Languages, automata and computation II
Tutorial 12 – Games

Winter semester 2024/2025

Exercise 1. We say that a game is *finite* if every play eventually reaches a dead end (no player can move). The player that cannot move loses. We do not assume that the game is finitely branching (i.e., the players may have infinitely many available moves at some positions). Show that finite games are determined.

Exercise 2. Show that one player parity games on finite graphs can be solved in polynomial time. Conclude that two player parity games on finite graphs can be solved in $\mathsf{NP} \cap \mathsf{coNP}$.

Exercise 3. Are Muller games on finite graphs positionally determined? Finite-memory determined?

Exercise 4. Are all games on finite graphs finite-memory determined?

Exercise 5 (The dwarfs)**.**

1. Consider $n \in \mathbb{N}_{\geq 1}$ dwarfs, each having either a red or a green hat on their head. Each dwarf can see all hats except their own. Dwarfs must correctly guess the color of their own hat. At the beginning of the game, starting from the first dwarf, they announce their guess. If a dwarf guesses correctly, he survives, otherwise he is killed. Show that there is a strategy for the dwarfs that guarantees the survival of all but one dwarf.
2. Generalise the previous game to countably many dwarfs. *Hint: Use the axiom of choice.*

Exercise 6. Consider the following *Banach-Mazur game*. There are two players that alternate playing nonempty finite words from a fixed finite alphabet Σ. The winning condition is given by a language of infinite words W over Σ. A play of the game $\pi \in \Sigma^\omega$ is obtained by concatenating the words played by the two players and the first player wins iff $\pi \in W$. Show that there a winning condition $W \subseteq \Sigma^\omega$ yielding an undetermined game.