Languages, automata and computation II
Tutorial 3 – Applications of well-quasi orders

Winter semester 2024/2025

Exercise 1. Consider the set of finite words well-quasi ordered by the subword relation $(\Sigma^*, \sqsubseteq)$. Show that *every* downward closed language over Σ is regular.

Exercise 2. Let $\mathcal{V}$ be a d-dimensional VASS and consider a target configuration $t \in P \times \mathbb{N}^d$, where P is the set of states. Show that one can compute the set of *all configurations* s which can cover t.

Exercise 3. Let $\mathcal{V}$ be a d-dimensional VAS and consider a source configuration $s \in \mathbb{N}^d$. Show that one decide whether there are only *finitely many* configurations reachable from s.

Exercise 4. Let $\mathcal{V}$ be a d-dimensional VAS and consider a source configuration $s \in \mathbb{N}^d$. Show that for any coordinate $k \in \{1, \ldots, d\}$ it is decidable whether there exists a number $n \in \mathbb{N}$ such that every configuration t reachable from s has the kth coordinate bounded by n.

Exercise 5. A *rewrite system* over a finite alphabet Σ is a finite set of pairs $u \to v$ with $u, v \in \Sigma^*$. Consider the least reflexive and transitive congruence $\to^*$ on Σ^* containing $\to$. A rewrite system is *lossy* if it contains transitions $a \to \varepsilon$ for every $a \in \Sigma$. Show that the relation $\to^*$ is decidable when $\to$ is lossy.

VASSes

This part is about VASSes but unrelated with well quasi orders. We show that reachability in VASSes is considerably simpler if we relax the requirement that counters cannot become negative.

Exercise 6. Show that a VASS of dimension d can be simulated by a VAS of dimension $d + 3$.

Exercise 7. Let a $\mathbb{Z}$-*VASS* of dimension $d \in \mathbb{N}$ be a finite set of location-vector pairs $V \subseteq_{\text{fin}} L \times \mathbb{Z}^d$. The semantics is as in VASS, except that now configurations are in $L \times \mathbb{Z}^d$ (instead of the more restrictive $L \times \mathbb{N}^d$). Show that reachability is decidable for $\mathbb{Z}$-VASSes.