

Languages, automata and computation II

Homework 3 (draft version)

Problems: deadline 31/01/2025

Problem 1. Show that the following problem is decidable:

- **Input.** A deterministic register automaton, defining a language $L \subseteq \mathbb{A}^*$;
- **Question.** Does the language satisfy

$$w \in L \iff \sigma(w) \in L$$

for *every* function $\sigma : \mathbb{A} \rightarrow \mathbb{A}$, not necessarily a permutation.

Problem 2. A language of infinite words $L \subseteq \Sigma^\omega$ is *closed* if the following condition is satisfied: For every infinite word $w = a_0a_1 \cdots \in \Sigma^\omega$, if every finite prefix $a_0a_1 \cdots a_n \in \Sigma^*$ of w can be extended to an ω -word $a_0a_1 \cdots a_n \cdot v$ in L , then $w \in L$.

1. Show that there is a ω -regular language which is not closed.
2. Show that there exists a closed language which is not ω -regular.
3. Show that the following problem is decidable: Given a nondeterministic Büchi automaton, decide whether the language it recognises is closed.

Star problems

The deadline for these problems is until the end of the exam session.

Problem 3. Show that the following problem is decidable:

- **Input.** Two nondeterministic Büchi automata A, B recognising ω -regular languages $L(A), L(B) \subseteq \Sigma^\omega$.
- **Question.** Does there exist a *deterministic* Büchi automaton C separating A, B , i.e., $L(A) \subseteq L(C)$ and $L(B) \cap L(C) = \emptyset$?