Languages, automata and computation II
Homework 3

Problems: deadline 31/01/2025

Problem 1. Show that the following problem is decidable:

- **Input.** A deterministic register automaton, defining a language $L \subseteq \mathbb{A}^*$;
- **Question.** Does the language satisfy, for every word $w \in \mathbb{A}^*$ and function $\sigma : \mathbb{A} \to \mathbb{A}$, not necessarily a permutation,

$$w \in L \quad \iff \quad \sigma(w) \in L.$$

Problem 2. A language of infinite words $L \subseteq \Sigma^\omega$ is *closed* if the following condition is satisfied: For every infinite word $w = a_0 a_1 \cdots \in \Sigma^\omega$, if every finite prefix $a_0 a_1 \cdots a_n \in \Sigma^*$ of w can be extended to an ω-word $a_0 a_1 \cdots a_n \cdot v$ in L, then $w \in L$.

1. Show that there is an ω-regular language which is not closed.
2. Show that there exists a closed language which is not ω-regular.
3. Show that the following problem is decidable: Given a nondeterministic Büchi automaton, decide whether the language it recognises is closed.

Star problems

The deadline for these problems is until the end of the exam session.

(*) Problem 3. Show that the following problem is decidable:

- **Input.** Two nondeterministic Büchi automata A, B recognising ω-regular languages $L(A), L(B) \subseteq \Sigma^\omega$.
- **Question.** Does there exist a *deterministic* Büchi automaton C separating A, B, i.e., $L(A) \subseteq L(C)$ and $L(B) \cap L(C) = \emptyset$?