Synchronization in MDPs

Mahsa Shirmohammadi

Laurent Doyen

Thierry Massart

Highlight 2013, September 19th, Paris

Spy and factory

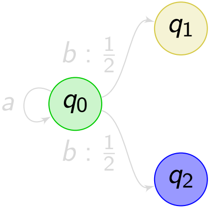
factory

thief

Spy and factory

factory

thief



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factory

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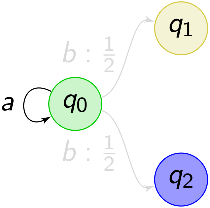

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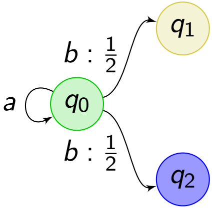

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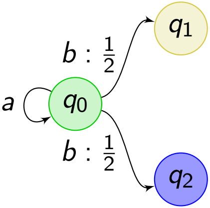
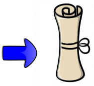
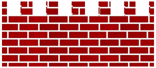




map

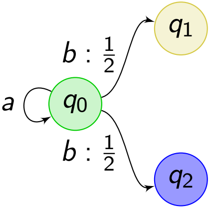
Spy and factory

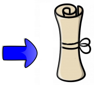
factory



impenetrable wall

thief





map

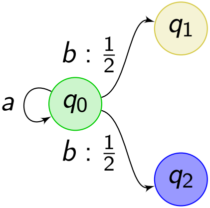
Spy and factory

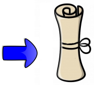
factory

request/reply

clients send physical requests,
and fix a "time" to pick it up

thief



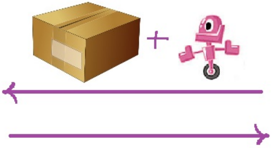


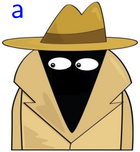
map

Spy and factory

factory

thief sends a request as a honest client with a hidden spy robot, plus the "time"





thief

the factory responses exactly at that "time"

It means that

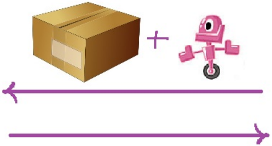
- ➪ when robot gets into the factory, it has to accomplish a "mission" at

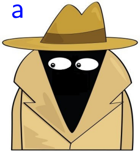
  that exact "time" fixed by thief

  no matter how the random behavior of factory responses

factory

thief sends a request as a honest client with a hidden spy robot, plus the "time"





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Spy and factory

- ϟ mission could be arriving in a target set T

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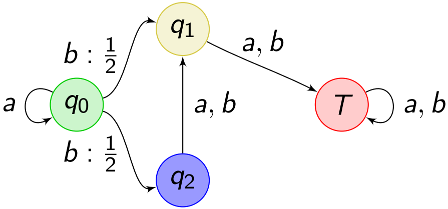

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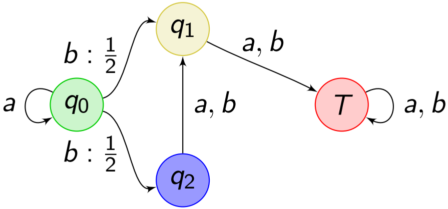

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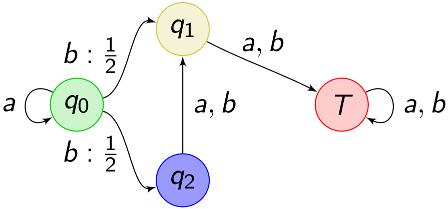

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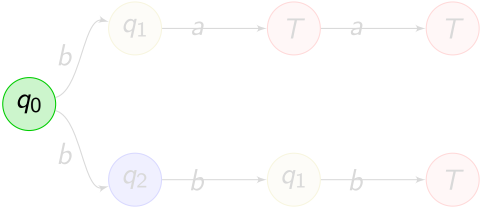

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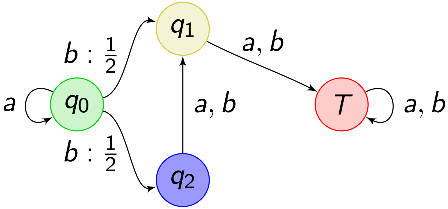

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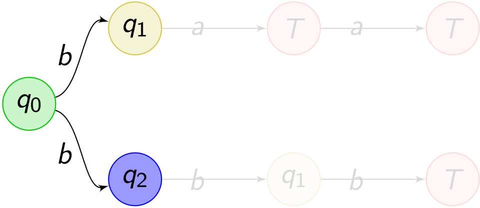

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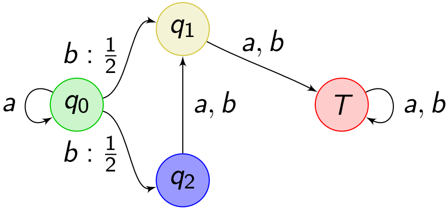

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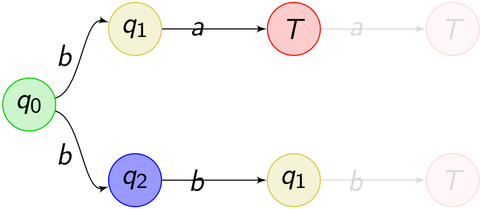

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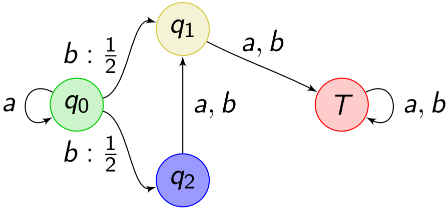

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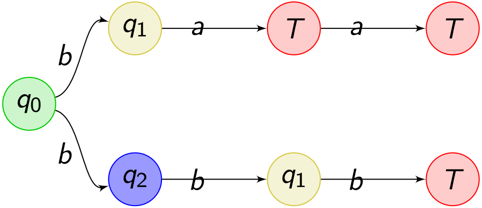

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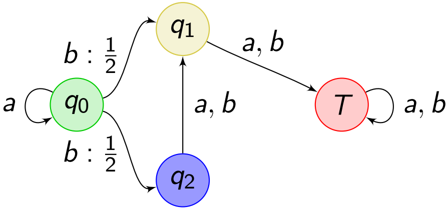

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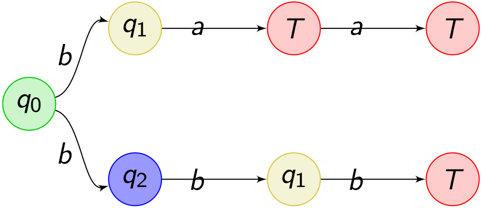

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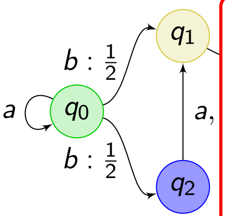

synchronization

The code for the spy robot  is a strategy

$$\alpha : Q^* \to A$$

MDP is synchronizing in T, under strategy α at the step 3:

$$Pr_{\alpha,3}(T) = 1$$

☺

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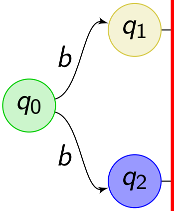

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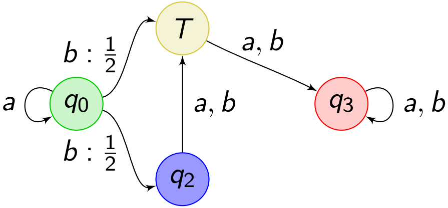
Synchronization vs. Reachability.

ϟ Different from classical reachability!

change the target set

☺ MDP can always reach T

☹ MDP is not synchronizing in T



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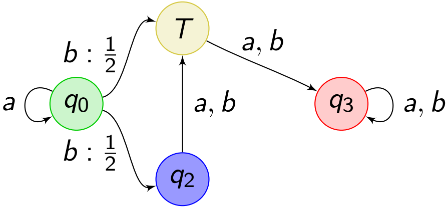

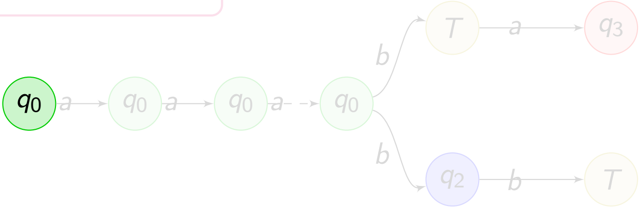

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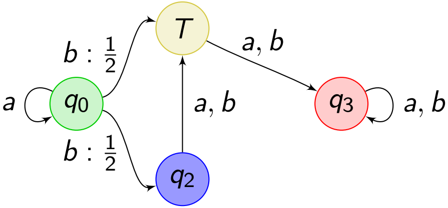

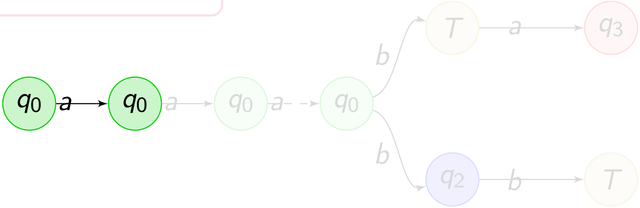

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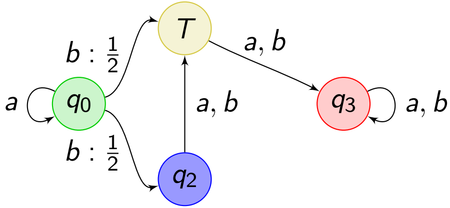

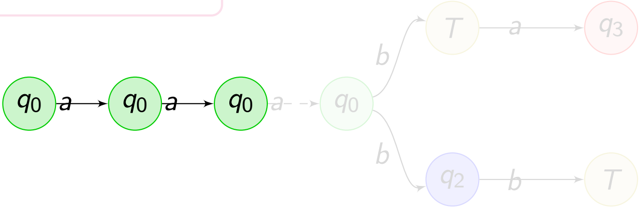

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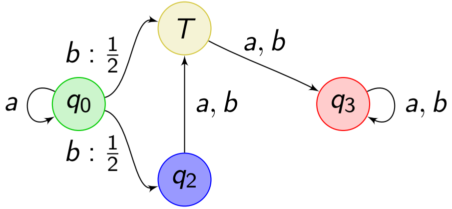

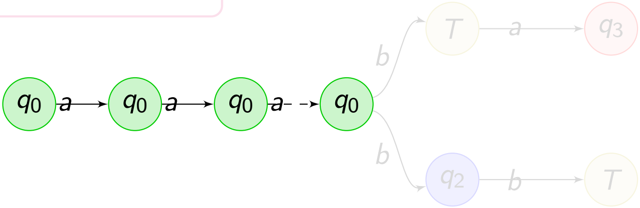

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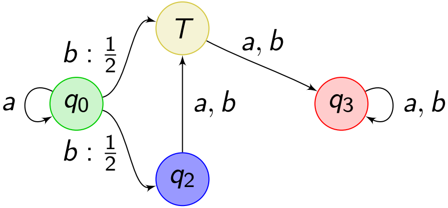

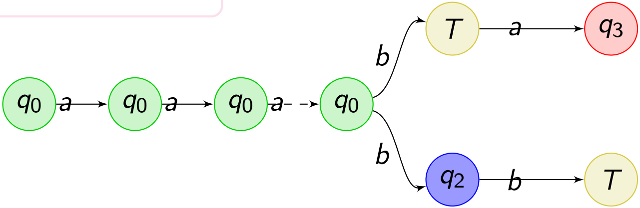

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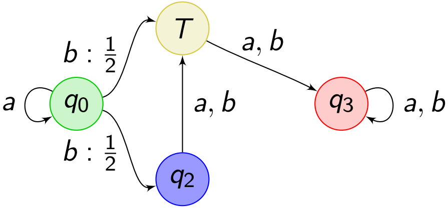

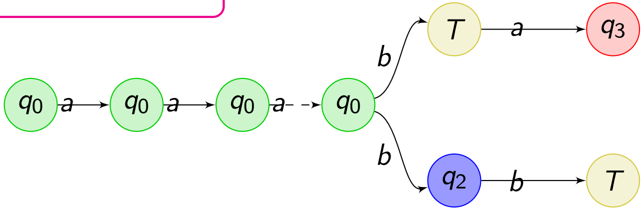

winning modes

For a target set T,

we say an MDP is synchronizing for the winning mode

	sure	almost-sure	limit-sure
Definition	there exists a strategy α and a certain step n $Pr_{\alpha,n}(T) = 1$	there exists a strategy α $\sup_n Pr_{\alpha,n}(T) = 1$	$\sup_{\alpha,n} Pr_{\alpha,n}(T) = 1$
Complexity	PSPACE-C	PSPACE-C	PSPACE-C
Memory	exponential	infinite	unbounded

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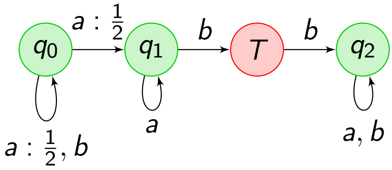
↯ Limit-sure synchronization requires unbounded memory!
(dependent on how close to 1: ϵ)

sync(T) :



Fix ϵ 1

Limit-sure synchronization

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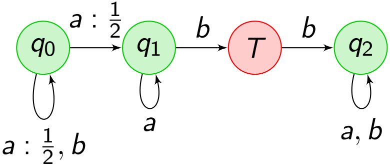

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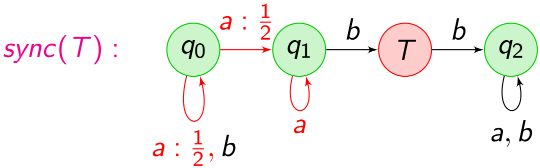

Fix ϵ	1	
playing a:	$\frac{1}{2}$	$\frac{1}{2}$
playing a:	$\frac{1}{4}$	$\frac{3}{4}$
playing a:	ϵ	$1-\epsilon$

too many times

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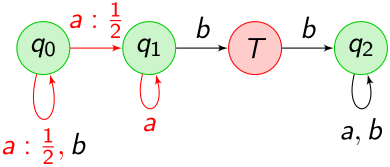

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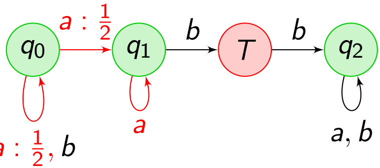

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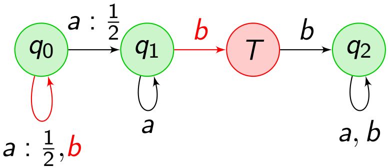

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too many times	playing a:	ϵ	$1-\epsilon$		
	playing b:	ϵ		$1-\epsilon$	

Almost-sure vs. Limit-sure synchronization

↯ Almost-sure and limit-sure synchronization are different!

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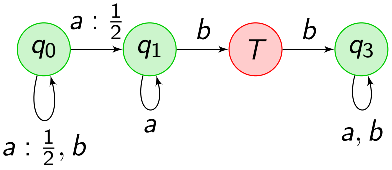

playing $a^* b$: ϵ $1 - \epsilon$

playing a or b: ϵ $\underbrace{1 - \epsilon}_{\text{trapped!}}$

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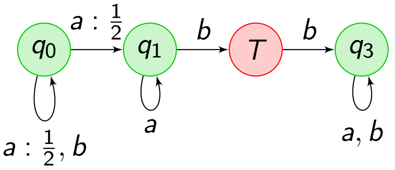

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Almost-sure synchronization

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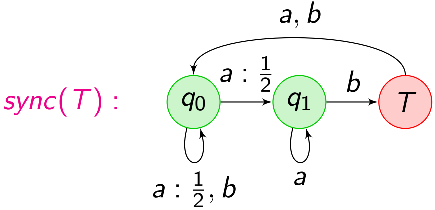

1

Fix ϵ (by $a^n b$): ϵ $1-\epsilon$

Playing b: 1 reset!!

Fix $\epsilon' < \epsilon$ ($a^{n+m} b$): ϵ' $1-\epsilon'$

Repeating this template forever => almost-sure

Almost-sure synchronization

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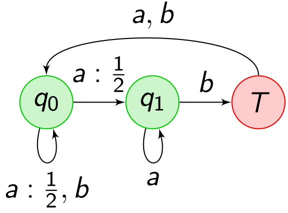

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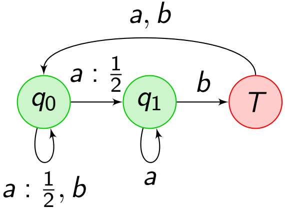

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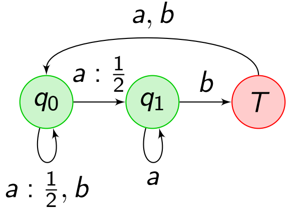

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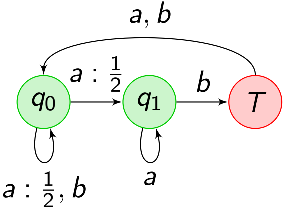

	q_0		T
	1		
Fix ϵ (by $a^n b$):	ϵ		$1 - \epsilon$
Playing b:	1	reset!!	
Fix $\epsilon' < \epsilon$ ($a^{n+m} b$):	ϵ'		$1 - \epsilon'$

Repeating this template forever => almost-sure

Contributions.

ϟ For a target set T, to decide whether an MDP is

synchronizing

for the winning mode

	sure	almost-sure	limit-sure
Complexity	PSPACE-C	PSPACE-C	PSPACE-C
Memory	exponential	infinite	unbounded

Questions! ☹ and Comments! ☺