The first chapter, by Scott, considers Boolean algebra as a model for combinational circuits. Chapter 2, by Cadden, deals with conversions of number bases and some of their number-theoretic properties. The third chapter, by Runyon, discusses the formulation of practical switching problems. Chapter 4, by Tanana, presents an introduction to the map method of minimization; especially the method of Karnaugh maps. The fifth chapter, by McCluskey, treats of minimization theory with emphasis on the realization of minimum-cost combinational circuits. The sixth chapter, by Bartee, mentions recent work on the use of digital computers to design combinational circuits. Chapter 7, by McCluskey, discusses, by example, the use of matrix methods to analyze sequential circuits. Chapter 8, by Runyon, deals with the derivation of completely and partially specified state tables in the senses of D. A. Huffman and E. F. Moore. The ninth chapter, by Unger, describes, by means of examples, a reduction process for minimizing the number of states in an incompletely specified state table. Chapter 10, by Phister, undertakes to synthesize circuits from their state matrices. The last chapter, by Reed, discusses the structure of sequential machines.

The papers are of an uneven quality and are not intended by the editors to constitute a research monograph. The book should be of interest to practicing engineers and to students interested in logical design, even though some of the topics are more thoroughly covered in S. H. Caldwell's book XXIII 433. For supplementary reading see, e.g., R. S. Ledley, editor, ***Switching circuit theory and logical design*** (1961) and T. C. Bartee et al., ***Theory and design of digital machines*** (1962).

ALBERT MULLIN

J. PAUL ROTH and R. M. KARP. *Minimization over Boolean graphs.* ***IBM journal of research and development,*** vol. 6 (1962), pp. 227–238.

The authors describe a new systematic approach to synthesis of gate-type combinational switching circuits with minimal cost. The discussion is based essentially on the solution of the following two problems concerning abstract decomposition of functions. Given finite sets X, Y, Z, W, E, $E \subset X \times Y$, and a function $F : E \to Z$. (i) Given the function $\alpha : X \to W$, does there exist a function $G : W \times Y \to Z$ such that $F(x, y) = G(\alpha(x), y)$ for all $(x, y) \in E$? (ii) Under what conditions do there exist functions $\alpha : X \to W$ and $G : W \times Y \to Z$ such that $F(x, y) = G(\alpha(x), y)$ for all $(x, y) \in E$? Necessary and sufficient conditions for positive solution of both problems are found, which are subsequently applied in decompositions of Boolean functions describing circuits to be designed. The latter functions are represented in the form of suitable tables from which all possible decompositions (corresponding to decompositions of Boolean graphs of circuits) can be found by means of algorithms too long to be described here. The algorithms, coupled with suitably formalized requirement of minimal cost, can be programmed for a digital computer. This is the fifth paper of a series by the first author.

PAWEŁ SZEPTYCKI

B. A. TRAHTÉNBROT. *Ob opératorah réalizuémyh v logičéskih sétah* (On operators realizable in logical nets). ***Doklady Akadémii Nauk SSSR,*** vol. 112 (1957), pp. 1005–1007.

Let X, Z, Q be finite sets with m, n, k elements; X is called the input, Z the output, and Q the auxiliary alphabet. We consider operators (i.e., certain finite automata) which transform finite or infinite words consisting of letters which belong to X into finite or infinite words consisting of letters which belong to Z. The alphabet Q may be considered as the set of internal states of the automaton. The memory of the automaton can retain exactly one letter of Z in every unit of time. If, for an automaton, the number k cannot be reduced without changing the functioning of the automaton, then we say that its weight is k.

The letters are represented as numbers in the binary scale. It follows therefore that the capacity of the memory must be equal to $\log_2 k$ binary places. If the automaton of weight k transforms only words of length w then the ratio $s = \log_2 k/(w \log_2 m)$ is called its "proper memory." Using these definitions the author proves the following theorems:

1. $s < 1$, but for every $\varepsilon > 0$ the number of non-equivalent automata for which $s < 1 - \varepsilon$ tends to 0 as w tends to infinity.

2. Every automaton of weight k transforms periodic words with period r into periodic words with period kr.

3. If two automata have weight k then they transform arbitrary infinite words in an identical manner if and only if they transform arbitrary words of length $2k - 1$ identically; the number $2k - 1$ cannot be reduced. ANDRZEJ BLIKLE