Automata toolbox - Homework 2

Winter semester 2023/2024

Problems: deadline 12/01/2024

Problem 1. Call a language of data words $L \subseteq \mathbb{A}^*$ *invariant* if for every $w \in \mathbb{A}^*$,

$$w \in L \Rightarrow \pi(w) \in L \quad \text{for every function } \pi : \mathbb{A} \to \mathbb{A} \text{ (not necessarily bijective).}$$

1. Find a language $L \subseteq \mathbb{A}^*$ which is recognised by a register automaton and is not invariant.
2. Find a language $L \subseteq \mathbb{A}^*$ which is invariant and it is not recognised by a register automaton. Provide a proof of this statement.

Problem 2. Consider a nondeterministic register automaton, with input alphabet $\mathbb{A}$, in which the transition function reads not only the current input letter, but the k most recent input letters. In other words, the transition relation is an equivariant subset

$$\delta \subseteq \text{configurations} \times \mathbb{A}^{\leq k} \times \text{configurations}, \quad \text{where } \mathbb{A}^{\leq k} := \mathbb{A} \cup \mathbb{A}^2 \cup \cdots \cup \mathbb{A}^k.$$

We put $\mathbb{A}^{\leq k}$ instead of $\mathbb{A}^k$, since when the automaton is a letter close to the beginning of the input string, i.e. the i-th letter for $i < k$, then there are only i most recent letters. Show that universality is decidable for this model when there is one register and without guessing, i.e. the space of configurations is

$$(\text{a finite set}) \times (\mathbb{A} + \perp).$$

Problem 3. Consider multisets of $\mathbb{A}^2$, ordered by

$$X \leq Y \qquad \text{if } \pi(X) \subseteq Y \text{ for some atom permutation } \pi.$$

In the above, $\subseteq$ is multiset inclusion. Show that this partially ordered set has an infinite antichain.