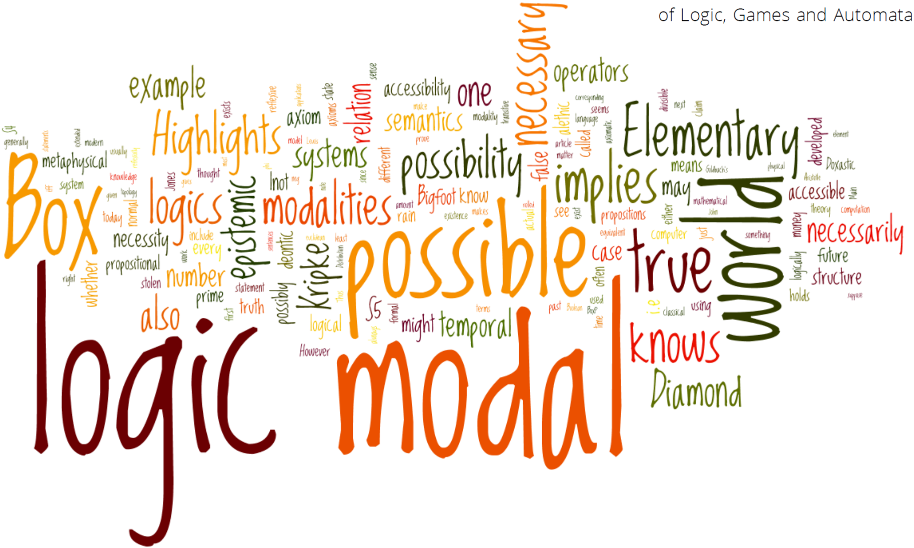
HIGHLIGHTS
of Logic, Games and Automata




Emanuel Kieroński, **Jakub Michaliszyn**, Jan Otop

# Modal logic

- Many different modal logics (K4, S5, CTL, LTL, ATL, HS, CTL$^*$K)
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- Many applications in verification, planing, linguistics
- Many proofs, many papers
- Our area of interest: a comprehensive study on the satisfiability problem.

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**Our ultimate goal**

For all first-order definable classes $\mathcal{K}$, determine the decidability and complexity of $\mathcal{K}$-SAT and $\mathcal{K}$-GSAT.

We are also interested in finite satisfiability.

# Negative results

- (E. Hemaspaandra, “The Price of Universality", 1996)
  $\mathcal{K}$-GSAT is undecidable for some $\forall$FO-definable $\mathcal{K}$.

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- (E. Kieroński, J. Michaliszyn, J. Otop, FSTTCS 2011)
$\mathcal{K}$-GSAT and $\mathcal{K}'$-SAT are undecidable for some $\forall\mathrm{FO}^3$-definable $\mathcal{K}$ and $\mathcal{K}'$ (holds also for finite satisfiability).

$$\neg xRy \vee \neg xRz \vee yRz \vee zRy \vee yRx \vee zRx$$

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Decidability in many interesting cases (even multimodal), including

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  - with one transitive relation (W. Szwast, L. Tendera, 2012),
  - with counting quantifiers (I. Pratt-Hartmann, 2005),
  - with two equivalence relations (E. Kieroński, J.Michaliszyn, I. Pratt-Hartmann, L. Tendera, 2012).
- Guarded Fragment:
  - with fixed points (E. Grädel, I. Walukiewicz, 1999),
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- Guarded Fragment:
  - with fixed points (E. Grädel, I. Walukiewicz, 1999),
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- High complexity.

# Positive results

*J. Michaliszyn, J. Otop*, LICS 2012

For any $\mathcal{K}$ definable by universal Horn formulas, $\mathcal{K}$-SAT and $\mathcal{K}$-GSAT are decidable.

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*J. Michaliszyn, E. Kieroński*, AIML 2012

Also finite satisfiability of modal logic is decidable w.r.t. the classes definable by universal Horn formulas.

# General satisfiability

| Type | $\mathcal{K}_\Phi$-GSAT | $\mathcal{K}_\Phi$-SAT |
|---|---|---|
| S1+ | EXPTIME-c | PSPACE-c |
| S1– | PSPACE-c | NP-c |
| S2+ | NP-c | PSPACE-c |
| S2– | NP-c | NP-c |
| S3+ | impossible | |
| S3– | NP-c | NP-c |

Except for some trivial formulas like $xRx \wedge (xRx \Rightarrow \bot)$.

# Finite satisfiability

| Type of $\Phi$ | $\mathcal{K}_\Phi$-GFINSAT | $\mathcal{K}_\Phi$-FINSAT |
|---|---|---|
| S3+, S3– | FMP, NP-c | |
| S2+, S2– | NEXPTIME | |
| S1+ & "merges" | Lack of FMP (always!), PSPACE-c | FMP, PSPACE-c |
| S1+ & not "merges" | FMP, EXPTIME-c | FMP, PSPACE-c |
| S1– | FMP, PSPACE-c | FMP, NP-c |

# Finite vs. General

**J. Michaliszyn, J. Otop, P. Witkowski, Gandalf 2012**

- There is an undecidable logic that is finitely decidable
- There is a decidable logic that is finitely undecidable

# Transitiveness

- Transitive modalities are popular in practice:
- $F$, $G$ of LTL
- $B$, $D$, $L$ of HS logic
- $K_i$, $C_G$ of epistemic logic

**J. Michaliszyn, J. Otop, CSL 2013**

For any $\mathcal{K}$ of transitive frames definable by universal formulas, $\mathcal{K}$-SAT and $\mathcal{K}$-GSAT are decidable. The same holds for the finite satisfiability problem.

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- Better understanding
- Easy modifications
- Unified theory

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## The "metaproblem"

Input: A first-order formula $\Phi$ that defines a class of frames $\mathcal{K}$.
Question: Is $\mathcal{K}$-SAT decidable?

**Is the metaproblem decidable?**

# Thank you for your attention!

## Summary

- We study the **satisfiability** problem of **modal logic** over **first-order** definable classes of **frames**.
- In some cases the problem is **undecidable**.
- There are wide classes of formulas that lead to **decidable** problems (Horn formulas, transitive formulas, $\mathrm{FO}^2$, $GF$).
- Our goal: to classify them all.

**Open: Is the "metaproblem" decidable?**

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