# Languages, automata and computation II
# Tutorial 3 – Applications of well-quasi orders

Winter semester 2024/2025

**Exercise 1.** Consider the set of finite words well-quasi ordered by the subword relation $(\Sigma^*, \sqsubseteq)$. Show that *every* downward closed language over $\Sigma$ is regular.

**Exercise 2.** Let $V$ be a $d$-dimensional VASS and consider a target configuration $t \in P \times \mathbb{N}^d$, where $P$ is a finite set of control locations. Show that one can compute the set of *all configurations* $s$ which can cover $t$.

**Exercise 3.** Let $V$ be a $d$-dimensional VASS and consider a source configuration $s \in \mathbb{N}^d$. Show that one decide whether there are only *finitely many* configurations reachable from $s$.

**Exercise 4.** A *rewrite system* over a finite alphabet $\Sigma$ is a finite set of pairs $u \to v$ with $u, v \in \Sigma^*$. Consider the least reflexive and transitive congruence $\to^*$ on $\Sigma^*$ containing $\to$. A rewrite system is *lossy* if it contains transitions $a \to \varepsilon$ for every $a \in \Sigma$. Show that the relation $\to^*$ is decidable when $\to$ is lossy.

## $\mathbb{Z}$-VASSes

This part is about VASSes but unrelated with well quasi orders. We show that reachability in VASSes is considerably simpler if we relax the requirement that counters cannot become negative.

**Exercise 5.** Let a $\mathbb{Z}$-*VASS* of dimension $d \in \mathbb{N}$ be a finite set of location-vector pairs $V \subseteq_{\text{fin}} L \times \mathbb{Z}^d$. The semantics is as in VASS, except that now configurations are in $L \times \mathbb{Z}^d$ (instead of the more restrictive $L \times \mathbb{N}^d$). Show that reachability is decidable for $\mathbb{Z}$-VASSes.