Energy Syst
DOI 10.1007/s12667-016-0228-2

ORIGINAL PAPER

Continuous optimal control approaches to microgrid energy management

Benjamin Heymann[1] · **J. Frédéric Bonnans**[1] · **Pierre Martinon**[1] · **Francisco J. Silva**[2] · **Fernando Lanas**[3] · **Guillermo Jiménez-Estévez**[3]

Received: 26 August 2016 / Accepted: 8 December 2016

Abstract We propose a novel method for the microgrid energy management problem by introducing a nonlinear, continuous-time, rolling horizon formulation. The method is linearization-free and gives a global optimal solution with closed loop controls. It allows for the modelling of switches. We formulate the energy management problem as a deterministic optimal control problem (OCP). We solve (OCP) with two classical approaches: the direct method and Bellman's Dynamic Programming Principle (DPP). In both cases we use the optimal control toolbox Bocop for the numerical simulations. For the DPP approach we implement a semi-Lagrangian scheme adapted to handle the optimization of switching times for the on/off modes of the diesel generator. The DPP approach allows for accurate modelling and is computationally cheap. It finds the global optimum in less than one second, a CPU time similar to the time needed with a Mixed Integer Linear Programming approach used in previous works. We achieve this result by introducing a 'trick' based on the Pontryagin Maximum Principle. The trick reduces the computation time by several orders and improves the precision of the solution. For validation purposes, we performed simulations on datasets from an actual isolated microgrid located in northern Chile. The result shows that the DPP method is very well suited for this type of problem.

✉ Benjamin Heymann
benjheymann@gmail.com

[1] CMAP, Inria, Ecole polytechnique, CNRS, Université Paris-Saclay, 91128 Palaiseau, France

[2] Institut de recherche XLIM-DMI, UMR-CNRS 7252, Faculté des sciences et techniques, Université de Limoges, 87060 Limoges, France

[3] Energy Center, Faculty of Mathematical and Physical Sciences, School of Engineering, Universidad de Chile, Santiago, Chile

Published online: 05 January 2017

Keywords Energy management system (EMS) · Microgrid · Optimal control · Direct method · Pontryagin Maximum Principle (PMP) · Semi-Lagrangian scheme

Mathematics Subject Classification 93C15 · 90C39 · 49L20 · 49J15

1 Introduction

Distributed Energy Resources (DER) play a key role as an energy supply alternative. Moreover, DER are in most of the cases renewable energy sources and bring positive environmental impacts and contribute to sustainability. In order to integrate in a massive way DER into interconnected power systems or make use of DER as a power source for isolated locations, microgrids appear as a suitable technical solution.

A microgrid is a group of interconnected loads and DER that acts as a single controllable entity. It can operate connected to the main network or autonomously (isolated) [1]. In either case, an Energy Management System (EMS) is required to coordinate the different units that compose it. The EMS solves an optimization problem and, as described in [2], this problem falls into the category of Mixed Integer Linear Programming (MILP).

Depending on the philosophy established for the EMS and the different components (generation units, loads, storage devices) incorporated into the microgrid, the objective function may be nonlinear. Moreover, the operation of some of these components involves start up / shut down set points that are typically represented as binary functions of time in the problem formulation. Constraints represent in particular operational limitations of storage devices and generation units (i.e., batteries (dis)charging patterns).

This problem is usually modeled by a MILP approach, for which the complexity mostly stems from the modelling of nonlinearities: battery charging/discharging pattern and the diesel engine efficiency [2] for instance. Heuristic techniques have been also applied to the microgrid EMS problem, such as Genetic Algorithms (GA) [3,4], Particle Swarm Optimization (PSO) [3], and Ant Colony Optimization (ACO) [5].

Finally, recent works which focus on microgrid energy management systems have incorporated a more detailed modelling of the energy storage system. This energy management system considers the importance of the cost associated with its replacement, so that extending the life span of the battery is part of the objective. In this context, GA have been implemented to solve the problem [6], and other predictive control approaches such as the ones described in [7–10].

Other authors have made use of the Dynamic Programming Principle (DPP) to solve the EMS problem. Kanchev et al. [11] use DPP but look for GHG emissions reductions, [12] focus its objective on the Energy Storage System (ESS) management. These cases do not consider the economic efficiency of the whole microgrid. In [13] the EMS problem is focused on buildings decision incorporating uncertainty modeled with Markov chains under a discrete approach. Babazadeh [14] makes use of DPP to handle the wind power management in a microgrid environment. In [15] DPP has been developed to solve out the maximum profit an owner might achieve from energy trading in a day, either in isolated or connected mode, without taking into account the management of batteries. Finally, [16] applies a multi path dynamic programming (MPDP) approach to solve a power scheduling considering load/generation changes and time of use (TOU) tariff for a low voltage DC microgrid incorporating energy storage battery, fuel cell and Photo Voltaic energy.

An important conceptual difference between previous works and the present study is that we start with the optimal control problem formulated in continuous time. This gives access to a broader set of theoretical and numerical tools (e.g. Pontryagin's Principle and the Direct Approach).

The microgrid model presented in this work handles some challenges involved with the microgrid EMS, such as units modelling, ESS management, CPU solving time for real applications and the switching of the generator mode (on or off) among others, with a continuous time optimal control formulation. This approach keeps the original non-linear model for the numerical optimization, which enhances the solutions accuracy. The proposal considers two solution methods:

The direct method starts with a time discretization to transform the continuous optimal control problem into a Nonlinear Programming (NLP) problem. The NLP is then solved with any usual technique (see for instance [17]).

The DPP method [18] relies on Bellman' s Principle and uses a discretization of both time and space to compute the value function. This information then allows the reconstruction of the optimal trajectory using feedback controls, see e.g. [19].

We perform numerical simulations for both methods using the optimal toolboxes Bocop and BocopHJB [20,21]. The proposed methods are validated with data from a real microgrid operating in Huatacondo, an isolated northern Chilean village that relies completely on the microgrid concept for its electricity supply, which is described in Sect. 2. The present study uses a similar model to the one presented in [22], so that the comparison is relevant. We show results for the three approaches: MILP, direct method and DPP.

Note that this work focuses on the comparison of the three techniques, but does not intend to deal with their implementation as building block of upper level algorithms, such as Model Predictive Control (MPC). Likewise, the demand and load modelling is out of the scope of this article. In addition, in all this work the microgrid is considered in disconnected mode, but a similar approach in connected mode could be envisioned (using a market price for instance).

The main contributions of this work are:

– the introduction of a continuous time non-linear framework for the microgrid energy management problem,
– for the dynamic programming approach, the modelling of the generator switching,
– the combination of the Pontryagin Maximum Principle and the Dynamic Programing Principle to get a surprising improvement of the computing time
– a comparison of the continuous time non-linear framework (with two resolution techniques, DPP and direct method) with the MILP formulation.

The paper is organized as follows. Section 2 describes the microgrid system and the optimal control formulation for its energy management. Section 3 explains the numerical methods we use to solve the optimal control problem. Section 4 presents the numerical simulations with the direct and DPP methods. Section 5 comments the results of the simulations. The conclusion sums up the main results and presents ongoing research.

2 Model presentation

2.1 General aspects

2.1.1 Description of the microgrid

The following model is based on a real microgrid operating in Huatacondo, an isolated village in northern Chile that relies entirely on the microgrid concept for its electricity supply. The microgrid we are considering includes a photovoltaic power plant (PV), a diesel generator and a battery energy storage system (BESS). It uses a mix of fuel and renewable energy sources. The solar panel and the wind turbine produce electricity without any additional cost, but the generation pattern cannot be controlled and depends on the daily weather. The BESS can store energy for later use, but has limited capacity and power. The diesel generator has a minimal and a maximal output levels, and has a fixed start-up cost. All these are local generation units, i.e. situated physically near the electric consumption point, and electric losses due to distribution are not considered.

The aim is to find the optimal planning that meets the power demand and minimizes the operational costs, which, in this case, mainly relates to the diesel consumption. We follow the problem description from [22].

2.1.2 Optimal control formulation

We consider a fixed horizon $T = 48$ h. This is practice among electrical engineers, due to the fact that the meteorological forecast is reliable over this horizon. It also eases comparisons with [22].

For $t \in [0, T]$, we denote by $P_S(t)$ the solar power from the photovoltaic panels, $P_D(t)$ the diesel generator power and $P_L(t)$ the electricity load. The state of charge $SOC(t)$ of the BESS evolves according to the dynamics

$$S\dot{O}C(t) = \frac{1}{Q_B}(P_I(t)\rho_I - P_O(t)/\rho_O), \tag{1}$$

where Q_B is the maximum capacity of the battery, P_I, $P_O > 0$ are the input and output power of the BESS, and ρ_I, ρ_o are the efficiency ratios for the charge and discharge processes, assumed constant. Observe that (1) writes equivalently

$$S\dot{O}C(t) = \frac{1}{\tilde{Q}_B}(P_I(t)\tilde{\rho} - P_O(t)), \tag{2}$$

where $\tilde{Q}_B = \rho_O Q_B$ and $\tilde{\rho} = \rho_I \rho_O$.

Table 1 Model parameters, CLP means Chilean Pesos

Name	Notation	Value	Unit
Min diesel power	P_{min}	5	kW
Max diesel power	P_{max}	120	kW
Unserved energy cost	C_{US}	250	CLP/kWh
Diesel start-up cost	–	1000	CLP
Diesel price	C_D	500	CLP

We introduce the slack variable P_{slack} that represents the excess power ($P_{slack} < 0$), which has to be shed, or the missing power in the microgrid ($P_{slack} > 0$), which turns into unmet demand. The addition of this variable ensures the mathematical feasibility of the problem. Positive P_{slack} will be penalized by $C_{US} P^{+}_{slack}$, where C_{US} is a positive constant (see Table 1).

The underlying power equilibrium equation is

$$P_D + P_O + P_S + P_{slack} - P_L - P_I = 0. \tag{3}$$

Taking into account the demand and the various power production devices, we obtain that P_O and P_I can be written as nonlinear functions of (t, P_D, P_{slack}):

$$\begin{aligned} P_O(t, P_D, P_{slack}) &= -\min(0, P_S(t) + P_D - P_L(t) + P_{slack}),\\ P_I(t, P_D, P_{slack}) &= \max(0, P_S(t) + P_D - P_L(t) + P_{slack}). \end{aligned} \tag{4}$$

We model the fuel consumption of the diesel generator by the following strictly concave function

$$\int_0^T K P_D(t)^{0.9} \mathrm{d}t, \tag{5}$$

with $K = 0.471$. The fuel consumption curve was extrapolated from the datasheet provided by the diesel generator manufacturer as in [22].

For physical reasons, the system is subject to the following constraints at every time $t \in [0, T]$:

$$SOC(t) \in [0.2, 1], \tag{6}$$

$$P_D(t) \in \{0\} \cup [5, 120], \tag{7}$$

$$\begin{cases} P_I(t, P_D(t), P_{slack}) \in [0, 13.2] & \text{if } SOC(t) < 0.9,\\ P_I(t, P_D(t), P_{slack}) \leq 13.2(1 - SOC(t))^2 & \text{otherwise,} \end{cases} \tag{8}$$

$$P_O(t) \in [0, 40]. \tag{9}$$

Note that (4) implies that (8) and (9) are constraints on P_D. The state constraint (6) expresses the maximum and minimum charge of the battery. Constraints (7)–(9) are control constraints. The minimal and maximal power for the diesel generator are given by (7). The charging and discharging limits for the battery are stated in (8) and (9). The

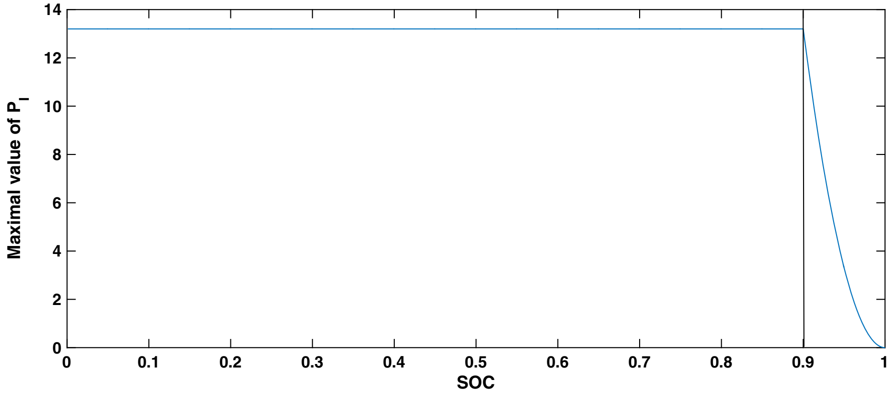

Fig. 1 Battery charge constraint

charging limit depends on the state of charge, and is therefore a mixed control-state constraint, as illustrated on Fig. 1.

Since the operations time frame is larger than the optimization horizon, we impose a constraint on the final time to avoid the battery depletion. We impose this constraint either with a periodicity condition $SOC(0) = SOC(T)$ (direct method) or a penalization term $g(SOC(T))$ (DPP method).

In summary the optimal control problem can be written under the following abstract formulation (see [23, Chapter 2])

$$(OCP) \begin{cases} \min\limits_{u} \displaystyle\int_0^T \ell(u(t))\mathrm{d}t + g(x(T)) \\ \dot{x}(t) = F(u(t), t) \\ x(0) = x_0 \\ u(t) \in U_{x(t)} \\ x(t) \in \mathcal{C}. \end{cases} \tag{10}$$

In the notation above, x, u, F, U_x and $\mathcal{C}$ correspond respectively to the state variable (SOC), the control variable (P_D and P_{slack}), the dynamics of the system (1) and (4), the control constraints on $u(t)$ (see (7)–(9)), and the state constraints (6). The time horizon T is 48 h. The objective function is the sum of a final cost g (introduced in Sect. 2.3 to impose a periodicity condition) and the integral of ℓ defined in (5). These functions are defined resp. over the state and control space.

2.2 Switching cost

Turning the diesel generator on consumes fuel. We model this by considering that the diesel generator has two modes: when off, the only admissible control is $P_D = 0$, whereas when it is on, $P_D \in [P_{\min}, P_{\max}]$. At any time, one can *switch* from one mode to the other by paying the corresponding switching cost. This cost is zero to turn the generator off, and is equal to C_D to turn the generator on. It should be stressed that while the modelling of the switching cost is straightforward in the DPP setting, it is challenging for the Direct Method approach.

2.3 Periodicity condition

To avoid the battery depletion at the end of the time horizon, we add a periodicity constraint on the state

$$SOC(0) = SOC(T). \tag{11}$$

The implementation of the constraint is straightforward for the MILP model and the Direct Method. The actual initial value is then optimized by the algorithm.

For the dynamic programming approach we model the periodicity condition by taking a similar approach to the "big M method" in linear programming:

$$\begin{aligned} g(SOC(T)) &= M \quad \text{if } SOC(T) < SOC_0, \\ g(SOC(T)) &= 0 \quad \text{if } SOC(T) \geq SOC_0. \end{aligned}$$

For the simulations, we set $SOC_0 = 0.7$.

3 Presentation of the numerical methods

We give here a brief presentation of the two resolution approaches we are considering and explain how to apply them in order to solve (10). The reader will find more on these approaches in [17,19].

3.1 The direct method approach

3.1.1 Presentation

The Direct Method consists in applying a nonlinear programming interior-point algorithm to a time discretization of the optimal control problem. The *decision* variables of this discretized problem are the values of the control variables at each time step. Since we solve the discretized problem by locally convergent algorithms, we cannot guarantee that the numerical solution (if any) is close to a global optimum. On the other hand, this approach often provides efficient solutions for large scale optimal control problems, with limited computing times. Here is a summary of the Euler type time discretization:

$$
\begin{array}{|lll|}
\hline
t \in [0,T] & \rightarrow \{t_0 = 0, \dots, t_N = T\} & \\
x(\cdot), u(\cdot) & \rightarrow Z = \{x_0, \dots, x_N, u_0, \dots, u_{N-1}\} & \\
\hline
Criterion & \rightarrow \min \ h \sum_{i=0}^{N-1} \ell(u_i) + G(x_N) & \\
Dynamics & \rightarrow x_{i+i} = x_i + hf(x_i, u_i) & i = 0, \dots, N \\
Controls & \rightarrow u_i \in U_{x_i} \quad i = 0, \dots, N-1 & \\
States & \rightarrow x_i \in \mathcal{C} \quad i = 0, \dots, N & \\
\hline
\end{array}
$$

We therefore obtain a nonlinear programming problem of the form

$$
(NLP) \begin{cases} \min_Z \ F(Z) \\ LB \leq C(Z) \leq UB. \end{cases}
$$

The optimal control toolbox BOCOP solves the discretized nonlinear optimization problem with the IPOPT solver [24] that implements a primal-dual interior point algorithm.

3.1.2 Modelling remarks

We come back to our setting. This method allows a periodicity constraint of the form $SOC(0) = SOC(T)$ where the actual value is optimized by the algorithm. On the other hand, the constraint (7) is changed into $P_D \in [0, 120]$ because switching costs are not easily modelled within this framework.

3.2 Dynamic programming approach

We propose a semi-Lagrangian scheme to solve the DPP, in particular because it is adapted for problems with switching modes. We refer the reader to the monograph [19] and the references therein for an introduction to semi-Lagrangian schemes applied to optimal control problems. In addition, the Pontryagin Maximum Principle (PMP), see [25], provides additional information on the optimal solution. The combination of the Dynamic Programming Principle and the Pontryagin Maximum Principle reduces the computing time of the method significantly.

3.2.1 Brief presentation of the theory

Let $V(t, x_0)$ denote the value of the variant of problem (10) with initial time t and initial state x_0. In Bellman's words [18] "An optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision." In mathematical terms, V satisfies for $h \in (0, T-t)$:

$$
V(t, x_0) = \inf \left\{ \int_t^{t+h} \ell(u_s) \mathrm{d}s + V(t+h, x(t+h)) \right\}, \tag{12}
$$

the infimum being taken over the set of admissible controls. In our case, we will use an extended version of the DPP approach that handles the switchings, see [19] for details.

3.2.2 Semi-Lagrangian scheme

The semi-Lagrangian scheme consists in solving a discretized version of (12) over the space backward in time (see [19] for an overview). We have chosen this scheme to solve the problem because it has good stability properties, it allows large time steps and it is easy to implement. Let us motivate the scheme by first discretizing in time (12). Given a time step h and N such that $Nh = T$, let us set $t_k = kh$ $(k = 0, \ldots, N)$. Denoting by V^k the "approximated" value function at t_k we have

$$V^k(x) = \min_{u \in U_x} \{h\ell(u) + V^{k+1}(x + hF(u, t_k))\}. \tag{13}$$

We derive the semi-Lagrangian scheme from (13) by discretizing in space the state variable x and introducing interpolation operators in order to approximate $V^{k+1}(x + hF(u, t_k))$ in terms of its values in the space grid. The scheme is solved backward in time and, under standard conditions, it converges to the solution V of (12). We use the implementation of BocopHJB (see [20,21]) for numerical experiments.

3.2.3 The PMP trick

Our problem has a property that greatly reduces the computing time. For the sake of simplicity we do not detail the aspects related to the state constraints. If $\bar{u}$ is the optimal control, denote by $\bar{x}$ the optimal state and by $\bar{p}$ the costate associated to the dynamics constraint $\dot{x}(t) = F(t, u(t))$. Defining the Hamiltonian $H(u, p, t) := pF(u, t) + \ell(u)$ the PMP says that for all $t \in [0, T]$ we have

$$H(\bar{u}(t), \bar{p}(t), t) \leq H(v, \bar{p}(t), t) \quad \text{for all } v \in U_{\bar{x}(t)}.$$

Since the dynamics is continuous and piecewise affine, the Hamiltonian is the sum of a continuous, piecewise affine and of a continuous strictly concave function, and therefore is continuous and piecewise strictly concave. Thus, it attains its minimum only at one of the extreme points of the pieces. Taking into account the constraints, we have at most five possible optimal controls, as illustrated in Fig. 2. Moreover, the values of these controls can be computed explicitly, since they do not depend on $\bar{p}$. Therefore, when doing the minimization in (13), we test only these controls instead of discretizing the control space, gaining both in speed and precision. So:

- if the Diesel is off (mode 0), we simply take $P_D = 0$.
- if the Diesel is on (mode 1), we test the five cases
  - $P_D = 5$ (minimum power),
  - $P_D = 120$ (maximum power),
  - P_D such that $\dot{SOC} = 0$ (battery unused),

!

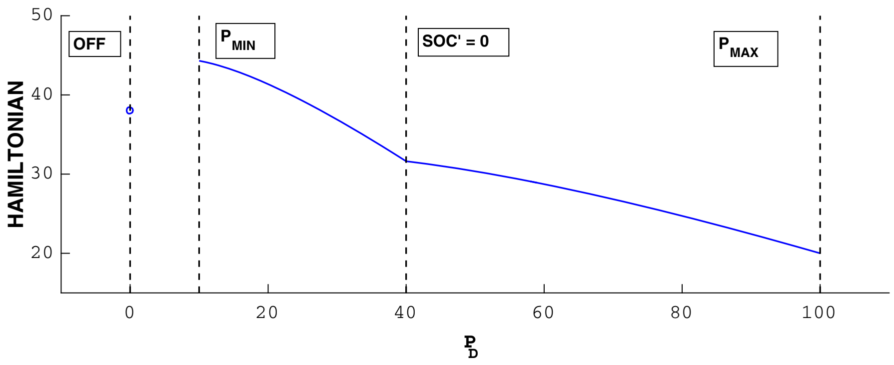

Fig. 2 The PMP trick illustrated

- P_D such that $P_i = P_i^{max}(SOC)$ (maximal charge),
- P_D such that $P_0 = 40$ (maximal discharge).

The specific structure of the problem permits to reduce the computing time. More precisely, the candidates for the optimal control do not depend on the costate $\bar{p}$ and therefore can be evaluated and tested when computing the value function. In the general case, the control that minimizes the Hamiltonian is expressed as a function of he state and costate, the latter being unavailable in the DPP approach (the costate actually corresponds to the gradient of the Value Function under suitable regularity assumptions).

Remark 1 (Slack variable) In the five cases mentioned above, we adjust the slack variable if needed to get an admissible Diesel output P_D.

We now propose a pseudo-algorithm for the numerical resolution:

Data: h, I_{SOC}
Result: $V^k, kh = 0 \dots T$
for $kh \in T \dots 0$ **do**
  for $m \in \{ON,OFF\}$ **do**
    for $SOC \in I_{SOC}$ **do**
      $V_m^k(SOC) = \min\{$
      $\min_{P_D \in G(SOC,m,kh)} h\ell(P_D) + V^{k+1}(x + hF(P_D, kh)),$
      $\min_{P_D \in G(SOC,\bar{m},kh)} h\ell(P_D) + V^{k+1}(x + hF(P_D, kh)) + C_{switch}(m)\}$
    end
  end
end

The parameter h corresponds to the time discretization size, and I_{SOC} is the state discretization grid. The result is the value function V^k for each time step k. The functions F and ℓ correspond to the dynamics and running cost as expressed in the abstract optimal control problem formulation (10). The mode $m \in$ {ON,OFF} corresponds to the fact that the diesel can be already working or turned off. We have denoted by $\bar{m}$

the negation of m. In case of switch, a cost $C_{switch}(m)$ has to be added to the cost to go function. This cost is the startup cost if the generator is turned on (see Table 1), and 0 otherwise. The set $G(SOC, m, t)$ corresponds to the potential optimal controls deduced from the *PMP trick*.

4 Numerical simulations

4.1 Comments on the inputs: solar power and power load

We test the algorithms with a 15′ time step that gives a good compromise between accuracy and numerical complexity, and allows comparisons with the MILP approach in [22]. We use two historical data sets. Both correspond to representative 48-h periods, One data set was obtained with winter data, the other one with summer data. Figures 3 and 4 show the load power and the solar power for the 2 days of each period.

Since the actual microgrid is situated in the Atacama desert, we assume the production from the photovoltaic panels to be reliably predictable. The demand, on the other hand, has a greater variability. While it is modeled as deterministic in this initial work, the extension of this model to a stochastic demand setting is the focus of [26].

4.2 Optimal solutions for the different methods

In addition to the direct and DPP methods, we present the results obtained with the MILP approach from [22] as baseline for comparison. The six solutions are illustrated in Figs. 5 and 6 for the DPP approach, in Figs. 7 and 8 for the direct approach and in Figs. 9 and 10 for MILP method. The numerical results are summarized in Tables 2 and 3.

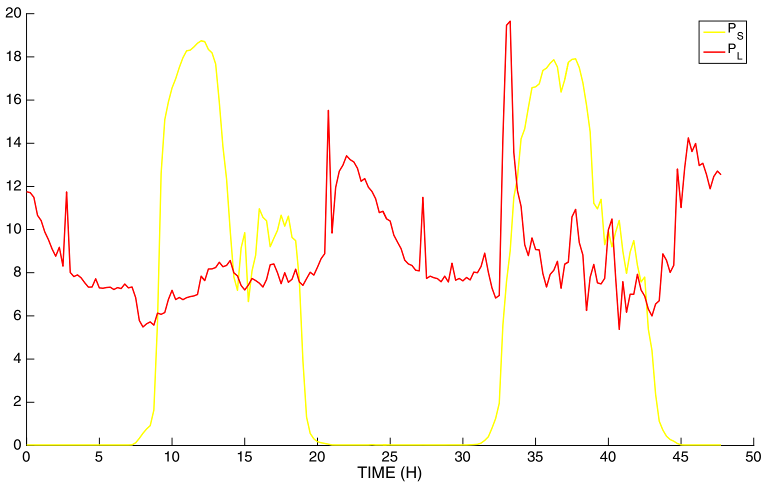

Fig. 3 Summer data in kW

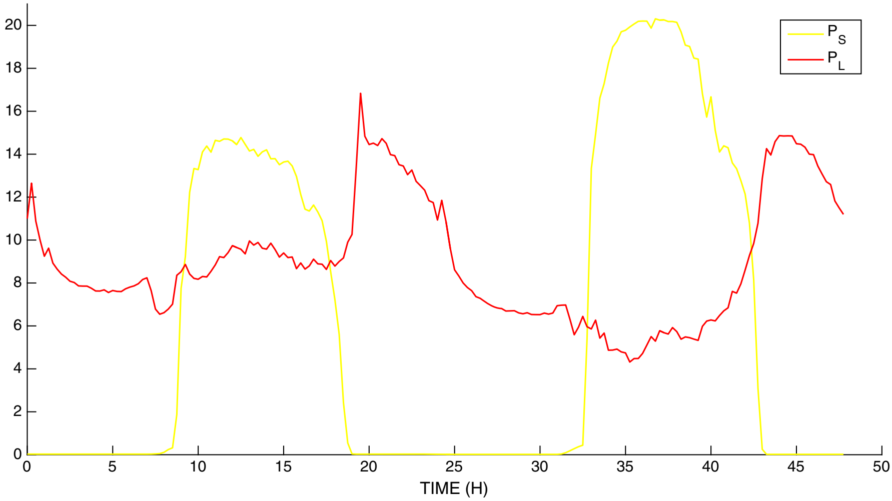

Fig. 4 Winter data in kW

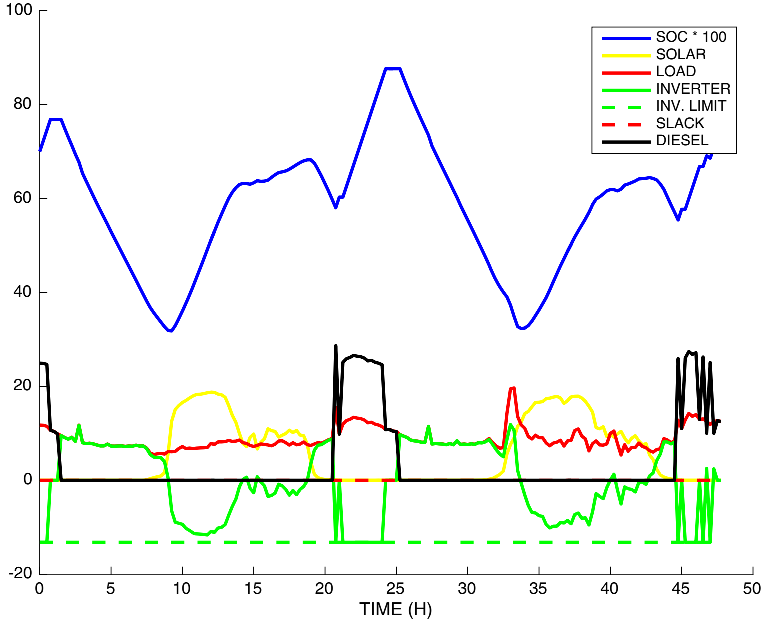

Fig. 5 Summer DPP simulation

4.2.1 General observations

– The Solar Power fills the demand, with any excess power used to charge the battery.
– The Diesel is always off when solar power is available, and is switched on once a day during the evening peak in demand. The Diesel output is often greater than power demand: it is also used to charge the battery.

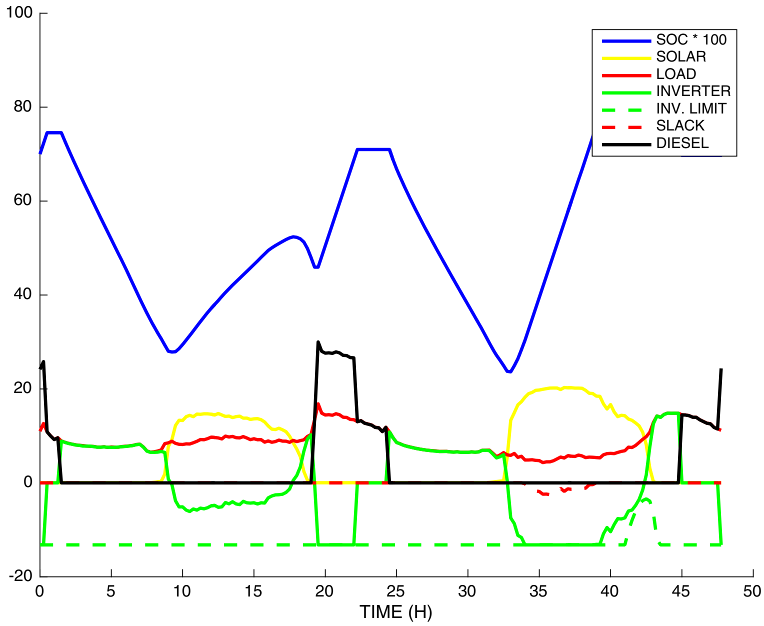

Fig. 6 Winter DPP simulation

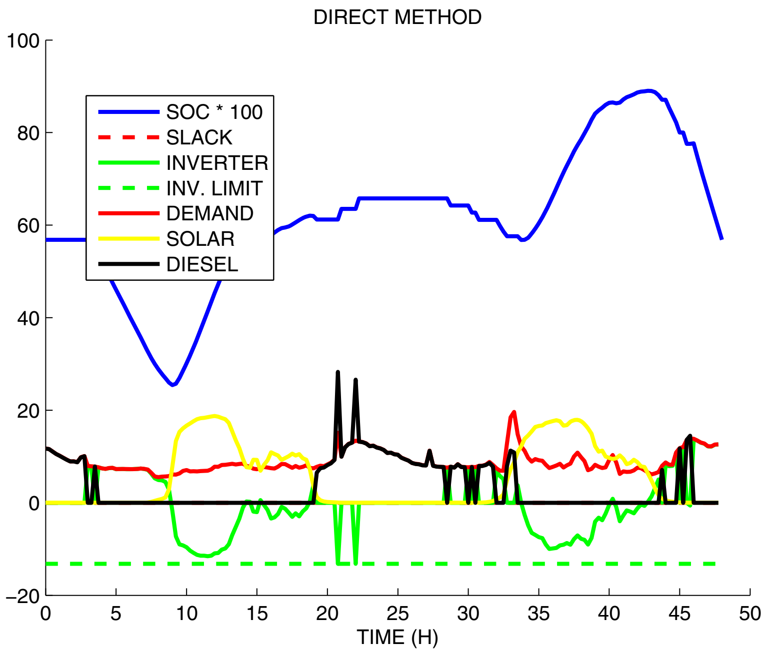

Fig. 7 Summer direct simulation

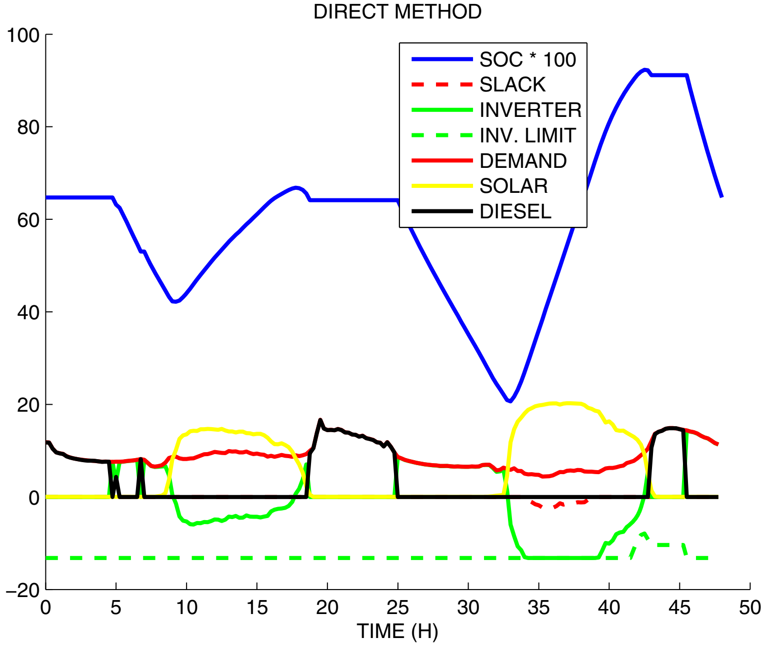

Fig. 8 Winter direct simulation

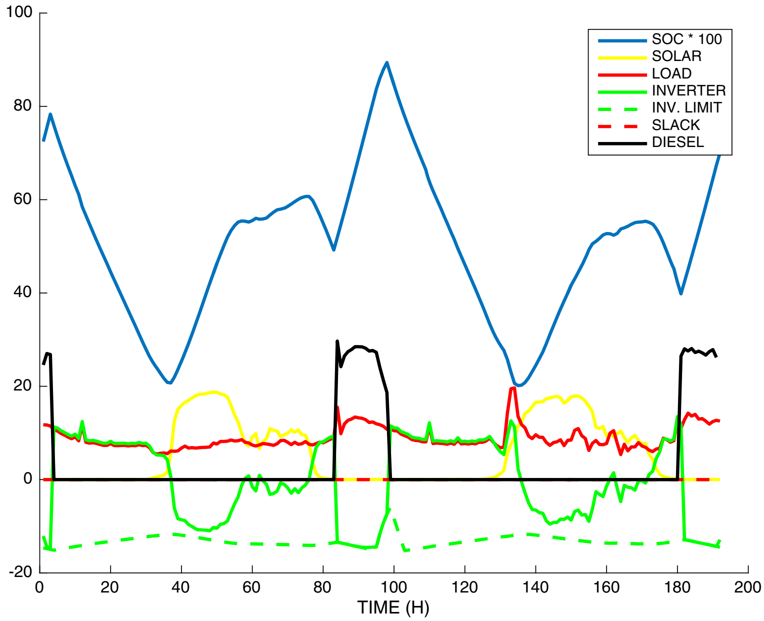

Fig. 9 Summer MILP simulation

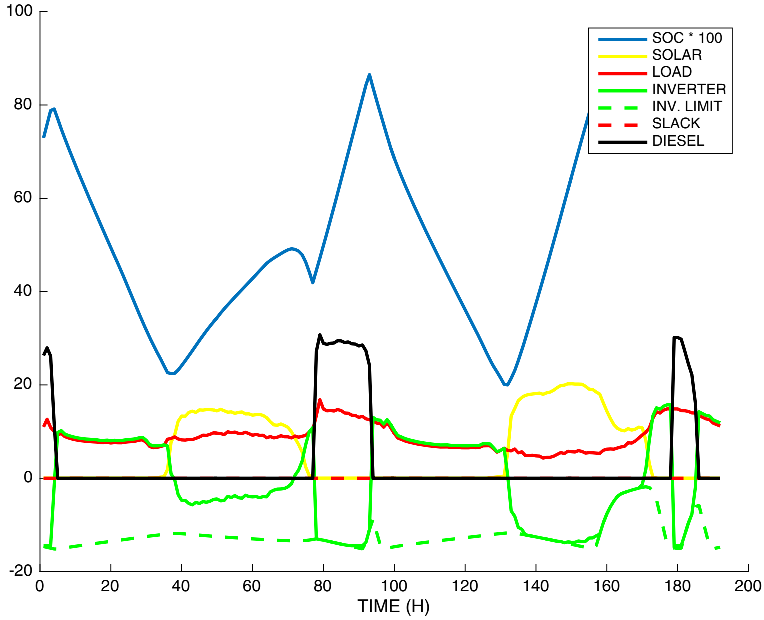

Fig. 10 Winter MILP simulation

– The battery fills the gaps between production and demand especially at night.
– MILP and DPP solutions are quite close, while the direct solution shows some clear differences: different initial/final SOC, no minimal power, and spurious switchings.

4.2.2 Diesel range

A qualitative difference between the MILP and DPP/direct approaches is, for the latter, the existence of time intervals with a constant SOC, while the diesel exactly matches the power load. In the MILP solutions the diesel is either off or saturating the charge limit.

We notice that there is a tradeoff between low average production cost and low storage cost per energy unit. On the one hand, since the diesel cost function is concave, the diesel generator should run at maximal capacity to minimize the average unit price of power produced. On the other hand one incurs on some losses when storing energy because the battery is not perfectly efficient. One has to set the diesel output equal to the net demand (so that nothing gets in or leaves the battery) to minimize those losses. This tradeoff explains why on the DPP and Direct solutions we observe two kinds of non zero diesel levels (low: just sufficient to satisfy the demand, and thus keep a constant SOC, or high: maximum physical production level). The question is then why we do not observe the low level on the MILP simulation. We do not have a straightforward answer, as different phenomena could contribute to this observation. First the MILP problem is a linearized version of the DPP problem, so it may happen that this linearized

Table 2 Results: MILP, direct and DP (summer case)

	MILP	DIRECT	DP
Diesel range	[18.7, 29.7]	[6.5, 28.3]	[9.8, 28.7]
Switchings	2	3 + spurious	2
Diesel cost (w/o switchings)	32,785	32,378	32,428
Cpu time (s)	3.92	3.41	0.52
SOC(0) = SOC(T)	0.72	0.57	0.70
SOC range	[0.20, 0.89]	[0.25, 0.89]	[0.32, 0.88]
Slack range	[0, 0]	[0, 0]	[0, 0]

Table 3 Results: MILP, direct and DP (winter case)

	MILP	DIRECT	DP
Diesel range	[11.7, 30.7]	[4.4, 16.6]	[9.2, 30.0]
Switchings	2	3 + spurious	2
Total cost	30,823	29,561	30,197
Cpu time (s)	3.28	3.92	0.55
SOC(0) = SOC(T)	0.73	0.65	0.70
SOC range	[0.20, 0.96]	[0.21, 0.92]	[0.24, 0.95]
Slack range	[0, 0]	[−2.48, 0]	[−2.51, 0]

version does not produce the same output. Second the MILP numerical solution is only locally optimal and so it may differ from the MILP actual solution.

4.3 Comparison of the methods

We highlight below the differences between the three optimization methods.

Global optimum Both the MILP and direct approaches are local methods and may converge to a local solution, depending on the provided starting point and the choice of the stopping criterion (gap). On the other hand, the DPP approach performs a global optimization over all possible (discretized) trajectories, and therefore always finds the global optimum. This is an advantage for the user since one does not have to find a "suitable" starting point. Also, the DPP solution provides a feedback optimal control, whereas MILP and direct solutions are open-loop.

Switching cost Both MILP and DPP approaches take into account the switching costs for the diesel generator. They typically find solutions with one switch per day, located during the peak of power demand in the evening. On the other hand, the direct approach has no cost for switchings, which explains why it may find solutions with many on-off oscillations.

Nonlinear model The MILP method requires a piecewise linear reformulation of the nonlinear functions in the model, here for example the charging power limit or the cost of diesel consumption. Both direct and DPP methods use the original nonlinear model. This simplifies the actual implementation, and may provide more accurate solutions.

Periodicity constraint and minimal diesel power Compared to MILP and DPP, the direct method optimizes the value of the initial/final SOC. On the other hand, it does not take into account the minimal power output for the diesel generator.

Computing time For this problem the computing time is a few seconds for MILP and direct method, and less than one second for the DPP approach. Note that DPP is outperforming the two other approaches thanks to the PMP trick and the fact that the state is one dimensional. An interesting question is how well each method would scale for higher dimensions. MILP and direct approach are iterative methods, so changing the problem size may lead to a different convergence, making it difficult to assess the evolution of the CPU time. For the DPP approach, on the other hand, the number of operations is always known and the CPU time can be predicted reliably. The CPU time should increase linearly in the number of time steps. Due to the state discretization, however, adding new state variables to the problem would have a significant impact on performance (the so-called *curse of dimensionality*). In terms of high performance computing, parallelization is possible with MILP and DPP methods, not so easily with the direct method.

5 Conclusion and perspectives

We applied two methods from the continuous optimal control field to the optimal energy management of a microgrid, namely the direct and DPP approaches. Numerical simulations indicate that the DPP method is very well suited to this problem as it is a linearization-free method that provides global optimal solutions in closed loop form. It allows for the modelling of the switches and it is as fast as the MILP approach. We were able to obtain the global optimum in less than one second of CPU time, while taking into account the switching cost for the diesel generator. Solutions are close to the ones obtained in [22] with a MILP formulation, the main difference being the existence of time intervals where the battery stays at a constant SOC level. In comparison with the two other approaches, the use of Pontryagins Maximum Principle combined with Dynamic Programming reduces the computing time. The numerical experiments were performed with the optimal control toolbox Bocop.

From a theoretical standpoint, the continuous model offers a very large collection of mathematical results. The PMP trick introduced here is an example of insight one can get from a continuous time mathematical analysis. (Observe that a Maximum Principle exists for the discrete case, but only for a convex Hamiltonian).

Ongoing works on this topic include the extension to a stochastic model for the power demand, see [26], and the study of the long-term aging of the battery, see the related work in [27].

Acknowledgements This work is product of collaboration between the COMMANDS (INRIA, France) and Centro de Energía teams (Universidad de Chile, Chile), it was also supported in part by CONICYT/FONDAP/15110019. FJS was supported by project iCODE: "Large-scale systems and Smart grids: distributed decision making" and from the Gaspar Monge Program for Optimization and Operation Research (PGMO). JFB was supported by the laboratory Dauphine CREST EDF R&D Finance des Marchés d'Energies.

References

1. U.S. Department of Energy.: The smart grid: an introduction. https://energy.gov/oe/downloads/smart-grid-introduction-0 (2010). Accessed 1 Sept 2016
2. Olivares, D.E., Mehrizi-Sani, A., Etemadi, A.H., Cañizares, C.A., Iravani, R., Kazerani, M., Hajimiragha, A.H., Gomis-Bellmunt, O., Saeedifard, M., Palma-Behnke, R., Jiménez-Estévez, G.A., Hatziargyriou, N.D.: Trends in microgrid control. IEEE Trans. Smart Grid **5**(6), 1905–1919 (2014). doi:10.1109/TSG.2013.2295514
3. Conti, S., Nicolosi, R., Rizzo, S.A.: Optimal dispatching of distributed generators in an MV autonomous micro-grid to minimize operating costs and emissions. In: IEEE International Symposium on Industrial Electronics, pp. 2542–2547. IEEE, Bari (2010)
4. Hatziargyriou, N., Contaxis, G., Matos, M., Lopes, J., Kariniotakis, G., Mayer, D., Halliday, J., Dutton, G., Dokopoulos, P., Bakirtzis, A., Stefanakis, J., Gigantidou, A., O'Donnell, P., McCoy, D., Fernandes, M., Cotrim, J., Figueira, A.: Energy management and control of island power systems with increased penetration from renewable sources. In: 2002 IEEE Power Engineering Society Winter Meeting. Conference Proceedings (Cat. No.02CH37309), vol. 1 (2002)
5. Colson, C.M., Nehrir, M.H., Wang, C.: Ant colony optimization for microgrid multi-objective power management. In: 2009 IEEE/PES Power Systems Conference and Exposition. PSCE, pp. 1–7. IEEE, Seattle, WA (2009)
6. Zhao, B., Zhang, X., Chen, J., Wang, C., Guo, L.: Operation optimization of standalone microgrids considering lifetime characteristics of battery energy storage system. IEEE Trans. Sustain. Energy **4**(4), 934–943 (2013)
7. Garcia, F., Bordons, C.: Optimal economic dispatch for renewable energy microgrids with hybrid storage using Model Predictive Control. In: IECON 2013—39th Annual Conference of the IEEE Industrial Electronics Society, pp. 7932–937. IEEE, Vienna (2013)
8. Pourmousavi, S.A., Sharma, R.K., Asghari, B.: A framework for real-time power management of a grid-tied microgrid to extend battery lifetime and reduce cost of energy. In: 2012 IEEE PES Innovative Smart Grid Technologies. ISGT 2012 (2012)
9. Hooshmand, A., Asghari, B., Sharma, R.: A novel cost-aware multi-objective energy management method for microgrids. In: Innovative Smart Grid Technologies (ISGT), 2013 IEEE PES, pp. 1–6. IEEE, Washington, DC (2013)
10. Khasawneh, H.J., Illindala, M.S.: Battery cycle life balancing in a microgrid through flexible distribution of energy and storage resources. J. Power Sources **261**, 378–388 (2014)
11. Kanchev, H., Francois, B., Lazarov, V.: Unit commitment by dynamic programming for microgrid operational planning optimization and emission reduction. In: 2011 International Aegean Conference on Electrical Machines and Power Electronics and 2011 Electromotion Joint Conference (ACEMP). IEEE (2011)
12. Liu, H., Wu, Y., Qian, C., Liu, X.: The application of dynamic programming in the stand-alone microgrid optimal operation. In: 2012 Asia-Pacific Power and Energy Engineering Conference, pp. 1–5. IEEE, Shanghai (2012)
13. Zhang, Y., Jia, Q.S.: Optimal operation for energy-efficient building micro-grid. In: 2015 34th Chinese Control Conference (CCC), pp. 8936–8940. IEEE, Hangzhou (2015)
14. Babazadeh, H., Gao, W., Wu, Z., Li, Y.: Optimal energy management of wind power generation system in islanded microgrid system. In: North American Power Symposium (NAPS), 2013. IEEE (2013)
15. Nguyen, M.Y., Yoon, Y.T., Choi, N.H.: Dynamic programming formulation of micro-grid operation with heat and electricity constraints. In: Transmission and Distribution Conference and Exposition: Asia and Pacific, 2009. IEEE (2009)
16. Huang, C.C., Chen, M.J., Liao, Y.T., Lu, C.N.: DC microgrid operation planning. In: 2012 International Conference on Renewable Energy Research and Applications (ICRERA). IEEE (2012)

17. Betts, J.T.: Practical Methods for Optimal Control and Estimation using Nonlinear Programming. Society for Industrial and Applied Mathematics (SIAM), Philadelphia (2010)
18. Bellman, R.: Dynamic Programming. Princeton University Press, Princeton (1957)
19. Falcone, M., Ferretti, R.: Semi-Lagrangian Approximation Schemes for Linear and Hamilton-Jacobi Equations. Society for Industrial and Applied Mathematics (SIAM), Philadelphia (2014)
20. Bonnans, F., Martinon, P., Giorgi, D., Grélard, V., Heymann, B., Jinyan, L., Maindrault, S., Tissot, O.: Bocop—a collection of examples. Technical report (2016). http://bocop.saclay.inria.fr/
21. Bonnans, F., Giorgi, D., Heymann, B., Martinon, P., Tissot, O.: Bocophjb 1.0. 1-User guide. Technical report (2015). https://hal.inria.fr/hal-01192610
22. Palma-Behnke, R., Benavides, C., Lanas, F., Severino, B., Reyes, L., Llanos, J., Saez, D.: A microgrid energy management system based on the rolling horizon strategy. IEEE Trans. Smart Grid **4**, 996–1006 (2013)
23. Bryson Jr., A.E., Ho, Y.C.: Applied Optimal Control. Hemisphere Publishing Corporation, Washington (1975)
24. Waechter, A., Biegler, L.T.: On the implementation of an interior-point filter line-search algorithm for large-scale nonlinear programming. Math. Program. **106**(1), 25–57 (2006). doi:10.1007/s10107-004-0559-y
25. Pontryagin, L., Boltyanski, V., Gamkrelidze, R., Michtchenko, E.: The Mathematical Theory of Optimal Processes. Wiley Interscience, New York (1962)
26. Heymann, B., Bonnans, J.F., Jiménez, G., Silva, F.: Stochastic continuous time model for microgrid energy management. In: European Control Conference (ECC), Aalborg, Denmark, June 29–July 1 (2016)
27. Heymann, B., Martinon, p., Bonnans, F.: Long term aging : an adaptative weights dynamic programming algorithm. https://hal.archives-ouvertes.fr/hal-01349932 (2016). Accessed 1 Sept 2016