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The accessibility of finitely presented groups

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§1. Introduction

Let K be a connected simplicial complex. The polyhedron $|K|$ is said to have more than one end (written $e(|K|)>1$) if there is a compact subset C of $|K|$ such that $|K|-C$ has more than one component with non-compact closure. Suppose a group G acts freely on a connected complex K so that $G\backslash K$ is finite. Such a complex exists if and only if G is finitely generated. For instance if G is finitely generated we could take K to be the Cayley graph of G with respect to a finite generating set. The property of whether or not $e(|K|)>1$ is independent of the particular complex K chosen. Thus we say that the finitely generated group G has more than one end, written $e(G)>1$, if $e(|K|)>1$ for any simplicial complex K on which G acts freely so that $G\backslash K$ is finite. An equivalent algebraic definition is that a finitely generated group G has more than one end if $H^1(G,\mathbb{Z}_2 G)\neq 0$. See Stallings [8] for proofs of the above statements and further background information.

In his Structure Theorem [9, Theorem 4.A.6.5 and Theorem 5.A.9] Stallings showed that for a finitely generated group G, $e(G)>1$ if and only if either G decomposes as a free product with amalgamation, $G=A*_C B$ where C is finite and $C\neq A$, $C\neq B$, or G is an HNN-group $G=\langle A,t|t^{-1}Ct=D\rangle$ where C and D are isomorphic finite subgroups of A. If either of these cases occur we say that G splits over a finite subgroup (with factors A, B in the first case and factor A in the second case).

If we now work within the framework of the Bass-Serre theory of groups acting on treees (see [2]), this theorem can be put in the following form.

Let G be a finitely generated group. Then $e(G)>1$, if and only if there is a G-tree T such that the stabilizer G_e is finite for each $e\in ET$ and $G\neq G_v$ for each $v\in VT$.

It can be shown that if G and T are as above, then G_v is finitely generated for each $v\in VT$. It is natural to ask if T can be chosen so that each G_v has at most one end.

Definition. A finitely generated group G is said to be accessible if there is a G-tree T such that G_e is finite for each $e\in ET$ and G_v has at most one end for each $v\in VT$.

Let G be finitely generated and suppose $e(G)>1$. By the Structure Theorem, G splits over a finite subgroup. If a factor has more than one end then it too splits over a finite subgroup. The group G is accessible if and only if the process of successively decomposing factors with more than one end terminates after a finite number of steps. For a proof of this equivalence see [2] or [3]. In [10] Wall conjectured that all finitely generated groups are accessible. In [3] (see also [2]) I showed that G is accessible if and only if either $H^1(G, RG)$ is finitely generated as an RG-module (R is an arbitrary ring) or if $\mathbb{Z} \otimes_{\mathbb{Z}G} \mathrm{Der}(G, \mathbb{Z}G)$ is a finitely generated abelian group. Linnell [5] has proved that a finitely generated group is accessible if its finite subgroups have bounded order.

In this paper I show that finitely presented groups are accessible. In fact accessibility is proved for a somewhat larger class of groups.

Definition. We say that a group G is almost finitely presented if there is a connected simplicial complex K, satisfying $H^1(K; \mathbb{Z}_2)=0$, on which G acts freely and $G \backslash K$ is finite.

This class of groups was considered by Stallings [8]. Every finitely presented group is almost finitely presented. To see this take L to be a finite simplicial complex such that $\pi_1(L) \cong G$ and then take K to be universal cover of L. In the terminology of Bieri and Strebel [1] a group is almost finitely presented if it is $(FP)_2$ over $\mathbb{Z}_2$. In a note added in proof they give an example of a group which is almost finitely presented but not finitely presented.

In this paper it is proved almost finitely presented groups are accessible.

The key idea behind the proof is an argument due to Kneser (see Hempel [4], p. 31). Kneser shows that in a compact 3-manifold M, there is an upper bound on the number of disjoint embedded 2-spheres that do not bound 3-balls and no two of which bound a region in M homeomorphic to $S^2 \times I$. If we shift the dimension down one, essentially the same argument shows that in a closed surface S there is an upper bound on the number of disjoint simple closed curves that do not bound discs and such that no two bound an embedded annulus in S. It is proved that a similar argument applies to any compact two-dimensional polyhedron if we generalize the idea of a simple closed curve in an appropriate way.

In another paper I will show that it is possible to deduce the equivariant loop and sphere theorems (Meeks, Yau [6] and [7]) using similar techniques.

The account given here is fairly self-contained. Thus the Structure Theorem is not assumed, and in fact can be deduced from Theorem 4.1 in the case when G is almost finitely presented.

I am grateful to Hamish Short for pointing out the quick proof of Theorem 3.3 included here.

§ 2. Tracks

Let L be a connected 2-dimensional complex. A *track* is a subset S of $|L|$ with the following properties.

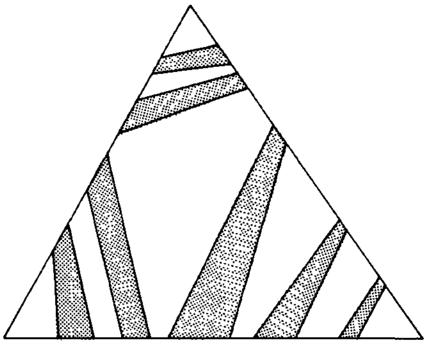

Fig. 1

(i) S is connected.

(ii) For each 2-simplex σ of $|L|$, $S \cap |\sigma|$ is a union of finitely many disjoint straight lines joining distinct edges of σ.

(iii) If γ is a 1-simplex of L and γ is not a face of a 2-simplex, then either $S \cap |\gamma| = \emptyset$ or S consists of a single point in the interior of $|\gamma|$.

If $|L|$ is a 2-manifold then S is a connected 1-dimensional submanifold.

A *band* is a subset B of $|L|$ with the following properties.

(i) B is connected.

(ii) For each 2-simplex σ of L, $B \cap |\sigma|$ is a union of finitely many components each of which is bounded by two closed intervals in distinct faces of σ and the disjoint lines joining the end points of these intervals (see Fig. 1).

(iii) If γ is a 1-simplex of L which is not a face of 2-simplex, then either $B \cap |\gamma| = \emptyset$ or B consists of a subset of $|\gamma|$ bounded by two points in the interior of $|\gamma|$.

If B is a band then we get a track $t(B)$ by choosing the midpoint of each component of $|\gamma| \cap B$ for every 1-simplex γ of L and joining these points in the appropriate components of $|\sigma| \cap B$ where σ is a 2-simplex of L.

We say that B is untwisted if B is homeomorphic to $t(B) \times [0,1]$ in which case ∂B has two components each homeomorphic to $t(B)$. Otherwise B is said to be twisted. In this case ∂B is a track which double covers $t(B)$. If S is a track then S is called twisted (or untwisted) if there is a twisted (untwisted) band B such that $t(B) = S$. Two tracks S_1 and S_2 are said to be parallel if there is an untwisted band B such that $\partial B = S_1 \cup S_2$.

Let S be a track. We define a 1-cochain $z(S)$ with coefficients in $\mathbb{Z}_2$ as follows. If γ is a 1-simplex of L $z(S)(\gamma) = k \pmod 2$ where $k = \#(|\gamma| \cap S)$. If σ is a 2-simplex, then $\partial |\sigma| \cap S$ has an even number of points. Hence $\delta z(S) = 0$, i.e. $z(S)$ is a 1-cocycle. Now $z(S)$ is a coboundary if and only if S separates $|L|$, i.e. $|L| - S$ has two components. A twisted track cannot separate $|L|$. Hence if S is twisted, $z(S)$ represents a non-trivial element of $H^1(L; \mathbb{Z}_2)$. In fact no non-empty union of disjoint twisted tracks separates $|L|$. Hence the corresponding elements of $H^1(L; \mathbb{Z}_2)$ are linearly independent. We have the following proposition.

Proposition 2.1. *Suppose $\beta = \operatorname{rank} H^1(L;\mathbb{Z}_2)$ is finite. Let $T=\{t_1, t_2, \dots, t_n\}$ be a set of disjoint tracks. Then $|L| - \bigcup t_i$ has at least $n-\beta$ components. The set T contains at most β twisted tracks.*

Proof. Let M be the subgroup of $H^1(L;\mathbb{Z}_2)$ generated by the elements corresponding to $z(t_1)$, $z(t_2), \dots, z(t_n)$. The kernel of the obvious epimorphism

$$\theta\colon \underbrace{\mathbb{Z}_2 \oplus \mathbb{Z}_2 \oplus \dots \oplus \mathbb{Z}_2}_{n \text{ copies}} \to M$$

has a basis corresponding to components of $|L| - \bigcup t_i$. The proposition follows immediately.

Suppose now that L is finite. Let

$$v_L = \text{number of vertices of } L,$$
$$f_L = \text{number of 2-simplexes of } L$$

and

$$n(L) = 2\beta + v_L + f_L.$$

Theorem 2.2. *Suppose $t_1, t_2, \dots, t_k$ are disjoint tracks in $|L|$. If $k > n(L)$, then there exists $i \neq j$ such that t_i and t_j are parallel.*

Proof. If σ is a 2-simplex of L and D is the closure of a component of $|\sigma| - \bigcup t_i$, then D is a disc. We say that D is good if $\partial D \cap \partial|\sigma|$ consists of two components in distinct faces of σ. For any σ there are at most three D's which contain a vertex of σ and at most one other component which is not good. Thus in Fig. 2 the shaded region is the only component which is not good and does not contain a vertex.

If $k > n(L)$ then $|L| - \bigcup t_i$ has more than $v_L + f_L + \beta$ components. It follows that there are at least $\beta + 1$ components whose closures are bands. Since $|L|$ contains at most β disjoint twisted bands, there is at least one component whose closure is an untwisted band. The Theorem follows immediately.

In the next proposition we no longer assume that L is finite.

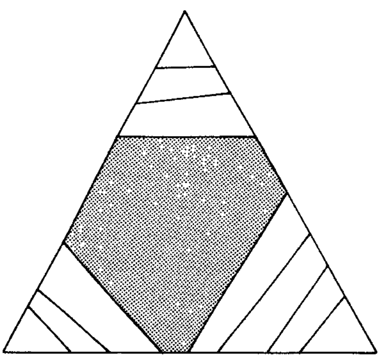

Fig. 2

Proposition 2.3. *Let $J \subset |L^1| - |L^0|$ and suppose for every 1-simplex γ of L, $j(\gamma) = \#(|\gamma| \cap J)$ is finite. Suppose also that for each 2-simplex σ of L with faces $\gamma_1, \gamma_2, \gamma_3$ we have $j(\gamma_1)+j(\gamma_2)+j(\gamma_3)=2m$ where m is an integer and $m-j(\gamma_i)\geqq 0$ for $i=1,2,3$. There exists a unique set T of disjoint tracks such that $J = |L^1| \cap (\bigcup_{t\in T} t)$.*

Proof. If such a set T exists then consider the set $Q=|\sigma|\cap(\bigcup_{t\in T} t)$. Suppose there are α_1 lines in Q joining γ_2 and γ_3, α_2 lines joining γ_1 and γ_3 and α_3 lines joining points in γ_1 and γ_2. Then $j(\gamma_1)=\alpha_2+\alpha_3$, $j(\gamma_2)=\alpha_1+\alpha_3$ and $j(\gamma_3)=\alpha_1+\alpha_2$. Hence $\alpha_i = m - j(\gamma_i)$, $i=1,2,3$. By our hypothesis $\alpha_i \geqq 0$ and so the required pattern of lines exists and is unique.

If J is a set satisfying the conditions of Proposition 2.3 let $S_J = \bigcup\{t\in T\}$ where T is the set of tracks constructed above.

§ 3. Minimal tracks

In this section K is a connected 2-complex for which $H^1(K;\mathbb{Z}_2)=0$. If t is a track, $z(t)$ is a coboundary and so t separates $|K|$.

We put $\|t\| = \#(|K^1|\cap t)$.

Definition. A track $t \subset |K|$ is called minimal if

$M1$ $\|t\|$ is finite.

$M2$ The two components of $|K|-t$ each contain infinitely many vertices of K.

$M3$ If the track $t_1 \subset |K|$ also satisfies $M1$ and $M2$ then $\|t_1\| \geqq \|t\|$.

If a minimal track $t\subset|K|$ exists then we put $m(K)=\|t\|$.

If U is a set of vertices of K, i.e. $U \subset |K^0|$, then put $U^* = |K^0| - U$. Let $w(U)$ denote the number (possibly ∞) of 1-simplexes of K which have one vertex in U and one in U^*.

Proposition 3.1. *There exist minimal tracks in $|K|$ if and only if there is a set $U\subset|K^0|$ such that both U and U^* are infinite and $w(U)$ is finite. If such a U exists then $m(K)\leqq w(U)$. If $w(U)=m(K)$ then there is a minimal track t such that U is the set of vertices in one component of $|K|-t$.*

Proof. If a minimal track t exists, then let U be the set of vertices in one component of $|K|-t$. It is easy to see that U has the required property.

Conversely suppose there exists $U\subset|K^0|$ such that both U and U^* are infinite and $w(U)$ is finite. Choose a set $J\subset|K^1|-|K^0|$ in the following way. For each 1-simplex γ of K, $J\cap|\gamma|$ consists of just one element if both U and U^* contain a vertex of γ, otherwise $J\cap|\gamma|=\emptyset$. It is easy to see that J satisfies the conditions of Proposition 2.3. Hence there is a set T of disjoint tracks such that $J=|K^1|\cap\bigcup_{s\in T} s$. Note that $w(U)=\sum_{s\in T}\|s\|$. Suppose for each $s\in T$, $|K|-s$ has a component with only finitely many vertices. Then $|K|-\bigcup_{s\in T} s$ has a component C such that if $V=|K^0|\cap C$ then V^* is finite. But $V\subset U$ or $V\subset U^*$ and we have a contradiction since U and U^* are both infinite. Thus for some $s\in T$, $|K|-s$

has two components each containing infinitely many vertices. It follows that minimal tracks exist and $w(U) \geqq \|s\| \geqq m(K)$.

If $w(U) = m(K)$ then T contains just one minimal track t, and U is the set of vertices in one component of $|K| - t$.

It is clear from Proposition 3.1 that a minimal track intersects a 1-simplex in at most one point, and hence meets a 2-simplex in at most one arc.

It is also clear from Proposition 3.1 that minimal tracks exist in $|K|$ if and only if $e(|K|) > 1$.

Proposition 3.2. *Let s, t be minimal tracks and suppose $s \cap t \cap |K^1| = \emptyset$. Let $J = (s \cup t) \cap |K^1|$. Then S_J is a union of two disjoint minimal tracks.*

Proof. Let L_s, R_s be the components of $|K| - s$ and let L_t, R_t be the components of $|K| - t$. Let $U = L_s \cap |K^0|$ and $V = L_t \cap |K^0|$. Now either $U \cap V$ and $U^* \cap V^*$ are both infinite or $U^* \cap V$ and $U \cap V^*$ are both infinite. For if say $U \cap V$ and $U^* \cap V$ are finite, then V is finite which contradicts $M2$ for t. A similar argument works for each of the other three possibilities. By relabelling, if necessary, we can assume that $U \cap V$ and $U^* \cap V^*$ are both infinite. Since $U \cap V$ and $U^* \cap V^*$ are disjoint, $w(U \cap V) \geqq m(K)$ and $w(U^* \cap V^*) \geqq m(K)$.

Let γ be a 1-simplex of K. If γ has one vertex in $U \cap V$ (or $U^* \cap V^*$) and one in $(U \cap V)^*$ (or $(U^* \cap V^*)^*$), then $|\gamma|$ contains a point of J. In the case when γ has one vertex in $U \cap V$ and one in $U^* \cap V^*$, then $|\gamma|$ contains two points of J. Thus

$$w(U \cap V) + w(U^* \cap V^*) \leqq \#(J) = 2m(K).$$

Hence $w(U \cap V) = w(U^* \cap V^*) = m(K)$. The above argument also shows that if $|\gamma|$ contains two points of J, then one vertex of γ is in $U \cap V$ and the other vertex is in $U^* \cap V^*$. For otherwise we would have $w(U \cap V) + w(U^* \cap V^*) < \#(J)$.

Let σ be a 2-simplex of K and suppose $s \cap t \cap |\sigma| \neq \emptyset$. Now $s \cap |\sigma|$ and $t \cap |\sigma|$ must consist of single line segments. The situation will be as in Fig. 3(a) or Fig. 3(b). In S_J these lines are replaced by the dotted lines. Now σ has an edge containing two points of J. One vertex of this edge will be in $U \cap V$ and one in $U^* \cap V^*$. It can be seen that there is a component of $S_J \cap |\sigma|$ which separates the vertices of σ which are in $U \cap V$ from those which are in $(U \cap V)^*$, and the other component of $S_J \cap |\sigma|$ separates the vertices of σ which are in $U^* \cap V^*$ from those which are in $(U^* \cap V^*)^*$. More generally in each 2-simplex of K

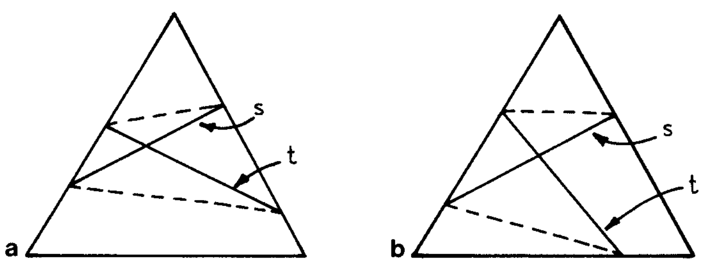

Fig. 3

which contains vertices of $U \cap V$ and $(U \cap V)^*$ there is a segment of S_J which separates these vertices. These segments can be chosen so that their union s' is a union of tracks and $S_J = s' \cup t'$, $s' \cap t' = \emptyset$, where t' is a union of segments separating vertices of $U^* \cap V^*$ and $(U^* \cap V^*)^*$. But $|s'| + |t'| = 2m(K)$. It follows as in the proof of Proposition 3.1 that s' and t' are minimal tracks.

The proof above also shows that either $U \cap V^*$ or $U^* \cap V$ is finite. For suppose $U \cap V$, $U^* \cap V$, $U \cap V^*$ and $U^* \cap V^*$ are all infinite. Then $s \cap t \neq \emptyset$. For if $s \cap t = \emptyset$, then a component of $|K| - s$ is contained in a component of $|K| - t$ and one of the four intersections is empty. Thus there must be a 1-simplex of K containing two points of J. But now the argument above shows that this simplex must have a vertex in each of four disjoint sets. Clearly this is impossible. The fact that $U \cap V^*$ or $U^* \cap V$ is finite is used in Stallings' original paper [8].

Theorem 3.3. *Let $S = \{s_1, s_2, \ldots, s_n\}$ be a set of minimal tracks in $|K|$ and suppose $|K^1| \cap s_i \cap s_j = \emptyset$ if $i \neq j$. Let $J = |K^1| \cap \bigcup \{s \mid s \in S\}$. Then S_J is a union of n disjoint minimal tracks.*

Proof. For $1 \leqq i < j \leqq n$, let $N_{ij} = \#(s_i \cap s_j)$. Let $N = \sum_{i,j} N_{ij}$. The theorem is proved by induction on N.

If $N = 0$, then the s_i's are disjoint, and the result is true. Suppose $N \neq 0$. Then $N_{pq} \neq 0$ for some $p < q$. By Proposition 3.2 there are disjoint minimal tracks s'_p and s'_q such that $(s'_p \cup s'_q) \cap |K^1| = (s_p \cup s_q) \cap |K^1|$. Put $s'_i = s_i$ if $i \neq p$ and $i \neq q$, and let $S' = \{s'_1, s'_2, \ldots, s'_n\}$. Put $N'_{ij} = \#(s'_i \cap s'_j)$ and $N' = \sum_{i,j} N'_{ij}$. We will show that N' is less than N. Clearly $N'_{pq} = 0$. Suppose $1 \leqq i \leqq n$, $i \neq p$, $i \neq q$. If s_i intersects $s'_p \cup s'_q$ exactly once inside $|\sigma|$, then it must intersect s_p or s_q inside $|\sigma|$. Also, by considering Fig. 3, it can be seen that if s_i intersects both s'_p and s'_q inside $|\sigma|$, then it must intersect both s_p and s_q. Hence $N'_{ip} + N'_{iq} \leqq N_{ip} + N_{iq}$ and $N' < N$. Since $J = |K^1| \cap \bigcup \{s' \mid s' \in S'\}$, the theorem follows by induction.

§ 4. Group actions

Again let K be a connected 2-dimensional complex such that $H^1(K; \mathbb{Z}_2) = 0$. Suppose the group G acts freely on K (on the left).

Theorem 4.1. *If there is a minimal track t in $|K|$, then there is a minimal track s in $|K|$ such that $gs = s$ or $gs \cap s = \emptyset$ for every $g \in G$.*

Proof. For each $g \in G$, gt is a minimal track. Now for each g choose a new minimal track t_g with the following properties.

(i) For each 1-simplex γ of K, $t_g \cap |\gamma| \neq \emptyset$ if and only if $gt \cap |\gamma| \neq \emptyset$.

(ii) If $g_1 \neq g_2$ then $t_{g_1} \cap t_{g_2} \cap |K^1| = \emptyset$.

Such a choice of the t_g's is always possible. Let $A = \{t_g \mid g \in G\}$. Let $J = |K^1| \cap \bigcup \{t_g \mid g \in G\}$. If γ is a 1-simplex of K then $\#(J \cap |\gamma|)$ is the number of intersections of t with edges in the G-orbit of γ. It follows that $\#(J \cap g|\gamma|) = \#(J \cap |\gamma|)$. This means that we can also choose the tracks t_g so that J is a G-

set, i.e. $gJ=J$ for every $g\in G$. Clearly J will satisfy the conditions of Proposition 2.3. Hence S_J exists. We show that S_J is a union of disjoint minimal tracks. If A is finite, then this is an immediate consequence of Theorem 3.3. In general A will not be finite. Note that if the 1-simplex γ intersects a minimal track t' then it is a face of only finitely many 2-simplexes. For otherwise $\|t'\|$ could not be finite. Thus there is a locally finite subcomplex K_f of K such that K_f contains every minimal track. Let γ_0 be a fixed 1-simplex of K_f. Let K_0 be the finite subcomplex of K_f consisting of all simplexes which contain a vertex whose distance from a vertex of γ_0 is not more than $m(K)$. Thus if a minimal track t' intersects $|\gamma_0|$ then $t'\subset|K_0|$. Let A_0 be the finite subset of A consisting of those tracks t_g for which $t_g\cap|K_0|\neq\emptyset$. Let

$$J_0=|K^1|\cap\bigcup\{t_g\,|\,t_g\in A_0\}.$$

Now Theorem 3.3 applies to J_0 and so S_{J_0} is a union of minimal tracks. In particular those components of S_{J_0} which intersect $|\gamma_0|$ are minimal tracks. But these tracks must be contained in $|K_0|$ and $J_0\cap|K_0|=J\cap|K|$. Thus any component of S_J which intersects γ_0 is a minimal track. Since the choice of γ_0 was arbitrary, S_J is a union of minimal tracks.

Now $gJ=J$ for every $g\in G$. Hence $gS_J=S_J$ for every $g\in G$. The theorem follows immediately.

§ 5. Accessibility

Theorem 5.1. *If G is an almost finitely presented group, then G is accessible.*

Proof. By hypothesis there is a 2-dimensional complex K for which $H^1(K;\mathbb{Z}_2)=0$ and G acts freely on K so that $G\backslash K=L$ is a finite 2-complex.

If $e(G)\leq 1$ there is nothing to prove. If $e(G)>1$, then $e(|K|)>1$ and there is a set $U\subset|K^0|$ such that both U and U^* are infinite and $w(U)$ is finite. By Proposition 3.1 there is a minimal track $t\subset|K|$. By Theorem 4.1 there is a minimal track s such that the set $T=\{gs\,|\,g\in G\}$ is a set of disjoint minimal tracks in $|K|$.

We construct a tree Γ which is in a sense dual to T as follows. Let $V\Gamma$ be the set of components of $|K|-\bigcup_{t\in T} t$. Let $E\Gamma=T$. If $t\in T$ then it is easy to see that there are precisely two elements of $V\Gamma$ whose closures in $|K|$ contain t. These are the vertices of t when t is regarded as an edge of Γ. Any finite edge path in K intersects only finitely many $t\in T$. It follows that Γ is connected. If $t\in T$ then t separates K. Hence removing any edge from Γ disconnects Γ. The only connected graphs with this property are trees.

Clearly Γ is a G-tree. If for every $v\in V\Gamma$, the stabilizer G_v has at most one end then G is accessible. If not, there is a vertex v such that G_v has more than one end. Now v is a component C of $|K|-\bigcup_{g\in G} gs$. Let K_v be the subcomplex of K consisting of those simplexes which meet C. If $x,y\in C$ and $gx=y$ for $g\in G$, then $g\in G_v$. Put $L=G\backslash K$ and $L_v=G_v\backslash K_v$ and let $\pi_v\colon K_v\to L_v$ be the natural map. Let $\iota_v\colon L_v\to L$ be the map induced by inclusion. Now ι_v may not be injective. However if σ_1,σ_2 are principal simplexes of K_v, then they must each contain points of C. (A principal simplex is one which is not a proper face of another simplex.) Thus if $\iota_v(\pi_v(\sigma_1))=\iota_v(\pi_v(\sigma_2))$ then $\pi_v(\sigma_1)=\pi_v(\sigma_2)$, i.e. ι_v is injective when restricted to principal simplexes. But this means that L_v is finite, since L is finite.

Let $gs\in T$ be an edge of Γ which has v as a vertex. Then $gs\subset|K_v|$. However $|K_v|-gs$ has a component C_g which contains only finitely many vertices. In fact if $x\in|K^0|\cap C_g$, then any principal simplex containing x must intersect gs. If G_v has more than one end, then $|K_v|$ has more than one end and there is a track t_v such that $|K_v|-t_v$ has two components each containing infinitely many vertices. Let U be the set of vertices in one of these components. Now for all but finitely many $gs\subset|K_v|$ either $C_g\cap|K_v^0|\subset U$ or $C_g\cap|K_v^0|\subset U^*$. There is a set $V\subset|K_v^0|$ which is almost equal to U (i.e. $(V-U)\cup(U-V)$ is finite) such that $C_g\cap|K_v^0|\subset V$ or $C_g\cap|K_v^0|\subset V^*$ for every $gs\subset|K_v|$. But $w(V)$ is finite since $w(U)$ is finite and K_v is locally finite. As in the proof of Proposition 3.1 there is a track t'_v which separates $|K_v|$ into two components with infinitely many vertices and which only intersects 1-simplexes with one vertex in V and one in V^*. But any such 1-simplex must contain a point of C, and so we can arrange that $t'_v\subset C$. We now apply Theorem 4.1 with K_v in place of K and G_v in place of G. We also only consider tracks contained in C. Thus we modify all the definitions and proofs so that the tracks considered have this extra property. We see then that there is a track $s_v\subset C$ such that $gs_v=s_v$ or $gs_v\cap s_v=\emptyset$ for every $g\in G_v$ and $|K_v|-s_v$ has two components each with infinitely many vertices. In fact $gs_v=s_v$ or $gs_v\cap s_v=\emptyset$ for every $g\in G$, since if $g\notin G_v$, $gC\cap C=\emptyset$. Clearly s_v is not parallel to any track in T.

Under the natural map π: $|K|\to|L|$, $T\cup\{gs_v|g\in G\}$ maps to a set of disjoint tracks. If $\pi(s_v)$ is parallel to $\pi(t)$ for $t\in T$ then s_v is parallel to some element of T. This is because the preimage of a band in $|L|$ must be a union of bands in $|K|$. Thus $\pi(s_v)$ cannot be parallel to $\pi(t)$ for $t\in T$. We now change T to $T\cup\{gs_v|g\in G\}$ and repeat the argument. This process cannot be repeated indefinitely by Theorem 2.2 and so G is accessible.

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