

# Automata toolbox - Homework 1

Winter semester 2023/2024

**Exercise 1** (Learning regular separators). Consider the following problem. Teacher knows two disjoint regular languages  $L, M \subseteq \Sigma^*$  and Learner wants to find a regular separator, i.e., a language  $S \subseteq \Sigma^*$  including  $L$  and disjoint from  $M$ . There are two kind of queries. 1) Learner gives a word  $w \in \Sigma^*$  to Teacher, who answers whether  $w \in L$  (accept),  $w \in M$  (reject), or “don’t care”. 2) Learner gives a (DFA recognising a) separator candidate  $S$  to Teacher, who answers either “yes” if it separates  $L, M$ , or in case it doesn’t Teacher answers “no” and provides either a counter-example to  $L \subseteq S$  or to  $M \cap S = \emptyset$ .

1. Is this problem more general than Angluin’s one?
2. Is there a learning protocol with polynomially many queries in the sizes of minimal DFAs for  $L, M$ ?

(\*) **Exercise 2** (Finite-valued rational functions). 1. Let  $L_1, \dots, L_k$  a regular partition of  $\Sigma^*$  and consider weights  $q_1, \dots, q_k \in \mathbb{Q}$ . Show that the following function  $f : \Sigma^* \rightarrow \mathbb{Q}$  is rational:

$$\text{for every } w \in \Sigma^*: \quad f(w) = \begin{cases} q_1 & \text{if } w \in L_1, \\ \vdots & \\ q_k & \text{if } w \in L_k. \end{cases}$$

2. Let  $f$  be a rational function taking only finitely many values. Show that for each value  $q \in \mathbb{Q}$ , the inverse image  $f^{-1}(q)$  (the set of words which  $f$  maps to  $q$ ) is a regular language.

*Hint: Consider the  $q$ -finite representation of  $f$ .*