Efficient Separability of Regular Languages by Subsequences and Suffixes

Wojciech Czerwiński

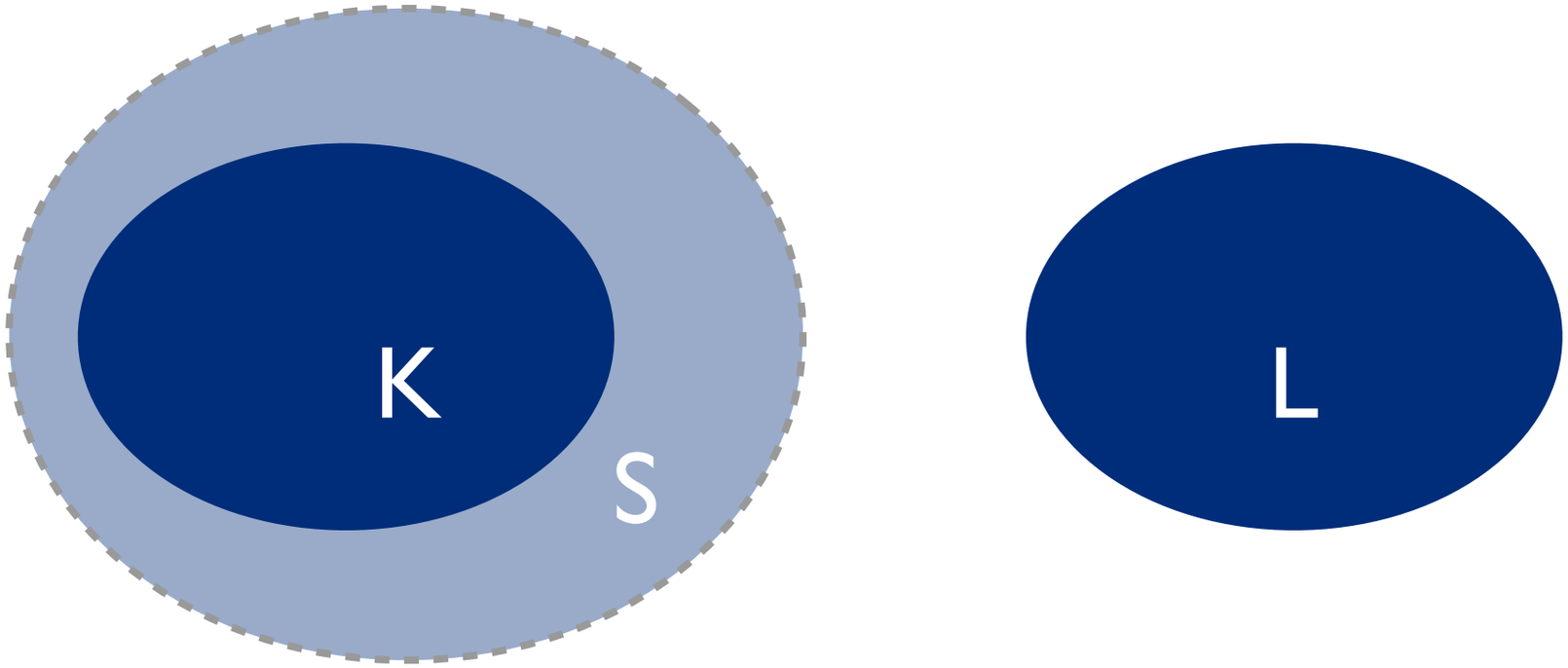
Tomáš Masopust

Wim Martens

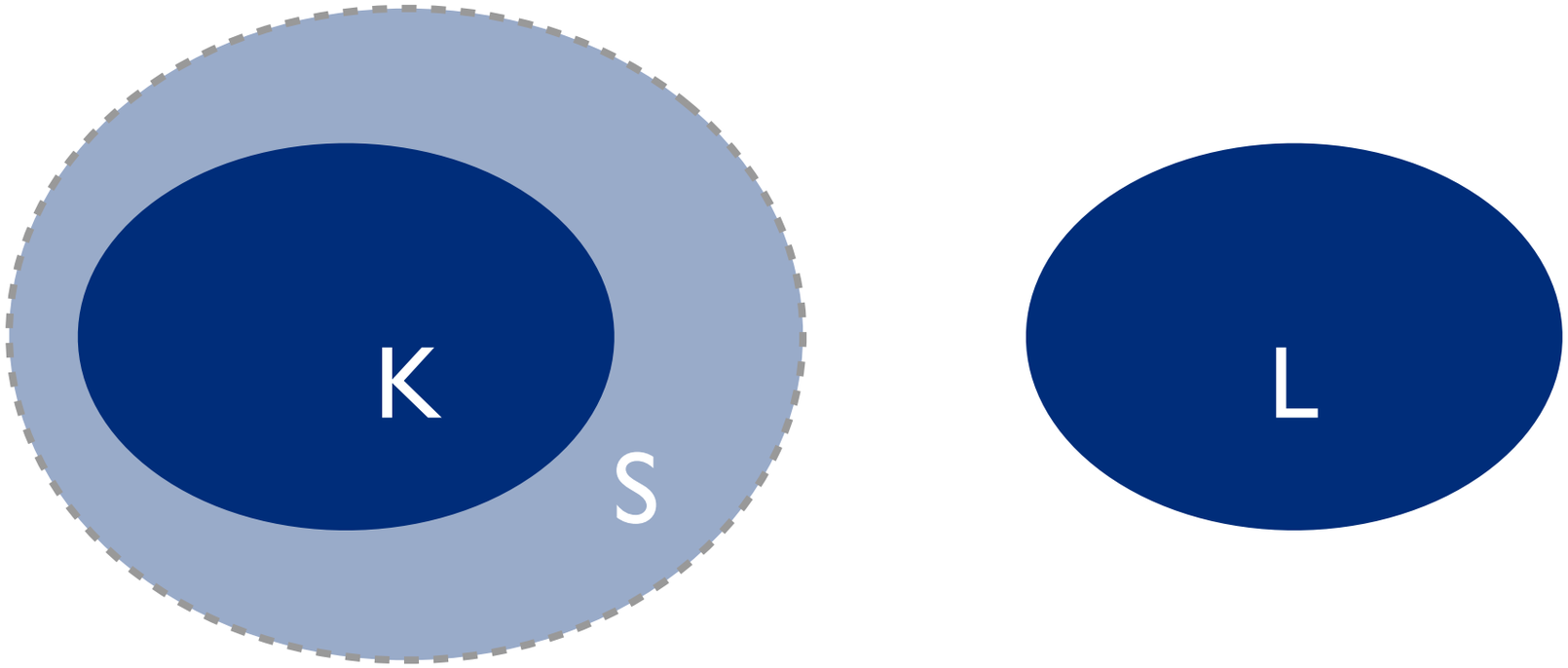
Separability

Separability

Separability

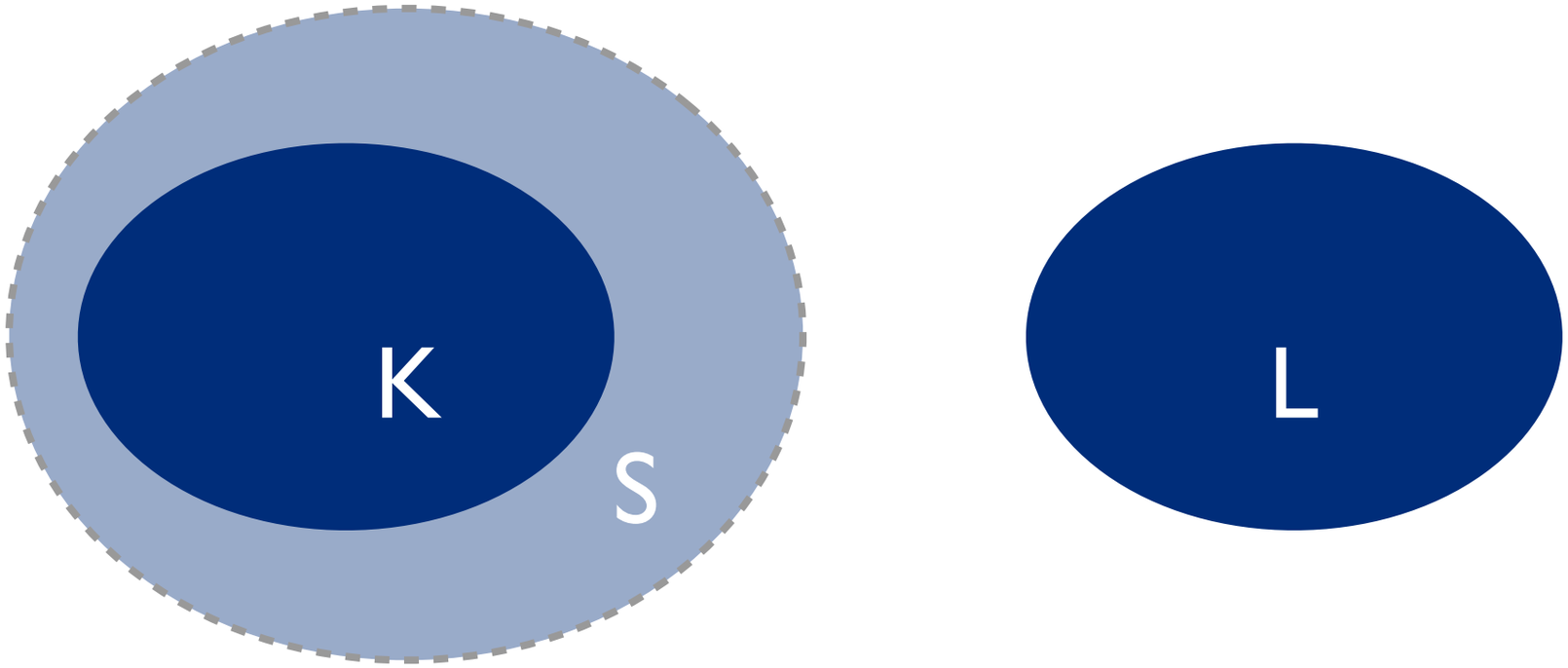


Separability



S *separates* K and L

Separability



S *separates* K and L

K and L are *separable by* family F
if some S from F separates them

Problem

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Given: nondeterministic automata for languages K and L

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Question: are K and L separable by Piecewise Testable Languages (PTL)?

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piece language

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piece language $\Sigma^* a_1 \Sigma^* a_2 \Sigma^* \ldots \Sigma^* a_n \Sigma^*$

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piecewise testable language

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piece language $\Sigma^* a_1 \Sigma^* a_2 \Sigma^* \ldots \Sigma^* a_n \Sigma^*$

piecewise testable language bool. comb. of pieces

What is known?

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- Simon 1975, piecewise testable = j-trivial

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- Almeida, Zeitoun 1997, exponential algorithm for separability by PTL

First main result

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Theorem:
Separability of Regular Languages
by Piecewise Testable Languages
can be decided in PTIME

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obtained independently
by Place, van Rooijen, Zeitoun
MFCS `13

First main result

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Separability of Regular Languages
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can be decided in PTIME

Second main result

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For any two word languages K and L the following conditions are equivalent:

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1) K and L are separable by piecewise testable languages

Second main result

For any two word languages K and L the following conditions are equivalent:

1) K and L are separable by piecewise testable languages

2) there is no infinite zigzag between K and L

Infinite zigzag

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$$abcd \preccurlyeq dbabacbcdb$$

Infinite zigzag

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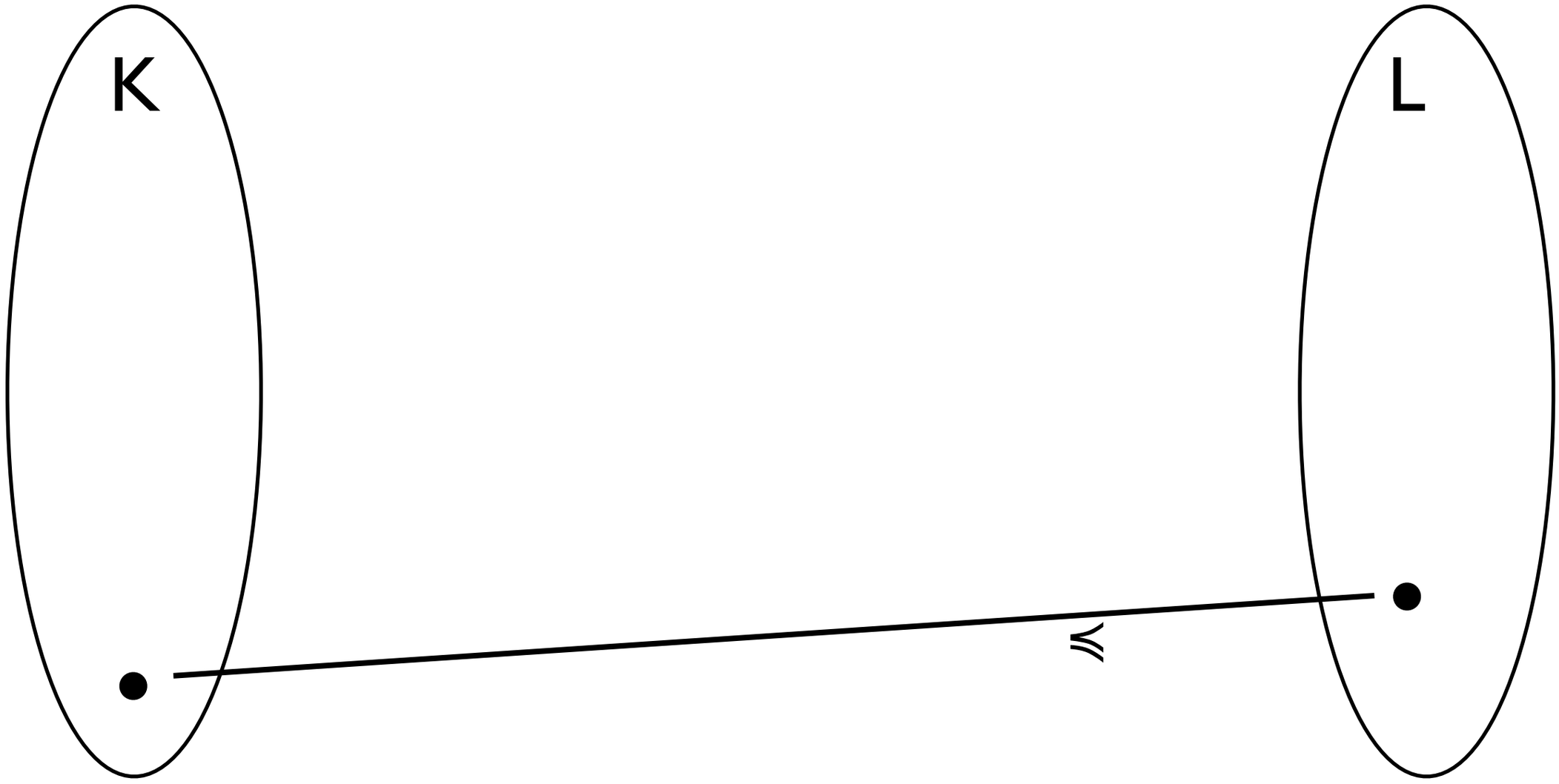
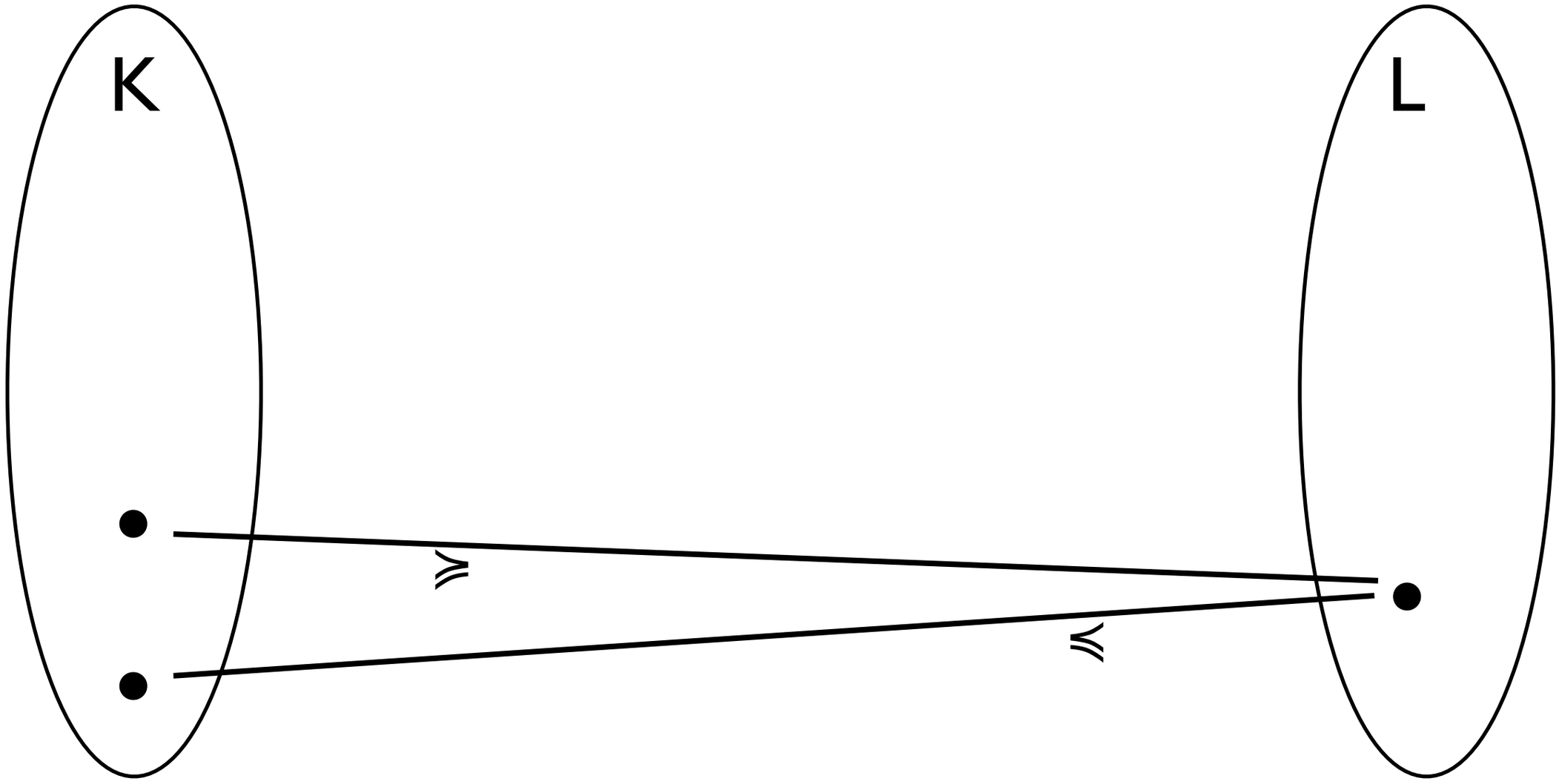
Infinite zigzag



Infinite zigzag



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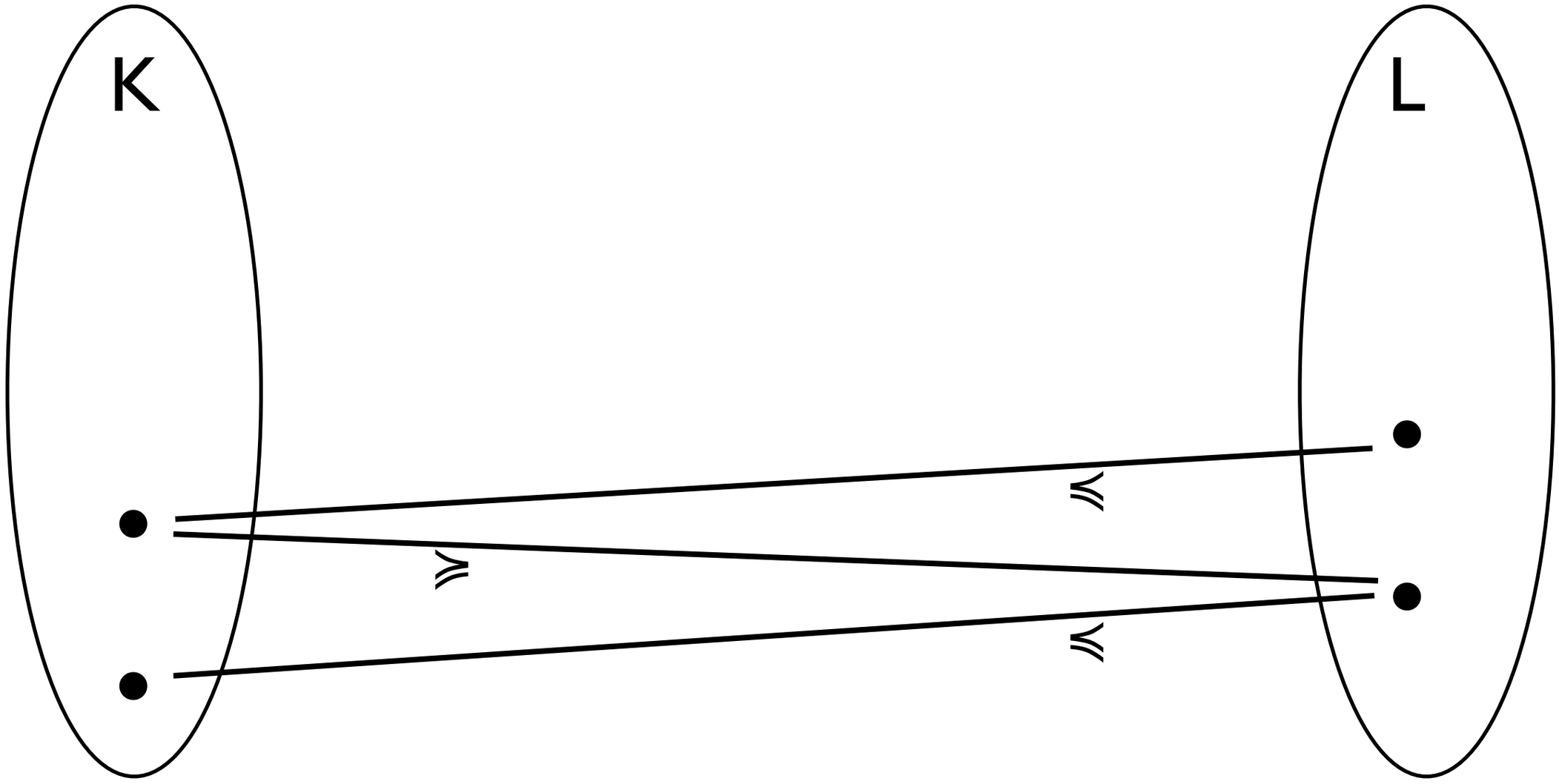
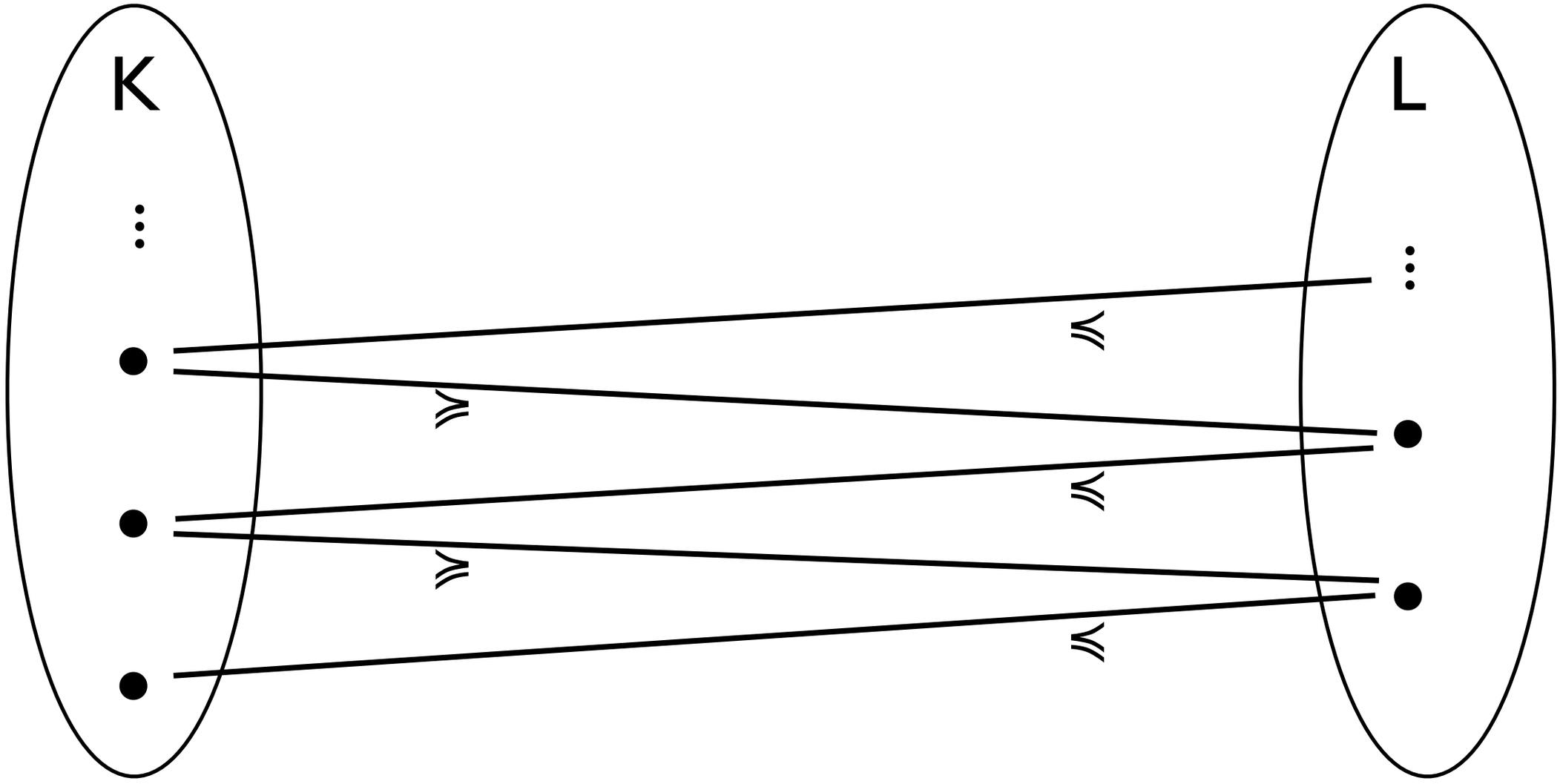


Infinite zigzag



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S_1

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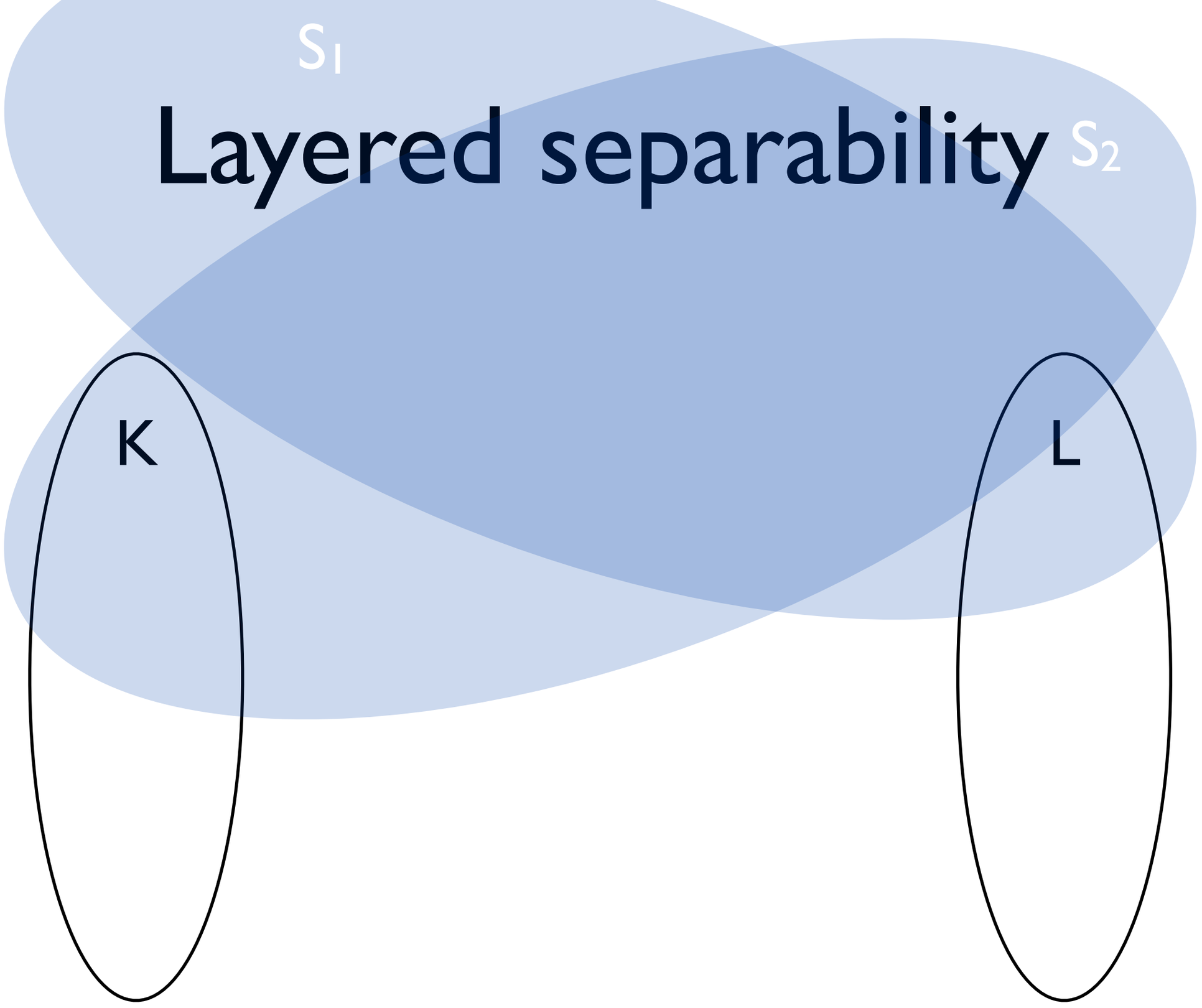
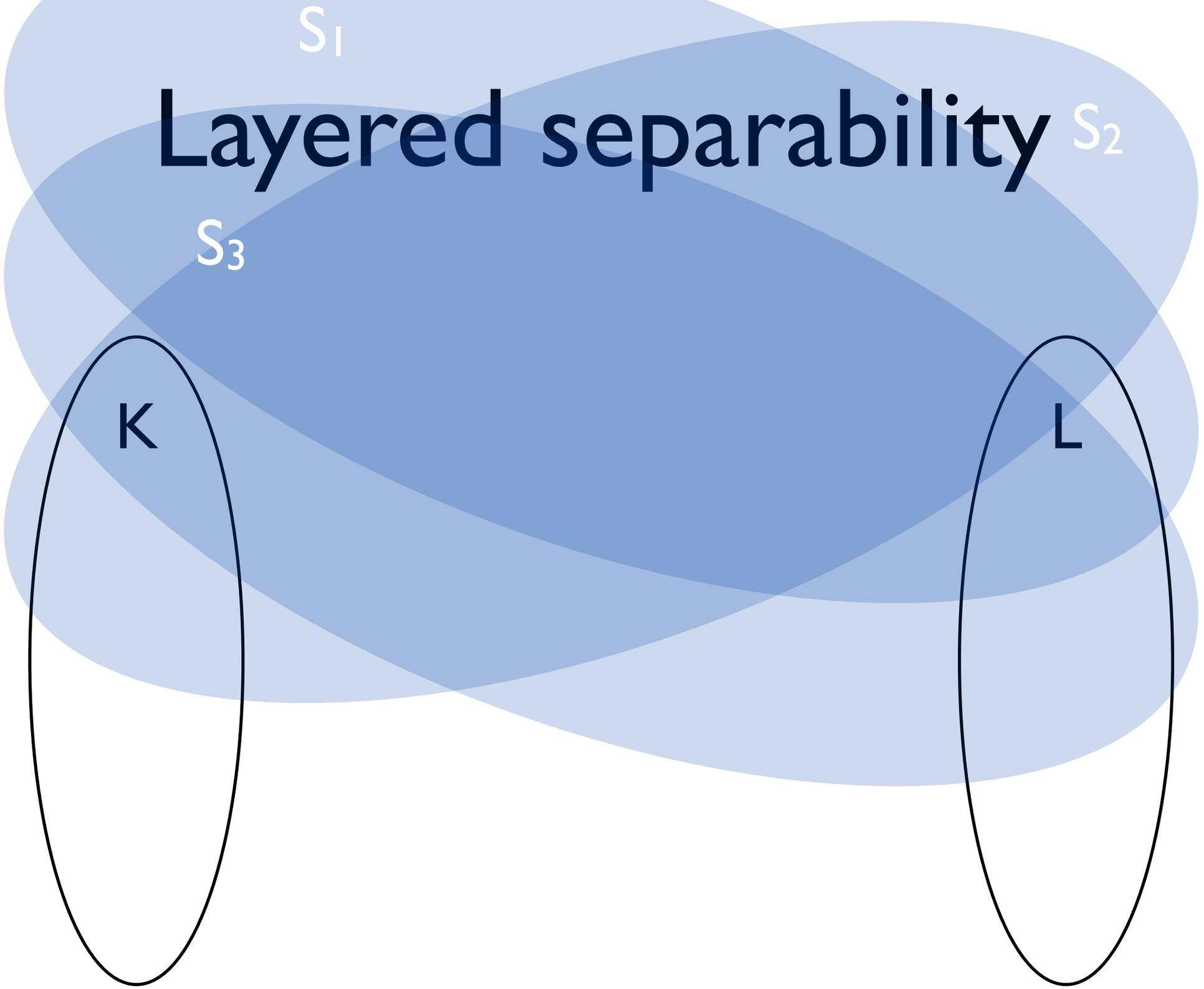


Layered separability



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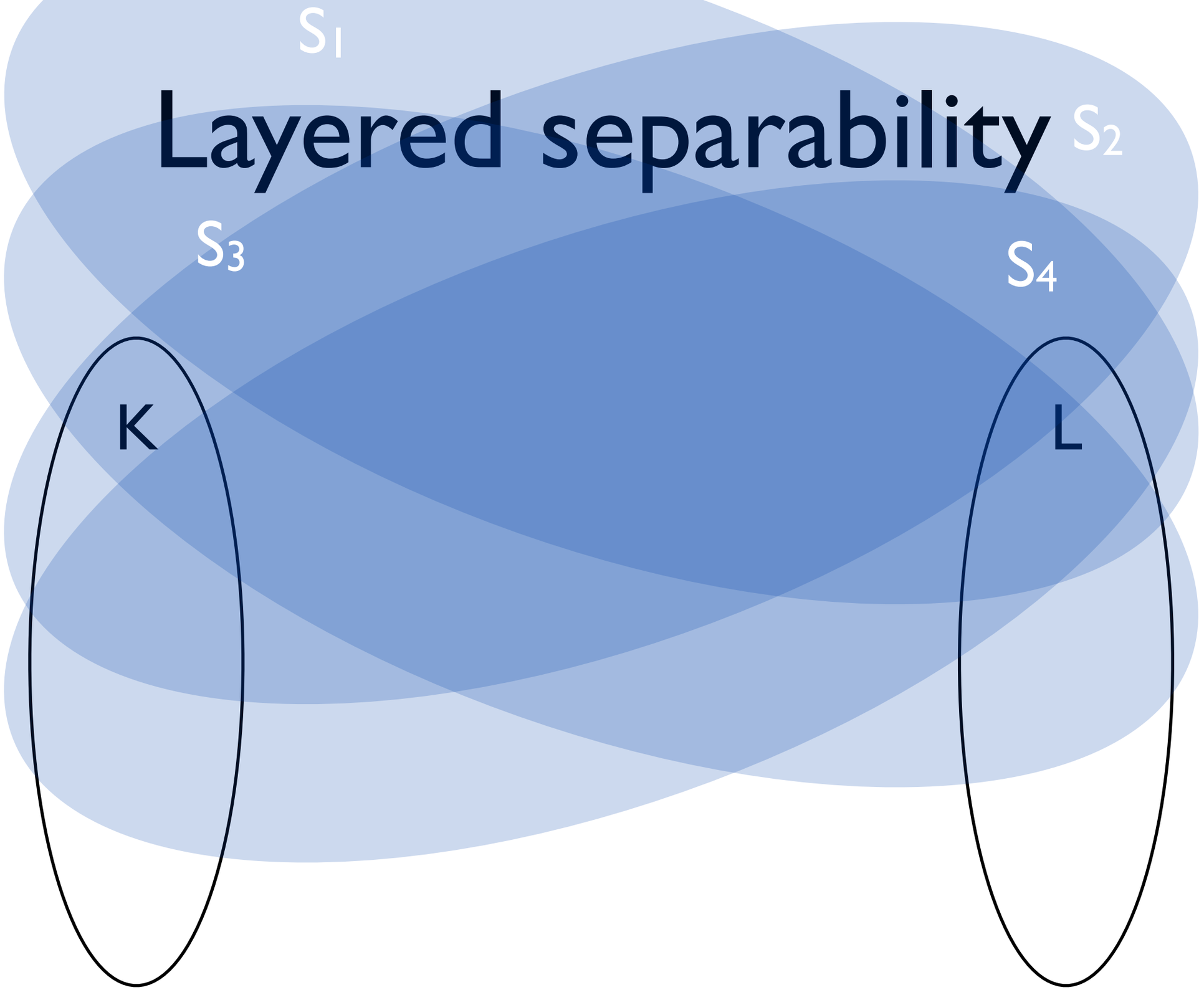

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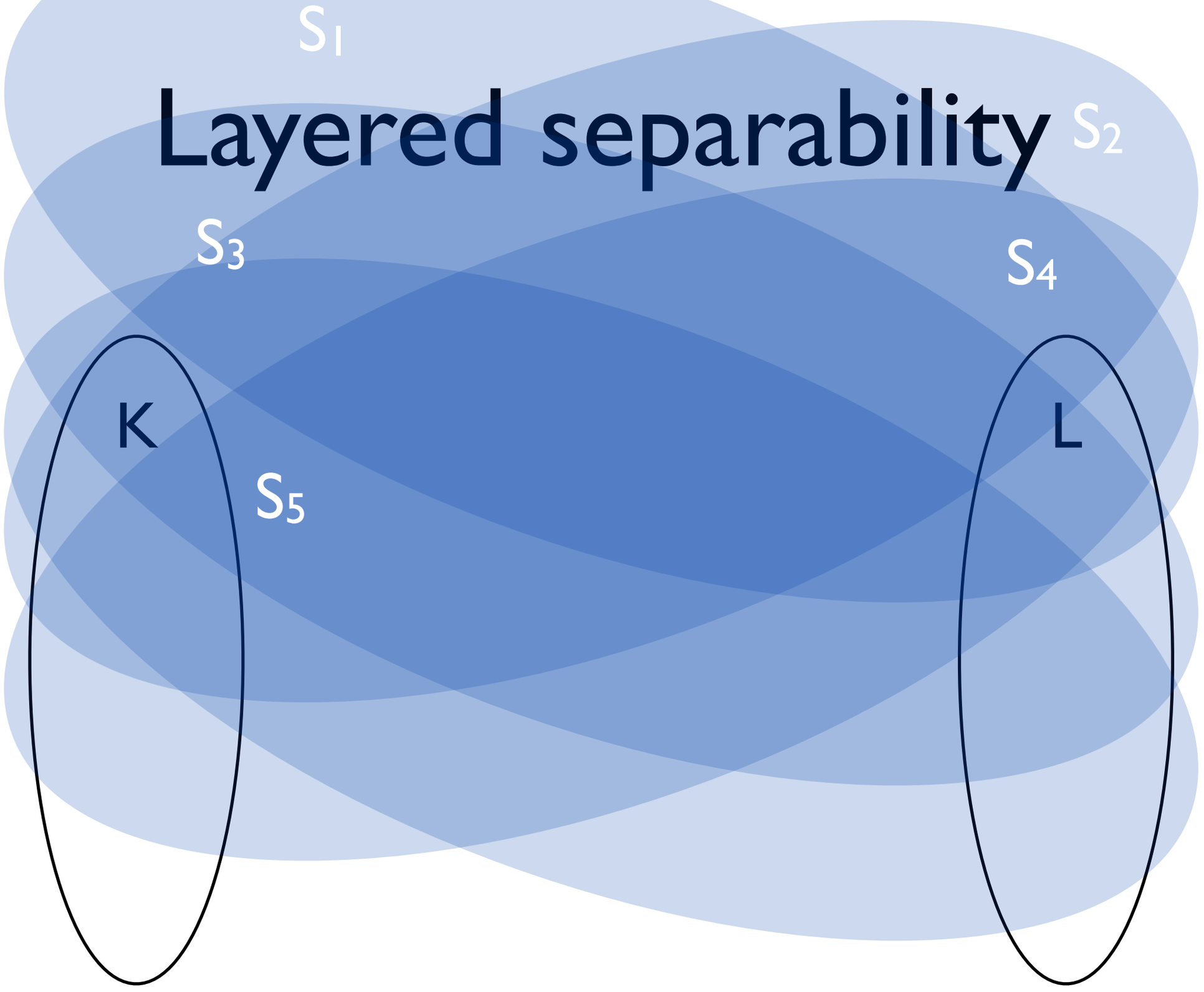

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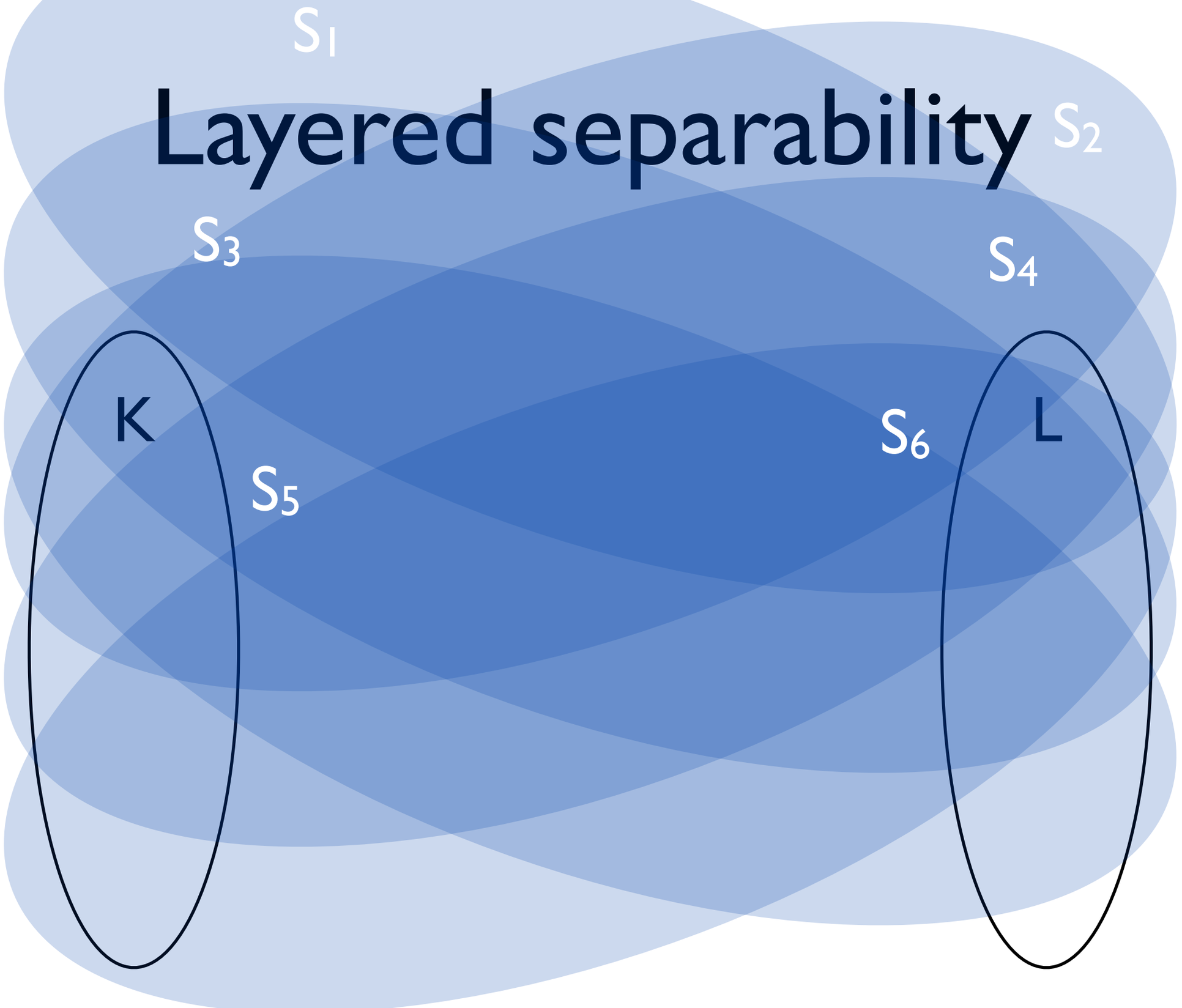

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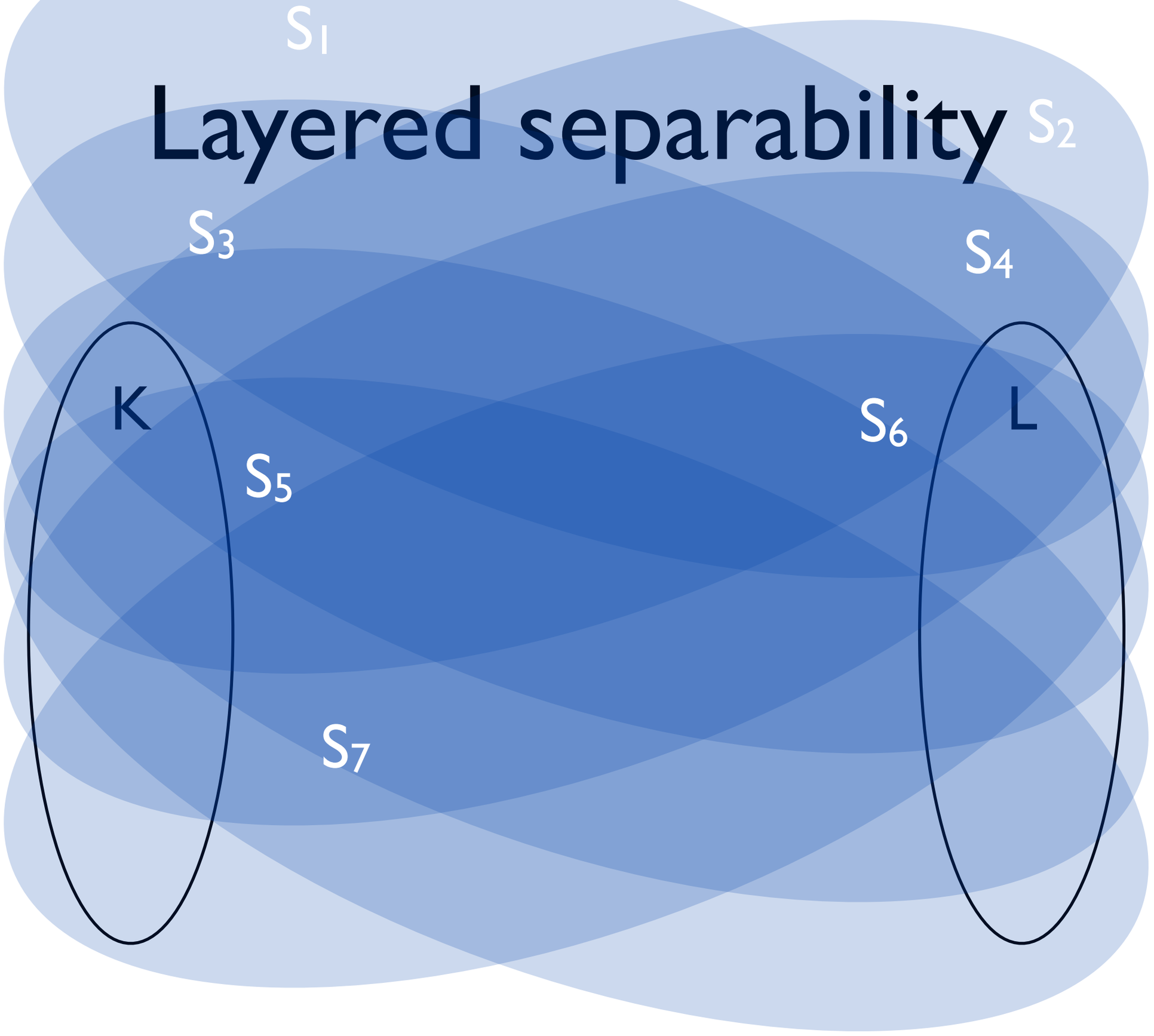

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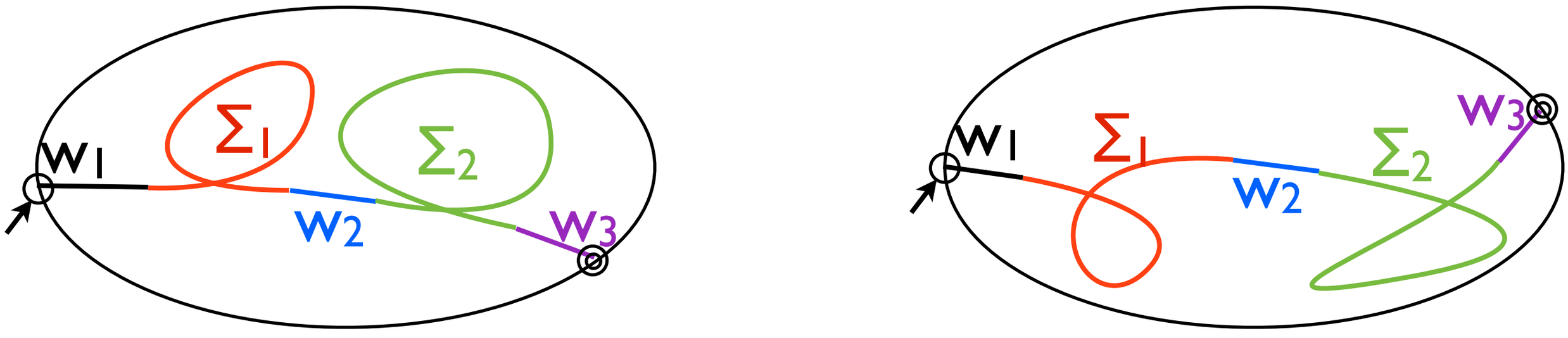

Effective characterization

For two regular languages K and L
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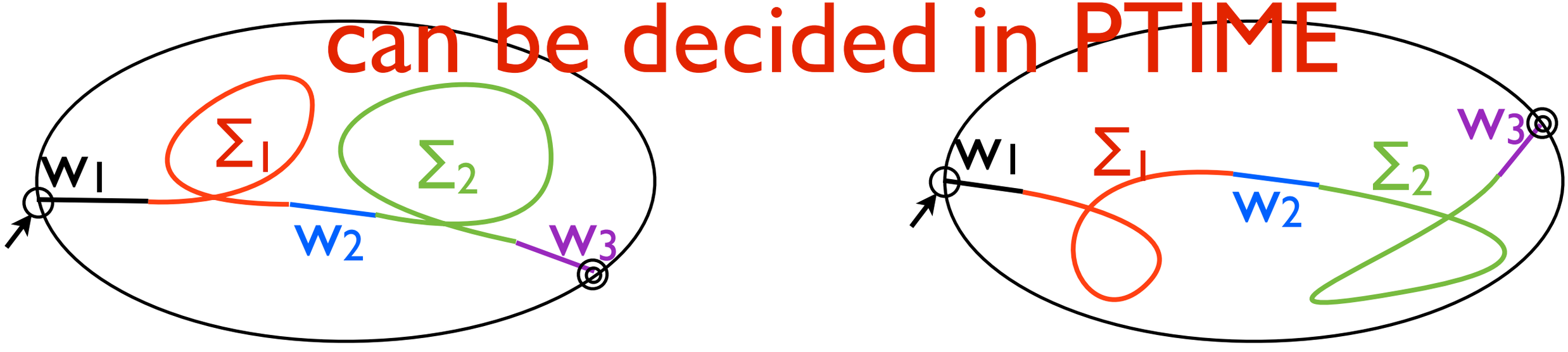
can be decided in PTIME



Other variants

Other variants

Separability Problem

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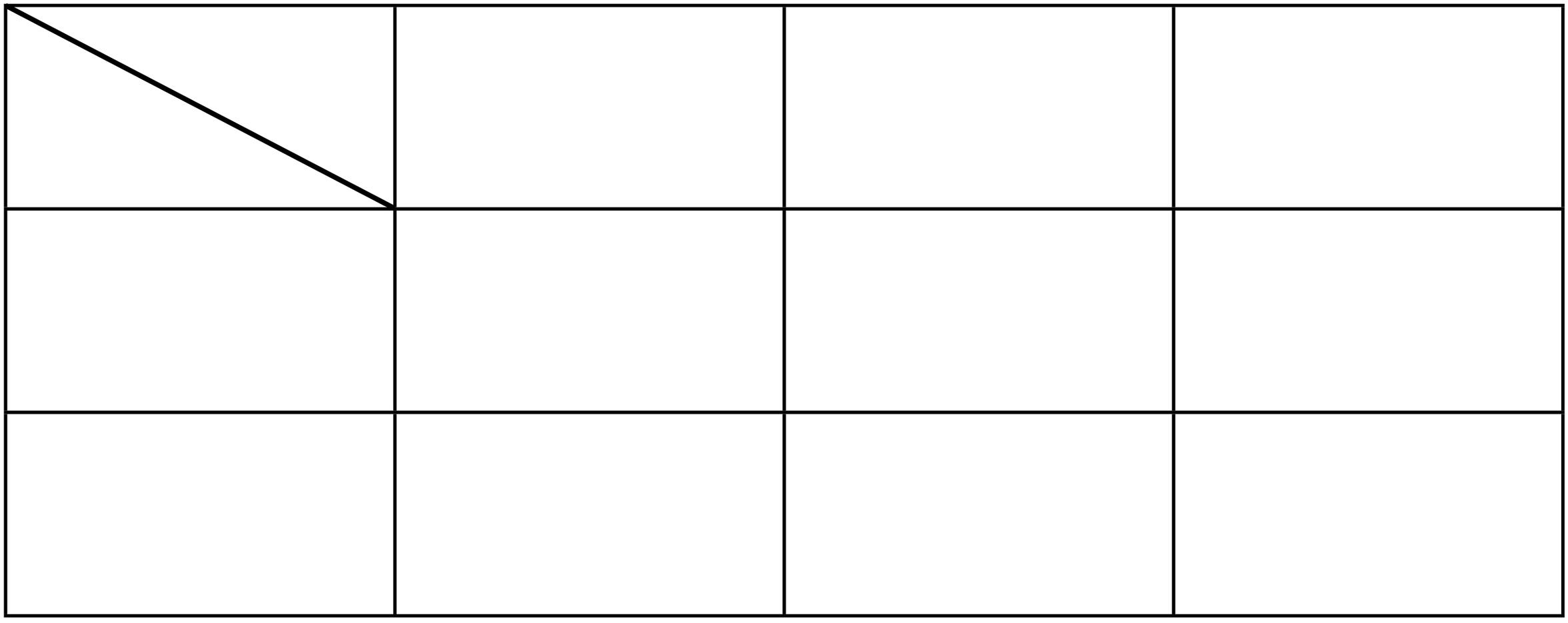
Separability Problem



Other variants

Separability Problem

pieces			

Other variants

Separability Problem

	single		
pieces			

Other variants

Separability Problem

	single	unions	
pieces			

Other variants

Separability Problem

	single	unions	boolean combinations
pieces			

Other variants

Separability Problem

	single	unions	boolean combinations
pieces			PTIME

Other variants

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	single	unions	boolean combinations
pieces		PTIME	PTIME

Other variants

Separability Problem

	single	unions	boolean combinations
pieces	NP-comp.	PTIME	PTIME

Other variants

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pieces	NP-comp.	PTIME	PTIME
suffixes			

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$\Sigma^* w$

Other variants

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Other variants

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Thank you!