Automata toolbox - Homework 2

Winter semester 2023/2024

Problems: deadline 12/01/2024

Problem 1. Call a language of data words $L \subseteq \mathbb{A}^*$ *invariant* if for every $w \in \mathbb{A}^*$,

$w \in L \Rightarrow \pi(w) \in L$ for every function $\pi : \mathbb{A} \to \mathbb{A}$ (not necessarily bijective).

1. Find a language $L \subseteq \mathbb{A}^*$ which is recognised by a register automaton and is not invariant.

2. Find a language $L \subseteq \mathbb{A}^*$ which is invariant and it is not recognised by a register automaton. Provide a proof of this statement.

Problem 2. Consider a nondeterministic register automaton, with input alphabet $\mathbb{A}$, in which the transition function reads not only the current input letter, but the k most recent input letters. In other words, the transition relation is an equivariant subset

$\delta \subseteq$ configurations $\times \, \mathbb{A}^{\leq k} \times$ configurations,   where $\mathbb{A}^{\leq k} := \mathbb{A} \cup \mathbb{A}^2 \cup \cdots \cup \mathbb{A}^k$.

We put $\mathbb{A}^{\leq k}$ instead of $\mathbb{A}^k$, since when the automaton is a letter close to the beginning of the input string, i.e. the i-th letter for $i < k$, then there are only i most recent letters. Show that universality is decidable for this model when there is one register and without guessing, i.e. the space of configurations is

$$(\text{a finite set}) \times (\mathbb{A} + \bot).$$

Problem 3. Consider multisets of $\mathbb{A}^2$, ordered by

$$X \leq Y \qquad \text{if } \pi(X) \subseteq Y \text{ for some atom permutation } \pi.$$

In the above, $\subseteq$ is multiset inclusion. Show that this partially ordered set has an infinite antichain.

(*) Problem 4. Consider a register automaton, in which the transition relation δ has the invariance property from Problem 1, i.e.

$x \in \delta \Rightarrow \pi(x) \in \delta$ for every function $\pi : \mathbb{A} \to \mathbb{A}$ (not necessarily bijective).

In the above, x is a triple of the form (configuration, input letter, configuration). Show that universality is decidable for such register automata, even with more than one register.