Rewriting Higher-Order Stack Trees

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Highlights, September 3^{rd}, 2014

Context

- Model Checking over some classes of infinite graphs: which logic theories (FO,FO[$\rightarrow^*$],MSO,...) are decidable ?

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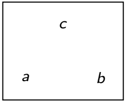
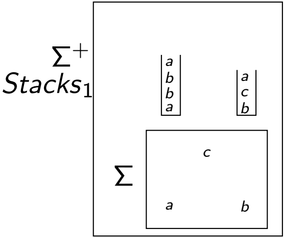
Configuration graphs of HOPDA [Caucal02] & [Carayol,Wohrle03]	?	Tree automatic of order *n* [Colcombet,Loding07]
Configuration graphs of PDA [Muller,Shupp85]	Ground tree rewriting graphs [Dauchet,Tison90]	Tree automatic [Khoussainov,Nerode94]
MSO	FO[$\to^*$]	FO

Stacks and Trees

Σ



Stacks and Trees



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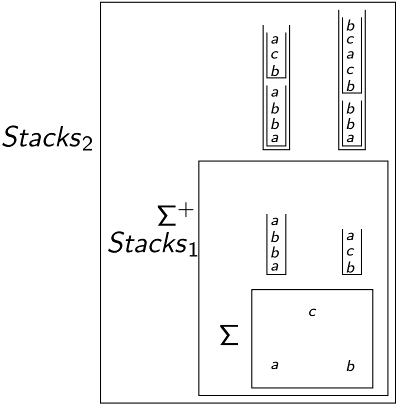

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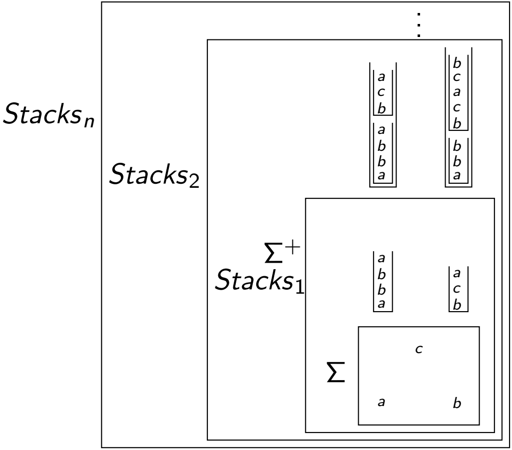

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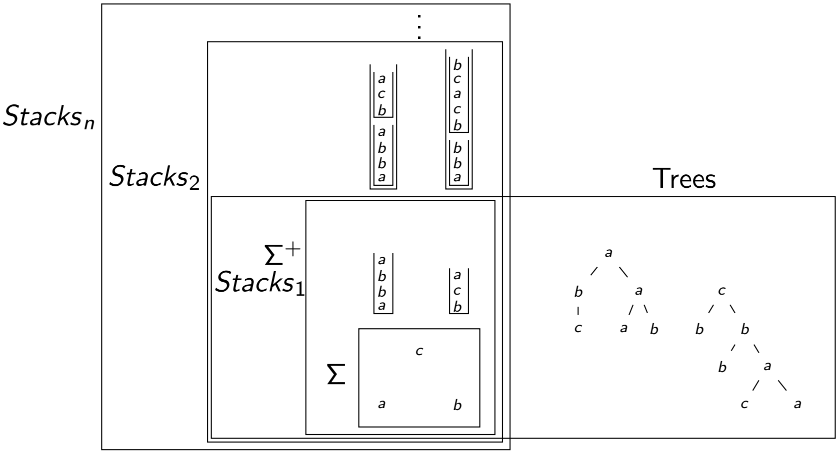

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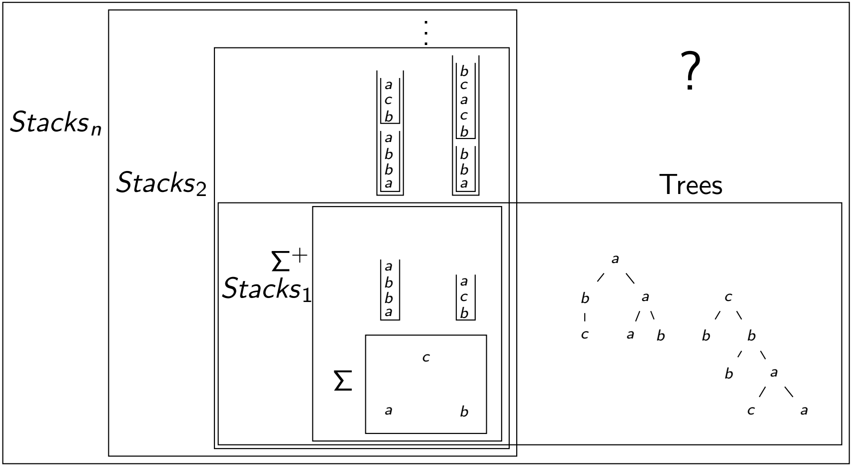
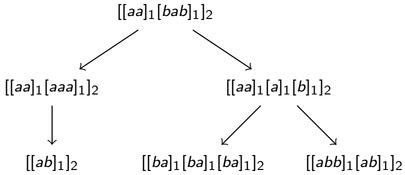


Higher-Order Stack Trees

A n-stack tree is a tree labelled by $(n-1)$-stacks.



Higher-Order Stack Trees

A n-stack tree is a tree labelled by $(n-1)$-stacks.

$$\begin{tikzcd}
 & & {[[aa]_1[bab]_1]_2} \arrow[dll] \arrow[dr] & & \\
{[[aa]_1[aaa]_1]_2} \arrow[d] & & & {[[aa]_1[a]_1[b]_1]_2} \arrow[dl] \arrow[dr] & \\
{[[ab]_1]_2} & & {[[ba]_1[ba]_1[ba]_1]_2} & & {[[abb]_1[ab]_1]_2}
\end{tikzcd}$$

A unary n-stack tree is a n-stack.
A 1-stack tree is a tree labelled by Σ.

Basic Operations: Level 0

Rewriting operations over Σ: $\mathrm{rew}_{\mathrm{b,a}}$

$$\begin{tikzcd}
 & {[[aa]_1[bab]_1]_2} \arrow[dl] \arrow[dr] & & \\
{[[aa]_1[aaa]_1]_2} \arrow[d] & & {[[aa]_1[a]_1[b]_1]_2} \arrow[dl] \arrow[dr] & \\
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\end{tikzcd}$$

Basic Operations: Level 0

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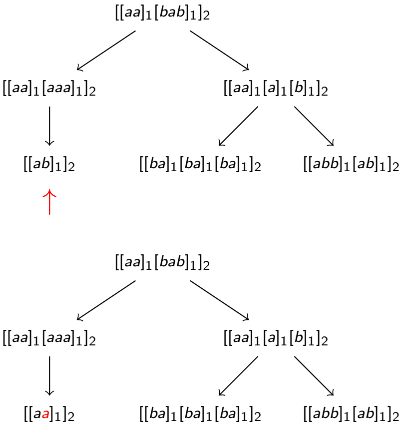
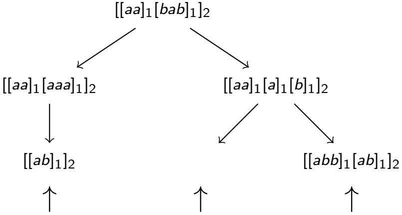


Basic Operations: Level $i < n$

Copy operations: $\mathrm{copy}_i, \overline{\mathrm{copy}}_i$

Copy operations: copy_2



Basic Operations: Level $i < n$

Copy operations: copy_2

$$
\begin{tikzcd}
& {[[aa]_1[bab]_1]_2} \arrow[dl] \arrow[dr] & & \\
{[[aa]_1[aaa]_1]_2} \arrow[d] & & {[[aa]_1[a]_1[b]_1]_2} \arrow[dl] \arrow[dr] & \\
{[[ab]_1]_2} & {} & & {[[abb]_1[ab]_1]_2}
\end{tikzcd}
$$

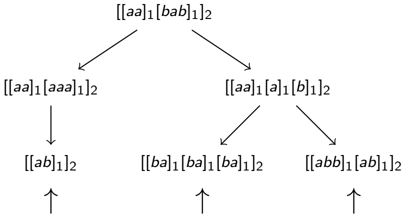
$\color{red}\uparrow$

$$
\begin{tikzcd}
& {[[aa]_1[bab]_1]_2} \arrow[dl] \arrow[dr] & & \\
{[[aa]_1[aaa]_1]_2} \arrow[d] & & {[[aa]_1[a]_1[b]_1]_2} \arrow[dl] \arrow[dr] & \\
{[[ab]_1\color{red}{[ab]_1}]_2} & {[[ba]_1[ba]_1[ba]_1]_2} & & {[[abb]_1[ab]_1]_2}
\end{tikzcd}
$$

Basic Operations: Level n

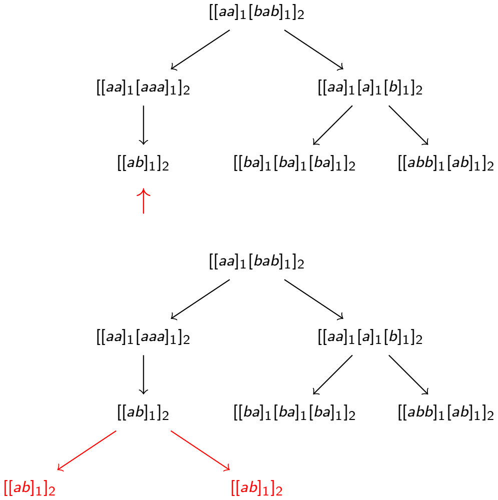
Tree copy operations: $\mathrm{copy}_n^i, \overline{\mathrm{copy}}_n^i$

Tree copy operations: copy_3^2



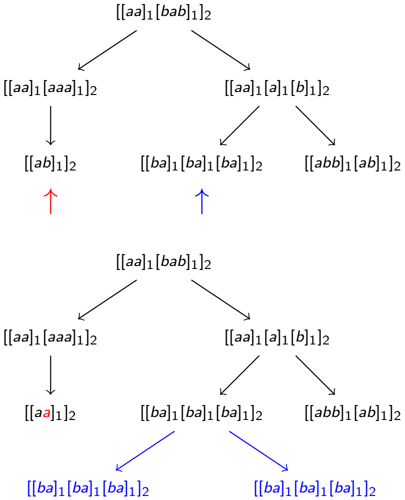
Basic Operations: Level n

Tree copy operations: copy_3^2



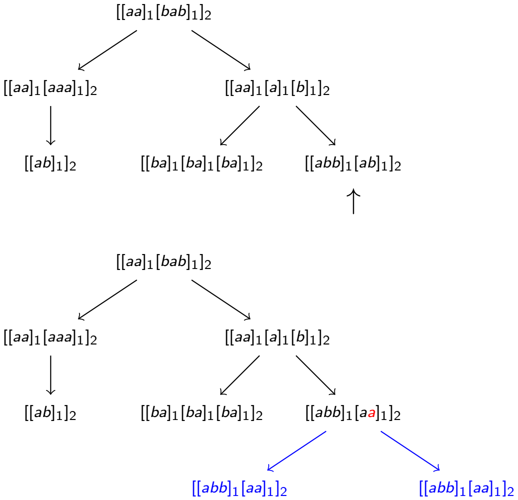
Composition

$\mathrm{rew}_{b,a} \circ \mathrm{copy}^2_3$:

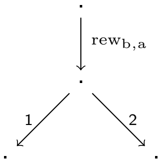


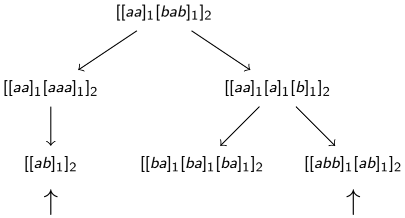
Composition

$\color{red}{\mathrm{rew}_{b,a}} \circ \color{blue}{\mathrm{copy}^2_{\bar{3}}}$:



Compound Operations





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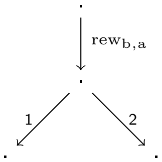

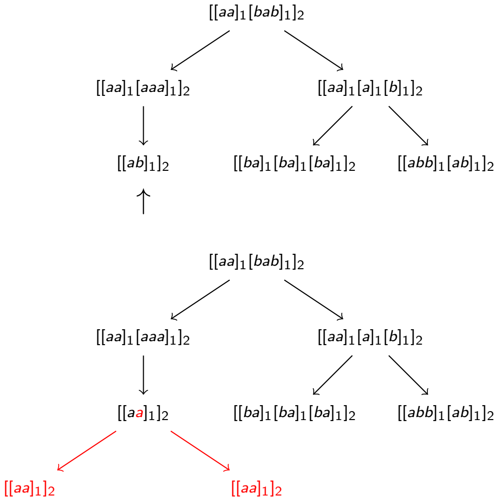

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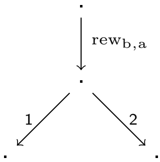

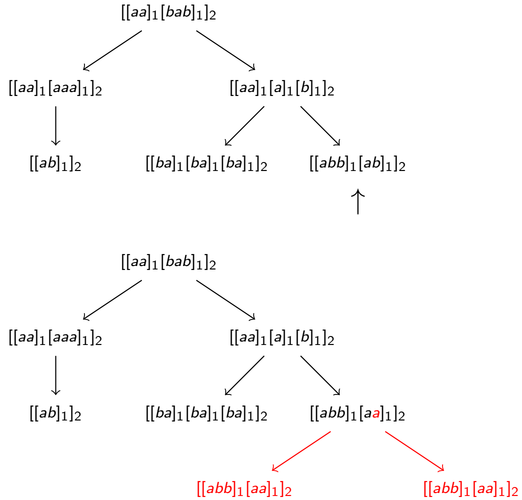

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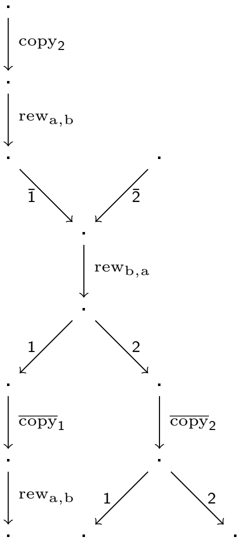
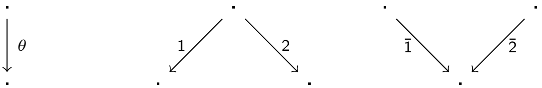
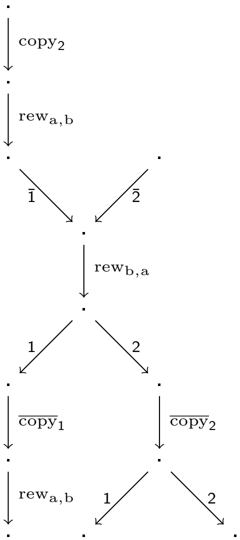


- Characterised by DAGs obtained by concatenations of DAGs whose edges represent basic operations:



- Only connected operations

Compound Operations



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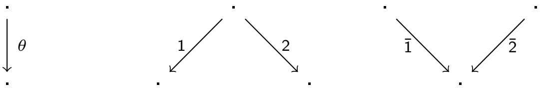

- Only connected operations
- Level 1 $\rightarrow$ ground tree rewriting rules
- Unary trees $\rightarrow$ finite composition of higher-order pushdown operations

Main Result

Given a set of compound operations R, its rewriting graph $\mathcal{G}_R$ is:

- $V_{\mathcal{G}_R} = ST_n$
- $E_{\mathcal{G}_R} = \{(t, r, t') \mid r \in R \wedge r(t, t')\}$

Theorem

Given a finite set of compound operations R, its rewriting graph has a decidable $FO[\xrightarrow{*}]$ theory.

Proof ingredients:

- Notion of recognisability over compound operations
- Finite set interpretation of every stack-tree rewriting graph into a graph with a decidable MSO-theory (the level n treegraph)

Perspectives

- Languages recognised by rewriting graphs of stack trees.
  Example $\{u \sqcup\!\sqcup u \mid u \in \Sigma^*\}$
- Strictness of the graph hierarchy
- Extension of the model to n-trees labelled by $(n-1)$-trees