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THE OCCURRENCE PROBLEM FOR EXTENSIONS OF ABELIAN GROUPS BY NILPOTENT GROUPS

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The occurrence problem, which is one of the classical algorithmic problems of group theory, consists of the following. Suppose a group G is defined in some constructive way. Given any finite set $\{g, h_1, \ldots, h_n\}$ of elements of G, how can we determine whether or not g belongs to the subgroup generated by $h_1, \ldots, h_n$? In the classical formulation, the group G is presented by means of a finite number of generators and defining relations. Since a group with algorithmically unsolvable word problem obviously has unsolvable occurrence problem, the theorem of Novikov [1] shows that, in general, the occurrence problem for finitely presented groups is algorithmically unsolvable. In this present paper we will consider groups that are finitely presented in certain varieties. We will say that a group G is presented in the variety $\mathfrak{M}$ by generators $x_1, \ldots, x_n$ and relations $r_1(x) = 1, \ldots, r_m(x) = 1$ if it is the factor group of the free group of this variety with basis $x_1, \ldots, x_n$ by the normal subgroup generated by the elements $r_1, \ldots, r_m$, or, in other words, if all relations in G among the elements $x_1, \ldots, x_n$ follow from the relations $r_1 = 1, \ldots, r_m = 1$ and the identical relations defining the given variety. Clearly, a group that is finitely presented in the usual sense is finitely presented in any variety containing it. It is known that the occurrence problem is solvable in the variety $\mathfrak{N}_c$ of nilpotent groups of class $\leq c$; this follows from the finite separability of subgroups of a finitely generated nilpotent group, which was proved by Mal'tsev [2]. On the other hand, Remeslennikov [3] constructed an example of a group, finitely presented in the variety $\mathfrak{A}^4$ ($\mathfrak{A}$ is the variety of Abelian groups), with unsolvable occurrence problem. Apparently, such an example also exists in the variety $\mathfrak{A}^3$. The author [4] proved the solvability of the occurrence problem for two-step solvable groups. In the present paper this result is generalized to groups in the variety $\mathfrak{A}\mathfrak{N}_c$ for any c. Note that in this variety, according to Theorem 3 of [5], any finitely generated group is finitely presented (in the variety). Note also that in the problem under consideration the method of finite separability is inapplicable, since it is easy to give examples of finitely generated, two-step solvable groups with finitely inseparable, finitely generated subgroups.

Let us state the main result.

THEOREM. For groups presented in the variety $\mathfrak{A}\mathfrak{N}_c$ by means of a finite number of generators and defining relations, the occurrence problem is solvable.

We will need the following assertion, which was proved by Kargapolov and others in [6].

Proposition 1. Suppose a group is presented in the variety $\mathfrak{N}_c$ by means of a finite number of generators and defining relations. Then for any subgroup presented by its generators we can effectively find defining relations in terms of these generators.

We will reduce the proof of our theorem to the solution of the problem of occurrence in a specific kind of subgroup of certain wreath products. Suppose a given group G is presented in the variety $\mathfrak{A}\mathfrak{N}_c$ by generators $x_1, \ldots, x_n$ and relations $r_1(x) = 1, \ldots, r_m(x) = 1$. As we have said, G can be represented as the factor group of the free group F of the variety $\mathfrak{A}\mathfrak{N}_c$ with basis $\{x_1, \ldots, x_n\}$ by the normal subgroup R generated by the elements $r_1, \ldots, r_m$. The theorem will be true if we can prove that there exists an algorithm enabling us to tell, for any finite set $\{g, h_1, \ldots, h_k\}$ of elements of F, whether the element g belongs to the subgroup HR, where $H = \mathrm{gr}(h_1, \ldots, h_k)$. Consider the Magnus embedding of F into the discrete wreath product $A \sim B = D$, where A

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is the free Abelian group with basis $\{a_1, \ldots, a_n\}$ and B is the free c-step nilpotent group with basis $\{b_1, \ldots, b_n\}$. Then $x_1 = b_1 a_1, \ldots, x_n = b_n a_n$. The group D can be represented as a semidirect product $D = B\bar{A}$, where $\bar{A}$ is the base subgroup of the wreath product. In accordance with this we decompose each element r_i into a product $r_i = r_i' r_i''$, where $r_i' \in B$, $r_i'' \in \bar{A}$. Let R' denote the normal subgroup generated in B by the elements $r_1', \ldots, r_m'$. It is known that the canonical homomorphism $B \to B/R' = B'$ and the identity mapping $A \to A$ can be extended to a homomorphism of wreath products $\varphi : A \sim B \to A \sim B' = D'$. Note that the group B' is presented in the variety $\mathfrak{N}_c$ by the generators $x_1, \ldots, x_n$ and relations $r_1 = 1, \ldots, r_m = 1$. It is easy to see that $R\varphi$ is a normal subgroup of D' that is contained in the base subgroup of the wreath product and is generated by the elements $r_1\varphi, \ldots, r_m\varphi$. Lemma 1 of [4] asserts that the canonical homomorphism $G \to F\varphi/R\varphi$ is an isomorphism, i.e., the group G is embedded in $D'/R\varphi$. Therefore, our theorem will be a consequence of the following.

Proposition 2. Suppose a nilpotent group B is presented in the variety $\mathfrak{N}_c$ by means of the generators $b_1, \ldots, b_n$ and a finite number of defining relations; A is the free Abelian group with basis $\{a_1, \ldots, a_n\}$; $G = A \sim B$ is the discrete wreath product of A and B; M is the normal subgroup of G generated by a given (in the form of words in the generators $a_1, \ldots, a_n, b_1, \ldots, b_n$) finite set S of elements of the base subgroup of the wreath product. Then there exists an algorithm enabling us to tell, for any set $\{g, h_1, \ldots, h_m\}$ of elements of G, whether g belongs to the subgroup HM, where $H = \mathrm{gr}(h_1, \ldots, h_m)$.

Proof. Suppose $\varphi : G \to B$ is the canonical projection. If $g\varphi \notin H\varphi$ (this can be effectively decided by a theorem of Mal'tsev), then $g \notin HM$ and our problem is settled. We will therefore assume that $g\varphi \in H\varphi$. Then we can find a representation of $g\varphi$ as a word $w(h_1\varphi, \ldots, h_m\varphi)$. Consider the element $g' = g \cdot w^{-1}(h_1, \ldots, h_m)$. Obviously, it belongs to HM if and only if $g \in HM$. Consequently, it suffices to prove Proposition 2 in the situation where g belongs to the base $\bar{A}$ of the wreath product. We will assume this in the sequel. Put $N = H \cap \bar{A}$. Clearly, $HM \cap \bar{A} = NM$ and we must solve the problem of the occurrence of an element of $\bar{A}$ in the subgroup NM.

It is well known that we can regard $\bar{A}$ as a free right $\mathbf{Z}[B]$-module with basis $\{a_1, \ldots, a_n\}$. Addition in the module corresponds to the group operation in $\bar{A}$, and the product of an element a of $\bar{A}$ by an element $\alpha_1 v_1 + \ldots + \alpha_k v_k$, $\alpha_i \in \mathbf{Z}$, $v_i \in B$, is understood to be $(v_1^{-1} a v_1)^{\alpha_1} \ldots (v_k^{-1} a v_k)^{\alpha_k}$. From this point of view, M is the submodule of $\bar{A}$ generated by the set S. By Proposition 1, we can find defining relations (in the class of all groups) of the group $H\varphi$ in the generators $h_1\varphi, \ldots, h_m\varphi$. Let these be $u_1(h_1\varphi, \ldots, h_m\varphi) = 1, \ldots, u_l(h_1\varphi, \ldots, h_m\varphi) = 1$. Then we can regard N as the $\mathbf{Z}[H\varphi]$-module generated by the elements $u_1 = u_1(h_1, \ldots, h_m), \ldots, u_l = u_l(h_1, \ldots, h_m)$. Thus, we must be able to solve, in the free module over the ring $\mathbf{Z}[B]$, the problem of occurrence in the set M + N, where M is the submodule generated by the given set S, and N is the $\mathbf{Z}[H\varphi]$-module generated by the set $U = \{u_1, \ldots, u_l\}$.

Note that in the group B we can effectively construct a polycyclic series passing through $H\varphi$. Indeed, we construct the subnormal series $B = \gamma_1(B) \supseteq \gamma_2(B) \cdot H\varphi \supseteq \ldots \supseteq \gamma_c(B) \cdot H\varphi \supseteq H\varphi \supseteq \gamma_2(H\varphi) \supseteq \ldots \supseteq \gamma_c(H\varphi) \supseteq 1$, where $\gamma_i(G)$ denotes the i-th term of the lower central series of G, and then, using Proposition 1, we refine this series to a polycyclic series and discard repetitions. Consider the resulting series:

$$B = B_1 > B_2 > \ldots > B_p > B_{p+1} = 1. \qquad (*)$$

Suppose the element x_k generates B_k modulo B_{k+1}, and let the order of this element modulo B_{k+1} be ω_k. Each element of B can be uniquely represented as a canonical product $x_1^{\alpha_1} \ldots x_p^{\alpha_p}$, where $\alpha_i \in \mathbf{Z}$ and $0 \le \alpha_i < \omega_i$ for finite ω_i. The canonical products of the form $x_1^{\alpha_1} \ldots x_{k-1}^{\alpha_{k-1}}$ constitute a set of representatives of all left cosets of B modulo B_k. Note also that, by Theorem 1 of [5], the ring $\mathbf{Z}[B]$ satisfies the maximum condition for right ideals, hence finitely generated right $\mathbf{Z}[B]$-modules satisfy the maximum condition for submodules. The assertion of Proposition 2 is easily deduced from the following lemma, in which we consider our group B with the series (*).

LEMMA. Suppose E is a free right $\mathbf{Z}[B]$-module of finite rank with basis $\{e_i / i \in I\}$. Fix natural numbers $\lambda_1, \ldots, \lambda_p$ and, for each k $(2 \le k \le p+1)$, let $E_{\lambda_1, \ldots, \lambda_{k-1}}$ denote the right $\mathbf{Z}[B_k]$-module generated in E by the elements of the form $e_i x_1^{\alpha_1} \ldots x_{k-1}^{\alpha_{k-1}}$, where $i \in I$, $|\alpha_j| \le \lambda_j$, $x_1^{\alpha_1} \ldots x_{k-1}^{\alpha_{k-1}}$ is a canonical product. Let M be the submodule of E generated by a given finite set of elements S. Then

a) we can effectively find a finite system of defining relations of the module M in the generators S;

b) there exists an algorithm enabling us to construct, starting from S, a new finite system of generators $\bar{S}$ of the module M (the elements of $\bar{S}$ will be expressed in terms of the elements of S, and conversely) such that $\bar{S} \cdot \mathbf{Z}[B_k] \supseteq M \cap E_{\lambda_1, \ldots, \lambda_{k-1}}$ for all k.

COROLLARY. In the module E the problem of occurrence in a submodule is algorithmically solvable.

Indeed, suppose we want to determine whether an element $a \in E$ lies in a submodule M. Choose $\lambda_1, \ldots, \lambda_p$ such that $a \in E_{\lambda_1,\ldots,\lambda_p}$. If $a \in M$, then, by the lemma, $a \in \bar{S} \cdot \mathbf{Z}$. It remains to observe that in a finitely generated $\mathbf{Z}$-module the problem of occurrence in a submodule is solvable.

Proof of the Lemma. Choose a natural number λ_1' such that $S \subseteq E_{\lambda_1'}$, $\lambda_1' \geqslant \lambda_2$ and $\lambda_1' \geq \omega_1$ if $\omega_1 < \infty$. Consider the $\mathbf{Z}[B_2]$-module $E_{\lambda_1'}$. It is a free module with basis $\{e_i x_1^{\alpha}\}$, where $i \in I$, $|\alpha| \leqslant \lambda_1'$, x_1^{α} is a canonical element. Using induction on p, we assume that the assertions of the lemma hold for the group B_2, the series $B_2 > B_3 > \ldots > B_p > B_{p+1} = 1$, and the module $E_{\lambda_1'}$. Note that $\left(E_{\lambda_1'}\right)_{\lambda_2,\ldots,\lambda_{k-1}} = E_{\lambda_1',\lambda_2,\ldots,\lambda_{k-1}} \supseteq E_{\lambda_1,\lambda_2,\ldots,\lambda_{k-1}}$.

1) Suppose first that $\omega_1 < \infty$. Then, in view of the choice of λ_1', it follows that $E = E_{\lambda_1'}$ and M is the $\mathbf{Z}[B_2]$-module generated by the set $S \cup Sx_1 \cup \ldots \cup Sx_1^{\omega_1 - 1}$. By the inductive assumption, in these generators we can effectively find a set of defining relations of the module M over the ring $\mathbf{Z}[B_2]$, which we can regard as relations among the elements of S over the ring $\mathbf{Z}[B]$. It is easy to see that they are defining relations. Then, by induction, we can effectively construct a set of generators $\bar{S}$ of the module M over the ring $\mathbf{Z}[B_2]$ such that $\bar{S} \cdot \mathbf{Z}[B_k] \supseteq M \cap \left(E_{\lambda_1'}\right)_{\lambda_2,\ldots,\lambda_{k-1}}$ for any $k \geq 2$. The set $\bar{S}$ obviously satisfies the lemma.

2) Now suppose $\omega_1 = \infty$. Consider the $\mathbf{Z}[B_2]$-module $Sx_1 \cdot \mathbf{Z}[B_2] \cap E_{\lambda_1'}$; we will find generators for it. Let $\sigma : E \to \sigma(E)$ denote the canonical projection of E onto the $\mathbf{Z}[B_2]$-module generated by the elements $e_i x_1^{\lambda_1'+1}$, $i \in I$. Note that $\sigma(E)$ is a free module. By the inductive assumption, we can effectively find generators of the module of relations over $\mathbf{Z}[B_2]$ among the elements of the set $\sigma(Sx_1)$. Let these relations be $\sum_{s \in S} \sigma(sx_1)\, t_s^{(j)} = 0$, $t_s^{(j)} \in \mathbf{Z}[B_2]$, $j \in J$. Then it is easy to see that the elements $\sum_{s \in S} sx_1 t_s^{(j)}$, $j \in J$, generate the module $Sx_1 \cdot \mathbf{Z}[B_2] \cap E_{\lambda_1'}$. In view of the corollary of the lemma and the inductive assumption, we can decide whether they are contained in $S \cdot \mathbf{Z}[B_2]$. If not, then we adjoin them to S and, replacing S by the resulting set, repeat this procedure. Since the modules under consideration are Noetherian, we will be able to construct a set $\tilde{S}$ containing S in which each element can be represented as a $\mathbf{Z}[B]$-combination of elements of S and such that $\tilde{S} \subseteq E_{\lambda_1'}$, $\tilde{S}x_1 \cdot \mathbf{Z}[B_2] \cap E_{\lambda_1'} \subseteq \tilde{S} \cdot \mathbf{Z}[B_2]$, $\tilde{S}x_1^{-1} \cdot \mathbf{Z}[B_2] \cap E_{\lambda_1'} \subseteq \tilde{S} \cdot \mathbf{Z}[B_2]$. We now claim that $\tilde{S} \cdot \mathbf{Z}[B_2] \supseteq M \cap E_{\lambda_1'}$. Indeed, suppose $a \in M \cap E_{\lambda_1'}$. Then there exist nonnegative integers m and n such that $a = a_{-m} + a_{-m+1} + \ldots a_{n-1} + a_n$, where $a_j \in \tilde{S}x_1^j \cdot \mathbf{Z}[B_2]$. If $n \geq 1$, then $a_n \in \tilde{S}x_1^n \cdot \mathbf{Z}[B_2] \cap E_{\lambda_1'+n-1} = \tilde{S}x_1^n \cdot \mathbf{Z}[B_2] \cap E_{\lambda_1'} \cdot x_1^{n-1}$. The latter set, according to the construction of $\tilde{S}$, is contained in $\tilde{S} \cdot x_1^{n-1} \cdot \mathbf{Z}[B_2]$. By induction, $a \in \tilde{S} \cdot \mathbf{Z}[B_2]$. Thus, $\tilde{S} \cdot \mathbf{Z}[B_2] = M \cap E_{\lambda_1'}$. In view of the assumption that the lemma holds for the module $E_{\lambda_1'}$, we can effectively construct, starting from $\tilde{S}$, a finite system of generators $\bar{S}$ of the module $\tilde{S} \cdot \mathbf{Z}[B_2]$ such that $\bar{S} \cdot \mathbf{Z}[B_k] \supseteq \tilde{S} \cdot \mathbf{Z}[B_2] \cap E_{\lambda_1',\lambda_2,\ldots,\lambda_{k-1}}$, $2 \leqslant k \leqslant p+1$. The set $\bar{S}$ satisfies the lemma, since $\tilde{S} \cdot \mathbf{Z}[B_2] \cap E_{\lambda_1',\lambda_2,\ldots,\lambda_{k-1}} \supseteq M \cap E_{\lambda_1'} \cap E_{\lambda_1',\lambda_2,\ldots,\lambda_{k-1}} \supseteq M \cap E_{\lambda_1,\lambda_2,\ldots,\lambda_{k-1}}$.

Finally, we will find defining relations of the module M in the generators S. Obviously, it suffices to give such relations for the generators $\tilde{S}$. They will be of three types. The first type consists of generators of the module of relations among the elements of $\tilde{S}$ over $\mathbf{Z}[B_2]$, which can be found effectively in view of the inductive assumption. Next, suppose $\sum_{s \in \tilde{S}} \sigma(sx_2)\, t_s^{(j)} = 0$, $t_s^{(j)} \in \mathbf{Z}[B_2]$, $j \in J$, are generators of the module of relations over $\mathbf{Z}[B_2]$ among the elements of the set $\sigma(\tilde{S}x_1)$, where σ is the mapping defined above. Then the elements $a_j = \sum_{s \in \tilde{S}} sx_1 t_s^{(j)}$ belong to the module $\tilde{S}\mathbf{Z}[B_2]$. Represent them in the form $a_j = \sum_{s \in \tilde{S}} sj_s^{(j)}$, $f_s^{(j)} \in \mathbf{Z}[B_2]$. The relations of the second type have the form $\sum_{s \in \tilde{S}} s\left(x_1 t_s^{(j)} - f_s^{(j)}\right) = 0$, $j \in J$. Analogously, let τ denote the canonical projection of E onto the $\mathbf{Z}[B_2]$-module generated by the elements $e_i x_1^{-\lambda_1'-1}$, $i \in I$, and let generators of the module of relations among the elements of the set $\tau(\tilde{S}x_1^{-1})$ over the ring $\mathbf{Z}[B_2]$ be $\sum_{s \in \tilde{S}} \tau(sx_1^{-1})\, v_s^{(l)} = 0$, $v_s^{(l)} \in \mathbf{Z}[B_2]$, $l \in L$. Represent the elements $a_l' = \sum_{s \in \tilde{S}} sx_1^{-1} v_s^{(l)}$ in the form $a_l' = \sum_{s \in \tilde{S}} sw_s^{(l)}$, $w_s^{l} \in \mathbf{Z}[B_2]$. The desired relations of the third type have the form $\sum_{s \in \tilde{S}} s\left(x_1^{-1} v_s^{(l)} - w_s^{(l)}\right) = 0$, $l \in L$. It can be verified directly that the relations of these types are indeed defining relations for the module M in the generators $\tilde{S}$. The lemma is proved.

Let us complete the proof of Proposition 2. We are considering the free module $\bar{A}$ over the ring $\mathbf{Z}[B]$. Suppose $H\varphi = B_k$. Then M is the submodule of $\bar{A}$ generated by the set S, and N is the $\mathbf{Z}[B_k]$-module generated by the set U. Suppose we are given an element $a \in \bar{A}$, and we want to know whether or not it belongs to M + N. Choose natural numbers $\lambda_1, \ldots, \lambda_p$ such that $a \in \bar{A}_{\lambda_1,\ldots,\lambda_{k-1}}$ and $N \subseteq \bar{A}_{\lambda_1,\ldots,\lambda_{k-1}}$. Construct a set $\bar{S}$ satisfying the lemma. We have $\bar{A}_{\lambda_1,\ldots,\lambda_{k-1}} \cap (M+N) = (\bar{A}_{\lambda_1,\ldots,\lambda_{k-1}} \cap M) + N \subseteq \bar{S}Z[B_k] + N = \bar{S}Z[B_k] + UZ[B_k]$. Thus, the element a belongs to M + N if and only if it belongs to the $\mathbf{Z}[B_k]$-module generated by the set $\bar{S} \cup U$. The latter can be effectively decided in view of the corollary to the lemma. Thus, Proposition 2, hence also the theorem, is proved.

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CONFORMAL MAPS OF MANIFOLDS OF BOUNDED CURVATURE

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1. Let $\langle M, \rho \rangle$ be a two-dimensional manifold of bounded curvature in the sense of Aleksandrov, ρ an inner metric for M [1]. A quasiconformal map $f: \langle M_1, \rho_1 \rangle \to \langle M_2, \rho_2 \rangle$ is a homeomorphism f possessing the properties:

a) for all $a \in M_1$ the limit $\lim_{x \to a} \rho_2(f(a), f(x))/\rho_1(a, x) = \lambda_f(a)$ exists; b) f preserves the upper and lower angles between curves. (In this paper we leave aside the question of the relations between conditions a) and b). The case when f maps the plane onto itself is considered in detail in [2].)

We say that a map $f: \langle M_1, \rho_1 \rangle \to \langle M_2, \rho_2 \rangle$ preserves minimal curves if the image of such a curve in M_1 is minimal in M_2. In this paper the following theorems are proved.

THEOREM 1. Let $f: \langle M_1, \rho_1 \rangle \to \langle M_2, \rho_2 \rangle$ be a bijective map of a manifold M_1 of nonnegative curvature onto a manifold M_2 of nonnegative curvature which preserves minimal curves and the angles between them. Then f is a similarity, i.e., there exists a constant $\lambda > 0$ such that for all $x, y \in M_1$

$$\rho_2(f(x), f(y)) = \lambda \rho_1(x, y).$$

THEOREM 2. Let $f: \langle M_1, \rho_1 \rangle \to \langle M_2, \rho_2 \rangle$ be a conformal map of M_1 of nonnegative curvature onto M_2 of nonnegative curvature. Assume that every minimal curve $\gamma \subset M_1$ satisfies the relation $\varkappa(f(\gamma)) \leq C \cdot d(\gamma)$, where C is a constant, $d(\gamma)$ is the length of γ, $\varkappa(K) = \max(\varkappa_l(K), \varkappa_r(K))$, where $\varkappa_l$ $(\varkappa_r)$ is the variation of the left (right) rotation of the curve K on M_2.

Then we have the estimate

$$|\ln \lambda_f(x) - \ln \lambda_f(y)| \leq 16C\rho_1(x, y)/\sqrt{3}.$$

2. Proof of Theorem 1. Let $\Delta = (a, b, c)$ be a triangle on M_1 with sides a, b, c given by minimal curves; let A, B, C be the angles opposite the faces a, b, c, respectively; we denote the length of side x of triangle Δ

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