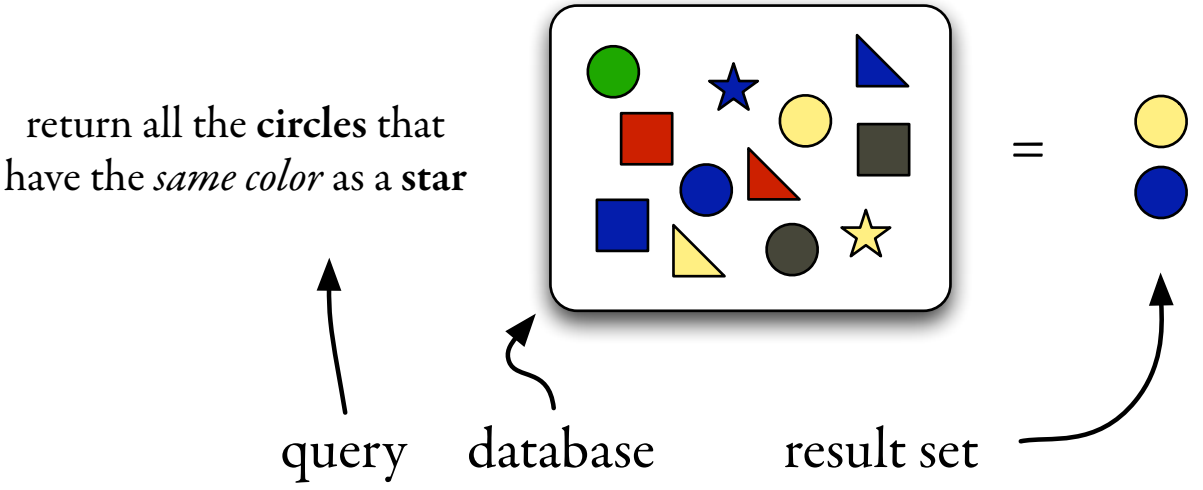
06-Dec-2010
Cachan, France

Reasoning on words and trees with data

On decidable automata on data words and data trees in relation to satisfiability of LTL and XPath.

Diego Figueira

verification of databases



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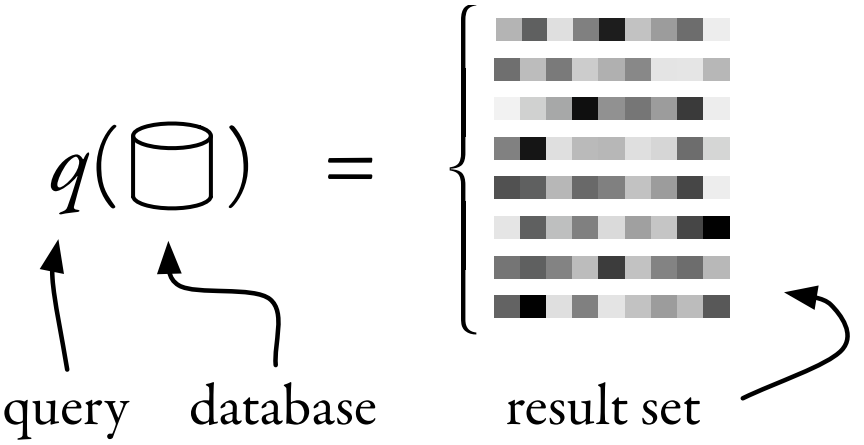

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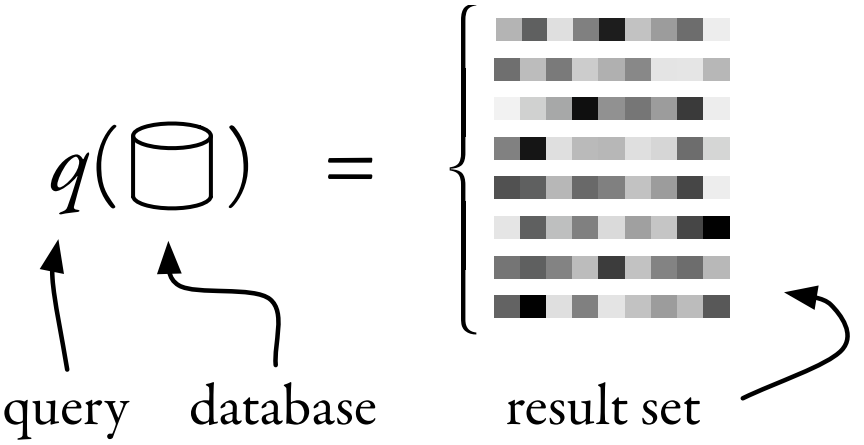

static analysis

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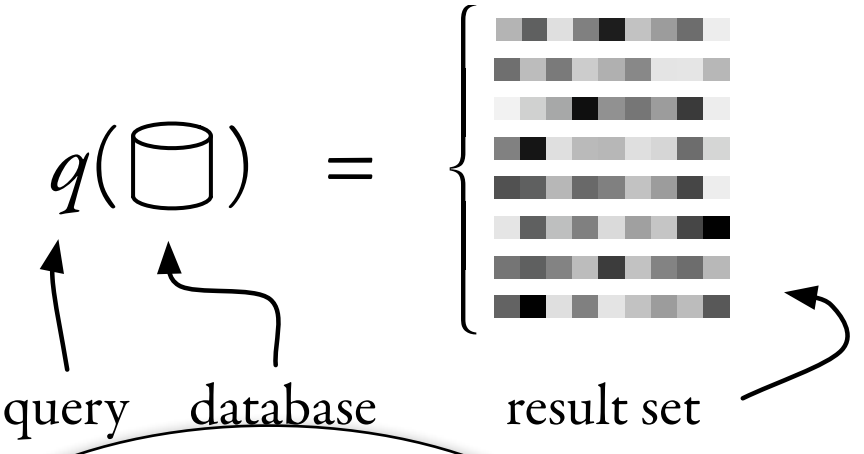


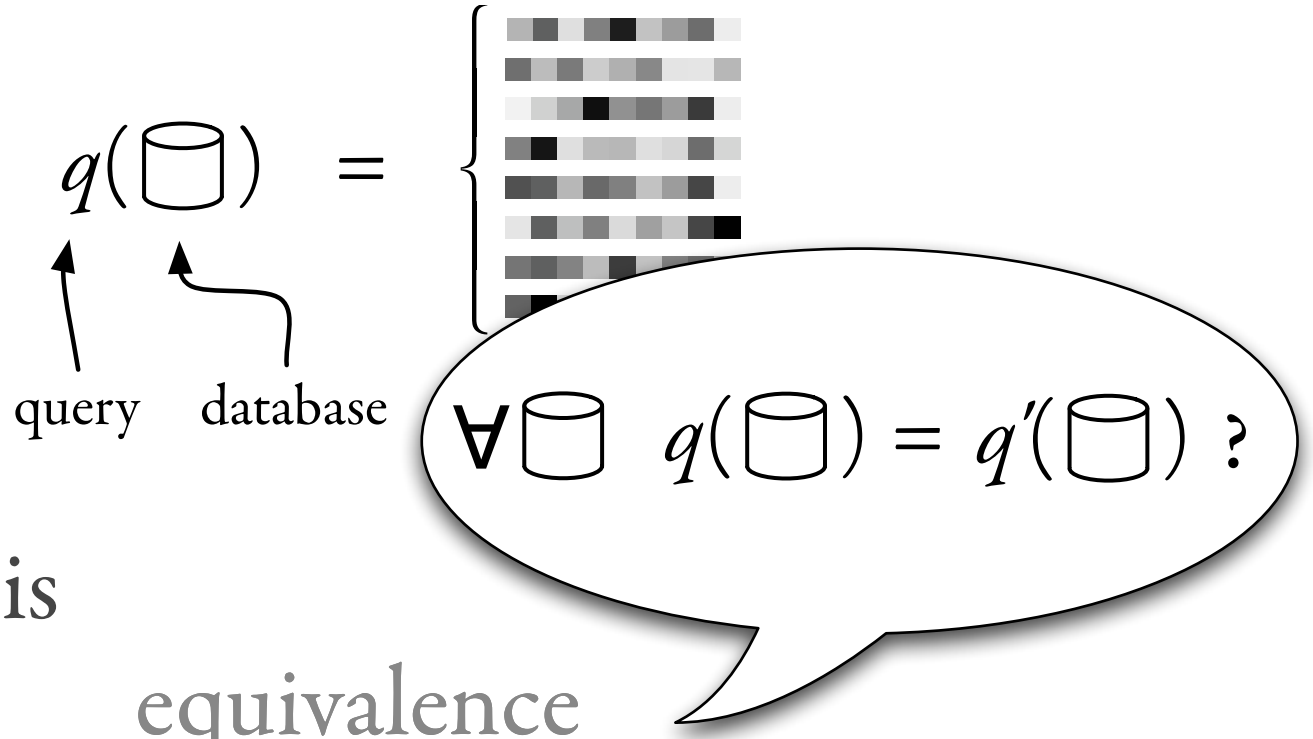
static anal

$\exists$ $q(\ ) \neq \varnothing$?

satisfiability

does *q* express a property?

verification of databases



static analysis

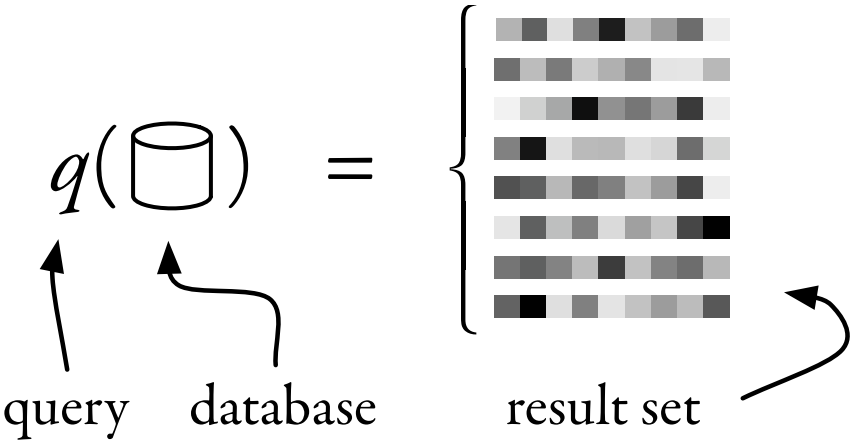
equivalence

can we use q' instead of q?

satisfiability

does q express a property?

verification of databases



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$\forall$ $q(\,) \subseteq q'(\,)$?

satisfiability
does q express a property?

containment
are q and q' comparable queries?

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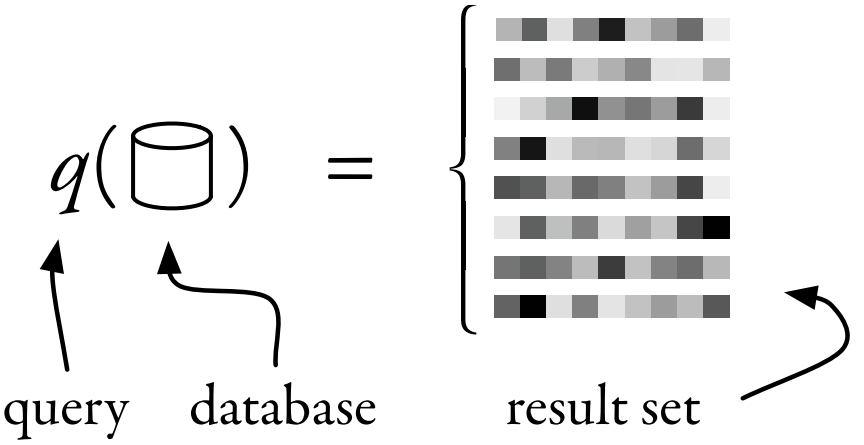

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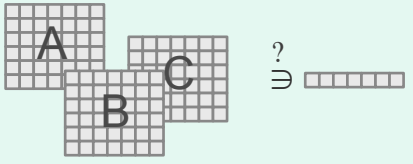
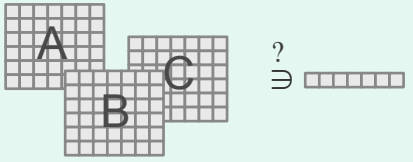
databases with structure



relational databases

databases with structure



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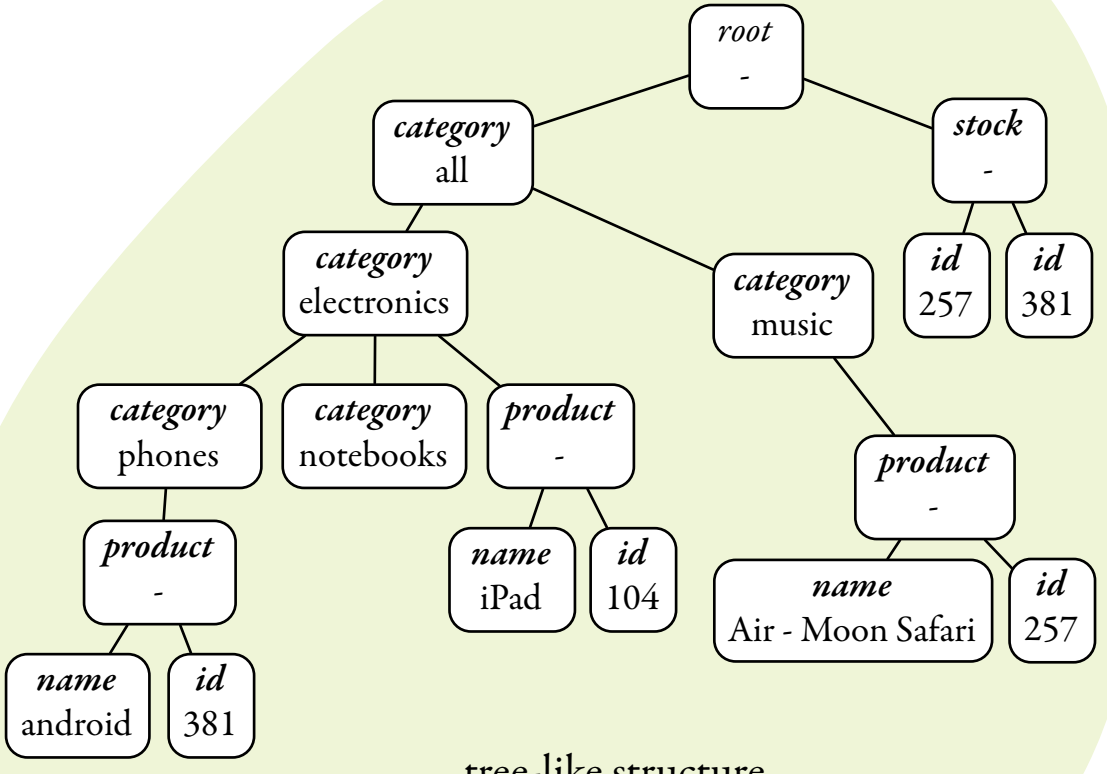
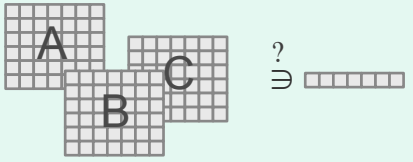


tree-like structure
unranked, ordered finite trees
XML documents
“data tree”

databases with structure



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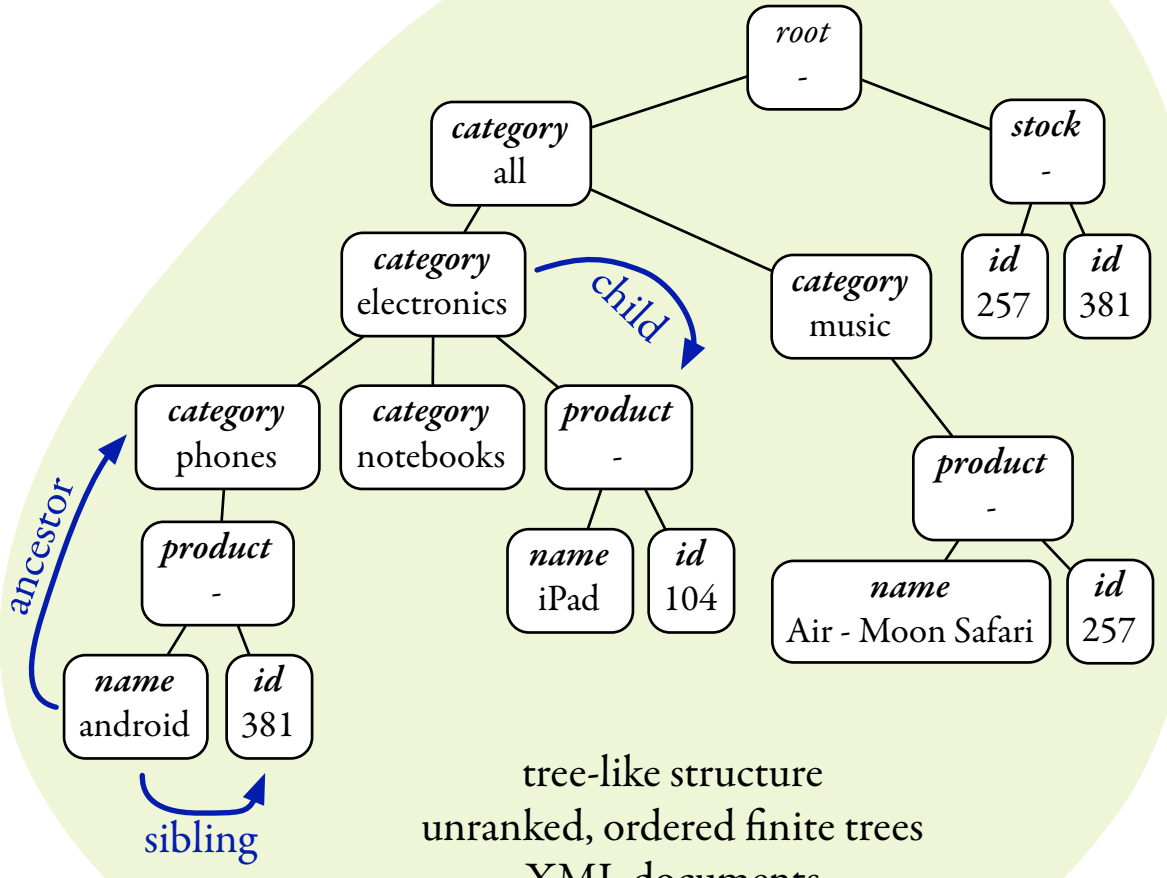

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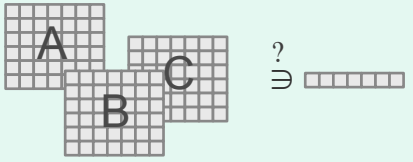

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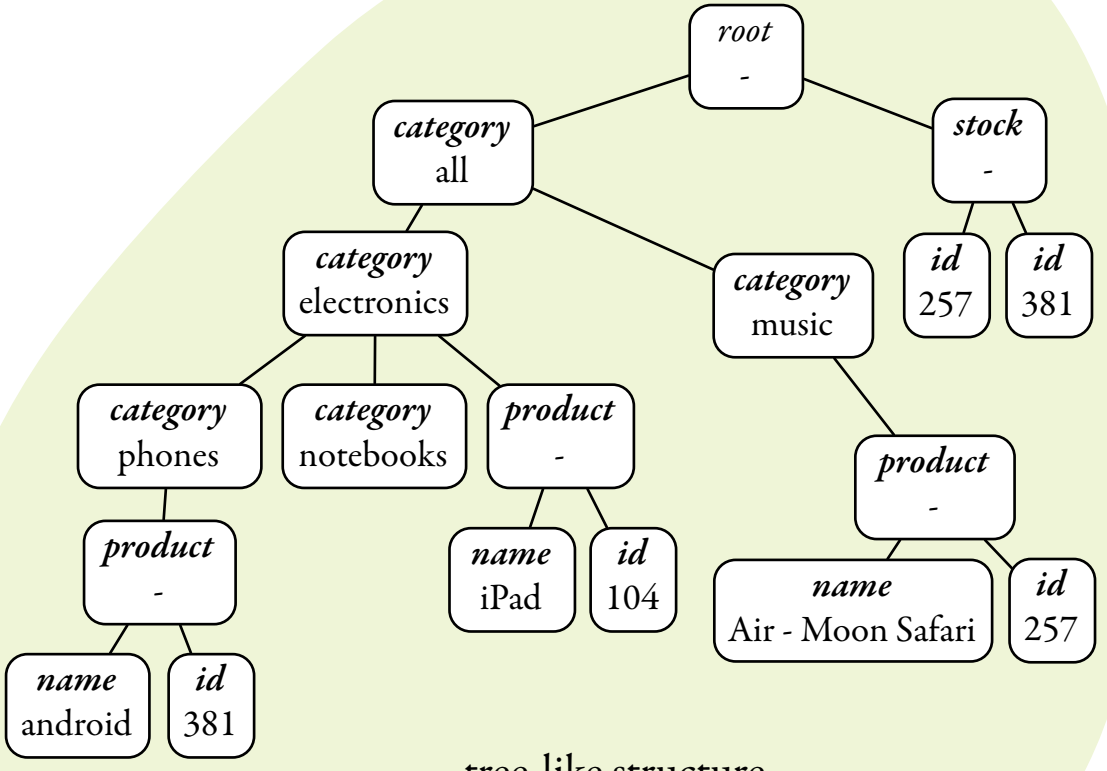

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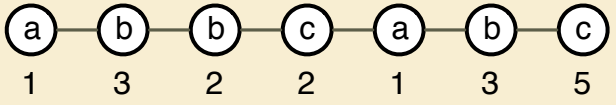
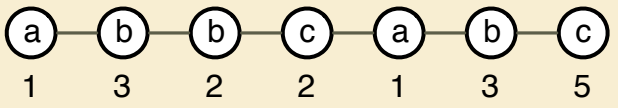


special case of a data tree
temporal databases
“data word”

databases with structure



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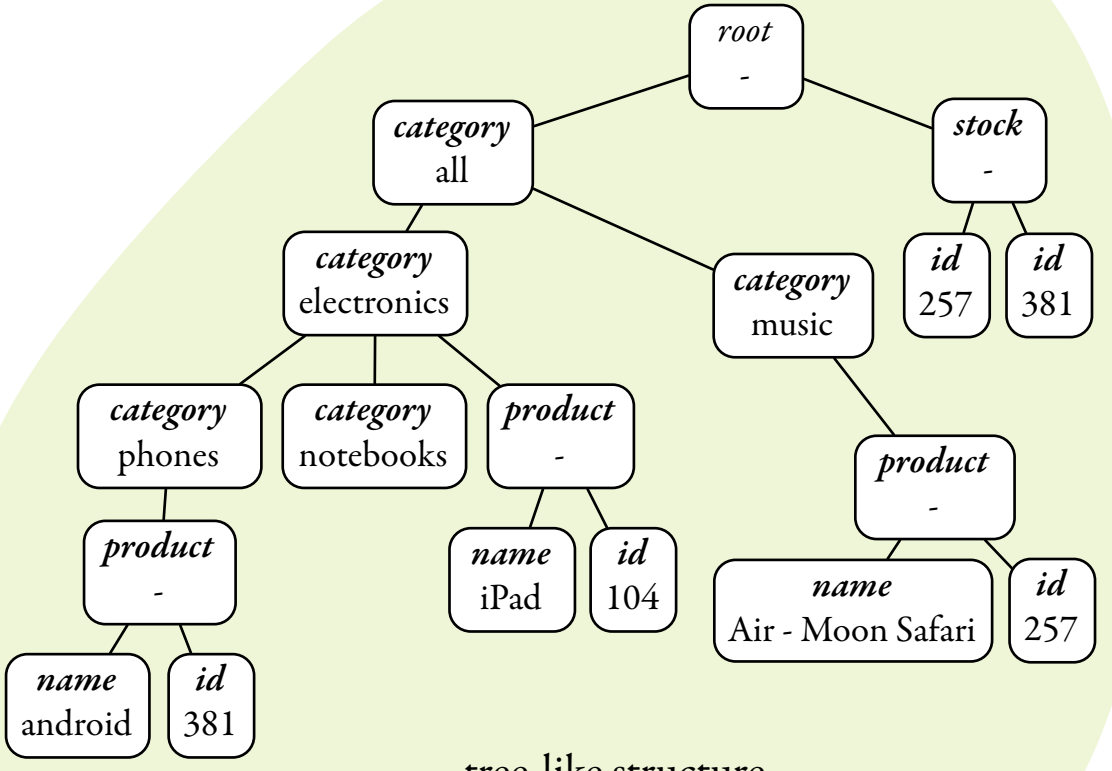

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XML documents
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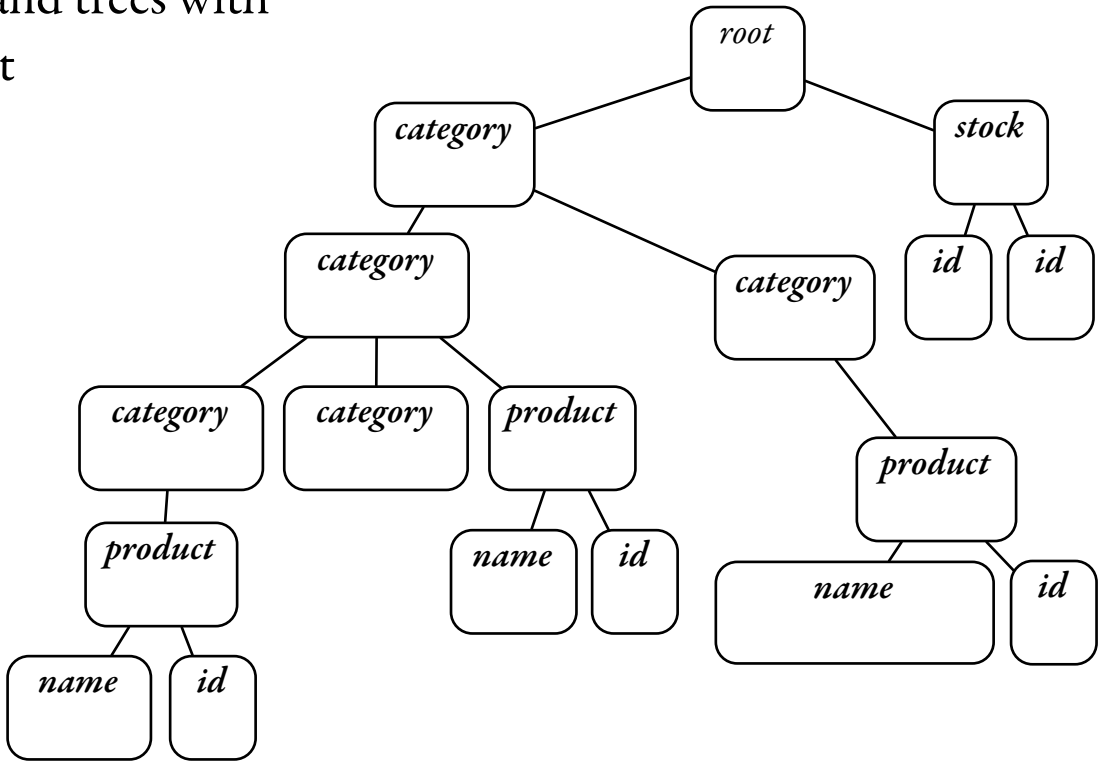
data values

work on words and trees with

- finite alphabet
- structure



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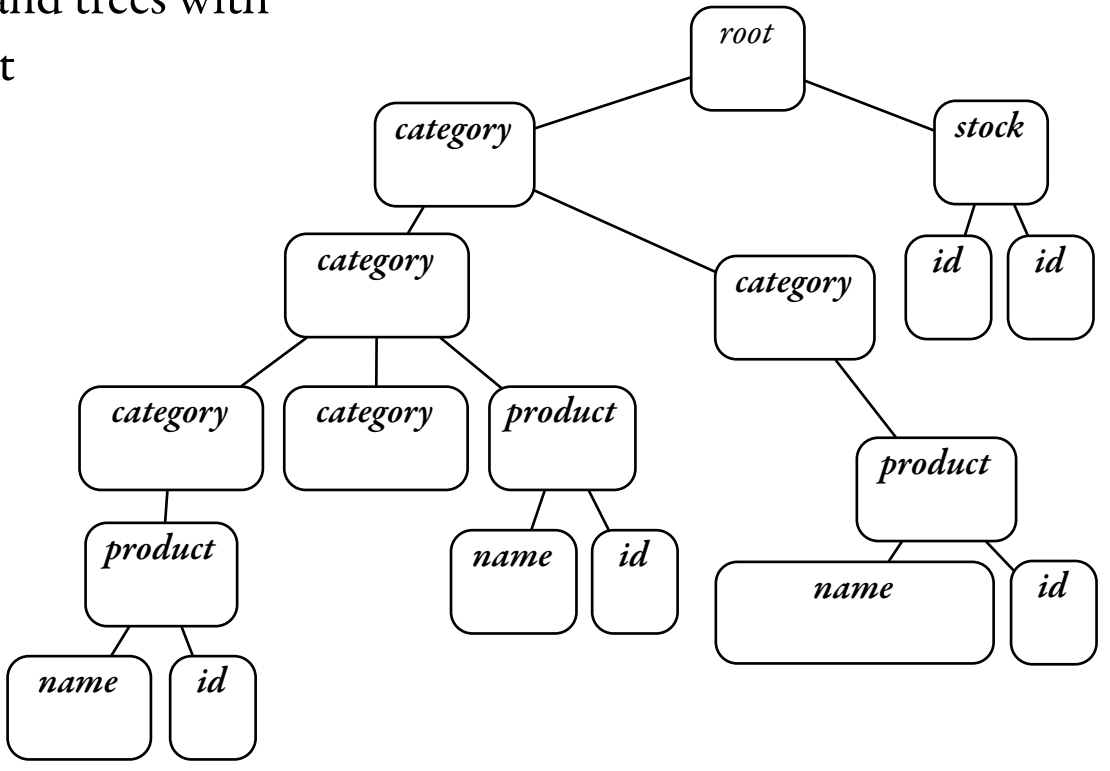

need to perform joins,
to test for equality of data values

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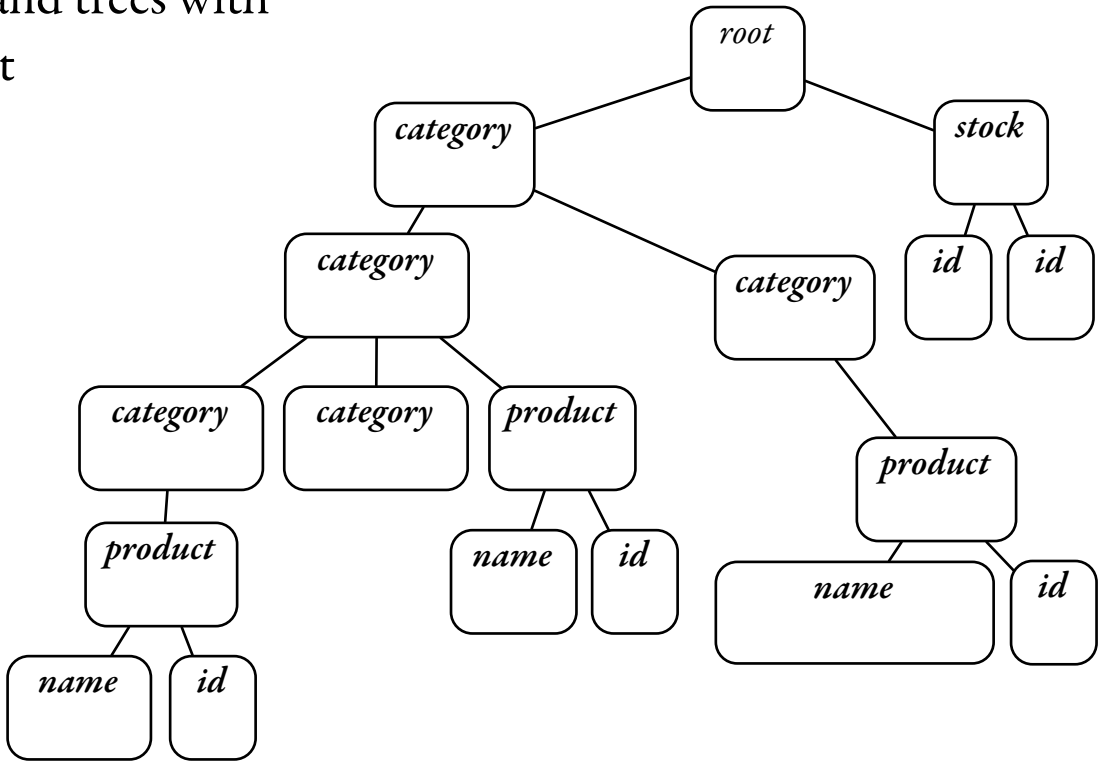

"return all products in stock"

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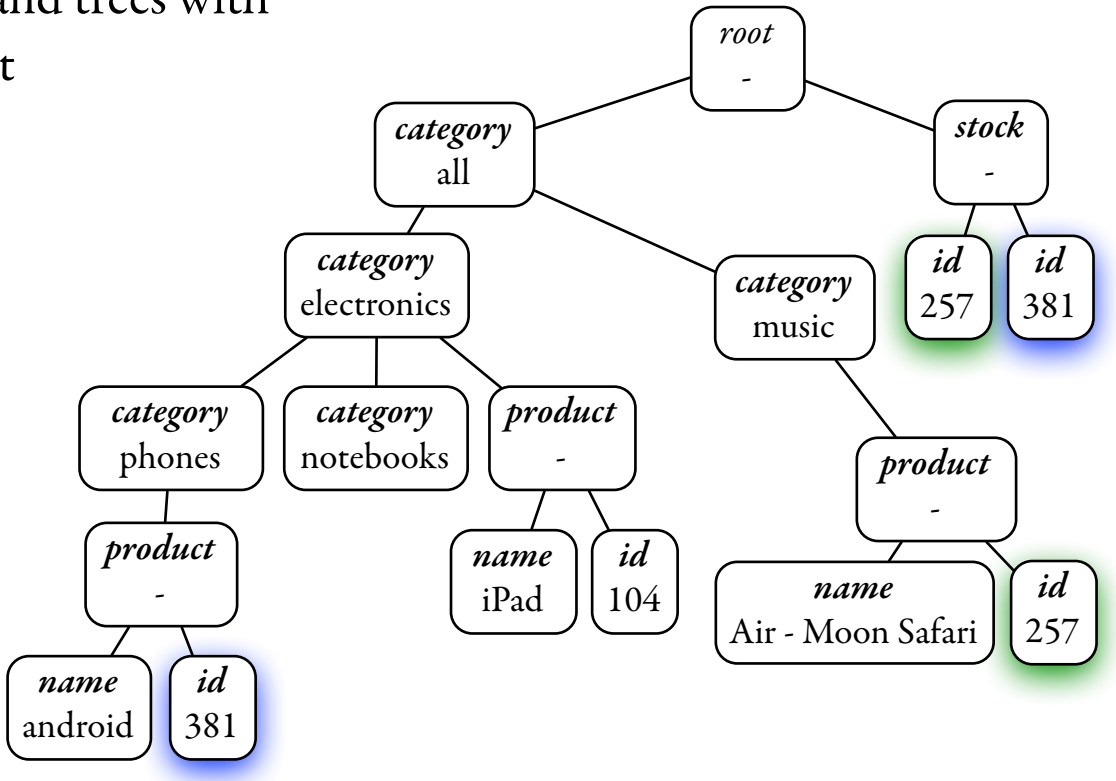

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what's out there?

(alternating) register automata
pebble automata
Kaminski, Francez, Neven, Schwentick, Vianu, Tan, Demri, Lazić, Jurdziński, . . .

FO^2
Bojańczyk, Muscholl, Schwentick, Segoufin, David, . . .

XPath
Benedikt, Fan, Geerts

patterns
conjunctive queries
David, Björklund, Martens, Schwentick

≈ [F, Hofman, Lasota, EXPRESS'10]

(alternating)
timed automata
Alur, Dill, . . .

LTL with
registers
Demri, Lazić, Nowak

real time logics
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Hybrid logics
Description Logics
with concrete domains

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objective

data-aware languages & automata for word- & tree- structured dbs

objective

data-aware languages & automata for word- & tree- structured dbs

rLTL
data-words

XPath
data-trees

objective

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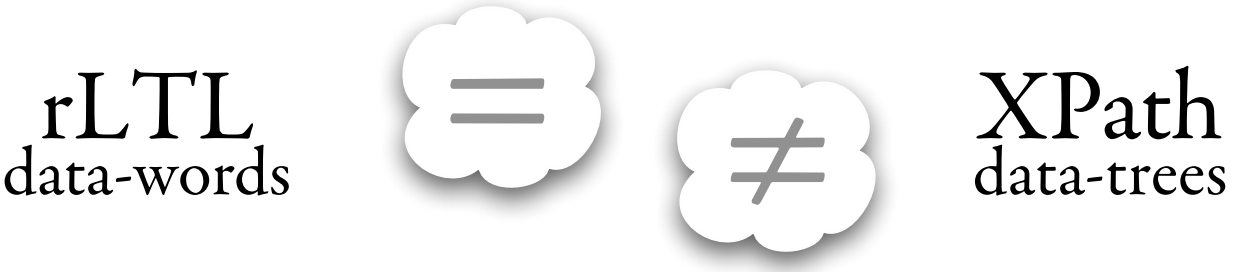

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equivalence satisfiability inclusion

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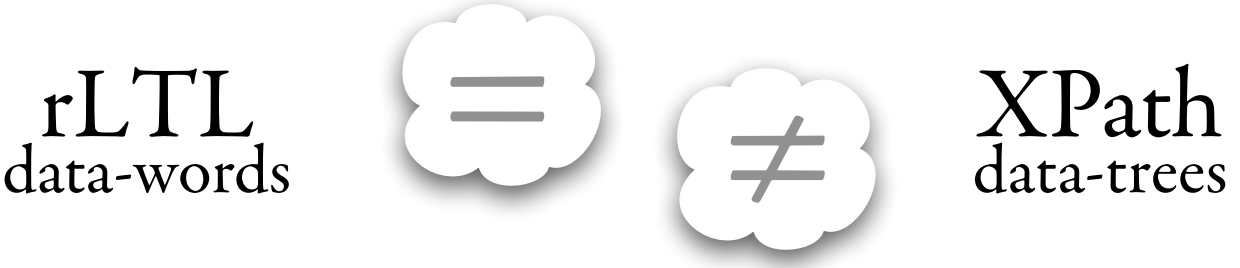

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rLTL
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=

≠

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equivalence satisfiability inclusion

rLTL(F,X)

register-LTL over data words

$\phi ::= a \mid \mathbf{F}\phi \mid \mathbf{X}\phi \mid \phi \vee \phi \mid \phi \wedge \phi \mid \neg \phi \mid \downarrow\phi \mid \uparrow$

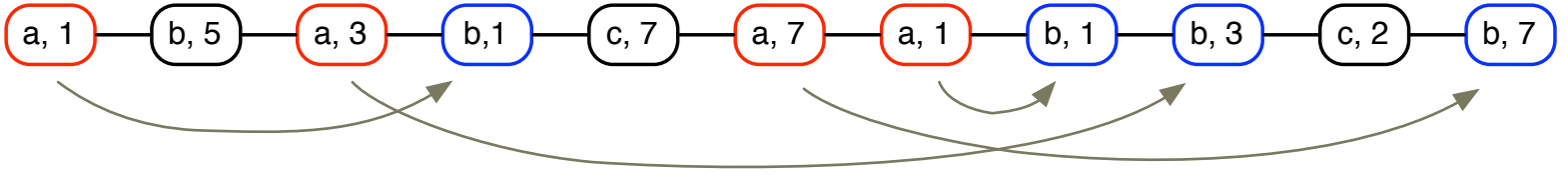
tests current datum against the stored

stores current datum

e.g.

"For every **a** there is a future **b** with the same value."

$$G\,(\ \neg \mathbf{a} \vee \downarrow F\,(\mathbf{b} \wedge \uparrow)\ \ )$$



lower bounds

We knew...

[Demri / Lazić]

rLTL(X, F) is decidable, **non primitive recursive**

rLTL(X, F, F^{-1}) is **undecidable**

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Now we know ...

The one-step is not necessary to obtain the lower-bounds!

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rLTL(X, F) is decidable, **non primitive recursive**

[F, Segoufin, MFCS'09]

rLTL(X, F, F^{-1}) is **undecidable**

even for a very simple fragment

rLTL(U,X)

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$\uparrow$ → tests current datum against the stored

$\downarrow \phi$ → stores current datum

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$$\phi ::= a \mid \phi \mathbf{U} \phi \mid \mathbf{X}\phi \mid \phi \vee \phi \mid \phi \wedge \phi \mid \neg \phi \mid \downarrow \phi \mid \uparrow \mid \forall_{\leq} \phi \mid \exists_{\geq} \phi$$

$\exists_{\geq}\phi$: Exists a future data value s.t. ϕ.

$\forall_{\leq}\phi$: For all past data value, ϕ.

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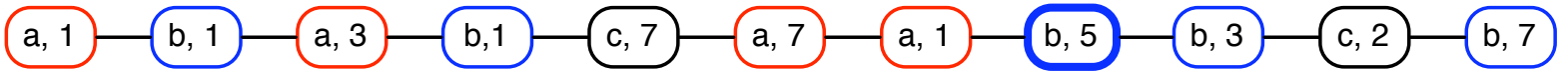
e.g.

"There is a **b** with a data value different from any previous element."

$$F\,(\ \neg \mathbf{b}\ \vee\ \forall_{\leq}\,(\neg \uparrow)\ )$$



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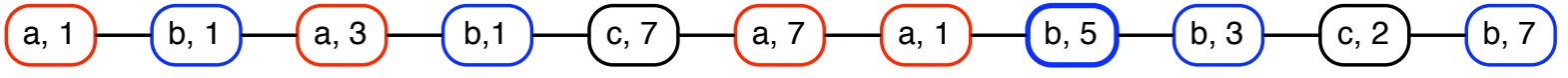
"There exists a data value that is contained in a **b** but not in an **a**."

$$\exists_{\geq} (\; F(\mathbf{b} \wedge \uparrow) \; \wedge \; \neg F(\mathbf{a} \wedge \uparrow) \;)$$



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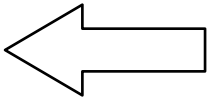
The satisfiability problem for rLTL(U,X) + $\forall_{\leq}$ + $\exists_{\geq}$ is **undecidable**.

ARA

[Demri / Lazić]

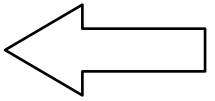


satisfiability of rLTL(U,X)

- ☺ Decidable emptiness problem
- ☹ With non-primitive recursive complexity
- ☺ Closed under complementation, intersection, union

ARA

[Demri / Lazić]

 satisfiability of rLTL(U,X)

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ARA + guess + spread

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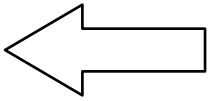

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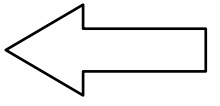

satisfiability of positive rLTL(U,X) + $\forall\leq$ + $\exists\geq$

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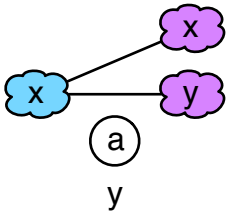
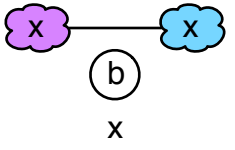
ARA
+guess +spread

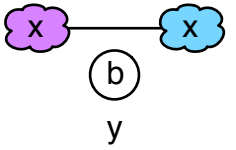
Alternating register automata
one-way
1 register

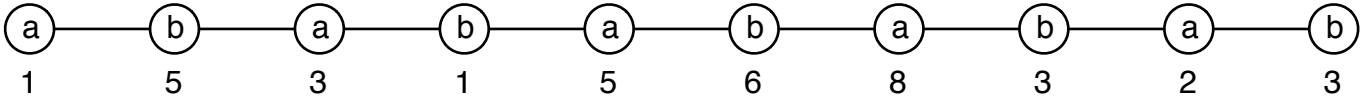
states:
initial state:
final states:

transitions









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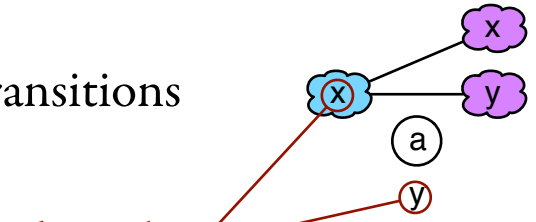
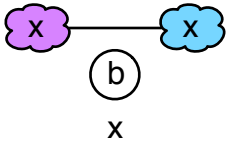
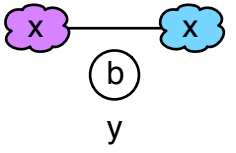


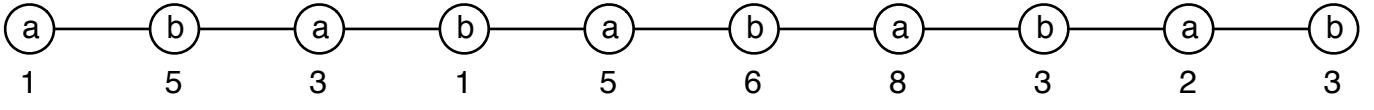
data values



x



y



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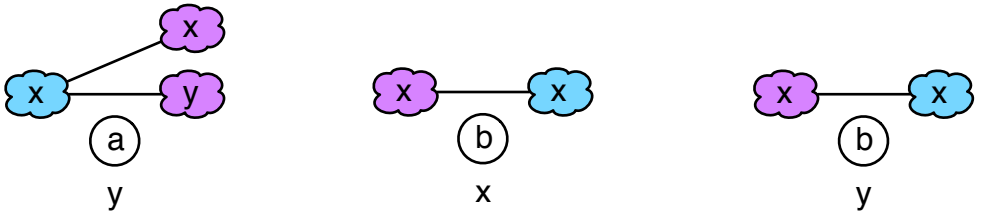
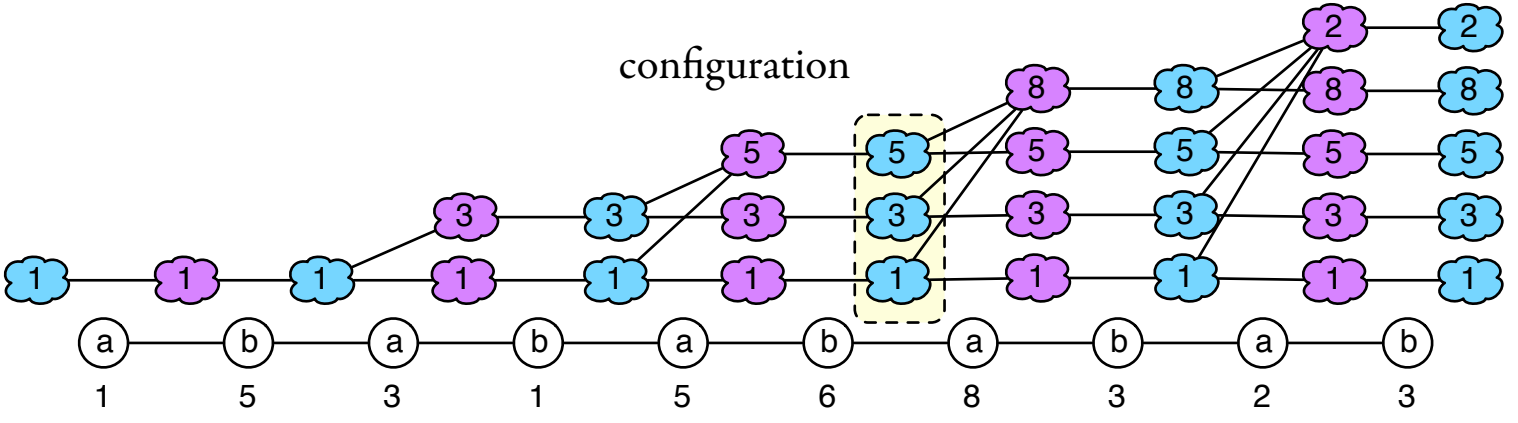


A configuration = a set of running threads (state, datum)



the string is $(ab)^*$, and all the a's have different data values

ARA
+guess +spread

Alternating register automata
one-way
1 register

states:
initial state:
final states:

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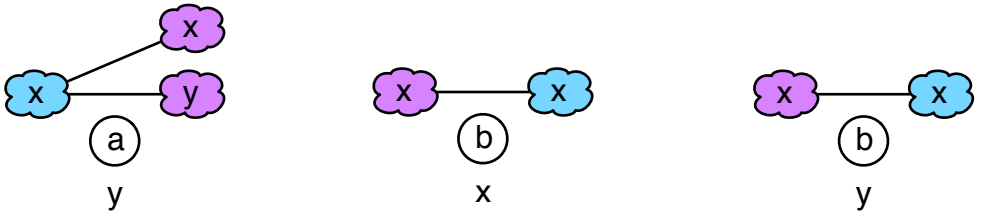

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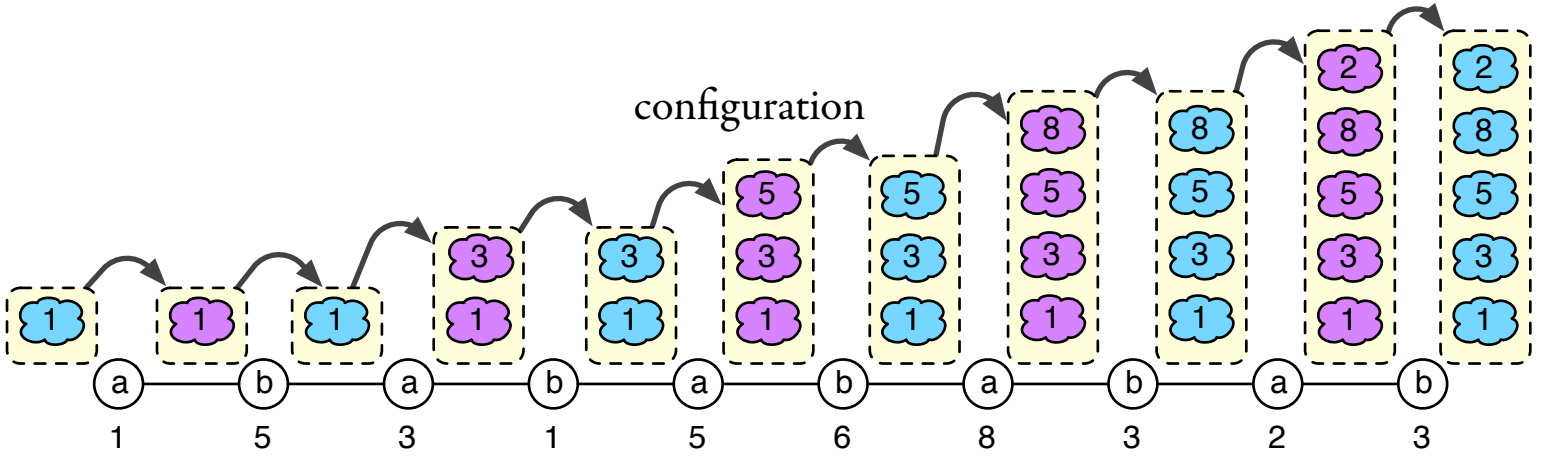

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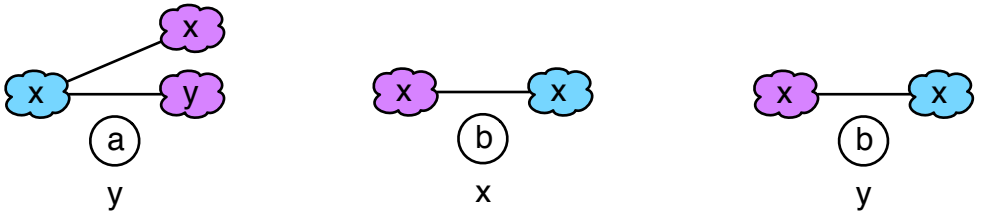
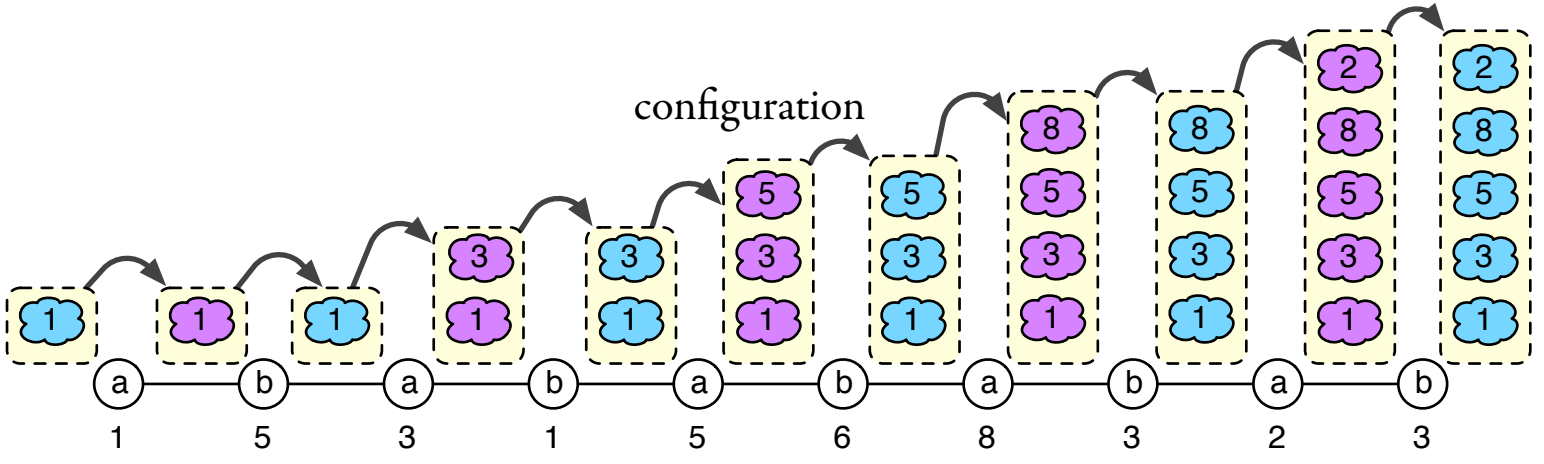


2 extensions

guess(q) Nondeterministically guess a data value and store it in the register.

spread(q, q') For every thread *(q,d)*, create a new one *(q',d)*.



the string is *(ab)**, and all the *a*'s have different data values

examples

"There exists a data value that is contained in a **b** but not in an **a**."

1. Guess a data value **d**
2. Create two threads that test
   a. there is a future position **b** with datum **d**
   b. there is not a future position **a** with datum **d**

examples

"There exists a data value that is contained in a b but not in an a."

1. Guess a data value d
2. Create two threads that test
   a. there is a future position b with datum d
   b. there is not a future position a with datum d

"There is a b with a data value different from any previous element."

Create two threads

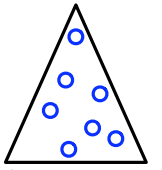
1. One that at every position create a thread that
   a. stores current data value, and
   b. advances until the last position using some state *q*
2. Other that
   a. guesses a b position of the word
   b. spreads all threads with *q* to threads with *p*

(state *p* checks that the stored data value is different from the current value)

XPath

node expressions

$\varphi, \psi ::= \mathbf{a} \mid \neg\varphi \mid \varphi \wedge \psi \mid \alpha{=}\beta \mid \alpha{\neq}\beta \mid \alpha? \qquad \mathbf{a} \in \mathcal{A}$

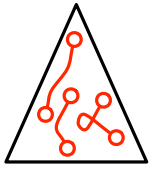


denote sets of nodes

path expressions

$\alpha, \beta ::= \varepsilon \mid \alpha\,\beta \mid \alpha \cup \beta \mid \alpha[\varphi] \mid o$

$o \in \{ \downarrow, \vdots, \rightarrow, \dashrightarrow, \uparrow, \vdots, \leftarrow, \dashleftarrow \}$



denote binary relations

‘$\rightarrow$’ : *one-step*

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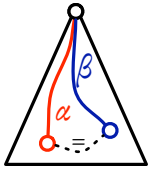

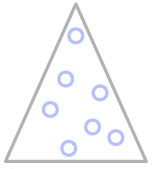

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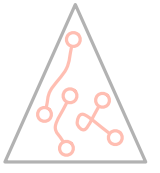

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XPath

we note "XPath(=, $\rightarrow$, $\dashrightarrow$, $\uparrow$)"

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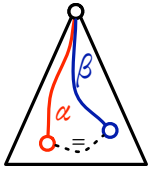

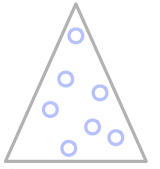

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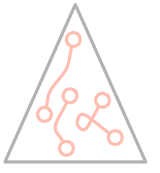

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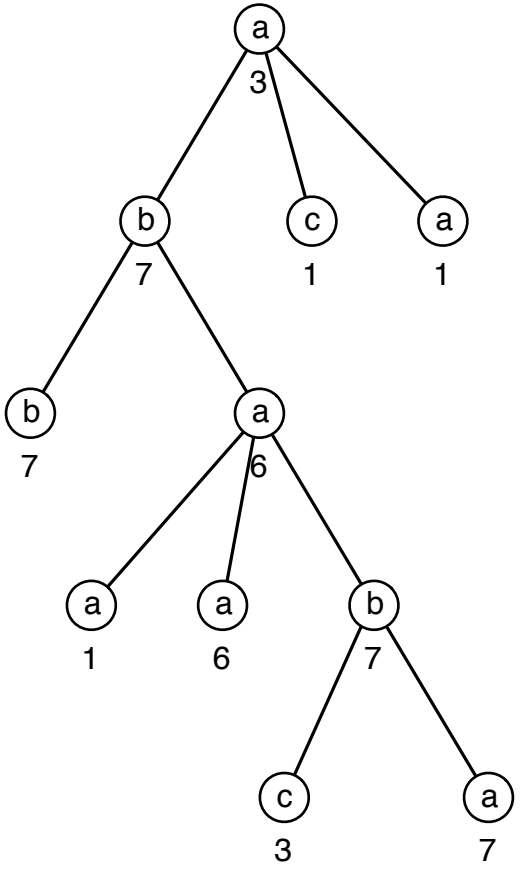

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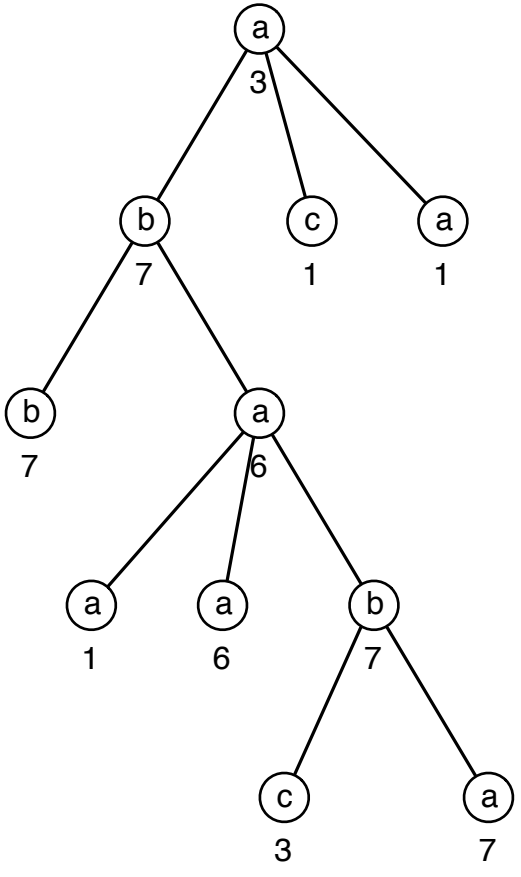

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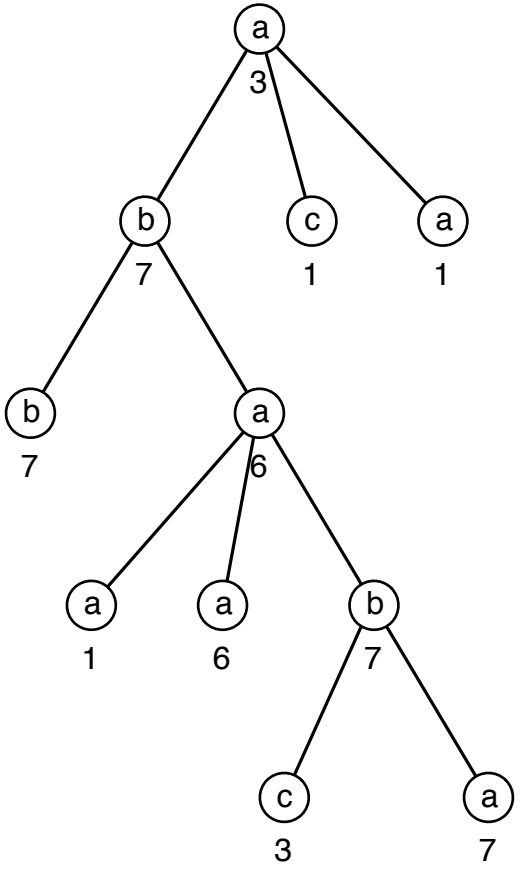

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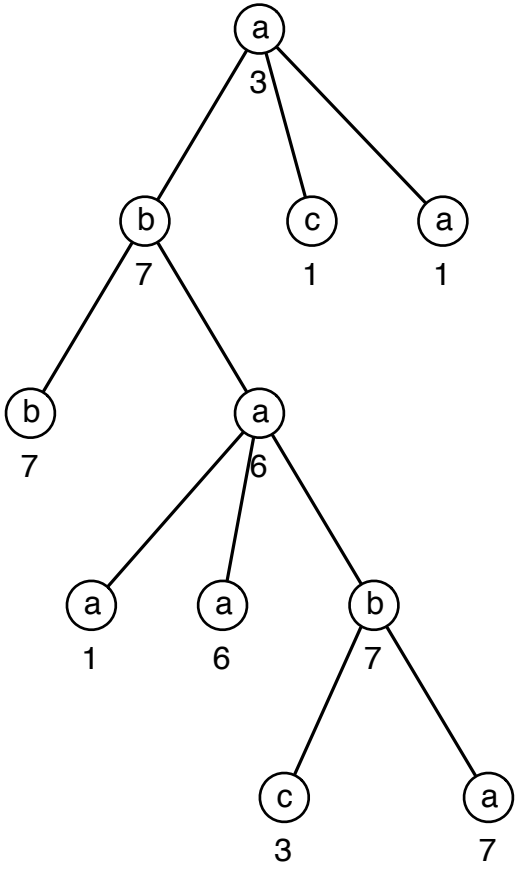

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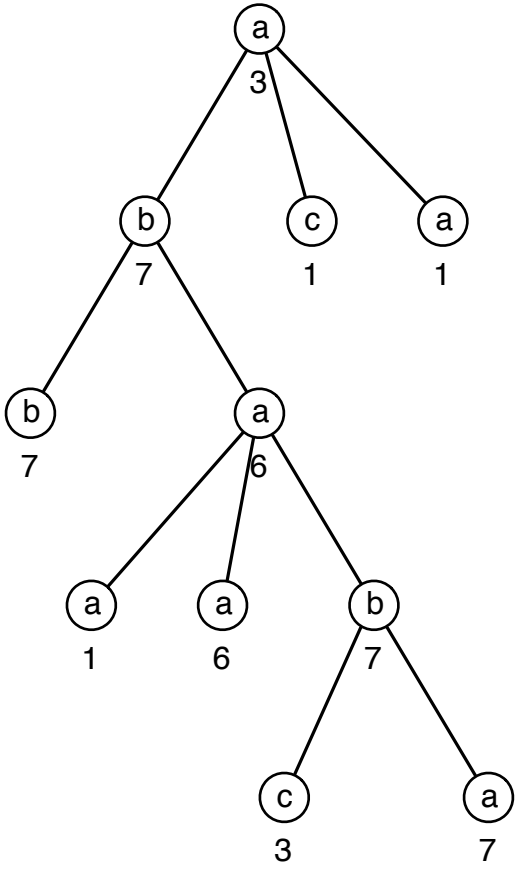

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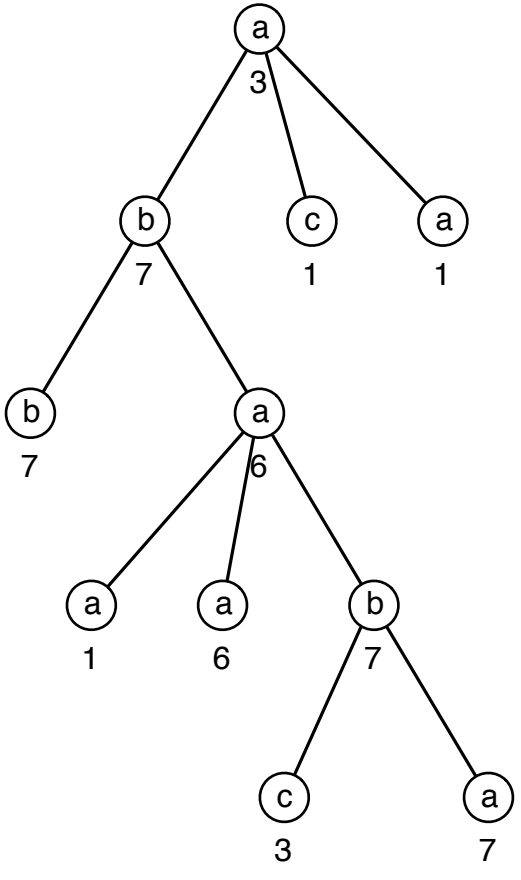

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The ***c*** constitutes a primary key

$\neg (\vdots [a \wedge \neg (\varepsilon \neq \updownarrow[b])])$

Every ***a*** has an ancestor ***b*** with different value

satisfiability of XPath

Satisfiability of XPath$(=, \downarrow, \vdots\downarrow, \rightarrow, \dashrightarrow, \uparrow, \vdots\uparrow, \leftarrow, \dashleftarrow)$ is undecidable.

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Our contribution:

Satisfiability of XPath(=, $\downarrow$, $\downarrow^*$, $\rightarrow$, $\rightarrow^*$, $\uparrow$, $\uparrow^*$, $\leftarrow$, $\leftarrow^*$) is undecidable. [F, Segoufin, MFCS'09]

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forward-XPath

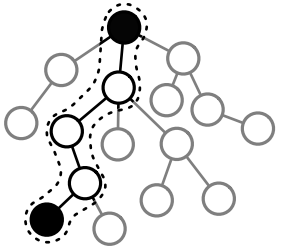
Satisfiability of $\mathrm{XPath}(=, \downarrow, \vdots \rightarrow, \cdots\blacktriangleright)$

forward-XPath

Satisfiability of ε-XPath$(=, \downarrow, \vdots \rightarrow, \cdots\blacktriangleright)$ is decidable.

[Lazić, Jurdziński, '07]

only tests: $\varepsilon = \alpha$



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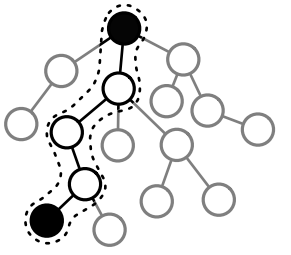

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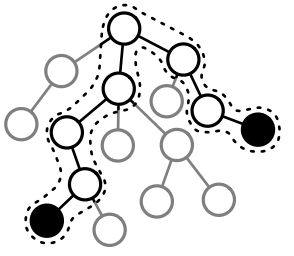
[F, ICDT'10] lower bound: non primitive recursive



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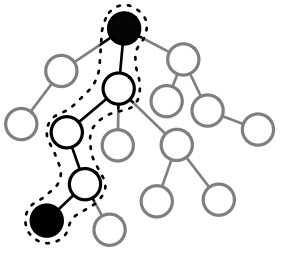

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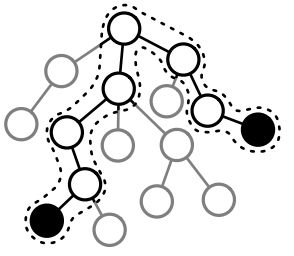

forward XPath ⟹ ATRA + guess + spread

- extension of ARA on trees
- alternating automata
- 1 register
- top-down, unranked
- **decidable** emptiness pb

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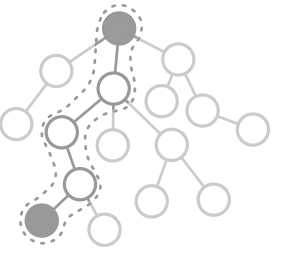

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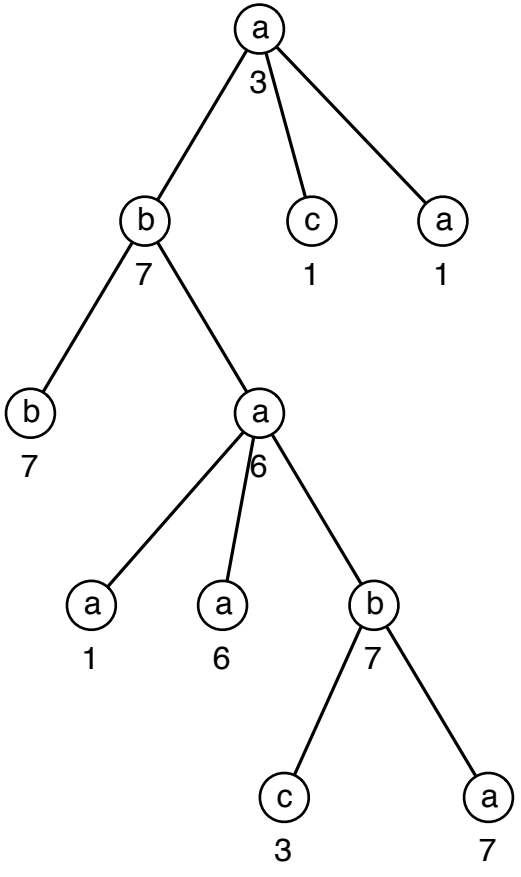
difficulty: ATRA$_{+\text{guess}+\text{spread}}$ are not closed under complementation

how do we code...?



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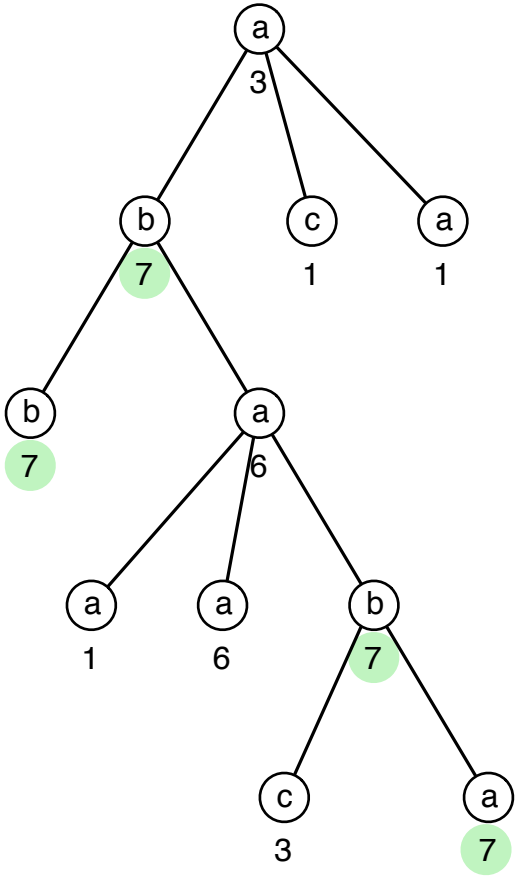

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Guess the data value 7.

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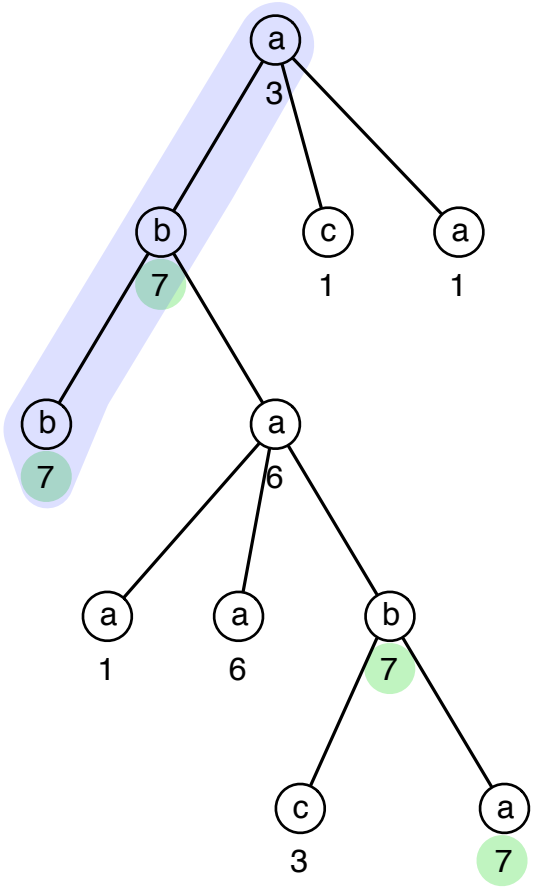

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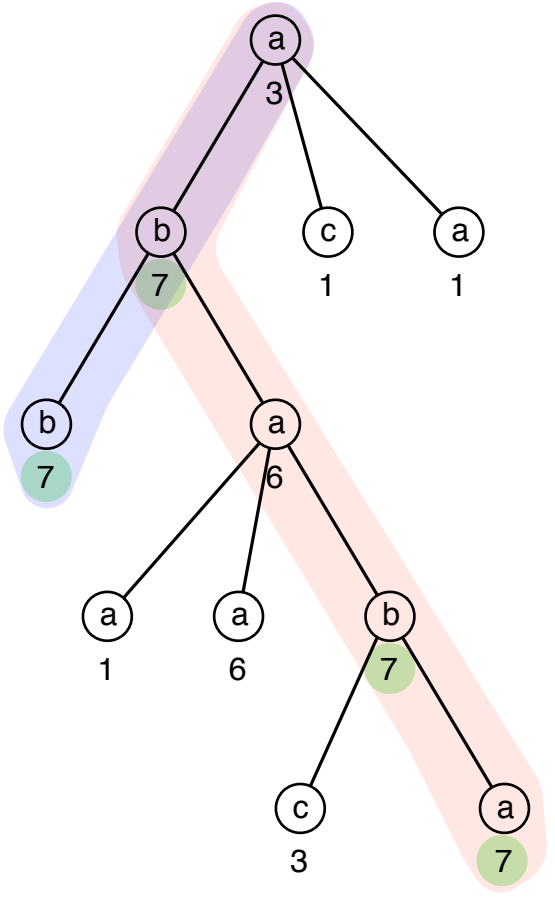

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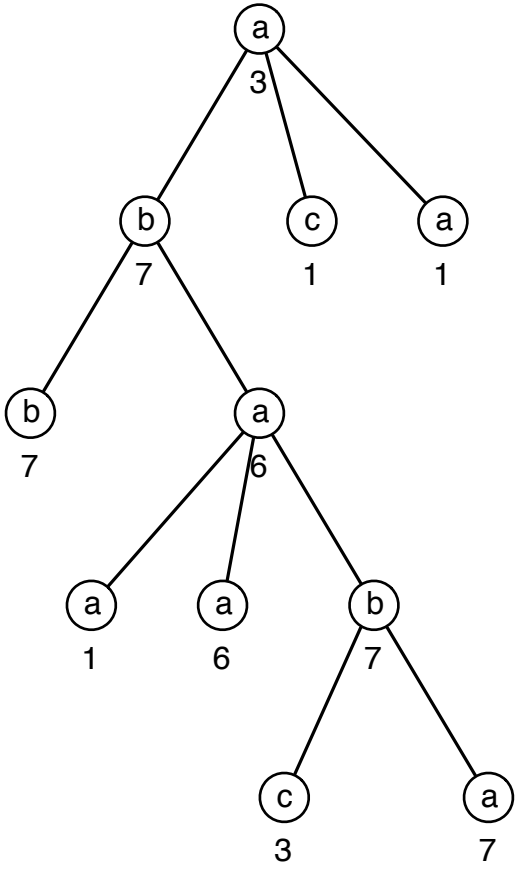

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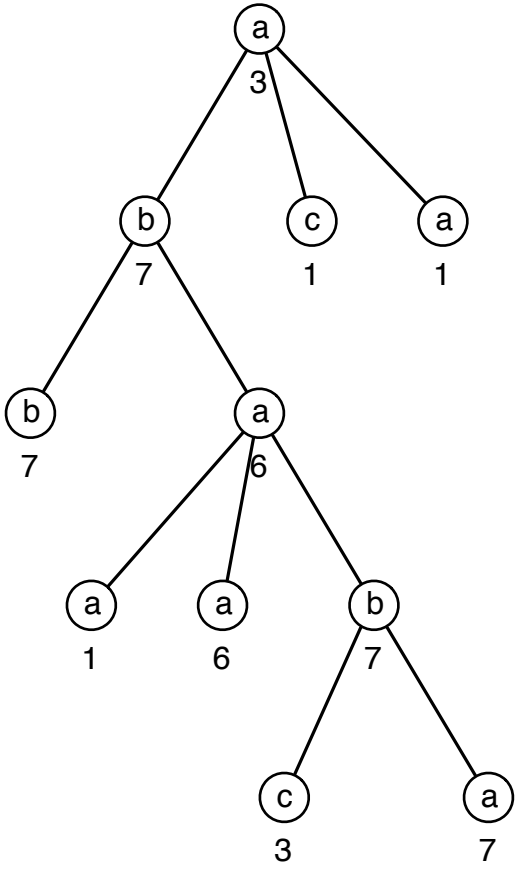

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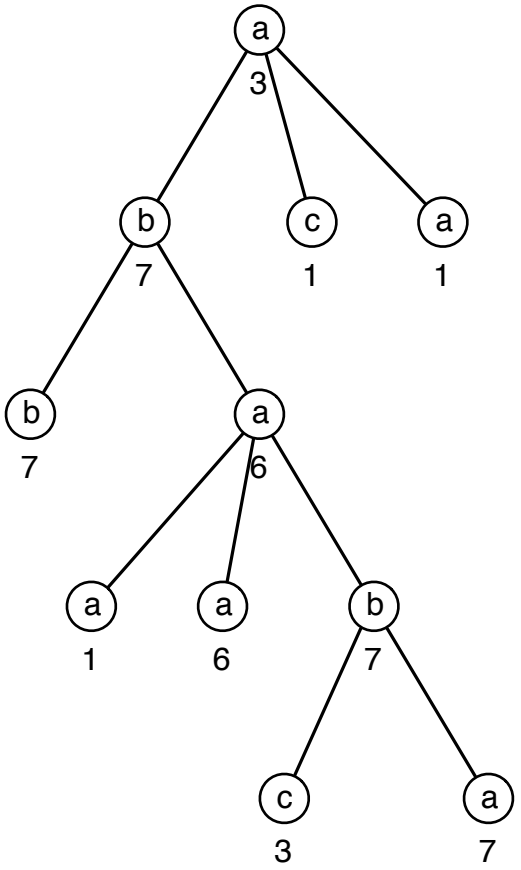

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All ***b*** have the same data value

$\neg(\vdots[b] = \vdots[c])$ ✘

There is no data value shared by a ***b*** and a ***c***

build automaton **A** such that

if $\mathbf{T} \models \varphi$, then **A** accepts **T**

if **A** accepts **T**, then there is $\mathbf{T}' = f(\mathbf{A},\mathbf{T})$ such that $\mathbf{T}' \models \varphi$

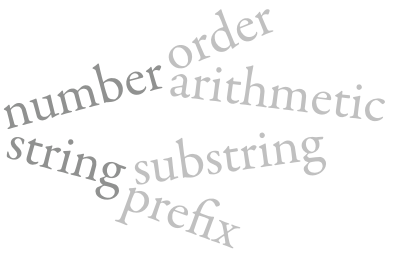
future work

XPath: inclusion&equivalence of *path expressions*

future work

XPath: inclusion&equivalence of *path expressions*

More domain specific operations



future work

XPath: inclusion&equivalence of *path expressions*

More domain specific operations number order arithmetic string substring prefix

Quest for an expressive formalism logic automaton
with *low* complexity

future work

XPath: inclusion&equivalence of *path expressions*

More domain specific operations number order arithmetic string substring prefix

Quest for an expressive formalism logic automaton
with *low* complexity

~~restricting the logics/automata~~
restricting the class of models to obtain decidable/tractable procedures

Results

1. Horizontal

rLTL + $\forall\leq$ + $\exists\geq$ is decidable
ARA(guess, spread) is decidable

rLTL(F) is non primitive recursive
rLTL(F, F^{-1}) is undecidable

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XPath(=, $\downarrow$, $\vdots$) is ExpTime-complete

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3. Forward

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XPath(=, $\downarrow$, $\downarrow^*$) is ExpTime-complete

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3. Forward

XPath(=, $\downarrow$, $\downarrow^*$,$\rightarrow$,$\rightarrow^*$) is decidable,
non primitive recursive

ATRA(guess, spread) is decidable

4. Vertical

XPath(=, $\downarrow$, $\downarrow^*$, $\uparrow$, $\uparrow^*$) is decidable,
non primitive recursive

BUDA is decidable

Results

1. Horizontal

rLTL + $\forall\leq$ + $\exists\geq$ is decidable
ARA(guess, spread) is decidable

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rLTL(F, F^{-1}) is undecidable

2. Downward

XPath(=, $\downarrow$, $\downarrow^+$) is ExpTime-complete

DD automaton is 2ExpTime

3. Forward

XPath(=, $\downarrow$, $\downarrow^+$,$\rightarrow$,$\rightarrow^+$) is decidable, non primitive recursive

ATRA(guess, spread) is decidable

4. Vertical

XPath(=, $\downarrow$, $\downarrow^+$, $\uparrow$, $\uparrow^+$) is decidable, non primitive recursive

BUDA is decidable

Tomorrow 16h45