32$$

$$G7(0, 1) = 0*-3 + 0*-3 + 0*-3 + 4*-3 + 6*0 + 7*-3 + 6*5 + 4*5 + 7*5 = 52$$

$$G7(0, 2) = 0*-3 + 0*-3 + 0*-3 + 6*-3 + 7*0 + 2*-3 + 4*5 + 7*5 + 2*5 = 41$$

$$G7(0, 3) = 0*-3 + 0*-3 + 0*-3 + 7*-3 + 2*0 + 2*-3 + 7*5 + 2*5 + 2*5 = 28$$

$$G7(0, 4) = 0*-3 + 0*-3 + 0*-3 + 2*-3 + 2*0 + 0*-3 + 2*5 + 2*5 + 0*5 = 14$$

$$G7(1, 0) = 0*-3 + 4*-3 + 6*-3 + 0*-3 + 6*0 + 4*-3 + 0*5 + 3*5 + 5*5 = -2$$

$$G7(1, 1) = 4*-3 + 6*-3 + 7*-3 + 6*-3 + 4*0 + 7*-3 + 3*5 + 5*5 + 3*5 = -35$$

$$G7(1, 2) = 6*-3 + 7*-3 + 2*-3 + 4*-3 + 7*0 + 2*-3 + 5*5 + 3*5 + 5*5 = 2$$

$$G7(1, 3) = 7*-3 + 2*-3 + 2*-3 + 7*-3 + 2*0 + 2*-3 + 3*5 + 5*5 + 4*5 = 0$$

$$G7(1, 4) = 2*-3 + 2*-3 + 0*-3 + 2*-3 + 2*0 + 0*-3 + 5*5 + 4*5 + 0*5 = 27$$

$$G7(2, 0) = 0*-3 + 6*-3 + 4*-3 + 0*-3 + 3*0 + 5*-3 + 0*5 + 4*5 + 6*5 = 5$$

$$G7(2, 1) = 6*-3 + 4*-3 + 7*-3 + 3*-3 + 5*0 + 3*-3 + 4*5 + 6*5 + 6*5 = 11$$

$$G7(2, 2) = 4*-3 + 7*-3 + 2*-3 + 5*-3 + 3*0 + 5*-3 + 6*5 + 6*5 + 4*5 = 11$$

$$G7(2, 3) = 7*-3 + 2*-3 + 2*-3 + 3*-3 + 5*0 + 4*-3 + 6*5 + 4*5 + 4*5 = 16$$

$$G7(2, 4) = 2*-3 + 2*-3 + 0*-3 + 5*-3 + 4*0 + 0*-3 + 4*5 + 4*5 + 0*5 = 13$$

$$G7(3, 0) = 0*-3 + 3*-3 + 5*-3 + 0*-3 + 4*0 + 6*-3 + 0*5 + 1*5 + 2*5 = -27$$

$$G7(3, 1) = 3*-3 + 5*-3 + 3*-3 + 4*-3 + 6*0 + 6*-3 + 1*5 + 2*5 + 3*5 = -33$$

$$G7(3, 2) = 5*-3 + 3*-3 + 5*-3 + 6*-3 + 6*0 + 4*-3 + 2*5 + 3*5 + 6*5 = -14$$

$$G7(3, 3) = 3*-3 + 5*-3 + 4*-3 + 6*-3 + 4*0 + 4*-3 + 3*5 + 6*5 + 6*5 = 9$$

$$G7(3, 4) = 5*-3 + 4*-3 + 0*-3 + 4*-3 + 4*0 + 0*-3 + 6*5 + 6*5 + 0*5 = 21$$

$$G7(4, 0) = 0*-3 + 4*-3 + 6*-3 + 0*-3 + 1*0 + 2*-3 + 0*5 + 0*5 + 0*5 = -36$$

$$G7(4, 1) = 4*-3 + 6*-3 + 6*-3 + 1*-3 + 2*0 + 3*-3 + 0*5 + 0*5 + 0*5 = -60$$

$$G7(4, 2) = 6*-3 + 6*-3 + 4*-3 + 2*-3 + 3*0 + 6*-3 + 0*5 + 0*5 + 0*5 = -72$$

$$G7(4, 3) = 6*-3 + 4*-3 + 4*-3 + 3*-3 + 6*0 + 6*-3 + 0*5 + 0*5 + 0*5 = -69$$

$$G7(4, 4) = 4*-3 + 4*-3 + 0*-3 + 6*-3 + 6*0 + 0*-3 + 0*5 + 0*5 + 0*5 = -42$$

G7 =

32	52	41	28	14
----	----	----	----	----

-2	-35	2	0	27
5	11	11	16	13
-27	-33	-14	9	21
-36	-60	-72	-69	-42

|G7| =

32	52	41	28	14
2	35	2	0	27
5	11	11	16	13
27	33	14	9	21
36	60	72	69	42

K8 =

-3	-3	-3
-3	0	5
-3	5	5

$$G8(0, 0) = 0*-3 + 0*-3 + 0*-3 + 0*-3 + 4*0 + 6*5 + 0*-3 + 6*5 + 4*5 = 80$$

$$G8(0, 1) = 0*-3 + 0*-3 + 0*-3 + 4*-3 + 6*0 + 7*5 + 6*-3 + 4*5 + 7*5 = 60$$

$$G8(0, 2) = 0*-3 + 0*-3 + 0*-3 + 6*-3 + 7*0 + 2*5 + 4*-3 + 7*5 + 2*5 = 25$$

$$G8(0, 3) = 0*-3 + 0*-3 + 0*-3 + 7*-3 + 2*0 + 2*5 + 7*-3 + 2*5 + 2*5 = -12$$

$$G8(0, 4) = 0*-3 + 0*-3 + 0*-3 + 2*-3 + 2*0 + 0*5 + 2*-3 + 2*5 + 0*5 = -2$$

$$G8(1, 0) = 0*-3 + 4*-3 + 6*-3 + 0*-3 + 6*0 + 4*5 + 0*-3 + 3*5 + 5*5 = 30$$

$$G8(1, 1) = 4*-3 + 6*-3 + 7*-3 + 6*-3 + 4*0 + 7*5 + 3*-3 + 5*5 + 3*5 = -3$$

$$G8(1, 2) = 6*-3 + 7*-3 + 2*-3 + 4*-3 + 7*0 + 2*5 + 5*-3 + 3*5 + 5*5 = -22$$

$$G8(1, 3) = 7*-3 + 2*-3 + 2*-3 + 7*-3 + 2*0 + 2*5 + 3*-3 + 5*5 + 4*5 = -8$$

$$G8(1, 4) = 2*-3 + 2*-3 + 0*-3 + 2*-3 + 2*0 + 0*5 + 5*-3 + 4*5 + 0*5 = -13$$

$$G8(2, 0) = 0*-3 + 6*-3 + 4*-3 + 0*-3 + 3*0 + 5*5 + 0*-3 + 4*5 + 6*5 = 45$$

$$G8(2, 1) = 6*-3 + 4*-3 + 7*-3 + 3*-3 + 5*0 + 3*5 + 4*-3 + 6*5 + 6*5 = 3$$

$$\begin{aligned}
G8(2, 2) &= 4*-3 + 7*-3 + 2*-3 + 5*-3 + 3*0 + 5*5 + 6*-3 + 6*5 + 4*5 = 3 \\
G8(2, 3) &= 7*-3 + 2*-3 + 2*-3 + 3*-3 + 5*0 + 4*5 + 6*-3 + 4*5 + 4*5 = 0 \\
G8(2, 4) &= 2*-3 + 2*-3 + 0*-3 + 5*-3 + 4*0 + 0*5 + 4*-3 + 4*5 + 0*5 = -19 \\
G8(3, 0) &= 0*-3 + 3*-3 + 5*-3 + 0*-3 + 4*0 + 6*5 + 0*-3 + 1*5 + 2*5 = 21 \\
G8(3, 1) &= 3*-3 + 5*-3 + 3*-3 + 4*-3 + 6*0 + 6*5 + 1*-3 + 2*5 + 3*5 = 7 \\
G8(3, 2) &= 5*-3 + 3*-3 + 5*-3 + 6*-3 + 6*0 + 4*5 + 2*-3 + 3*5 + 6*5 = 2 \\
G8(3, 3) &= 3*-3 + 5*-3 + 4*-3 + 6*-3 + 4*0 + 4*5 + 3*-3 + 6*5 + 6*5 = 17 \\
G8(3, 4) &= 5*-3 + 4*-3 + 0*-3 + 4*-3 + 4*0 + 0*5 + 6*-3 + 6*5 + 0*5 = -27 \\
G8(4, 0) &= 0*-3 + 4*-3 + 6*-3 + 0*-3 + 1*0 + 2*5 + 0*-3 + 0*5 + 0*5 = -20 \\
G8(4, 1) &= 4*-3 + 6*-3 + 6*-3 + 1*-3 + 2*0 + 3*5 + 0*-3 + 0*5 + 0*5 = -36 \\
G8(4, 2) &= 6*-3 + 6*-3 + 4*-3 + 2*-3 + 3*0 + 6*5 + 0*-3 + 0*5 + 0*5 = -24 \\
G8(4, 3) &= 6*-3 + 4*-3 + 4*-3 + 3*-3 + 6*0 + 6*5 + 0*-3 + 0*5 + 0*5 = -21 \\
G8(4, 4) &= 4*-3 + 4*-3 + 0*-3 + 6*-3 + 6*0 + 0*5 + 0*-3 + 0*5 + 0*5 = -42
\end{aligned}$$

G8 =

80	60	25	-12	-2
30	-3	-22	-8	-13
45	3	3	0	-19
21	7	2	17	-27
-20	-36	-24	-21	-42

|G8| =

80	60	25	12	2
30	3	22	8	13
45	3	3	0	19
21	7	2	17	27
20	36	24	21	42

G = max { |G1|, |G2|, |G3|, |G4|, |G5|, |G6|, |G7|, |G8| }:

80	84	73	68	30
66	37	34	48	45
75	19	29	32	53
61	33	18	17	75
60	68	72	69	70

Câu 4. Thực hiện phát hiện biên bằng bộ lọc Laplacian:

I =	4	6	7	2	2
	6	4	7	2	2
	3	5	3	5	4
	4	6	6	4	4
	1	2	3	6	6

Thực hiện dò biên theo phương pháp Laplacian. Sử dụng mặt nạ:

-1	-1	-1
-1	4	-1
-1	-1	-1

$$G(0, 0) = 0*-1 + 0*-1 + 0*-1 + 0*-1 + 4*4 + 6*-1 + 0*-1 + 6*-1 + 4*-1 = 0$$

$$G(0, 1) = 0*-1 + 0*-1 + 0*-1 + 4*-1 + 6*4 + 7*-1 + 6*-1 + 4*-1 + 7*-1 = -4$$

$$G(0, 2) = 0*-1 + 0*-1 + 0*-1 + 6*-1 + 7*4 + 2*-1 + 4*-1 + 7*-1 + 2*-1 = 7$$

$$G(0, 3) = 0*-1 + 0*-1 + 0*-1 + 7*-1 + 2*4 + 2*-1 + 7*-1 + 2*-1 + 2*-1 = -12$$

$$G(0, 4) = 0*-1 + 0*-1 + 0*-1 + 2*-1 + 2*4 + 0*-1 + 2*-1 + 2*-1 + 0*-1 = 2$$

$$G(1, 0) = 0*-1 + 4*-1 + 6*-1 + 0*-1 + 6*4 + 4*-1 + 0*-1 + 3*-1 + 5*-1 = 2$$

$$G(1, 1) = 4*-1 + 6*-1 + 7*-1 + 6*-1 + 4*4 + 7*-1 + 3*-1 + 5*-1 + 3*-1 = -25$$

$$G(1, 2) = 6*-1 + 7*-1 + 2*-1 + 4*-1 + 7*4 + 2*-1 + 5*-1 + 3*-1 + 5*-1 = -6$$

$$G(1, 3) = 7*-1 + 2*-1 + 2*-1 + 7*-1 + 2*4 + 2*-1 + 3*-1 + 5*-1 + 4*-1 = -24$$

$$G(1, 4) = 2*-1 + 2*-1 + 0*-1 + 2*-1 + 2*4 + 0*-1 + 5*-1 + 4*-1 + 0*-1 = -7$$

$$G(2, 0) = 0*-1 + 6*-1 + 4*-1 + 0*-1 + 3*4 + 5*-1 + 0*-1 + 4*-1 + 6*-1 = -13$$

$$G(2, 1) = 6*-1 + 4*-1 + 7*-1 + 3*-1 + 5*4 + 3*-1 + 4*-1 + 6*-1 + 6*-1 = -19$$

$$G(2, 2) = 4^{*-1} + 7^{*-1} + 2^{*-1} + 5^{*-1} + 3^*4 + 5^{*-1} + 6^{*-1} + 6^{*-1} + 4^{*-1} = -27$$

$$G(2, 3) = 7^{*-1} + 2^{*-1} + 2^{*-1} + 3^{*-1} + 5^*4 + 4^{*-1} + 6^{*-1} + 4^{*-1} + 4^{*-1} = -12$$

$$G(2, 4) = 2^{*-1} + 2^{*-1} + 0^{*-1} + 5^{*-1} + 4^*4 + 0^{*-1} + 4^{*-1} + 4^{*-1} + 0^{*-1} = -1$$

$$G(3, 0) = 0^{*-1} + 3^{*-1} + 5^{*-1} + 0^{*-1} + 4^*4 + 6^{*-1} + 0^{*-1} + 1^{*-1} + 2^{*-1} = -1$$

$$G(3, 1) = 3^{*-1} + 5^{*-1} + 3^{*-1} + 4^{*-1} + 6^*4 + 6^{*-1} + 1^{*-1} + 2^{*-1} + 3^{*-1} = -3$$

$$G(3, 2) = 5^{*-1} + 3^{*-1} + 5^{*-1} + 6^{*-1} + 6^*4 + 4^{*-1} + 2^{*-1} + 3^{*-1} + 6^{*-1} = -10$$

$$G(3, 3) = 3^{*-1} + 5^{*-1} + 4^{*-1} + 6^{*-1} + 4^*4 + 4^{*-1} + 3^{*-1} + 6^{*-1} + 6^{*-1} = -21$$

$$G(3, 4) = 5^{*-1} + 4^{*-1} + 0^{*-1} + 4^{*-1} + 4^*4 + 0^{*-1} + 6^{*-1} + 6^{*-1} + 0^{*-1} = -9$$

$$G(4, 0) = 0^{*-1} + 4^{*-1} + 6^{*-1} + 0^{*-1} + 1^*4 + 2^{*-1} + 0^{*-1} + 0^{*-1} + 0^{*-1} = -8$$

$$G(4, 1) = 4^{*-1} + 6^{*-1} + 6^{*-1} + 1^{*-1} + 2^*4 + 3^{*-1} + 0^{*-1} + 0^{*-1} + 0^{*-1} = -12$$

$$G(4, 2) = 6^{*-1} + 6^{*-1} + 4^{*-1} + 2^{*-1} + 3^*4 + 6^{*-1} + 0^{*-1} + 0^{*-1} + 0^{*-1} = -12$$

$$G(4, 3) = 6^{*-1} + 4^{*-1} + 4^{*-1} + 3^{*-1} + 6^*4 + 6^{*-1} + 0^{*-1} + 0^{*-1} + 0^{*-1} = 1$$

$$G(4, 4) = 4^{*-1} + 4^{*-1} + 0^{*-1} + 6^{*-1} + 6^*4 + 0^{*-1} + 0^{*-1} + 0^{*-1} + 0^{*-1} = 10$$

Kết quả:

$G = I \oplus H =$	0	-4	7	-12	2
	2	-25	-6	-24	-7
	-13	-19	-27	-12	-1
	-1	-3	-10	-21	-9
	-8	-12	-12	1	10