A Note on Decidable Separability by Piecewise Testable Languages

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Separability

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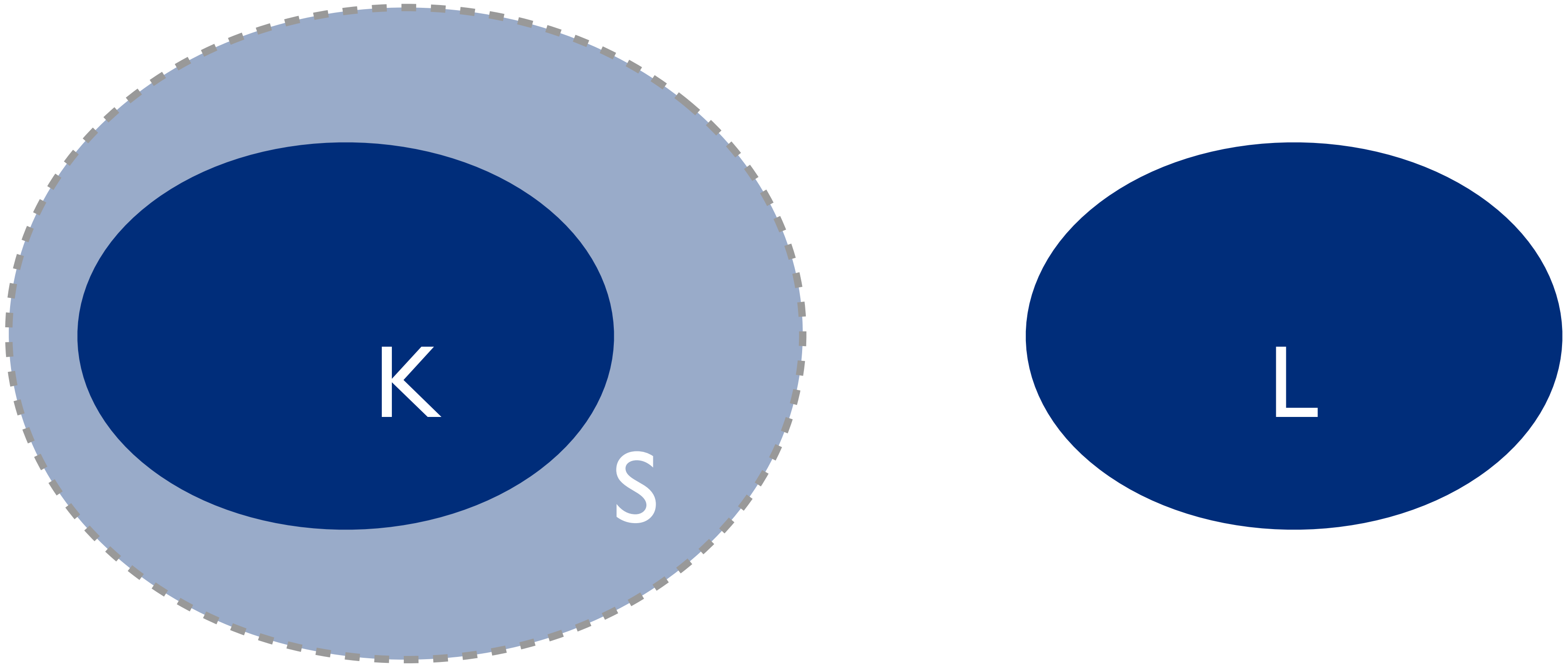

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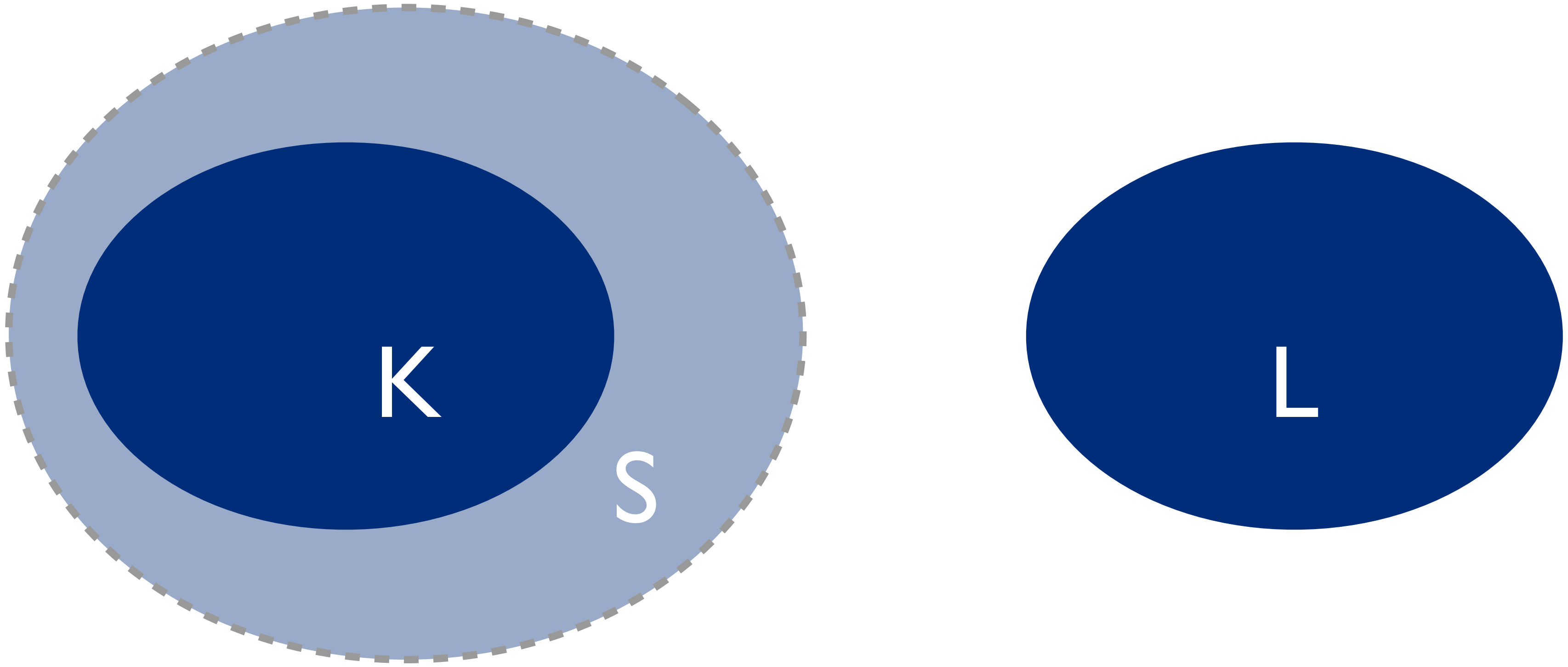
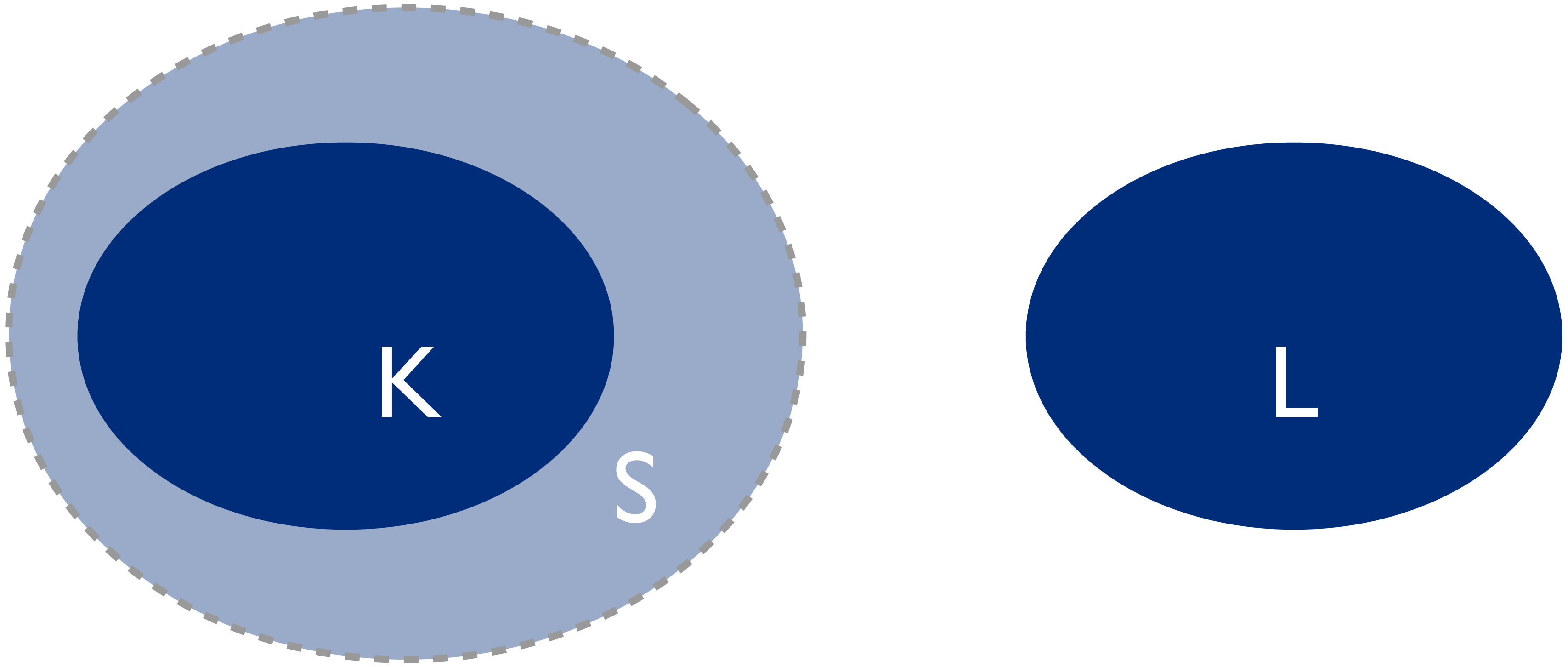


S *separates* K and L

Separability



S *separates* K and L

K and L are *separable by* family F
if some S from F separates them

General problem

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Given: two languages K and L from family F_1

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Question: are K and L separable by some language from family F_2

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Separability of F_1 by F_2

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Separability of F_1 by F_2

If F_1 effectively closed under complement - generalization of membership

Main problem

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Given: context-free grammars for languages K and L

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Question: are K and L separable by piecewise testable languages (PTL)?

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piece language

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piece language $\Sigma^* a_1 \Sigma^* a_2 \Sigma^* \ldots \Sigma^* a_n \Sigma^*$

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piece language $\Sigma^* a_1 \Sigma^* a_2 \Sigma^* \ldots \Sigma^* a_n \Sigma^*$

piecewise testable language

Main problem

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Question: are K and L separable by piecewise testable languages (PTL)?

piece language $\Sigma^* a_1 \Sigma^* a_2 \Sigma^* \ldots \Sigma^* a_n \Sigma^*$

piecewise testable language bool. comb. of pieces

What is known?

Separability of CFL by

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- CFL - undecidable (intersection problem)

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- regular languages - undecidable

What is known?

Separability of CFL by

- CFL - undecidable (intersection problem)
- regular languages - undecidable
- any family containing $w\Sigma^*$ and closed under boolean combination - undecidable

Our main result

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Theorem:
Separability of context free languages
by piecewise testable languages
is decidable

Our main message

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- something nontrivial possible for separability of CFL

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- piecewise testable languages are special

Our main message

- something nontrivial possible for separability of CFL
- no algebra needed
- piecewise testable languages are special
- separability problem is special (deciding whether CFL is a PTL is undecidable)

Proof (sketch)

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Two semi-procedures

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One tries to show
separability

Proof (sketch)

Two semi-procedures

One tries to show separability

One tries to show non-separability

Proof (sketch)

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One tries to show separability

One tries to show non-separability

Enumerates all piecewise testable languages and test them

Proof (sketch)

Two semi-procedures

One tries to show separability

One tries to show non-separability

Enumerates all piecewise testable languages and test them

Enumerates all patterns and test them

Second main result

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Theorem

Languages K and L are non-separable by PTL
if and only if
there exists a pattern p,
that fits both to K and L

Second main result

Theorem

Languages K and L are non-separable by PTL
if and only if
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It is decidable whether
pattern p fits to CFL L

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Pattern p over Σ consists of:

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words $w_0, w_1, \ldots, w_n$ in Σ^*

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$B^{\otimes}$ = words from B^* that contain all the letters from B

Patterns

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$B^{\otimes}$ = words from B^* that contain all the letters from B

Pattern p fits to a language L if for all $k \geq 0$ intersection of L and

$$w_0 \, (B_1^{\otimes})^k \, w_1 \ldots w_{n-1} \, (B_n^{\otimes})^k \, w_n$$

is nonempty

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- languages of Higher Order Pushdown Automata of order 2

Generalization

The same construction works for separating:

- languages of Petri Nets
- languages of Higher Order Pushdown Automata of order 2
- every well-behaving family of languages

Well-behaving languages

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Family of languages over Σ is a full-trio
if it is effectively closed under:

- removing letters from subalphabet $B \subseteq \Sigma$
- adding letters from subalphabet $B \subseteq \Sigma$
- intersection with regular languages

Diagonal problem

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Given: word language L over alphabet Σ

Diagonal problem

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Question: does there exists for every n a word in L containing each letter from Σ at least n times?

Generalized theorem

Generalized theorem

Theorem:
For every full-trio F with
decidable diagonal problem
separability of F by PTL is decidable

Further research

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- complexity of separability of CFL by PTL

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- complexity of separability of CFL by PTL
- is separability of CFL by some other nontrivial family decidable?

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- complexity of separability of CFL by PTL
- is separability of CFL by some other nontrivial family decidable?
  - group languages?

Further research

- complexity of separability of CFL by PTL
- is separability of CFL by some other nontrivial family decidable?
    - group languages?
    - solvable group languages?

Further research

- complexity of separability of CFL by PTL
- is separability of CFL by some other nontrivial family decidable?
  - group languages?
  - solvable group languages?
- connections with other problems

Thank you!