Automata toolbox - Homework 1

Winter semester 2023/2024

Exercise 1 (Learning regular separators)**.** Consider the following problem. Teacher knows two disjoint regular languages $L, M \subseteq \Sigma^*$ and Learner wants to find a regular separator, i.e., a language $S \subseteq \Sigma^*$ including L and disjoint from M. There are two kind of queries. 1) Learner gives a word $w \in \Sigma^*$ to Teacher, who answers whether $w \in L$ (accept), $w \in M$ (reject), or "don't care". 2) Learner gives a (DFA recognising a) separator candidate S to Teacher, who answers either "yes" if it separates L, M, or in case it doesn't Teacher answers "no" and provides either a counter-example to $L \subseteq S$ or to $M \cap S = \emptyset$.

1. Is this problem more general than Angluin's one?
2. Is there a learning protocol with polynomially many queries in the sizes of minimal DFAs for L, M?

(*) Exercise 2 (Finite-valued rational functions)**.** 1. Let $L_1, \ldots, L_k$ a regular partition of Σ^* and consider weights $q_1, \ldots, q_k \in \mathbb{Q}$. Show that the following function $f : \Sigma^* \to \mathbb{Q}$ is rational:

$$\text{for every } w \in \Sigma^*: \quad f(w) = \begin{cases} q_1 & \text{if } w \in L_1, \\ \vdots & \\ q_k & \text{if } w \in L_k. \end{cases}$$

2. Let f be a rational function taking only finitely many values. Show that for each value $q \in \mathbb{Q}$, the inverse image $f^{-1}(q)$ (the set of words which f maps to q) is a regular language.

   Hint: Consider the q-finite representation of f.