Stochastic Optimization of Grid to Vehicle Frequency Regulation Capacity Bids

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***Abstract*—This paper investigates the application of stochastic dynamic programming to the optimization of charging and frequency regulation capacity bids of an electric vehicle (EV) in a smart electric grid environment. We formulate a Markov decision problem to minimize an EV's expected cost over a fixed charging horizon. We account for both Markov random prices and a Markov random regulation signal. We also propose an enhancement to the classical discrete stochastic dynamic programming method. This enhancement allows optimization over a continuous space of decision variables via linear programming at each state. Simple stochastic process models are built from real data and used to simulate the implementation of the proposed method. The proposed method is shown to outperform deterministic model predictive control in terms of average EV charging cost.**

***Index Terms*—Approximation algorithms, dynamic programming, electric vehicles, frequency regulation, linear programming, Markov decision problem (MDP), smart grid, stochastic optimization, vehicle-to-grid (V2G).**

I. Introduction

In ORDER TO reduce harmful emissions from electric power generation, significant amounts of wind and solar power are being installed in many countries. The power supplied by these renewable energy resources is intermittent and cannot be predicted perfectly, creating new challenges in safely and efficiently operating electric power grids. In electric power grids where a high percentage of power is produced by wind and solar, the Independent System Operator (ISO) must procure additional ancillary services capacity to compensate for production forecast errors. Additional ancillary services can be provided by keeping more generation capacity online and idling. Alternatively, adjustment of flexible power consumption could provide ancillary services without fuel costs or emissions.

Electric vehicles (EVs) are a flexible load that could be controlled in order to provide ancillary services to the electric grid. Consumers may begin to purchase EVs in large numbers in the near future. EV drivers want to drive when and where they desire without waiting for the battery to charge, but would also like to charge their batteries for minimum cost. In liberalized electric power systems, such as PJM, ancillary services are procured through a market. If an EV provides ancillary services to the electric grid, the EV owner would earn revenue at the market price, offsetting the cost of charging.

The ancillary service that EVs are best suited to provide is secondary frequency regulation, also simply referred to as regulation. An EV could effectively provide regulation service without discharging into the electric grid by committing to a baseline charge rate and a capacity for regulation before the start of a regulation service period, usually lasting one hour. For the duration of the contracted period, the EV would receive an automatic generation control (AGC) signal, which is broadcast by the ISO. The EV would then vary its charging power according to the AGC signal and its commitments. This scheme is known as grid to vehicle (G2V) regulation. If the EV also discharges into the grid, the scheme would be called vehicle to grid (V2G) regulation.

Recent literature has investigated the possible system-wide benefits of V2G [1], [2]. The feasibility of the concept has also been demonstrated in hardware [3]. Other work has focused on implementable methods for optimizing EV charging and regulation capacity bids. Some research has focused on the charging and regulation capacity bids of an EV aggregator. Reference [4] investigates various optimization models to be used by an EV aggregator. The models are formulated as deterministic model predictive control (MPC) problems and are solved using linear programming. The problems incorporate constraints that might be imposed on the aggregate consumption of a fleet of EVs. Reference [5] also takes a deterministic MPC approach to the V2G EV aggregator problem.

An EV providing ancillary services in a deregulated market environment faces many forms of uncertainty. Market prices for energy and regulation service as well as the AGC signal are unknown before an EV would commit to providing regulation for some contract period. Providing regulation service would make the battery's future state of charge (SOC) uncertain. This motivates the use of stochastic optimization methods when determining an EV's baseline charge rate and capacity for regulation. Stochastic optimization, which directly considers parameter uncertainty, results in lower expected realized costs than deterministic methods, which simply optimize using the expected values of parameters [7]. Optimization and control under uncertainty is a mature field with rich theory and various applications as introduced in [6]–[8] as well as many other references.

Recently, attempts have been made to apply stochastic optimization methods to V2G and G2V charging problems for single EVs. [9] describes the V2G problem as a Markov decision

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problem (MDP) with uncertain prices and a small set of available control actions. Q-learning is used to create a control policy in an online and model-free way. It is not clear if this method would outperform other methods in the literature or how long it would take to train good control policies in the real world. [10] formulates a G2V MDP for a single EV, and also provides an approximate solution method based on mixed integer linear programming and stochastic dynamic programming (SDP). The model presented in [10] directly considers that the integrated energy of the AGC signal has a distribution of possible outcomes. Some advantages of a stochastic approach over a deterministic dynamic programming approach are demonstrated in [10].

This paper extends the existing literature by formulating the G2V charging problem for a single EV as an MDP with multiple sources of uncertainty. The work presented here could easily be modified for the V2G situation. We optimize EV charging and regulation capacity bids assuming that the hourly price of energy, the hourly price of regulation service, and the integrated hourly energy of the AGC signal follow Markov random processes. It is important to account for Markovian dependence when optimizing decisions. For instance, if the most recently observed price is unusually high, a good charging strategy should wait until the price spike subsides before purchasing more energy. Additionally, we introduce an intuitive approximate SDP method which allows for optimization over a continuous space of charging and regulation capacity bids. This approximate SDP method is applicable to any convex sequential decision making problem under uncertainty. A parallel implementation of the proposed method is described. Finally, we demonstrate that the proposed method results in lower expected charging costs than the deterministic MPC method. Although the proposed method is more computationally demanding than the deterministic MPC method, we show that it could still be practical to use for hourly decision making.

In this paragraph we present an outline of the remainder of the paper. In the next section we formulate the G2V problem as an MDP with multiple sources of uncertainty. In Section III, we briefly review the available approaches for solving multi-stage stochastic optimization problems. This is followed by the development of the proposed approximate SDP approach in Section IV. In order to simulate the implementation of the proposed method, we analyze real data and construct models of the random processes of the prices and AGC signal in Section V. In Section VI we describe our simulation experiments and interpret our results. Section VII concludes the paper with a summary of the main results.

II. Problem Formulation

In this section, we detail the presumed sequence of events and other assumptions which lead to our G2V MDP formulation. In liberalized energy markets, generators and loads determine the amount of regulation capacity which they are willing to provide. It is the responsibility of the local electrical grid ISO to ensure that an adequate amount of regulation capacity has been procured. The ISO must also determine an AGC signal which maintains the system's alternating current frequency. The resources that have committed to providing regulation capacity are then obligated to respond to the ISO's AGC signal. In this paper we analyze a decision optimization problem from the perspective of an EV as a market participant. As in the related works [4], [5], [9] and [10], we do not concern ourselves with determining the total system capacity for regulation, determining the appropriate AGC signal, or verifying that the system frequency is maintained.

A. Decision Making Scenario

Our goal is to minimize the charging costs for a single EV owner who is able to provide G2V service. We assume that an EV bids as a price taker in both the energy and ancillary services markets. We assume that individual EVs are either operating in a smart electric grid where they can bid directly into markets, or that they communicate their bids to a 3rd party aggregator, who then bids the sum of many EVs' decisions into the markets. This aggregation step would be necessary to provide G2V service in today's markets because an individual EV does not consume enough power to participate in today's markets directly. The focus of this work is a decision making method for a single EV, and not on aggregation or system level effects.

We are assuming that EVs make decisions without coordination. There are situations that would warrant the coordination of charging and regulation capacity bids, such as low voltage in distribution networks or market power. In this work, we assume that the disadvantages of uncoordinated charging in comparison with coordinated charging are negligible. It is important to note that the act of maintaining the system frequency is still coordinated by the ISO through the AGC signal once regulation capacity commitments have been made.

We assume that when an EV is plugged in at home or work the EV driver communicates a planned unplugging time to a smart EV charger. We assume that the EV driver agrees to not unplug the EV before this scheduled time, or else he might face some financial penalty. At the beginning of each hour, h, while the EV is plugged in, the smart charger decides on and commits to a baseline charge rate, P_h(kW), and capacity for regulation, B_h (kW), before knowing the true energy price, regulation service price, or AGC signal. The smart EV charger should choose P_h and B_h in such a way that it minimizes the total expected cost to charge the EV. In the following subsections we will develop the MDP that a smart charger must solve in order to determine the best bids, P_h and B_h, to commit to at each hourly decision epoch.

B. Decision Constraints

The baseline charge rate and regulation capacity bids, P_h and B_h, are the decision variables available for the smart charger to optimize. These decision variables are subject to constraints based on the physical design of the smart charger and the battery SOC. Over the course of the hour, the instantaneous charge rate may take any value between $P_h + B_h$ and $P_h - B_h$, as will be explained in the next subsection. Since we are analyzing a G2V scenario where the EV does not discharge into the grid, the decisions should not allow the possibility of a negative charge rate, giving constraints (1)–(3). The choice of P_h and B_h is also constrained by the maximum charge rate of the EV and smart charger, P_{max} (4). If the battery SOC, E_h, were to reach maximum SOC, E_{max}, before the end of the hour, the EV would stop charging and would not be able to modify its charge rate in a way that effectively provides regulation service to the grid. This would violate the contract to provide regulation service for the whole hour. We add constraint (5) so that it is impossible to violate the regulation service contract. Δ_h is the time-step length of 1 hr.

Some EV drivers are afraid of their batteries running out of energy and being stranded. This fear is known as "range anxiety." Because of range anxiety, we will assume that the EV driver requires the battery SOC to be $E_{\max}$ by the scheduled unplugging time $h = H$. In order to guarantee that the EV will be fully charged by the known unplugging time, we add constraint (6). Here, $E_{\min,h+1}$ is the minimum SOC needed at the start of the next hour to ensure that E_H will be able to reach $E_{\max}$. $E_{\min,h+1}$ can be calculated for each hour based on $P_{\max}$ and the number of hours remaining until the EV unplugs.

$$0 \leq B_h \tag{1}$$
$$0 \leq P_h \tag{2}$$
$$B_h \leq P_h \tag{3}$$
$$B_h + P_h \leq P_{\max} \tag{4}$$
$$E_h + \Delta_h B_h + \Delta_h P_h \leq E_{\max} \tag{5}$$
$$E_h + \Delta_h P_h - \Delta_h B_h \geq E_{\min,h+1} \tag{6}$$

C. Battery SOC Dynamics

In this section we describe how decision variables P_h and B_h and the AGC signal determine the dynamics of the battery SOC. We assume that the ISO knows the total capacity for regulation in the system and computes an AGC signal for restoring the electric grid's AC frequency to nominal. The AGC signal can then be normalized to the total regulation capacity and broadcast to all participating generators or flexible loads. When a generator or load specifies a regulation capacity, it specifies how much it is willing to vary its power generation or consumption. A generator would then change its power output by the normalized AGC signal multiplied by its agreed upon regulation capacity. Loads would respond to the negative of the normalized AGC signal, which is intended for generators.

Given the hourly baseline charge rate P_h and the hourly regulation capacity B_h, the charge rate at any time within the hour is given by $\boldsymbol{P}_t$ as determined in (7), where $\boldsymbol{R}_t$ (unitless) is the latest normalized AGC signal broadcast by the ISO. In this paper, bold letters will indicate random variables. When random variables are shown without bolding, this indicates observations of the random variable that are known with certainty.

$$\boldsymbol{P}_t = P_h - \boldsymbol{R}_t B_h \tag{7}$$

On an hourly basis, the battery SOC evolves according to (8), which is referred to as the state transition function. $\boldsymbol{R}_h$ (hrs) is the hourly time integral of the normalized AGC signal, $\boldsymbol{R}_t$, which is uncertain when the decisions are made. Realizations of $\boldsymbol{R}_h$ can take on any value between -1 and 1 with some probability distribution. This makes the next battery SOC, $\boldsymbol{E}_{h+1}$, uncertain with a distribution that depends on our decisions as well as the probability distribution of the AGC signal.

$$\boldsymbol{E}_{h+1} = E_h + \Delta_h P_h - \boldsymbol{R}_h B_h \tag{8}$$

D. Exogenous State Dynamics

The exogenous state vector $\boldsymbol{S}_h = (\boldsymbol{R}_h, \boldsymbol{r}_h, \boldsymbol{c}_h)$, is composed of the hourly integral of the normalized AGC signal $\boldsymbol{R}_h$, the hourly price of regulation capacity $\boldsymbol{r}_h$ (\$/MW), and the hourly price of energy $\boldsymbol{c}_h$ (\$/MWh). We refer to these random variables as state variables because we model them as following Markov random processes with distributions as in (9).

$$\boldsymbol{S}_h \sim f_h(S_{h-1}) \tag{9}$$

Hourly electric energy prices and some AGC signals are known to have a statistical dependence on recently observed values [11], [12]. A Markov model is simple way to model this dependence. In reality, electric energy prices depend on a variety of seasonal and exogenous factors not included in our state vector. Our problem has a finite horizon on the scale of half a day, so the state distributions can be estimated conditionally given the state of slower seasonal factors. Although we are likely ignoring other useful exogenous information, we assume that the state captures the majority of the information that would be useful in predicting the next hourly prices and AGC signal.

E. Cost Functions

Because the SOC is guaranteed to be $E_{\max}$ when $h = H$, there is no cost incurred at the unplugging time. One hour before the EV is scheduled to unplug, epoch $h = H - 1$, providing any amount of regulation service would result in the expected final SOC being less than $E_{\max}$. To guarantee the EV reaches a full SOC, the only action available is to finish charging the battery. This gives the terminal cost function (10), where $\boldsymbol{c}_{H-1}$ (\$/kWh) is the random energy price.

$$g_{H-1}(E_{H-1}, \boldsymbol{c}_{H-1}) = \boldsymbol{c}_{H-1}(E_{\max} - E_{H-1}) \tag{10}$$

For earlier hours, when the EV will not unplug at the end of the hour, the cost function of our decisions is given by (11). The first term of the cost function comes from the fact that we pay for energy charged into the battery. The actual amount of energy charged into the battery will depend on the realization of the AGC signal and our decisions. The EV driver also earns money for providing regulation service, giving the second term.

$$g_h(\boldsymbol{S}_h, P_h, B_h) = \boldsymbol{c}_h(\Delta_h P_h - \boldsymbol{R}_h B_h) - \boldsymbol{r}_h B_h \tag{11}$$

F. Markov Decision Problem

The minimum expected cost of charging an EV that can provide G2V service, given some initial battery SOC, E_1, and the previous observation of the exogenous state vector, S_0, is denoted as $J_1(E_1, S_0)$. $J_h(E_h, S_{h-1})$, commonly called the "cost to go," is the expected total future cost of having battery SOC E_h, having observed the last exogenous state S_{h-1}, and making optimal decisions for the remainder of the problem horizon. The minimum expected cost for our system with state transition equations (8) and (9) is the result of solving the finite horizon MDP given below as Problem 1. $\mathbb{E}^{\pi}$ is the expectation over the continuous joint distribution of the exogenous random processes when using decision making policy π. A policy π is a set of hourly decision functions which take in the current state and return a decision in the feasible region given by constraints (1)–(6).

Problem 1:

$$J_1(E_1, S_0) = \min_{\pi} \mathbb{E}^{\pi} \left\{ g_{H-1}(\boldsymbol{E}_{H-1}, \boldsymbol{S}_{H-2}) + \sum_{h=1}^{H-2} g_h(\boldsymbol{S}_h, P_h, B_h) \middle| S_0, E_1 \right\}$$

III. Possible Solution Approaches

Problem 1 is the true problem that we would like to solve. In reality, Problem 1 involves continuous random variables and a continuous space of available decisions given by (1)–(6). Such problems are very difficult to solve or even approximate. Existing solution approaches are examined in this section. Ultimately, each examined method leaves something to be desired, so we develop a new heuristic approach in the next section.

A simple sub-optimal approach to decision making is to approximate Problem 1 with a deterministic MPC problem, Problem 2, where uncertain parameters are replaced by their joint expected values given S_0. Conditionally expected values, given the most recently observed state, are denoted as $\bar{x}$ for some random variable $\boldsymbol{x}$. The state transition equation, (8) is simply replaced by the deterministic state transition equation in (12). This method was used in [4] and [5] for the V2G aggregator problem. The method is simple and fast, but it results in sub-optimal control decisions. In fact, the optimal decisions for a deterministic optimization problem are not guaranteed to be anywhere near the optimal decisions for the true stochastic problem [7].

Problem 2:

$$J_1^{MPC}(E_1, S_0) = \min_{\substack{P_h, B_h, \bar{E}_{h+1} \forall h \\ s.t. (1)-(6), (12)}} g_{H-1}\left(\bar{E}_{H-1}, \bar{S}_{H-1}\right) + \sum_{h=1}^{H-2} g_h(\bar{S}_h, P_h, B_h)$$

$$\bar{E}_{h+1} = E_h + \Delta_h P_h - \bar{R}_h B_h, \forall h \tag{12}$$

Problem 1 also fits into the framework of stochastic linear programming (SLP). The SLP framework is most powerful when there are many state and decision variables, but only a few decision stages. When uncertain data have correlation over time, solving multi-stage SLP problems is impractical. This is because SLP problems grow exponentially in the number of decision stages. They become very difficult to even approximately solve by using scenario reduction [13] or nested Bender's decomposition [7], [14].

It is common practice to approximate an MDP that has continuous states and decision variables with a discrete MDP, having only discrete sets of possible states and decisions. This approach is used in [15] for an infinite horizon MDP. Reference [15] also shows that it is often necessary to develop approximation schemes when applying SDP to real problems. This reference cannot apply the discrete methods of [8] directly because the state transition equations do not necessarily lead to states that have been evaluated. Instead they rely on linear interpolation for approximating the cost to go at the next state. This encourages us to find an SDP approximation scheme which enables us to optimize over a continuous space of actions.

MDPs with discrete states and compact action spaces can also be solved if state transition probabilities are available as continuous functions of the actions [8]. Estimating such discrete transition functions is not appealing because of the difficulty and because our battery SOC is truly continuous.

The management of hydropower reservoirs has motivated practical solution methods for solving multistage stochastic optimization problems with continuous control variables. These solution methods have been developed by recognizing the shared theory of the MDP and SLP frameworks. Reference [16] describes how hydropower management problems can be decomposed by decision stages in a fashion similar to discrete MDPs. It also shows that continuous cost to go functions can be created and updated using SLP theory. The stochastic dual dynamic programming (SDDP) method of [16] can be thought of as a specific implementation of nested Benders decomposition, which avoids the discretization of controlled endogenous state variables. Reference [17] and more recently [18] extend [16] and propose solution methods which consider uncertain and Markovian energy prices. These methods approximate the exogenous random process for price with a finite set of states and transition probabilities, and are described as a combination of SDP and SDDP.

However, the methods proposed in [16]–[18] cannot be directly applied to the G2V problem. These methods do not allow uncertain decision variable coefficients in the state transition equation. For the G2V problem, (8) shows a random variable, $\boldsymbol{R}_h$, multiplying a decision variable, B_h, in the state transition equation. Therefore, we develop an approximate SDP scheme which is inspired by the hydropower operations literature to leverage SLP theory.

IV. Proposed Stochastic Dynamic Programming Approach

We propose an SDP algorithm for minimizing an approximation of the expected future costs given by Problem 1. The proposed algorithm consists of a single backwards recursion over a discrete set of possible states. We propose an intuitive way to optimize over a continuous space of decisions given each state. We begin by approximating Problem 1 with a discrete MDP and then extending it.

A. Discrete MDP Recursion

Since our random variables are truly continuous we first assume that the stochastic processes for the energy price, regulation service price, and AGC signal are well approximated as discrete and Markovian. This allows us to discretize the random exogenous state vector $\boldsymbol{S}_h$ into a set of possible values in each hour, Ω_h, with elements ω. To create a discrete MDP, the range of possible values for the SOC, E_h, must also be discretized into a set I_h with elements i for each hour. The energy price, regulation service price, AGC signal, and battery SOC give us a 4-dimensional state to evaluate in the SDP backwards recursion. Discretization gives the state transition equation (13), where E^{ω}_{h+1} must also be an element of I_{h+1} in order to directly apply the SDP backwards recursion as presented in [8].

$$E^{\omega}_{h+1} = E_h + \Delta_h P_h - R^{\omega}_h B_h, \quad \forall \omega \tag{13}$$

A discrete MDP also limits our optimization to a search over a discrete set of decisions, $D(E^i_h)$. This discrete MDP can then be decomposed into hourly decision problems by the principle of optimality, yielding the recursive equation in Problem 3. Expected costs within the current hour, $\mathbb{E}[g_h(\cdot)]$, can be calculated using $\bar{S}_h$ if the random processes are independent. $Pr(E^{\omega}_{h+1}, S^{\omega}_h \mid S_{h-1}, E_h, P_h, B_h)$ is a conditional probability mass function, which is also called a state transition probability. The discrete SDP backwards recursion recursively solves for $J^{\text{Disc}}_h(E_h, S_{h-1})$, the discrete MDP cost to go. It proceeds by starting at the final decision epoch and evaluating $J^{\text{Disc}}_h(E^i_h, S^{\omega}_{h-1}) \forall (i, \omega) \in I_h \times \Omega_h$. Then the states of the next earlier decision epoch can be evaluated. This recursion continues until the current state and decision epoch has been evaluated. This procedure also yields the best decision available in $D(E^i_h)$ for each evaluated state. Also note that in our problem at decision epoch $h = H - 1$, $J^{Disc}_{H-1}(E^i_{H-1}, S^{\omega}_{H-2})$ will be the conditional expectation of (10).

Problem 3:

$$J^{Disc}_h(E_h, S_{h-1}) = \min_{\substack{P_h, B_h \in D(E_h) \\ s.t. (1)-(6),(13)}} g_h\left(\bar{S}_h, P_h, B_h\right) + F$$

where

$$F = \sum_{\omega \in \Omega_h} Pr\left(E^{\omega}_{h+1}, S^{\omega}_h \mid S_{h-1}, E_h, P_h, B_h\right) \times J^{Disc}_{h+1}\left(E^{\omega}_{h+1}, S^{\omega}_h\right)$$

B. Approximate MDP With a Continuous Space of Decisions

We now turn our attention to the case where decisions can take any value in the feasible region of (1)–(6). In this case E^{ω}_{h+1} is not necessarily an element of I_{h+1}, the set of SOCs that were evaluated in the previous step of the backwards recursion. We propose using a piecewise linear convex function of the SOC as an approximation of the cost to go. This approach will allow the approximation of expected future costs for any decision in the feasible space and will allow the use of linear programming methods to find the best decision given some state. This approximation method can be used more generally to create an approximate cost to go function of controlled endogenous states, such as E_h, whenever the recursive equation of an MDP is a convex optimization problem.

The recursion in Problem 4 is a modification of that in Problem 3 which enables optimization over a continuous space of decisions. $\hat{J}_h(E_h, S_{h-1})$ is the "approximate cost to go" which approximates expected future costs given some state. The constraints in (14) can be thought of as a set of optimality cuts which form a continuous approximate cost to go function of SOC at the next decision epoch. In order to solve for $\hat{J}_h(E_h, S_{h-1})$ given some state, one must know the inequality constraint coefficients of (14), $\alpha^{\omega,i}_{h+1}$ and $\beta^{\omega,i}_{h+1}$, the next conditional expected values of the states, $\bar{S}_h$, and the state transition probabilities, $Pr(S^{\omega}_h \mid S_{h-1})$, from the last observed exogenous state to the next hour's possible exogenous states. This information can be estimated from historical data as will be described in Section V. Given some state, the recursive equation in Problem 4 can be solved over a continuous space of decisions using linear programming methods. At each decision epoch of the proposed SDP backwards recursion, one solves for $\hat{J}_h(E^i_h, S^{\omega}_{h-1}) \forall (i, \omega) \in I_h \times \Omega_h$. This also yields the inequality coefficients, $\alpha^{\omega,i}_h$ and $\beta^{\omega,i}_h$, which are associated with state $(E^i_h, S^{\omega}_{h-1})$ as will be explained next.

Problem 4:

$$\hat{J}_h(E_h, S_{h-1}) = \min_{\substack{P_h, B_h, E^{\omega}_{h+1}, J^{\omega}_{h+1} \\ s.t. (1)-(6),(13),(14)}} g_h(\bar{S}_h, P_h, B_h) + \sum_{\omega \in \Omega_h} Pr(S^{\omega}_h \mid S_{h-1}) J^{\omega}_{h+1}$$

$$J^{\omega}_{h+1} \geq \alpha^{\omega,i}_{h+1} - \beta^{\omega,i}_{h+1} E^{\omega}_{h+1}, \quad \forall i, \forall \omega \tag{14}$$

C. Derivation of Inequality Coefficients

In order to calculate the coefficients $\alpha^{\omega,i}_h$ and $\beta^{\omega,i}_h$ we first mention that Problem 4 can be rewritten as a linear program in the general form of (15) with parameter matrix A, parameter vectors W and T, and price vector q. The current stage decision vector is shown as y, and the given SOC, E^i_h, is represented as z^i. Solution of (15) given some z^i results in cost $Q(z^i)$ and the optimal dual variable vector λ^i.

$$Q(z^i) = \min_y \{q^T y \mid Ay = W - T z^i : \lambda^i\} \tag{15}$$

A lower bound for the primal function $Q(z)$ for all z can be constructed from the optimal dual variables of (15). Construction of this lower bound is shown in (16)–(18) [7].

$$\beta^i = \lambda^{i^T} T \tag{16}$$

$$\alpha^i = \lambda^{i^T} W \tag{17}$$

$$Q(z) \geq \alpha^i - \beta^i z \tag{18}$$

We can calculate the set of coefficients $\alpha^{\omega,i}_h$ and $\beta^{\omega,i}_h$ for all of the constraints in (14) after solving Problem 4 for all states in the set of discrete exogenous states, $\forall \omega \in \Omega_{h-1}$, and for all elements in a discretization of battery SOC $\forall i \in I_h$. The constraints in (14) form a piecewise linear convex function of battery SOC for each scenario ω. In the next, earlier decision epoch of the backwards recursion, future costs in state ω are approximated by a decision variable, J^{ω}_{h+1}. When solving Problem 4, $J^{\omega'}_{h+1}$ is minimized and is simultaneously greater than the constraints in (14) where $\omega = \omega'$ and for all i. Because the constraints in (14) are an under approximation of $\hat{J}_{h+1}(E^{\omega}_{h+1}, S^{\omega}_h)$, J^{ω}_{h+1} will be less than or equal to $\hat{J}_{h+1}(E^{\omega}_{h+1}, S^{\omega}_h)$. The transition probability weighted sum of J^{ω}_{h+1} calculates the expectation of approximate future costs with a continuous space of decisions.

In order to solve for the initial decision when $h = 1$, approximate cost to go functions must be constructed recursively for $h = H-2, \ldots, 2$. We also note that when solving for $\hat{J}_{H-2}(E_{H-2}, S_{H-3})$, the coefficients $\alpha_{H-1}^{\omega,i}$ and $\beta_{H-1}^{\omega,i}$ can be calculated for each exogenous state by taking the conditional expectation of (10) and rearranging terms.

D. Reduction of Problem Size

Problem 4 can grow in size very quickly with the refinement of the discretization of states. If each exogenous state is discretized into k values, then the number of possible next states is $|\Omega_h| = k^3$. Constraint set (14) consists of $|\Omega_h| \times |I_h|$ constraints. This motivates finding some way to reduce the size of Problem 4.

First, we assume that the random processes of the exogenous states, $\boldsymbol{R}_h, \boldsymbol{r}_h, \boldsymbol{c}_h$, are independent, although simple modifications to the method presented here could account for cross-correlation. The discrete Markov random process for $\boldsymbol{c}_h$ has possible states $n \in N_h$, the process for $\boldsymbol{r}_h$ has possible states $o \in O_h$, and the process for $\boldsymbol{R}_h$ has possible states $m \in M_h$, such that $\Omega_h = N_h \times O_h \times M_h$. In order to reduce the size of Problem 4, we propose taking the expectation of the cost to go function with respect to $\boldsymbol{c}_h$ and $\boldsymbol{r}_h$, reducing the dimensionality of the cost to go function from four to two. We use the state transition probabilities of the Markov random processes to compute the expectation of the inequality constraint coefficients of (14) as shown in (19) and (20).

$$\alpha_{h+1}^{m,i} = \sum_{n \in N_h} \sum_{o \in O_h} Pr\left(c_h^n \,|\, c_{h-1}\right) Pr\left(r_h^o | r_{h-1}\right) \alpha_{h+1}^{\omega,i} \tag{19}$$

$$\beta_{h+1}^{m,i} = \sum_{n \in N_h} \sum_{o \in O_h} Pr\left(c_h^n | c_{h-1}\right) Pr\left(r_h^o | r_{h-1}\right) \beta_{h+1}^{\omega,i} \tag{20}$$

Due to linearity of expectation, using the reduced dimension inequality constraints of (22) will give the same expected future cost as the constraints shown in (14). This property is shown for the L-Shaped method in [7]. This allows us to reduce Problem 4 to Problem 5, which minimizes expected costs over possible state transitions of the AGC signal, $m \in M_h$. We replace state transition equation (13) and approximate cost to go constraints (14) of Problem 4, with state transition equation (21) and the cost to go constraints shown in (22).

Problem 5:

$$\hat{J}_h(E_h, S_{h-1}) = \min_{\substack{P_h, B_h, E_{h+1}^m, J_{h+1}^m \\ s.t. (1)-(6), (21), (22)}} g_h(\bar{S}_h, P_h, B_h) + \sum_{m \in M_h} Pr\left(R_h^m | R_{h-1}\right) J_{h+1}^m$$

$$E_{h+1}^m = E_h + \Delta_h P_h - R_h^m B_h, \quad \forall m \tag{21}$$

$$J_{h+1}^m \geq \alpha_{h+1}^{m,i} - \beta_{h+1}^{m,i} E_{h+1}^m, \quad \forall i, \forall m \tag{22}$$

E. Proposed SDP Algorithm

Assuming that each exogenous state is discretized into k values, the proposed SDP algorithm is a backwards recursion that successively solves $|I| \times k^3$ instances of the linear programming problem Problem 5 for each decision epoch. Using a modern multi-core desktop computer, we can solve the batch of problems for each decision epoch in a parallel fashion. The approximate cost to go constraint coefficients of (22) are then calculated and incorporated into the next batch of earlier decision problems, back propagating expected future costs. This is repeated until the recursive procedure reaches the first decision epoch. At the first decision epoch, only one instance of Problem 5 must be solved, as the current SOC and previously observed exogenous states are known. This decision can then be submitted to an aggregator or ISO. The backwards recursion starts at decision stage $H-2$ because the approximate cost to go function $\hat{J}_{H-1}(E_{H-1}, S_{H-2})$ can be computed directly as described in Subsection IV.C.

Proposed Stochastic Dynamic Programming Algorithm

For $h = H-2, \ldots, 2$

  Parallel For $(i, \omega) \in I_h \times \Omega_h$

    Solve Problem 5 for $\hat{J}_h(E_h^i, S_{h-1}^\omega)$

    Calculate $\alpha_h^{\omega,i}, \beta_h^{\omega,i}$

  End Parallel For

  Calculate $\alpha_h^{m,i}, \beta_h^{m,i}$

End For

Solve Problem 5 for $\hat{J}_1(E_1, S_0)$

V. Stochastic Process Models

In order to simulate an EV charging by our proposed method, we construct models of the stochastic processes of the exogenous data. These models will allow us to generate random realizations from continuous distributions. Models for energy price and regulation service price were built from PJM ISO market data from the year 2011. 5 weeks of AGC signal data was also gathered from PJM ISO [19]. We made slightly different assumptions in modeling each stochastic process.

In our simulation experiments, we assume that the EV pays the real-time hourly energy market price for energy consumed. In order to model real-time energy market prices, we first assume that day-ahead energy markets are the best predictors of real-time prices available a day in advance. This motivates building a model of the differences between real-time and day-ahead prices, which can then be added back to the known day-ahead prices. Real-time energy prices are known to exhibit higher variance during peak load hours, when it is more likely that the electric grid is bound by capacity constraints, than during low load hours. And so, we modeled the energy price differences using a unique marginal distribution for each hour of the day. We use empirical CDFs to model each marginal distribution [20]. In order to model Markovian dependence in the price differences, a Gaussian copula model was fit to each pair of distributions for adjacent hours of the day. The Gaussian copula uses rank correlation to model the dependence between random variables of arbitrary marginal distributions [21]. By fitting a different copula to each pair of distributions, we model the dependence as varying with hour of the day. This model allows us to simulate realizations of the next price difference given the difference of the current hour. We can then add this difference to the day-ahead price and call it a realization from the real-time price distribution.

The hourly regulation service price appears to exhibit cyclical daily patterns. This motivates modeling the regulation service price with a unique marginal probability distribution for each hour of the day. An empirical CDF was used to estimate the unique marginal distributions in each hour of the day. A Gaussian copula model was fit to each pair of distributions for adjacent hours of the day.

Due to the limited amount of data available, we modeled the AGC signal's normalized energy, $\boldsymbol{R}_h$, with the same marginal probability distribution in all hours. The distribution used is the empirical CDF of the collected data. $\boldsymbol{R}_h$ is restricted to take values between -1 and 1. A single Gaussian copula was fit to model the dependence between $\boldsymbol{R}_h$ and its last observed value.

VI. Simulation Experiments

We investigated the value of using the proposed SDP approach for determining G2V charging and regulation bids as opposed to using a deterministic MPC approach. Using both approaches, we simulated 20 000 trials of an EV charging overnight given the same vehicle parameters, initial state, and day-ahead energy prices. We also compared the SDP algorithm solution times when using Problem 5 instead of Problem 4. The simulation procedure and results are described below.

A. Input Data

In Section V, we described how we estimate stochastic process models that can be used to generate realizations of the random states from continuous distributions. However, the SDP algorithm presented in this paper requires a discrete Markov model for each exogenous random state. In order to create a discrete Markov model, we first discretize each random variable's distribution in each hour. This is done by evaluating each distribution's inverse CDF at k equally spaced values between 0 and 1, exclusive. For the energy price differences, we simply add these values to the day-ahead prices to get real-time prices.

State transition probabilities, $Pr(S_h^\omega \mid S_{h-1})$, and the conditional expected value of the next exogenous state vector, $\bar{S}_h$, are needed as input to each instance of Problem 4 or Problem 5 solved in the proposed SDP recursion. In order to estimate transition probabilities, we used a Monte Carlo approach. Given each possible state of the discrete random processes, we generate 500 000 realizations of the next hour's data with the continuous distribution stochastic process models of Section V. These realizations are then binned according to which discretized state has the closest CDF value. Bin counts are divided by the total number of generated realizations to yield estimated state transition probabilities. In order to calculate the expected values for the next state, which are used as input for the cost functions (10) and (11), we simply take the average of the generated realizations. Once this input data has been computed, we can execute the proposed SDP algorithm.

To optimize bids using the MPC approach, expected exogenous state values must be available for the entire remaining charging horizon. Using the continuous distribution stochastic process models of Section V and the known current state, we generated 500 000 sample paths of the exogenous random processes evolving over the remaining horizon. We then average the values of each state in each hour and use these as our expected state values.

B. Simulation Procedure

After the initial input data is calculated, we use the procedure presented in this subsection to simulate each of the 20 000 EV charging trials. For the proposed SDP algorithm, a single backwards recursion computes all of the approximate cost to go inequality coefficients needed to solve for any charging or regulation bid of the fixed charging horizon. Given the known current state, we then solve an instance of Problem 4 or Problem 5 for the first SDP based bids. An instance of Problem 2, which does not require a backwards recursion to be completed, is solved for the first MPC based bids. After the bids are submitted, the continuous distribution stochastic process models of Section V are used to generate the next realization of each random state variable. Then running total incurred costs and current states are updated.

Given the newly observed current state, data for the next decision epoch's instance of Problem 2, Problem 4 or Problem 5 must be created. For the SDP method, new state transition probabilities and expected exogenous state values are estimated for only the current decision epoch using the procedure described in Subsection VI.A above. Using these new state transition probabilities, the weighted average constraint coefficients must be recalculated by (19) and (20). For the MPC method, expected values of the exogenous states are estimated for the remainder of the charging horizon given the current state as was described in Subsection VI.A above. Given the new state and data, Problem 2, Problem 4 or Problem 5 is solved for new bids. This fixed horizon simulation procedure continues in each simulation trial until either the EV reaches maximum SOC or the unplugging time is reached.

All experiments were conducted on a desktop PC with an Intel 3930k 6 core processor and 12 GB of RAM. Simulations were implemented in MATLAB, and CPLEX was used to solve linear programs. During each decision stage of the SDP algorithm's backwards recursion, the many instances of linear program Problem 4 or Problem 5 were solved in parallel. This is done using MATLAB's Parallel Computing Toolbox. All other steps of the SDP algorithm run in serial.

In our experiments, we solve an example problem consisting of 12 hourly decision stages, with $H = 13$. Our EV plugs in at 8 p.m. and unplugs at 8 a.m. The EV's initial SOC is 8 kWh and must charge to $E_{\max} = 24$ kWh. The maximum charge rate is $P_{\max} = 7$ kW. These values are typical of EVs currently on the market. Typical January Day-ahead energy prices are used in simulating energy prices.

The SDP based methods were implemented with $|I| = 7$ and $k = 12$. Due to memory limitations, this is the finest discretization we successfully implemented for the SDP routine using Problem 4. During the backwards recursion, 12 processing

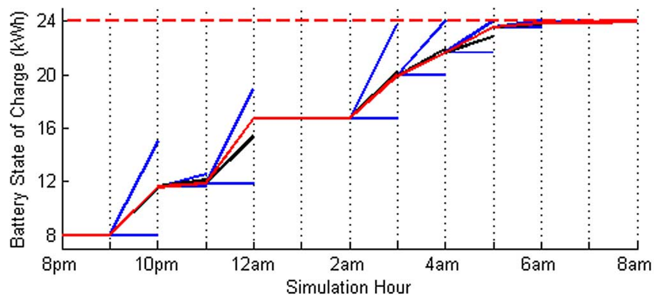

Fig. 1. Example charging simulation results.

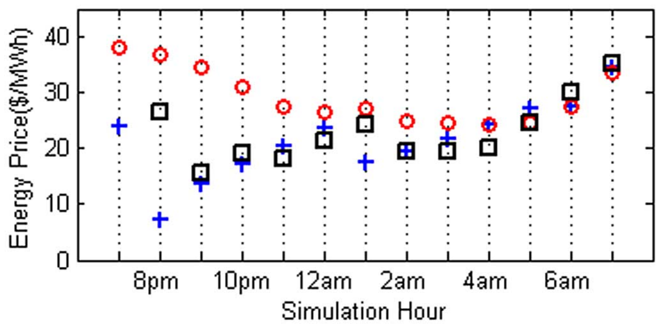

Fig. 2. Example energy price simulation.

threads are used to solve linear programs based on Problem 5 in parallel. Again, due to memory limitations, the number of processing threads was limited to 4 when executing the backwards recursion with Problem 4. We suspect that the memory limitations are partially due to the amount of simulation data we saved for analysis and partially due to how the Parallel Computing Toolbox clones data for parallel jobs.

C. Results

Fig. 1 shows the battery SOC trajectory resulting from a single simulation trial of an EV charging and providing frequency regulation service. The solid black line shows how the SOC would evolve if the battery charged at a rate of P_h, while the solid red line shows the actual SOC trajectory including the effect of following the AGC signal. The random effect of following the AGC signal is most noticeable following 11 p.m. and 4 a.m. The blue lines show the maximum and minimum trajectories that the SOC could take over the hours when the EV provides frequency regulation. The dashed red line at 24 kWh shows $E_{\max}$.

Figs. 2 and 3 show the evolution of prices during this simulation trial. Day-ahead energy prices are shown as red circles in Fig. 2, while the hourly means of the regulation service price distributions are shown as red circles in Fig. 3. The actual prices observed are shown with blue cross marks while the expected price, given the last hour's value, is shown with a black square.

Table I summarizes the results of the three sets of simulation trials. Because the costs are the result of a stochastic simulation, the mean cost varies with each batch of simulations. In order to make a stronger statement about the relative performance of the evaluated methods, we used basic statistics to estimate a 95% confidence interval for the true mean cost of using each method. The upper and lower bounds of this confidence

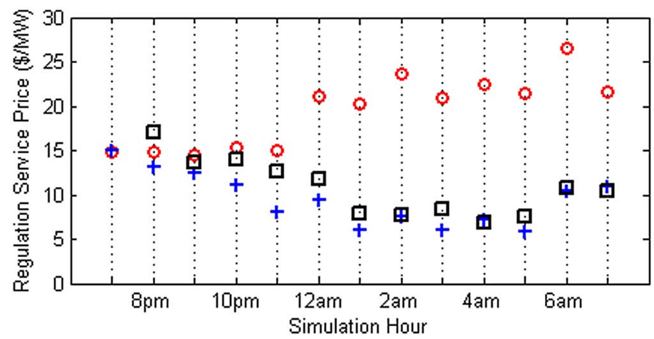

Fig. 3. Example regulation service price simulation.

TABLE I
SUMMARIZED SIMULATION RESULTS

Method	SDP	SDP	MPC
Problem #	5	4	2
Mean cost ($)	0.057	0.056	0.072
Cost variance	0.097	0.094	0.113
C.I. Upper Bound	0.061	0.060	0.077
C.I. Lower Bound	0.052	0.052	0.067
Recursion time (s)	212	15,590	N/A
Average instance solution time (ms)	33	750	3

interval are given in Table I in the rows labeled "C.I. upper bound" and "C.I. lower bound." For this example problem, the mean EV charging cost when using the proposed SDP method with Problem 5 is 22% lower than the mean cost incurred when using MPC. The 95% confidence interval for the mean cost when using SDP is completely below the confidence interval for the mean cost when using MPC.

Performing the SDP routine with Problem 5 instead of Problem 4 results in a negligible difference in mean cost as is expected. However, replacing Problem 4 with Problem 5 results in a significant reduction in computation time as shown in the rows of Table I labeled "Recursion time" and "Average instance solution time." The recursion time refers to the time required to execute the entire proposed SDP backwards recursion, while the Average instance solution time refers to the average amount of time required to solve for hourly charging and regulation bids after the backwards recursion has completed. This highlights the importance of leveraging theory to reduce the problem size as done in Section IV.D.

Our results show that the proposed approximate SDP method could be a practical hourly decision making strategy for EVs. An EV smart charger would need to perform the proposed SDP backwards recursion once each time the EV plugs in for the night. The required recursion time is less than four minutes in our implementation. At the beginning of each hour while the EV is charging, the smart charger would optimize its bids by solving a single linear program, requiring 33ms on average. The proposed method requires much more computation than the

MPC method, but it is still well within the abilities of today's multi-core personal computers. However, we have not investigated the practicality of implementing this method on today's commercially available EV smart chargers or on-board vehicle computers.

VII. Conclusion

In this paper we present an MDP with three sources of uncertainty for the G2V charging problem. We then develop an approximate SDP algorithm for the optimization of an EV's charging and frequency regulation bids over a continuous space of decisions. The methods developed here minimize an approximation of expected future costs. The proposed SDP solution method results in lower average EV charging costs than deterministic MPC. Although the improvement in mean charging cost is large in relative terms, it is still very small in absolute terms. We also demonstrate that the proposed method can be solved in a practical amount of time on a personal computer.

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