Ehrenfeucht-Fraïssé Games for Metric Temporal Logic on Data Words

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Extensions of Linear Temporal Logic

$$
\begin{array}{ccc}
 & \mathrm{LTL} & \\
\rotatebox{45}{$\subseteq$} & & \rotatebox{-45}{$\supseteq$} \\
\mathrm{MTL} & \subseteq & \mathrm{TPTL}
\end{array}
$$

Theorem: MTL($\mathbb{N}$) $=$ TPTL($\mathbb{N}$) [Alur&Henzinger, 1993]

Theorem: MTL($\mathbb{R}_{\geq 0}$) $\subsetneq$ TPTL($\mathbb{R}_{\geq 0}$) [Bouyer et al, 2010]

What is the relative expressiveness of MTL($\mathbb{Z}$) and TPTL($\mathbb{Z}$)?

Nonmonotonic Data Words

Data words over finite set **P** of propositional variables.

$(P_0, d_0)(P_1, d_1)(P_2, d_2)(P_3, d_3) \cdots \in (2^{\mathbf{P}} \times \mathbb{N})^\omega$

$d_1, d_2, d_3, \ldots$ nonmonotonic sequence

e.g., computation of a one-counter machine

Metric Temporal Logic

$\varphi ::= p \mid \neg\varphi \mid \varphi \wedge \psi \mid \varphi U_I \psi \quad (p \in P,\ I \subseteq \mathbb{Z}$ is an interval)

$w = (P_0, d_0)(P_1, d_1)(P_2, d_2)\ldots$

$$\begin{aligned}(w,i) \models \varphi_1 U_I \varphi_2 \quad \text{iff} \quad & \exists j > i \text{ such that } (w,j) \models \varphi_2 \text{ and} \\ & \forall i < k < j, (w,k) \models \varphi_1 \text{ and } d_j - d_i \in I\end{aligned}$$

Timed Propositional Temporal Logic

$\varphi ::= p \mid \neg\varphi \mid \varphi \wedge \psi \mid \varphi U \psi \mid x.\varphi \mid x \sim c$

(x is a register variable, $c \in \mathbb{Z}$, $\sim \in \{\leq, <, =, >, \geq\}$)

$$\begin{array}{lll} (w,i,v) \models x.\varphi & \text{iff} & (w,i,v[x \mapsto d_i]) \models \varphi \\ (w,i,v) \models x \sim c & \text{iff} & d_i - v(x) \sim c \end{array}$$

($v : X \to \mathbb{N}$ a function from register variables to natural numbers,

$v[x \mapsto n]$ changes the value of $v(x)$ to n.)

k-round EF game for MTL on w_0 and w_1

Played by: Spoiler ● and Duplicator ●

Constraints set **I**

k-round EF game for MTL on w_0 and w_1

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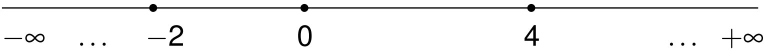
Constraints set $\mathbf{I}$

Example: $\mathbf{I} = \{-\infty, -2, 0, 4, +\infty\}$



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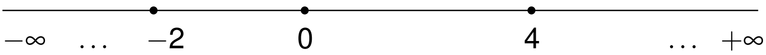

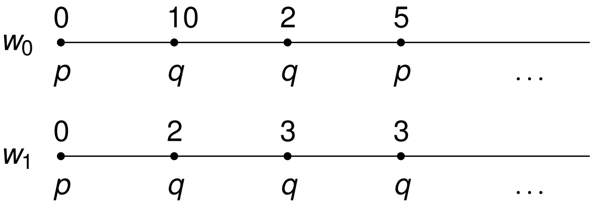

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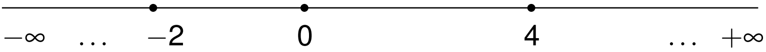

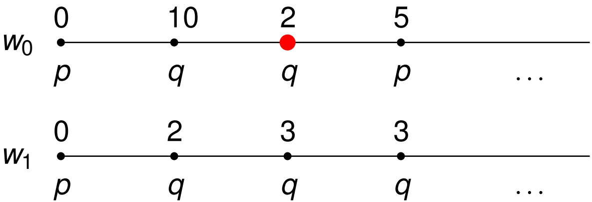

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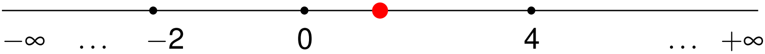

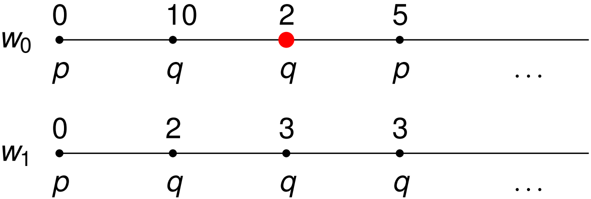

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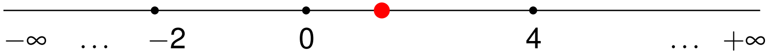

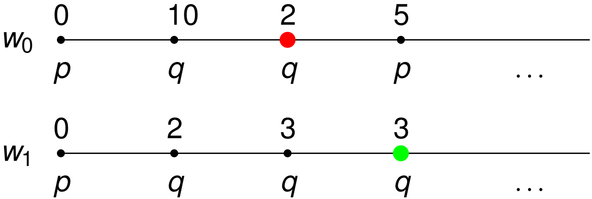

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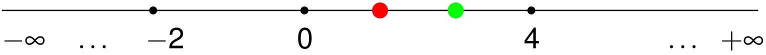

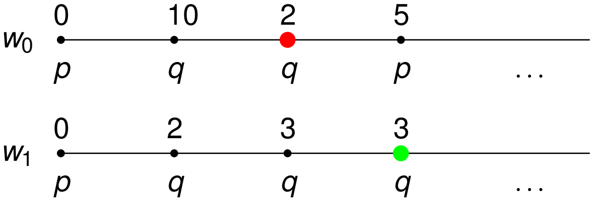

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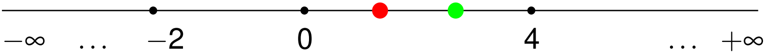

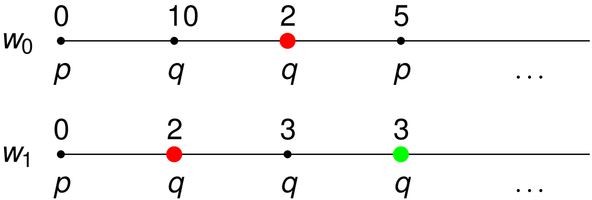

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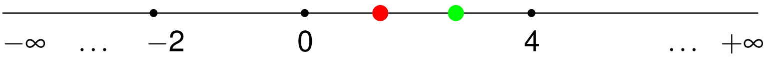

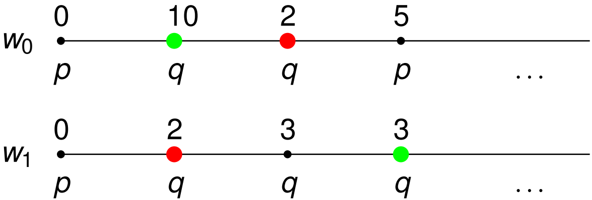

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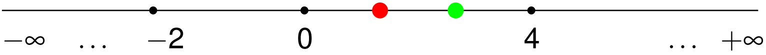
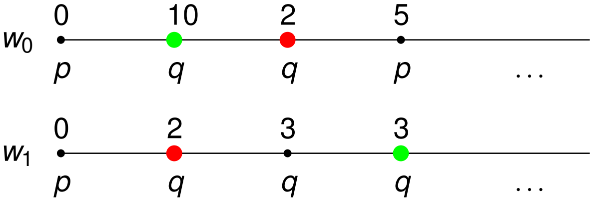




$\mathrm{GAME}_k^{\mathbf{I}}(w_0, w_1)$

Main Theorems

Theorem 1

The following are equivalent:

1. Duplicator has a winning strategy for $\mathrm{GAME}_k^{\mathbf{I}}(w_0, w_1)$.
2. w_0 and w_1 satisfy the same formulas in $\mathrm{MTL}_k^{\mathbf{I}}(\mathbb{Z})$.

$\mathrm{MTL}_k^{\mathbf{I}}(\mathbb{Z})$: • k nested until operators,

• $\varphi_1 U_{[a,b]} \varphi_2 \Rightarrow a, b \in \mathbf{I}$

$\mathrm{MTL}(\mathbb{Z}) = \bigcup_{k,\mathbf{I}} \mathrm{MTL}_k^{\mathbf{I}}(\mathbb{Z})$

Main Theorems

Theorem 2

Let $\varphi \in \mathrm{TPTL}(\mathbb{Z})$, the following are equivalent:

1. φ is not definable in $\mathrm{MTL}(\mathbb{Z})$.
2. $\forall \mathbf{I}, k,\ \exists w_0 \models \varphi, w_1 \not\models \varphi$ such that Duplicator can win $\mathrm{GAME}_k^{\mathbf{I}}(w_0, w_1)$.

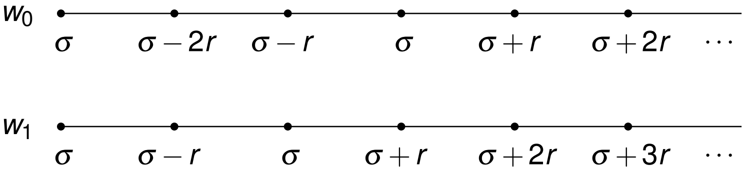
Main Theorems

Theorem 3

MTL($\mathbb{Z}$) $\subsetneq$ TPTL($\mathbb{Z}$)

Proof:

$x.FFF(x=0) \in$ TPTL($\mathbb{Z}$) cannot be expressed in MTL($\mathbb{Z}$)



Further Applications of the EF Game on MTL

- Theorem 4: The following MTL membership decision problem is undecidable:

  GIVEN: $\varphi \in$ TPTL($\mathbb{Z}$)

  QUESTION: Is φ definable in MTL($\mathbb{Z}$)?

- EF game for MTL can also be used for proving results for timed words over $\mathbb{R}_{\geq 0}$.

- We define also an EF game for TPTL.