

# Unambiguity in Automata Theory

Edited by

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## Abstract

This report documents the program and the outcomes of Dagstuhl Seminar 21452 “Unambiguity in Automata Theory”. The aim of the seminar was to improve the understanding of the notion of unambiguity in automata theory, especially with respect to questions related to the expressive power, succinctness, and the tractability of unambiguous devices. The main motivation behind these studies is the hope that unambiguous machines can provide a golden balance between efficiency – sometimes not worse than for deterministic devices – and expressibility / succinctness, which often is similar to the general nondeterministic machines. These trade-offs become especially important in the models where the expressiveness or the decidability status of unambiguous machines is different from that of nondeterministic ones, as it is the case, e.g., for register automata.

**Seminar** November 07-12, 2021 – <https://www.dagstuhl.de/21452>

**2012 ACM Subject Classification** Theory of computation → Regular languages; Theory of computation → Automata over infinite objects; Theory of computation → Algebraic language theory; Theory of computation → Tree languages

**Keywords and phrases** Unambiguity in Automata Theory, Dagstuhl Seminar

**Digital Object Identifier** 10.4230/DagRep.11.10.57

## 1 Executive Summary

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The Dagstuhl Seminar 21452 “Unambiguity in Automata Theory” was a seminar of five days that took place from November 7th to 12th, 2021, organized by Thomas Colcombet, Karin Quaas, and Michał Skrzypczak. A general goal of the seminar was to bring together experts from different fields of automata theory, to stimulate an exchange of recent results and new proof techniques concerning unambiguity and related topics from automata theory. There were 26 on-site participants from nine different countries (Belgium, Czech Republic, France, Germany, India, Italy, Poland, UK), and further 10 remote participants from seven countries (France, Germany, Poland, Sweden, Switzerland, UK, USA).

The central topic of the seminar was *unambiguous automata*. An automaton is unambiguous if it can make nondeterministic choices, but it is guaranteed that for every input there is *at most one accepting run*. There have recently been numerous new results concerning



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Unambiguity in Automata Theory, *Dagstuhl Reports*, Vol. 11, Issue 10, pp. 57–71  
Editors: Thomas Colcombet and Karin Quaas and Michał Skrzypczak



Dagstuhl Reports  
Schloss Dagstuhl – Leibniz-Zentrum für Informatik, Dagstuhl Publishing, Germany

unambiguous automata; at the same time, a lot of natural and interesting problems have been open for decades. Before the seminar, we identified the following key topics/open problems:

- **Unambiguous Finite Automata** What is the state complexity of the complementation of unambiguous automata? Here, the state complexity refers to how the number of states of the resulting automaton depends on the number of states of the original automaton.
- **Unambiguous versions of infinite state systems, such as vector addition systems with states (VASS) or register automata** Open problems concerning such systems are, for instance: What can be new techniques for proving lower bounds for the containment problem? Are languages accepted by unambiguous register automata with guessing closed under complement?
- **Unambiguous tree automata** One of the most important open questions is how to decide whether a given tree-regular language is recognizable by an unambiguous automaton.
- **Büchi automata and probabilistic automata** What is the computational complexity of the containment of unambiguous Büchi automata?
- **Tropical automata** For this class of weighted automata one of the most important and long standing open questions is whether a given series is polynomially ambiguous.

The seminar was planned to consist of talks and working group sessions, where participants could work on-site on open problems. In order to integrate all participants and to initiate new collaborations, we started the seminar on Monday with introductory talks, where every participant shortly introduced herself to the group. In these introductory sessions, it was also possible to announce open problems the participants were interested to work on during the seminar. We had additionally collected such open problems before the seminar to make them available to the participants in advance.

The second day of the seminar (Tuesday) was dedicated to presentations given by the participants. This day started with an invited talk by Denis Kuperberg on good-for-games automata. Later the day, eight participants of the seminar presented short contributed talks on topics related to unambiguity.

Wednesday began with the invited talk by Gabriele Puppis on register automata. Later, a single contributed talk was given and the whole afternoon was devoted to an excursion and group work.

On Thursday morning, Wojtek Czerwiński gave an invited talk on future-determinisation. After that, four contributed talks were given, and the late afternoon was devoted to work in subgroups.

Finally, on Friday morning we held a closing ceremony. The rest of the day was left to participants to summarise their discussions in subgroups and prepare for departure.

During all days, we have used Schloss Dagstuhl's excellent technical facilities to connect and communicate to remote participants of the seminar. Our experiences regarding such a hybrid Dagstuhl seminar are twofold. On the one hand, it is practical to give remote participants the opportunity to follow the on-site presentations (and Sylvain Lombardi also gave a remote talk). On the other hand, our main aim was to bring together researchers to actually work on concrete problems. It was difficult to integrate participants in group work, when groups gather at different places in the facilities, or when important discussions are led during the excursion or the dinner. We appreciated very much the opportunity to gather on-site at Schloss Dagstuhl after a long time of only non-physical meetings due to the Covid pandemics. As summarized in Session 4, several new collaborations between participants of the seminar have been initiated. We hope that the seminar has inspired new ideas, and interesting new results will be published by the participants.

We would like to warmly thank Schloss Dagstuhl for making this seminar possible. We especially would like to thank for the great help and support in the organization before and during the seminar.

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### 3 Overview of Talks

#### 3.1 Regular Tree Algebras

*Achim Blumensath (Masaryk University – Brno, CZ)*


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Joint work of Mikołaj Bojańczyk, Thomas Colcombet, Bartek Klin

We introduce a class of algebras that can be used as recognisers for regular tree languages. We show that it is the only such class that forms a pseudo-variety and we prove the existence of syntactic algebras.

#### 3.2 Between Deterministic and Nondeterministic Quantitative Automata

*Udi Boker (Reichman University – Herzliya, IL)*

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Joint work of Udi Boker, Karoliina Lehtinen

There is a challenging trade-off between deterministic and nondeterministic automata, where the former suit various applications better, however at the cost of being exponentially larger or even less expressive.

This gave birth to many notions in between determinism and nondeterminism, aiming at enjoying, sometimes, the best of both worlds. Some of the notions are yes/no ones, for example initial nondeterminism (restricting nondeterminism to allowing several initial states), and some provide a measure of nondeterminism, for example the ambiguity level.

We analyze the possible generalization of such notions from Boolean to quantitative automata, and suggest that it depends on the following key characteristics of the considered notion  $N$  – whether it is syntactic or semantic, and if semantic, whether it is word-based or language-based.

A syntactic notion, such as initial nondeterminism, applies as is to a quantitative automaton  $A$ , namely  $N(A)$ . A word-based semantic notion, such as unambiguity, applies as is to a Boolean automaton  $t - A$  that is derived from  $A$  by accompanying it with some threshold value  $t$ , namely  $N(t - A)$ . A language-based notion, such as history determinism, also applies as is to  $A$ , while in addition, it naturally generalizes into two different notions with respect to  $A$  itself, by either: i) taking the supremum of  $N(t - A)$  over all thresholds  $t$ , denoted by  $Th - N(A)$ ; or ii) generalizing the basis of the notion from a language to a function, denoted simply by  $N(A)$ . While in general  $N(A)$  implies  $Th - N(A)$  implies  $N(t - A)$ , we have for some notions that  $N(A)$  and  $Th - N(A)$  are equivalent and for some not. (For measure notions, “implies” stands for  $\leq$  with respect to the nondeterminism level.)

We classify numerous notions known in the Boolean setting according to their characterization above, generalize them to the quantitative setting and look into relations between them. The generalized notions open new research directions with respect to quantitative automata, and provide insights on the original notions with respect to Boolean automata.

### 3.3 Unambiguous automata acceptance?

*Dmitri Chistikov (University of Warwick – Coventry, GB)*

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Given a nondeterministic finite automaton (NFA) with  $m$  transitions and an input word of length  $\ell$ , one can decide in time  $O(m\ell)$  if the word is accepted. If  $m \approx n^2$  (where  $n$  is the number of states) and  $\ell \approx n$ , this running time is essentially cubic in  $n$ . I don't know if significantly faster algorithms exist for this and several related problems. Can we obtain speed-ups if the automaton is known to be unambiguous?

### 3.4 Computational complexity of universality and related problems for unambiguous context-free grammars

*Lorenzo Clemente (University of Warsaw, PL)*

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In this talk I recall a classic approach to decide universality of unambiguous context-free grammars. It originates in the work of Chomsky and Schutzenberger, who showed that the (commutative) power series of an unambiguous grammar is algebraic. Based on this fact, one can reduce in PTIME the universality problem to the zeroness problem for a related algebraic power series, and in turn the latter problem can be shown to be PTIME reducible to the existential fragment of the first-order theory of the reals. Since the last problem is in PSPACE by the result of Canny, it follows that universality of unambiguous grammars is in PSPACE. Whether the latter problem actually belongs to a lower complexity class is advertised as an open problem.

### 3.5 On Future-Determinization of Unambiguous Systems (Invited Talk)


*Wojciech Czerwiński (University of Warsaw, PL)*

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Joint work of Wojciech Czerwiński, Piotr Hofman

I will present you a result based on an on-going work jointed with Piotr Hofman. We have shown that language equivalence is decidable for unambiguous vector addition systems with states (VASS) (acceptance is by state). I'd like to focus more on our technique: we have proven that each unambiguous VASS can be determinized in a certain sense (with a use of some additional information about the future), which we call future-determinization. This result makes use of some known regular-separability results. There is a hope that similar techniques can be possible for other unambiguous systems and maybe even point to some high-level connection between separability and unambiguity notions.

### 3.6 Alternation as a tool for disambiguation

*Simon Jantsch (TU Dresden, DE)*

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**Joint work of** Simon Jantsch, David Müller, Christel Baier, Joachim Klein, Yufei Liu

In this talk we show how alternating automata can be used as a tool to devise disambiguation algorithms for nondeterministic automata over finite and infinite words. The main idea is to use conjunction and complementation, both of which can be naturally implemented in alternating automata, to restrict nondeterministic branching in a way that preserves the language and makes sure that for any given word only one choice leads to acceptance. A notion of unambiguity for alternating automata is introduced, and we show that standard alternation removal techniques preserve it. The approach works well for automata on finite words and restricted forms of automata (namely very weak ones) but we show that it fails for arbitrary nondeterministic Büchi automata (NBA), and discuss the issues that arise. Finally, we speculate about the relationship between complementation and disambiguation and possible consequences for the state complexity of disambiguating NBA.

### 3.7 Good-for-Games Automata: State of the Art and Perspectives (Invited Talk)

*Denis Kuperberg (ENS – Lyon, FR)*


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**Joint work of** Denis Kuperberg, Marc Bagnol, Udi Boker, Emile Hazard, Michał Skrzypczak

In the setting of regular languages of infinite words, Good-for-Games (GFG) automata can be seen as an intermediate formalism between determinism and nondeterminism, with advantages from both worlds. Indeed, like deterministic automata, GFG automata enjoy good compositional properties (useful for solving games and composing automata and trees) and easy inclusion checks. Like nondeterministic automata, they can be exponentially more succinct than deterministic automata. Since their introduction in 2006 by Henzinger and Piterman, there has been a steady research effort to uncover the properties of GFG automata, with some surprises along the way. I will give an overview of the results obtained in this line of research, the proof techniques typically used, and the remaining open problems and conjectures.

### 3.8 Quotients, Coverings and Conjugacy of Unambiguous Automata

*Sylvain Lombardy (University of Bordeaux, FR)*

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**Joint work of** Sylvain Lombardy, Marie-Pierre Bal, Jacques Sakarovitch

In this talk, I shall recall the definitions of quotients and coverings, which are useful tools to transform the structure of an automaton while preserving the unambiguity. We shall see that it is always possible to turn an unambiguous automaton to any equivalent one using



these tools. The construction of this transformation is based on a more algebraic concept, that is the conjugacy of automata. An open question concerning the transformation of an automaton to another one is the state complexity of the transitional automata. This talk is based on a work with Marie-Pierre Bal and Jacques Sakarovitch.

### 3.9 Active learning sound negotiations

Anca Muscholl (*University of Bordeaux, FR*)

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**Joint work of** Anca Muscholl, Igor Walukiewicz

Sound deterministic negotiations are models of distributed systems, a kind of Petri nets or Zielonka automata with additional structure. We show that the additional structure allows to minimize such negotiations. Based on minimisation we present two Angluin-style learning algorithms for sound deterministic negotiations. The two algorithms differ in the kind of membership queries they use, and both have similar (polynomial) complexity as Angluins algorithm.

### 3.10 Lower bound for unambiguous arithmetic circuits via Hankel matrix

Pierre Ohlmann (*CNRS – Paris, FR*)

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**Joint work of** Pierre Ohlmann, Nathanaël Fijalkow, Guillaume Lagarde, Olivier Serre

This talk is about arithmetic circuits, for which a major goal is to devise lower bounds: can one find polynomials such that any arithmetic circuit computing them has to be large. I will present a new characterization of the size of the smallest arithmetic circuit computing a given non-associative polynomial, in term of the rank of a so-called Hankel matrix. This generalizes an important result of Nisan (1992); it is based on a result for weighted tree-automata due to Bozapalidis and Loscou-Bozapalidou (1984).

We will then show how the characterization can be used to establish an exponential lower bound for (associative) unambiguous circuits computing the permanent polynomial.

### 3.11 Unambiguous Automata for Data Languages (Invited Talk)

Gabriele Puppis (*University of Udine, IT*)

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**Joint work of** Gabriele Puppis, Thomas Colcombet, Michał Skrzypczak

I will present the status of an ongoing research work with Thomas Colcombet and Michał Skrzypczak about unambiguity in register automata (register automata, or finite memory automata, are automata that can describe languages over an infinite alphabet). Differently from finite state automata, the amount of non-determinism allowed in register automata has an impact on the expressive power and the closure properties of the recognized class of

languages, as well as on the complexity of some fundamental decision problems. For example, deterministic register automata are strictly less expressive than non-deterministic ones, they are closed under complement, but not under mirroring. On the other hand, non-deterministic register automata (with guessing) are closed under mirroring, but not under complement. It comes natural then to study the intermediate class of unambiguous register automata with guessing. Recently (LICS'21), this class has been shown to enjoy a decidable equivalence problem and is believed to be effectively closed under complement. However, proving this closure property turned out to be more difficult than expected. I will present some ideas and partial results along this goal, mentioning a few other conjectures related to the expressive power of unambiguous register automata.

### 3.12 On Uniformization in the Full Binary Tree

*Alexander Rabinovich (Tel Aviv University, IL)*

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Joint work of Alexander Rabinovich, Amit Shomrat

Gurevich and Shelah proved that the uniformization property fails for Monadic Second-Order logic (MSO) over the full binary tree, i.e., there is a formula  $A(X, Y)$  in MSO such that no MSO formula uniformizes it (over the full binary tree).

The cross-section of a relation  $R(X, Y)$  at  $d$  is the set of all  $e$  such that  $R(d, e)$  holds. We prove:

**Theorem (Finite-cross Section):** If every cross-section of an MSO definable relation is finite then it has an MSO definable uniformizer.

**Theorem (Uncountable-cross Section):** There is an MSO definable relation  $R$  such that every MSO definable relation included in  $R$  and with the same domain as  $R$  has an uncountable cross-section.

### 3.13 State complexity of complementing unambiguous automata

*Mikhail Raskin (TU München, DE)*

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Not so long ago, even a polynomial upper bound on state complexity of recognising the complement of the language of an unambiguous finite automaton felt plausible. Now it does not, but what else do we know? Not so much. In this talk I plan to briefly show the approaches that give the best currently known lower and upper bounds for the state complexity of complementation in the unary and binary alphabets; and draw a (straightforward) game reformulation of the large-alphabet problem in the hope it will inspire someone to prove the exponential lower bound in that case.

### 3.14 Problems on unambiguous WAs and PAs

Mahsa Shirmohammadi (University Paris Diderot, FR)

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In this survey talk we recall a proof of the classical results on weighted automata (WAs) over fields, that given a weighted function  $f$  realisable with WAs, the size of a minimal canonical WA computing  $f$  is equivalent to the rank of the Hankle matrix of  $f$ . We also briefly talk about recent results of Bell and Smertnig showing that every weighted function taking values in a finitely generated subgroup of a field (and zero) can be realised with an unambiguous WA. We conclude the talk with open problems and directions for future research.

### 3.15 Unambiguity in Transducer Theory

Sarah Winter (UL – Brussels, BE)

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Joint work of Sarah Winter, Emmanuel Filliot

This talk surveys some introductory results regarding unambiguity in transducer theory. Transducers are automata with output; they recognize relations. A transducer is unambiguous if for each word  $u$  from its domain there is a unique accepting run with input  $u$ .

In more detail, we show that the classes of functions recognized by functional transducers and unambiguous transducers coincide. We also show that unambiguity, while necessary for one-way transducers, can be traded for determinism at the price of two-wayness.

This is a joint work with Emmanuel Filliot.

## 4 Working Groups

The participants were not formally decomposed into working groups, though many small groups have been interacting and evolving during the program.

- Achim Blumensath and Michał Skrzypczak worked on the *Thin Tree Conjecture*.
- Emmanuel Filiot, Karin Quaas, and Sarah Winter started a new collaboration on synthesis for register automata. A collaboration on a standing open problem related to unambiguity in transducer models emerged during the various discussions. Specifically, the problem concerns the possibility of transforming any streaming string transducer with boundedly many outputs per input into an equivalent finite union of unambiguous functional transducers. The collaboration involved the researchers Emmanuel Filiot, Ismaël Jecker, Christof Löding, Anca Muscholl, Gabriele Puppis, and Sarah Winter, and it is still active. A paper with the outcome of this collaboration will likely be produced in the near future.
- Another research collaboration emerged between Anca Muscholl and Gabriele Puppis on the possibility of having minimal and canonical forms of streaming string transducers, as well as an Angluin-style learning algorithm for these types of transducers.
- Wojciech Czerwiński, Diego Figueira, Gabriele Puppis, Mikhail Raskin, and Georg Zetsche have collaborated on a decision problem concerning the separability of synchronous relations (i.e. relations represented by letter-to-letter transducers) by means of recognizable relations (i.e. relations obtained as finite unions of products of regular languages).

- Thomas Colcombet and Alexander Rabinovich have been working on the uniformization questions for monadic second-order logic over countable ordinals. The open problems regarding this topic are solved and a paper is under writing.
- Karin Quaas and Narayanan Krishna Shankara have initiated a new collaboration on temporal logics for real-timed systems.

## 5 Open Problems

### 5.1 Characterizing the counter hierarchy of unambiguous automata


*Georg Zetsche (MPI Kaiserslautern, DE, [georg@mpi-sws.org](mailto:georg@mpi-sws.org))*

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Given a counter language, how many counters does it require to be recognizable by an unambiguous counter machine? This problem is undecidable for non-deterministic counter machines.

### 5.2 Program synthesis for unambiguous devices


*Emmanuel Filiot (UL Bruxelles, BE, [efiliot@gmail.com](mailto:efiliot@gmail.com))*

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Is it the case that for every regular specification  $\varphi \in \text{REG}(\Sigma^* \times \Sigma^*)$  there exists an unambiguous transducer which realises this specification?

### 5.3 Deciding efficiently history-determinism for $\omega$ -automata

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The  $G2$  conjecture states that a parity automaton over  $\omega$ -words is history-deterministic if and only if there is a winning strategy in a specific two pebbles game (hence the name  $G2$ ). The conjecture is only known to hold for very low levels of the parity hierarchy.

### 5.4 What is the complexity of constructing unambiguous automata

*Denis Kuperberg (LIP, ENS Lyon, FR, [denis.kuperberg@ens-lyon.fr](mailto:denis.kuperberg@ens-lyon.fr))*

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Joint work of Denis Kuperberg, Shaull Amalgor

What is the complexity of the following problem: given a non-deterministic finite automaton  $A$  and an integer  $n$  in binary, does there exist a deterministic (unambiguous) finite automaton  $B$  that accepts  $L(A)$  and has less than  $n$  states?

## 5.5 Characterizing classes of languages with atoms from internal closure operations

*Antoine Mottet (Charles Univ. Prague, CZ, mottet@karlin.mff.cuni.cz)*

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We say that an operation  $f$  of finite arity over the set of data words preserves a language  $L$  if  $f(L, \dots, L)$  is a subset of  $L$ . For example, if  $L$  is recognizable by a register automaton with an atom structure  $A$ , then every automorphism of  $A$  preserves  $L$ . The internal closure properties of data languages have not been considered so far. In particular, can one understand the complexity of a language (i.e., deterministically recognizable, unambiguously recognizable, recognizable, with/without guessing) in terms of the operations preserving a language? This question was answered positively for Turing machines (recognizing several variants of constraint satisfaction problems) where closure properties have been central in characterizing the (descriptive) complexity of problems

## 5.6 The zeroness problem

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What is the complexity and decidability of the zeroness problem, ie deciding if a machine representing a function computes the everywhere null constant. The question is of interesting, in particular, for weighted grammars over a field, unary polynomial automata, weighted Parikh automata, weighted vector addition systems with states.

## 5.7 Stronger versions of inclusion of probabilistic automata

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Can one prove decidability of the language containment problem for probabilistic automata with bounded ambiguity without having to assume Schanuel's conjecture?

## 5.8 The state complexity of unambiguous Büchi automata


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The first asks the general question: can we probe the  $2^n$  lower bound for the problem in the infinite words case? The second and third problems focus on specific LTL formulae over infinite words and asks about the lower bound for unambiguous automata recognising their languages.

## 5.9 Universality of register automata over ordered domains

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Is the universality problem for unambiguous register automata over the integers with order and constants decidable? If yes, what is the complexity?

## 5.10 Decomposition of finitely unambiguous automata

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Is it possible to decompose finitely ambiguous register automata into finitely many unambiguous ones?

## 5.11 Better bounds on complementing unambiguous automata

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It is now known that complementing the language of an  $n$ -state unambiguous finite automaton might yield a language not recognisable by some nondeterministic finite automata with fewer than  $n^{(\log \log \log n)^{\Omega(1)}}$  for unary alphabet and there is an upper bound of  $n^{O(\log n)}$ . In the binary case the lower bound is  $n^{\Omega(\log n)}$  but the upper bound is still exponential; same for large alphabets. How do we close the non-unary complement gap?

## 6 Panel Discussions

No panel discussions were organised.

## Participants

- |   |   |   |
|---|---|---|
| ■ Achim Blumensath<br>Masaryk University – Brno, CZ           | ■ Ismaël Jecker<br>University of Warsaw, PL                                   | ■ Jakob Piribauer<br>TU Dresden, DE                   |
| ■ Udi Boker<br>Reichman University –<br>Herzliya, IL          | ■ Stefan Kiefer<br>University of Oxford, GB                                   | ■ Gabriele Puppis<br>University of Udine, IT          |
| ■ Dmitry Chistikov<br>University of Warwick –<br>Coventry, GB | ■ Shankaranarayanan Krishna<br>Indian Institute of Technology –<br>Mumbai, IN | ■ Karin Quaas<br>Universität Leipzig, DE              |
| ■ Lorenzo Clemente<br>University of Warsaw, PL                | ■ Denis Kuperberg<br>ENS – Lyon, FR   | ■ Alexander Rabinovich<br>Tel Aviv University, IL     |
| ■ Thomas Colcombet<br>CNRS – Paris, FR                        | ■ Karoliina Lehtinen<br>Aix-Marseille University, FR                          | ■ Michael Raskin<br>TU München, DE                    |
| ■ Wojciech Czerwinski<br>University of Warsaw, PL             | ■ Antoine Mottet<br>Charles University – Prague, CZ                           | ■ Mahsa Shirmohammadi<br>University Paris Diderot, FR |
| ■ Diego Figueira<br>CNRS & Université de<br>Bordeaux, FR      | ■ Anca Muscholl<br>University of Bordeaux, FR                                 | ■ Michal Skrzypczak<br>University of Warsaw, PL       |
| ■ Emmanuel Filiot<br>UL – Brussels, BE                        | ■ Pierre Ohlmann<br>CNRS – Paris, FR  | ■ Sarah Winter<br>UL – Brussels, BE                   |
| ■ Simon Jantsch<br>TU Dresden, DE                             | ■ Guillermo A. Pérez<br>University of Antwerp, BE                             | ■ Georg Zetsche<br>MPI-SWS – Kaiserslautern, DE       |



## Remote Participants

- |   |  |  |
|---|--|--|
| ■ Christel Baier<br>TU Dresden, DE                    | ■ Nathanael Fijalkow<br>University of Bordeaux, FR | ■ Christof Löding<br>RWTH Aachen, DE                                 |
| ■ Johanna Björklund<br>University of Umeå, SE         | ■ Mika Göös<br>EPFL Lausanne, CH                   | ■ Sylvain Lombardy<br>University of Bordeaux, FR                     |
| ■ Michaël Cadilhac<br>DePaul University – Chicago, US | ■ Arthur Jaquard<br>CNRS – Paris, FR               | ■ Radek Piórkowski<br>University of Warsaw, PL                       |
| ■ Antonio Casares<br>University of Bordeaux, FR       | ■ Stefan Kiefer<br>University of Oxford, GB        | ■ Mikhail V. Volkov<br>Ural Federal University –<br>Ekaterinburg, RU |