Resource reachability games on pushdown graphs

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Games with Resources

- Idea: Two-player games on graphs with non-negative integer counters
- Counters support:
  (i) `n` - no resource used / leave counter unchanged
  (ii) `i` - resource usage / increment
  (iii) `r` - refill resource / set the resource counter to zero

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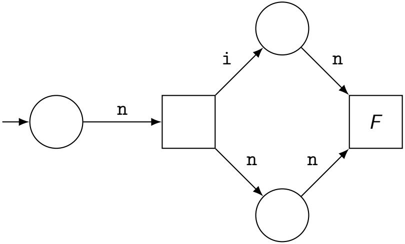

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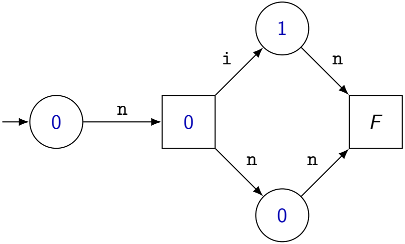

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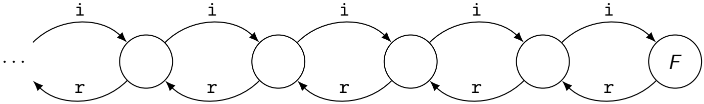
How much memory is needed to win?



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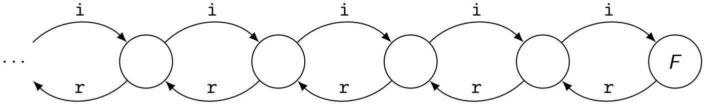
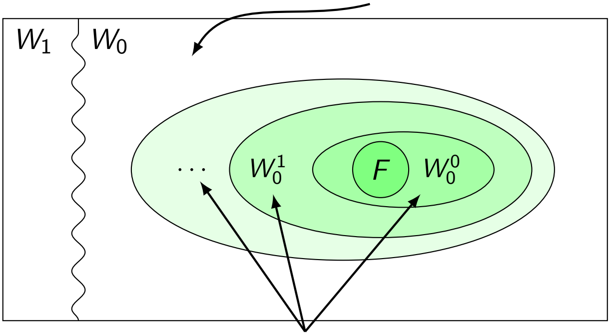


- Memoryless does not suffice for a fixed bound.
- Yet: For a fixed k one can reduce the games to normal reachability games.

Bounded Winning Problem

Winning region of Eve when ignoring resources



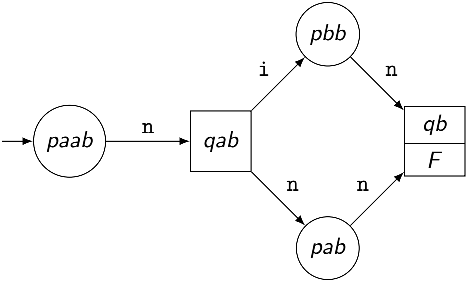
$W_0^i = \{v \in V \mid \text{Eve wins with resource-cost at most } i\}$

Problem

Is there a $k \in \mathbb{N}$ such that $W_0^k = W_0$?

Play on pushdown graphs

- We consider pushdown systems with such counters and play on their configuration graphs.



- Induced by a pushdown system $(\{p\} \uplus \{q\}, \Delta, \{c_0\})$ where Δ is

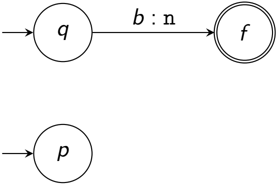
$$pa \xrightarrow{\mathtt{n}} q\varepsilon \qquad qa \xrightarrow{\mathtt{i}} pb$$

$$pb \xrightarrow{\mathtt{n}} q\varepsilon \qquad qa \xrightarrow{\mathtt{n}} pa$$

Apply saturation

Idea: Construct alternating B-automaton that recognizes configuration with the cost of winning.

Start with automaton for F



Apply saturation

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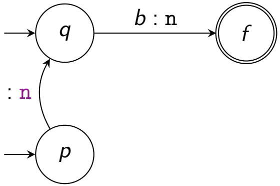
Saturate with $pa \xrightarrow{\mathtt{n}} q\varepsilon$ (p is player 0 controlled)



Apply saturation

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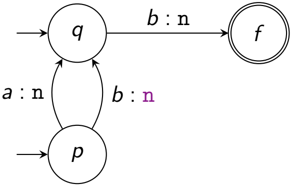
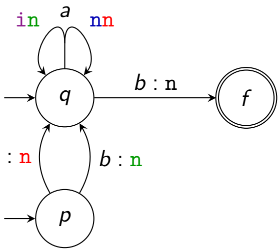
Saturate with $pb \xrightarrow{\mathtt{n}} q\varepsilon$ (p is player 0 controlled)



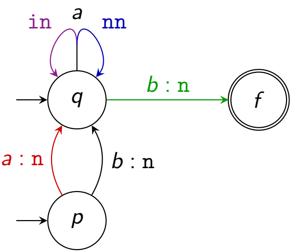
Apply saturation

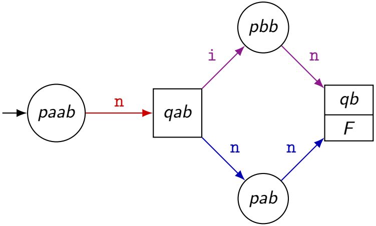
Idea: Construct alternating B-automaton that recognizes configuration with the cost of winning.

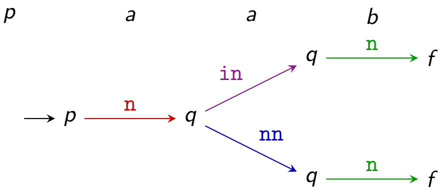
Saturate with $qa \xrightarrow{\mathtt{i}} pb$ and $qa \xrightarrow{\mathtt{n}} pa$ (q is player 1 controlled)



Saturation Result







Solve Bounded Winning Problem

Theorem

For a resource pushdown system $\mathcal{P}$ and a regular goal set F, one can effectively compute an alternating B-automaton $\mathfrak{A}$ such that $[\![\mathfrak{A}]\!]((p, w)) \leq k \Leftrightarrow$ Eve wins the resource reachability game from (p, w) with resource-limit k.

Theorem (Colcombet, Löding, 2010)

The boundedness problem for alternating B-automata is decidable.