

# DDA Software

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This software can be found on Github (<https://github.com/lclaudia/DDA>). The code is written in JULIA and works on Linux, Mac, and Windows. The underlying C codes are compiled using `cosmocc` from <https://github.com/jart/cosmopolitan>.

**LINUX:** If you get the error “run-detectors: unable to find an interpreter”, you can fix that by running these commands:

(see <https://github.com/jart/cosmopolitan/blob/master/tool/cosmocc/README.md> for more details).

```
sudo wget -O /usr/bin/ape https://cosmo.zip/pub/cosmos/bin/ape-$(uname -m).elf
sudo chmod +x /usr/bin/ape
sudo sh -c "echo ':APE:M::MZqFpD::/usr/bin/ape:' >/proc/sys/fs/binfmt_misc/register"
sudo sh -c "echo ':APE-jart:M::jartsr::/usr/bin/ape:' >/proc/sys/fs/binfmt_misc/register"
```

**WINDOWS:** Microsoft might be seeing `run_DDA_ASCII.exe` as a virus and is deleting it. To fix this problem, turn the “Real-time protection” (temporarily) off to execute the codes.

## 1 Data for DDA

Before we start with the DDA software we will first generate some simulated data from a 3D nonlinear system of ODEs (ordinary differential equations), namely the Rössler system [9]:

$$\begin{aligned}\dot{u}_1 &= -u_2 - u_3 \\ \dot{u}_2 &= u_1 + a u_2 \\ \dot{u}_3 &= b - c u_3 + u_1 u_3\end{aligned}\tag{1}$$

with  $a = 0.2$  and  $c = 5.7$  and  $\delta t = 0.05$ . This system can be encoded as

system	equation #	variable		coefficients
$\dot{u}_1 = -u_2 - u_3$	0	0	2	-1
$\dot{u}_1 = -u_2 - u_3$	0	0	3	-1
$\dot{u}_2 = u_1 + a u_2$	1	0	1	1
$\dot{u}_2 = u_1 + a u_2$	1	0	2	$a$
$\dot{u}_3 = b - c u_3 + u_1 u_3$	2	0	0	$b$
$\dot{u}_3 = b - c u_3 + u_1 u_3$	2	0	3	$-c$
$\dot{u}_3 = b - c u_3 + u_1 u_3$	2	1	3	1

Note, that the equation numbers are (0,1,2) for the three equations. This defines DIM=3 (the number of equations). There are two “variable” columns which define the order of nonlinearity `ODEorder=2`. The numbers in the two columns are 1 for  $u_1$ , 2 for  $u_2$ , and 3 for  $u_3$ . A line with only zeros denotes a constant term. All other entries are filled with zeros.

This encoding can be used to numerically integrate the Rössler system. The plots are shown in Fig. 1.

```
include("DDAfunctions.jl"); # set of Julia functions
NrSyst=1; # 1 single system
ROS=[0 0 2]; # single Roessler system
```

```

[0 0 3];
[1 0 1];
[1 0 2];
[2 0 0];
[2 0 3];
[2 1 3]
];
(MOD_nr,DIM,ODEorder,P) = make_MOD_nr(ROS,NrSyst);

a=.2; c=5.7;
dt=.05; X0=rand(DIM,1);
L=10000; TRANS=5000;

b=0.45;
MOD_par=[-1 -1 1 a b -c 1];

X = integrate_ODE_general_BIG(MOD_nr,MOD_par,dt,
                              L,DIM,ODEorder,X0,"",1:3,1,TRANS);

plot(X[:,1],X[:,2],X[:,3],
      color=:blue,legend=false,
      xlabel=L"x",ylabel=L"y",zlabel=L"z")
plot!(size=(500,500))

display(current());
print("Make pdf file and continue? ");
readline()

savefig("Roessler_0.45.pdf")

b=1;
MOD_par=[-1 -1 1 a b -c 1];
X = integrate_ODE_general_BIG(MOD_nr,MOD_par,dt,
                              L,DIM,ODEorder,X0,"",1:3,1,TRANS);

plot(X[:,1],X[:,2],X[:,3],
      color=:blue,legend=false,
      xlabel=L"x",ylabel=L"y",zlabel=L"z")
plot!(size=(500,500))

display(current());
print("Make pdf file and continue? ");
readline()

savefig("Roessler_1.pdf")

```

# encoding of the Roessler system  
# function defined in DDFunctions.jl

# choice of parameters  
# integration length and transient

# chaotic attractor  
# parameters

# integrate system  
# function defined in DDFunctions.jl  
# plot the attractor

# periodic attractor  
# parameters

# integrate system  
# function defined in DDFunctions.jl  
# plot the attractor

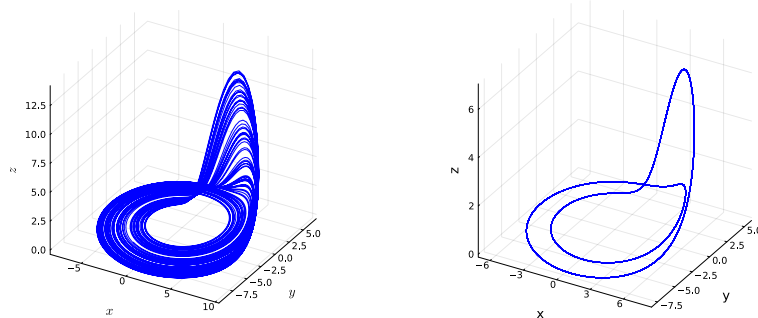


Figure 1: Rössler attractor with  $b = 0.45$  (left) and  $b = 0.1$  (right)

These are data from a single system

## 2 Delay Differential Analysis

DDA is a detection/classification framework that combines differential embeddings with linear and non-linear nonuniform functional delay embeddings [11, 7, 10] to relate the current derivatives of a system to the current and past values of the system variables [1, 6, 3].

More traditional analyses that are often based on spectral features have hundreds of features per data segment and approaches based on artificial neural networks increase the feature space even further. Therefore, such techniques rely on dimensionality reduction techniques to achieve a viable number of features. DDA, on the other hand, achieves a reduced feature space by mapping the data on a “natural” nonlinear basis (inspired by Max Planck’s “natural units” [8]) that is selected according to the classification problem. Therefore, DDA is efficient at embedding the meaningful dynamics of the data in a low-dimensional DDA model of only three terms.

### 2.1 Flavors of DDA

The different versions (flavors) of DDA are summarized in Tab. 1. See the papers in the reference column for more information.











Flavor	Reference	Description
ST-DDA	[5]	$u(t)$  $\dot{\mathbf{u}} = \mathbf{M}_u \mathbf{A}_u \Rightarrow \rho_u$
CT-DDA	[4]	$u(t)$  $v(t)$  $\begin{pmatrix} \dot{\mathbf{u}} \\ \dot{\mathbf{v}} \end{pmatrix} = \begin{pmatrix} \mathbf{M}_u \\ \mathbf{M}_v \end{pmatrix} \mathbf{B} \Rightarrow \rho_B$
CD-DDA	[3]	<div style="display: flex; align-items: center;"> <div style="margin-right: 20px;"> <math>u(t)</math>   <math>\mathcal{C}_{uv}</math>   <math>\mathcal{C}_{vu}</math>  <math>v(t)</math>  </div> <div> <div style="background-color: #fce4ec; padding: 5px; margin-bottom: 5px;"> <math>\dot{\mathbf{u}} = \mathbf{M}_u \mathbf{A}_u \Rightarrow \rho_u</math>  <math>\dot{\mathbf{u}} = (\mathbf{M}_u, \mathbf{M}_v) \mathbf{D} \Rightarrow \rho_{uv}</math> </div> <div style="background-color: #e0f7fa; padding: 5px;"> <math>\dot{\mathbf{v}} = \mathbf{M}_v \mathbf{A}_v \Rightarrow \rho_v</math>  <math>\dot{\mathbf{v}} = (\mathbf{M}_v, \mathbf{M}_u) \mathbf{G} \Rightarrow \rho_{vu}</math> </div> </div> </div> <div style="display: flex; justify-content: space-around; margin-top: 10px;"> <math>\mathcal{C}_{uv} =  \rho_u - \rho_{uv} </math> <math>\mathcal{C}_{vu} =  \rho_v - \rho_{vu} </math> </div>
DE-DDA	[2]	$u(t)$   $\mathcal{E}$ $v(t)$  <div style="margin-left: 20px;"> <math>\mathcal{E} = \left  \frac{\overline{\rho_u \rho_v}}{\rho_B} - 1 \right </math> </div>

Table 1: **The flavors of DDA:** Single-Timeseries DDA (ST-DDA) is the classical variant developed for analyzing single time series. Cross-Timeseries DDA (CT-DDA) determines the overall dynamics of multiple time series simultaneously. Cross-Dynamical DDA (CD-DDA) measures causality between two time series. Dynamical-Ergodicity DDA (DE-DDA) is a combination of ST-DDA and CT-DDA to assess dynamical ergodicity or similarity from data.

## 2.2 Data

We generate data from the Rössler system in two different dynamical regimes:

```
include("DDAfunctions.jl");

NrSyst=4;
ROS=[ [0 0 2];
      [0 0 3];
      [1 0 1];
      [1 0 2];
      [2 0 0];
      [2 0 3];
      [2 1 3]
    ];
(MOD_nr,DIM,ODEorder,P) = make_MOD_nr(ROS,NrSyst);

a123= 0.21;
a456= 0.20;

b1  = 0.2150;
b2  = 0.2020;
b4  = 0.4050;
b5  = 0.3991;

c   = 5.7;

MOD_par=[
      -1 -1 1 a123 b1 -c 1
      -1 -1 1 a123 b2 -c 1
      -1 -1 1 a456 b4 -c 1
      -1 -1 1 a456 b5 -c 1
    ];
MOD_par=reshape(MOD_par',size(ROS,1)*NrSyst)';

TRANS=20000;
dt=0.05;
X0=rand(DIM*NrSyst,1);
CH_list = 1:DIM:DIM*NrSyst;
DELTA=2;
FN_DATA = "ROS_4.ascii";
L=20000;
if !isfile(FN_DATA)
    integrate_ODE_general_BIG(MOD_nr,MOD_par,dt,L,DIM*NrSyst,ODEorder,X0,
                              FN_DATA,CH_list,DELTA,TRANS);
end
```

## 2.3 Single Timeseries DDA (ST-DDA)

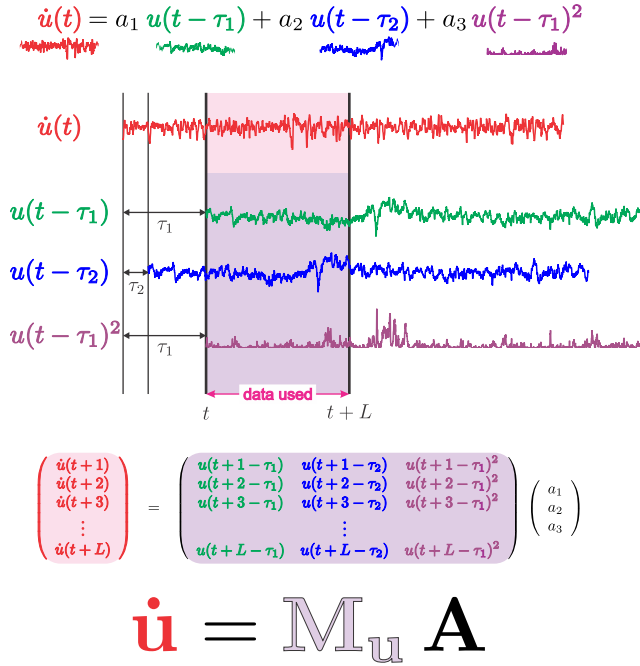


Figure 2: Estimation of the DDA coefficients  $A = (a_1, a_2, a_3)$  from data.

We will first generate data from the Rössler system and then apply DDA to these data. To understand the basics of DDA we first code DDA ourselves in Julia before we use the DDA software.

```
### single time series
Y=readldm(FN_DATA); Y=Y[:,1];

ST = fill(NaN, WN, 4);
for wn=0:WN-1
    anf=wn*WS; ende=anf+WL+TM+2*dm-1;

    data=Y[anf+1:ende+1]; ddata=deriv_all(data, dm); data=data[dm+1:end-dm];

    STD=std(data);
    DATA = (data ./ mean(data)) ./ STD; dDATA = ddata / STD;

    M=hcata(DATA[(TM+1:end) .- TAU[1]] ,
            DATA[(TM+1:end) .- TAU[2]] ,
            DATA[(TM+1:end) .- TAU[1]].^3 );
    dDATA=dDATA[TM+1:end];

    ST[wn+1,1:3] = (M \ dDATA);
    ST[wn+1,4] = sqrt(mean((dDATA ./ M*ST[wn+1,1:3]).^2));
end
```

```
### all four time series
ST = fill(NaN, WN, 4, size(Y, 2));
for n_Y=1:size(Y, 2)
    for wn=0:WN-1
        anf=wn*WS; ende=anf+WL+TM+2*dm-1;

        data=Y[anf+1:ende+1, n_Y]; ddata=deriv_all(data, dm); data=data[dm+1:end-dm];
```

```

STD=std(data);
DATA = (data .- mean(data)) ./ STD; dDATA = ddata / STD;

M=hcat (DATA[ (TM+1:end) .- TAU[1]] ,
        DATA[ (TM+1:end) .- TAU[2]] ,
        DATA[ (TM+1:end) .- TAU[1]].^3 );
dDATA=dDATA[TM+1:end];

ST[wn+1,1:3,n_Y] = (M \ dDATA);
ST[wn+1,4,n_Y] = sqrt(mean((dDATA .- M*ST[wn+1,1:3,n_Y]).^2));
end
end

```

## 2.4 Cross Timeseries DDA (CT-DDA)

```

\
NrCH=size(Y,2); CH=collect(1:NrCH);
LIST=collect(combinations(CH,2));
LL1=vcat(LIST...);
LIST=reduce(hcat,LIST)';

CT = fill(NaN,WN,4,size(LIST,1));
for n_LIST=1:size(LIST,1)
    ch1=LIST[n_LIST,1]; ch2=LIST[n_LIST,2];
    for wn=0:WN-1
        anf=wn*WS; ende=anf+WL+TM+2*dm-1;

        data1=Y[anf+1:ende+1,ch1]; ddata1=deriv_all(data1,dm); data1=data1[dm+1:end-dm];
        data2=Y[anf+1:ende+1,ch2]; ddata2=deriv_all(data2,dm); data2=data2[dm+1:end-dm];

        STD=std(data1); DATA1 = (data1 .- mean(data1)) ./ STD; dDATA1 = ddata1 / STD;
        STD=std(data2); DATA2 = (data2 .- mean(data2)) ./ STD; dDATA2 = ddata2 / STD;

        M1=hcat (DATA1[ (TM+1:end) .- TAU[1]] ,
                  DATA1[ (TM+1:end) .- TAU[2]] ,
                  DATA1[ (TM+1:end) .- TAU[1]].^3 );
        M2=hcat (DATA2[ (TM+1:end) .- TAU[1]] ,
                  DATA2[ (TM+1:end) .- TAU[2]] ,
                  DATA2[ (TM+1:end) .- TAU[1]].^3 );
        M=vcat (M1,M2); dDATA=vcat (dDATA1[TM+1:end],dDATA2[TM+1:end]);

        CT[wn+1,1:3,n_LIST] = (M \ dDATA);
        CT[wn+1,4,n_LIST] = sqrt(mean((dDATA .- M*CT[wn+1,1:3,n_LIST]).^2));
    end
end
end{lstlisting}

\subsection{DDA software run_DDA_AsciiEdf}

The same results can be achieved using \texttt{run_DDA_AsciiEdf}

\begin{lstlisting}
nr_delays=2;
DDAmodel=[[0 0 1];
          [0 0 2];
          [1 1 1]];
(MODEL, L_AF, DDAorder)=make_MODEL(DDAmodel);

FN_DDA="ROS_4.DDA";

if Sys.iswindows()
    if !isfile("run_DDA_AsciiEdf.exe")
        run(`cp run_DDA_AsciiEdf run_DDA_AsciiEdf.exe`);
    end
    CMD=".\\run_DDA_AsciiEdf.exe";
else

```

```

CMD="./run_DDA_AsciiEdf";
end
CMD = "$CMD -ASCII";
CMD = "$CMD -MODEL $(join(MODEL, " "))"
CMD = "$CMD -TAU $(join(TAU, " "))"
CMD = "$CMD -dm $dm -order $DDAorder -nr_tau $nr_delays"
CMD = "$CMD -DATA_FN $FN_DATA -OUT_FN $FN_DDA"
CMD = "$CMD -WL $WL -WS $WS";
CMD = "$CMD -SELECT 1 1 0 0";
CMD = "$CMD -CH_list $(join(LIST[:, " "]))";
CMD = "$CMD -WL_CT 2 -WS_CT 2";
if Sys.iswindows()
    run(Cmd(string.(split(CMD, " ")));
else
    run(`sh -c $CMD`);
end

ST2=readdlm("ROS_4.DDA_ST"); ST2=ST2[:,3:end];
CT2=readdlm("ROS_4.DDA_CT"); CT2=CT2[:,3:end];

mean(reshape(ST, WN, size(ST, 2)*size(ST, 3)) .- ST2)
mean(reshape(CT, WN, size(CT, 2)*size(CT, 3)) .- CT2)

```

## References

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