

# CFA2: Pushdown Flow Analysis for Higher-Order Languages

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Flow analysis is instrumental in building good software.



Optimization



Debugging



Verification



Development

# Overview

Finite-state analyses and their limitations

CFA2 by example

Applications to JavaScript

Open problems

# Finite-state analyses

Program as a graph whose nodes are the program points.

⇒ executions are strings in a regular language.

⇒ approximate program with finite-state machine.

⇒ call/return mismatch.

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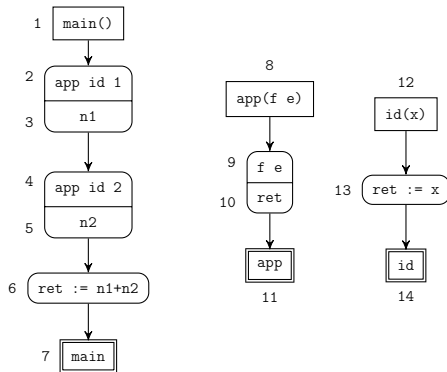
Fine for conditionals and loops (think Fortran).

Call/return is the fundamental control-flow mechanism in HOLs.

Finite-state analyses, such as  $k$ -CFA, have several limitations.

```
(define app (λ (f e) (f e)))  
(define id (λ (x) x))  
  
(let* ((n1 (app id 1))  
       (n2 (app id 2)))  
  (+ n1 n2))
```

## OCFA example





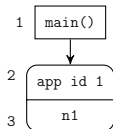
# OCFA example

1

main()

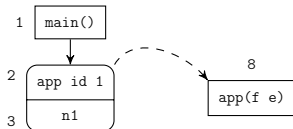
Global environment:

# OCFA example



Global environment:

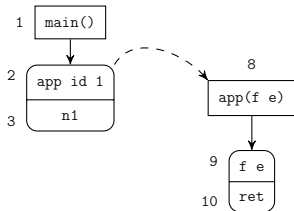
# OCFA example



Global environment:

f	id
e	1

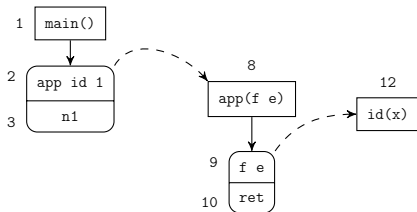
## OCFA example



Global environment:

f	id
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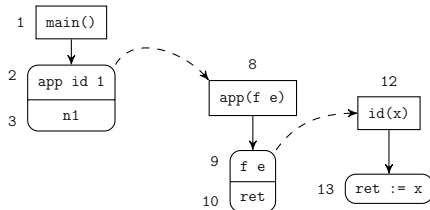
## OCFA example



Global environment:

f	id
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# OCFA example

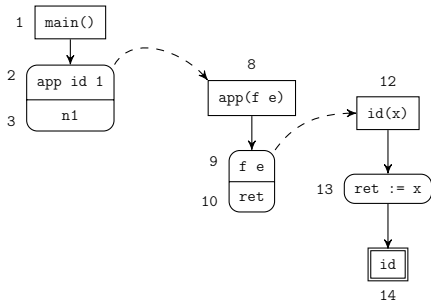


Global environment:

f	id
e	1

x	1
ret-id	1

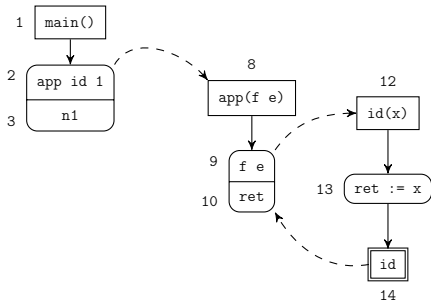
## OCFA example



Global environment:

f	id
e	1
x	1
ret-id	1

## OCFA example

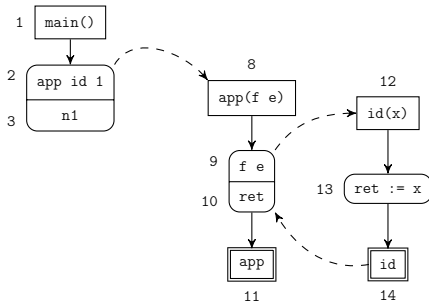


Global environment:

f	id
e	1
ret-app	1
x	1
ret-id	1



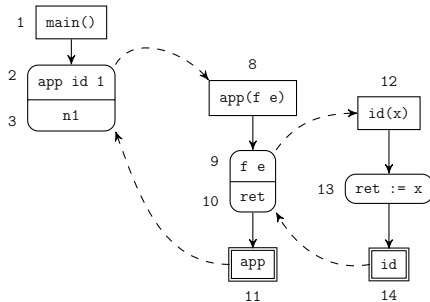
## OCFA example



Global environment:

f	id
e	1
ret-app	1
x	1
ret-id	1

## OCFA example



Global environment:

n1 1

f id

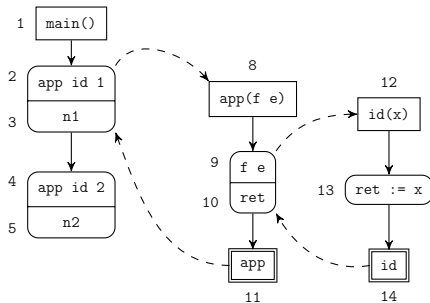
e 1

ret-app 1

x 1

ret-id 1

## OCFA example



Global environment:

n1 1

f id

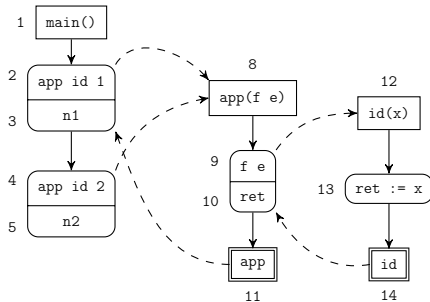
e 1

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x 1

ret-id 1

# OCFA example



Global environment:

n1 1

f id

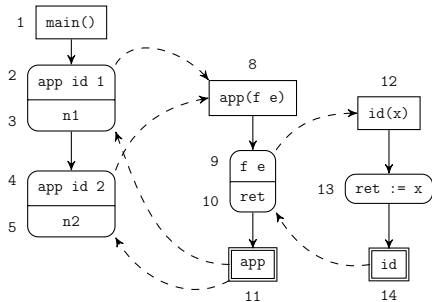
e 1 2

ret-app 1 2

x 1 2

ret-id 1 2

# OCFA example



Global environment:

n1            1 2

n2            1 2

f            id

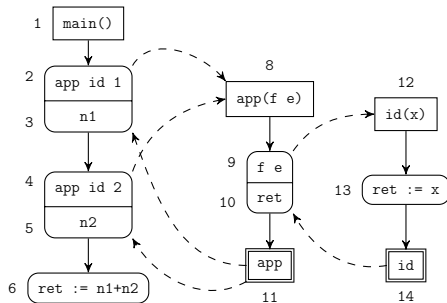
e            1 2

ret-app      1 2

x            1 2

ret-id       1 2

# OCFA example



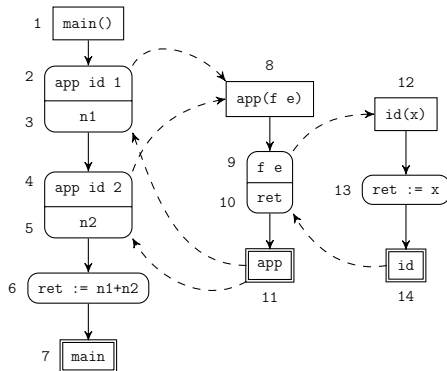
Global environment:

n1	1	2
n2	1	2
ret-main	2	3 4

f	id
e	1 2
ret-app	1 2

x	1 2
ret-id	1 2

# OCFA example



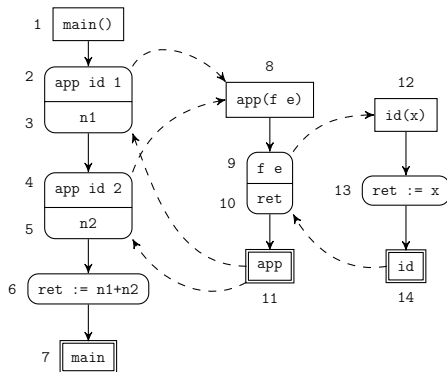
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# OCFA example



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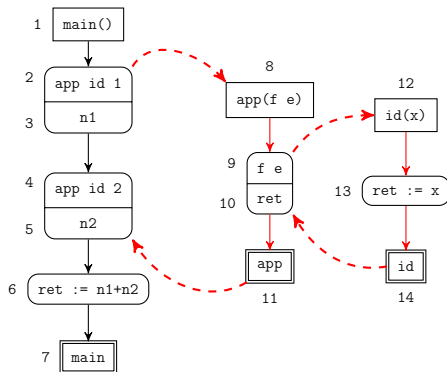
f	id
e	1 2
ret-app	1 2

x	1 2
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Call/return mismatch causes spurious flow of data  
⇒ commonly called functions pollute the analysis.



# OCFA example



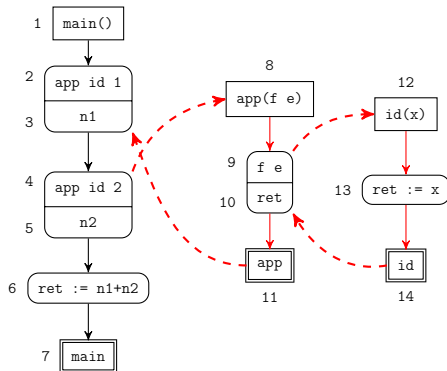
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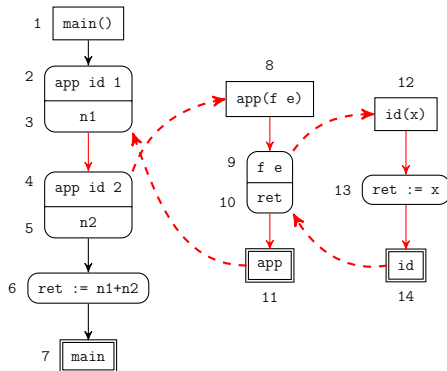
Global environment:

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# OCFA example



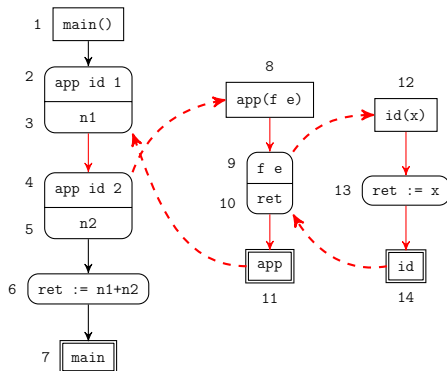
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ret-main	2	3 4

f	id
e	1 2
ret-app	1 2

x	1 2
ret-id	1 2

Call/return mismatch causes spurious control flow  
⇒ cannot accurately calculate stack change.

## Fake rebinding

```
(define (compose-same f x)
  (f (f x)))
```

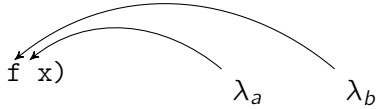
## Fake rebinding

```
(define (compose-same f x)
  (f (f x)))
```

The diagram illustrates the concept of 'fake rebinding'. It shows a function definition `(define (compose-same f x) (f (f x)))`. To the right of the definition are two lambda expressions,  $\lambda_a$  and  $\lambda_b$ . Two curved arrows originate from these labels and point to the parameter `f` in the function definition. The arrow from  $\lambda_a$  is the upper curve, and the arrow from  $\lambda_b$  is the lower curve. This indicates that both  $\lambda_a$  and  $\lambda_b$  are associated with the same parameter `f`, despite being distinct lambda expressions.

## Fake rebinding

```
(define (compose-same f x)
```



$\lambda_a$   $\lambda_b$

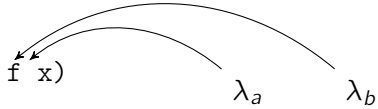
```
  (f (f x)))
```

Flows:

```
(f (f x))
```

## Fake rebinding

```
(define (compose-same f x)
```



The diagram illustrates the concept of 'fake rebinding'. It shows two variables,  $\lambda_a$  and  $\lambda_b$ , positioned to the right of the code. Two curved arrows originate from these variables and point to the variable `f` in the code snippet. The arrow from  $\lambda_a$  points to the first occurrence of `f` (the function argument), and the arrow from  $\lambda_b$  points to the second occurrence of `f` (the function body). This visualizes how different lambda expressions can bind to the same variable in different contexts, even when the variable is used multiple times within the same scope.

```
  (f (f x)))
```

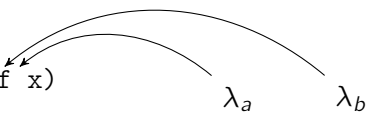
Flows:

```
(f ( $\lambda_a$  x))
```



## Fake rebinding

```
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```



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```
  (f (f x)))
```

Flows:

$(\lambda_a (\lambda_a x))$  ✓

# Fake rebinding

```
(define (compose-same f x)
```



$\lambda_a$   $\lambda_b$

```
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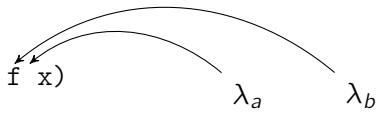
Flows:

$(\lambda_a (\lambda_a x))$  ✓

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# Fake rebinding

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Flows:

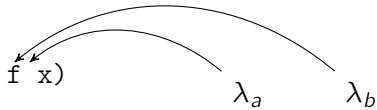
$(\lambda_a (\lambda_a x))$  ✓

$(\lambda_b (\lambda_b x))$  ✓

$(\lambda_b (\lambda_a x))$  ✗

# Fake rebinding

`(define (compose-same f x)`  
    `(f (f x)))`



The diagram illustrates the concept of 'fake rebinding'. It shows two lambda expressions,  $\lambda_a$  and  $\lambda_b$ , positioned to the right of the code. Two curved arrows originate from  $\lambda_a$  and  $\lambda_b$  and both point to the variable `f` in the code snippet `(define (compose-same f x) (f (f x)))`. This indicates that both  $\lambda_a$  and  $\lambda_b$  are bound to the same environment where `f` is defined, despite the code suggesting a local binding.

Flows:

$(\lambda_a (\lambda_a x))$  ✓

$(\lambda_b (\lambda_b x))$  ✓

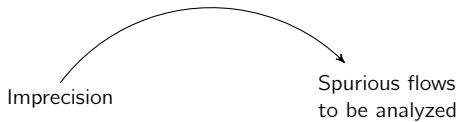
$(\lambda_b (\lambda_a x))$  ✗

$(\lambda_a (\lambda_b x))$  ✗

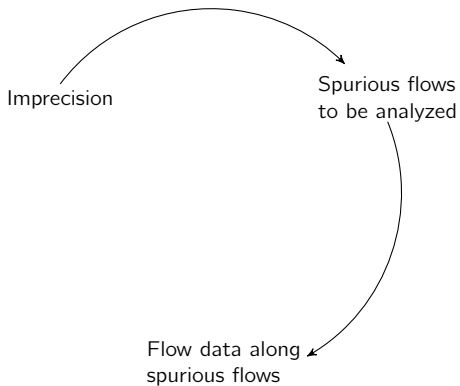
# Imprecision slows down the analysis

Imprecision

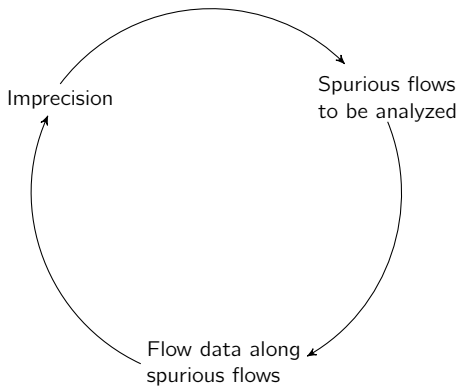
## Imprecision slows down the analysis



# Imprecision slows down the analysis



## Imprecision slows down the analysis





# The root cause: call/return mismatch

Causes spurious data flow.

Causes spurious control flow.

Leads to imprecision which slows down the analysis.

Fake rebinding?

## CFA2 in a nutshell

Approximate a program as a PDA.

Use the stack for return-point information.

Unbounded call/return matching.

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A pushdown flow analysis [Sharir–Pnueli 81, Reps et al. 95].

First-class functions, tail calls.

Recursion causes stacks of unbounded size

⇒ infinite state space.

# What we hope to achieve

Advanced reasoning about stack and environment:

- ▶ escape analysis for stack allocation
- ▶ super- $\beta$  inlining
- ▶ transducer fusion

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Advanced reasoning about stack and environment:

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Do old things better.

0CFA too imprecise.

Polyvariance didn't help  $k$ -CFA much  
and slowed it down a lot [Van Horn–Mairson 08].

# Variable binding in CFA2

Binding environments:

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- ▶ stack



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Bound in the top frame.

Stack references of same variable bound in same environment.

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- ▶ stack

Stack references:  $(\lambda(x) (\lambda(y) (\textcolor{red}{y} (\textcolor{red}{y} x))))$

Bound in the top frame.

Stack references of same variable bound in same environment.

Heap references:  $(\lambda(x) (\lambda(y) (y (y \textcolor{red}{x}))))$

Either deeper in stack or in heap.

## CFA2: pushdown automaton

```
(define merger
  ( $\lambda_1$  (x) ( $\lambda_2$  () x)))

(merger ( $\lambda_3$  (y) y))

(define id
  (merger ( $\lambda_4$  (z) z))())

(define comp-same
  ( $\lambda_5$  (f w) (f (f w))))

(define n1
  (comp-same id 1))

(define n2
  (comp-same id 2))
```

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### Heap:

```
merger
x
id
comp-same
n1
n2
```

### Stack:

## CFA2: pushdown automaton

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```

### Heap:

merger  $\lambda_1$   
x  
id  
comp-same  
n1  
n2

### Stack:

## CFA2: pushdown automaton

```
(define merger  
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  ( $\lambda_5$  (f w) (f (f w))))  
  
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### Heap:

merger  $\lambda_1$   
x  
id  
comp-same  
n1  
n2

### Stack:

## CFA2: pushdown automaton

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(define n2  
  (comp-same id 2))
```

### Heap:

merger	$\lambda_1$
x	$\lambda_3$
id	
comp-same	
n1	
n2	

### Stack:

$x \mapsto \lambda_3$
-----------------------

## CFA2: pushdown automaton

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### Heap:

merger	$\lambda_1$
x	$\lambda_3$
id	
comp-same	
n1	
n2	

### Stack:



## CFA2: pushdown automaton

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```

### Heap:

merger	$\lambda_1$
x	$\lambda_3, \lambda_4$
id	
comp-same	
n1	
n2	

### Stack:

$x \mapsto \lambda_4$
-----------------------

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(define merger  
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### Heap:

merger	$\lambda_1$
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comp-same	
n1	
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### Stack:

## CFA2: pushdown automaton

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### Heap:

merger	$\lambda_1$
x	$\lambda_3, \lambda_4$
id	
comp-same	
n1	
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### Stack:

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```

### Heap:

merger	$\lambda_1$
x	$\lambda_3, \lambda_4$
id	$\lambda_3, \lambda_4$
comp-same	
n1	
n2	

### Stack:

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```

### Heap:

merger	$\lambda_1$
x	$\lambda_3, \lambda_4$
id	$\lambda_3, \lambda_4$
comp-same	$\lambda_5$
n1	
n2	

### Stack:

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(define n1
  (comp-same id 1))

(define n2
  (comp-same id 2))
```

### Heap:

merger	$\lambda_1$
x	$\lambda_3, \lambda_4$
id	$\lambda_3, \lambda_4$
comp-same	$\lambda_5$
n1	
n2	

### Stack:

## CFA2: pushdown automaton

```
(define merger  
  ( $\lambda_1$  (x) ( $\lambda_2$  () x)))  
  
(merger ( $\lambda_3$  (y) y))  
  
(define id  
  (merger ( $\lambda_4$  (z) z))())  
  
(define comp-same  
  ( $\lambda_5$  (f w) (f ( $\textcolor{red}{f}$   $\textcolor{red}{w}$ ))))  
  
(define n1  
  (comp-same id 1))  
  
(define n2  
  (comp-same id 2))
```

### Heap:

merger	$\lambda_1$
x	$\lambda_3, \lambda_4$
id	$\lambda_3, \lambda_4$
comp-same	$\lambda_5$
n1	
n2	

### Stack:

$f \mapsto \{\lambda_3, \lambda_4\}, w \mapsto 1$
---

## CFA2: pushdown automaton

```
(define merger  
  ( $\lambda_1$  (x) ( $\lambda_2$  () x)))  
  
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  ( $\lambda_5$  (f w) (f ( $\textcolor{red}{f}$   $\textcolor{red}{w}$ ))))  
  
(define n1  
  (comp-same id 1))  
  
(define n2  
  (comp-same id 2))
```

### Heap:

merger	$\lambda_1$
x	$\lambda_3, \lambda_4$
id	$\lambda_3, \lambda_4$
comp-same	$\lambda_5$
n1	
n2	

### Stack:

$f \mapsto \lambda_3, w \mapsto 1$
------------------------------------



## CFA2: pushdown automaton

```
(define merger
  ( $\lambda_1$  (x) ( $\lambda_2$  () x)))

(merger ( $\lambda_3$  (y)  $y$ ))

(define id
  (merger ( $\lambda_4$  (z) z))())

(define comp-same
  ( $\lambda_5$  (f w) (f (f w))))

(define n1
  (comp-same id 1))

(define n2
  (comp-same id 2))
```

### Heap:

merger	$\lambda_1$
x	$\lambda_3, \lambda_4$
id	$\lambda_3, \lambda_4$
comp-same	$\lambda_5$
n1	
n2	

### Stack:

$y \mapsto 1$
$f \mapsto \lambda_3, w \mapsto 1$

## CFA2: pushdown automaton

```
(define merger
  ( $\lambda_1$  (x) ( $\lambda_2$  () x)))

(merger ( $\lambda_3$  (y) y))

(define id
  (merger ( $\lambda_4$  (z) z))())

(define comp-same
  ( $\lambda_5$  (f w) ( $\textcolor{red}{f}$  ( $\textcolor{red}{f}$   $\textcolor{red}{w}$ ))))

(define n1
  (comp-same id 1))

(define n2
  (comp-same id 2))
```

### Heap:

merger	$\lambda_1$
x	$\lambda_3, \lambda_4$
id	$\lambda_3, \lambda_4$
comp-same	$\lambda_5$
n1	
n2	

### Stack:

$f \mapsto \lambda_3, w \mapsto 1$
------------------------------------

## CFA2: pushdown automaton

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(define merger  
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n2	

### Stack:

$y \mapsto 1$
$f \mapsto \lambda_3, w \mapsto 1$

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```
(define n1  
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```

```
(define n2  
  (comp-same id 2))
```

### Heap:

merger	$\lambda_1$
x	$\lambda_3, \lambda_4$
id	$\lambda_3, \lambda_4$
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n1	1
n2	

### Stack:

## CFA2: pushdown automaton

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(define merger
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```

### Heap:

merger	$\lambda_1$
x	$\lambda_3, \lambda_4$
id	$\lambda_3, \lambda_4$
comp-same	$\lambda_5$
n1	1
n2	

### Stack:

$f \mapsto \lambda_4, w \mapsto 1$
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## CFA2: pushdown automaton

```
(define merger  
  ( $\lambda_1$  (x) ( $\lambda_2$  () x)))  
  
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(define id  
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(define comp-same  
  ( $\lambda_5$  (f w) (f (f w))))  
  
(define n1  
  (comp-same id 1))  
  
(define n2  
  (comp-same id 2))
```

### Heap:

merger	$\lambda_1$
x	$\lambda_3, \lambda_4$
id	$\lambda_3, \lambda_4$
comp-same	$\lambda_5$
n1	1
n2	

### Stack:

$z \mapsto 1$
$f \mapsto \lambda_4, w \mapsto 1$

## CFA2: pushdown automaton

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(define merger
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### Heap:

merger	$\lambda_1$
x	$\lambda_3, \lambda_4$
id	$\lambda_3, \lambda_4$
comp-same	$\lambda_5$
n1	1
n2	

### Stack:

$f \mapsto \lambda_4, w \mapsto 1$
------------------------------------

## CFA2: pushdown automaton

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(define merger
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### Heap:

merger	$\lambda_1$
x	$\lambda_3, \lambda_4$
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### Stack:

$z \mapsto 1$
$f \mapsto \lambda_4, w \mapsto 1$



## CFA2: pushdown automaton

```
(define merger  
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(define id  
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(define comp-same  
  ( $\lambda_5$  (f w) (f (f w))))
```

```
(define n1  
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```

```
(define n2  
  (comp-same id 2))
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### Heap:

merger	$\lambda_1$
x	$\lambda_3, \lambda_4$
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n1	1
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### Stack:

## CFA2: pushdown automaton

```
(define merger
  ( $\lambda_1$  (x) ( $\lambda_2$  () x)))

(merger ( $\lambda_3$  (y) y))

(define id
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(define n1
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  (comp-same id 2))
```

### Heap:

merger	$\lambda_1$
x	$\lambda_3, \lambda_4$
id	$\lambda_3, \lambda_4$
comp-same	$\lambda_5$
n1	1
n2	2

### Stack:

## Resilience to syntax changes

```
(define id (λ (x) x))
```

```
(let* ((n1 (id 1))  
      (n2 (id 2)))  
  (+ n1 n2))
```

## Resilience to syntax changes

```
(define id (λ (y) ((λ (x) x) y)))
```

```
(let* ((n1 (id 1))  
       (n2 (id 2)))  
  (+ n1 n2))
```

## Resilience to syntax changes

```
((λ (id)
  (let* ((n1 (app id 1))
         (n2 (app id 2)))
    (+ n1 n2)))
(λ (x) x))
```

## Resilience to syntax changes

```
(define id (λ (x) (λ () x)()))
```

```
(let* ((n1 (id 1))  
       (n2 (id 2)))  
  (+ n1 n2))
```

# Summarization

Functions don't care about their return point.

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Don't keep track of the stack explicitly.

Inside a function, remember top frame only.



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Don't keep track of the stack explicitly.

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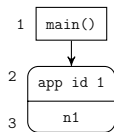
Record summaries, which express in/out relations.

Use summaries at call sites to simulate the effect of the call.

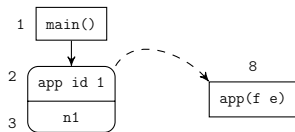
## CFA2: summarization

1 `main()`

## CFA2: summarization



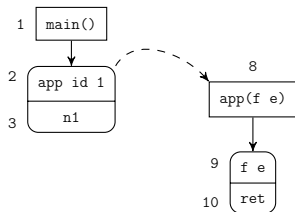
## CFA2: summarization



Callers:

2 calls 8[ $e \mapsto 1$ ]

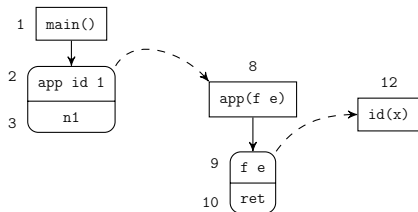
## CFA2: summarization



Callers:

2 calls 8[ $e \mapsto 1$ ]

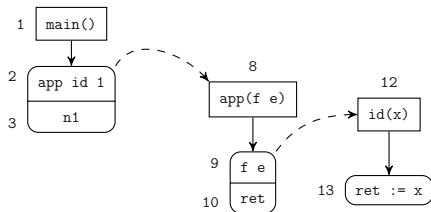
## CFA2: summarization



Callers:

2	calls	$8[e \mapsto 1]$
$9[e \mapsto 1]$	calls	$12[x \mapsto 1]$

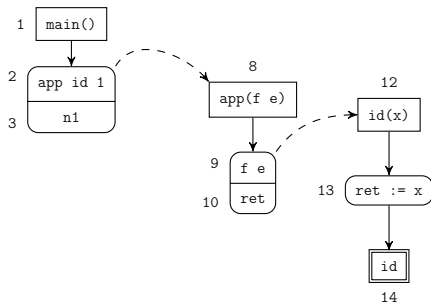
## CFA2: summarization



Callers:

2	calls	8[e ↦ 1]
9[e ↦ 1]	calls	12[x ↦ 1]

## CFA2: summarization



Callers:

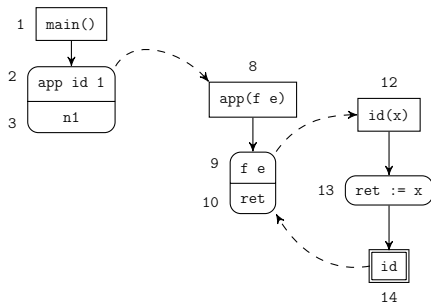
2	calls	8[ $e \mapsto 1$ ]
9[ $e \mapsto 1$ ]	calls	12[ $x \mapsto 1$ ]

Entry/exit summaries:

12[ $x \mapsto 1$ ] reaches 14[ $x \mapsto 1, \text{ret} \mapsto 1$ ]



## CFA2: summarization



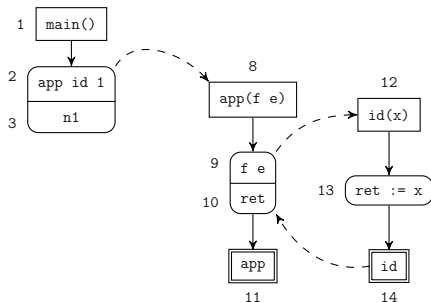
Callers:

2	calls	8[ $e \mapsto 1$ ]
9[ $e \mapsto 1$ ]	calls	12[ $x \mapsto 1$ ]

Entry/exit summaries:

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## CFA2: summarization



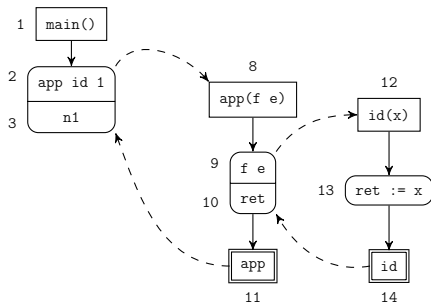
Callers:

2	calls	8[ $e \mapsto 1$ ]
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Entry/exit summaries:

12[ $x \mapsto 1$ ]	reaches	14[ $x \mapsto 1, \text{ret} \mapsto 1$ ]
8[ $e \mapsto 1$ ]	reaches	11[ $e \mapsto 1, \text{ret} \mapsto 1$ ]

## CFA2: summarization



Callers:

2            calls    8[ $e \mapsto 1$ ]  
9[ $e \mapsto 1$ ]    calls    12[ $x \mapsto 1$ ]

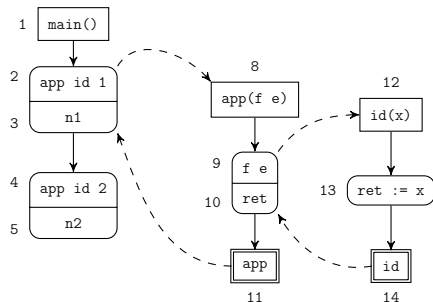
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Top level:

n1            1

## CFA2: summarization



Callers:

2	calls	8	$[e \mapsto 1]$	
9	$[e \mapsto 1]$	calls	12	$[x \mapsto 1]$

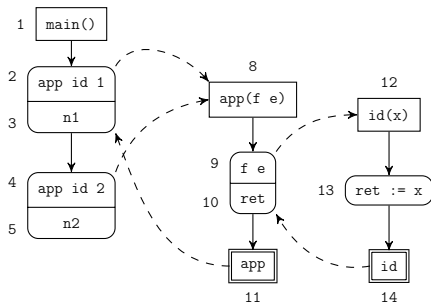
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12	$[x \mapsto 1]$	reaches	14	$[x \mapsto 1, \text{ret} \mapsto 1]$
8	$[e \mapsto 1]$	reaches	11	$[e \mapsto 1, \text{ret} \mapsto 1]$

Top level:

n1	1
----	---

## CFA2: summarization



Callers:

2	calls	8[ $e \mapsto 1$ ]
9[ $e \mapsto 1$ ]	calls	12[ $x \mapsto 1$ ]
4	calls	8[ $e \mapsto 2$ ]

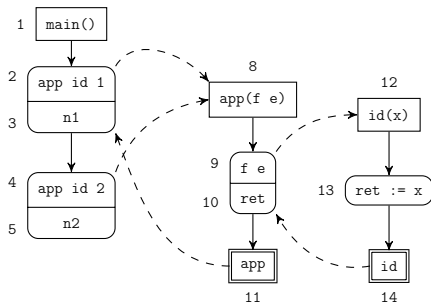
Entry/exit summaries:

12[ $x \mapsto 1$ ]	reaches	14[ $x \mapsto 1, \text{ret} \mapsto 1$ ]
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Top level:

n1	1
----	---

## CFA2: summarization



Callers:

2	calls	8[e ↦ 1]
9[e ↦ 1]	calls	12[x ↦ 1]
4	calls	8[e ↦ 2]
9[e ↦ 2]	calls	12[x ↦ 2]

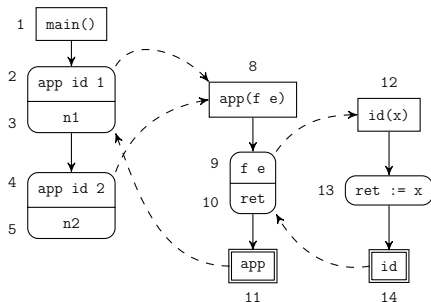
Entry/exit summaries:

12[x ↦ 1]	reaches	14[x ↦ 1, ret ↦ 1]
8[e ↦ 1]	reaches	11[e ↦ 1, ret ↦ 1]
12[x ↦ 2]	reaches	14[x ↦ 2, ret ↦ 2]

Top level:

n1	1
----	---

## CFA2: summarization



Callers:

2	calls	8[e ↦ 1]
9[e ↦ 1]	calls	12[x ↦ 1]
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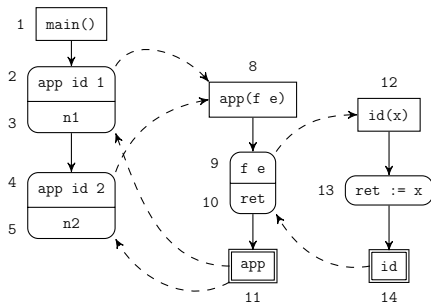
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12[x ↦ 2]	reaches	14[x ↦ 2, ret ↦ 2]
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Top level:

n1	1
----	---

## CFA2: summarization



Callers:

2	calls	8[e ↦ 1]
9[e ↦ 1]	calls	12[x ↦ 1]
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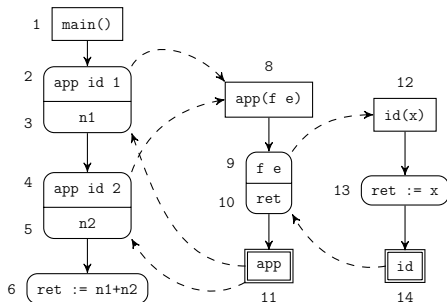
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8[e ↦ 2]	reaches	11[e ↦ 2, ret ↦ 2]

Top level:

n1	1
n2	2



## CFA2: summarization



Callers:

2	calls	8[e ↦ 1]
9[e ↦ 1]	calls	12[x ↦ 1]
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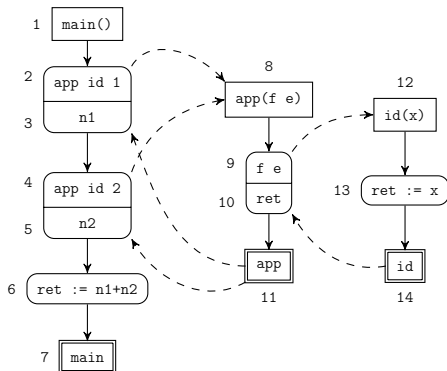
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8[e ↦ 2]	reaches	11[e ↦ 2, ret ↦ 2]

Top level:

n1	1
n2	2
ret	3

## CFA2: summarization



Callers:

2	calls	8[e ↦ 1]
9[e ↦ 1]	calls	12[x ↦ 1]
4	calls	8[e ↦ 2]
9[e ↦ 2]	calls	12[x ↦ 2]

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12[x ↦ 1]	reaches	14[x ↦ 1, ret ↦ 1]
8[e ↦ 1]	reaches	11[e ↦ 1, ret ↦ 1]
12[x ↦ 2]	reaches	14[x ↦ 2, ret ↦ 2]
8[e ↦ 2]	reaches	11[e ↦ 2, ret ↦ 2]

Top level:

n1	1
n2	2
ret	3

## Handling tail calls

```
(define app (λ (f e) (f e)))
```

```
(define id (λ (x) x))
```

```
(let* ((n1 (app id 1))  
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With tail calls, call site and return point in different procedures.

## Handling tail calls

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```
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```

With tail calls, call site and return point in different procedures.

Cross-procedure summaries:

From entry of `app` to exit of `id`.

## Handling first-class control

Summarization relies on call/return nesting.

As a result, it can't handle generators, coroutines, `call/cc`.

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Restricted CPS:

$(\lambda_1 (f \ cc) (f (\lambda_2 (u \ k) (\text{cc } u)) \ cc))$



$(\lambda_1 (f \ cc) (f (\lambda_2 (u \ k) (u \ 123 \ \text{cc})) \ cc))$





## Handling first-class control

Summarization relies on call/return nesting.

As a result, it can't handle generators, coroutines, `call/cc`.

Restricted CPS:

$(\lambda_1 (f \ cc) (f (\lambda_2 (u \ k) (\text{cc } u)) \ cc))$  ✓

$(\lambda_1 (f \ cc) (f (\lambda_2 (u \ k) (u \ 123 \ \text{cc})) \ cc))$  ✗

Effective stack reasoning in the presence of first-class control.

Summaries for `call/cc`: connect entry of  $\lambda_1$  with `call (cc u)`.

All kinds of summaries (normal call/return, tail calls, exceptions, first-class control) connect a continuation passed to a user function with the state that calls it.

# Theoretical formulation of CFA2

Abstract interpretation of CPS programs (1st-class control).

Concrete semantics [Might 07]

⇓ expose stack structure

Abstract semantics

- Orbit stack policy
- Stack and heap environments
- Stack and heap references

⇓ nothing tricky here

Local semantics

No stack.

+ summarization

- Generalized summaries (tail calls, call/cc).
- Record callers as you find them.

# Correctness

## Simulation

The abstract semantics is a safe approximation of the runtime behavior of the program.

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## Soundness

The summarization algorithm doesn't miss any flows of the abstract semantics . . .

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The abstract semantics is a safe approximation of the runtime behavior of the program.

## Soundness

The summarization algorithm doesn't miss any flows of the abstract semantics . . .

## Completeness

. . . and it doesn't add spurious flows.

# JavaScript

The only composite piece of data is the object.  
Functions, arrays are objects.

Object: map from strings (property names) to values.  
Properties can be added/deleted at runtime.  
Full field sensitivity undecidable.

Inheritance: each object has one prototype object.  
No cycles in the prototype chain.

# Static analysis for JavaScript

Array access: `a[i]`

General computed-property access: `obj[prop]`

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The prototype chain can have cycles.

Exceptions are included in the summaries.

Recursive implementation of call/return matching (ask me).

# Analyzing Firefox add-ons

Core JavaScript manageable in a summer.  
Inferred types for Sunspider, V8 benchmarks.  
DOM is huge.

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Content: Webpage elements

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Events can be generated from chrome/content.  
Listeners can be attached to chrome/content.  
New architecture prevents listening on chrome for content.

# Results

	LOC	time (ms)	safe/total
Commentblocker	537	248	3/10
Flashblock	935	357	3/5
Imtranslator	1263	406	2/4
Flagfox	2081	896	5/12
Greasemonkey	4809	1716	13/23
Flashgot	9741	4524	10/21
Video download helper	12749	4621	13/19
Web developer	22018	12603	9/63
Stumbleupon	32594	18235	13/44

# ToDo list

CFA2:

- ▶ Complexity? Polytime variant?
- ▶ Completeness for first-class control?

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Declarative specification of an analysis (Jones–Muchnick vision)

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Polyvariant CFA should be very efficient

- ▶ if not much recursion/loops in  $p$ , then a bit slower than  $p$ .
- ▶ if recursion/loops in  $p$ , then much faster than  $p$ .

## More info

Slides: [www.ccs.neu.edu/home/dimvar/cfa2-shonan.pdf](http://www.ccs.neu.edu/home/dimvar/cfa2-shonan.pdf)

CFA2 w/out first-class control [ESOP 10, LMCS 11]

Restricted CPS [PEPM 11]

CFA2 w/ first-class control [ICFP 11]

DoctorJS: [github.com/mozilla/doctorjs](https://github.com/mozilla/doctorjs)