Comparison of Several Periodic Operations of a Continuous Fermentation Process

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A continuous fermentation process is modeled numerically with five periodic input functions of substrate feed rate. The results show that, for the same average feed concentration of substrate, some of the periodic input functions can improve the performance of the process, giving increased productivities. The reduced ramp-wave function has been found to give the highest productivity.

Introduction

The forced periodic operation of continuous reactions can give superior time-averaged performance compared to that obtained under a conventional steady-state operation (Bailey, 1977). For bioreactor operation, under certain specific growth rate, the phenomena of input multiplicities can be observed. For instance, more than one value of input feed substrate concentration gives identical productivity due to the presence of competing effects in the process or due to recycle structure (Henson and Seborg, 1992; Liou and Chien, 1991).

Recently, the periodic square-wave operation of the feed substrate concentration of a continuous fermentation process has been modeled by Kumar et al. (1993). In this particular simulated process, the same productivities can be obtained at two different feed substrate concentrations and there exists one feed substrate concentration, $S_{\rm fc}$, which gives the maximum production rate. For periodic operation, it has been pointed out that an average substrate concentration of greater than S_{fc} gives a superior performance to that of a lower one. However, this does not mean that the periodic operation at this higher feed substrate concentration would always perform better than the steady-state operation. For example, under the same conditions predicted by Kumar et al. (1993), $S_f = 32.99$ g/L and $S_f = 15.55$ g/L both give a productivity of $Q = 3 \text{ g/(L} \cdot \text{h)}$ under the steady-state operation. Under the periodic operation (as shown in Figure 1), however, $\hat{S}_f = 32.99$ g/L gives a poorer productivity when used with an operating period of more than 10 h. Figure 1 shows the same result as that produced by Kumar et al. (1993), which is reproduced using the current simulation program.

To extend the work by Kumar et al. (1993), we have examined a series of interesting simple periodic functions numerically to investigate their performance in continuous fermentation.

Mathematical Model and Parameters

The following unstructured model of continuous culture fermentation (Agarwal et al., 1989) has been used in this study:

$$\frac{\mathrm{d}X}{\mathrm{d}t} = -DX + \mu X \tag{1}$$

$$\frac{\mathrm{d}S}{\mathrm{d}t} = D(S_{\mathrm{f}} - S) - \frac{\mu X}{Y_{\mathrm{x/s}}} \tag{2}$$

$$\frac{\mathrm{d}P}{\mathrm{d}t} = -DP + (\alpha\mu + \beta)X\tag{3}$$

where μ is the specific growth rate, $Y_{x/s}$ is the yield of

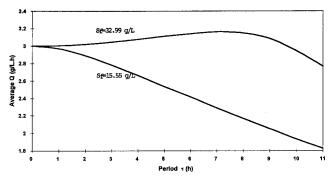


Figure 1. Average productivity versus period of oscillation under a rectangular pulse of S_t cycling (pulse width = 0.1).

Table 1. Nominal Fermenter Parameters Used in Current Work

param	nominal value	param	nominal value
$Y_{\mathrm{x/s}}$	0.4 g/g	$P_{ m m}$	50 g/L
α	$\begin{array}{c} 2.2 \ \text{g/g} \\ 0.2 \ \text{h}^{-1} \end{array}$	$K_{ m m}$	1.2 g/L
β	$0.2 h^{-1}$	$K_{ m i}$	22 g/L
$\mu_{ m m}$	$0.48 \ h^{-1}$		

cell mass, and α and β are yield parameters for the product. Because $Y_{x/s}$, α , and β are assumed to be independent of the operating conditions, eqs 1-3 are called as a "constant yield model". The specific growth rate equation is structured to allow both substrate and product inhibition:

$$\mu = \mu_{\rm m} [1 - (P/P_{\rm m})] S/[K_{\rm m} + S + (S^2/K_{\rm i})]$$
 (4)

This model contains four model parameters, i.e. the maximum specific growth rate $\mu_{\rm m}$, the product saturation constant $P_{\rm m}$, the substrate saturation constant $K_{\rm m}$, and the substrate inhibition constant $K_{\rm i}$. Many types of fermentation can be modeled by choosing these model parameters appropriately (Agarwal et al., 1989). For instance, if the product is growth associated, then one sets $\alpha \neq 0$ and $\beta = 0$. In order to compare with the published work by Kumar et al. (1993), the nominal model parameters and operating conditions used throughout this paper are listed in Table 1, which are identical to those in Kumar et al. (1993).

As can be seen, this model consists of five important variables, i.e. X (cell mass concentration, g/L), S (substrate concentration, g/L), P(product concentration, g/L), S_f (feed substrate concentration, g/L), and D (dilution rate, h^{-1}). If the quantities of cell mass and substrate are negligible in comparison to that of product, the productivity Q can be defined simply as the quantity of

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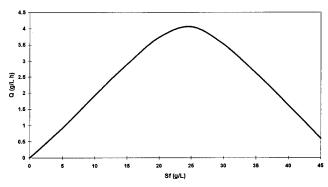


Figure 2. Productivity versus feed substrate concentration of steady-state operation ($D = 0.15 \text{ h}^{-1}$).

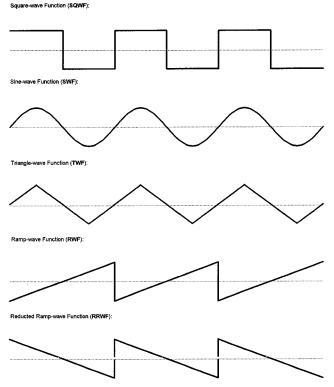


Figure 3. Five periodic functions to feed substrate concentration used in the current study (the dotted lines show the average feed substrate concentrations).

the cells produced per unit time:

$$Q = PD \tag{5}$$

When D is assumed to be $0.15~h^{-1}$ in the case suggested by Henson and Seborg (1992), the steady-state solution of eqs 1-5 can be calculated as shown in Figure 2. For simulations under periodic operation, initial values of three variables (X, S, P) of eqs 1-3 are assumed to be equal to the steady-state values at the same average $S_{\rm f}$. In this study, $S_{\rm f}$ is chosen as the input variable, i.e. the singular-input/singular-output system of periodic operation is considered, as the periodic operation with varying dilution rates (D) gives inferior performance compared to that of the steady-state operation according to Kumar et al. (1993).

Five different periodic functions have been applied in this study (as shown in Figure 3), i.e. (1) square-wave function (SQWF), (2) sine-wave function (SWF), (3) triangle-wave function (TWF), (4) ramp-wave function (RWF), and (5) reduced ramp-wave function (RRWF), where the dotted lines show the average feed substrate concentration. Equations 1–5 have been solved numerically by the improved-Euler method. Each of the average

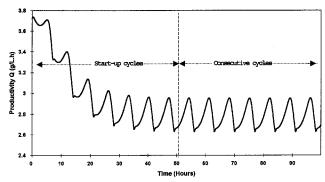


Figure 4. Productivity response to S_f square-wave input ($\tau = 7$ h, pulse width = 0.1, $S_f = 20$ g/L).

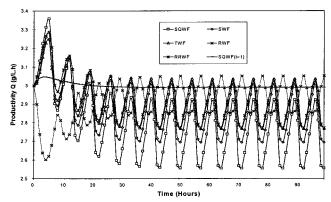


Figure 5. Productivity response to five periodic functions ($\tau = 7$ h, pulse width = 0.5, $S_f = 15.55$ g/L).

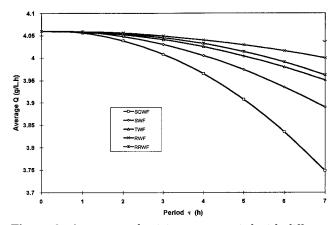


Figure 6. Average productivity versus period with different periodic inputs ($S_{\rm f}=24.296~g/L$).

productivities (*Q*) is calculated only after identical productivity profiles are obtained for consecutive cycles (see Figure 4). The typical operating period chosen was from 1 to 7 h; finite difference analysis has been carried out using a time increment of 0.01 h, which has been found to give adequate accuracy. Initial simulations were carried out to repeat all of the situations predicted by Kumar et al. (1993) to ensure the validity of the current computer program.

Results and Discussion

Start-up Cycles. Instead of the 5–10 cycles suggested by Kumar et al. (1993), in fact, the start-up times do not finish until the 40th–50th hour no matter what function types and how long the operating period (typically shown in Figure 5). In the following analysis, the average productivity for each function is calculated after the 50th hour of operation where a steady periodic response begins.

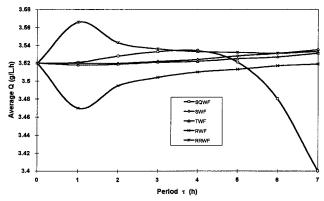


Figure 7. Average productivity versus period with different periodic inputs ($S_f = 30 \text{ g/L}$).

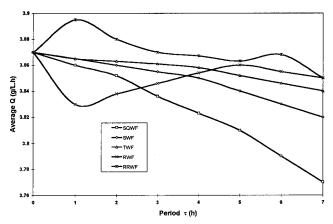


Figure 8. Average productivity versus period with different periodic inputs ($S_{\rm f}=27.5~{\rm g/L}$).

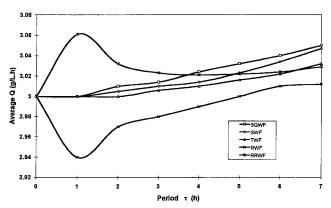


Figure 9. Average productivity versus period with different periodic inputs ($S_f = 32.99$ g/L).

Average Productivity. The calculated average productivity (Q) can be plotted against period τ . Figure 6 shows that an optimal average value of 24.296 g/L $S_{\rm f}$ gives a productivity of 4.06 g/(L·h) under steady-state operation. Under periodic operation, however, it always exhibits a deteriorating performance. Calculations also indicate that other types of periodic input functions can enhance productivity more than the steady-state operation if S_f value is sufficiently large. It has been found that $S_f = 30$ g/L is a good starting point beyond which improved productivities can be developed in all five periodic functions used (see Figure 7). In Figure 8, the periodic operation of average $S_f = 27.5$ g/L (a value between 24.296 and 30) exhibits poorer performance than steady-state operation in four periodic functions except the reduced ramp-wave function. When $S_f = 32.99 \text{ g/L}$

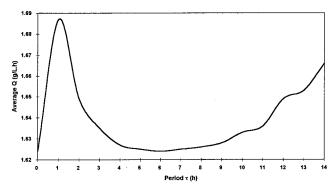


Figure 10. Average productivity versus period with reduced ramp-wave periodic input ($S_f = 40 \ g/L$).

is chosen as a typical operation, the reduced ramp-wave (RRWF) appears to be superior to the other four types of periodic functions while the ramp-wave (RWF) is inferior (refer to Figure 9). The average productivity of three symmetric periodic input functions (square, sine, and triangle wave) depends on their projection coverage areas. The larger the area, the higher the productivity. The square-wave (SQWF) operation is better than that of the sine and the triangle waves. Starting off from the point $S_{\rm f}=40$ g/L, the reduced ramp-wave (RRWF) operation gives consistently better performance than that of the steady-state operation at the same average $S_{\rm f}$ value in a wide range of operating periods ($\tau=0$ –14 h) as shown in Figure 10. The period of 1 h is seen to give the highest rise in average productivity.

Conclusions

The productivity of continuous flow fermentation can be increased by several periodic operations as long as the feed substrate concentration is higher than the value corresponding to the maximum steady-state production rate, according to the current predictions. Among the five feed substrate periodic functions, the reduced rampwave appears to give a higher productivity than any other periodic function over a wide range of the period of 0-14 h used in this study. The square, sine, and triangle-wave operation give similar trends, but the square-wave is better than the other two under the condition of small operating period.

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