

Automata toolbox - Homework 2

Winter semester 2023/2024

Exercise 1. Recall that a word $w = a_0 \cdots a_n \in \{0, 1\}^*$ encodes a number $[w]_2$ under the least binary digit first encoding

$$[w]_2 = a_0 + a_1 \cdot 2 + \cdots + a_n \cdot 2^n.$$

Let $f : \mathbb{N} \rightarrow \mathbb{Q}$ be a rational function. Show that the following function $g : \{0, 1\}^* \rightarrow \mathbb{Q}$ is recognisable by a polynomial automaton:

$$g(w) = f([w]_2), \quad \text{for every } w \in \{0, 1\}^*.$$

(*) **Exercise 2.** The *reversal* of a function $f : \Sigma^* \rightarrow \mathbb{Q}$ is the function $f^R : \Sigma^* \rightarrow \mathbb{Q}$ s.t. for every input $w = a_1 \cdots a_n$, $f^R(a_1 \cdots a_n) = f(a_n \cdots a_1)$. Show that functions $\Sigma^* \rightarrow \mathbb{Q}$ recognised by polynomial automata are not closed under reversal.