Logic for Communicating Automata with Parameterized Topology

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27 janvier 2014

Introduction

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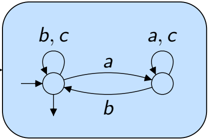

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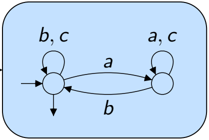
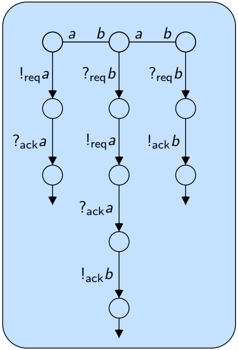


Has been extended to trees, graphs, weighted automata, ...

… and communicating automata (CA)



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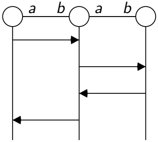

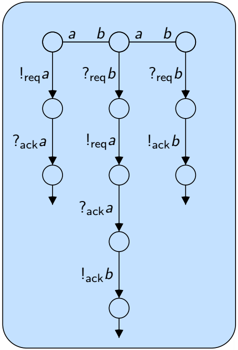

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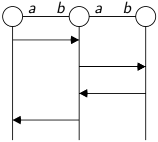

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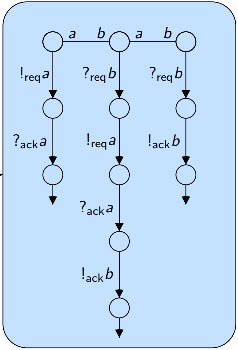

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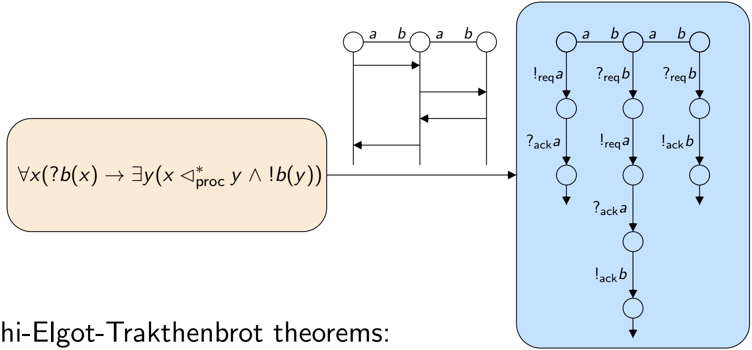

Büchi-Elgot-Trakthenbrot theorems:

- $\forall$-bounded channels [Henriksen-Mukund-Kumar-Sohoni-Thiagarajan 2000]
- $\forall$-bounded channels [Kuske 2002]
- $\exists$-bounded channels [Genest-Kuske-Muscholl 2004]
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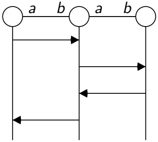

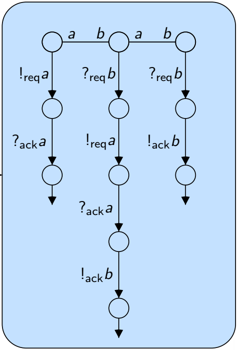

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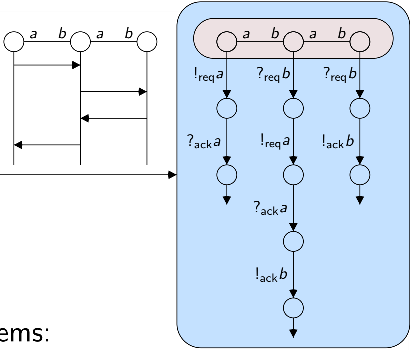

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Towards a parameterized version

Parameterized realizability

Let φ be a formula and $\mathfrak{T}$ be a class of topologies.

Is there a CA that is equivalent to φ on all topologies $\mathcal{T} \in \mathfrak{T}$?

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⇨ Need for new notions

- Topologies (of bounded degree)
- Parameterized communicating automata (PCA)

Outline

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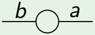
- Positive results:

  All formulas $\varphi \in \mathcal{C}$ are realizable for $\mathfrak{T}$.

Topologies and MSCs

Topologies

Topology



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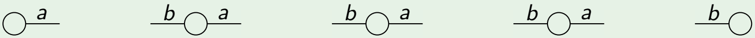

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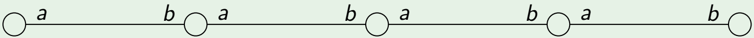

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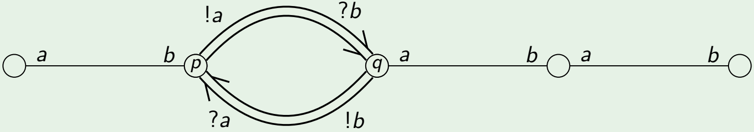
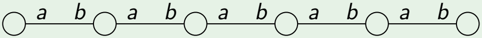


Topologies

Pipeline



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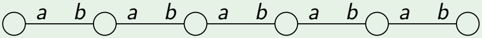
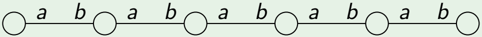


Fix finite set $\mathcal{N} = \{a, b, c, \ldots\}$ of interface names.

Topologies

Pipeline



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A topology over $\mathcal{N}$ is a pair $\mathcal{T} = (P, \longmapsto)$ where

- P is the nonempty finite set of processes
- $\longmapsto \subseteq P \times \mathcal{N} \times \mathcal{N} \times P$ is the edge relation

Topologies

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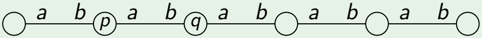

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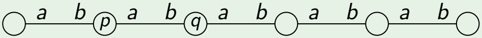

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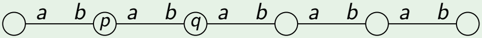

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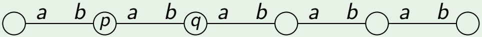

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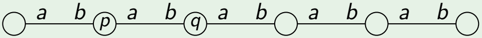

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Topologies

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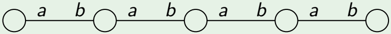

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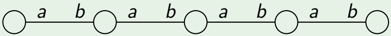
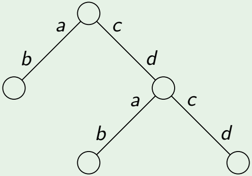
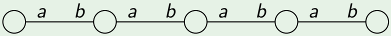


Tree

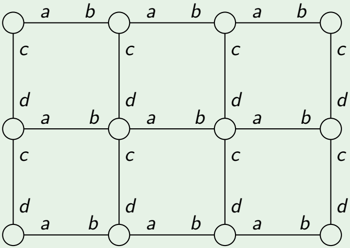


Topologies

Pipeline $\mathcal{T}_{\text{lin}}^{5}$



Grid $\mathcal{T}_{\text{grid}}^{3,4}$



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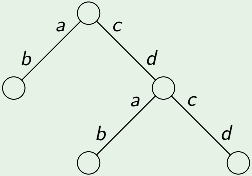

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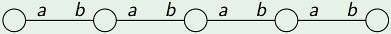

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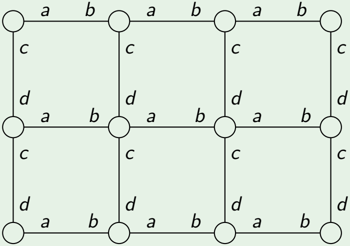

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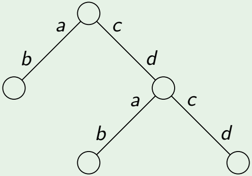
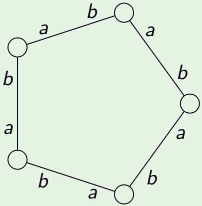
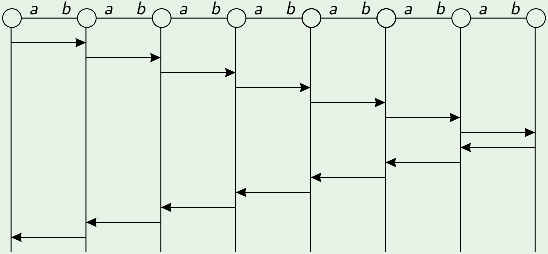


Ring $\mathcal{T}^5_{\mathrm{ring}}$



Message Sequence Charts (MSCs)

MSC



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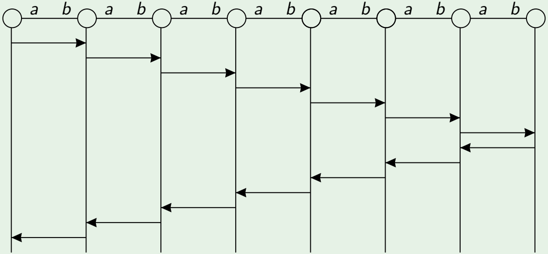

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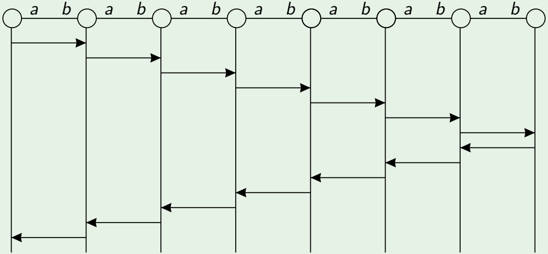

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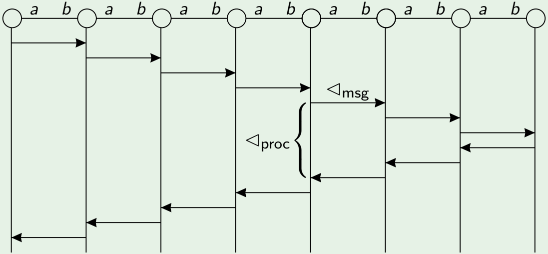

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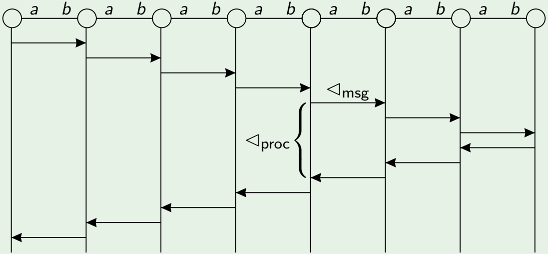

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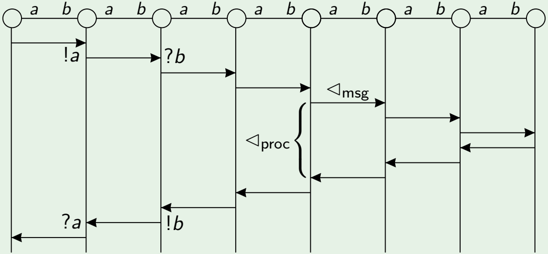

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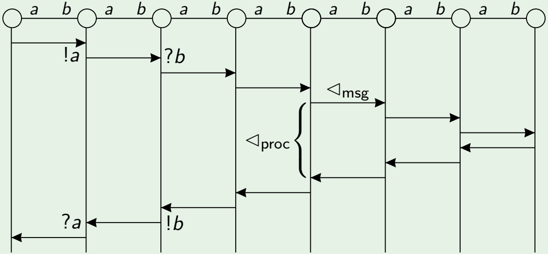

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$+$ some extra conditions

Parameterized Communicating Automata (PCA)

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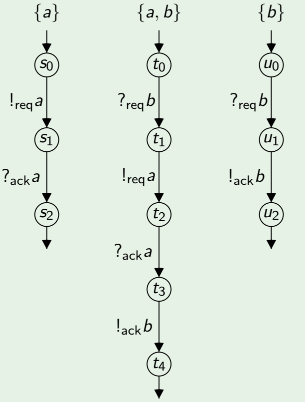
PCA $\mathcal{A}$ over $\{a, b\}$



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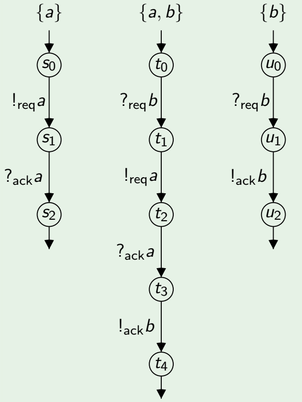
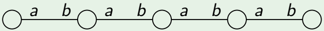


PCA $\mathcal{A}$ running on $\mathcal{T}^5_{\text{lin}}$



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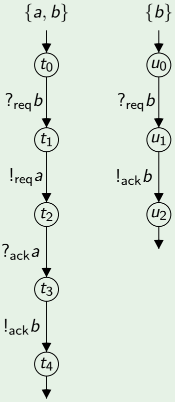

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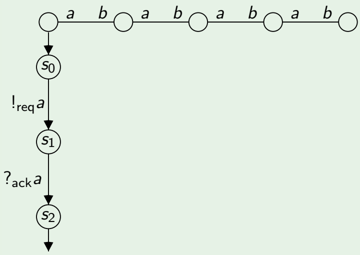

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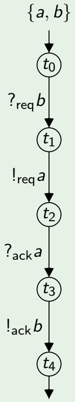

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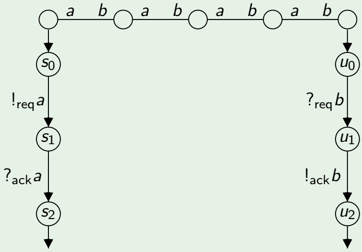
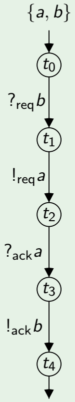


Parameterized communicating automata (PCA)

PCA $\mathcal{A}$ over $\{a, b\}$



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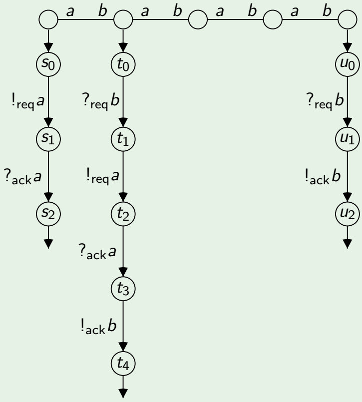

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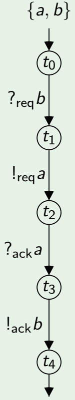

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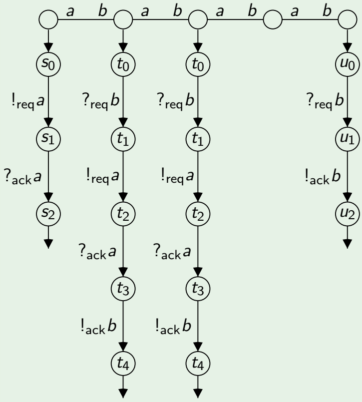

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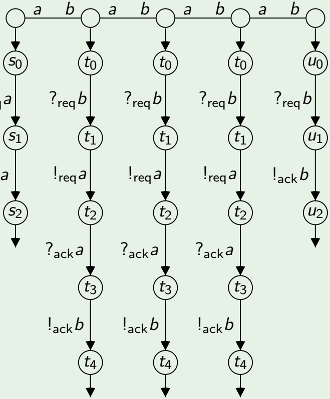
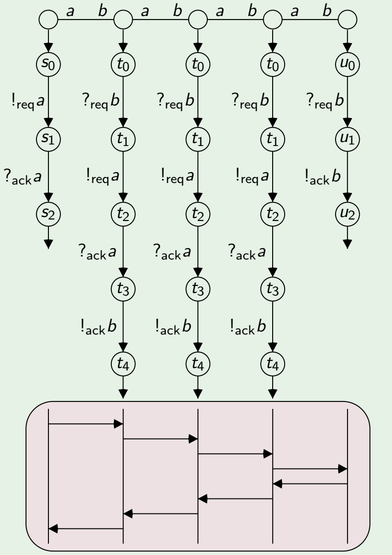


Parameterized communicating automata (PCA)

PCA $\mathcal{A}$ running on $\mathcal{T}_{\text{lin}}^5$



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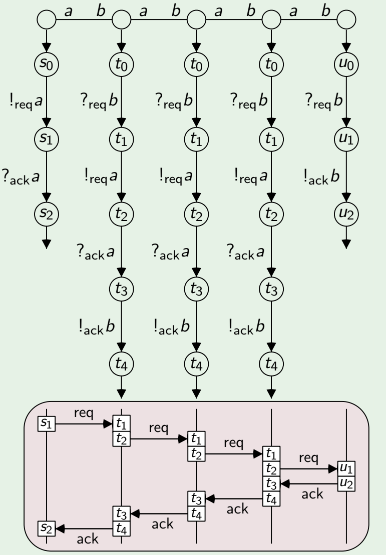
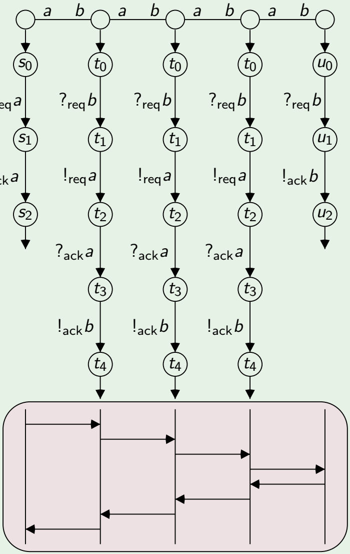


Parameterized communicating automata (PCA)

PCA $\mathcal{A}$ running on $\mathcal{T}_{\text{lin}}^5$

$M_{\text{lin}}^5 =$



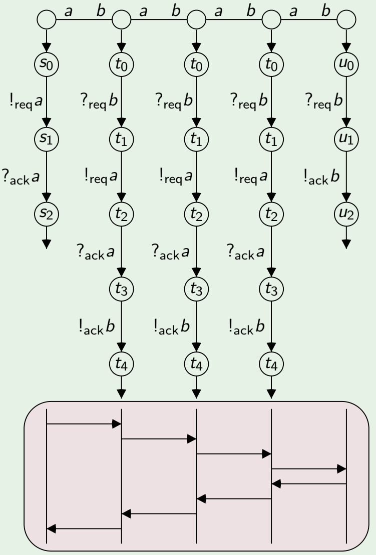
$\in L_{\mathcal{T}_{\text{lin}}^5}(\mathcal{A})$

Parameterized communicating automata (PCA)

Accepted language

$L_{\mathcal{T}^n_{\text{lin}}}(\mathcal{A}) = \{M^n_{\text{lin}}\}$
for all $n \geq 2$

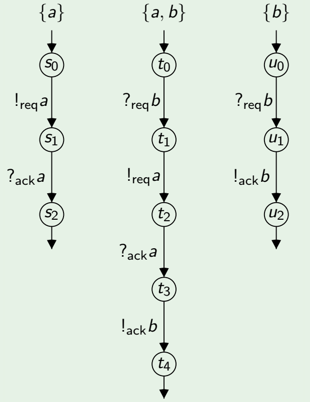
PCA $\mathcal{A}$ running on $\mathcal{T}^5_{\text{lin}}$



$M^5_{\text{lin}} =$ [diagram above] $\in L_{\mathcal{T}^5_{\text{lin}}}(\mathcal{A})$

Parameterized communicating automata (PCA)

PCA $\mathcal{A}$ over $\{a, b\}$



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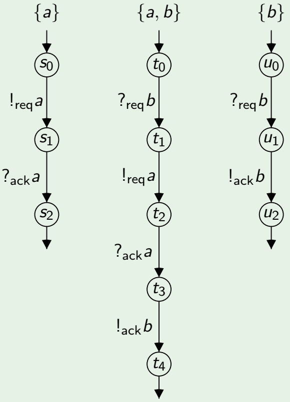

Definition

A PCA over $\mathcal{N}$ is a tuple (S, Msg, Δ, I, F):

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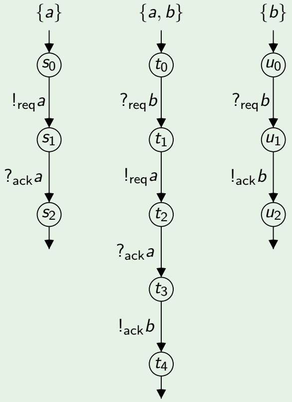

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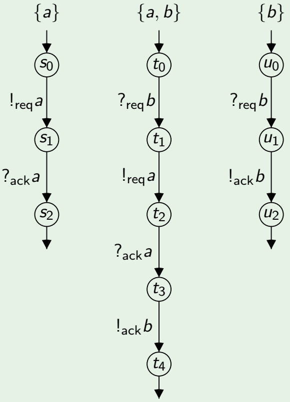

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Parameterized communicating automata (PCA)

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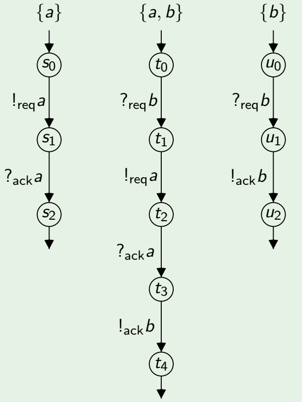

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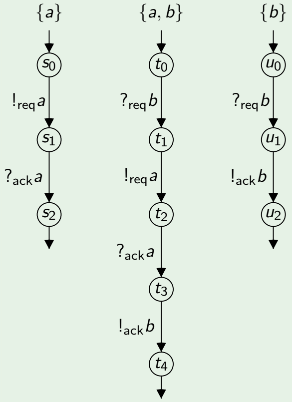

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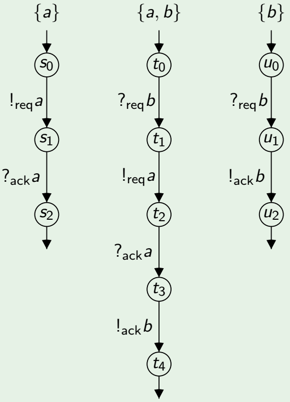

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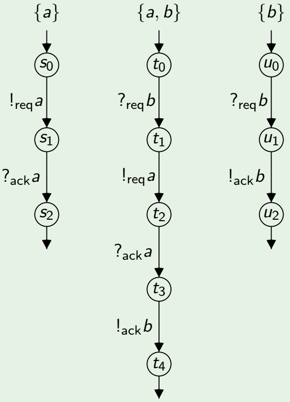

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$$F = \bigwedge_{s \in S \setminus \{s_2, t_4, u_2\}} \neg \langle \#(s) \geq 1 \rangle$$

Parameterized communicating automata (PCA)

✗ A PCA cannot say "the topology has at least 5 processes"

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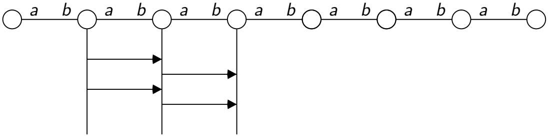

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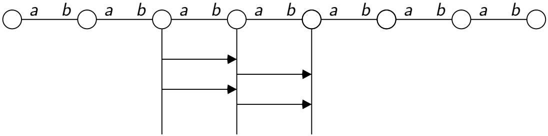

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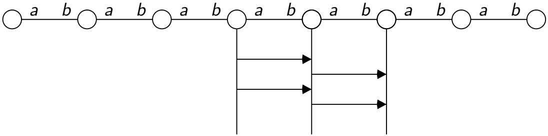

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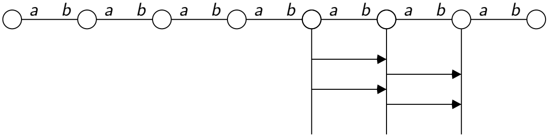

MSO Logic

MSO logic

MSO logic

$$\begin{aligned}\varphi ::= \quad & !a(x) \mid ?a(x) \mid a \in \mathit{type}(x) \mid \\ & x \lhd_{\mathrm{proc}} y \mid x \lhd_{\mathrm{proc}}^{*} y \mid x \lhd_{\mathrm{msg}} y \mid x \lhd^{*} y \mid x \sim y \mid \\ & x = y \mid x \in X \mid \neg\varphi \mid \varphi \vee \varphi \mid \exists x \varphi \mid \exists X \varphi\end{aligned}$$

where $a \in \mathcal{N}$

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Define fragments:

- FO: first-order logic, without $\exists X\varphi$

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MSO logic

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$$\sigma \subseteq \quad x \lhd_{\text{proc}} y \mid x \lhd^*_{\text{proc}} y \mid x \lhd_{\text{msg}} y \mid x \lhd^* y \mid x \sim y \mid$$

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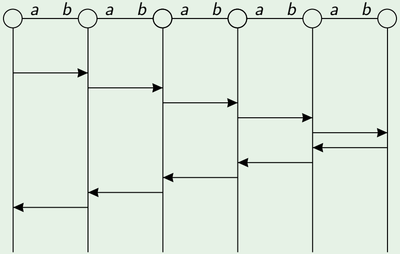
Let $L_{\mathcal{T}}(\varphi)$ be the set of MSCs over $\mathcal{T}$ that are a model of φ.

MSO logic

MSC M^6_{lin}



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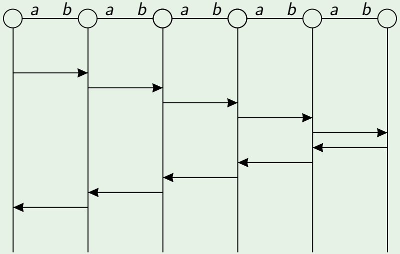

- $M^6_{\text{lin}} \models \forall x(?b(x) \;\rightarrow\; \exists y(x \lhd^*_{\text{proc}} y \wedge !b(y)))$

MSO logic

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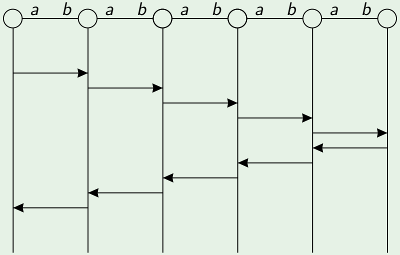

- $M^6_{\text{lin}} \models \forall x(?b(x) \;\rightarrow\; \exists y(x \lhd^*_{\text{proc}} y \wedge !b(y))) \in \text{FO}[\lhd^*_{\text{proc}}]$

MSO logic

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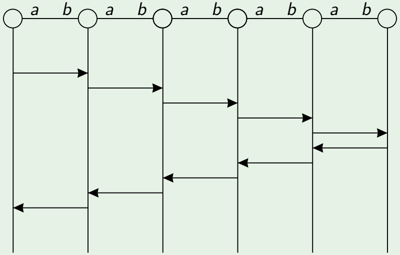

- $M^6_{\text{lin}} \models \forall x(?b(x) \ \rightarrow \ \exists y(x \lhd^*_{\text{proc}} y \wedge !b(y))) \in \text{FO}[\lhd^*_{\text{proc}}]$
- $M^6_{\text{lin}} \models \forall x \forall y(x \lhd^* y \vee y \lhd^* x)$

MSO logic

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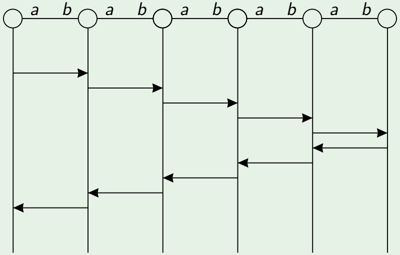

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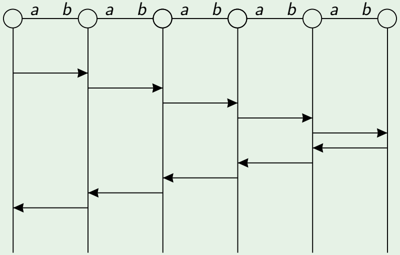
MSC M^6_{lin}



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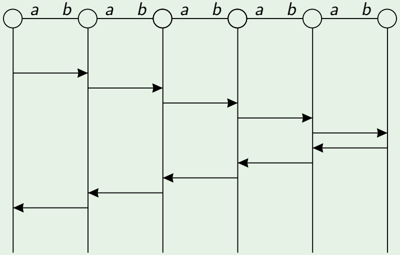

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- $M^n_{\text{lin}} \models \varphi$ iff $n = 2$

Positive results

Theorem

For every PCA $\mathcal{A}$, there is a formula $\varphi \in \mathsf{EMSO}[\lhd_{\mathsf{proc}}, \lhd_{\mathsf{msg}}]$ that is equivalent to $\mathcal{A}$ on all topologies.

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Proof

Standard. $\square$

Negative Results

Restrictions on topologies are necessary

Theorem

There exists a sentence $\varphi \in \mathsf{FO}[\lhd_{\mathsf{proc}}, \lhd_{\mathsf{msg}}]$ over $\{a, b\}$ such that, for all PCA $\mathcal{A}$, there is a ring forest $\mathcal{T}$ with $L_{\mathcal{T}}(\mathcal{A}) \neq L_{\mathcal{T}}(\varphi)$.

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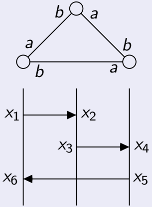
Proof

$$\varphi = \forall x \exists x_1, \ldots, x_6 (x \in \{x_1, \ldots, x_6\} \wedge \mathit{cycle}(x_1, \ldots, x_6))$$



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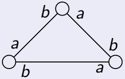

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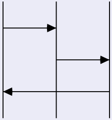

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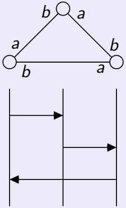
$M =$ 

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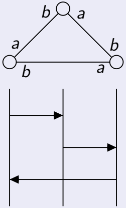
$M =$ 

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- We have $\underbrace{M \uplus \ldots \uplus M}_{n} \in L_{\mathcal{T}^3_{\text{ring}} \uplus \ldots \uplus \mathcal{T}^3_{\text{ring}}}(\mathcal{A})$ for all $n \geq 1$.

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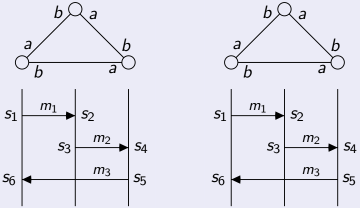
$M =$ 

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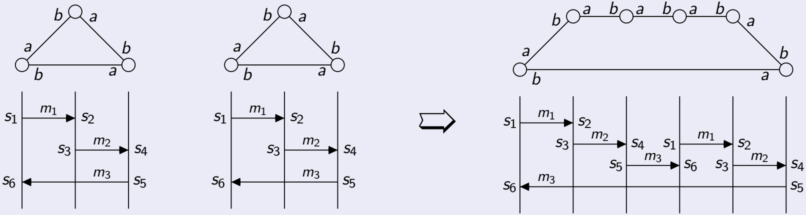
- We have $\underbrace{M \uplus \ldots \uplus M}_{n} \in L_{\mathcal{T}^3_{\text{ring}} \uplus \ldots \uplus \mathcal{T}^3_{\text{ring}}}(\mathcal{A})$ for all $n \geq 1$.
- For large enough n, there is a run of $\mathcal{A}$ that behaves the same on two copies of M.

Restrictions on topologies are necessary

$M =$ 

- Suppose there is $\mathcal{A}$ such that $L_{\mathcal{T}}(\mathcal{A}) = L_{\mathcal{T}}(\varphi)$ for all ring forests $\mathcal{T}$.
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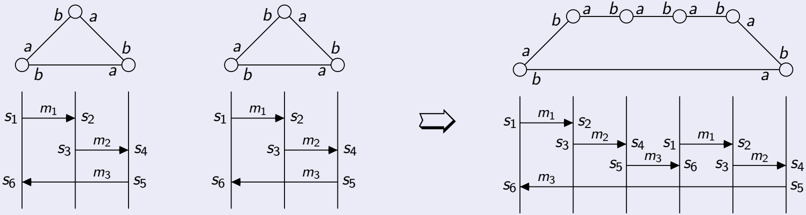
$M =$ 

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Lesson learned

PCA have limited ability to "detect" cycles.

Restrictions on logic are necessary

Theorem

There exists a sentence $\varphi \in \mathrm{FO}[\lhd^*_{\mathrm{proc}}, \lhd_{\mathrm{msg}}, \lhd^*]$ over $\{a, b, c, d\}$ such that, for all PCA $\mathcal{A}$, there is a tree $\mathcal{T}$ with $L_{\mathcal{T}}(\mathcal{A}) \neq L_{\mathcal{T}}(\varphi)$.

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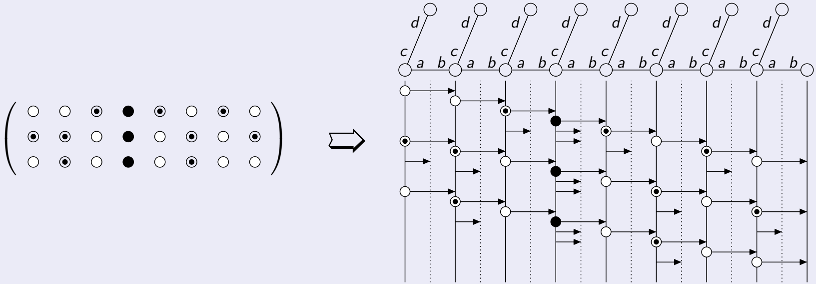

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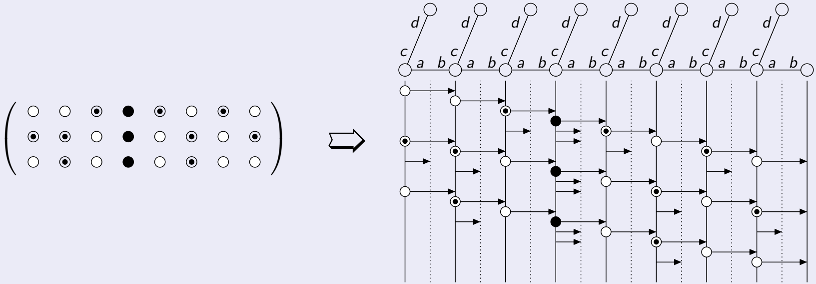

Lesson learned

Look at more "local" logics.

Positive Results

Locality of FO logic

Theorem [Schwentick-Barthelmann 1999]

Every formula $\varphi \in \mathsf{FO}[\sigma]$ is equivalent to a formula of the form $\exists x_1 \ldots \exists x_n \forall y \psi \in \mathsf{FO}[\sigma]$ where ψ is *r*-local around y, for some $r \geq 1$ (quantification is restricted to elements of distance $\leq r$ from y).

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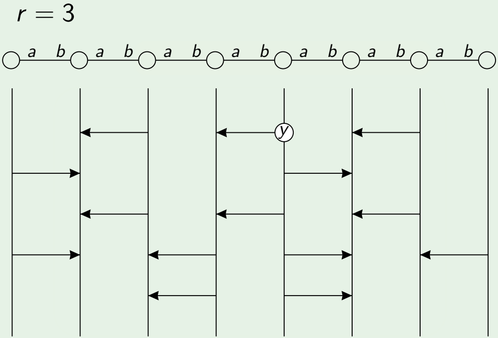

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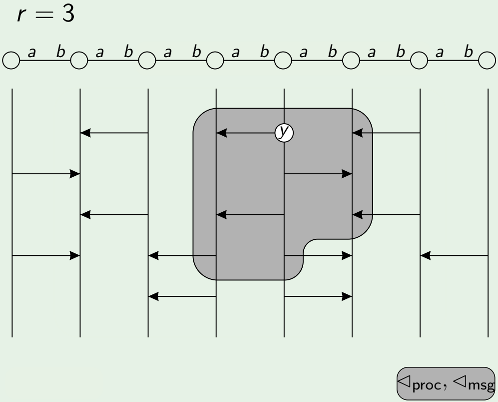

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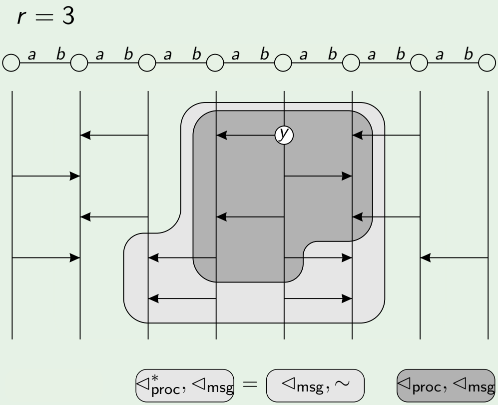

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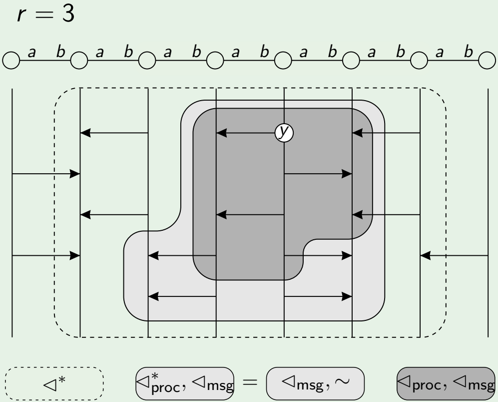

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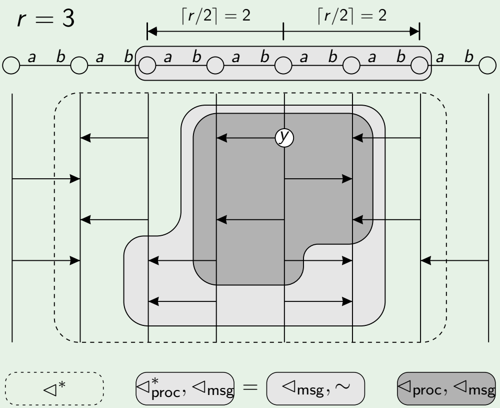

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Let $\varphi \in \text{EMSO}[\lhd^*_{\text{proc}}, \lhd_{\text{msg}}]$, $B \geq 1$, and $\mathfrak{T}$ be a $(r_\varphi + 2)$-unambiguous set of topologies. There is a PCA $\mathcal{A}$ such that, for all $\mathcal{T} \in \mathfrak{T}$, we have $L^B_{\mathcal{T}}(\mathcal{A}) = L^B_{\mathcal{T}}(\varphi)$.

Here, r_φ is the radius associated with the first-order kernel of φ.

Unambiguous topology classes

Definition

Let $k \in \mathbb{N}$. A class $\mathfrak{T}$ of topologies is k-unambiguous if, for all $w \in (\mathcal{N} \times \mathcal{N})^*$ with $|w| \leq k$, all $(P, \longmapsto), (P', \longmapsto') \in \mathfrak{T}$, and all processes $p, q \in P$ and $p', q' \in P'$ such that $p \overset{w}{\longmapsto} q$ and $p' \overset{w}{\longmapsto}' q'$, we have $p = q$ iff $p' = q'$.

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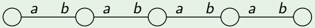
In other words:

If w forms a cycle in a topology from $\mathfrak{T}$, then it forms a cycle anywhere, in any topology of $\mathfrak{T}$ (if it is applicable).

Unambiguous topology classes

Pipelines ✔ k-unambiguous for all $k \in \mathbb{N}$



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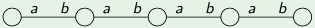
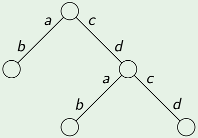


Trees ✔ k-unambiguous for all $k \in \mathbb{N}$



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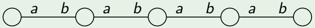
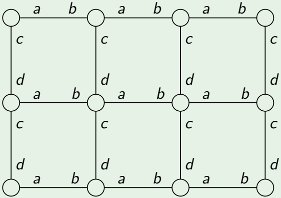


Grids ✔ k-unambiguous for all $k \in \mathbb{N}$



$w = (a, b)(c, d)(b, a)(d, c)$

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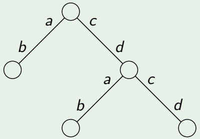

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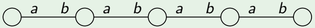

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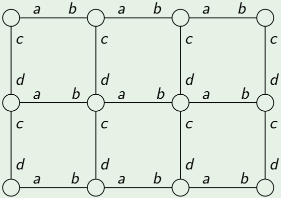

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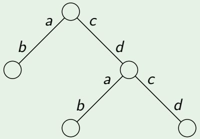
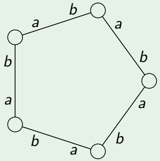


Rings ✗ not k-unambiguous for all $k \geq 3$



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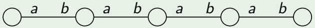

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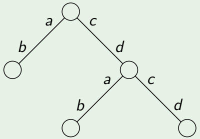

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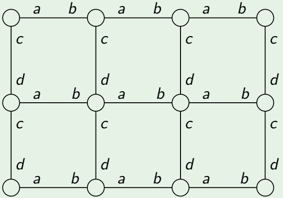

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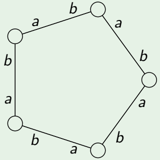

But:

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Unambiguous topology classes

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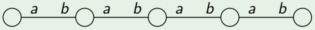

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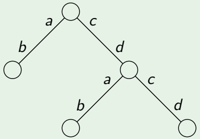

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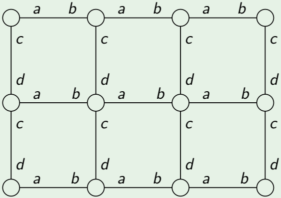

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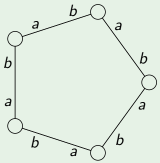

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- Every single ring is k-unambiguous, for all $k \in \mathbb{N}$.

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Proof

- Translate φ into normal form

$$\exists X_1 \ldots \exists X_m \exists x_1 \ldots \exists x_n \forall y \psi$$

where ψ is r_φ-local around y.

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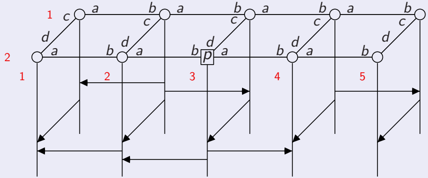
- Glue fixed-topology CA together to obtain a PCA for $\forall y \psi$.

Unambiguous topology classes

Proof (cntd.) suppose $r_\varphi = 3$ so that $\lceil r_\varphi/2 \rceil = 2$



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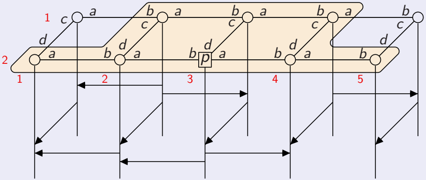

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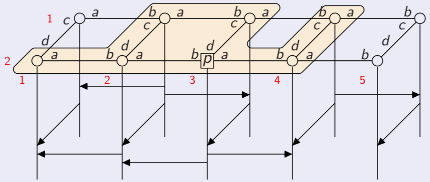

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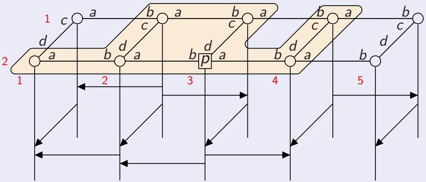

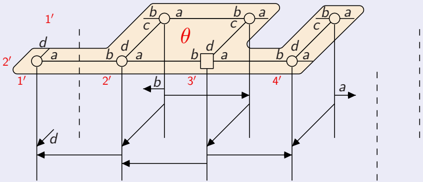

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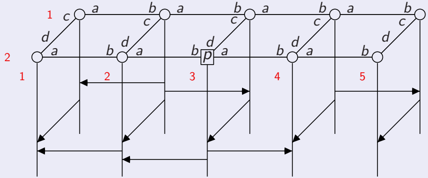

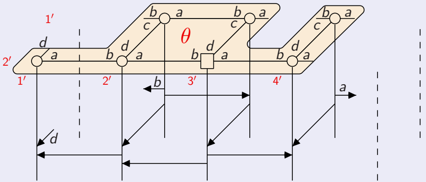

- Process $p = (2, 3)$ guesses topology neighborhood θ and simulates local automaton $\mathcal{A}_\theta[2', 3']$.

Unambiguous topology classes

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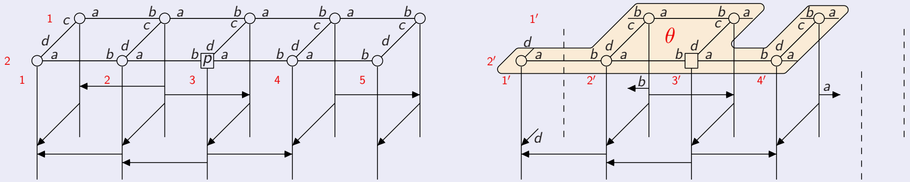

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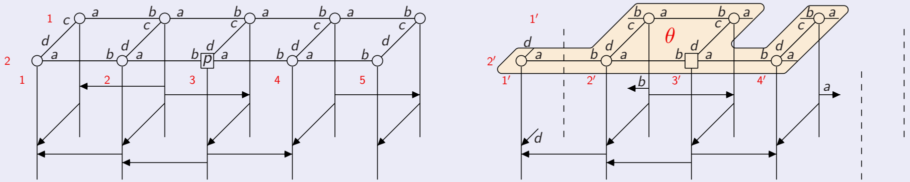

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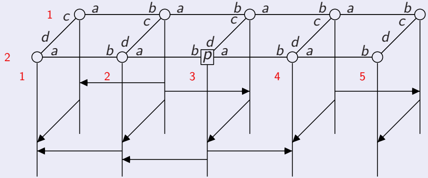

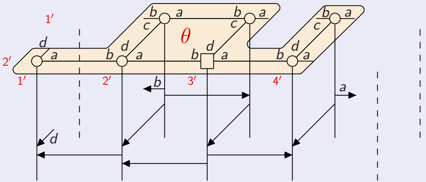

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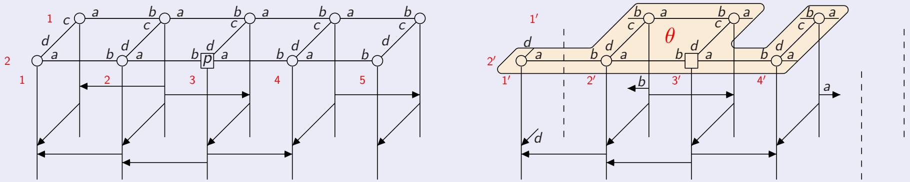

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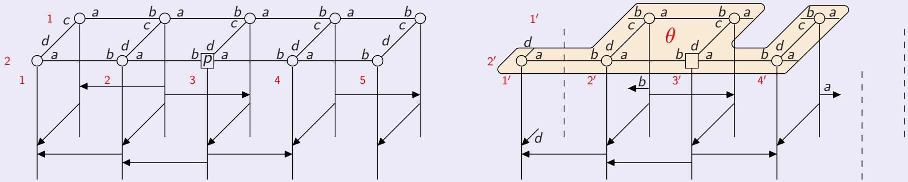

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- Every process has to simulate several automata.

Unambiguous topology classes

Almost the same proof works for a weaker logic without channel bound:

Theorem

Let $\varphi \in \mathrm{EMSO}[\lhd_{\mathsf{proc}}, \lhd_{\mathsf{msg}}, \sim]$, and $\mathfrak{T}$ be a $(r_\varphi + 2)$-unambiguous set of topologies. There is a PCA $\mathcal{A}$ such that, for all $\mathcal{T} \in \mathfrak{T}$, $L_{\mathcal{T}}(\mathcal{A}) = L_{\mathcal{T}}(\varphi)$.

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- Translate φ into normal form

$$\exists X_1 \ldots \exists X_m \exists x_1 \ldots \exists x_n \forall y \psi$$

  where ψ is r_φ-local around y.
- Existential quantification $\Rightarrow$ projection & guessing of truth values for propositions involving only free variables of $\forall y \psi$ [Gastin-Kuske 2010].
- Construct fixed-topology CA $\mathcal{A}_\theta$ for formula $\forall y \psi$ over all topology neighborhoods θ of radius $\lceil r_\varphi / 2 \rceil$ [Genest-Kuske-Muscholl 2004].
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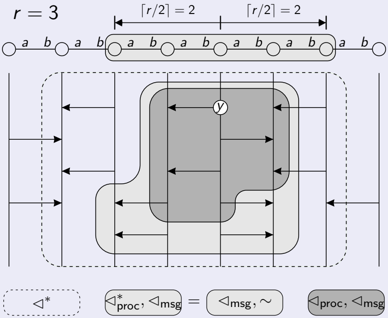

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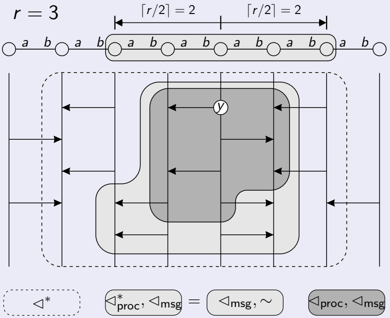

Exploit sphere automaton from [B.-Leucker] to compute $\{\lhd_{\text{proc}}, \lhd_{\text{msg}}\}$-neighborhoods.

Summary of results

Negative results

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Positive results

- Under a channel bound, every FO[$\lessdot^*_{\mathsf{proc}}, \lessdot_{\mathsf{msg}}$]-formula is realizable for the classes of pipelines, trees, grids, and rings.
- Every FO[$\lessdot_{\mathsf{proc}}, \lessdot_{\mathsf{msg}}, \sim$]-formula is realizable for the classes of pipelines, trees, grids, and rings.

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Positive results

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- Every $\mathrm{FO}[\lhd_{\mathrm{proc}}, \lhd_{\mathrm{msg}}, \sim]$-formula is realizable for the classes of pipelines, trees, grids, and rings.

Open problems

- Is every $\mathrm{FO}[\lhd^*_{\mathrm{proc}}, \lhd_{\mathrm{msg}}]$-formula realizable without channel bound?
- Is every $\mathrm{FO}[\lhd^*]$-formula realizable (for interesting classes of topologies)?

Related work

- Parameterized synthesis [Jacobs-Bloem 2012]
- Parameterized verification [Browne-Clarke-Grumberg 1989], [Emerson-Namjoshi 2003], [Bouajjani-Habermehl-Vojnar 2008], [Delzanno-Sangnier-Zavattaro 2010]
- Distributed algorithms [Grumbach-Wu 2010], [Chalopin-Das-Kosowski 2010]
- Automata from normal forms [Schwentick-Barthelmann 1999], [Gastin-Kuske 2010]

Conclusion

Contribution

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Future work

- Topologies of unbounded degree (unranked trees, star architectures)
- Parameterized verification

Thank You!