Unambiguous Finite Automata

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In general, a nondeterministic automaton or machine (for example a finite automaton, pushdown automaton or Turing machine) is called unambiguous if each input is accepted by at most one run or computation. Each deterministic automaton is obviously unambiguous. However, in many settings, unambiguous automata are more expressive or admit more succinct automata than deterministic models, while preserving some good algorithmic properties. The aim of this talk is to survey some classical and some more recent results on unambiguous finite automata over different kind of input structures, namely finite words, infinite words, finite trees, and infinite trees.

A typical example is the inclusion problem for automata on finite words, which is solvable in polynomial time for unambiguous automata [9], while it is PSPACE-complete for general nondeterministic automata (see Section 10.6 of [1]). This result can be lifted to unambiguous automata on finite ranked trees [8], and can, for example, be used to derive efficient inclusion tests for certain classes of automata on unranked trees [7].

The method of [9] uses a counting argument for the number of accepting runs for words up to a certain length. This method cannot be used for unambiguous Büchi automata in the setting of infinite input words. As a consequence, the question whether inclusion testing for unambiguous Büchi automata can be done efficiently, is still open. Partial results for a stronger notion of unambiguity taken from [5], and for subclasses of Büchi automata have been obtained in [3] and [6].

Concerning infinite trees, the situation becomes different because unambiguous automata are not expressively equivalent to unrestricted nondeterministic automata anymore. The proof of this result, presented in [4], relies on the fact that it is not possible to define in monadic second-order logic a choice function on the infinite binary tree. Since not all regular languages of infinite trees are unambiguous, a natural decision problem arises: "Given a regular language of an infinite tree, does there exist an unambiguous automaton for it?" It is still unknown whether this problem is decidable, only partial solutions for specific cases have been obtained [2].

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