Boundedness games

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Nathanaël Fijalkow Florian Horn Denis Kuperberg
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LIAFA, Université Paris 7 Denis Diderot – France

Highlights, September 19th, 2013

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This talk is about our *joint* effort to understand boundedness games.

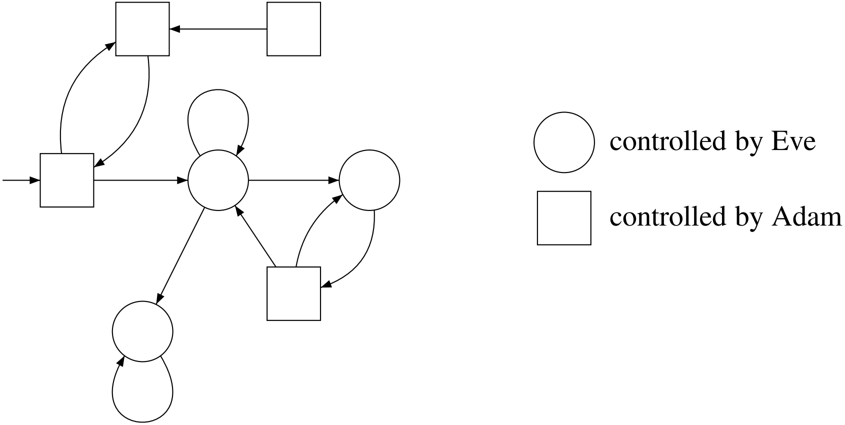
Motivation: expressing boundedness properties

MSO + $\mathbb{U}$

cost MSO

A lot is known, and even more is not known about those two logics!

Definition of boundedness games



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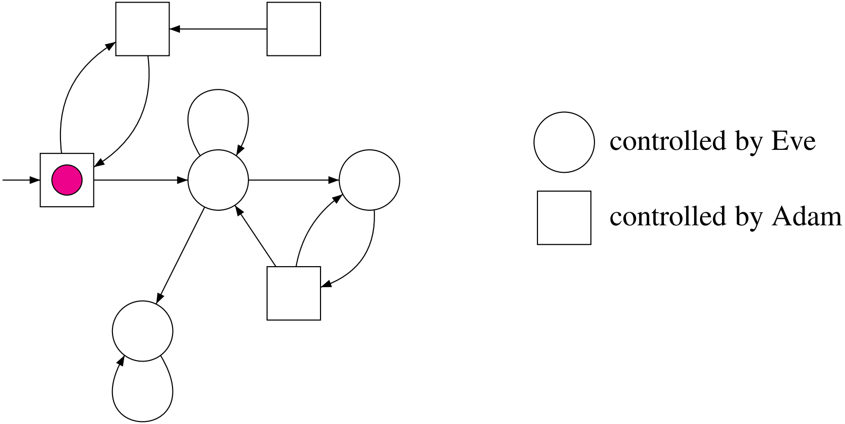

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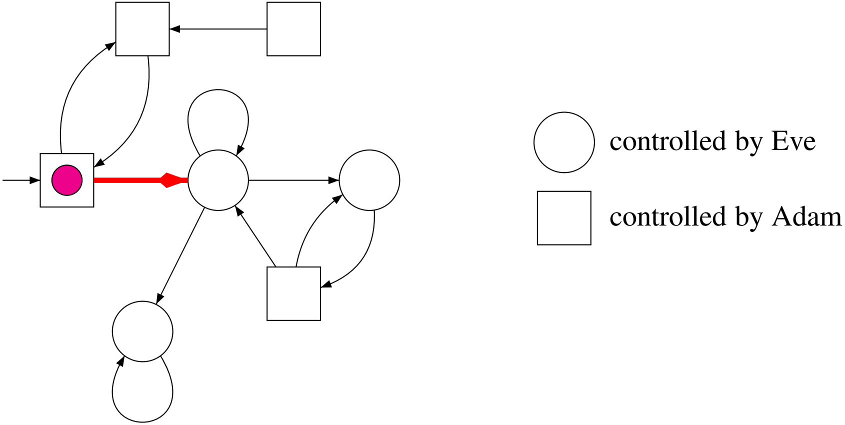

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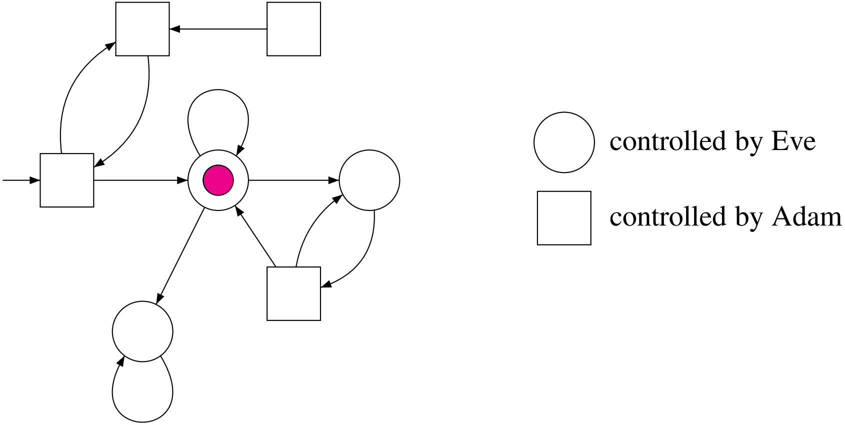

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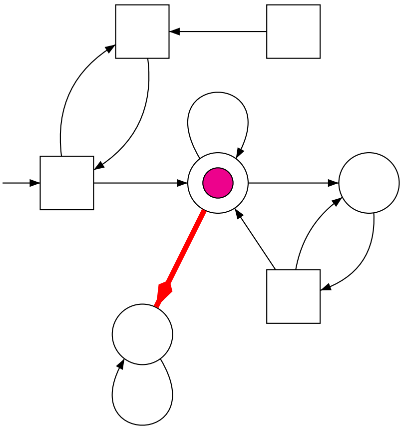

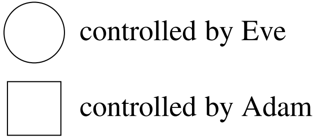

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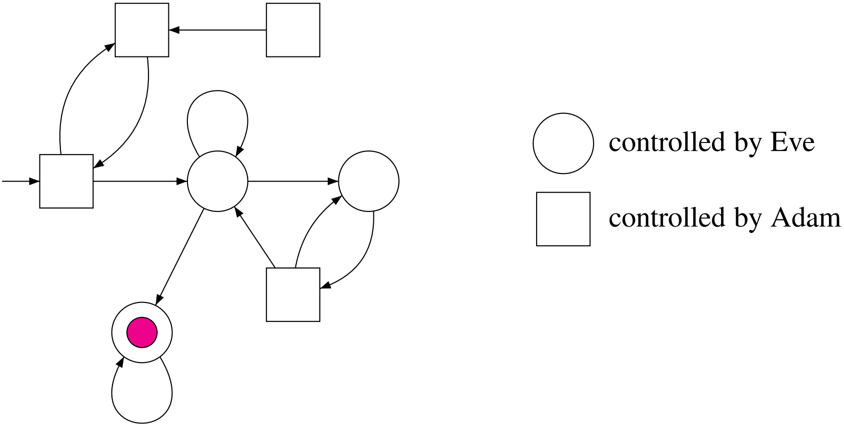
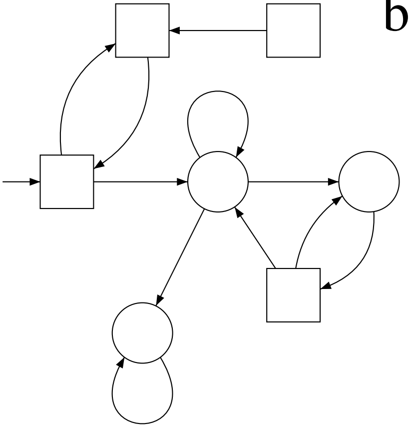
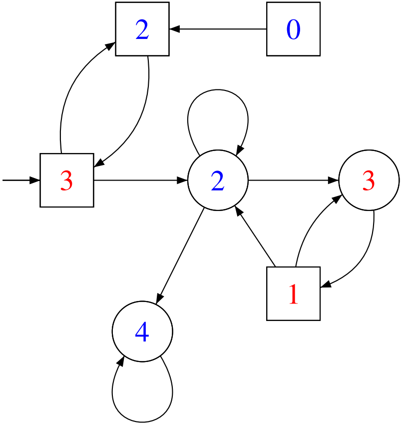




boundedness condition:

parity
and
all counters
are bounded

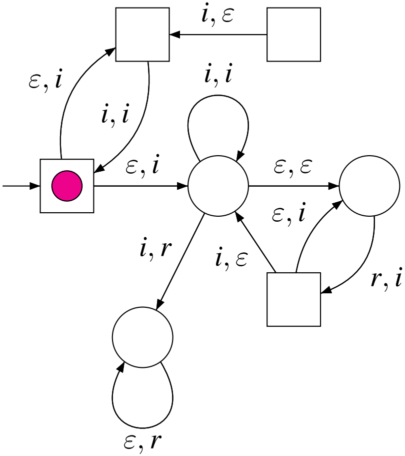
Definition of boundedness games



parity condition:

the minimal priority seen infinitely often is even

Definition of boundedness games



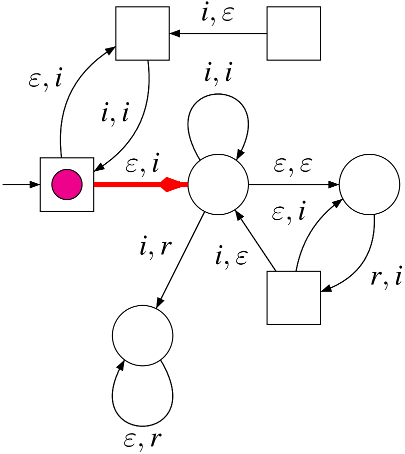
$$c_1 = 0$$
$$c_2 = 0$$

ε : nothing

i : increment

r : reset

Definition of boundedness games



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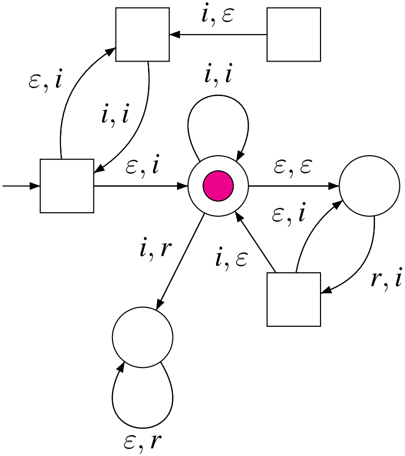

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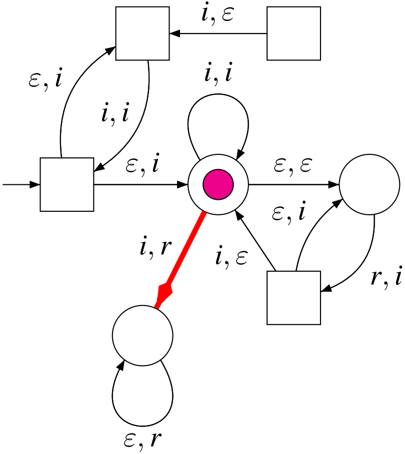

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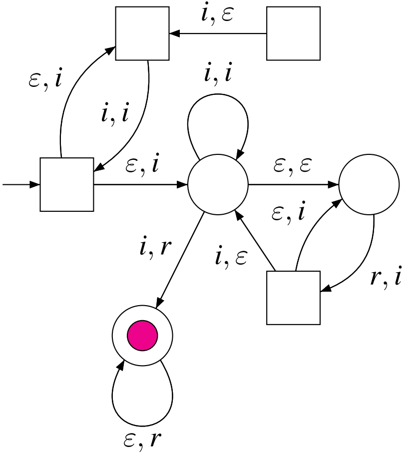

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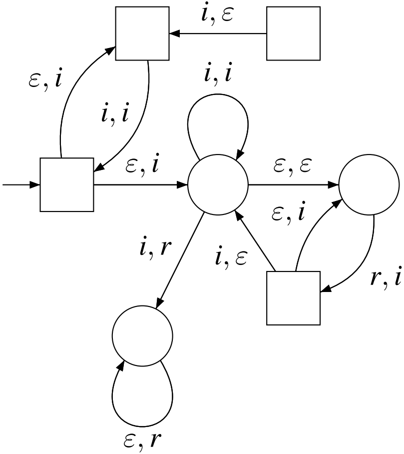

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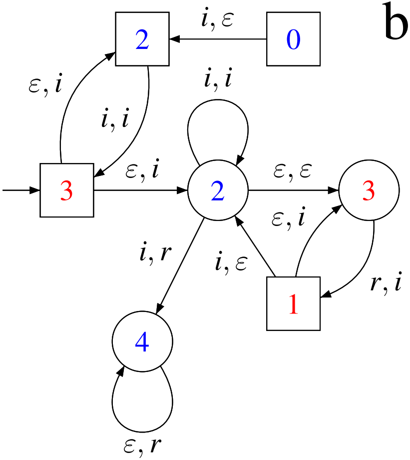

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Quantification

Eve wins means:

$\exists \sigma$ (strategy for Eve),
$\forall \pi$ (paths),
$\exists N \in \mathbb{N}$,

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non-uniform
$(\text{MSO} + \mathbb{U})$

uniform
(cost MSO)

Research questions and some answers

- When are the two quantifications equivalent?

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- When does Eve has finite-memory winning strategies?
  ↬ Uniform quantifications, the Büchi case over infinite chronological arenas [Vanden Boom, 2011].
  ↬ Uniform quantifications, the parity case over thin tree arenas [F., Horn, Kuperberg, Skrzypczak, unpublished].

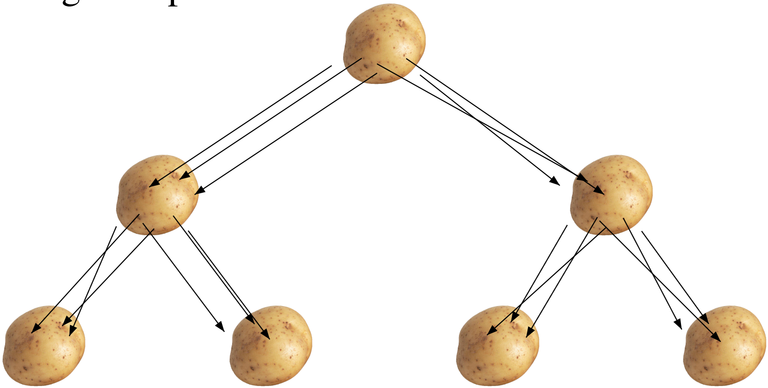
Why finite-memory strategies?

Thomas Colcombet's habilitation:

Conjecture 9.3. *Les objectifs* $\mathtt{hB} \wedge \mathtt{parité}$ *et* $\neg\mathtt{B} \wedge \mathtt{parité}$ *sont à* $\approx$*-mémoire finie, sur toutes les arènes/sur les arènes chronologiques/sur les arènes «arborescentes».*

Existence of finite-memory strategies in (some) boundedness games
$\Longrightarrow$ Decidability of cost MSO over infinite trees
$\Longrightarrow$ Decidability of the index of the non-deterministic Mostowski's hierarchy (open for 40 years)!

Working with potato trees



Theorem (F., Horn, Kuperberg, Skrzypczak)

The Colcombet's conjecture holds for thin tree arenas!

Corollary

The cost MSO logic over thin trees is decidable.