Languages, automata and computation II
Tutorial 8 – Weighted automata

Winter semester 2024/2025

Exercise 1. Construct weighted automata over unary alphabet, which for a word of length n output

1. n^2;
2. $n^2 + 2n$;
3. n^3;
4. n^k for $k \in \mathbb{N}$;
5. $p(n)$ for any polynomial $\mathbb{Q}[x]$;
6. The nth Fibonacci number F_n.

Exercise 2. Construct a weighted automaton that computes an injective function from $\{a, b\}$ to the positive rational numbers.

Exercise 3. Call an NFA unambiguous if for every input there is at most one accepting run. Show that equivalence problem for unambiguous automata can be decided in polynomial time.

Exercise 4. Show that for weighted automata with 2 states over a unary alphabet, it is decidable whether the automaton assigns value 0 to some word.

Remark: for weighted automata over a unary alphabet with an arbitrary number of states, this is an important open problem, called the Skolem Problem.

Exercise 5. Show that for every weighted automaton there is an isomorphic (using the notion of isomorphism inherited from vector space automata) one which has one initial and one final state.