

Supplementary Material: *Robust Trading Strategies through Topological Features and Deep Reinforcement Learning*

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Abstract

We introduce a novel strategy that incorporates Topological Data Analysis (TDA) into a Deep Reinforcement Learning (DRL) system tailored for financial trading tasks. Standard econometric techniques and deep learning models frequently struggle to capture the nonlinear dependencies and regime shifts characteristic of asset price behavior. Although DRL has gained popularity in financial applications, it typically depends on statistical or technical signals, which may limit interpretability and resilience. TDA, by contrast, provides a mathematical framework for capturing the evolving geometric patterns in time series through persistent homology. Our method employs time-delay embedding with sliding windows to convert one-dimensional price sequences into point clouds. These are transformed into persistence diagrams, from which quantitative descriptors—such as entropy, amplitude, and feature counts—are extracted. These topological features are incorporated into the state representation of a Deep Q-Network (DQN) agent, enabling it to leverage the structural properties of financial time series. We test the proposed DQN_{TDA} model across multiple datasets spanning both cryptocurrencies and traditional financial markets, and under varying regime conditions. Comparative experiments show that augmenting the agent’s input space with TDA-derived metrics improves its performance in terms of Sharpe ratio, Sortino ratio, cumulative returns, and reward efficiency, outperforming baselines that use purely technical or statistical inputs. The code is available at https://github.com/lcjr86/RTS_TDA_DRL

Supplementary Material

This document provides the complete experimental results for the manuscript *Robust Trading Strategies through Topological Features and Deep Reinforcement Learning*. Below, we report the full results for the ten datasets and all tested methods. Each table entry is formatted as μ_σ , where μ is the mean and σ is the standard deviation over the different random seeds tested. For each dataset column and metric, the best method is highlighted in **bold** and the second-best in *italics*.

Method	BNBUSDT	BTCUSD	ETHUSD	XRPUSD	ADAUSD	LTCUSD	FCHI	USO	EURUSD	IEUR
DQN _{SMA}	0.362 _{0.967}	-0.054 _{0.200}	1.295 _{0.761}	2.481 _{1.005}	0.673 _{2.314}	-0.051 _{0.397}	-0.053 _{0.193}	0.054 _{0.186}	0.019 _{0.107}	0.068 _{0.177}
DQN _{EMA}	0.207 _{0.857}	0.573 _{0.131}	0.538 _{0.428}	-0.193 _{0.476}	3.769 _{5.633}	0.086 _{0.742}	-0.113 _{0.175}	1.028_{0.517}	-0.057 _{0.016}	0.049 _{0.231}
DQN _{MACD}	0.743 _{0.431}	0.265 _{0.666}	-0.168 _{0.384}	0.158 _{0.580}	<i>3.771_{6.490}</i>	-0.036 _{0.415}	-0.054 _{0.091}	0.607 _{0.381}	-0.004 _{0.180}	0.009 _{0.056}
DQN _{RSI}	0.417 _{0.345}	-0.051 _{0.548}	0.370 _{0.877}	-0.211 _{0.815}	-0.089 _{1.182}	0.201 _{0.571}	0.025 _{0.118}	0.366 _{0.664}	-0.033 _{0.119}	-0.058 _{0.219}
DQN _{SO}	0.135 _{0.340}	0.479 _{0.473}	-0.674 _{0.098}	1.561 _{1.618}	0.446 _{2.038}	-0.006 _{1.195}	-0.032 _{0.117}	0.255 _{0.329}	0.066 _{0.076}	-0.054 _{0.185}
DQN _{BB}	-0.119 _{0.463}	0.587 _{0.950}	0.111 _{0.679}	3.050 _{2.980}	-0.529 _{0.126}	1.666 _{1.751}	0.043 _{0.053}	0.082 _{0.659}	0.039 _{0.034}	0.077 _{0.184}
DQN _{ATR}	0.387 _{0.725}	0.359 _{0.530}	1.021 _{0.596}	0.959 _{1.598}	-0.121 _{0.858}	0.947 _{1.194}	-0.053 _{0.087}	0.258 _{0.582}	-0.029 _{0.074}	-0.009 _{0.358}
DQN _{LF}	0.372 _{0.578}	-0.167 _{0.163}	-0.302 _{0.323}	0.822 _{1.102}	-0.886 _{0.120}	1.115 _{2.301}	-0.073 _{0.044}	0.185 _{0.312}	0.049 _{0.088}	0.031 _{0.248}
DQN _{DF}	-0.208 _{0.395}	-0.339 _{0.128}	-0.685 _{0.143}	0.380 _{0.827}	2.316 _{2.625}	0.307 _{0.224}	-0.024 _{0.167}	0.100 _{0.540}	0.027 _{0.041}	0.107 _{0.218}
DQN _{DCF}	0.357 _{0.000}	0.757_{0.000}	1.131 _{1.818}	<i>3.516_{6.688}</i>	0.624 _{2.410}	8.332_{4.089}	<i>0.172_{0.145}</i>	0.088 _{0.000}	0.082 _{0.019}	0.018 _{0.601}
DQN _{TD}	0.889_{1.492}	<i>0.737_{0.092}</i>	2.878_{1.982}	1.002 _{2.442}	0.110 _{0.092}	<i>4.493_{4.200}</i>	0.024 _{0.310}	<i>0.982_{1.083}</i>	0.105_{0.115}	<i>0.356_{0.208}</i>
DQN _{TDE}	0.623 _{0.979}	-0.286 _{0.289}	-0.416 _{0.066}	0.690 _{1.016}	-0.734 _{0.167}	0.516 _{1.289}	0.093 _{0.178}	0.261 _{0.372}	<i>0.085_{0.041}</i>	-0.154 _{0.351}
DQN _{FRUITS}	0.088 _{0.571}	0.155 _{0.633}	1.281 _{0.516}	-0.423 _{0.386}	1.268 _{3.699}	0.122 _{0.291}	-0.157 _{0.059}	-0.131 _{0.195}	-0.058 _{0.023}	-0.185 _{0.045}
DQN _{NetF}	-0.244 _{0.047}	-0.217 _{0.313}	-0.344 _{0.428}	-0.369 _{0.382}	-0.512 _{0.351}	-0.318 _{0.503}	0.202_{0.046}	0.076 _{0.470}	-0.072 _{0.029}	0.090 _{0.034}
DQN _{TDA}	<i>0.853_{0.629}</i>	0.287 _{0.751}	<i>1.912_{1.352}</i>	5.926_{4.690}	4.739_{7.944}	1.179 _{2.213}	0.165 _{0.285}	0.582 _{0.952}	0.079 _{0.072}	0.362_{0.164}

Table 1: Total Return across the 10 datasets. Each entry reports the mean and standard deviation over the different random seeds tested, formatted as μ_σ . For each dataset column, the best method is highlighted in **bold**, and the second-best in *italics*.

Method	BNBUSDT	BTCUSD	ETHUSD	XRPUSD	ADAUSD	LTCUSD	FCHI	USO	EURUSD	IEUR
DQN _{SMA}	0.426 _{1.248}	0.080 _{0.345}	1.168 _{0.418}	<i>1.414_{0.262}</i>	0.405 _{0.945}	0.208 _{0.506}	-0.327 _{1.184}	0.292 _{0.565}	0.224 _{1.205}	0.404 _{0.983}
DQN _{EMA}	0.238 _{1.188}	0.966 _{0.138}	0.749 _{0.292}	0.089 _{0.647}	1.114_{1.012}	0.120 _{1.095}	-0.742 _{1.293}	2.260_{0.817}	-0.644 _{0.196}	0.261 _{1.303}
DQN _{MACD}	<i>1.108_{0.399}</i>	0.458 _{0.823}	0.026 _{0.507}	0.454 _{0.418}	<i>1.110_{0.936}</i>	0.229 _{0.477}	-0.279 _{0.594}	1.546 _{0.775}	-0.123 _{0.069}	0.132 _{0.316}
DQN _{RSI}	0.757 _{0.392}	-0.099 _{1.055}	0.513 _{0.660}	-0.061 _{0.794}	0.100 _{0.782}	0.426 _{0.559}	0.201 _{0.711}	0.827 _{1.713}	-0.425 _{1.515}	-0.340 _{1.236}
DQN _{SO}	0.355 _{0.557}	0.806 _{0.554}	-0.954 _{0.327}	1.028 _{0.655}	0.176 _{1.112}	-0.221 _{1.299}	-0.147 _{0.734}	0.787 _{0.783}	0.771 _{0.848}	-0.288 _{1.055}
DQN _{BB}	-0.219 _{1.083}	0.796 _{0.968}	0.218 _{0.892}	1.288 _{0.958}	-0.109 _{0.200}	1.155 _{0.757}	0.333 _{0.308}	0.070 _{1.708}	0.487 _{0.386}	0.449 _{1.015}
DQN _{ATR}	0.615 _{0.815}	0.613 _{0.772}	1.042 _{0.345}	0.543 _{1.225}	0.095 _{0.789}	0.809 _{0.865}	-0.267 _{0.578}	0.665 _{1.329}	-0.331 _{0.912}	-0.204 _{2.035}
DQN _{LF}	0.645 _{0.664}	-0.136 _{0.356}	-0.171 _{0.519}	0.740 _{0.696}	-1.075 _{0.704}	0.576 _{1.345}	-0.385 _{0.291}	0.613 _{0.780}	0.579 _{0.965}	0.140 _{1.466}
DQN _{DF}	-0.397 _{1.090}	-0.528 _{0.342}	-1.036 _{0.497}	0.565 _{0.538}	0.961 _{0.948}	0.591 _{0.166}	-0.123 _{1.005}	0.204 _{1.539}	0.351 _{0.468}	0.587 _{1.122}
DQN _{DCF}	0.716 _{0.000}	1.157_{0.000}	0.883 _{0.902}	0.603 _{1.863}	0.273 _{1.036}	2.191_{0.528}	<i>0.969_{0.713}</i>	0.423 _{0.000}	0.994 _{0.205}	-0.387 _{3.094}
DQN _{TD}	0.940 _{1.320}	<i>1.138_{0.088}</i>	1.681_{0.682}	0.498 _{1.153}	0.762 _{0.057}	<i>1.885_{0.740}</i>	0.048 _{1.815}	<i>1.985_{1.641}</i>	0.958 _{1.140}	<i>1.769_{0.874}</i>
DQN _{TDE}	0.832 _{0.964}	-0.461 _{0.650}	-0.291 _{0.123}	0.750 _{0.463}	-0.369 _{0.606}	0.531 _{0.796}	0.572 _{0.968}	0.770 _{0.989}	<i>0.996_{0.442}</i>	-1.168 _{2.267}
DQN _{FRUITS}	0.116 _{1.161}	0.271 _{0.941}	1.191 _{0.266}	-0.202 _{0.700}	0.064 _{1.482}	0.429 _{0.252}	-0.969 _{0.437}	-0.305 _{0.661}	-0.657 _{0.281}	-1.061 _{0.310}
DQN _{NetF}	-0.276 _{0.109}	-0.333 _{0.812}	-0.292 _{0.655}	-0.059 _{0.457}	-0.232 _{0.698}	-0.274 _{0.867}	1.199_{0.235}	0.178 _{1.415}	-0.840 _{0.362}	0.573 _{0.178}
DQN _{TDA}	1.183_{0.566}	0.355 _{1.285}	<i>1.389_{0.584}</i>	1.841_{0.687}	0.883 _{1.492}	0.757 _{1.060}	0.884 _{1.619}	1.232 _{1.752}	1.000_{0.785}	1.775_{0.683}

Table 2: Sharpe Ratio across the 10 datasets. Each entry reports the mean and standard deviation over the different random seeds tested, formatted as μ_σ . For each dataset column, the best method is highlighted in **bold**, and the second-best in *italics*.

Method	BNBUSDT	BTCUSD	ETHUSD	XRPUSD	ADAUSD	LTCUSD	FCHI	USO	EURUSD	IEUR
DQN _{SMA}	0.721 _{1.963}	0.130 _{0.525}	2.003 _{0.787}	<i>2.107_{0.634}</i>	0.875 _{1.749}	0.320 _{0.683}	-0.442 _{1.516}	0.478 _{0.886}	0.389 _{1.903}	0.533 _{1.395}
DQN _{EMA}	0.453 _{1.894}	1.391 _{0.317}	1.225 _{0.525}	0.161 _{0.827}	2.242_{2.252}	0.260 _{1.484}	-0.928 _{1.605}	4.404_{2.053}	-0.957 _{0.316}	0.461 _{1.662}
DQN _{MACD}	<i>1.765_{0.665}</i>	0.712 _{1.274}	0.094 _{0.746}	0.645 _{0.595}	1.924 _{2.034}	0.315 _{0.660}	-0.374 _{0.819}	2.555 _{1.351}	-0.031 _{0.096}	0.140 _{0.411}
DQN _{RSI}	1.219 _{0.636}	-0.104 _{1.518}	0.755 _{1.005}	-0.021 _{1.030}	0.518 _{1.510}	0.612 _{0.805}	0.413 _{1.115}	1.682 _{3.034}	-0.593 _{2.203}	-0.103 _{2.017}
DQN _{SO}	0.563 _{0.858}	1.112 _{0.782}	-1.239 _{0.363}	1.612 _{1.161}	0.421 _{2.088}	-0.158 _{1.911}	-0.127 _{1.093}	1.210 _{1.160}	1.306 _{1.419}	-0.219 _{1.335}
DQN _{BB}	-0.277 _{1.628}	1.177 _{1.402}	0.367 _{1.307}	1.971 _{1.612}	-0.182 _{0.329}	1.843 _{1.333}	0.434 _{0.402}	0.287 _{2.961}	0.828 _{0.659}	0.554 _{1.454}
DQN _{ATR}	1.021 _{1.406}	0.996 _{1.255}	1.674 _{0.561}	0.933 _{1.731}	0.409 _{1.182}	1.273 _{1.329}	-0.340 _{0.748}	1.168 _{2.283}	-0.425 _{1.266}	0.236 _{3.131}
DQN _{LF}	0.939 _{0.963}	-0.179 _{0.479}	-0.272 _{0.784}	1.096 _{1.013}	-1.059 _{0.699}	1.022 _{2.198}	-0.572 _{0.458}	1.021 _{1.307}	1.020 _{1.704}	0.557 _{2.249}
DQN _{DF}	-0.586 _{1.607}	-0.763 _{0.496}	-1.324 _{0.560}	0.806 _{0.744}	1.908 _{1.920}	0.964 _{0.306}	-0.203 _{1.344}	0.350 _{2.331}	0.555 _{0.718}	1.066 _{1.935}
DQN _{DCF}	1.070 _{0.000}	1.862_{0.000}	1.338 _{1.360}	1.264 _{2.969}	0.651 _{1.686}	<i>2.347_{1.069}</i>	1.428 _{1.352}	0.620 _{0.000}	1.567 _{0.334}	0.392 _{5.002}
DQN _{TD}	1.433 _{1.989}	<i>1.733_{0.294}</i>	2.620_{1.089}	0.681 _{1.484}	0.740 _{0.064}	3.097_{1.500}	0.323 _{2.747}	<i>3.099_{2.380}</i>	<i>1.817_{2.150}</i>	<i>2.705_{1.465}</i>
DQN _{TDE}	1.327 _{1.547}	-0.669 _{0.950}	-0.428 _{0.187}	1.020 _{0.648}	-0.370 _{0.613}	0.815 _{1.226}	0.781 _{1.288}	1.301 _{1.720}	1.849_{0.825}	-1.209 _{2.741}
DQN _{FRUITS}	0.235 _{1.836}	0.386 _{1.416}	1.865 _{0.461}	-0.199 _{0.817}	0.633 _{2.445}	0.616 _{0.316}	-1.370 _{0.539}	-0.510 _{1.032}	-1.129 _{0.611}	-1.374 _{0.422}
DQN _{NetF}	-0.433 _{0.220}	-0.506 _{1.219}	-0.331 _{0.862}	-0.014 _{0.606}	-0.130 _{0.761}	-0.341 _{1.172}	1.952_{0.579}	0.415 _{2.313}	-1.255 _{0.564}	0.959 _{0.483}
DQN _{TDA}	1.793_{0.854}	0.533 _{1.783}	<i>2.234_{0.967}</i>	3.048_{1.241}	<i>2.090_{2.870}</i>	1.165 _{1.624}	<i>1.756_{2.655}</i>	1.983 _{2.794}	1.551 _{1.334}	2.822_{0.480}

Table 3: Sortino Ratio across the 10 datasets. Each entry reports the mean and standard deviation over the different random seeds tested, formatted as μ_σ . For each dataset column, the best method is highlighted in **bold**, and the second-best in *italics*.

Method	BNBUSDT	BTUSD	ETHUSD	XRPUSD	ADAUSD	LTCUSD	FCHI	USO	EURUSD	IEUR
DQN _{SMA}	1362.275 _{967.241}	946.270 _{199.756}	2294.652 _{760.553}	3480.976 _{1005.077}	1673.474 _{2314.218}	948.939 _{397.234}	947.467 _{192.811}	1054.065 _{185.821}	1019.174 _{107.371}	1067.910 _{177.181}
DQN _{EMA}	1207.412 _{856.686}	1573.445 _{131.058}	1537.787 _{427.589}	806.803 _{475.514}	4769.114 _{5633.361}	1086.268 _{741.716}	886.949 _{175.232}	2027.687 _{516.972}	943.286 _{15.735}	1048.818 _{231.268}
DQN _{MACD}	1742.716 _{430.978}	1264.598 _{666.159}	831.780 _{383.924}	1158.337 _{579.907}	<i>4770.865</i> _{6490.366}	963.851 _{415.291}	945.566 _{91.255}	1606.548 _{381.332}	996.116 _{180.040}	1008.535 _{55.947}
DQN _{RSI}	1416.955 _{345.181}	948.751 _{547.937}	1369.552 _{877.479}	789.349 _{815.456}	910.906 _{1181.970}	1201.074 _{570.551}	1024.532 _{117.533}	1366.391 _{664.305}	966.506 _{119.456}	941.753 _{218.909}
DQN _{Sto}	1134.719 _{339.732}	1478.724 _{472.911}	325.871 _{198.060}	2561.492 _{1618.374}	1446.322 _{2037.595}	994.444 _{195.444}	967.923 _{117.484}	1254.651 _{328.659}	1065.808 _{75.554}	946.092 _{184.962}
DQN _{BB}	881.310 _{463.075}	1586.911 _{1949.524}	1110.513 _{679.431}	4049.650 _{2979.931}	470.778 _{126.199}	2665.702 _{1750.836}	1043.080 _{52.965}	1082.190 _{658.841}	1039.116 _{33.861}	1077.250 _{183.992}
DQN _{ATR}	1386.948 _{725.399}	1358.567 _{530.470}	2020.654 _{595.778}	1959.127 _{1597.600}	878.536 _{857.506}	1947.488 _{1193.940}	947.198 _{87.477}	1257.686 _{581.710}	970.645 _{73.843}	991.413 _{357.758}
DQN _{LF}	1371.687 _{577.829}	832.610 _{163.254}	698.476 _{323.158}	1821.793 _{102.305}	113.546 _{120.433}	2115.360 _{2300.846}	926.914 _{44.257}	1184.966 _{312.366}	1049.068 _{87.729}	1030.908 _{247.787}
DQN _{DF}	791.516 _{394.997}	661.158 _{128.426}	314.651 _{142.608}	1379.566 _{827.272}	3315.942 _{2624.777}	1306.743 _{223.762}	975.851 _{167.270}	1100.253 _{540.471}	1027.214 _{41.285}	1106.667 _{217.934}
DQN _{DCF}	1357.444 _{0.000}	1756.789 _{0.000}	2130.526 _{1817.956}	<i>4516.319</i> _{6687.764}	1624.231 _{2409.740}	7331.506 _{7088.784}	<i>1171.883</i> _{145.211}	1088.370 _{0.000}	1081.640 _{18.690}	1018.052 _{600.718}
DQN _{TD}	1889.182 _{1491.925}	<i>1736.618</i> _{91.516}	3878.148 _{1981.613}	1999.898 _{2441.605}	1110.005 _{92.015}	<i>5493.160</i> _{4199.694}	1023.960 _{309.997}	<i>1982.266</i> _{1083.198}	1105.453 _{214.972}	<i>1355.984</i> _{207.984}
DQN _{TDE}	1623.434 _{979.035}	714.240 _{289.014}	583.694 _{65.644}	1690.325 _{1015.929}	266.358 _{166.644}	1516.486 _{1289.498}	1092.678 _{178.067}	1261.336 _{371.666}	<i>1085.081</i> _{10.863}	846.373 _{350.757}
DQN _{FRUTTS}	1087.886 _{570.600}	1154.677 _{632.903}	2280.691 _{515.928}	577.359 _{386.312}	2268.374 _{3698.862}	1122.205 _{291.009}	843.025 _{59.455}	869.411 _{194.570}	942.245 _{22.526}	815.346 _{45.355}
DQN _{NetF}	756.272 _{47.173}	783.363 _{313.409}	656.087 _{427.625}	630.966 _{382.055}	488.085 _{350.889}	682.249 _{503.276}	1201.535 _{45.751}	1075.691 _{470.100}	927.864 _{28.944}	1090.384 _{34.289}
DQN _{TDA}	<i>1852.609</i> _{629.074}	1286.661 _{751.204}	<i>2911.577</i> _{1352.411}	6926.017 _{4689.636}	5739.003 _{7943.822}	2179.183 _{2213.033}	1165.204 _{284.507}	1581.984 _{952.156}	1079.301 _{71.571}	1372.189 _{164.471}

Table 4: Final Net Worth across the 10 datasets. Each entry reports the mean and standard deviation over the different random seeds tested, formatted as μ_σ . For each dataset column, the best method is highlighted in **bold**, and the second-best in *italics*.