# Verification of ***Open*** Interactive Markov Chains

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# Our contribution

The first assume-guarantee reasoning
for systems with stochastic continuous time

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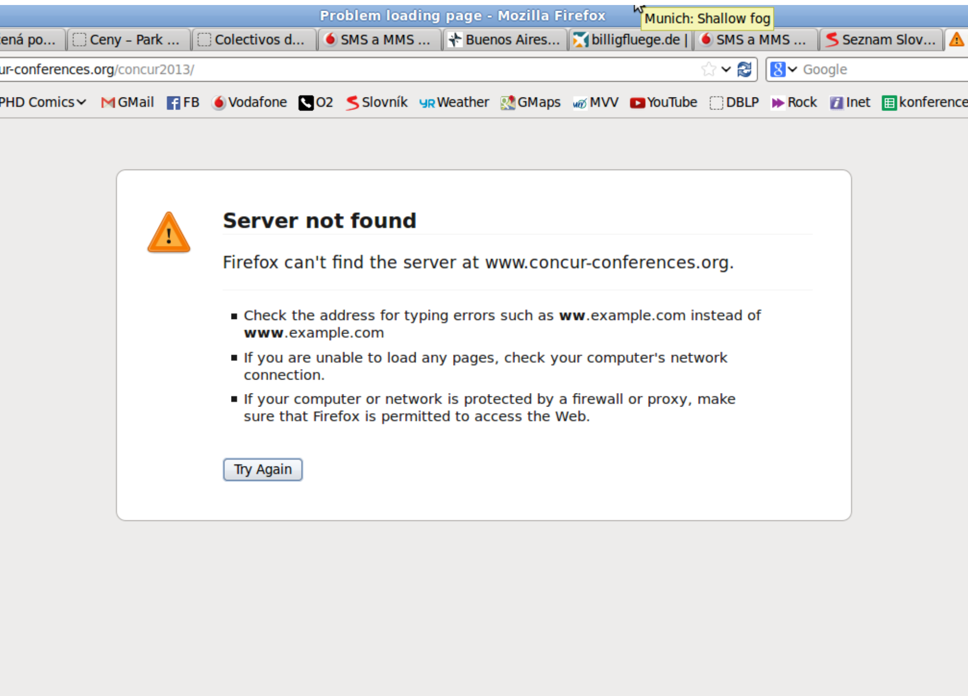
The first assume-guarantee reasoning
for systems with stochastic continuous time

- given a system $S$,
  we compute guarantees on $S \parallel ?$
- we give a specification formalism to express assumptions
- given a system $S$ and assumptions $\varphi$ on its environment,
  we compute guarantees on $S \parallel \varphi$



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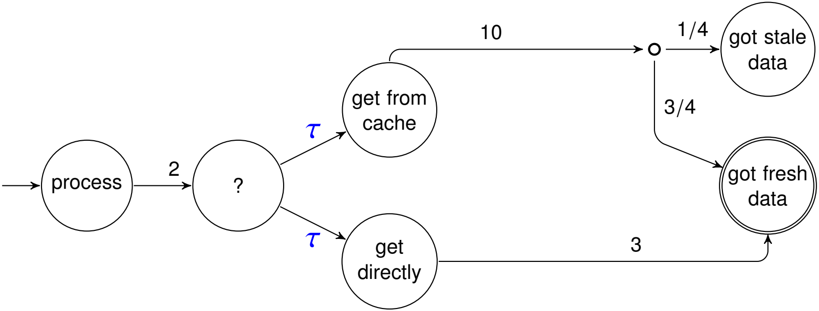
# Guarantees in continuous-time stochastic systems

Synthesize optimal scheduler $\sigma$ of system $S$

$$\sup_{\sigma} \quad Pr_S^{\sigma} \quad [Reach \leq 1.5] = ?$$



Interactive Markov chains

- similar to continuous-time Markov decision processes
- *compositional* process-algebraic framework

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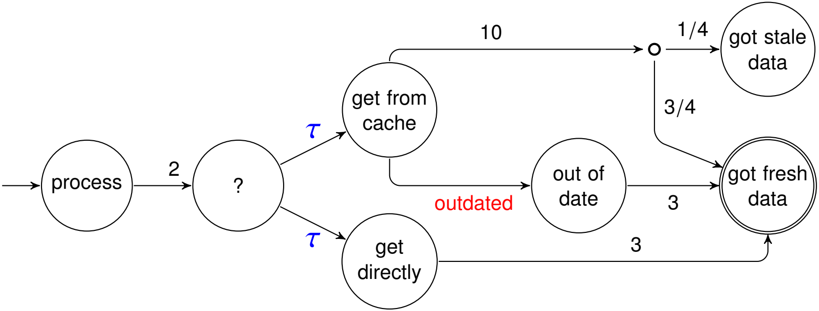

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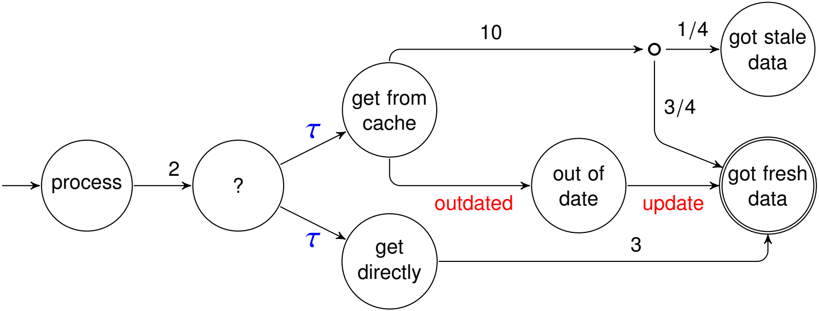

# Guarantees in continuous-time stochastic systems

Synthesize optimal scheduler $\sigma$ of system $S$

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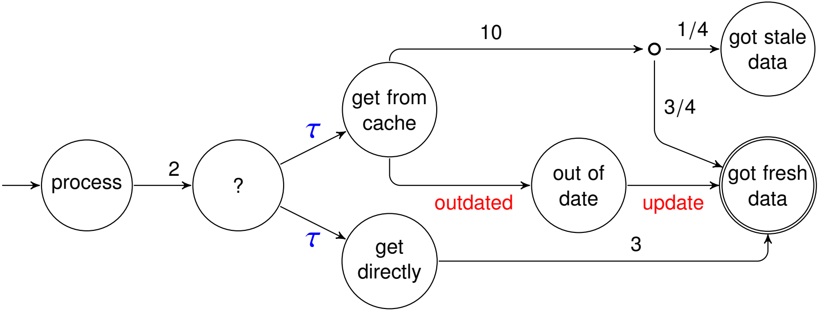
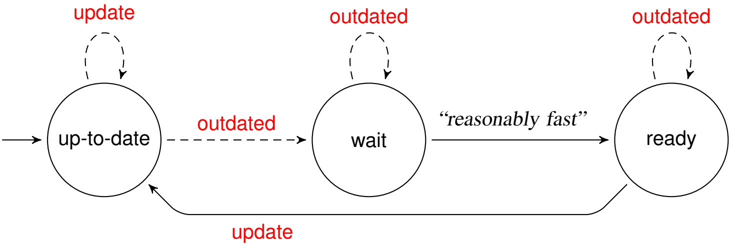


# Specification formalism

We introduce modal continuous-time automata (MCA)

- may/must transitions as in modal transition systems [Larsen&Thomsen'88]

**Example:** after each outdated an update is ready *reasonably fast*



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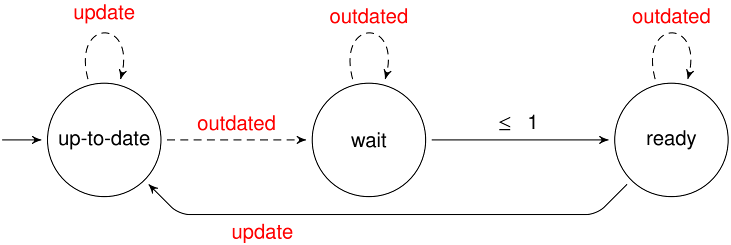

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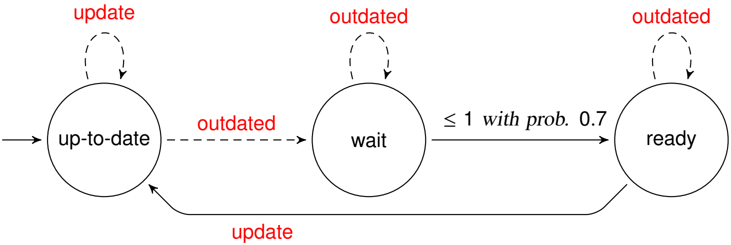

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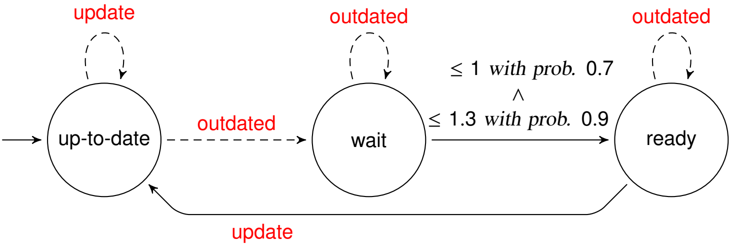

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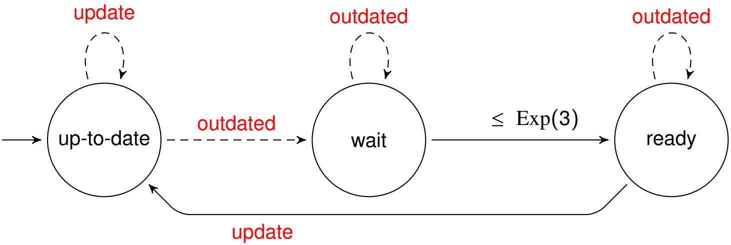
**Example:** after each outdated an update is ready within time ~Exp(3)

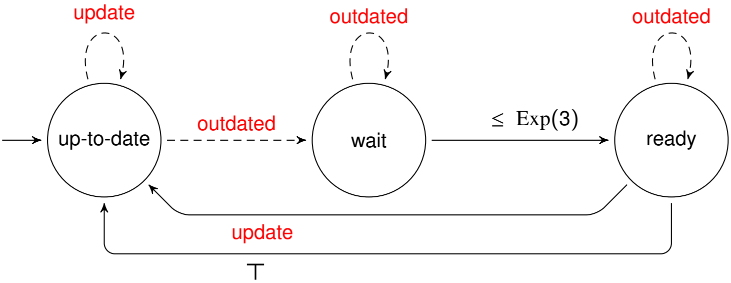


# Specification formalism

We introduce modal continuous-time automata (MCA)

- may/must transitions as in modal transition systems [Larsen&Thomsen'88]
- continuous time constraints extending timed automata [Alur,Courcoubetis&Dill'91]

**Example:** after each outdated an update is ready within time ∼Exp(3)



# Main results

**Theorem**

*For IMC $S$ without internal and external edges enabled at once, the guarantee $\sup_{\sigma} \inf_{E} Pr^{\sigma}_{S\|E}[Reach \leq T]$ can be $\varepsilon$-approximated in polynomial time.*

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*For IMC $S$ and a modal continuous-time automaton $\varphi$, the guarantee $\sup_{\sigma} \inf_{E \models \varphi} Pr^{\sigma}_{S\|E}[Reach \leq T]$ can be $\varepsilon$-approximated in exponential time.*

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*For IMC $S$ without internal and external edges enabled at once, the guarantee $\sup_{\sigma} \inf_{E} Pr^{\sigma}_{S\|E}[Reach \leq T]$ can be $\varepsilon$-approximated in polynomial time.*

**Idea:** games with stochastic and non-deterministic time

*more on Saturday 16:45 (the very last talk!)*

**Theorem**

*For IMC $S$ and a modal continuous-time automaton $\varphi$, the guarantee $\sup_{\sigma} \inf_{E \models \varphi} Pr^{\sigma}_{S\|E}[Reach \leq T]$ can be $\varepsilon$-approximated in exponential time.*

**Idea:** reduce to $\sup_{\sigma} \inf_{E} Pr^{\sigma}_{(S\times\varphi)\|E}[Reach \leq T]$

games with partial information

# Summary and conclusions

- The first assume-guarantee reasoning on stochastic continuous-time systems
- Specification language **modal continuous-time automata** with *continuous time constraints*

For IMC $S$ without internal and external edges enabled at once, the guarantee $\sup_{\sigma} \inf_{E} Pr^{\sigma}_{S\|E}[Reach \leq T]$ can be $\varepsilon$-approximated in polynomial time.

For IMC $S$ and a modal continuous-time automaton $\varphi$, the guarantee $\sup_{\sigma} \inf_{E \models \varphi} Pr^{\sigma}_{S\|E}[Reach \leq T]$ can be $\varepsilon$-approximated in exponential time.

Future work

- lowering the theoretical/practical complexity
- logical specification language