

Reduction QBF  $\rightarrow$  zeroness of a polyrec sequence

$\exists x_1 \forall x_2 \exists x_3 \dots \forall x_k \varphi(x_1, \dots, x_k) \rightsquigarrow$  set of sequences  $\text{poly}(k)$

All sequences are  $\{0, 2\}$ -sequences

Counters  $c^1 = 010101 \dots$

$c^2 = 00110011 \dots$

$c^3 = 00001111 \dots$

$\vdots$

$c^k = \dots$

$$c_n^1 = 1 - c_{n-1}^1$$

$$c_n^2 = \begin{cases} 1 - c_{n-1}^2 & \text{if } c_{n-1}^1 = 1 \text{ and } c_n^1 = 0 \\ c_{n-1}^2 & \text{otherwise} \end{cases}$$

$$c_n^2 = (1 - c_{n-1}^2) \cdot c_{n-1}^1 \cdot (1 - c_n^1) + c_{n-1}^2 \cdot (1 - (1 - c_{n-1}^1) c_n^1)$$

$c^3 \rightsquigarrow c^2$  and  $c^2 \rightarrow 1 \rightsquigarrow c^1, c^1 \rightarrow 1, c^1 \rightarrow 2$

$c^4 \rightsquigarrow c^3, c^3 \rightarrow 1, c^3 \rightarrow 2, c^3 \rightarrow 3$

$\dots$  We defined  $c^1, c^2, \dots, c^k$  using  $\text{poly}(k)$  sequences

$x_k \rightsquigarrow c^k$   
 $x_{k-1} \rightsquigarrow c^{k-1}$   
 $\vdots$   
 $010101$

$$d_n^1 := \begin{cases} 1 & \text{iff } (c_n^1, c_n^2, \dots, c_n^k) \models \varphi \\ 0 & \text{otherwise} \end{cases}$$

$\uparrow$  polynomial  $\wedge \rightarrow \cdot$  roughly  $\rightarrow$   
 $\vee \rightarrow +$

$d_n^{k-1} \rightsquigarrow d_n^k$

$d_n^k$

1 iff  $\exists x_k \forall x_{k-1} \dots \forall x_1 \varphi$

1. iff  $\forall x_{k-1} \exists \dots \varphi$   
 is true for  $x_k = 0$

multiplication of  $2^{k-1}$

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0000000000 0000000000 0000000000 0000000000

$c = a \vee b$

1 iff ... for  $x_k = 1$

$d_n^{k-1}$

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$f_n^{k-1}$

$$f_n^{k-1} = \begin{cases} \text{if } c_n^{k-1} \text{ shifts from 0 to 1 then store } d_n^{k-1} \\ \text{if } c_n^{k-1} \text{ shifts from 1 to 0 then reset} \end{cases}$$

otherwise copy.

$$d_n^k = \begin{cases} d_n^{k-1} \vee f_n^{k-1} & \text{if } c_n^k \text{ changes value} \\ 0 & \text{otherwise} \end{cases}$$