Automata toolbox - Homework 2

Winter semester 2023/2024

Exercise 1. How many subsets $X \subseteq \mathbb{A}^3$ satisfiy the following condition

$$x \in X \Rightarrow \pi(x) \in X \qquad \text{for every } \pi : \mathbb{A} \to \mathbb{A}, \text{ not necessarily bijective.}$$

Exercise 2. Consider a nondeterministic register automaton, with input alphabet $\mathbb{A}$, in which the transition function reads not only the current input letter, but the k most recent input letters. In other words, the transition relation is an equivariant subset

$$\delta \subseteq \text{configurations} \times \mathbb{A}^{\leq k} \times \text{configurations}.$$

We put $\mathbb{A}^{\leq k}$ instead of $\mathbb{A}^k$, since when the automaton is a letter close to the beginning of the input string, i.e. the i-th letter for $i < k$, then there are only i most recent letters. Show that universality is decidable for this model when there is one register, i.e. the space of configurations is

$$(\text{a finite set}) \times (\mathbb{A} + \perp).$$

Exercise 3. Consider multisets of $\mathbb{A}^2$, ordered by

$$X \leq Y \qquad \text{if } \pi(X) \subseteq Y \text{ for some atom permutation } \pi.$$

In the above, $\subseteq$ is multiset inclusion. Show that this partially ordered set has an infinite antichain.