Experimental Test of the Kochen-Specker Theorem with Single Photons

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Using the spontaneous parametric down-conversion process in a type-I phase matching beta-barium-borate crystal as a single photon source, we perform an all-or-nothing–type Kochen-Specker experiment proposed by Simon *et al.* [Phys. Rev. Lett. **85**, 1783 (2000)] to verify whether noncontextual hidden variables or quantum mechanics is right. The results strongly agree with quantum mechanics.

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The problem of hidden variables in quantum mechanics has been discussed for many years. In 1964, Bell derived his inequality [1] to show the contradiction between local hidden variables (LHV) and quantum mechanics (QM). Many experiments have been performed to test Bell-type inequalities [2,3].

LHV assumes that the predetermined value of an observable does not depend on what other observables are simultaneously measured in a spatially separated region. But for noncontextual hidden variables (NCHV), the predetermined value of an observable does not depend on what other observables are measured simultaneously and spacelike separation is not required here. Without this requirement, NCHV theories reveal a basic opinion in hidden variable theories more directly; that is, the value we get from one measurement of an observable must be predetermined in spite of any other observables being measured simultaneously. While for the same reason, one can still make up LHV theories after the NCHV theories have been disproved by experiments.

The Kochen-Specker (KS) theorem [4–6] designs a rather complex formulation to show that NCHV is not compatible with quantum mechanics. But for a long time, there is not any experimental disproof of NCHV theories using the KS theorem. The reasons are pointed out by Cabello and Garcia-Alcaine (CG) [7]: (i) The proof of the KS theorem refers to a single individual system but involves noncompatible observables that cannot be measured in the same individual system. (ii) The proof also refers to NCHV theories which share some properties with quantum mechanics. So they are not entirely independent of the formal structure of quantum mechanics.

Up to now, most experiments testing LHV theories have two loopholes — "light-cone" loophole and detection efficiency loophole. A recent experiment by Rowe *et al.* [8] has closed the detection loophole using massive particles. In another experiment [9], the light-cone loophole is closed. For NCHV theories, recently two experiments were completed [10], using the three particle Greenberger-Horne-Zeilinger (GHZ) theorem and the Bell-like inequality. In Ref. [7], CG proposed an experimental scheme to test KS theorem based on two spin-$\frac{1}{2}$ particles and the proof is completely independent of the formal structure of quantum mechanics. Inspired by this scheme, Simon *et al.* [11] present a rather simple scheme to test NCHV theories. Simon's scheme is feasible with single particles, using both the path and the spin degrees of freedom to form a two-qubit system. Comparing with experiments [3] that have been exhibited to disprove the LHV theories (except for a recent experiment [12] which tests LHV in a nonstatistical way using three-photon GHZ entanglement), the above two schemes are both nonstatistical and they provide a very direct all-or-nothing–type test of NCHV. Because the scheme by Simon *et al.* [11] uses single particles which makes it easier to be performed, we choose it to test the NCHV theories.

In our experiment we use the polarization and path degrees of a single photon to form a two-qubit system that is equivalent to the original proposal in Ref. [11] which uses spin-$\frac{1}{2}$ particles.

In this scheme four observables Z_1, X_1, Z_2, and X_2 are considered, where the subscripts 1 and 2 denote the path and polarization qubits, respectively. Each of the observables has possible values of $+1$ or -1. In the language of quantum mechanics, we prepare a single photon in a two-qubit state $|\Psi\rangle$ using its path and polarization degree,

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|u\rangle|z+\rangle + |d\rangle|z-\rangle), \tag{1}$$

where $|u\rangle$ and $|d\rangle$ denote the "up" and "down" paths of the photon after a beam splitter, $|z+\rangle$ and $|z-\rangle$ denote the "vertical" and "horizontal" polarization states of the photon. The four observables Z_1, X_1, Z_2, and X_2 are represented by [11]

$$\begin{aligned} Z_1 &= |u\rangle\langle u| - |d\rangle\langle d|, \qquad X_1 = |u'\rangle\langle u'| - |d'\rangle\langle d'|, \\ Z_2 &= |z+\rangle\langle z+| - |z-\rangle\langle z-|, \\ X_2 &= |x+\rangle\langle x+| - |x-\rangle\langle x-|, \end{aligned} \tag{2}$$

where $|u'\rangle = (1/\sqrt{2})(|u\rangle + |d\rangle)$, $|d'\rangle = (1/\sqrt{2}) \times (|u\rangle - |d\rangle)$, $|x+\rangle = (1/\sqrt{2})(|z+\rangle + |z-\rangle)$, $|x-\rangle = (1/\sqrt{2})(|z+\rangle - |z-\rangle)$.

We can learn from $|\Psi\rangle$ that the measurement of Z_1Z_2 and X_1X_2 will both get the results of $+1$. And when we perform the joint measurement of Z_1X_2 and X_1Z_2, QM predicts that the results of them will definitely be opposite [11].

On the other hand, in the language of NCHV theories, the scheme can be described as below: First we prepare many systems (the single photons) in a certain way and show that for four observables Z_1, Z_2, X_1, andX_2 (defined by certain experimental operations) the results of the measurements of Z_1Z_2 and X_1X_2 both equal $+1$. Then, for systems prepared in the same way, NCHV theories predict that the results of the joint measurement of Z_1X_2 and X_1Z_2 will surely be equal [11] and this leads to the contradiction with QM. So our task is to prepare such systems and perform a joint measurement of Z_1X_2 and X_1Z_2 to determine whether QM or NCHV theories give the right answer.

Our experimental setup is shown in Fig. 1. The single photon source is provided by one photon (the signal photon) of the emitted photon pair produced through the spontaneous parametric down-conversion process in a 1-mm thick type-I phase matching beta-barium-borate crystal, which is pumped by a 351.1 nm laser beam (100 mW) produced by an Ar^+ laser (Coherent, Sabre, model DBW25/7). The other photon of the pair (the idle photon) is detected by $D0$ as a trigger, and the coincidence rates are recorded as the experimental data. This makes sure that the recorded data are provided by single photons from the emitted photon pairs, not by other noises. A time window of 5 ns is chosen to capture true coincidences and photons are detected by single photon detectors ($D0$–$D8$)—silicon avalanche photodiodes (EG&G, SPCM-AQR), with efficiencies of ~70% at 702.2 nm and dark counts of order 25 s^{-1}, each placed after a 4.6 nm interference filter and a 40× lens. In this experiment, the signal photon is prepared in the required state (described as $|\Psi\rangle$ in QM) by a properly rotated half-wave plate (HWP0), and a polarizing beam splitter (PBS0)—each PBS in Fig. 1 is set to reflect vertical polarization photons. For the symmetry of the setup in Fig. 1, in the following we first discuss the interferometer formed by PBS0 and the beam splitter (BS1) in detail and then briefly discuss the other interferometer formed by PBS0 and BS2.

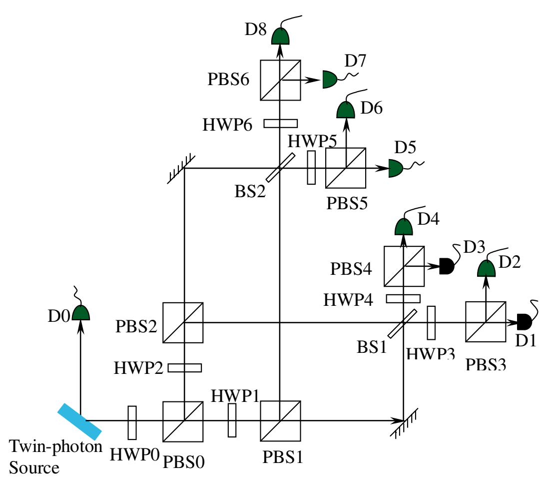

FIG. 1 (color online). Experimental setup of setup 1 (setup 1′) and setup 2. For setup 1 (setup 1′), HWP1 and HWP2 are both set at 0° (+45°). For setup 2, HWP1 is set at +22.5°, and HWP2 is set at −67.5°.

Now we give the definitions of Z_1, Z_2, X_1, and X_2 in an operational way. Corresponding to the definitions in Eq. (2), we can easily get that (i) Z_1 means in which path after PBS0 we find the photon, up (+ 1) or down (−1). (ii) Z_2 is the measurement of polarization of the photon, "vertical (+1)" or "horizontal (−1)." (iii) X_1 means that when the up and the down path lengths between PBS0 and BS1 equal, we find the photon in the up or the down path after BS1 (because the interference on a BS performs a Hadamard transformation of the path qubit). (iv) X_2 means that the photon is polarized at +45° (+1) or −45° (−1) away from the horizontal direction.

From the above definitions, we can easily see that the property $Z_1Z_2 = +1$ of the prepared photons is decided by the performance of HWP0 and PBS0. Because of the rather good quality of our PBS's (extinction ratio of the order 10^{-5}) and HWPs ($\Delta\theta = 0.2°$), we can regard the fact that the prepared photons just have the property $Z_1Z_2 = +1$.

To decide whether $X_1X_2 = +1$ or -1, an equal-arm interferometer formed by PBS0 and BS1 is used to measure X_1; HWP3 (set at +22.5°) followed by PBS3 both with HWP4 (set at +22.5°) followed by PBS4 are used to measure X_2. While working together, they can measure X_1X_2. When measuring X_1X_2, HWP1 and HWP2 are set at 0° to just let the photons pass without changing its polarization state. We name the above setup as setup 1 in the following.

The setup of the joint measurement of Z_1X_2 and X_1Z_2 is similar to setup 1, the only difference is that HWP1 and HWP2 are, respectively, set at +22.5° and −67.5° to perform the measurement of X_2 (with PBS1 and PBS2). In the original setup [11] two more HWPs should be placed, one between PBS1 and BS1, the other between PBS2 and BS1 to rotate the polarization of the photon to +45° and −45°, respectively, for the measurement of X_2. (In fact, we also need two more HWPs between PBS1, PBS2, and BS2 for the same reason. But we will not discuss it because of their similarity.) On the other hand, HWP3 and HWP4 should be set at an angle of 0° to measure Z_2. But in the concept of a +45° rotation of our polarization measurement basis, the setup that HWP3 and HWP4 at +22.5° (without two more HWPs) will not result in any change in the recorded data compared with the original setup. So, the setup of HWP1 and HWP2, respectively, at $+22.5°$ and $-67.5°$ will complete the last step in our task, that is, to decide whether Z_1X_2 equals to X_1Z_2 or not. We name this setup as setup 2. We note here that it is important to make sure that in setup 2 the two interferometers formed by PBS0 and BS1 (BS2) are both "equal arm." The measurement of X_1X_2 using setup 1 can tell us whether the interferometer formed by PBS0 and BS1 is equal arm in setup 1, so we need only to smoothly change the setup from setup 1 to setup 2 to make sure of the equal-arm property of the interferometer of PBS0 and BS1. For the other interferometer (PBS0 and BS2), we can use a similar method, i.e., we set HWP1 and HWP2 both at $+45°$ to make all the photons change their way to BS2. Then we can use the interferometer (BS2) to measure X_1X_2 (HWP5 and HWP6 both set at $+22.5°$) and make sure of its equal-arm property in setup 2. We call this setup $1'$.

It can be verified [11] that if QM is right, only $D1$, $D3$, $D5$, and $D7$ will detect photons in setup 2 when only $D2$, $D4$ ($D6$, $D8$) have detected photons in setup 1 (setup $1'$). While for NCHV theories only $D2$, $D4$, $D6$, and $D8$ will detect photons in setup 2. Thus this is an all-or-nothing–type experiment.

Because of the limitation of our devices, we did not record all the coincidence rates of $D0$ and $D1$–$D8$ at the same time. Instead, each time we recorded only one of them. But the experimental processes of the eight cases are almost the same.

Now we will describe the experiment process. For the first step we use setup 1 (setup $1'$). The interferometer's arm length is tuned so that it reaches its minimal value at $D1$, $D3$, $D5$, and $D7$ (while it reaches its maximal value at $D2$, $D4$, $D6$, and $D8$). This result shows that the prepared photons have the property of $X_1X_2 = +1$, which can be easily deduced from the operational definitions for the four observables above. In the second step, we smoothly change setup 1 (or setup $1'$ for the case of $D5$, $D6$, $D7$, and $D8$) to setup 2, just by rotating HWP1 and HWP2 to $+22.5°$ and $-67.5°$, respectively, with a specially designed mechanical device, and record the corresponding coincidence rates. If now the interferometer reached its maximal (minimal) value in $D1$, $D3$, $D5$, and $D7$ ($D2$, $D4$, $D6$, and $D8$), we know that $Z_1X_2 = -1$ $(+1)$ and $X_1Z_2 = +1$ (-1). (In detail, when $D1$ gets maximum value, we can conclude the values of the four observables: $Z_1 = +1$ ($|u\rangle$), $X_2 = -1$ ($-45°$) or $Z_1 = -1$ ($|d\rangle$), $X_2 = +1$ ($+45°$), that is, $Z_1X_2 = -1$; and $X_1 = -1$ ($|d'\rangle$), $Z_2 = -1$ (horizontal), which means $X_1Z_2 = +1$, that means QM is right.

The recorded coincidence rates between $D0$ and $D1$–$D8$ are shown in Fig. 2, from $D1$ to $D8$. We can see that each figure in Fig. 2 has three stages along the time axis. In the first stage, the setup is in setup 1 (or setup $1'$ for $D5$, $D6$, $D7$, and $D8$) and the coincidence rates are stable at the interferometer's minimal value for $D1$, $D3$, $D5$, and $D7$, or maximal value for $D2$, $D4$, $D6$, and $D8$. That shows $X_1X_2 = +1$ for the prepared photons. In the second stage, the setup has been changed to setup 2 (the time of changing the setup is only about 2 s), and the recorded coincidence rates are stable at the interferometer's maximal value. [Note that in setup 2 the maximal value in $D1$, $D3$ ($D5$, $D7$) of the interferometer is only half of that in setup 1 (setup $1'$), because half of the photons are detected by $D5$–$D8$ ($D1$–$D4$).] The maximal value of $D1$, $D3$ ($D5$, $D7$) in setup 1 (setup $1'$) is about $750\ \mathrm{s}^{-1}$, so the maximal value in setup 2 is about $375\ \mathrm{s}^{-1}$ for $D1$, $D3$, $D5$, and $D7$ or a minimal value for $D2$, $D4$, $D6$, and $D8$ that shows Z_1X_2 is opposite to X_1Z_2. In the third stage, the setup is again in setup 1 (setup $1'$) and the recorded rates recover to the same level as those in the first stage. That means our system is stable and controllable during the experiment (in fact, the stable time of the interferometer is about 5 min).

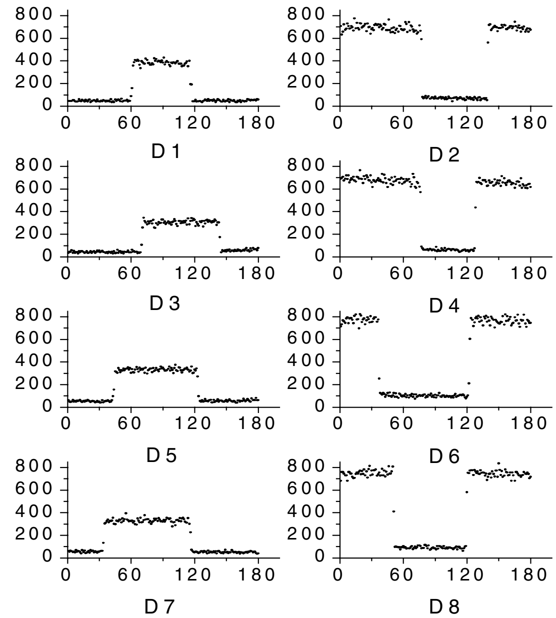

FIG. 2. Recorded coincidence rates of $D0$ and $D1$–$D8$ in the experiment. The vertical axis is the coincidence rate (s^{-1}) and the horizontal axis is the time axis (s) of performing the experiment.

Figure 3 is the analyzed experiment results. Result 1 is the fraction that agrees with NCHV and result 2 is the fraction that agrees with QM in the joint measurement of Z_1X_2 and X_1Z_2. In the fraction number $\varepsilon = (S_{\min}/S_{\max} + S_{\min}) = 0.19$, $S_{\min}$ is of the sum of the averaged minimum coincidences datas of $D2$, $D4$, $D6$, and $D8$, while $S_{\max}$ is the sum of the averaged maximum coincidence datas of $D1$, $D3$, $D5$, and $D7$. It shows to what extent our results violate the prediction of NCHV theories.

As Simon *et al.* pointed out [11], the appearance of the paradox in this experiment is related to the superposition principle, so choosing the right angle of HWP1 and

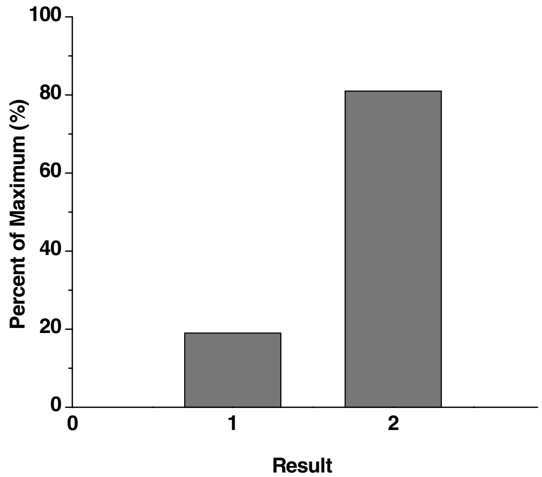

FIG. 3. Analyzed experiment results. Result 1 is the fraction that agrees with NCHV. Result 2 is the fraction of total coincidence rates that agrees with QM. Results 1 and 2 show the violation against NCHV theories in our experiment.

HWP2 is very important. If we set HWP1 and HWP2 both at $+22.5°$ or $-67.5°$ in setup 2, theoretical analysis and further experiment show that the results would not lead to a contradiction between QM and NCHV theories. For the low collection efficiency of photon pairs in our experiment, we have to invoke the fair sampling assumption as most other experiments testing the hidden variable theories.

An important problem which may lead to argument on this experiment is whether the finite precision measurement would nullify the demonstration in this paper. This kind of problem in KS-type experiments was first argued by Meyer [13], who claimed that the Kochen-Specker theorem was "nullified" in real experiments because of the unavoidably finite measurement precision. Kent [14] generalized his work and came to a similar conclusion.

In response, Simon *et al.* [15] demonstrated in an entirely operational way that even when the finite measurement precision is taken into account, NCHV theories still can be excluded from the KS theorem. Cabello [16] also proved that only finite measurement precision is needed to disprove NCHV theories in real KS experiments. In our experiment, we may treat this problem such that the error fraction of the measurement of Z_1Z_2 (the cases in which $Z_1Z_2 \neq 1$) can be neglected because of the good performance of our PBS and HWP as mentioned above; the error fraction of the measurement of X_1X_2 (the cases in which $X_1X_2 \neq 1$) is $p1 = 0.09$, mainly coming from the imperfect interference in setup 1 and setup $1'$; and the estimated error fraction in the joint measurement of Z_1X_2 and X_1Z_2 caused by imperfect experimental setup is $p2 = 0.18$, mainly coming from the imperfect interference effect in setup 2. So for a NCHV theory, the measured fraction of cases which have Z_1X_2 different from X_1Z_2 can be at most $(p1 + p2)$. But in our results, this fraction number is $1 - \varepsilon = 0.81 \succ (p1 + p2)$. In this way, we can still come to our conclusion that this experiment disproves the NCHV theories. Note that though this experimental test of noncontextuality is in principle an all-or-nothing–type test, the consideration of a precision reveals an inessential statistical property of the experimental test [15].

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[1] J. S. Bell, Physics (Long Island City, N.Y.) **1**, 195 (1965).
[2] A. Aspect, J. Dalibard, and G. Roger, Phys. Rev. Lett. **49**, 1804 (1982).
[3] Y. H. Shih and C. O. Alley, Phys. Rev. Lett. **61**, 2921 (1988); J. G. Rarity and P. R. Tapster, *ibid.* **64**, 2495 (1990); P. G. Kwiat, K. Mattle, H. Weinfurter, A. Zeilinger, A. V. Sergienko, and Y. H. Shih, *ibid.* **75**, 4337 (1995); G. Weihs, T. Jennewein, C. Simon, H. Weinfurter, and A. Zeilinger, *ibid.* **81**, 5039 (1998).
[4] S. Kochen and E. P. Specker, J. Math. Mech. **17**, 59 (1967).
[5] N. D. Mermin, Phys. Rev. Lett. **65**, 3373 (1990); A. Peres, Phys. Lett. A **151**, 107 (1990); N. D. Mermin, Rev. Mod. Phys. **65**, 803 (1993); A. Cabello and G. Garcia-Alcaine, J. Phys. A **29**, 1025 (1996); A. Cabello, J. M. Estebaranz, and G. Garcia-Alcaine, Phys. Lett. A **212**, 183 (1996); **218**, 115 (1996).
[6] A. Cabello, Phys. Rev. Lett. **87**, 010403 (2001).
[7] A. Cabello and G. Garcia-Alcaine, Phys. Rev. Lett. **80**, 1797 (1998).
[8] M. A. Rowe *et al.*, Nature (London) **409**, 791 (2001).
[9] G. Weihs *et al.*, Phys. Rev. Lett. **81**, 5039 (1998).
[10] M. Michler, H. Weinfurter, and M. Zukowski, Phys. Rev. Lett. **84**, 5457 (2000).
[11] C. Simon, M. Zukowski, H. Weinfurter, and A. Zeilinger, Phys. Rev. Lett. **85**, 1783 (2000).
[12] J.-W. Pan, D. Bouwmeester, M. Daniell, H. Weinfurter, and A. Zeilinger, Nature (London) **403**, 515 (2000).
[13] D. Meyer, Phys. Rev. Lett. **83**, 3751 (1999).
[14] A. Kent, Phys. Rev. Lett. **83**, 3755 (1999).
[15] C. Simon, C. Brukner, and A. Zeilinger, Phys. Rev. Lett. **86**, 4427 (2001).
[16] A. Cabello, Phys. Rev. A **65**, 052101 (2002).