# Reachability for Dynamic Parametric Processes

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TU München, U. Bordeaux

Dagstuhl, Nov. 25, 2016





## Reachability for Dynamic Parametric Processes w/o Weak Memory

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```
root() {
     spawn(p);
     switch(x) {
        case 2 : result = #;
     }
}
```





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```

thread creation





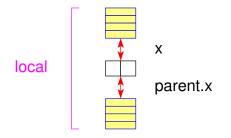
```
root() {
     spawn(p);
     switch(x) {
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}
```

- thread creation
- access to data





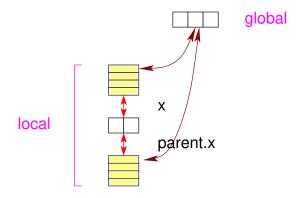
### **Data Access**







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## Dynamic Process (cont.)

```
p()
    switch (parent.x) {
     case 0 : spawn(p);
               if (*) parent.x = 1
               else switch (x) {
                    case 1: parent.x = 1; break;
                    case 2: break:
               break:
     case 1 : spawn(p);
               if (*) parent.x = 0
               else switch (x) {
                    case 1: parent.x = 2; break;
                    case 2: parent.x = 2; break;
```



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arbitrary locking

Kahlon et al. 2005





#### **Good News**

Restricting locking disciplines helps! Kahlon et al. 2005





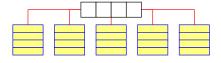
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Restricting locking disciplines helps!

Kahlon et al. 2005

Parametrization helps!

Kahlon 2008







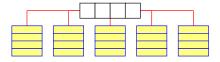
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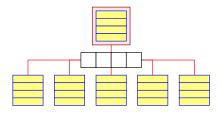




#### More Good News

 $Parametric\ contributors\ +\ one\ master$ 

Hague 2011 Esparza et al. 2015



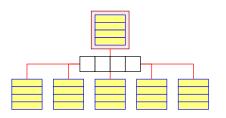




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# Dynamic Processes no data, no locking

Bouajjani et al. 2005







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no data, restricted locking

Müller-Olm and friends





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- recursion
- dynamic parametric creation of child processes
- asynchronous execution (no test-and-set)





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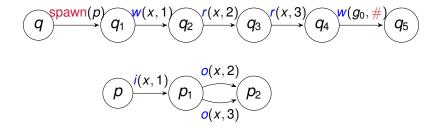


- recursion
- dynamic parametric creation of child processes
- asynchronous execution (no test-and-set)
- Atomic global variables
- Atomic local variables

## Dynamic parametric processes

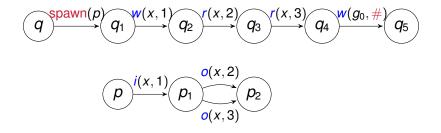








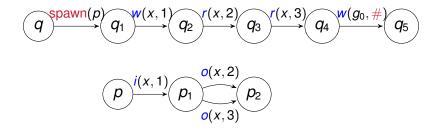




- spawn creates new process;
- r, w access own variable x;
- *i*, *o* access parent variable *x*.





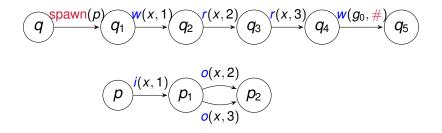


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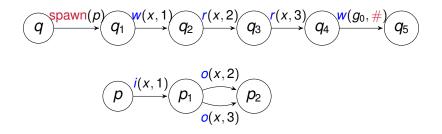
**Reachability:** may # ever be written? No







## Dynamic Parametric Process



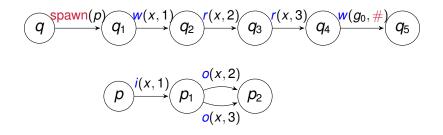
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## Dynamic Parametric Process

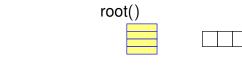


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**Reachability:** may # ever be written? **Yes** 

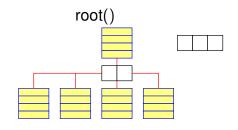






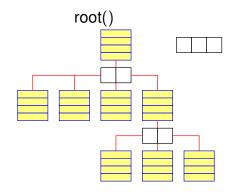






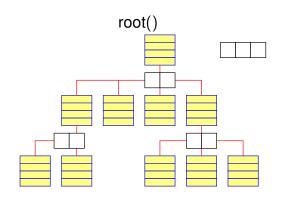
















#### **Our Results**

#### Theorem 1

Reachability for dynamic processes is decidable.





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In absence of locals, reachability is PSPACE complete.





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In absence of locals, reachability is PSPACE complete.

#### Theorem 3

For generalized futures, reachability is DEXPTIME complete.





#### **Multiset Configuration**

$$\langle p, ABBC, \{x \mapsto 1, y \mapsto 2\}, \{5 \cdot t_1, 2 \cdot t_2, 17 \cdot t_3\} \rangle$$





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\langle p, ABBC, \{x \mapsto 1, y \mapsto 2\}, \{5 \cdot t_1, 2 \cdot t_2, 17 \cdot t_3\} \rangle
```

#### **Set Configuration**

```
\langle \textit{p}, \textit{ABBC}, \{\textit{x} \mapsto \textit{1}, \textit{y} \mapsto \textit{2}\}, \ \{ \quad \textit{t}_{\textit{1}} \ , \quad \textit{t}_{\textit{2}} \ , \quad \textit{t}_{\textit{3}} \ \} \rangle
```



#### **Multiset Configuration**

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\langle p, ABBC, \{x \mapsto 1, y \mapsto 2\}, \{5 \cdot t_1, 2 \cdot t_2, 17 \cdot t_3\} \rangle
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\langle \textit{p}, \textit{ABBC}, \{\textit{x} \mapsto \textit{1}, \textit{y} \mapsto \textit{2}\}, \ \{\textit{set}(\textit{t}_{\textit{1}}), \textit{set}(\textit{t}_{\textit{2}}), \textit{set}(\textit{t}_{\textit{3}})\} \rangle
```





Intuition

Child actions may be repeated arbitrarily.





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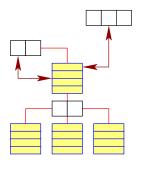
Proposition

Multset Reachability — Set Reachability





## **External Behavior**

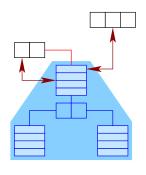


Forget about internal actions!





## **External Behavior**



Forget about internal actions!





#### **Hypothesis**

```
{ spawn(p),
spawn(p) i(x, 1),
spawn(p) i(x, 1) o(x, 2),
spawn(p) i(x, 1) o(x, 3) }
```

— all external behaviors of child processes.





#### Hypothesis

— all external behaviors of child processes.

#### **Abstraction**

Set of child processes  $\rightarrow$  Set of external behaviors





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Internal activities of child processes are irrelevant.





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Internal activities of child processes are irrelevant.

#### **Problem**

Hypotheses may be

infinite





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Internal activities of child processes are irrelevant.

#### **Problem**

Hypotheses may be

- infinite
- complicated





Arbitrarily insert earlier actions ...





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```
\begin{array}{ll} \operatorname{spawn}(\mathsf{p}) \, \mathit{i}(x,1) \, \mathit{o}(x,2) & \in \\ & \operatorname{lift} \left\{ \begin{array}{ll} \operatorname{spawn}(\mathsf{p}), \\ \operatorname{spawn}(\mathsf{p}) \, \mathit{i}(x,1), \\ \operatorname{spawn}(\mathsf{p}) \, \mathit{i}(x,1) \, \mathit{o}(x,2), \\ \operatorname{spawn}(\mathsf{p}) \, \mathit{i}(x,1) \, \mathit{o}(x,3) \end{array} \right\} \end{array}
```





Arbitrarily insert earlier actions ...

```
\begin{aligned} \mathsf{spawn}(\mathsf{p}) \, & \mathit{i}(x,1) \, \mathit{o}(x,2) \, \mathit{i}(x,1) \quad \in \\ & \mathsf{lift} \, \big\{ & \mathsf{spawn}(\mathsf{p}), \\ & \mathsf{spawn}(\mathsf{p}) \, \mathit{i}(x,1), \\ & \mathsf{spawn}(\mathsf{p}) \, \mathit{i}(x,1) \, \mathit{o}(x,2), \\ & \mathsf{spawn}(\mathsf{p}) \, \mathit{i}(x,1) \, \mathit{o}(x,3) \, \big\} \end{aligned}
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#### **Proposition**

 $H_1$  is equivalent to  $H_2$  iff  $Iift(H_1) = Iift(H_2)$ .





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#### Observation

► Consider minimal set  $core(H) \subseteq H$  with lift(core(H)) = lift(H)



#### **Proposition**

 $H_1$  is equivalent to  $H_2$  iff  $lift(H_1) = lift(H_2)$ .

#### Observation

► Consider minimal set  $core(H) \subseteq H$  with

$$lift(core(H)) = lift(H)$$

► core(*H*) is finite.



 $\mathsf{Ext}_{k} = \{ \mathsf{external} \; \mathsf{behaviors} \; \mathsf{of} \; \mathsf{conf.} \; \mathsf{of} \; \mathsf{depth} \leq {\color{red} k} \}$ 





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▶ Determine  $core(Ext_0)$ .





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▶ Determine core(Ext₀). // plain pushdown system





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- ▶ Determine core(Ext₀). // plain pushdown system
- ▶ Determine core(Ext<sub>k</sub>) for k > 0. // plain pushdown system
- ► Stop when  $core(Ext_k) = core(Ext_{k-1})$ .





# Wrap-up

#### Theorem 1

Reachability for dynamic parametric processes is decidable.





## Wrap-up

#### Theorem 1

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The same holds true for higher-level dynamic processes.



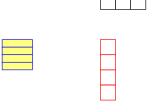






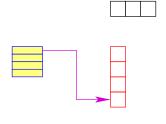






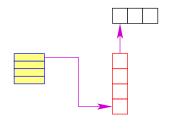
















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- channels
- finite hypothesis on children





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Conjecture: Reachability is decidable





## Summary

 Parametrization is a useful abstraction to deal with unbounded concurrency.





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- It is applicable to PSO.





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- Parametrization is a useful abstraction to deal with unbounded concurrency.
- It is applicable to PSO.
- How to parameterize model / results with various kinds of WMs?





# Thank you!



