Turing Machines with Atoms

(LICS 2013)

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University of Warsaw

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Summary

- TMs with atoms model limited access to data

Theorem. In sets with atoms, there is a language that is decidable in nondeterministic polynomial time, but not deterministically semi-decidable.

(proof technique: Cai-Fürer-Immerman graphs)

Sorting

Comparison model

- numbers given as units
- compared in one step
- nothing else is allowed

$3 < 7$

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Turing machines

- numbers represented as strings
- arbitrary manipulation allowed

0	1	1

1	1	1

Sets with atoms

(nominal sets, sets with urelements, permutation models)

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$\mathbb{A}$

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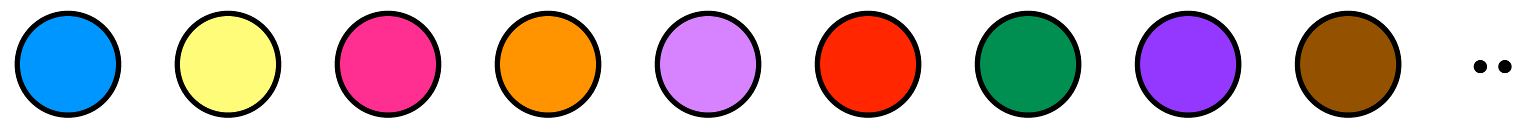

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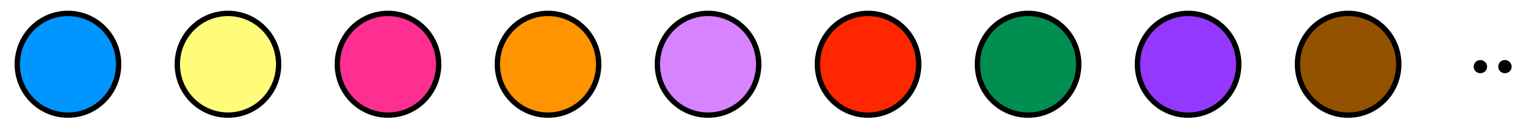
$\mathbb{A}$ 

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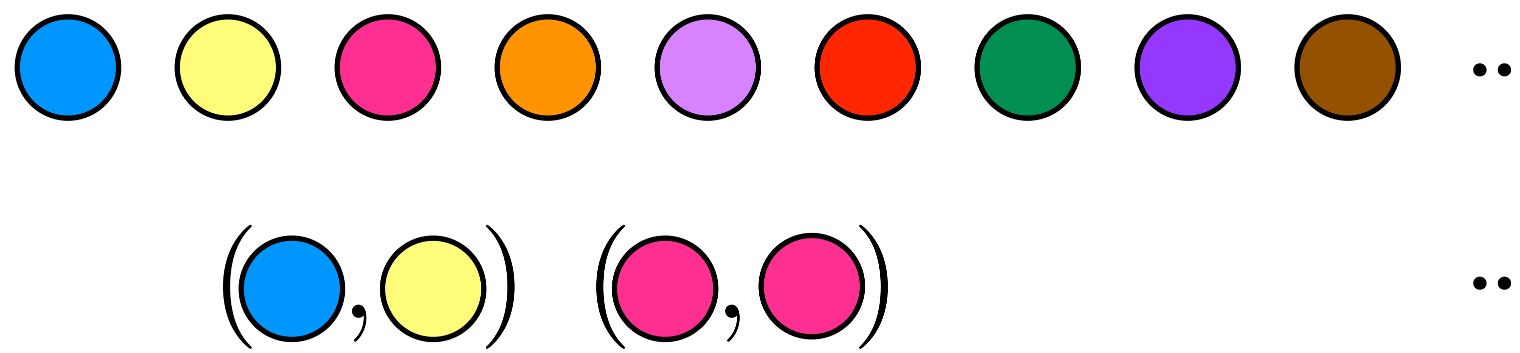

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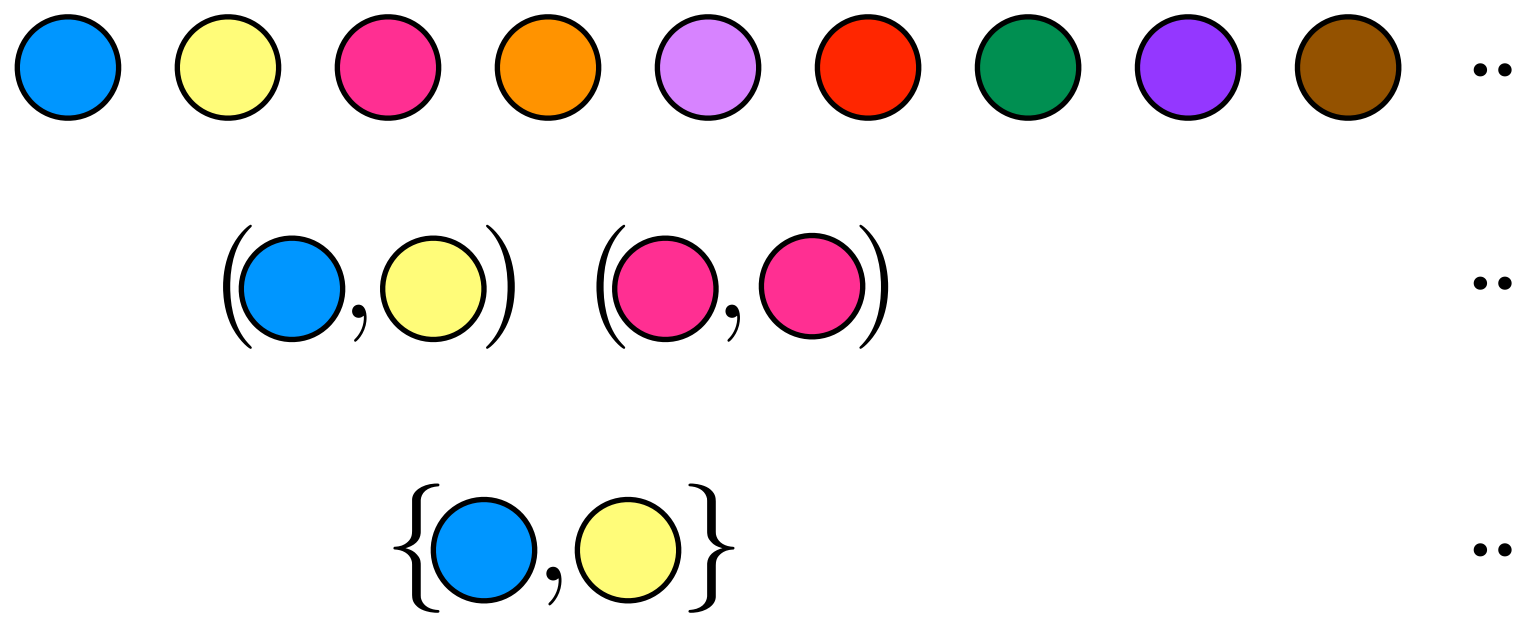

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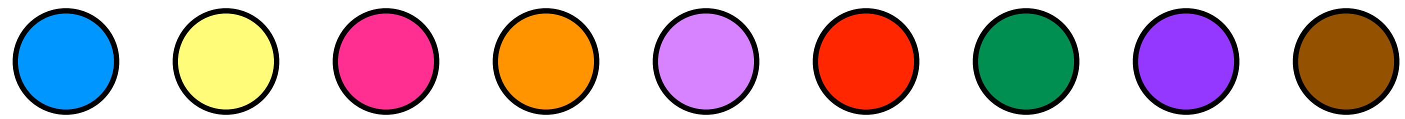
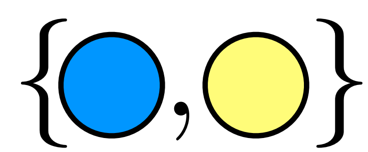
$\mathbb{A}$  ...

$\mathbb{A}^2$  ...

$\binom{\mathbb{A}}{2}$  ...

$$\mathbb{A}^{\triangleleft} = \{\{(a,b,c),(b,c,a),(c,a,b)\} \mid a,b,c \in \mathbb{A}\}$$

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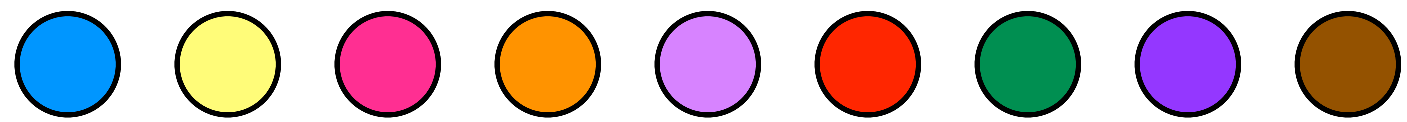

$\cdots$

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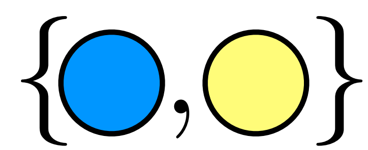

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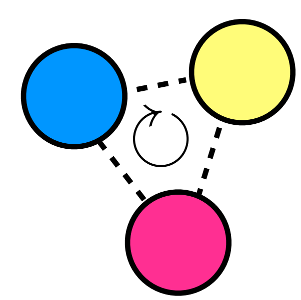

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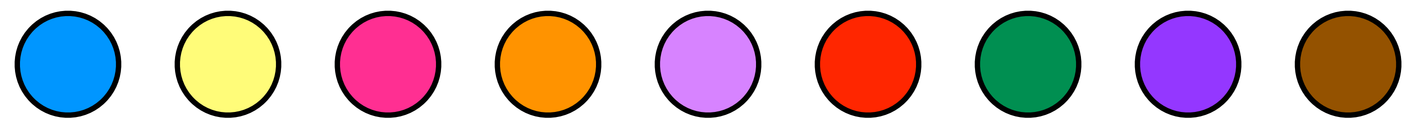
$\mathbb{A}$  $\ldots$

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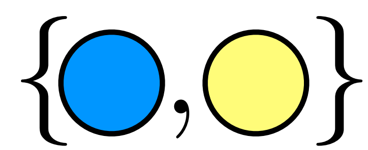
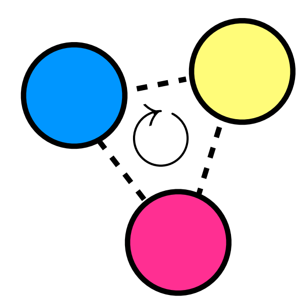
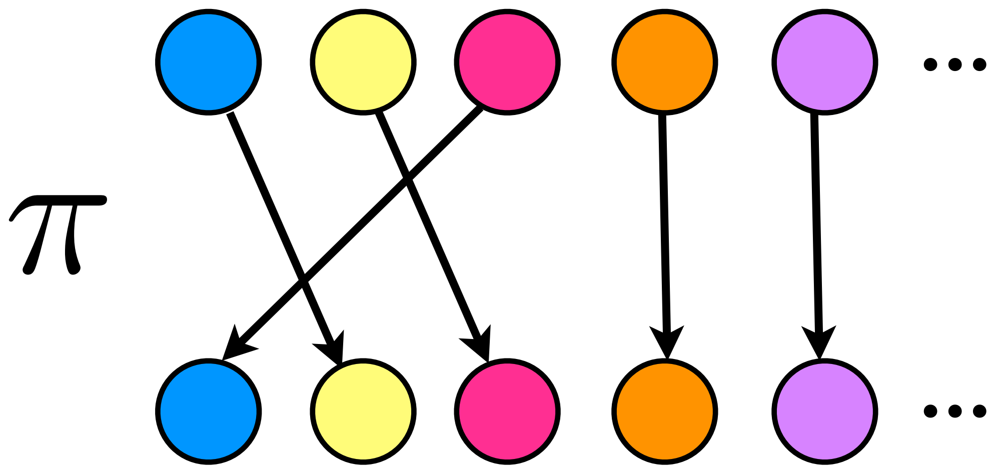
$\binom{\mathbb{A}}{2}$  $\ldots$

$\mathbb{A}^{\triangleleft}$ 

(must be “hereditarily finitely supported”)

Actions and orbits



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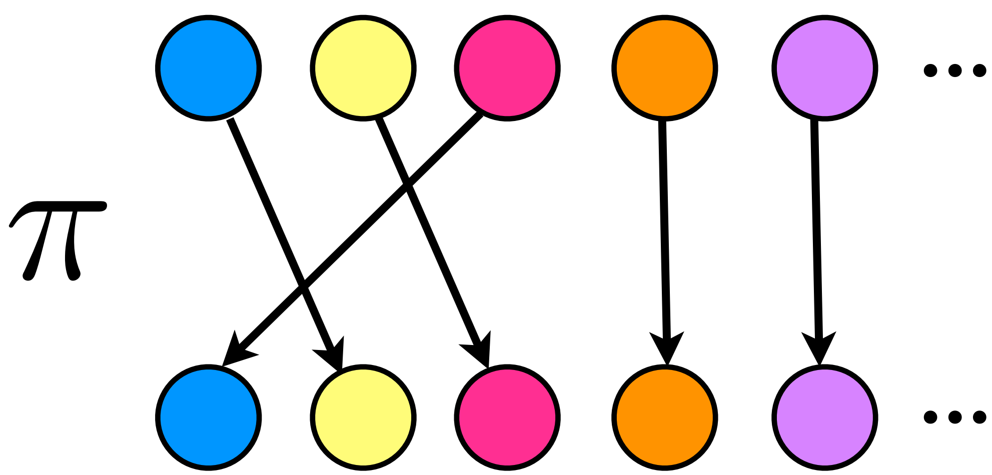

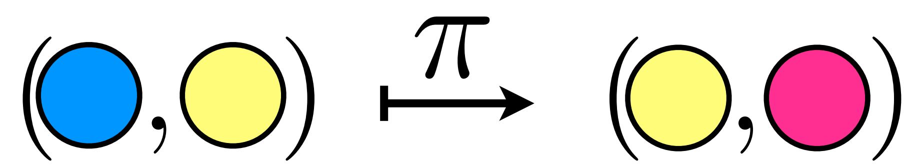

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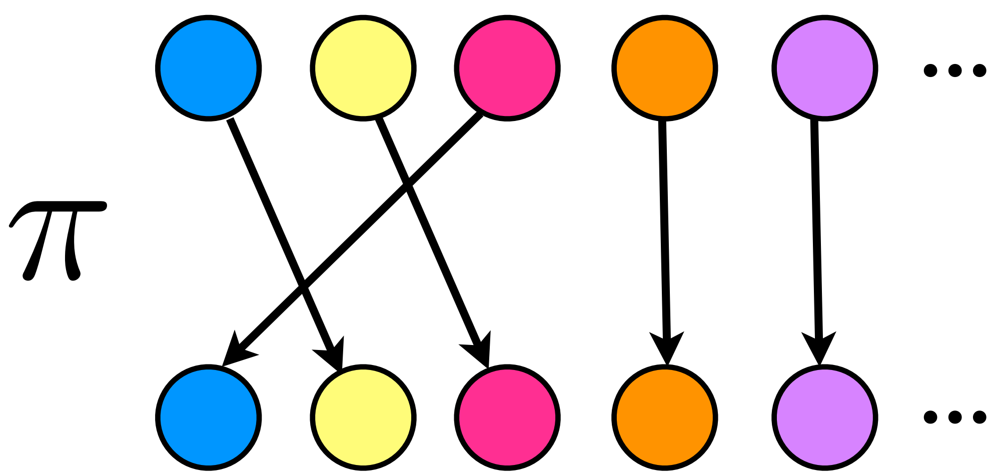

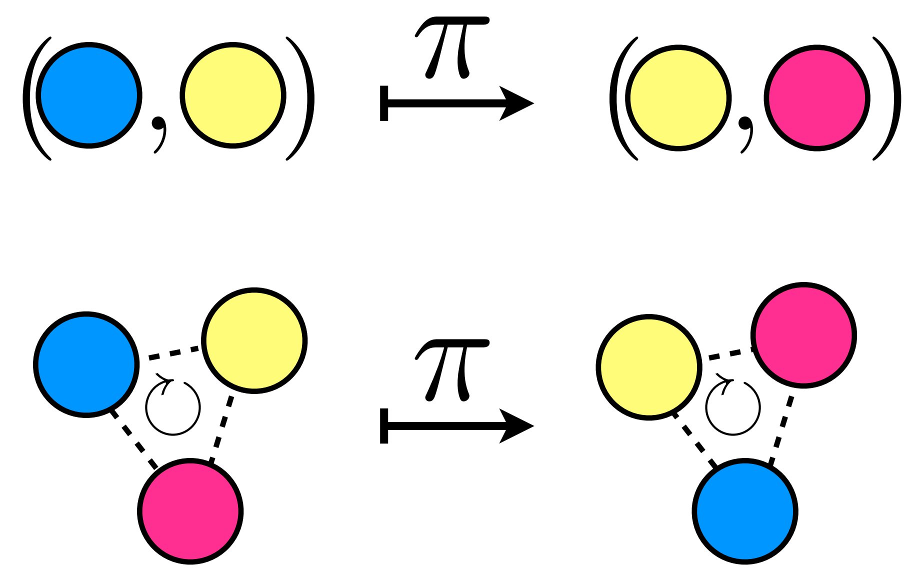

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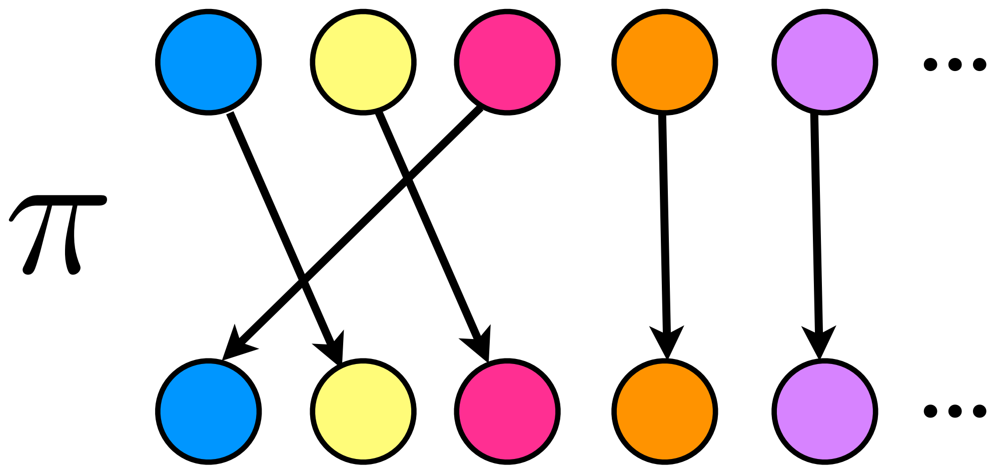

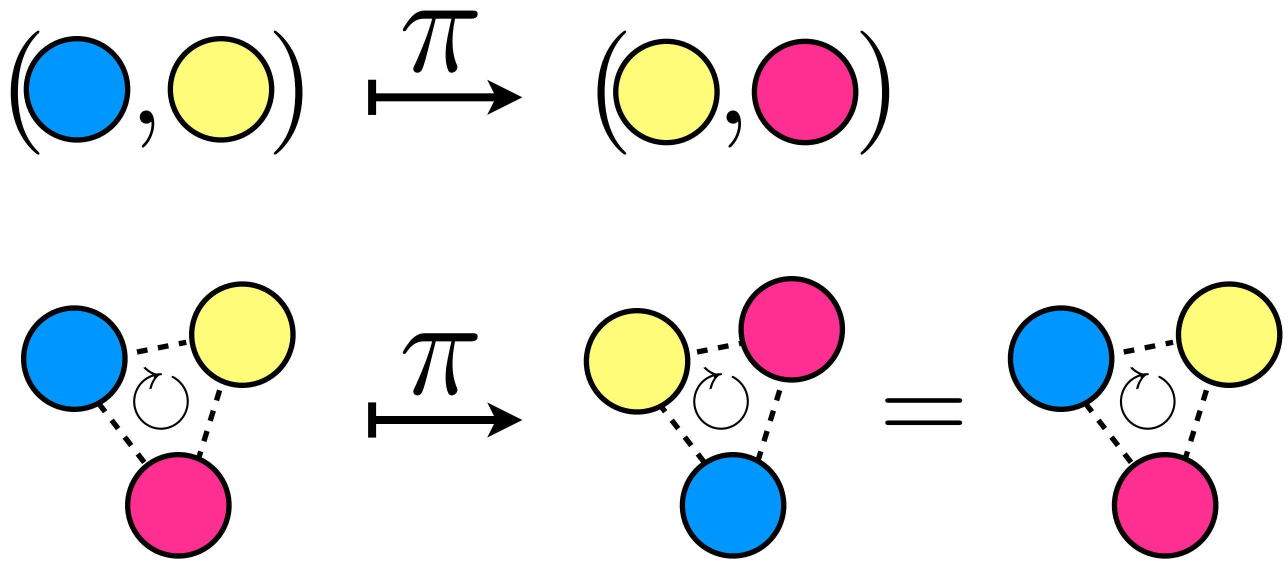

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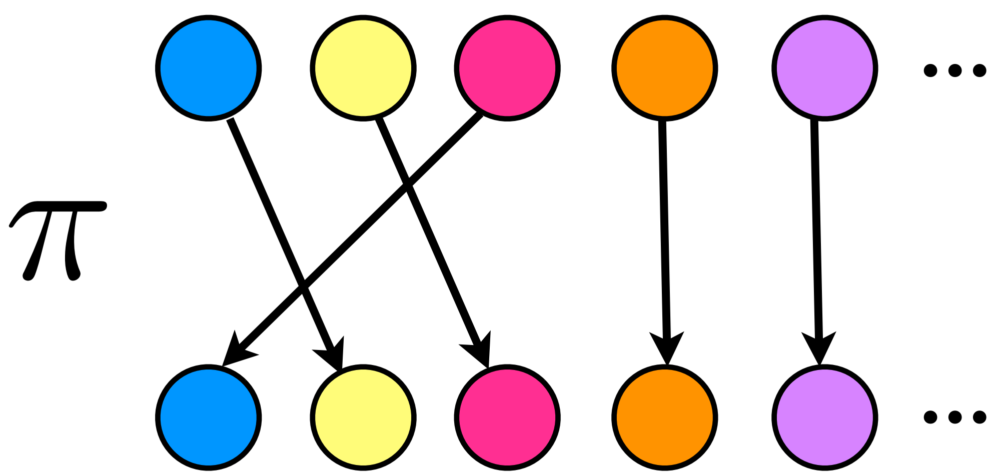

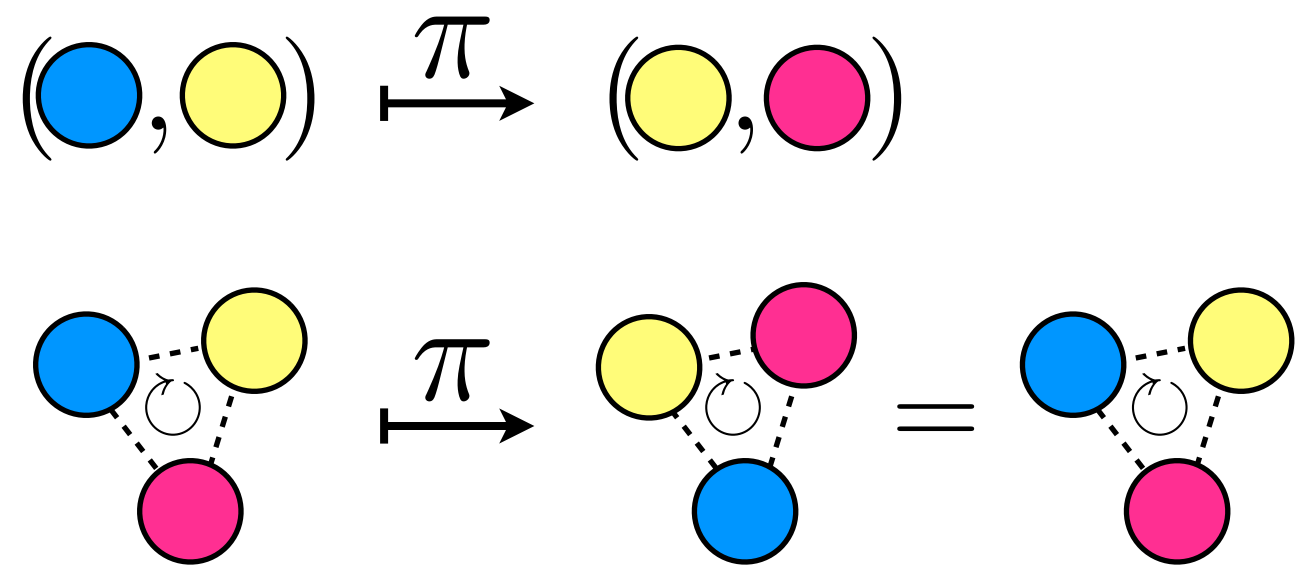

$$\mathrm{Orbit}(x) = \{\pi(x) \mid \pi \in \mathrm{Perm}(\mathbb{A})\}$$

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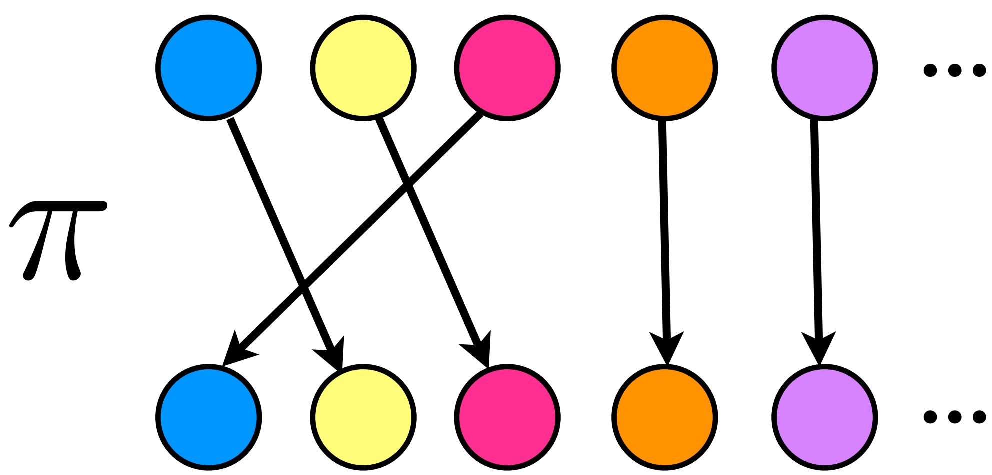

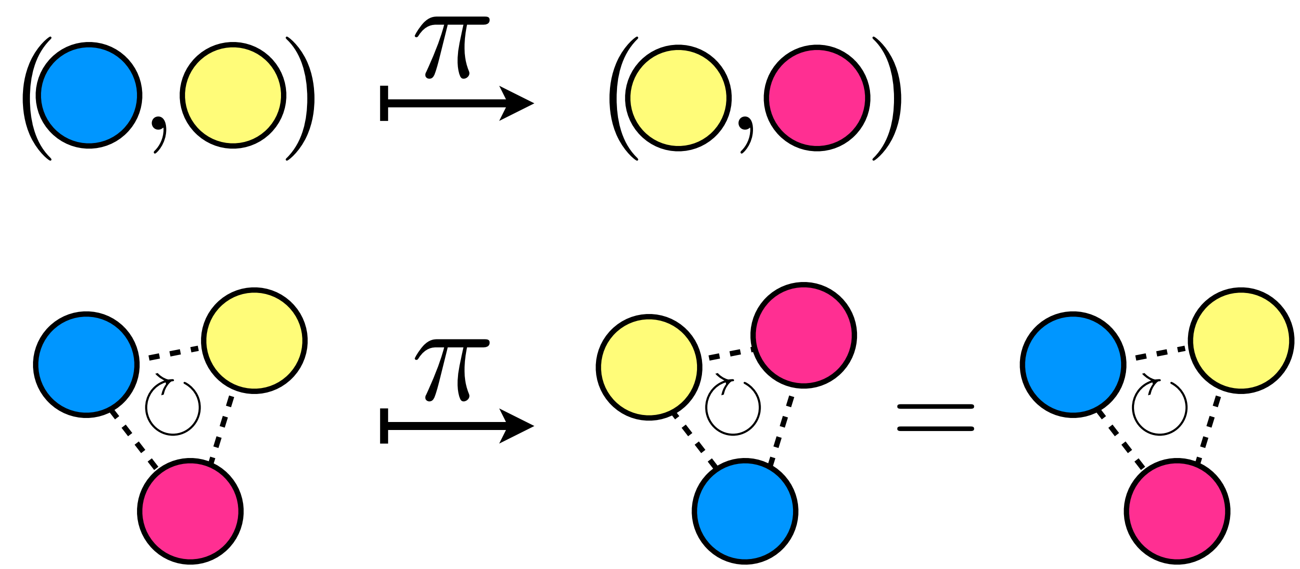

$$\mathrm{Orbit}(x) = \{\pi(x) \mid \pi \in \mathrm{Perm}(\mathbb{A})\}$$

Equivariant function $f : X \to Y$:

$$f(\pi(x)) = \pi(f(x))$$

for all $x \in X, \pi \in \mathrm{Perm}(\mathbb{A})$

Turing machines with atoms

- orbit-finite tape alphabet Γ
- orbit-finite input alphabet $\Sigma \subseteq \Gamma$
- orbit-finite set of states Q
- initial state $q_0 \in Q$, final states $F \subseteq Q$
- equivariant transition function

$$\delta : Q \times \Gamma \to Q \times \Gamma \times \{-1, 0, 1\}$$

Example 1

"last letter appears before" $\Sigma = \mathbb{A}$

Nondeterministic:

- read left-to-right
- guess that a current letter will be last
- read to the end
- check the last letter

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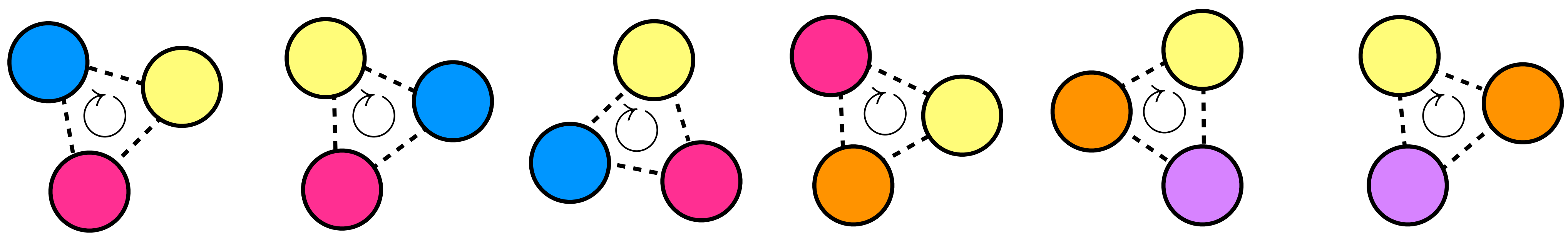
Deterministic:

- read to the end
- store last letter
- come back and check

Example 2

“compatible chains of rotating triangles”



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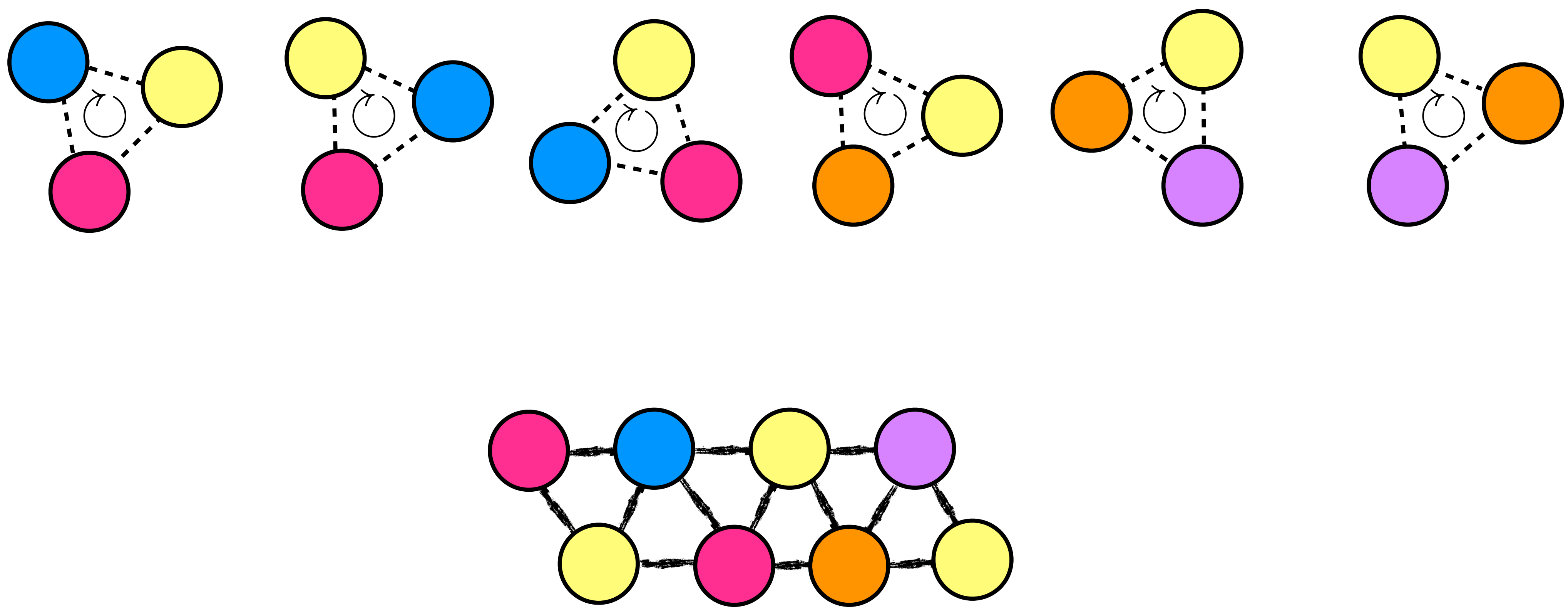

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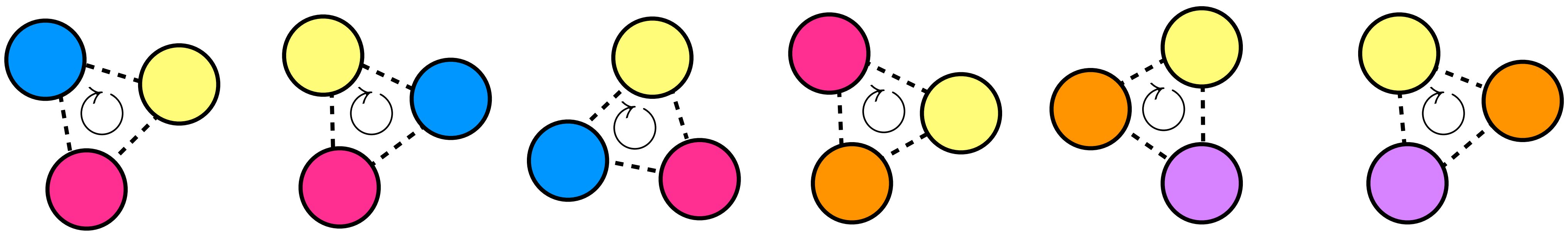

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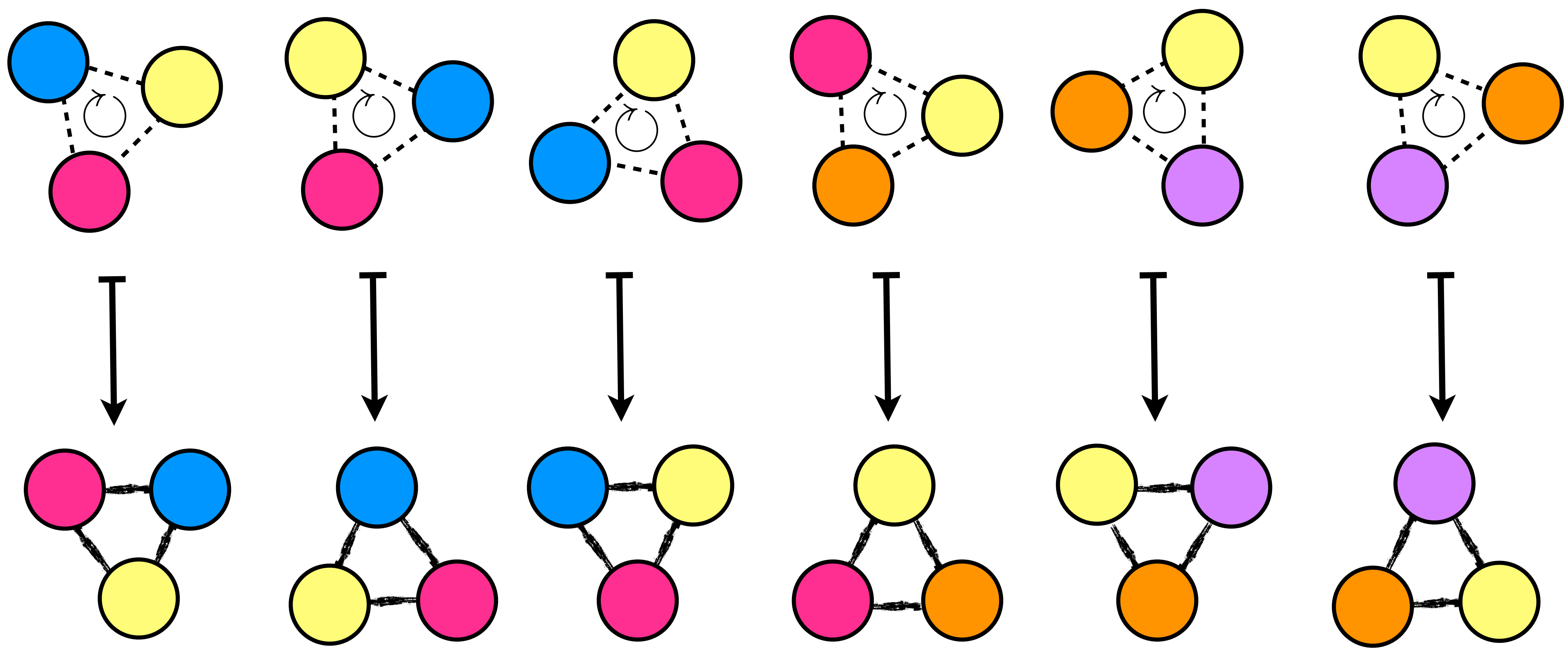

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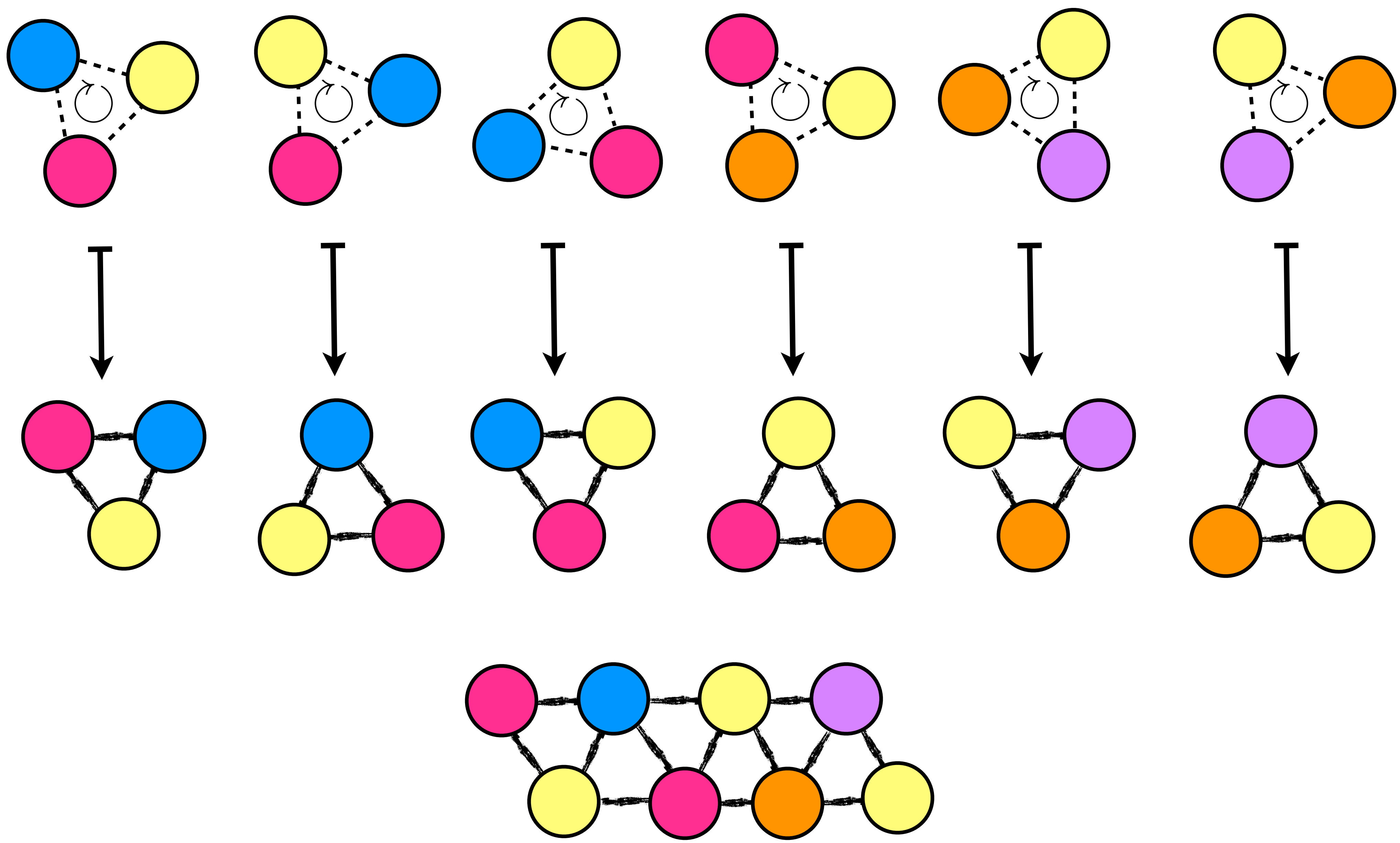

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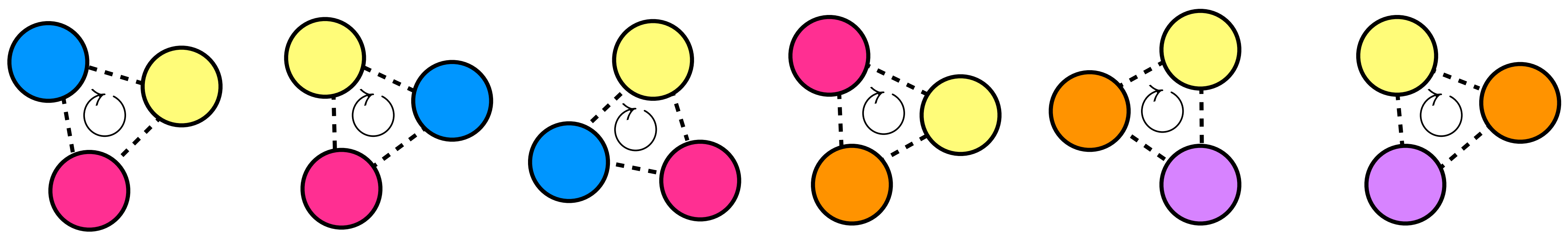

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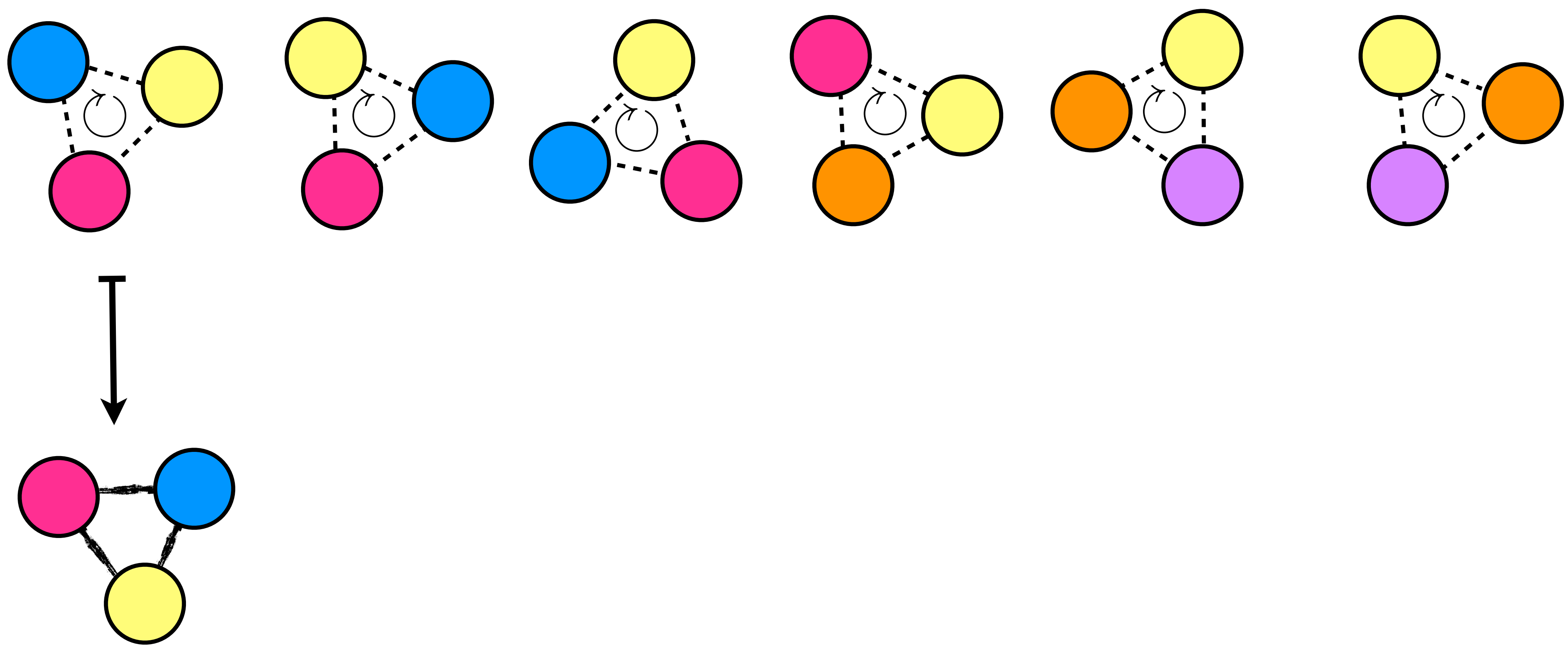

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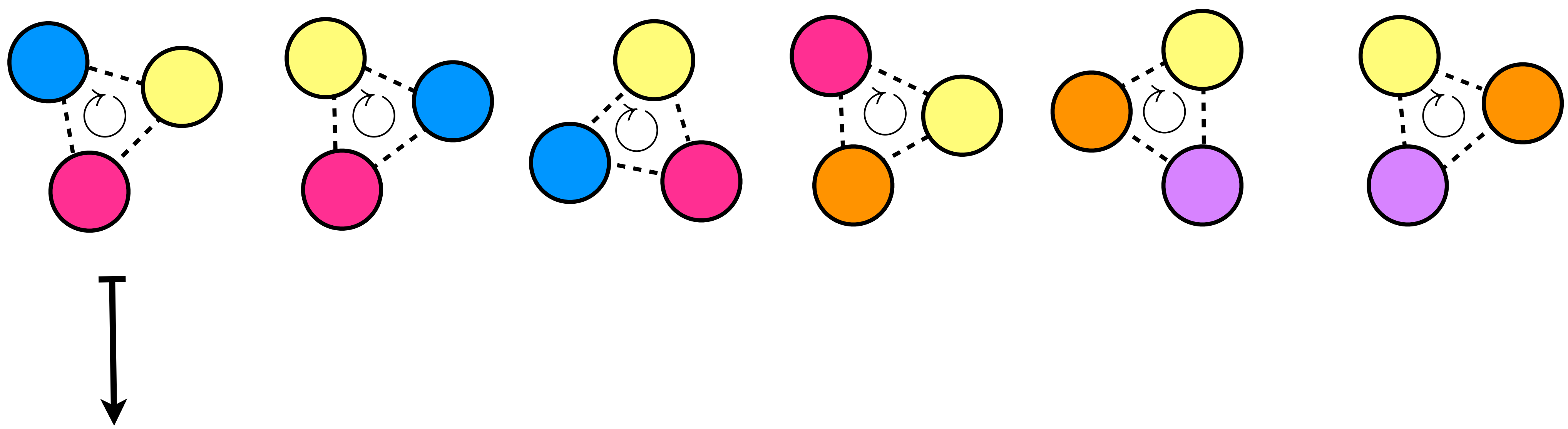

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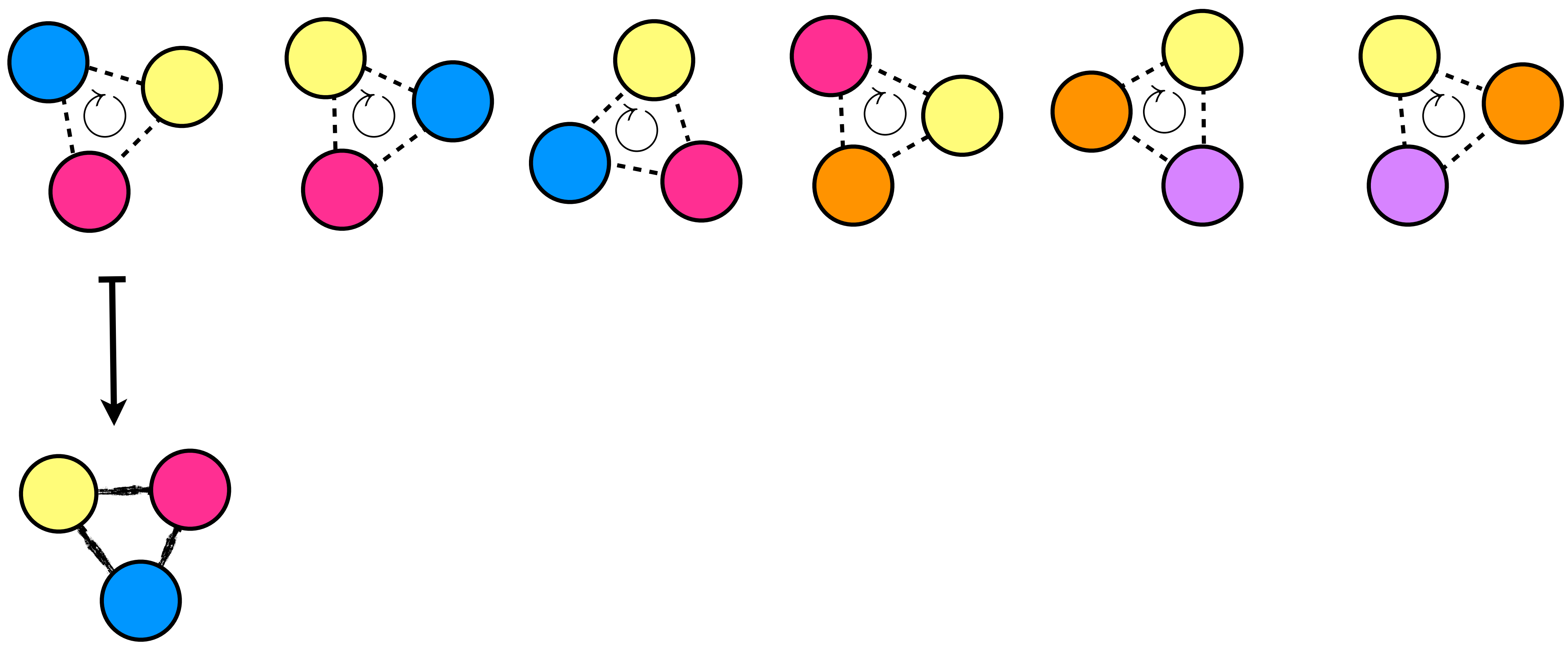

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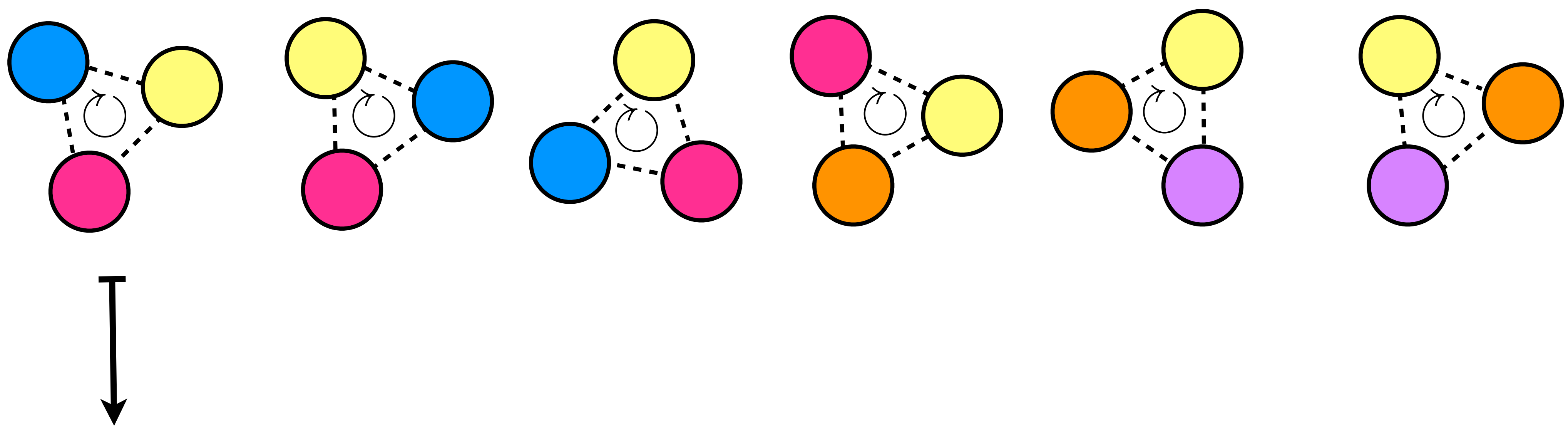

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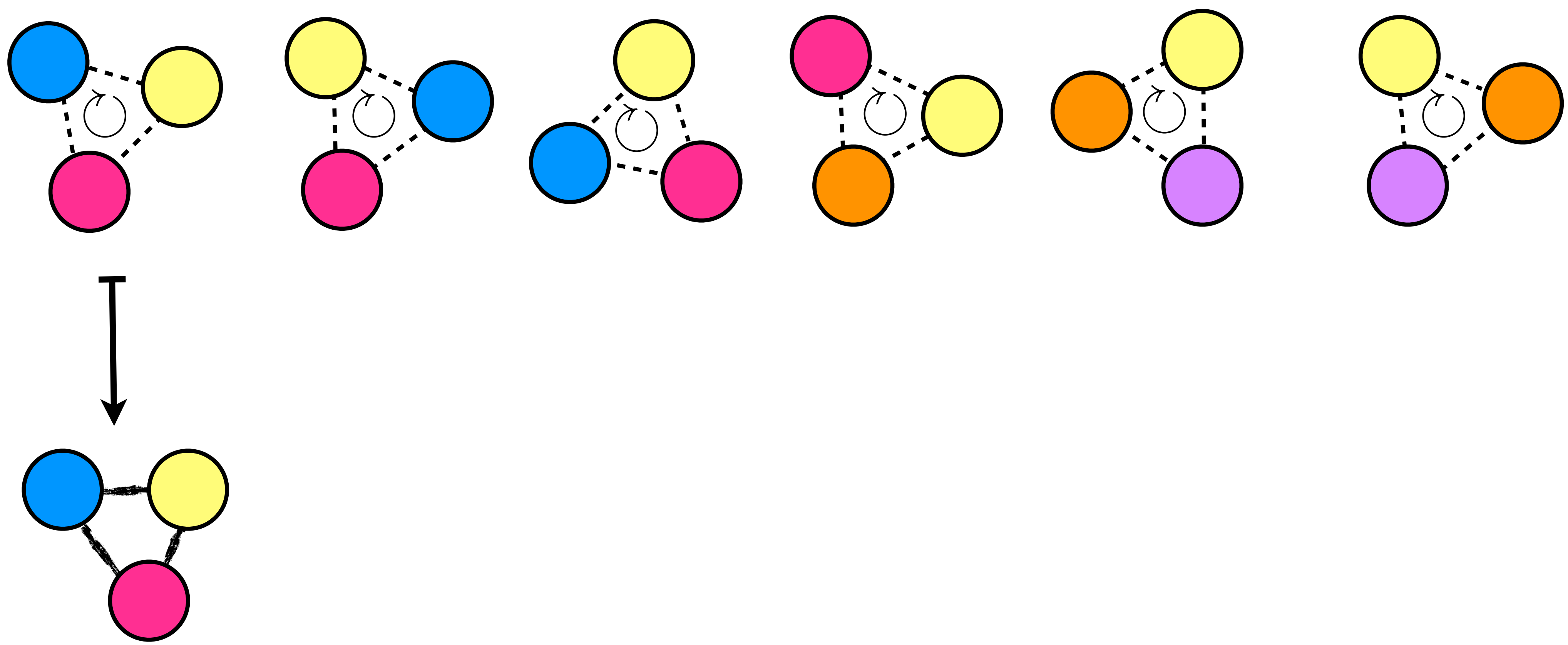

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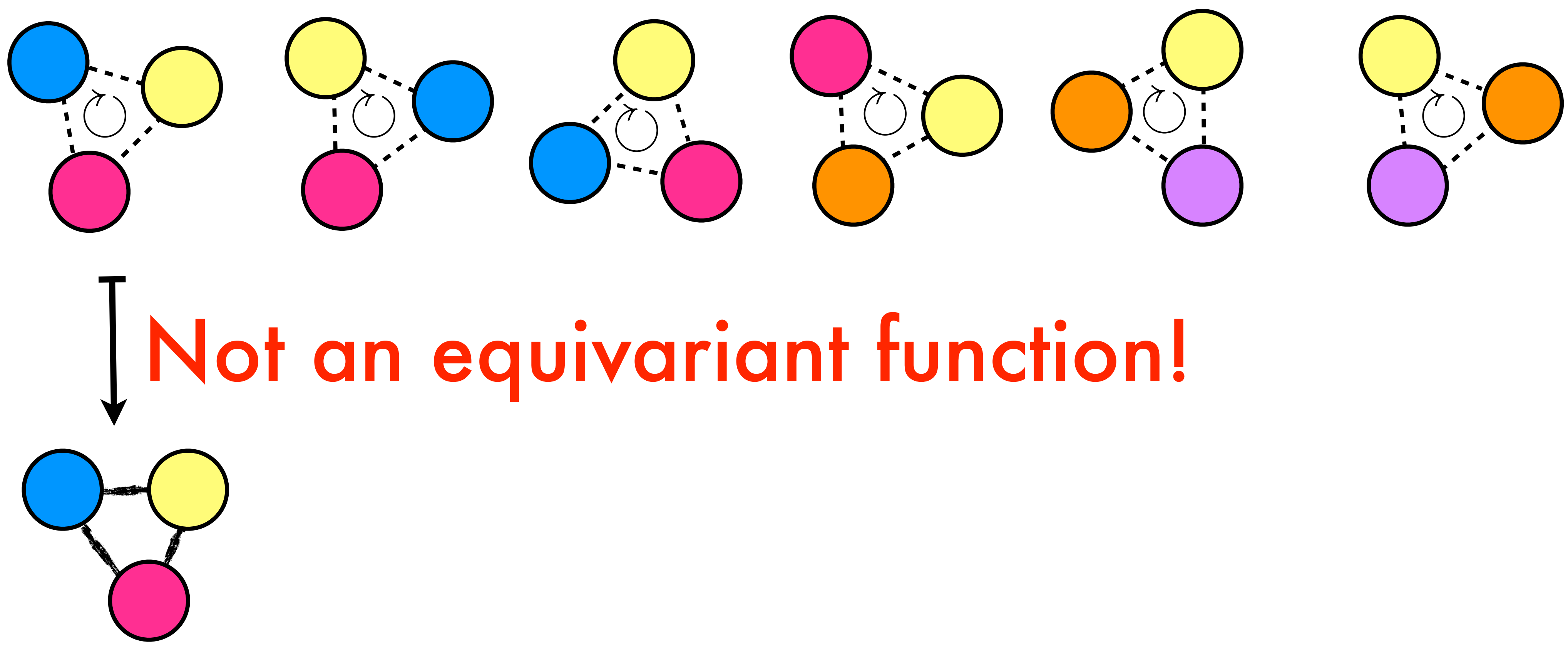

Not an equivariant function!

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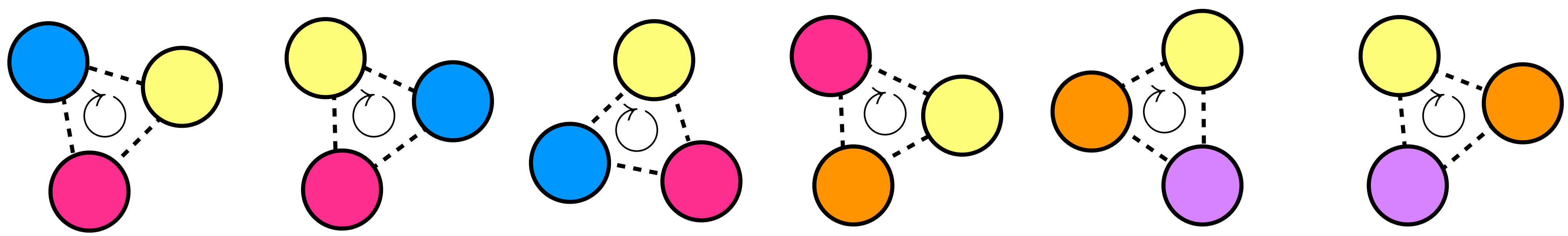

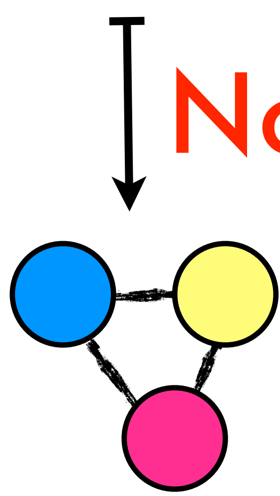

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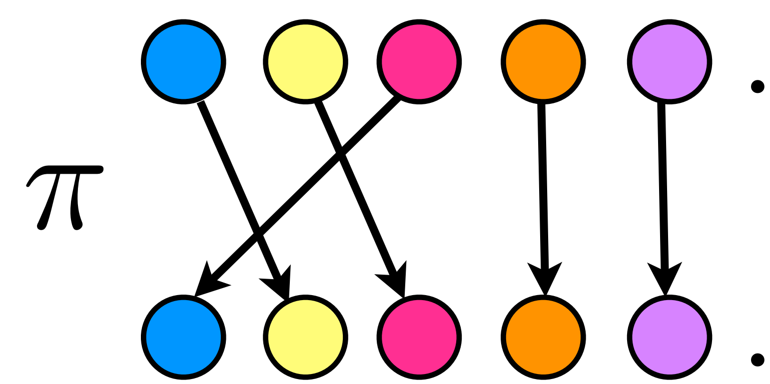

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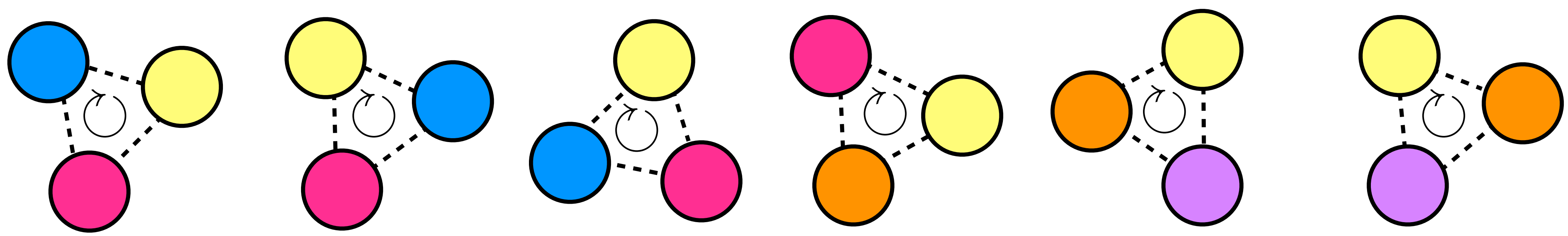

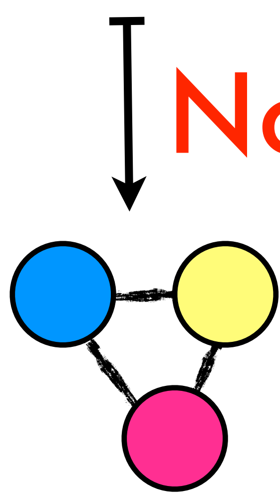

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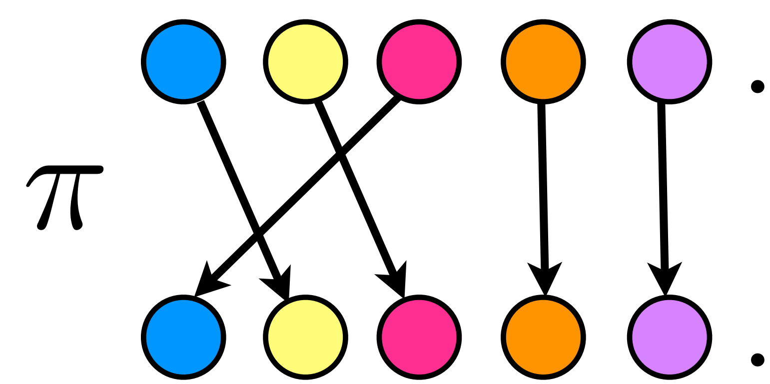

Determinisation fails.

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