

Supplementary Material: *Robust Trading Strategies through Topological Features and Deep Reinforcement Learning*

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Abstract

We introduce a novel strategy that incorporates Topological Data Analysis (TDA) into a Deep Reinforcement Learning (DRL) system tailored for financial trading tasks. Standard econometric techniques and deep learning models frequently struggle to capture the nonlinear dependencies and regime shifts characteristic of asset price behavior. Although DRL has gained popularity in financial applications, it typically depends on statistical or technical signals, which may limit interpretability and resilience. TDA, by contrast, provides a mathematical framework for capturing the evolving geometric patterns in time series through persistent homology. Our method employs time-delay embedding with sliding windows to convert one-dimensional price sequences into point clouds. These are transformed into persistence diagrams, from which quantitative descriptors—such as entropy, amplitude, and feature counts—are extracted. These topological features are incorporated into the state representation of a Deep Q-Network (DQN) agent, enabling it to leverage the structural properties of financial time series. We test the proposed DQN_{TDA} model across multiple datasets spanning both cryptocurrencies and traditional financial markets, and under varying regime conditions. Comparative experiments show that augmenting the agent’s input space with TDA-derived metrics improves its performance in terms of Sharpe ratio, Sortino ratio, cumulative returns, and reward efficiency, outperforming baselines that use purely technical or statistical inputs. The code is available at https://github.com/lcj86/RTS_TDA_DRL

Supplementary Material

This document provides the complete experimental results for the manuscript *Robust Trading Strategies through Topological Features and Deep Reinforcement Learning*. Below, we report the full results for the ten datasets and all tested methods. Each table entry is formatted as μ_σ , where μ is the mean and σ is the standard deviation over the different random seeds tested. For each dataset column and metric, the best method is highlighted in **bold** and the second-best in *italics*.

Method	BNBUSDT	BTCUSDT	ETHUSDT	XRPUSDT	ADAUSDT	LTCUSDT	FCHI	USO	EURUSD	IEUR
DQN _{SMA}	0.362 _{0.967}	-0.054 _{0.200}	1.295 _{0.761}	2.481 _{1.005}	0.673 _{2.314}	-0.051 _{0.397}	-0.053 _{0.193}	0.054 _{0.186}	0.019 _{0.107}	0.068 _{0.177}
DQN _{EMA}	0.207 _{0.857}	0.573 _{0.131}	0.538 _{0.428}	-0.193 _{0.476}	3.769 _{5.633}	0.086 _{0.742}	-0.113 _{0.175}	1.028_{0.517}	-0.057 _{0.016}	0.049 _{0.231}
DQN _{MACD}	0.743 _{0.431}	0.265 _{0.666}	-0.168 _{0.384}	0.158 _{0.580}	3.771 _{6.490}	-0.036 _{0.415}	-0.054 _{0.091}	0.607 _{0.381}	-0.004 _{0.180}	0.009 _{0.056}
DQN _{RSI}	0.417 _{0.345}	-0.051 _{0.548}	0.370 _{0.877}	-0.211 _{0.815}	-0.089 _{1.182}	0.201 _{0.571}	0.025 _{0.118}	0.366 _{0.664}	-0.033 _{0.119}	-0.058 _{0.219}
DQN _{SO}	0.135 _{0.340}	0.479 _{0.473}	-0.674 _{0.098}	1.561 _{1.618}	0.446 _{2.038}	-0.006 _{1.195}	-0.032 _{0.117}	0.255 _{0.329}	0.066 _{0.076}	-0.054 _{0.185}
DQN _{BB}	-0.119 _{0.463}	0.587 _{0.950}	0.111 _{0.679}	3.050 _{2.980}	-0.529 _{0.126}	1.666 _{1.751}	0.043 _{0.053}	0.082 _{0.659}	0.039 _{0.034}	0.077 _{0.184}
DQN _{ATR}	0.387 _{0.725}	0.359 _{0.530}	1.021 _{0.596}	0.959 _{1.598}	-0.121 _{0.858}	0.947 _{1.194}	-0.053 _{0.087}	0.258 _{0.582}	-0.029 _{0.074}	-0.009 _{0.358}
DQN _{LF}	0.372 _{0.578}	-0.167 _{0.163}	-0.302 _{0.323}	0.822 _{1.102}	-0.886 _{0.120}	1.115 _{2.301}	-0.073 _{0.044}	0.185 _{0.312}	0.049 _{0.088}	0.031 _{0.248}
DQN _{DF}	-0.208 _{0.395}	-0.339 _{0.128}	-0.685 _{0.143}	0.380 _{0.827}	2.316 _{2.625}	0.307 _{0.224}	-0.024 _{0.167}	0.100 _{0.540}	0.027 _{0.041}	0.107 _{0.218}
DQN _{DCF}	0.357 _{0.000}	0.757_{0.000}	1.131 _{1.818}	3.516 _{6.688}	0.624 _{2.410}	8.332_{4.089}	0.172 _{0.145}	0.088 _{0.000}	0.082 _{0.019}	0.018 _{0.601}
DQN _{TD}	0.889_{1.492}	0.737 _{0.092}	2.878_{1.982}	1.000 _{2.442}	0.110 _{0.092}	4.493 _{4.200}	0.024 _{0.310}	0.982 _{1.083}	0.105_{0.115}	0.356 _{0.208}
DQN _{TDE}	0.623 _{0.979}	-0.2860 _{0.289}	-0.416 _{0.066}	0.690 _{1.016}	-0.734 _{0.167}	0.5161 _{2.89}	0.0930 _{0.178}	0.2610 _{0.372}	0.085 _{0.041}	-0.154 _{0.351}
DQN _{FRUITS}	0.088 _{0.571}	0.1550 _{0.633}	1.281 _{0.516}	-0.423 _{0.386}	1.268 _{3.699}	0.1220 _{0.291}	-0.157 _{0.059}	-0.131 _{0.195}	-0.058 _{0.023}	-0.185 _{0.045}
DQN _{NetF}	-0.244 _{0.047}	-0.217 _{0.313}	-0.344 _{0.428}	-0.369 _{0.382}	-0.512 _{0.351}	-0.318 _{0.503}	0.202_{0.046}	0.0760 _{0.470}	-0.072 _{0.029}	0.090 _{0.034}
DQN _{TDA}	0.853 _{0.629}	0.2870 _{0.751}	1.912 _{1.352}	5.926_{4.690}	4.739_{7.944}	1.179 _{2.213}	0.1650 _{0.285}	0.582 _{0.952}	0.0790 _{0.072}	0.362_{0.164}

Table 1: Total Return across the 10 datasets. Each entry reports the mean and standard deviation over the different random seeds tested, formatted as μ_σ . For each dataset column, the best method is highlighted in **bold**, and the second-best in *italics*.

Method	BNBUSDT	BTCUSDT	ETHUSDT	XRPUSDT	ADAUSDT	LTCUSDT	FCHI	USO	EURUSD	IEUR
DQN _{SMA}	0.426 _{1.248}	0.080 _{0.345}	1.168 _{0.418}	4.141_{0.262}	0.405 _{0.945}	0.208 _{0.506}	-0.327 _{1.184}	0.292 _{0.565}	0.224 _{1.205}	0.404 _{0.983}
DQN _{EMA}	0.238 _{1.188}	0.966 _{0.138}	0.749 _{0.292}	0.089 _{0.647}	1.114_{1.012}	0.120 _{1.095}	-0.742 _{1.293}	2.260_{0.817}	-0.644 _{0.196}	0.261 _{1.303}
DQN _{MACD}	1.108_{0.399}	0.458 _{0.823}	0.026 _{0.507}	0.454 _{0.418}	1.110_{0.936}	0.229 _{0.477}	-0.279 _{0.594}	1.546 _{0.775}	-0.123 _{0.209}	0.132 _{0.316}
DQN _{RSI}	0.757 _{0.392}	-0.099 _{1.055}	0.513 _{0.660}	-0.061 _{0.794}	0.100 _{0.782}	0.426 _{0.559}	0.201 _{0.711}	0.827 _{1.713}	-0.425 _{1.515}	-0.340 _{1.236}
DQN _{SO}	0.355 _{0.557}	0.806 _{0.554}	-0.954 _{0.327}	1.028 _{0.655}	0.176 _{1.112}	-0.221 _{1.299}	-0.147 _{0.734}	0.787 _{0.783}	0.771 _{0.848}	-0.288 _{1.055}
DQN _{BB}	-0.219 _{1.083}	0.796 _{0.968}	0.218 _{0.892}	1.288 _{0.958}	-0.109 _{0.200}	1.155 _{0.757}	0.333 _{0.308}	0.070 _{1.708}	0.487 _{0.386}	0.449 _{1.015}
DQN _{ATR}	0.615 _{0.815}	0.613 _{0.772}	1.042 _{0.345}	0.543 _{1.225}	0.095 _{0.789}	0.809 _{0.865}	-0.267 _{0.578}	0.665 _{1.329}	-0.331 _{0.912}	-0.204 _{2.035}
DQN _{LF}	0.645 _{0.664}	-0.136 _{0.356}	-0.171 _{0.519}	0.740 _{0.696}	-1.075 _{0.704}	0.5761 _{3.45}	-0.385 _{0.291}	0.613 _{0.780}	0.579 _{0.965}	0.140 _{1.466}
DQN _{DF}	-0.397 _{1.090}	-0.528 _{0.342}	-1.036 _{0.497}	0.565 _{0.538}	0.961 _{0.948}	0.591 _{0.166}	-0.123 _{0.005}	0.204 _{1.539}	0.351 _{0.468}	0.587 _{1.122}
DQN _{DCF}	0.716 _{0.000}	1.157_{0.000}	0.883 _{0.902}	0.603 _{1.863}	0.273 _{1.036}	2.191_{0.528}	0.969_{0.713}	0.423 _{0.000}	0.994 _{0.205}	-0.387 _{3.094}
DQN _{TD}	0.940 _{1.320}	1.138_{0.088}	1.681_{0.682}	0.498 _{1.153}	0.762 _{0.057}	1.885_{0.740}	0.048 _{1.815}	1.985_{1.641}	0.958 _{1.140}	1.769_{0.874}
DQN _{TDE}	0.832 _{0.964}	-0.461 _{0.650}	-0.291 _{0.123}	0.750 _{0.463}	-0.369 _{0.606}	0.531 _{0.796}	0.572 _{0.968}	0.770 _{0.989}	0.996_{0.442}	-1.168 _{2.267}
DQN _{FRUITS}	0.116 _{1.161}	0.271 _{0.941}	1.191 _{0.266}	-0.202 _{0.700}	0.064 _{1.482}	0.429 _{0.252}	-0.969 _{0.437}	-0.305 _{0.661}	-0.657 _{0.281}	-1.061 _{0.310}
DQN _{NetF}	-0.276 _{0.109}	-0.333 _{0.812}	-0.292 _{0.655}	-0.059 _{0.457}	-0.232 _{0.698}	-0.274 _{0.867}	1.199_{0.235}	0.178 _{1.415}	-0.840 _{0.362}	0.573 _{0.178}
DQN _{TDA}	1.183_{0.566}	0.355 _{1.285}	1.989 _{0.584}	1.841_{0.687}	0.883 _{1.492}	0.757 _{1.060}	0.884 _{1.619}	1.232 _{1.752}	1.000_{0.785}	1.775_{0.683}

Table 2: Sharpe Ratio across the 10 datasets. Each entry reports the mean and standard deviation over the different random seeds tested, formatted as μ_σ . For each dataset column, the best method is highlighted in **bold**, and the second-best in *italics*.

Method	BNBUSDT	BTCUSDT	ETHUSDT	XRPUSDT	ADAUSDT	LTCUSDT	FCHI	USO	EURUSD	IEUR
DQN _{SMA}	0.721 _{1.963}	0.130 _{0.525}	2.003 _{0.787}	2.107_{0.634}	0.875 _{1.749}	0.320 _{0.683}	-0.442 _{1.516}	0.478 _{0.886}	0.389 _{1.903}	0.533 _{1.395}
DQN _{EMA}	0.453 _{1.894}	1.391 _{0.317}	1.225 _{0.525}	0.161 _{0.827}	2.242_{2.252}	0.260 _{1.484}	-0.928 _{1.605}	4.404_{2.053}	-0.957 _{0.316}	0.461 _{1.662}
DQN _{MACD}	1.765_{0.665}	0.712 _{1.274}	0.094 _{0.746}	0.645 _{0.595}	1.924 _{2.034}	0.315 _{0.660}	-0.374 _{0.819}	2.555 _{1.351}	-0.031 _{3.096}	0.140 _{4.411}
DQN _{RSI}	1.219 _{0.636}	-0.104 _{1.518}	0.755 _{1.005}	-0.021 _{1.030}	0.518 _{1.510}	0.612 _{0.805}	0.413 _{1.115}	1.682 _{3.034}	-0.593 _{2.203}	-0.103 _{2.017}
DQN _{SO}	0.563 _{0.858}	1.112 _{0.782}	-1.239 _{0.363}	1.612 _{1.611}	0.421 _{2.088}	-0.158 _{1.911}	-0.127 _{1.093}	1.210 _{1.160}	1.306 _{1.419}	-0.219 _{1.335}
DQN _{BB}	-0.2771 _{1.628}	1.177 _{1.402}	0.367 _{1.307}	1.971 _{1.612}	-0.182 _{0.329}	1.843 _{1.333}	0.434 _{0.402}	0.287 _{2.961}	0.828 _{0.659}	0.554 _{1.454}
DQN _{ATR}	1.021 _{1.406}	0.996 _{1.255}	1.674 _{0.561}	0.933 _{1.731}	0.409 _{1.182}	1.273 _{1.329}	-0.340 _{0.748}	1.168 _{2.283}	-0.425 _{1.266}	0.236 _{3.131}
DQN _{LF}	0.939 _{0.963}	-0.179 _{0.479}	-0.272 _{0.784}	1.096 _{1.013}	-1.059 _{0.699}	1.022 _{2.198}	-0.572 _{0.458}	1.022 _{1.307}	1.020 _{1.704}	0.557 _{2.249}
DQN _{DF}	-0.586 _{1.607}	-0.763 _{0.496}	-1.324 _{0.560}	0.806 _{0.744}	1.908 _{1.920}	0.964 _{0.306}	-0.203 _{0.344}	0.350 _{2.331}	0.555 _{0.718}	1.066 _{1.935}
DQN _{DCF}	1.070 _{0.000}	1.862_{0.000}	1.338 _{1.360}	1.264 _{2.969}	0.651 _{1.686}	2.347_{1.069}	1.428 _{1.352}	0.620 _{0.000}	1.567 _{0.334}	0.392 _{5.002}
DQN _{TD}	1.433 _{1.989}	1.733_{0.294}	2.620_{1.089}	0.681 _{1.484}	0.740 _{0.064}	3.097_{1.500}	0.323 _{2.747}	3.099_{2.380}	1.817_{2.150}	2.705_{1.465}
DQN _{TDE}	1.327 _{1.547}	-0.669 _{0.950}	-0.428 _{0.187}	1.020 _{0.648}	-0.370 _{0.613}	0.815 _{1.226}	0.781 _{1.288}	1.301 _{1.720}	1.849_{0.825}	-1.209 _{2.741}
DQN _{FRUITS}	0.235 _{1.836}	0.386 _{1.416}	1.865 _{0.461}	-0.199 _{0.817}	0.633 _{2.445}	0.616 _{0.316}	-1.370 _{0.539}	-0.510 _{1.032}	-1.129 _{0.611}	-1.374 _{0.422}
DQN _{NetF}	-0.433 _{0.220}	-0.506 _{1.219}	-0.331 _{0.862}	-0.014 _{0.606}	-0.130 _{0.761}	-0.341 _{1.172}	1.952_{0.579}	0.415 _{2.313}	-1.255 _{0.564}	0.959 _{0.483}
DQN _{TDA}	1.793_{0.854}	0.533 _{1.783}	2.234 _{0.967}	3.048_{1.241}	2.090 _{2.870}	1.165 _{1.624}	1.756 _{2.655}	1.983 _{2.794}	1.551 _{1.334}	2.822_{0.480}

Method	BNBUSDT	BTCCUSDT	ETHUSDT	XRPUSDT	ADAUSDT	LTCUSDT	FCHI	USO	EURUSD	IEUR
DQN _{SMA}	1362.275967.241	946.270199.756	2294.652760.553	3480.9761005.077	1673.4742314.218	948.939397.234	947.467192.811	1054.065185.821	1019.174107.371	1067.910177.181
DQN _{EMA}	1207.412456.686	1573.445131.058	1537.787427.589	806.803475.514	4769.1145633.361	1086.268741.716	886.949175.232	2027.687 <i>516.972</i>	943.28615.735	1048.818231.268
DQN _{MACD}	1742.716430.978	1264.598666.159	831.780383.924	1158.337579.907	<i>4770.865</i> 64.90.366	963.851415.29	945.56911.255	1606.548381.332	996.116180.040	1008.53555.947
DQN _{RSI}	1416.955345.181	948.751547.937	1369.552877.479	789.34915.456	910.9061181.970	1201.074570.551	1024.532117.533	1366.391664.305	966.506119.456	941.753218.909
DQN _{SO}	1134.719339.732	1478.724472.911	325.87198.060	2561.4921618.374	1446.3222037.595	994.444195.444	967.923117.484	1254.651328.659	1065.80875.554	946.092184.962
DQN _{BB}	881.310463.075	1586.911949.524	1110.513679.431	4049.6502979.931	470.778126.199	2665.7021750.836	1043.08052.965	1082.190658.841	1039.11633.861	1077.250183.992
DQN _{ATR}	1386.948725.399	1358.567530.470	2020.654595.778	1959.1271597.600	878.536857.506	1947.4881193.940	947.19887.477	1257.686581.710	970.64573.843	991.413357.758
DQN _{LF}	1371.687577.829	832.610163.254	698.476323.158	1821.793102.305	113.546120.433	2115.3602300.846	926.9144.257	1184.966312.366	1049.06887.729	1030.908247.787
DQN _{DP}	791.51694.997	661.158128.426	314.651142.608	1379.560827.272	3315.9422624.777	1306.743223.762	975.851167.270	1100.253540.471	1027.21441.285	1106.667217.934
DQN _{DCF}	1357.4440.000	1756.7890.000	2130.5261817.956	<i>4516.3196687.764</i>	1624.2312409.740	7331.5067088.784	<i>1171.883145.211</i>	1088.3700.000	1081.64618.690	1018.052600.718
DQN _{TID}	1889.1821.491.925	<i>1736.61891.516</i>	3878.1481981.613	1999.8982441.605	1110.00592.015	<i>54.93.160199.694</i>	1023.960309.997	<i>1982.2661083.194</i>	1105.453214.972	<i>1355.984207.924</i>
DQN _{TDE}	1623.434079.035	714.240289.014	583.69465.644	1690.3251015.929	266.358166.644	1516.4861289.498	1092.678178.067	1261.336371.666	<i>1085.08110.863</i>	846.373350.757
DQN _{Fruits}	1087.886570.600	1154.677632.903	2280.691515.928	577.359386.312	2268.3743698.862	1122.205291.009	843.02559.455	869.411194.570	942.24522.526	815.34645.355
DQN _{NetF}	756.27247.173	783.363313.409	656.087427.625	630.966382.055	488.085350.889	682.249503.276	1201.53545.751	1075.691470.100	927.86428.944	1090.38434.289
DQN _{TDA}	<i>1852.609629.074</i>	1286.661751.204	<i>2911.5771352.411</i>	6926.0174689.636	<i>5739.0037943.822</i>	2179.1832213.033	1165.204284.507	1581.984952.156	1079.30171.571	1372.189164.471

Table 4: Final Net Worth across the 10 datasets. Each entry reports the mean and standard deviation over the different random seeds tested, formatted as μ_σ . For each dataset column, the best method is highlighted in **bold**, and the second-best in *italics*.