

Automata toolbox - Homework 2

Winter semester 2023/2024

Problems: deadline 12/01/2024

Problem 1. Call a language of data words $L \subseteq \mathbb{A}^*$ *invariant* if for every $w \in \mathbb{A}^*$,

$w \in L \Rightarrow \pi(w) \in L$ for every function $\pi : \mathbb{A} \rightarrow \mathbb{A}$ (not necessarily bijective).

1. Find a language $L \subseteq \mathbb{A}^*$ which is recognised by a register automaton and is not invariant.
2. Find a language $L \subseteq \mathbb{A}^*$ which is invariant and it is not recognised by a register automaton. Provide a proof of this statement.

Problem 2. Consider a nondeterministic register automaton, with input alphabet \mathbb{A} , in which the transition function reads not only the current input letter, but the k most recent input letters. In other words, the transition relation is an equivariant subset

$\delta \subseteq \text{configurations} \times \mathbb{A}^{\leq k} \times \text{configurations}$, where $\mathbb{A}^{\leq k} := \mathbb{A} \cup \mathbb{A}^2 \cup \dots \cup \mathbb{A}^k$.

We put $\mathbb{A}^{\leq k}$ instead of \mathbb{A}^k , since when the automaton is a letter close to the beginning of the input string, i.e. the i -th letter for $i < k$, then there are only i most recent letters. Show that universality is decidable for this model when there is one register and without guessing, i.e. the space of configurations is

$(\text{a finite set}) \times (\mathbb{A} + \perp)$.

Problem 3. Consider multisets of \mathbb{A}^2 , ordered by

$X \leq Y$ if $\pi(X) \subseteq Y$ for some atom permutation π .

In the above, \subseteq is multiset inclusion. Show that this preordered set has an infinite antichain.

Star problem: deadline 19/01/2024

(*) **Problem 4.** Consider a register automaton in which the transition relation δ has the invariance property from Problem 1, i.e.

$x \in \delta \Rightarrow \pi(x) \in \delta$ for every function $\pi : \mathbb{A} \rightarrow \mathbb{A}$ (not necessarily bijective).

In the above, x is a triple of the form (configuration, input letter, configuration). Show that universality is decidable for such register automata, even with more than one register.