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Undecidability of performance equivalence of Petri nets

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ABSTRACT

We investigate bisimulation equivalence on Petri nets under durational semantics. Our motivation was to verify the conjecture that in durational setting, the bisimulation equivalence checking problem becomes more tractable than in ordinary setting (which is the case, e.g., over communication-free nets). We disprove this conjecture in three of four proposed variants of durational semantics. The fourth variant remains an intriguing open problem.

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1. Introduction

Bisimulation equivalence [18,17] is one of the most relevant semantical equivalences of concurrent systems. One of its advantages is that it often allows for an efficient verification algorithms in settings where other approaches (see, e.g., [8]), like language equality, lead to undecidable verification problems. There is now a wide range of results about decidability and complexity of different variants of bisimulation equivalences in different classes of infinite-state systems (see e.g. [3]).

Bisimulation equivalence relates processes exhibiting the same behaviour. In this paper we investigate *performance equivalence*, a variant of bisimulation equivalence that aims at relating not only purely functional behaviour, but also effectiveness of processes. A basic assumption is that each action of a process has assigned a positive *duration*, that is amount of time (or other resource) necessary to complete this action. Performance equivalence is then a variant of bisimulation that respects amount of time (resource) requested in both processes during execution. This notion was introduced in [9] and then studied among the others in [5,6,1,12,14].

A starting point for our investigations was an observation made in [1] that the complexity of performance equivalence may be substantially lower than complexity of ordinary bisimulation equivalence. The authors of [1] investigated so called Basic Parallel Processes (BPP in short), a natural and simple fragment of process algebra CCS [17] (it is expressively equivalent to CCS without communication). BPP, when transformed to a normal form [4], is equivalent to communication-free Petri nets. While bisimulation equivalence on BPP in normal form is PSPACE-complete [11,21], in [1] it was shown that performance equivalence may be decided in polynomial time. An intuitive justification of this is that the latter equivalence, being more discriminating, satisfies stronger decomposition properties and hence is more tractable. Later on, it was shown that the polynomial time procedure exists for the whole BPP (not necessarily in the normal form studied in [1]) and that it coincides with distributed equivalence [14].

Performance equivalence is computationally more tractable than bisimulation equivalence on BPP, i.e., on communication-free Petri nets, hence a natural question arises: is it also more tractable in the case of general Petri nets? As bisimulation

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equivalence is undecidable in this case [10], the crucial question is whether performance equivalence is decidable or not. This is the main problem investigated in this paper.

However, when one tries to define the durational semantics over Petri nets, necessary to host the notion of performance equivalence, it quickly turns out that there is no such unique semantics. We made a systematic research of possible ways to define it and come up with four different variants of durational semantics. The distinction depends on the choice between global-time or local-time approach, and on the way of synchronisation (patient and impatient approach).

As our main result, we proved undecidability of performance equivalence under three of the four semantics. While the patient variants are easily undecidable, the proof for global-time impatient semantics is nontrivial and constitutes the main technical contribution of the paper. The proof builds on the method of Jančar [10]; however, substantial new insight was necessary to come over new difficulties and subtleties appearing in the durational setting.

Under the impatient local-time semantics, the question is still open. If decidable, performance equivalence would be one of very few notions of equivalence of general Petri nets exhibiting a decision procedure. This is actually the main motivation of this paper:

Motivation: basing on the positive impact of durational semantics on complexity of equivalence-checking for BPP-nets, attempt to prove decidability for general nets, thus discovering a decidable bisimulation-like equivalence of general nets.

There is a wide range of research on timed extensions of Petri nets, like time nets or timed nets [16,19]. In most of these extensions, some timing restrictions are posed on transition, places, or arcs. However, we would like to stress that our durational setting is different from the timed ones. The principal difference is that we do not aim at modelling timed behaviour; instead, our aim is to *measure* effectiveness of processes. In particular, we allow for a local-time semantics, where the time-stamps observed during an execution of a net need not be a monotonic sequence.

Another distinguishing aspect is that the timed settings usually properly extend ordinary untimed nets, therefore decision problems become never easier, and typically harder. As a relevant example, the reachability problem, decidable for ordinary nets, becomes undecidable both for time nets and timed nets [7].

Our durational setting *does not* subsume ordinary nets. This gives hope for decidability of performance equivalence, and this stands behind the fact that the reachability problem is decidable in all four variants (in contrast to time nets and timed nets). This topic is discussed in detail in the last section. The durational setting does properly extend ordinary nets only when we allow for zero durations; see [12] for decidability results about performance equivalence over BPP.

In Section 2 we introduce the background material and formalise the durational setting. We also mention quickly undecidability in both patient variants. Then Section 3 is devoted to the proof of undecidability under global-time impatient variant. The last section contains a brief discussion on decidability of reachability problem for durational Petri nets and some final remarks.

2. Preliminaries

We start with the necessary background and notation (Sections 2.1 and 2.2), and then in Section 2.3 we define durational Petri nets, the model investigated in this paper. Finally, in Sections 2.4 and 2.5 we discuss some easy cases, as a warmup before the main technical result to be presented in the next section.

2.1. Bisimulation equivalence

A labelled transition system consists of a set of states S and a family of binary relations $\{\xrightarrow{a}\}_{a \in \mathcal{L}}$ indexed by a labelling set \mathcal{L} . We will write $s_1 \xrightarrow{a} s_2$ instead of $(s_1, s_2) \in \xrightarrow{a}$.

A relation $\mathcal{R} \subseteq S \times S$ is a *bisimulation* if for each $(s_1, s_2) \in \mathcal{R}$ the following two conditions hold:

- if $s_1 \xrightarrow{a} t_1$ for some a, t_1 then there is some t_2 such that $s_2 \xrightarrow{a} t_2$ and $(t_1, t_2) \in \mathcal{R}$;
- if $s_2 \xrightarrow{a} t_2$ for some a, t_2 then there is some t_1 such that $s_1 \xrightarrow{a} t_1$ and $(t_1, t_2) \in \mathcal{R}$.

States s_1, s_2 are *bisimulation equivalent* (*bisimilar*), if there is a bisimulation \mathcal{R} containing (s_1, s_2) .

It is instructive to recall an alternative definition of bisimilarity, in a setting of games. A Bisimulation Game is played between two players Spoiler and Duplicator. For convenience of presentation, we view Spoiler as “him” and Duplicator as “her”. The *positions* in the game are pairs $(s_1, s_2) \in S \times S$. In a position (s_1, s_2) , Spoiler chooses $i \in \{1, 2\}$ and a transition from s_i , say $s_i \xrightarrow{a} t_i$; Duplicator must respond by choosing some transition with the same label a from the other element of (s_1, s_2) , i.e., a transition $s_{3-i} \xrightarrow{a} t_{3-i}$. The play then continues from the position (t_1, t_2) . If one of the players gets stuck (there is no appropriate transition), the other player wins. If the play continues forever, Duplicator wins unconditionally.

Generally speaking, a *strategy* for a player P in a game is a (partial) function which determines a concrete P -move for each sequence of so far played moves after which it is P 's turn. A strategy is *winning* if player P wins each play when he/she uses the strategy. In what follows, by a strategy we always mean a *memoryless strategy*: each prescribed move depends on

the current position only, not on the whole sequence of so-far played moves. By standard results, for each position (s_1, s_2) , precisely one of the players has a memoryless winning strategy; moreover we have:

Proposition 1. (See [22].) *Two states s_1 and s_2 are bisimilar iff Duplicator has a winning strategy in Bisimulation Game starting from position (s_1, s_2) .*

Hence Spoiler has a winning strategy iff s_1 and s_2 are not bisimilar.

2.2. Labelled Petri nets

A finite multiset over a set \mathcal{A} is formally a mapping M from \mathcal{A} to \mathbb{N} , the set of natural numbers, such that $M(a) > 0$ for only a finite number of elements a . E.g., the empty multiset \emptyset maps all $a \in \mathcal{A}$ to 0. We apply the usual arithmetical operations to multisets in a point-wise manner. E.g., point-wise addition is the union operation of multisets; and $M \leq M'$ means that $M(a) \leq M'(a)$ for each $a \in \mathcal{A}$. We will often write finite multisets by enumerating its elements, e.g., aab will denote the function mapping a to 2, b to 1, and all other elements of \mathcal{A} to 0.

A labelled Petri net (net in short) N is given by a finite set of places \mathcal{P} , a finite set \mathcal{L} of labels, and a finite set of transition rules of the form $X \xrightarrow{a} Y$, where X and Y are finite nonempty multisets over \mathcal{P} and $a \in \mathcal{L}$. In the sequel it will be sufficient to consider only those nets, where X and Y are always sets (i.e., multisets mapping each element to 0 or 1). This restriction corresponds to the class of pure Place/Transition labelled Petri nets.

A net naturally induces a labelled transition system. Its states are all finite multisets over \mathcal{P} (traditionally called *markings*). There is a transition from marking M to M' , labelled by a , if there is a transition rule $X \xrightarrow{a} Y$ such that $M \geq X$ (point-wise) and $M' = M - X + Y$. In traditional terminology, one says that the transition rule $X \xrightarrow{a} Y$ is *fired* at marking M ; the transition rule is *fireable* in M if $M \geq X$. Note that we do not fix an *initial marking* of a net.

There is a natural operational interpretation of the induced transition system. A marking may be understood as an assignment of a number of *tokens* to each place. And each transition rule specifies the necessary condition on the number of tokens on some places. For instance, a rule $ppq \xrightarrow{a} ps$ would mean that an a -transition is allowed assumed at least two tokens on place p and at least one on place q ; as an outcome of the transition, the three tokens are removed, then one token is placed back on p and one on s .

The bisimulation equivalence problem for labelled Petri nets is defined as follows: given a net N and two markings M_1, M_2 , decide whether they are bisimilar as the states in the labelled transition system induced by N . Undecidability of this problem was proved by Jančar in [10]. Note that equivalently we could formulate the problem for the (initial) markings in two distinct nets.

Bisimulation equivalence relates two states (or two systems, in general) with the same functional behaviour. We will now extend this setting to allow to compare, in addition to pure functional behaviour, also its effectiveness (performance). The idea comes from [9] and amounts to (1) assigning a *duration* to each transition; (2) respecting the durations in the bisimulation equivalence.

2.3. Durational labelled Petri nets

Following [9], we choose a discrete time domain, represented by natural numbers. From now on we will assume that each transition rule r of any labelled Petri net N has assigned a positive natural number, its *duration*, written $\text{dur}(r)$. Such nets, enriched by a duration function, are called *durational nets* in the sequel. Note that we do not allow for $\text{dur}(r) = 0$, as in [9] and in the following papers [5,6,1]. If we allowed for zero durations our model would trivially subsume ordinary (non-durational) Petri nets.

A *durational marking* is a finite multiset over $\mathcal{P} \times \mathbb{N}$; intuitively, each token in a marking has now a *time-stamp*. In an initial marking, the time-stamps of all the tokens will be usually 0. We will write $t \triangleright p$ instead of (p, t) . A durational marking M may be naturally mapped to an ordinary marking $\text{untime}(M)$ by removing all the time-stamps (untiming) but preserving the number of tokens at each place. E.g., the durational marking $M = (0 \triangleright p)(3 \triangleright p)(3 \triangleright p)(2 \triangleright q)$ would be mapped to $\text{untime}(M) = ppq$.

For notational convenience we will extend the $t \triangleright _$ notation to non-durational markings: by $t \triangleright M$ we will mean the multiset $\{t \triangleright p : p \in M\}$. E.g., $(0 \triangleright p)(3 \triangleright p)(3 \triangleright p)(2 \triangleright q)$ may be equivalently written as $(0 \triangleright p)(3 \triangleright pp)(2 \triangleright q)$.

We will define the durational semantics for Petri nets by specifying (i) when a transition rule is fireable in a durational marking M , and (ii) what is the effect of its firing. The labels in the induced transition system will be now pairs $(a, t) \in \mathcal{L} \times \mathbb{N}$. The intuitive meaning of a transition $M \xrightarrow{a,t} M'$ is that t units of time had to elapse *before* this transition became fireable. The amount of time t is always measured relative to the starting moment of an execution; in particular, the very first transition will be usually fired at $t = 0$.

Before making our durational semantics explicit, we need to introduce some notation. For a durational marking M , by $\text{stamps}(M)$ denote the set (not multiset) of all time-stamps of tokens in M , and by $\text{max-stamp}(M)$ the greatest time-stamp in M . Formally:

$$\text{stamps}(M) = \{t : t \triangleright p \in M\} \quad \text{max-stamp}(M) = \begin{cases} \max(\text{stamps}(M)) & \text{if } M \neq \emptyset \\ 0 & \text{otherwise} \end{cases}$$

We distinguish four different durational semantics, depending on whether it is patient or impatient, and whether it is global-time or local-time. In each of the four variants, we will make it precise when a *transition rule* $X \xrightarrow{a} Y$ is *fireable* in a *durational marking* M , at time t due to a *submarking* \bar{X} . In each of the four variants, it requires that $\bar{X} \leq M$ (\bar{X} is a *durational submarking* of M) and $X = \text{untime}(\bar{X})$ (the rule actually applies to \bar{X}). In addition, the following is required, depending on the variant of semantics:

- local-time, patient semantics: $\text{max-stamp}(\bar{X}) = t$;
- local-time, impatient semantics: $\text{stamps}(\bar{X}) = \{t\}$.

Intuitively, in the patient semantics, a token with an “earlier” (smaller) time-stamp may “wait” for other tokens with “later” time-stamps; but this is not allowed in the impatient variant, where the tokens must agree on their time-stamps to be able to fire synchronously a transition rule.

Note that we did not assume that a transition with the smallest possible time-stamp is chosen to be fired. In the global-time patient (impatient) semantics we additionally require this: a transition rule $X \xrightarrow{a} Y$ is fireable in M , at time t due to \bar{X} , if it is fireable according to the local-time patient (impatient) semantics, and t is the smallest among all possible choices of a rule $X \xrightarrow{a} Y$ and a submarking \bar{X} .

Uniformly for all four variants, whenever a transition rule $r = (X \xrightarrow{a} Y)$ is fireable in M , at time t due to \bar{X} , then in the induced labelled transition system there is a transition $M \xrightarrow{a,t} M'$, where $M' = M - \bar{X} + \{(t + \text{dur}(r)) \triangleright p : p \in Y\}$. I.e., fresh tokens are produced as specified by Y , and their time-stamps are equal to $t + \text{dur}(r)$. We call t a *time label* of transition $M \xrightarrow{a,t} M'$.

When convenient, we will identify a Petri net with its induced transition system, e.g., we will speak of ‘transitions’ of a net. Note that the induced transition system is always acyclic, due to the assumption that durations are strictly positive. By an execution of a net we mean a finite or infinite sequence of transitions:

$$M_0 \xrightarrow{a_0, t_0} M_1 \xrightarrow{a_1, t_1} M_2 \xrightarrow{a_2, t_2} \dots$$

Note that the initial marking M_0 is not fixed for the net. For two durational markings M, M' , we say that M' is *reachable* from M if there is a finite execution that starts in M and ends in M' .

We say that two durational markings M, M' of a given durational net N are *performance equivalent* if they are bisimulation equivalent in the induced labelled transition system. The main topic of this paper is undecidability of the following problem: given N, M and M' , decide whether M and M' are performance equivalent.

The aim of performance equivalence is not only to relate functionalities of two systems, but also their *performance*, i.e. amount of resources necessary for these functionalities. We illustrate this aspect with a simple example.

Example 1. Consider a durational Petri net with the following transition rules (essentially a finite system):

$$\begin{array}{lll} p \xrightarrow{a} p_a & p_a \xrightarrow{b} p_{ab} & q_a \xrightarrow{a} q' \\ p \xrightarrow{b} p_b & p_b \xrightarrow{a} p_{ab} & q_b \xrightarrow{b} q' \end{array}$$

and two its markings, $0 \triangleright p$ and $0 \triangleright q_a q_b$. According to the standard bisimulation equivalence the (untimings of) two markings are equivalent, as both can perform actions a and b in any order. On the other hand the two marking are distinguished by performance equivalence, as they differ in performance. For instance, the second transition in the transition sequence

$$0 \triangleright q_a q_b \xrightarrow{a,0} (0 \triangleright q_b)(1 \triangleright q') \xrightarrow{b,0} 1 \triangleright q' q'$$

can not be matched properly from $0 \triangleright p$, as the only possible option is

$$0 \triangleright p \xrightarrow{a,0} 1 \triangleright p_a \xrightarrow{1,b} 2 \triangleright p_{ab}.$$

Intuitively speaking, one reason behind the distinguishing power of performance equivalence is that it can observe (amount of) concurrency in a process. In the example above, the actions a and b are concurrent when performed from $0 \triangleright q_a q_b$, but not when performed from $0 \triangleright p$. In this respect, performance equivalence resembles non-interleaving process equivalences. In fact, in the simple subclass of communication-free Petri nets (aka Basic Parallel Processes, BPP), performance equivalence coincides with distributed bisimilarity, but also with causal equivalence, history-preserving bisimilarity, and location equivalence (see [13] and further references therein).

It is often not obvious how to define the non-interleaving equivalences mentioned above for classes of systems significantly larger than the simple class of BPP (or maybe BPP extended with communication). For instance, history-preserving bisimilarity requires explicit notion of concurrency (independence) between transitions; and distributed bisimilarity requires

a clear separation into local and non-local part, which is typically impossible in Petri nets. In contract to this, performance equivalence is a reasonable and robust notion of equivalence easy to define in different settings and for different process classes, as it only requires durations assigned to transitions.

2.4. Reachability

It is an easy exercise to show that reachability is decidable in all the four variants. Moreover, the durational semantics makes the problem drastically easier than for the ordinary nets.

Formally, the reachability problem is formulated as follows: given a net N and two durational markings M_0, M , we ask whether M is reachable from M_0 in N . This problem is easily decidable, as it is sufficient to search, roughly speaking, through a finite part of the induced transition system containing transitions with time label smaller than $\max\text{-stamp}(M)$ (the other transitions need not be used if we aim at reaching exactly marking M).

From the point of view of complexity, a more interesting variant of reachability is obtained when the given source and destination markings M_0 and M are non-durational. We ask now whether for some markings \bar{M}_0 and \bar{M} satisfying

$$\text{untime}(\bar{M}_0) = M_0 \quad \text{and} \quad \text{untime}(\bar{M}) = M,$$

\bar{M} is reachable in N from \bar{M}_0 . Hence, we specify only the number of tokens on each place in the source and destination marking, and leave their time-stamps unspecified.

As an easy observation, this variant of reachability is decidable in the local-time patient semantics. Indeed, local-time patient semantics does not differ significantly from non-durational semantics in the context of the latter variant of the reachability problem. Hence, a durational net may be faithfully simulated by a non-durational one, and therefore reachability is decidable.

On the other hand, all the other three variants of semantics are different from the local-time patient (and hence also from the non-durational setting) in that in a given reachable marking, some of the transitions may be non-fireable in the former while fireable in the latter. That is to say, our durational settings pose some additional restriction of fireability of a transition in a marking. We leave investigation of decidability of reachability as the possible further work.

2.5. Patient semantics

It is an easy observation that patient semantics can faithfully simulate ordinary (non-durational) semantics, and therefore we have:

Proposition 2. *The problem of performance equivalence is undecidable for Petri nets under (global-time or local-time) patient semantics.*

The easy proof is by reduction from bisimulation equivalence of ordinary Petri nets. Given such a net N , we extend it by one special place p and add this place both to pre-places and post-places of any transition. In the initial marking, p is marked with a single token. Therefore during a run, place p will always have precisely one token, with the “latest” time-stamp; and this time-stamp will be always observable at the next transition. Hence, performance equivalence between so extended nets coincides with bisimulation equivalence of the original ordinary nets.

The case of impatient semantics is more difficult. We were able to show undecidability only in the global-time variant (the whole Section 3 is devoted to the undecidability proof). Decidability of performance equivalence in the last, local-time impatient variant, remains still open.

3. Undecidability under global-time impatient semantics

In this section, we concentrate on proving the following result:

Theorem 1. *The problem of performance equivalence is undecidable for Petri nets under global-time impatient semantics.*

The proof is by reduction from the (undecidable) halting problem of Minsky machines, and occupies the rest of this section. It is motivated by the Jančar’s proof for ordinary Petri nets [10]; however, a new insight was necessary to adapt this proof to the durational setting.

A Minsky deterministic 2-counter machine \mathbb{M} consists of two counters c_0, c_1 and a set of n labelled instructions

```
1 : instr1
...
n : instrn
```

each instruction in one of the following forms:

$i : c_b := c_b + 1; \text{goto } j$ (increment)
 $i : \text{if } c_b = 0 \text{ then goto } k$ (zero-test or decrement)
 $\quad \text{else } c_b := c_b - 1; \text{goto } j$
 $n : \text{halt}$ (halting instruction)

Variables i, j and k range over $\{1 \dots n\}$ and b over $\{0, 1\}$. We say that \mathbb{M} *halts* if starting from $1 : \text{instr}_1$ and $c_0 = c_1 = 0$, the unique run of \mathbb{M} ends in instruction $n : \text{halt}$.

Given a Minsky machine \mathbb{M} , we will construct a Petri net $N_{\mathbb{M}}$ and two markings in such a way that the markings are performance equivalent iff \mathbb{M} does not halt.

There will be places in $N_{\mathbb{M}}$ corresponding to particular instructions of \mathbb{M} and to particular counters. Formally, the set of places will be

$$\mathcal{P} = \{p_i, q_i, p'_i, q'_i\}_{i=1 \dots n} \cup \{b', b'', Z'_b, Z''_b\}_{b=0,1}$$

(actually we only use those p'_i and q'_i places where $1 : \text{instr}_1$ is 'zero-test or decrement' instruction) and the labelling set

$$\mathcal{L} = \{\text{inc}, \text{dec}, \text{zero}, \overline{\text{zero}}, \tau_0, \tau_1, \omega\}.$$

Transition rules of $N_{\mathbb{M}}$ are defined as follows. For every increment instruction we define the transition rules:

$$(I) \quad p_i \xrightarrow{\text{inc}} p_j b' b'' \quad q_i \xrightarrow{\text{inc}} q_j b' b''$$

where $b \in \{0, 1\}$. For every 'zero-test or decrement' instruction we define the following transition rules:

$$\begin{array}{ll}
 (D) & p_i b' b'' \xrightarrow{\text{dec}} p_j \quad q_i b' b'' \xrightarrow{\text{dec}} q_j \\
 (Z) & p_i \xrightarrow{\text{zero}} p'_i Z'_b Z''_b \quad q_i \xrightarrow{\text{zero}} q'_i Z'_b Z''_b \\
 (Z_I) & p'_i Z'_b Z''_b \xrightarrow{\overline{\text{zero}}} p_k \quad q'_i Z'_b Z''_b \xrightarrow{\overline{\text{zero}}} q_k \\
 (Z_{II}) & p'_i b'' Z'_b \xrightarrow{\overline{\text{zero}}} p_k \quad q'_i b'' Z'_b \xrightarrow{\overline{\text{zero}}} q_k \\
 (Z_{III}) & p'_i b'' Z''_b \xrightarrow{\overline{\text{zero}}} p_k \quad q'_i b'' Z''_b \xrightarrow{\overline{\text{zero}}} p_k
 \end{array}$$

And for the halting instruction we define:

$$(O) \quad p_n \xrightarrow{\omega} p_n$$

Finally, we define additional transition rules:

$$\begin{array}{ll}
 (T_I) & b' b'' \xrightarrow{\tau_b} b' b'' \\
 (T_{II}) & b' Z''_b \xrightarrow{\tau_b} b' b'' \\
 (T_{III}) & b' Z'_b \xrightarrow{\tau_b} b' b''
 \end{array}$$

and set the duration of every transition rule to 1. Note that each of the rules, except (O), appears actually in two instances, for b equal to 0 or 1. We hope that it will be always clear from the context which of the two instances is considered.

As in the proof of Jančar [10], we will show that machine \mathbb{M} halts if and only if two (singleton) durational markings $0 \triangleright p_1$ and $0 \triangleright q_1$ are not performance equivalent. By inspecting the rules one observes an invariant: every marking reachable from any of the two markings contains exactly one token on places in $\{p_i, p'_i, q_i, q'_i\}_{i=1 \dots n}$. We write $(0'0'')^x$ for x copies of $0'0''$. In the following, a marking of one of the forms (after untiming):

$$p_i (0'0'')^x (1'1'')^y \quad \text{or} \quad q_i (0'0'')^x (1'1'')^y$$

will be used to represent the machine \mathbb{M} being in a state that enables to execute instruction instr_i , with the counter values $c_0 = x$ and $c_1 = y$. In addition, the auxiliary markings of the forms (after untiming):

$$p'_i (0'0'')^x (1'1'')^y, \quad \text{or} \quad q'_i (0'0'')^x (1'1'')^y$$

will be used as intermediate ones in the faithful simulation of zero tests of \mathbb{M} . Note that not all reachable markings are of this forms, but every marking has a maximal sub-marking of this form, and it is this sub-marking that we use to determine the state of machine \mathbb{M} .

According to the global-time semantics, a transition rule may be fired with time label $t \in \mathbb{N}$ only when no transition rule is fireable with time label smaller than t . Consider now any execution (a sequence of transitions) of $N_{\mathbb{M}}$ starting from $0 \triangleright p_1$ or $0 \triangleright q_1$, and the first transition $M \xrightarrow{a,t} M'$ with time label t during this execution. All the tokens in M with time-stamp smaller than t are therefore not able to engage in a transition from M . We say these tokens are *dead* in M ; formally, a token is dead in a marking M if some transition is fireable in M with time label bigger than the time-stamp of this token. Note that a token which is dead in some M , remains dead in each marking reachable from M , since the semantics is impatient.

We make the following observation:

Claim. For every $t \in \mathbb{N}$, every execution of $N_{\mathbb{M}}$ from $0 \triangleright p_1$ or $0 \triangleright q_1$ contains a unique marking such that the time-stamps of all non-dead tokens are equal to t (such a marking is called t -equimarking).

Indeed, if a transition with time label t appears in the execution, then the source marking M of the first such transition satisfies the condition (M contains no tokens with time-stamp larger than t as the duration of each transition rule is 1). Or no transition with time label t appears at all, which implies that the execution is finite and ends in a deadlock marking, in which all tokens are dead and have time-stamps less or equal t . It clearly satisfies the condition of the claim.

All transitions executed between two consecutive equimarkings (t - and $(t+1)$ -equimarking, for some t) we call a *large step*.

Now we are prepared for the reduction. As the first case, assume that machine \mathbb{M} halts; we will show a winning strategy for Spoiler. This strategy corresponds to the actual execution of \mathbb{M} , hence we call it a *correct simulation*. An execution of each instruction of the machine is represented (simulated) by one or two large steps.

A position (M, M') in a play is *conforming* if M and M' are identical, except that one of them may have a token $t \triangleright p_i$ and the other $t \triangleright q_i$, or one of M, M' may have $t \triangleright p'_i$ while the other $t \triangleright q'_i$. The starting position $(0 \triangleright p_1, 0 \triangleright q_1)$ is conforming. Consider a conforming pair of t -equimarkings:

$$t \triangleright p_i(0'0'')^x(1'1'')^y \quad | \quad t \triangleright q_i(0'0'')^x(1'1'')^y$$

As mentioned above, when we write $t \triangleright p_i(0'0'')^x(1'1'')^y$ we mean that time-stamps of all tokens are t . In the sequel, to succinctly write markings containing tokens with different time-stamps, we will use a union operation on multisets, denoted by \parallel . We will show how Spoiler can enforce a conforming pair of $(t+1)$ -equimarkings during the play, at the same time simulating the execution of \mathbb{M} . Assume, without loss of generality, that instr_i acts on counter c_0 . We consider all possible types of instruction instr_i . In each case, Spoiler plays on the left-hand side and Duplicator is constantly forced to copy Spoiler's moves on the other side.

(*increment*) Spoiler executes transition (I), to which Duplicator must respond with (I). Next, Spoiler fires $x+y$ times transition rule (T_I) (first all τ_0 , then all τ_1); for future reference, we call such a sequence of (T_I) transitions a *default step completion*. During the completion, the responses of Duplicator are uniquely determined, as transitions rules (T_{II}) and (T_{III}) are not fireable.

$$\begin{array}{c|c} \begin{array}{l} t \triangleright p_i(0'0'')^x(1'1'')^y \\ \downarrow (I) \\ (t+1) \triangleright p_j(0'0'') \parallel t \triangleright (0'0'')^x(1'1'')^y \\ \downarrow (T_I)^x \\ (t+1) \triangleright p_j(0'0'')^{x+1} \parallel t \triangleright (1'1'')^y \\ \downarrow (T_I)^y \\ (t+1) \triangleright p_j(0'0'')^{x+1}(1'1'')^y \end{array} & \begin{array}{l} t \triangleright q_i(0'0'')^x(1'1'')^y \\ \downarrow (I) \\ (t+1) \triangleright q_j(0'0'') \parallel t \triangleright (0'0'')^x(1'1'')^y \\ \downarrow (T_I)^x \\ (t+1) \triangleright q_j(0'0'')^{x+1} \parallel t \triangleright (1'1'')^y \\ \downarrow (T_I)^y \\ (t+1) \triangleright q_j(0'0'')^{x+1}(1'1'')^y \end{array} \end{array}$$

For convenience, we decorate each transition by the identifier of the relevant transition rule, in place of its label.

Both the resulting markings are $(t+1)$ -equimarkings which completes the simulation of instruction instr_i . They represent the machine \mathbb{M} in a state that enables to execute instruction instr_j , and the counters values are as expected.

(*zero-test or decrement*) In this case, the behaviour of Spoiler depends on the value of counter c_0 , i.e. the value of x .

If $x > 0$, Spoiler start with transition (D) and completes the step in the default manner:

$$\begin{array}{c|c} \begin{array}{l} t \triangleright p_i(0'0'')^x(1'1'')^y \\ \downarrow (D) \\ (t+1) \triangleright p_j \parallel t \triangleright (0'0'')^{x-1}(1'1'')^y \\ \downarrow (T_I)^{x+y-1} \\ (t+1) \triangleright p_j(0'0'')^{x-1}(1'1'')^y \end{array} & \begin{array}{l} t \triangleright q_i(0'0'')^x(1'1'')^y \\ \downarrow (D) \\ (t+1) \triangleright q_j \parallel t \triangleright (0'0'')^{x-1}(1'1'')^y \\ \downarrow (T_I)^{x+y-1} \\ (t+1) \triangleright q_j(0'0'')^{x-1}(1'1'')^y \end{array} \end{array}$$

In the other case, when $x = 0$, Spoiler executes transition (Z), to which Duplicator must respond with (Z). Then Spoiler completes the step in the default manner.

$$\begin{array}{c|c} \begin{array}{l} t \triangleright p_i(1'1'')^y \\ \downarrow (Z) \\ (t+1) \triangleright p'_i(Z'_0Z''_0) \parallel t \triangleright (1'1'')^y \\ \downarrow (T_I)^y \\ (t+1) \triangleright p'_i(Z'_0Z''_0)(1'1'')^y \end{array} & \begin{array}{l} t \triangleright q_i(1'1'')^y \\ \downarrow (Z) \\ (t+1) \triangleright q'_i(Z'_0Z''_0) \parallel t \triangleright (1'1'')^y \\ \downarrow (T_I)^y \\ (t+1) \triangleright q'_i(Z'_0Z''_0)(1'1'')^y \end{array} \end{array}$$

Then Spoiler starts the next large step with transition (Z_I) . As there are no $0''$ tokens in the marking, transitions (Z_{II}) and (Z_{III}) are not fireable and Duplicator must respond with (Z_I) . Next, Spoiler completes the step in the default manner, reaching $(t+2)$ -equimarkings, thus finalising the simulation of instruction instr_i as expected.

$$\begin{array}{c|c}
\begin{array}{l}
(t+1) \triangleright p'_i(Z'_0 Z''_0)(1'1'')^y \\
\downarrow (Z_I) \\
(t+2) \triangleright p_k \parallel (t+1) \triangleright (1'1'')^y \\
\downarrow (T_I)^y \\
(t+2) \triangleright p_k(1'1'')^y
\end{array} &
\begin{array}{l}
(t+1) \triangleright q'_i(Z'_0 Z''_0)(1'1'')^y \\
\downarrow (Z_I) \\
(t+2) \triangleright q_k \parallel (t+1) \triangleright (1'1'')^y \\
\downarrow (T_I)^y \\
(t+2) \triangleright q_k(1'1'')^y
\end{array}
\end{array}$$

In both cases the final position is a conforming pair of equimarkings.

(*halting instruction*) Player Spoiler executes transition (O); Duplicator has no response and losses the game.

As the second case, assume that machine \mathbb{M} does not halt; we will show a winning strategy for Duplicator. As long as Spoiler plays a correct simulation, Duplicator's behaviour is determined. However, a position enabling firing of transition rule (O) is never reached, and the game never ends – Duplicator wins.

Spoiler can diverge from the correct simulation (*cheat*) in two ways: *insignificant* or *significant*. The former means that the equimarkings are the same as in the correct simulation. This kind of cheating involves shuffling the order of transitions within one large step and changing the side on which Spoiler is playing. With such behaviour, he still essentially simulates the execution of \mathbb{M} : he will never fire transition rule (O), thus Duplicator wins.

Spoiler can also cheat in a significant manner, simulating a positive zero test, when it should decrease the counter. However, Duplicator can respond to such behaviour in a way which will make the next equimarkings identical, modulo dead tokens. Afterwards, Duplicator can copy Spoiler's behaviour exactly, and hence win the game.

In what follows, we give an exhaustive overview of possible Spoiler's cheating behaviour, and show how Duplicator can respond to them to ensure her win. As before, we start with a conforming equimarking position and we assume that instruction instr_i uses counter c_0 .

$$t \triangleright p_i(0'0'')^x(1'1'')^y \quad | \quad t \triangleright q_i(0'0'')^x(1'1'')^y$$

We consider below all possible types of instruction instr_i . In each case we assume that the first move of Spoiler is on the left-hand side.

(*increment*) Spoiler can, in any order, execute transitions (I) and (T_I) , which Duplicator must (and can) copy. The equimarkings reached in the end cannot be different then those appearing in the correct simulation. It is hence an insignificant cheating.

(*zero-test or decrement*) If counter c_0 used by the instruction is equal to 0, Spoiler can only cheat insignificantly. During the first large step involved in the simulation, Spoiler can choose an arbitrary ordering of (Z) and (T_I) transitions, as it does not influence the resulting equimarkings:

$$(t+1) \triangleright p'_i(Z'_0 Z''_0)(1'1'')^y \quad | \quad (t+1) \triangleright q'_i(Z'_0 Z''_0)(1'1'')^y$$

In the second large step, Spoiler's choice is again limited to shuffling of (Z_I) and (T_I) transitions, as none of the (Z_{III}) , (T_{II}) or (T_{III}) transition rules is fireable. It has no influence on the resulting equimarkings, which completes the simulation of instr_i :

$$(t+2) \triangleright p_k(1'1'')^y \quad | \quad (t+2) \triangleright q_k(1'1'')^y$$

Now assume that c_0 is non-zero, $x > 0$. Spoiler can cheat insignificantly, if he decides to decrease the counter. As before, he will be limited to shuffling around the (D) and (T_I) transitions, with no influence on the resulting equimarkings:

$$(t+1) \triangleright p_j(0'0'')^{x-1}(1'1'')^y \quad | \quad (t+1) \triangleright q_j(0'0'')^{x-1}(1'1'')^y$$

In all the cases above, Duplicator was forced to copy the moves of Spoiler. A more interesting case is when Spoiler decides to cheat significantly, and executes transition (Z) despite that $x > 0$. Duplicator is forced to respond with (Z). Then, after completion of the step, irrespectively of the shuffling of the (T_I) and (Z) transitions (no other transition rules are fireable), the equimarkings will have the form:

$$(t+1) \triangleright p'_i(Z'_0 Z''_0)(0'0'')^x(1'1'')^y \quad | \quad (t+1) \triangleright q'_i(Z'_0 Z''_0)(0'0'')^x(1'1'')^y$$

Note that the only fireable τ_1 -labelled transition rule is (T_I) , activated by tokens $(t+1) \triangleright 1'$ and $(t+1) \triangleright 1''$ (as there are no Z'_1 or Z''_1 tokens). It does not matter for the next $(t+2)$ -equimarkings how firings of (T_I) are interleaved with the other transitions. Hence, we omit it in the presentation below and proceed as if we were in the position:

$$(t+1) \triangleright p'_i(Z'_0 Z''_0)(0'0'')^x \quad | \quad (t+1) \triangleright q'_i(Z'_0 Z''_0)(0'0'')^x \quad (\dagger)$$

In all of the cases below (except for one, which is explained separately), Duplicator will respond in such a way that the resulting equimarkings are identical. From that point on she may copy exactly the Spoiler's moves and hence win the game.

1. If Spoiler executes (Z_I) , Duplicator responds with (Z_{III}) . The step can be completed only by x executions of the (T_I) transition on the left and $x-1$ executions of (T_I) and one execution of (T_{III}) on the right. These transitions have identical labels, hence it is irrelevant in what order they are executed and on which side Spoiler is playing.

$$\begin{array}{c|c}
 \begin{array}{l}
 (t+1) \triangleright p'_i(Z'_0 Z''_0)(O' O'')^x \\
 \downarrow (Z_I) \\
 (t+2) \triangleright p_k \parallel (t+1) \triangleright (O' O'')^x \\
 \downarrow (T_I)^x \\
 (t+2) \triangleright p_k(O' O'')^x
 \end{array} &
 \begin{array}{l}
 (t+1) \triangleright q'_i(Z'_0 Z''_0)(O' O'')^x \\
 \downarrow (Z_{III}) \\
 (t+2) \triangleright p_k \parallel (t+1) \triangleright (O' Z'_0)(O' O'')^{x-1} \\
 \downarrow (T_I)^u (T_{III})(T_I)^{x-u-1} \\
 (t+2) \triangleright p_k(O' O'')^x
 \end{array}
 \end{array}$$

2. If Spoiler executes transition (Z_{II}) , Duplicator responds with (Z_{III}) . Completing the step involves $x-1$ firings of (T_I) and one firing of (T_{II}) on the left, and $x-1$ firings of (T_I) and one firing of (T_{III}) on the right. As above, the Spoiler's choices at this stage have no influence on the resulting equimarkings.

$$\begin{array}{c|c}
 \begin{array}{l}
 (t+1) \triangleright p'_i(Z'_0 Z''_0)(O' O'')^x \\
 \downarrow (Z_{II}) \\
 (t+2) \triangleright p_k \parallel (t+1) \triangleright (O' Z'_0)(O' O'')^{x-1} \\
 \downarrow (T_I)^u (T_{II})(T_I)^{x-u-1} \\
 (t+2) \triangleright p_k(O' O'')^x
 \end{array} &
 \begin{array}{l}
 (t+1) \triangleright q'_i(Z'_0 Z''_0)(O' O'')^x \\
 \downarrow (Z_{III}) \\
 (t+2) \triangleright p_k \parallel (t+1) \triangleright (O' Z'_0)(O' O'')^{x-1} \\
 \downarrow (T_I)^u (T_{III})(T_I)^{x-u-1} \\
 (t+2) \triangleright p_k(O' O'')^x
 \end{array}
 \end{array}$$

3. If Spoiler executes transition (Z_{III}) , Duplicator responds with (Z_{II}) . This case is similar to the one above.
4. If Spoiler executes transition (T_I) , Duplicator responds depending on the value of x . If $x > 1$, she executes (T_I) . Notice that the tokens $(t+2) \triangleright (O' O'')$ have no influence on the $(t+1)$ -time-stamped tokens. Hence Duplicator can continue as if the configuration was (\dagger) , but with one less O' and O'' tokens.

$$\begin{array}{c|c}
 \begin{array}{l}
 (t+1) \triangleright p'_i(Z'_0 Z''_0)(O' O'')^x \\
 \downarrow (T_I) \\
 (t+1) \triangleright p'_i(Z'_0 Z''_0)(O' O'')^{x-1} \parallel \\
 (t+2) \triangleright (O' O'')
 \end{array} &
 \begin{array}{l}
 (t+1) \triangleright q'_i(Z'_0 Z''_0)(O' O'')^x \\
 \downarrow (T_I) \\
 (t+1) \triangleright q'_i(Z'_0 Z''_0)(O' O'')^{x-1} \parallel \\
 (t+2) \triangleright (O' O'')
 \end{array}
 \end{array}$$

If $x = 1$, Duplicator executes (T_{III}) . Completing the step involves firing of (Z_I) on the left and (Z_{III}) on the right.

$$\begin{array}{c|c}
 \begin{array}{l}
 (t+1) \triangleright p'_i(Z'_0 Z''_0)(O' O'') \\
 \downarrow (T_I) \\
 (t+1) \triangleright p'_i(Z'_0 Z''_0) \parallel (t+2) \triangleright (O' O'') \\
 \downarrow (Z_I) \\
 (t+2) \triangleright p_k(O' O'')
 \end{array} &
 \begin{array}{l}
 (t+1) \triangleright q'_i(Z'_0 Z''_0)(O' O'') \\
 \downarrow (T_{III}) \\
 (t+1) \triangleright q'_i(O' Z'_0) \parallel (t+2) \triangleright (O' O'') \\
 \downarrow (Z_{III}) \\
 (t+2) \triangleright p_k(O' O'')
 \end{array}
 \end{array}$$

5. If Spoiler executes (T_{II}) , Duplicator responds with (T_{III}) .

$$\begin{array}{c|c}
 \begin{array}{l}
 (t+1) \triangleright p'_i(Z'_0 Z''_0)(O' O'')^x \\
 \downarrow (T_{II}) \\
 (t+1) \triangleright p'_i(O' Z'_0)(O' O'')^{x-1} \parallel (t+2) \triangleright (O' O'')
 \end{array} &
 \begin{array}{l}
 (t+1) \triangleright q'_i(Z'_0 Z''_0)(O' O'')^x \\
 \downarrow (T_{III}) \\
 (t+1) \triangleright q'_i(O' Z'_0)(O' O'')^{x-1} \parallel (t+2) \triangleright (O' O'')
 \end{array}
 \end{array}$$

Now the play can continue in one of two ways, depending on the behaviour of Spoiler. Before it is decided, as long as Spoiler executes (T_I) , Duplicator responds with (T_I) :

$$\begin{array}{c|c}
 \begin{array}{l}
 (t+1) \triangleright p'_i(O' Z'_0)(O' O'')^{x-1} \parallel (t+2) \triangleright (O' O'') \\
 \downarrow (T_I)^u \\
 (t+1) \triangleright p'_i(O' Z'_0)(O' O'')^{x-u-1} \parallel \\
 (t+2) \triangleright (O' O'')^{u+1}
 \end{array} &
 \begin{array}{l}
 (t+1) \triangleright q'_i(O' Z'_0)(O' O'')^{x-1} \parallel (t+2) \triangleright (O' O'') \\
 \downarrow (T_I)^u \\
 (t+1) \triangleright q'_i(O' Z'_0)(O' O'')^{x-u-1} \parallel \\
 (t+2) \triangleright (O' O'')^{u+1}
 \end{array}
 \end{array}$$

At some point Spoiler must decide between executing, on the left-hand side, of (Z_{II}) (equivalently, (Z_{III}) on the right) or (T_{III}) (equivalently, (T_{II}) on the right). In the first case, Duplicator responds as follows:

$$\begin{array}{c|c}
 \begin{array}{l}
 (t+1) \triangleright p'_i(O' Z'_0)(O' O'')^{x-u-1} \parallel \\
 (t+2) \triangleright (O' O'')^{u+1} \\
 \downarrow (Z_{II}) \\
 (t+1) \triangleright (O' O'')^{x-u-1} \parallel (t+2) \triangleright p_k(O' O'')^{u+1} \\
 \downarrow (T_I)^{x-u-1} \\
 (t+2) \triangleright p_k(O' O'')^x
 \end{array} &
 \begin{array}{l}
 (t+1) \triangleright q'_i(O' Z'_0)(O' O'')^{x-u-1} \parallel \\
 (t+2) \triangleright (O' O'')^{u+1} \\
 \downarrow (Z_{III}) \\
 (t+1) \triangleright (O' O'')^{x-u-1} \parallel (t+2) \triangleright p_k(O' O'')^{u+1} \\
 \downarrow (T_I)^{x-u-1} \\
 (t+2) \triangleright p_k(O' O'')^x
 \end{array}
 \end{array}$$

whereas in the latter case:

$$\begin{array}{c|c}
\begin{array}{l}
(t+1) \triangleright p'_i(0''Z'_0)(0'O'')^{x-u-1} \parallel \\
(t+2) \triangleright (0'O'')^{u+1} \\
\downarrow (\mathbb{T}_{III}) \\
(t+1) \triangleright p'_i0''0''(0'O'')^{x-u-2} \parallel (t+2) \triangleright (0'O'')^{u+2} \\
\downarrow (\mathbb{T}_I)^{x-u-2} \\
(t+1) \triangleright p'_i0''0'' \parallel (t+2) \triangleright (0'O'')^x
\end{array}
&
\begin{array}{l}
(t+1) \triangleright q'_i(0''Z''_0)(0'O'')^{x-u-1} \parallel \\
(t+2) \triangleright (0'O'')^{u+1} \\
\downarrow (\mathbb{T}_{II}) \\
(t+1) \triangleright q'_i0''0''(0'O'')^{x-u-2} \parallel (t+2) \triangleright (0'O'')^{u+2} \\
\downarrow (\mathbb{T}_I)^{x-u-2} \\
(t+1) \triangleright q'_i0''0'' \parallel (t+2) \triangleright (0'O'')^x
\end{array}
\end{array}$$

This is the only case so far in which the resulting markings are not identical. However, the $(t+1)$ -time-stamped tokens on both sides are dead, whereas the $(t+2)$ -time-stamped ones are identical. Hence, *Duplicator* can still copy in future all the transitions that *Spoiler* makes, even if the configurations do not represent a proper machine state anymore.

6. If *Spoiler* executes (\mathbb{T}_{III}) , *Duplicator* responds with (\mathbb{T}_{II}) . What follows is similar to the previous point.

(*halting instruction*) Simulation of instruction $n:\text{halt}$ may happen only if *Spoiler* have cheated earlier in a significant manner. As we have shown above, in such a case the markings on both sides are identical and *Duplicator* still wins.

We have shown that the initial markings $0 \triangleright p_1$ and $0 \triangleright q_1$ are performance equivalent if and only if machine \mathbb{M} does not halt. Hence performance equivalence is undecidable.

4. Concluding remarks

We have investigated Petri nets under four different durational semantics, and the corresponding variant of bisimulation equivalence, called performance equivalence. We have proved that in three of the four cases, performance equivalence is undecidable. Hence, unfortunately, our results do not confirm a conjecture that performance equivalence might be easier to decide than classical bisimulation equivalence. However in the fourth variant, i.e., local-time impatient semantics, we were able neither to confirm nor to falsify the conjecture. There is hence still a hope that this last variant might admit an effective decision procedure and we conjecture that this is really the case. Verifying this conjecture is the main open problem left.

As an encouraging observation we mention a partial decomposition property which holds in case of local-time impatient semantics. Consider a pair of performance equivalent markings, $t \triangleright M$ and $t \triangleright M'$, and some transition of one of them $t \triangleright M \xrightarrow{a,t} t \triangleright M_1 \parallel (t+1) \triangleright M_2$, assuming $\text{dur}(a) = 1$, which may be answered by some $t \triangleright M' \xrightarrow{a,t} t \triangleright M'_1 \parallel (t+1) \triangleright M'_2$. An easy observation is that $(t+1) \triangleright M_2$ and $(t+1) \triangleright M'_2$ must be necessarily equivalent, as *Spoiler* may decide to use only time labels greater than t from this point on. More generally, if M and M' are equivalent, then for any t , their submarkings containing tokens with time-stamps greater than t are equivalent too.

The durational nets may be considered as a special case of *data nets* [15], once one slightly relaxes this notion by allowing data domain different than equality data $(\mathbb{N}, =)$ or ordered data (\mathbb{Q}, \leq) . Fix a logical structure called *data domain*. Data Petri nets are very much like ordinary Petri nets, with a proviso that every token carries a data value (an element of data domain), and every transition rule has a constraint on data values of tokens involved. The transition constraints are specified by formulas in the language of data domain; for our purposes, it is enough to restrict to quantifier-free formulas.

Within this generic setting, durational Petri nets can be seen as a syntactic restriction of data nets over data domain $(\mathbb{Z}, +1, =)$, i.e. integers with the successor function $+1$ and equality. In particular, transition constraints of durational nets are specified using equality and $+1$. In this perspective, performance equivalence is exactly the natural interpretation of the standard bisimulation equivalence in data nets. As data nets always generalise ordinary nets, the bisimulation equivalence will be always undecidable in general. The present paper may be therefore seen as a search for interesting syntactic restrictions that would guarantee decidability of bisimulation equivalence for data nets over data domain $(\mathbb{Z}, +1, =)$.

For the sake of comparison, if data universe is chosen to be $(\mathbb{Z}, +1, \leq)$ then data nets generalise discrete-timed Petri nets; and for data universe $(\mathbb{Q}, +1, \leq)$ we obtain a generalisation of dense-timed Petri nets.

Now we turn to the reachability problem. In Section 2 we have identified two different variants of this problem: one in which source and destination markings are specified exactly (easy to solve in all the four variants of semantics), and another one in which source and destination markings are given as non-durational ones. While the latter variant of the problem is easily decidable for local-time patient semantics, we do not know its status for the other variants. Investigation of this question is a natural continuation of the work presented in this paper. We suspect that there may be a reduction from durational nets to nets with hierarchical inhibitor arcs, a model with decidable reachability problem according to [20] (see also [2] for a self-contained proof of reachability for nets with one zero test).

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