

# Languages, automata and computation II

## Homework 3

### Problems: deadline 31/01/2025

**Problem 1.** Show that the following problem is decidable:

- **Input.** A deterministic register automaton, defining a language  $L \subseteq \mathbb{A}^*$ ;
- **Question.** Does the language satisfy

$$w \in L \iff \sigma(w) \in L$$

for *every* function  $\sigma : \mathbb{A} \rightarrow \mathbb{A}$ , not necessarily a permutation.

**Problem 2.** A language of infinite words  $L \subseteq \Sigma^\omega$  is *closed* if the following condition is satisfied: For every infinite word  $w = a_0a_1 \cdots \in \Sigma^\omega$ , if every finite prefix  $a_0a_1 \cdots a_n \in \Sigma^*$  of  $w$  can be extended to an  $\omega$ -word  $a_0a_1 \cdots a_n \cdot v$  in  $L$ , then  $w \in L$ .

1. Show that there is a  $\omega$ -regular language which is not closed.
2. Show that there exists a closed language which is not  $\omega$ -regular.
3. Show that the following problem is decidable: Given a nondeterministic Büchi automaton, decide whether the language it recognises is closed.

### Star problems

The deadline for these problems is until the end of the exam session.

(\*) **Problem 3.** Show that the following problem is decidable:

- **Input.** Two nondeterministic Büchi automata  $A, B$  recognising  $\omega$ -regular languages  $L(A), L(B) \subseteq \Sigma^\omega$ .
- **Question.** Does there exist a *deterministic* Büchi automaton  $C$  separating  $A, B$ , i.e.,  $L(A) \subseteq L(C)$  and  $L(B) \cap L(C) = \emptyset$ ?