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Note

On the separability of sparse context-free languages and of bounded rational relations[☆]

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Abstract

This paper proves two results. (1) Given two bounded context-free languages, it is recursively decidable whether or not there exists a regular language which includes the first and is disjoint with the second and (2) given two rational k-ary bounded relations it is recursively decidable whether or not there exists a recognizable relation which includes the first and is disjoint with the second.

Keywords: Bounded languages; Context-free languages; Rational relations; Decisional problems

1. Introduction

In the most general terms, the problem we tackle can be stated as follows. Given two families $\mathcal{F}_0, \mathcal{F}_1$ of subsets of a given set E, is it possible, given two subsets X, Y in $\mathcal{F}_1$, to determine whether or not there exists a subset Z in $\mathcal{F}_0$ that *separates* them in the sense that $X \subseteq Z$ and $Y \cap Z = \emptyset$ holds? The problem is addressed in [2] where E is the direct product $A^* \times \mathbb{N}^k$ (where A^* is the free monoid generated by A and $\mathbb{N}$ is the additive monoid of nonnegative integers), $\mathcal{F}_1$ is the family $\mathrm{Rat}(A^* \times \mathbb{N}^k)$ of rational subsets of $A^* \times \mathbb{N}^k$ and $\mathcal{F}_0$ is the family $\mathrm{Rec}(A^* \times \mathbb{N}^k)$ of recognizable subsets of $A^* \times \mathbb{N}^k$.

Here we consider two cases for which we give a positive answer based on the results of [2]. In the first case $\mathcal{F}_1$ is the family of bounded context-free languages and $\mathcal{F}_0$ is the family of regular languages. In the second case $\mathcal{F}_1$ is the family of bounded rational subsets of a direct product of finitely generated free monoids and $\mathcal{F}_0$ is their family of recognizable relations.

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To our knowledge the general problem where $\mathcal{F}_1$ is the unrestricted family of context-free languages is open and does not seem to be easy to solve. Indeed, if we were to consider $\mathcal{F}_1$ to be the family of deterministic context-free languages which is closed under complement, the decidability of the separability problem would entail the decidability of the question of whether or not given a subset in $\mathcal{F}_1$ belongs to $\mathcal{F}_0$, which amounts to asking whether or not a deterministic context-free language is regular, a problem whose solution given by Stearns [13] and then improved by Valiant [14] is nontrivial.

2. Preliminaries

We assume that the reader is familiar with the basic notions of rational and recognizable subsets of an arbitrary monoid M, respectively denoted by $\mathrm{Rat}(M)$ and $\mathrm{Rec}(M)$ and with the notion of context-free languages. The reader is referred to the various textbooks on the topic [1,6,5,8,9]. In order to prevent any misunderstanding due to the not yet normalized use of these terms, we recall that a rational subset is expressed by a rational expression containing the operations of set union, set product and taking the submonoid generated, while a recognizable subset is a union of classes of a congruence of finite index on M. When M is the additive monoid $\mathbb{N}^k$, the family of rational subsets of $\mathbb{N}^k$ coincides with the family of *semilinear sets*, i.e., finite unions of *linear sets* (cf. [12]).

2.1. Basic definitions

The basic notion underlying this work is the following.

Definition 1. Let M be a monoid. Two subsets X and Y of M are said to be *separable* if there exists a recognizable set Z of M such that:

$$X \subseteq Z, \qquad Y \cap Z = \emptyset.$$

The subset Z *separates* X and Y.

Actually, the monoid that we are interested in is the free monoid. Given a finite *alphabet* Σ of *letters*, Σ^* denotes the free monoid that it generates. Its elements are called *words*.

The following theorem has been recently proven [2].

Theorem 1. *Let $M = \Sigma^* \times \mathbb{N}^k$ be the direct product of the monoids Σ^* and $\mathbb{N}^k$, where Σ is a finite nonempty alphabet and $\mathbb{N}$ is the additive monoid of nonnegative integers. Then it is decidable whether or not two rational sets of M are separable.*

2.2. Bounded languages

In Section 3 we deal with context-free languages. The idea is to apply Theorem 1 by ignoring the component Σ^* and to convert rational subsets of $\mathbb{N}^k$ into so-called k-bounded context-free languages of the free monoid. We are thus led to the following definition.

Definition 2. Let L be a language of a free monoid. For any positive integer k, L is called *k-bounded* if there exist nonempty words $u_1, \ldots, u_k$ such that

$$L \subseteq u_1^* \cdots u_k^*.$$

Moreover we say that L is *bounded* if there exists some integer $k \geq 1$ such that L is k-bounded.

We recall that bounded context-free languages are exactly the context-free languages for which the number of words belonging to the language and of a given length n is bounded by a polynomial in n [10,11]. These languages are thus also known as *sparse*. The counting function of sparse context-free languages and some related decision problems have been considered in [3,4].

Since the words $u_1, \ldots, u_k \in \Sigma^*$ in the previous definition are fixed in the rest of the paper except if otherwise stated, the following proves to be useful.

Definition 3. Define $\phi(x_1, \ldots, x_k) = u_1^{x_1} \cdots u_k^{x_k}$ for all $(x_1, \ldots, x_k) \in \mathbb{N}^k$. Next let $A = \{a_1, \ldots, a_k\}$ be a new alphabet of cardinality k. Consider the morphism defined by $h(a_i) = u_i$ for all $i = 1, \ldots, k$ and the mapping $\theta : \mathbb{N}^k \to A^*$ defined as $\theta(x_1, \ldots, x_k) = a_1^{x_1} \cdots a_k^{x_k}$. Then we have $\phi = h \circ \theta$.

The two main results on bounded languages used in this work are the following; see [8, Theorem 5.4.2] (actually a stronger result is proved) and [7, Theorem 1.2] respectively.

Theorem 2. *Let $L \subseteq \Sigma^*$ be a bounded context-free language. Then $\phi^{-1}(L)$ is a rational subset of $\mathbb{N}^k$.*

Theorem 3. *Let $L \subseteq \Sigma^*$ be a bounded language. Let us have $k \in \mathbb{N}$ and let $u_1, \ldots, u_k \in \Sigma^*$ such that $L \subseteq u_1^* \cdots u_k^*$. Then $L \in \mathrm{Rec}(\Sigma^*)$ if and only if $\phi^{-1}(L) \in \mathrm{Rec}(\mathbb{N}^k)$.*

This theorem requires the subset of $\mathbb{N}^k$ to be the inverse image of some subset in Σ^*. The next result, which is a consequence of the theorem, weakens the hypothesis.

Proposition 1. *Let $R \in \mathrm{Rec}(\mathbb{N}^k)$ and let $u_1, \ldots, u_k \in \Sigma^*$. Then $\phi(R) \in \mathrm{Rec}(\Sigma^*)$.*

Proof. We use the notation of Definition 3. Because $R = \theta^{-1}(\theta(R))$ holds, we have $\theta(R) \in \mathrm{Rec}(A^*)$ by the previous theorem. This yields $\phi(R) = h(\theta(R))$, which completes the proof. $\square$

2.3. Recognizable relations

Since the second result (Section 4) concerns relations of a direct product of free monoids, say $M = M_1 \times \cdots \times M_k$, we recall the characterization of recognizable relations of M in terms of the recognizable subsets of each component M_i (this result is attributed to Elgot and Mezei by Eilenberg in [6]).

Theorem 4. *A subset of $M_1 \times \cdots \times M_k$ is recognizable if and only if it is a finite union of subsets of the form $X_1 \times \cdots \times X_k$ where each X_i is a recognizable subset of M_i, for $i = 1, \ldots, k$.*

3. Separating bounded context-free languages

We now have all the ingredients to prove our main result concerning separability of bounded context-free languages.

Theorem 5. *It is decidable whether two context-free, bounded languages of the free monoid Σ^* are separable or not.*

Proof. Let L_1 and L_2 be two bounded context-free languages of Σ^*. Since the family of bounded languages is closed with respect to the operations of product and union of sets, we can always suppose that there exist words $u_1, \ldots, u_k \in \Sigma^+$ such that $L_1, L_2 \subseteq u_1^* \cdots u_k^*$. Let ϕ be the mapping defined by $\phi(x_1, \ldots, x_k) = u_1^{x_1} \cdots u_k^{x_k}$. We claim that L_1 and L_2 are separable if and only if so are $\phi^{-1}(L_1)$ and $\phi^{-1}(L_2)$ which are rational subsets of $\mathbb{N}^k$ by Theorem 2.

Indeed, if there exists a recognizable subset R of Σ^* satisfying $L_1 \subseteq R$ and $L_2 \cap R = \emptyset$, then by Theorem 3 the subset $\phi^{-1}(R)$ is recognizable in $\mathbb{N}^k$. Now, $L_1 \subseteq R$ implies $\phi^{-1}(L_1) \subseteq \phi^{-1}(R)$ and $L_2 \cap R = \emptyset$ implies $\phi^{-1}(L_2) \cap \phi^{-1}(R) = \phi^{-1}(L_2 \cap R) = \emptyset$.

Conversely, if $\phi^{-1}(L_1)$ and $\phi^{-1}(L_2)$ are separable by a recognizable subset $R \subseteq \mathbb{N}^k$, then by the previous proposition we have $\phi(R) \in \mathrm{Rec}(\Sigma^*)$. Furthermore, $\phi^{-1}(L_1) \subseteq R$ implies $L_1 = \phi(\phi^{-1}(L_1)) \subseteq \phi(R)$. Finally, if $L_2 \cap \phi(R) = \phi(\phi^{-1}(L_2)) \cap \phi(R) \neq \emptyset$ then there exists an element $x \in R$ which maps into L_2, implying $x \in \phi^{-1}(L_2)$, a contradiction.

The reduction to the result in [2] goes as follows. Let L_1 and L_2 be two bounded context-free languages. By a result of S. Ginsburg [8, Theorem 5.5.2], one can effectively compute nonempty words $v_1, \ldots, v_p, w_1, \ldots w_r$, such that $L_1 \subseteq v_1^* \cdots v_p^*$ and $L_2 \subseteq w_1^* \cdots w_r^*$. Let $k = p + r$ and define

$$u_i = \begin{cases} v_i & \text{for} \quad i = 1, \ldots, p, \\ w_{i-p} & \text{for} \quad i = p+1, \ldots, k. \end{cases}$$

The languages L_1 and L_2 may be viewed as bounded languages in $u_1^* \cdots u_k^*$. We now use the notation of Definition 3. Consider the Parikh map $\psi : A^* \to \mathbb{N}^k$ which assigns to each $u \in A^*$ the k-tuple $(|u|_{a_1}, \ldots, |u|_{a_k})$ of number of occurrences of each letter of A in u. Obviously $\phi^{-1}(L_1) = \psi(h^{-1}(L_1) \cap a_1^* \cdots a_k^*)$ and $\phi^{-1}(L_2) = \psi(h^{-1}(L_2) \cap a_1^* \cdots a_k^*)$. Since the languages $h^{-1}(L_1) \cap a_1^* \cdots a_k^*$ and $h^{-1}(L_2) \cap a_1^* \cdots a_k^*$ are context-free languages, we may resort to the well known Parikh theorem, which implies that the sets $\phi^{-1}(L_1)$ and $\phi^{-1}(L_2)$ are effective semilinear subsets of $\mathbb{N}^k$. Then apply the decision procedure to $\phi^{-1}(L_1)$ and $\phi^{-1}(L_2)$. $\square$

Lemma 1. *Let $\mathcal{F}$ be a family of subsets of Σ^* closed under intersection with the recognizable subsets. Let $L_1, L_2 \in \mathcal{F}$ and assume $L_1 \subseteq R$ for some recognizable subset R. Then L_1 and L_2 are separable if and only if there exists a recognizable subset $S \subseteq R$ separating L_1 and $L_2 \cap R$.*

Proof. The condition is sufficient since if it holds then we have $L_1 \subseteq S$ and $L_2 \cap S = (L_2 \cap R) \cap S = \emptyset$. It is necessary since if $L_1 \subseteq S$ and $L_2 \cap S = \emptyset$ holds, then $L_1 \subseteq S \cap R$ and $(L_2 \cap R) \cap (S \cap R) = L_2 \cap (S \cap R) = \emptyset$ holds. $\square$

As a consequence, we have

Corollary 1. *Let L_1, L_2 be context-free languages of Σ^* and assume that L_1 is bounded. Then it is decidable whether L_1 and L_2 are separable or not.*

4. Separating bounded rational relations

In this last section we consider finite direct products of finitely generated free monoids, i.e., $A_1^* \times \cdots \times A_k^*$. It is well known that the family of recognizable subsets is strictly included in the family of rational subsets whenever at least two alphabets are non-empty. The problem posed in the introduction therefore makes sense in this setting. Here also, we show how the decidability is a consequence of the result in [2].

The following is a formal definition of bounded relations.

Definition 4. A relation $R \subseteq A_1^* \times \cdots \times A_k^*$ is *bounded* if there exist n_1 words $u_{1,1} \cdots u_{1,n_1} \in A_1^*$, etc ..., n_k words $u_{k,1} \cdots u_{k,n_k} \in A_k^*$ such that $R \subseteq u_{1,1}^* \cdots u_{1,n_1}^* \times \cdots \times u_{k,1}^* \cdots u_{k,n_k}^*$. Define the mapping $\phi : \mathbb{N}^{n_1+\cdots+n_k} \to A_1^* \times \cdots \times A_k^*$ as

$$\phi(x_{1,1}, \ldots, x_{1,n_1}, \ldots, x_{k,1}, \ldots, x_{k,n_k}) = (u_{1,1}^{x_{1,1}} \cdots u_{1,n_1}^{x_{1,n_1}}, \ldots, u_{k,1}^{x_{k,1}} \cdots u_{k,n_k}^{x_{k,n_k}}).$$

The restriction of ϕ to $\mathbb{N}^{n_i}$ is denoted by ϕ_i.

4.1. Closure properties of rational and recognizable subsets

Given two monoids M and N and a morphism $h : M \to N$, it is well known that the image under h of a rational subset of M is a rational subset of N and that the inverse image of a recognizable subset of N is a recognizable subset of M. Loosely speaking, this means that the family of rational subsets is closed under direct morphism and that the family of recognizable subsets is closed under inverse morphism: $h(\mathrm{Rat}(M)) \subseteq \mathrm{Rat}(N)$ and $h^{-1}(\mathrm{Rec}(N)) \subseteq \mathrm{Rec}(M)$. The inclusions $h(\mathrm{Rec}(M)) \subseteq \mathrm{Rec}(N)$ and $h^{-1}(\mathrm{Rat}(N)) \subseteq \mathrm{Rat}(M)$ do not hold in general. Here we show that they do hold under specific conditions on the monoids and the morphisms. Indeed, consider two direct products of free monoids $M = B_1^* \times \cdots \times B_k^*$ and $N = A_1^* \times \cdots \times A_k^*$ and morphisms $h : M \to N$ defined as follows. Let $h_i : B_i^* \to A_i^*$ be a morphism for $i = 1, \ldots, k$ and define $h(w_1, \ldots, w_k) = (h_1(w_1), \ldots, h_k(w_k))$.

Proposition 2. *With the morphism defined as previously we have: If $R \in \mathrm{Rec}(M)$ then $h(R) \in \mathrm{Rec}(N)$. If $R \in \mathrm{Rat}(N)$ then $h^{-1}(R) \in \mathrm{Rat}(M)$.*

Proof. We show that if $R \in \mathrm{Rat}(A_1^* \times \cdots \times A_k^*)$ then $h^{-1}(R) \in \mathrm{Rat}(B_1^* \times \cdots \times B_k^*)$. By composition we may assume that the morphism leaves unchanged all components except one, e.g., that $h(u_1, u_2, \ldots, u_k) = (h_1(u_1), u_2, \ldots, u_k)$ holds.

Let $\mathcal{A}$ be a k-tape automaton which accepts a (rational) relation $R \subseteq A_1^* \times \cdots \times A_k^*$. We may assume that the transitions of $\mathcal{A}$ are of the kind $(q, (x_1, x_2, \ldots, x_k), p)$, where for any i, $x_i \in A_i \cup \varepsilon$, and there exists at most one j such that $x_j \neq \varepsilon$. The k-tape automaton $\mathcal{B}$ which accepts the inverse image of R under the morphism h is defined as follows. The set $Q_{\mathcal{B}}$ of the states of $\mathcal{B}$ contains the set $Q_{\mathcal{A}}$ and new states of the kind $[q, u]$, where q is a state of $Q_{\mathcal{A}}$ and u is a nonempty suffix of some word of $h_1(B_1)$. Any transition of $\mathcal{A}$ of the form $(q, (\varepsilon, x_2, \ldots, x_k), p)$ is a transition of $\mathcal{B}$ as well as the transition $(q, (y, x_2, \ldots, x_k), p)$ if $h_1(y) = \varepsilon$. Furthermore, it yields the new transitions $([q, u], (\varepsilon, x_2, \ldots, x_k), [p, u])$.

For any $y \in B_1$ with $h_1(y) \neq \varepsilon$ and $q \in Q_{\mathcal{A}}$, we add to $\mathcal{B}$ the transition

$$(q, (y, \varepsilon, \ldots, \varepsilon), [q, h_1(y)]).$$

Finally, for any transition of $\mathcal{A}$ of the form $(q, (a_1, \varepsilon, \ldots, \varepsilon), p)$ we add the following ε-transitions to $\mathcal{B}$:

$$([q, a_1 \ldots a_n], (\varepsilon, \varepsilon, \ldots, \varepsilon), [p, a_2 \ldots a_n]),$$

with $n \geq 2$, and

$$([q, a_1], (\varepsilon, \varepsilon, \ldots, \varepsilon), p).$$

The initial state and the final states of $\mathcal{B}$ are the same as those of $\mathcal{A}$. It is not difficult to see that the k-tape automaton $\mathcal{B}$ accepts the set $h^{-1}(R)$.

We now show that if $R \in \mathrm{Rec}(B_1^* \times \cdots \times B_k^*)$ then $h(R) \in \mathrm{Rec}(A_1^* \times \cdots \times A_k^*)$. By the characterization of Elgot and Mezei, R is a finite union of direct products $X_1 \times \cdots \times X_k$ where for $i = 1, \ldots, k$, X_i is a recognizable set of B_i^*. It clearly suffices to consider the case where R is reduced to this product. But then we obtain $h(R) = h_1(X_1) \times \cdots \times h_k(X_k)$ which is recognizable. □

Proposition 3. *Let* $R \subseteq u_{1,1}^* \cdots u_{1,n_1}^* \times \cdots \times u_{k,1}^* \cdots u_{k,n_k}^*$.

(1) *If R is rational then the set $\phi^{-1}(R)$ is rational.*
(2) *If $S \subseteq \mathbb{N}^{n_1+\cdots+n_k}$ is recognizable then $\phi(S)$ is recognizable.*
(3) *R is recognizable if and only if $\phi^{-1}(R)$ is recognizable.*

Proof. Claim 1. Consider for all $i = 1, \ldots, k$ the alphabets $B_i = \{a_{i,1}, \ldots, a_{i,n_i}\}$ of new symbols, the morphisms $h_i : B_i^* \to A_i^*$ defined by $h_i(a_{i,j}) = u_{i,j}$ and the Parikh mappings $g_i : B_i^* \to \mathbb{N}^{n_i}$. Set $g(w_1, \ldots, w_k) = (g_1(w_1), \ldots, g_k(w_k))$. Then we have

$$\phi^{-1}(R) = g\left(h^{-1}(R) \cap a_{1,1}^* \cdots a_{1,n_1}^* \times \cdots \times a_{k,1}^* \cdots a_{k,n_k}^*\right).$$

The claim is a consequence of the previous proposition and the general closure properties of rational subsets.

Claim 2. If S is recognizable then, by the characterization of Elgot and Mezei, it is a finite union of direct products $X_1 \times \cdots \times X_k$, where X_i is a recognizable set of $\mathbb{N}^{n_i}$, for $i = 1, \ldots, k$. Then, $\phi(S)$ is a finite union of direct products $\phi_1(X_1) \times \cdots \times \phi_k(X_k)$. By Proposition 1 each subset $\phi_i(X_i)$ is recognizable in A_i^*. This completes the proof.

Claim 3. If R is recognizable then, by the characterization of Elgot and Mezei, it is a finite union of direct products $Z = X_1 \times \cdots \times X_k$, where for $i = 1, \ldots, k$, X_i is a recognizable set of A_i^* included in $u_{i,1}^* \cdots u_{i,n_i}^*$. Then, the subset $\phi_i^{-1}(X_i)$ is a recognizable subset of $\mathbb{N}^{n_i}$ by Theorem 3. Therefore, since $\phi^{-1}(Z) = \phi_1^{-1}(X_1) \times \cdots \times \phi_k^{-1}(X_k)$, then $\phi^{-1}(Z)$ is recognizable.

Conversely, if $\phi^{-1}(R)$ is recognizable in $\mathbb{N}^{n_1+\cdots+n_k}$, then by claim 2 we have $R = \phi(\phi^{-1}(R))$ is recognizable in $A_1^* \times \cdots \times A_k^*$. □

We come to the main result of this section.

Theorem 6. *Given two bounded rational subsets of a direct product of free monoids, it is recursively decidable whether or not they are separable.*

Proof. The proof follows the same pattern as that for bounded context-free languages. The only point which requires some care concerns the effectiveness of the computation of the various words $u_{i,j}$. In the monoid which is a direct product of free monoids, it is recursively decidable whether or not a rational set is contained in a recognizable set [1] since this reduces to the emptiness problem for rational sets. Therefore, for fixed words $u_{1,1} \cdots u_{1,n_1} \in A_1^*$, etc ..., $u_{k,1} \cdots u_{k,n_k} \in A_k^*$, given a rational relation R, one can check the inclusion

$$R \subseteq u_{1,1}^* \cdots u_{1,n_1}^* \times \cdots \times u_{k,1}^* \cdots u_{k,n_k}^*.$$

Since we know that these words exist, an exhaustive search can find them. □

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