Review of
The Golden Ratio and Fibonacci Numbers
by Richard A. Dunlap
World Scientific, 1997
172 pages, Hardcover $58, ebook $46

Review by
Michaël Cadilhac <michael@cadilhac.name>
WSI, Universität Tübingen

1 Overview

Professor Dunlap, a physicist specialized in the study of materials, presents a few tokens of the pervasiveness of the golden ratio and the Fibonacci sequence in our mathematical, physical, and biological world. Mainly focused on geometrical facts and their applications, the book contains a fantasy mix of the author's favorite curiosities around this topic. If the book is by no means exhaustive—and it has no pretension to be—it is a nice potpourri that anyone can enjoy, while claiming to not be addressed to any particular profile.

2 Summary of Contents

Chapter 1: Introduction. Using simple mathematical facts as well as historical and artistic considerations, the author introduces the golden ratio as a natural and compelling value.

Chapter 2: Basic properties of the golden ratio. Some links between the golden ratio and sequences, in particular those of Lucas and Fibonacci, are presented. More importantly, the name "ratio" is explained, using geometry. Indeed, if a segment AB of length 1 is divided into two segments AC and CA such that $AB/AC = AC/CA$, then this ratio is the golden ratio.

Chapter 3: Geometric problems in two dimensions. Regular polygons are shown to have some dimensions related to the golden ratio. The golden gnomons[3] are introduced.

Chapter 4: Geometric problems in three dimensions. A similar study is carried in three dimensions.

Chapter 5: Fibonacci numbers. The classical rabbit problem is introduced, and some natural occurrences of the Fibonacci numbers are explored. Binet's formula is presented:

$$F_n = \frac{1}{\sqrt{5}} \times \left(\tau^n - (-\tau)^{-n}\right) \ ,$$

with F_n the n-th Fibonacci number, and τ the golden ratio. Necessary and sufficient conditions are given to "recognize" a Fibonacci sequence of rabbits (given by the rules "Adult $\mapsto$ Adult, Baby" and "Baby $\mapsto$ Adult").

[3] According to the Oxford dictionary, a gnomon is the part of a parallelogram left when a similar parallelogram has been taken from its corner.

Chapter 6: Lucas numbers and generalized Fibonacci numbers. Some generalizations of the Fibonacci recurrence relation are studied, leading to Lucas numbers, the Tribonacci sequence, and the like.

Chapter 7: Continued fractions and rational approximants. Continued fractions are introduced, in particular that of the golden ratio.

Chapter 8: Generalized Fibonacci representation theorems. This chapter focuses on representing numbers in "bases" that are extracted from Fibonacci-like sequences. For instance, using $F_2 = 1, F_3 = 2, F_4 = 3, F_5 = 5, \ldots$ as a base, the number 10 can be written 10010 or 1110, as $10 = F_6 + F_3 = F_5 + F_4 + F_3$. Using some extra rules, this representation can be chosen to be unique.

Chapter 9: Optimal spacing and search algorithms. Golden ratio search, an algorithm to search for an extremum of unimodal functions, is presented. This relies in part on the properties of iterating rotations around the unit circle by transcendental multiples of π.

Chapter 10: Commensurate and incommensurate projections. Herein, two-dimensional latices are projected onto lines, and the cut angle determines the nature of the segments defined by the projections on the line. In particular, when the cut angle is an irrational tangent ($\tan\theta = 1/\tau$), this is an *incommensurate projection*, and the grid is projected down the line in such a way that the line is cut in long (L) and short (s) segments that reproduce the sequence of Adult and Baby rabbits.

Chapter 11: Penrose tilings. These tilings are created by incommensurate projections of higher dimensions. Generalizations of the concepts of the previous chapter are presented. This gives rise to *quasiperiodic* tilings, i.e., tilings that can fill the space without exhibiting translational symmetry.

Chapter 12: Quasicrystallography. Quasiperiodic tilings are used to arrange atoms to create crystal-like structures. The main difference with crystals is the lack of translational symmetry. Their refraction properties are studied.

Chapter 13: Biological applications. In this chapter, the classical links between the Fibonacci sequence and flowers, marine animals, viruses, pine cones, pineapples, sunflowers, and mollusks are presented.

3 Opinion

On the intended audience. I have a hard time deciphering to whom this book is written. On the one hand, I learned a few things reading it, both in geometry and physics. I feel that the writing style is very dry, if not technical, and the book lacks the usual entertaining side of scientific popularization. The lack of a compelling driving force makes it hard to stay interested, save for being a scientist at heart. The reader is expected to be versed in some terminology that were way beyond my (arguably limited) knowledge.

On the other hand, the lack of proofs is blatant, appealing to the nonscientist. There is, altogether, no prior knowledge required, and the mathematical facts are usually within grasp (provided the reader knows about basic geometry, trigonometry, transcendental numbers, ...). The sheer number of tables giving numerical values may help the reader to understand how values go, although it is in my opinion generally overdone (e.g., Chapter 8 is 6.5 pages, 3 of which are just numerical values). The succinctness of the chapters allows the reader to pick up the book, read a chapter, and let it sit for a few days, to digest the content.

On the content. The choice of topics is the author's own, and I believe, not to be argued against. It is certainly a diverse and healthy mix, that may have aged a tad bit since its publication some twenty years ago. In a way, the content can be seen as being steered toward quasicrystals, stopping along the way to discover some nuggets of mathematical knowledge.

On the form. The book contains a large number of drawings that do come as a relief in the driest parts. Their position in the text, on the physical pages, is often amiss, forcing the reader to go back and forth, and search for the correct figure number. This is a more general problem in fact, since equations are cross referenced throughout the book without a single aid to find them (e.g., when reaching page 56, referencing "Eq. (2.1)," appearing page 7, is quite a stretch). More than once was I tempted not to look at a figure or an equation as I felt it was "too far."

Final opinion. I would recommend this book to physics students, not as an object of study, but as mathematical entertainment. I do not think laypeople would enjoy it, because of the writing style and the choice of topics. I feel that the lack of proofs and the over abundance of numerical values would discourage a mathematician. All in all, the text end up occupying a small niche, despite a (overly?) general title.