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On the Complexity of Deciding Soundness of Acyclic Workflow Nets

Ferucio Laurenţiu Ţiplea, Corina Bocăneală, and Raluca Chiroşcă

***Abstract*—This paper focuses on the complexity of the (weak) soundness problem of acyclic workflow (WF) nets, and two main results are established: 1) soundness of 1-bounded acyclic WF nets is co-NP-complete and 2) weak soundness of 3-bounded acyclic asymmetric-choice WF nets is co-NP-complete.**

***Index Terms*—Complexity, decidability, Petri net, soundness, workflow (WF) net, WF system.**

I. Introduction

One of the most successful techniques for workflow (WF) specification and modeling is the one based on Petri nets [1]–[13]. This is amply justified by the fact that Petri nets are expressive, have a well-defined semantics, provide a suggestive graphical language, and many properties and analysis techniques for them are now available.

A Petri net model of a WF, called WF net, represents tasks by transitions and conditions by places. Cases, which are enactments of processes, are modeled by tokens. Proper termination (also called weak soundness) of a WF model means that the procedure can terminate for any case, and quasi-liveness means the absence of dead tasks. Together, these two properties define soundness [14], which is an important correctness criterion a WF net should satisfy. Various extensions of the soundness concept, which give the practitioners and researchers a broad spectrum of correctness criteria for WF nets, have also been proposed (for details the reader is referred to [1], [11], and [15]).

Both from a theoretical and a practical point of view, we are interested in answering the following two questions regarding WF nets: is it decidable whether a WF net is sound. if soundness of WF nets is decidable, how complex is to decide it. A standard result obtained in [14] shows that soundness of WF nets is equivalent to boundedness and liveness of Petri nets. As both boundedness and liveness of Petri nets are decidable properties [16], we conclude that soundness of WF nets is decidable too. Soundness can also be characterized in terms of home markings [3], which is a decidable Petri net property [16].

As with respect to the complexity of deciding soundness, not too many results are known. As this paper focuses on this topic, we will survey the most important results obtained so far. The characterization of soundness based on home markings does not currently lead to precise complexity results because the precise complexity of the home marking problem is not known. The characterization based on boundedness and liveness reduces the complexity of the soundness problem to the complexity of boundedness and liveness which have been intensively studied by researchers. For general Petri nets, boundedness can be solved in $2^{2ck\log k}(l+\log n)$ space for some constant c, where k is the number of places, l is the maximum number of inputs or outputs of a transition, and n is the number of transitions [17]. The liveness problem is recursively equivalent to the reachability problem [18] whose precise complexity is still unknown. For special subclasses of Petri nets, boundedness and liveness can be decided more efficiently. This is, for instance, the case of free-choice Petri nets for which boundedness and liveness can together be decided in polynomial time [19], [20] (interestingly, boundedness and liveness for free-choice Petri nets, when taken separately, require higher complexity). As the conclusion, the soundness problem of free-choice WF nets can be decided in polynomial time. Recently, new results regarding the complexity of the soundness problem have been obtained. Thus, it was shown that the soundness problem of bounded WF nets is PSPACE-complete [21], [22], while weak soundness of bounded asymmetric-choice workflow (ACWF) nets is co-NP-hard [23].

A. Paper Contribution and Related Work

The study of the complexity of the soundness problem of WF nets has both theoretical and practical relevance. We are interested in knowing how to position the soundness problem in the hierarchy of complexity classes, how difficult is to check soundness of classes of WF nets of practical relevance, or how to extend the class of free-choice WF nets to be more expressive while keeping the polynomial complexity of checking its soundness.

This paper aims to contribute new results to the study of the complexity of the soundness problem of the class of acyclic WF nets. The starting point is given in some of the latest results obtained by [21] and [23]. This paper refines and completes them, as explained below.

[21, Th. 2] shows that the soundness problem of bounded WF nets is co-NP-hard. Looking more carefully at the proof of this theorem, one can see that this result is specific to 1-bounded acyclic WF nets (in fact, soundness of bounded WF nets is PSPACE-complete [22]). This remark is completed in this paper by a proof of the membership of soundness to the complexity class co-NP and, therefore, we obtain that the soundness problem of 1-bounded acyclic WF nets is co-NP-complete.

Another interesting result, obtained in [23], shows that weak soundness of 3-bounded ACWF nets is co-NP-hard. We refine this result to hold for acyclic ACWF nets too, and then complete it with the membership to the complexity class co-NP. Thus, we obtain that the weak soundness problem of 3-bounded acyclic ACWF nets is co-NP-complete.

The construction used to obtain our first result cannot be applied to ACWF nets, while the construction used to obtain our second result cannot be applied to 1- or 2-bounded WF nets. Therefore, the status of the complexity of soundness of 1- or 2-bounded acyclic ACWF nets remains open.

B. Paper Organization

This paper is organized into five sections. The next section recalls basic concepts on Petri nets, WF nets, and complexity theory. Section III discusses soundness of 1-bounded acyclic WF nets, and Section IV discusses soundness of 3-bounded acyclic ACWF nets. We conclude this paper in Section V.

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II. Preliminaries

We fix in this section the basic notation and terminology used in the rest of this paper.

The set of non-negative integers is denoted by $\mathbb{N}$. A *Petri net* [24] is a tuple $\Sigma = (S, T, F)$, where S and T are two finite sets (of *places* and *transitions*, respectively), $S \cap T = \emptyset$, and $F \subseteq (S \times T) \cup (T \times S)$ is the *flow relation*. Given $x \in S \cup T$, its *preset* is ${}^\bullet x = \{y|(y, x) \in F\}$ and its *post-set* is $x^\bullet = \{y|(x, y) \in F\}$. It is also useful, from a technical point of view, to define the function $W_\Sigma\colon (S \times T) \cup (T \times S) \to \mathbb{N}$ by $W_\Sigma(x, y) = 1$ for all $(x, y) \in F$, and $W_\Sigma(x, y) = 0$, otherwise.

A *marking* of Σ is any function $M \in \mathbb{N}^S$ from S into $\mathbb{N}$. The transition relation of a Petri net Σ states that a transition t is *enabled* at a marking M, denoted $M[t\rangle_\Sigma$, if $M(s) \geq W_\Sigma(s, t)$ for all $s \in S$. If t is enabled at M, then it can fire yielding a new marking M' given by $M'(s) = M(s) - W_\Sigma(s, t) + W_\Sigma(t, s)$, for all $s \in S$; we denote this by $M[t\rangle_\Sigma M'$. The transition relation is usually extended to sequences of transitions. When there is a sequence w of transitions such that $M[w\rangle_\Sigma M'$, we say that M' is *reachable* (from M in Σ). We denote by $[M\rangle_\Sigma$ the set of all reachable markings (from M) in Σ. When no confusion may arise, we simplify the notation $[\cdot\rangle_\Sigma$ to $[\cdot\rangle$.

The Petri nets are pictorially represented by directed graphs. The places and transitions of a Petri net are the nodes of the graph, and the flow relation is representing by drawing an arc from x to y for each pair (x, y) of it. It is customary to represent the nodes which are places by circles and those which are transitions by boxes. The markings are pictorially represented by drawing $M(s)$ black tokens into the circle representing the place s, for any place s.

Let Σ be a Petri net, M_0 a marking of it, and k a positive integer. Σ is called:

1) Acyclic, if it does not have structural cycles, that is, there is no sequence $s_1, t_1, s_2, \ldots, s_n, t_n, s_{n+1}$ in Σ with $s_{n+1} = s_1 \in S$, $(s_i, t_i) \in F$, and $(t_i, s_{i+1}) \in F$, for all $1 \leq i \leq n$;
2) Asymmetric-choice (AC), if $s_1^\bullet \subseteq s_2^\bullet$ or $s_2^\bullet \subseteq s_1^\bullet$, for any two places s_1 and s_2 with $s_1^\bullet \cap s_2^\bullet \neq \emptyset$;
3) k-bounded with respect to M_0, if $M(s) \leq k$ for all markings $M \in [M_0\rangle$ and all places s;
4) Bounded with respect to M_0, if it is n-bounded with respect to M_0 for some n;
5) Quasi-live with respect to M_0, if all its transitions are quasi-live with respect to M_0, that is, for any transition t there exists $M \in [M_0\rangle$ such that $M[t\rangle$.

The first two properties above (acyclicity, ACness) are syntactic and can be easily decided by inspecting the Petri net Σ; the other properties (boundedness, quasi-liveness) are behavioral and decidable too [16], [25].

A *WF net* [26] is a Petri net Σ with the following two properties.

1) Σ has two special places i and o called the *input* and the *output* place of Σ, respectively. They satisfy ${}^\bullet i = \emptyset$ and $o^\bullet = \emptyset$.
2) Any node $x \in S \cup T$ in the graph of Σ is on a path from i to o.

Given a WF net Σ and a place s of it, denote by M_s the marking which marks s by one token and leaves unmarked all the other places.

A k-bounded (bounded) WF net with respect to M_i will be simply called a *k-bounded (bounded) WF net*.

A WF net should satisfy some "behavioral correctness criteria." The first such criterion (for WF nets) was formulated in [26] and it was called soundness. It corresponds to the one case handling by a WF net and consists of the following:

1) Proper termination: $M_o \in [M\rangle$ for all $M \in [M_i\rangle$.
2) Quasi-liveness: Σ is quasi-live.

(The original soundness concept [14] requires one additional constraint which, however, can be obtained from proper termination and quasi-liveness [3]).

The proper termination property is also called *weak soundness* and it was used by many researchers as the only correctness criterion of WF nets. It is clear that the soundness property implies the weak soundness property.

We close the section by recalling a few concepts from the complexity theory (for details the reader is refereed to [27]). NP stands for the class of the decision problems decidable in polynomial time by nondeterministic algorithms. The set of complements of the decision problems in the complexity class NP is denoted co-NP. A decision problem A is called *NP-hard (co-NP-hard)* if any problem in NP (co-NP) is polynomial time reducible to A. Given a complexity class $\mathcal{C}$, we say that the decision problem A is $\mathcal{C}$-complete or complete for $\mathcal{C}$ if it is in $\mathcal{C}$ and is $\mathcal{C}$-hard.

One of the most prominent decision problems is the *satisfiability problem* (SAT) of the Boolean formulas. Recall that the *Boolean formulas* are built from Boolean variables by using the operators of conjunction ($\wedge$), *disjunction* ($\vee$), and *negation* (the negation of a Boolean variable x is denoted by $\bar{x}$). A *literal* over a set of Boolean variables is either a variable in the set or its negation.

The SAT asks to decide whether a Boolean formula is satisfiable or not (a Boolean formula is satisfiable if there exists an assignment of the Boolean variables which makes the formula evaluate to the truth value true). This problem is NP-complete, and remains so even if the Boolean formulas are in the *3-conjunctive normal form* (3-CNF). In this form, the Boolean formulas are conjunctions of *clauses*, where each clause is a disjunction of exactly three literals. This simplified form of SAT is known as *3SAT*.

A closely related problem to the satisfiability is the *validity problem* (VAL) which asks to decide whether a Boolean formula is true under all assignments of the Boolean variables. This problem is co-NP-complete [27] even if the Boolean formulas are in the 3-disjunctive normal form (3-DNF). A Boolean formula is in 3-DNF if it is a disjunction of clauses, where each clause is a conjunction of three literals. This simplified form of VAL is known as *3VAL*.

It is customary to assume that no clause of a Boolean formula contains literals with more than one occurrence or both a variable and its negation at the same time.

III. Soundness of (1-Bounded) Acyclic WF Nets

The problem of deciding the soundness of bounded or even 1-bounded WF nets is PSPACE-complete [22]. We will show in this section that this problem still remains very complex even if it is restricted to (1-bounded) acyclic WF net instances.

Theorem 1: The soundness problem of acyclic WF nets is in co-NP.

Proof: Remark first that the acyclic WF nets are bounded with respect to their initial marking. Moreover, given an acyclic WF net Σ one can compute in polynomial time an integer k such that Σ is k-bounded with respect to M_i.

The nondeterministic Algorithm 1 solves the nonsoundness problem of acyclic WF nets. It checks first if the WF net has nonquasi-live transitions. If this is the case, it outputs "Yes" because the WF net is not sound. Otherwise, it nondeterministically selects a marking M of Σ and outputs Yes if M is reachable from M_i but M_o is not reachable from M (Σ is not sound in such a case either). In all the other cases, the algorithm outputs "No."

According to the definition of the nondeterministic algorithms, if Algorithm 1 outputs Yes on at least one computation branch when it takes as input an acyclic WF net Σ, then Σ is not sound (it has at least one nonquasi-live transition or there exists at least one marking M reachable from M_i which does not lead to M_o); otherwise, Σ is sound.

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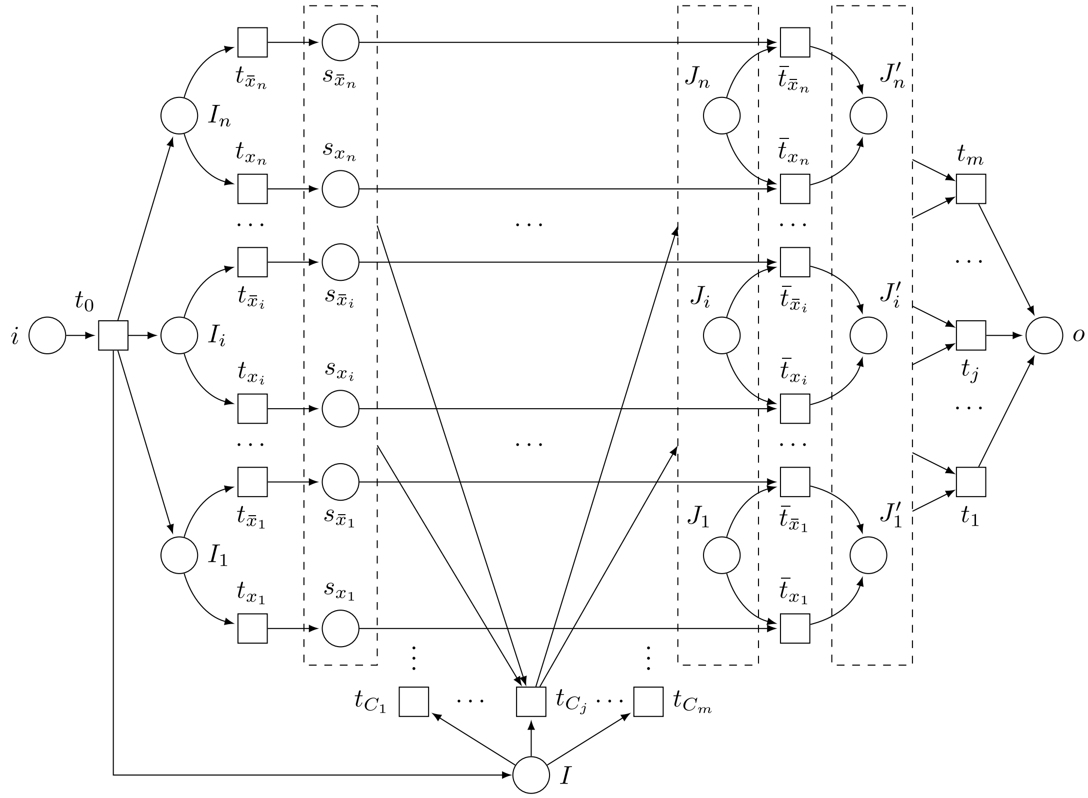

Fig. 1. WF net Σ_C associated to a boolean formula C in 3-DNF.

Algorithm 1: Nonsoundness of Acyclic WF Nets

input : acyclic WF net Σ
output: "Yes" if Σ is not sound, and "No" otherwise
if *there are non-quasi-live transitions of* Σ **then**
  "Yes"
else
  compute k such that Σ is k-bounded;
  non-deterministically select a marking $M \in \mathbb{N}^k$ of Σ;
  if $(M \in [M_i\rangle \ \wedge \ M_o \notin [M\rangle)$ **then**
    "Yes"
  else
    "No"

The quasi-liveness of acyclic Petri nets can be decided in polynomial time, while the complexity of the reachability problem of acyclic Petri nets is NP-complete [28], [29]. Therefore, Algorithm 1 is a nondeterministic algorithm working in polynomial time and, as the conclusion, the soundness problem of acyclic WF nets is in co-NP. ■

According to the above theorem, the soundness problem of 1-bounded acyclic WF nets is in co-NP. One of the results in [21, Th. 2] shows that the soundness problem of bounded WF nets is co-NP-hard (this result was improved in [22] where it was shown that the soundness of bounded WF nets is PSPACE-complete). The main idea of the proof of [21, Th. 2] is to use a characterization result of the soundness by means of the boundedness and liveness of some Petri net associated to the given WF net [14]. Then, a polynomial time reduction from 3VAL to the liveness problem is exhibited (see Section II for a description of 3VAL). In fact, by a careful analysis of this reduction in [21], one can see that it is specific to 1-bounded acyclic WF nets. Moreover, one can see that this reduction

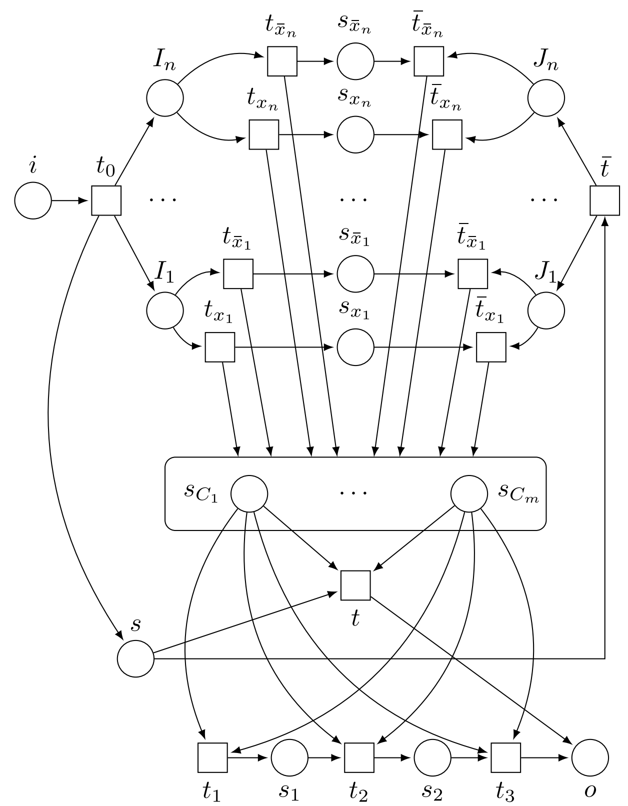

Fig. 2. WF net Σ_C.

can directly be performed from 3VAL to the soundness problem of 1-bounded acyclic WF nets: given an instance C of 3VAL, one can construct a 1-bounded acyclic WF net Σ_C such that C is valid if and

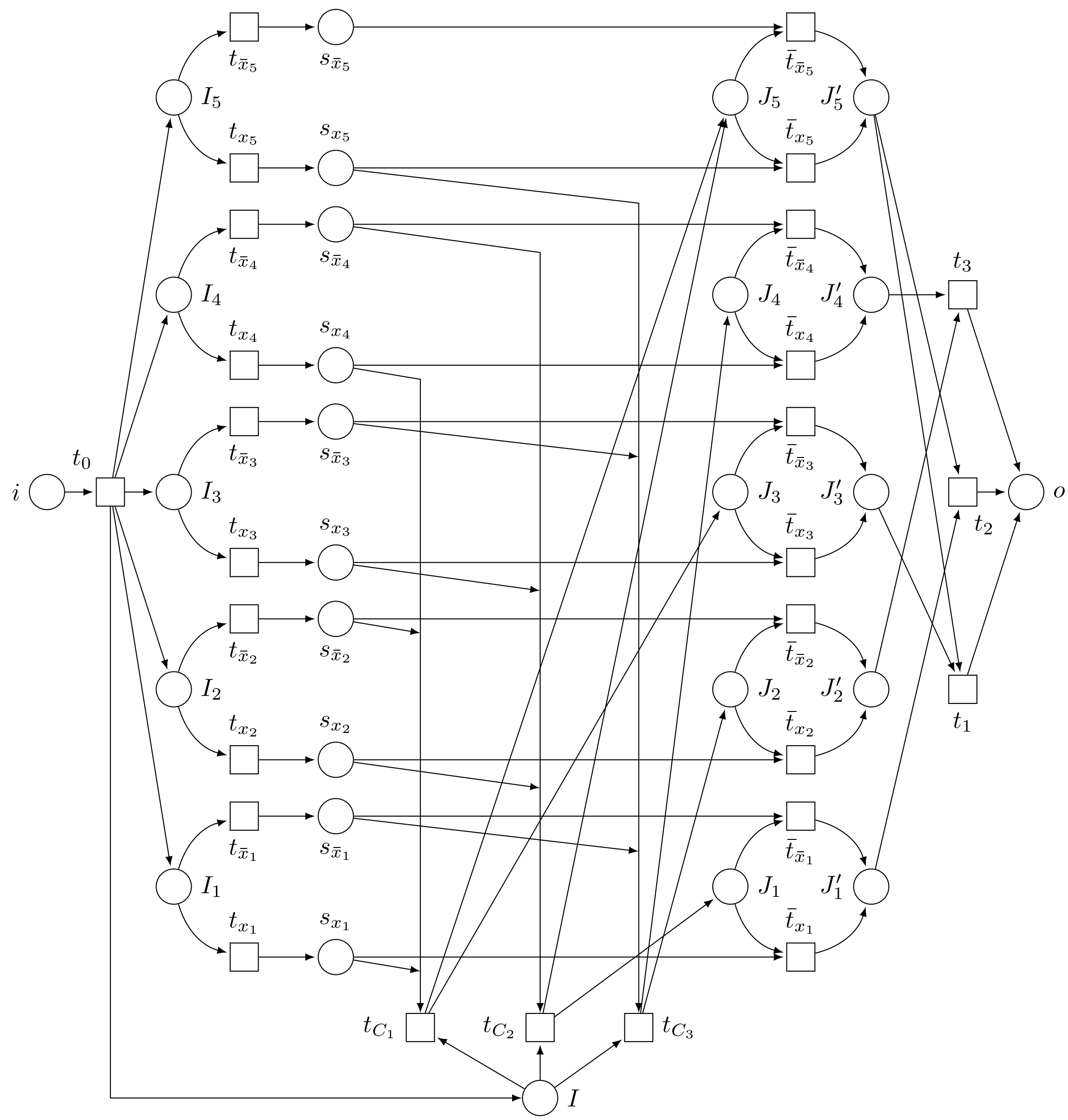

Fig. 3. One-bounded acyclic WF net Σ_C associated to $C = (x_1 \wedge \bar{x}_2 \wedge x_4) \vee (x_2 \wedge x_3 \wedge \bar{x}_4) \vee (\bar{x}_1 \wedge \bar{x}_3 \wedge x_5)$.

only if Σ_C is sound. For the sake of clarity, we recall the construction here in a "slightly" different way (an example is provided in the Appendix). First, given a Boolean formula $C = C_1 \vee \cdots \vee C_m$ in 3-DNF over a set $X = \{x_1 \ldots, x_n\}$ of Boolean variables, we may assume without loss of generality that the following holds (see also [21] and the last part of the Section II): for each variable there exists a clause such that neither the variable nor its negation occurs in the clause.

Consider now the WF net Σ_C in Fig. 1. The meaning of the elements of this WF net are as follows (given a literal ℓ over X and a clause C_j, "$\ell \in C_j$" means that ℓ occurs in C_j; the negation of ℓ, denoted $\bar{\ell}$, is $\bar{x}$ if $\ell = x \in X$, and it is x if $\ell = \bar{x}$ and $x \in X$).

1) s_ℓ is a place associated to the literal ℓ, for each ℓ.
2) t_0 is a transition which initiates the assignment process.
3) t_ℓ and $\bar{t}_\ell$ are transitions associated to the literal ℓ, for each ℓ. When t_ℓ fires, it assigns the literal ℓ to the truth value true. When $\bar{t}_\ell$ fires, it puts on zero the place s_ℓ.
4) t_{C_j} and t_j are transition associated to the clause C_j, $1 \leq j \leq m$, defined as follows.
   a) ${}^\bullet t_{C_j} = \{s_\ell \mid \ell \in C_j\}$.
   b) $t_{C_j}^\bullet = \{J_i \mid 1 \leq i \leq n \ \wedge \ x_i, \bar{x}_i \notin C_j\}$.
   c) ${}^\bullet t_j = \{J'_i \mid 1 \leq i \leq n \ \wedge \ x_i, \bar{x}_i \notin C_j\}$.
   d) $t_j^\bullet = \{o\}$.

The transition t_0 initiates the assignment process. If all the places in the preset of some transition t_{C_j} get marked (which means that the corresponding assignments makes the formula C true), then t_{C_j} can fire. It puts on zero the places in its preset and enables the transitions $\bar{t}_\ell$ associated to literals ℓ for which $\ell, \bar{\ell} \notin C_j$. These transitions will empty all the other places associated to literals and, when all of them fired, t_j is activated to fire and the marking M_o is reached. If none of $t_{C_1}, \ldots, t_{C_m}$ is enabled when the transitions associated to literals have fired and simulated an entire assignment of the variables (which means that the corresponding assignment makes the formula C false), the marking M_o cannot be reached.

According to our assumption above, the WF net Σ_C is quasi-live. Indeed, remark first that t_0 and t_ℓ are quasi-live, for any literal ℓ. For each $1 \leq j \leq m$, there exists an assignment which enables t_{C_j} and, therefore, t_{C_j} is quasi-live. For each literal ℓ, there exists a clause C_j which contains neither ℓ nor $\bar{\ell}$. There exists an assignment then which enables t_{C_j} and marks s_ℓ. When t_{C_j} fires, $\bar{t}_\ell$ will be enabled which shows that $\bar{t}_\ell$ is quasi-live.

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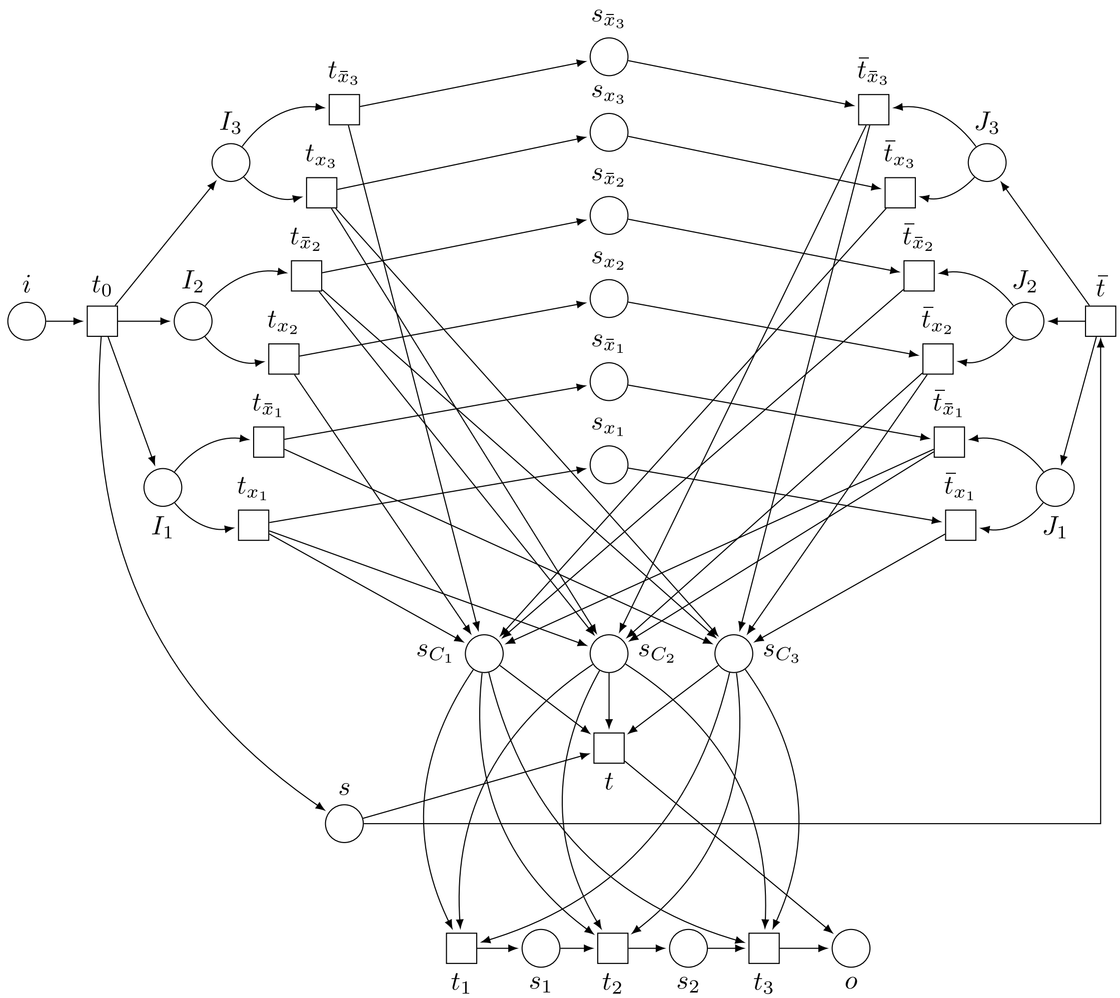

Fig. 4. Three-bounded acyclic ACWF net Σ_C associated to $C = (x_1 \vee x_2 \vee \bar{x}_3) \wedge (x_1 \vee \bar{x}_2 \vee x_3) \wedge (\bar{x}_1 \vee \bar{x}_2 \vee x_3)$.

We have obtained that C is valid if and only if Σ_C is sound. Moreover, Σ_C is 1-bounded and acyclic. Combining this with Theorem 1, we have the following result.

Corollary 1: The soundness problem of (1-bounded) acyclic WF nets is co-NP-complete.

Proof: The construction provided above the corollary shows that the 3VAL problem can be reduced in polynomial time to the soundness problem of (1-bounded) acyclic WF nets. As the 3VAL problem is co-NP-complete, it follows that the soundness problem of (1-bounded) acyclic WF nets is co-NP-hard. Combining this with the membership of the soundness problem of (1-bounded) acyclic WF nets to the complexity class co-NP (proved in Theorem 1), it follows that the soundness problem of (1-bounded) acyclic WF nets is co-NP-complete. ■

IV. Soundness of Acyclic ACWF Nets

It was shown in [23] that the soundness problem of 3-bounded ACWF nets is co-NP-hard. We sharpen this result by showing that the same holds for 3-bounded acyclic ACWF nets. The main idea is to use a reduction from the complement of 3SAT (see Section II for a description of 3SAT) in a different way than in [23] in order to obtain acyclicity as well. It is worthy to mention that a reduction from 3SAT to the reachability problem of some subclass of Petri nets was also exhibited in [30].

Theorem 2: The weak soundness problem of 3-bounded acyclic ACWF nets is co-NP-hard.

Proof: We prove the result in the theorem in two steps. First, we show that any instance of the complement of 3SAT can be reduced in polynomial time to an instance of the weak soundness problem of 3-bounded acyclic WF nets (our construction is different from the one provided in [23]).

Let $X = \{x_1, x_2, \ldots, x_n\}$ be a set of Boolean variables, and $C = C_1 \wedge C_2 \wedge \ldots \wedge C_m$ be a Boolean formula over X in 3-CNF, where each C_j, $1 \leq j \leq m$, is a disjunction of three literals. Without loss of generality, we may assume that each literal over X occurs in some clause of C.

Consider the WF net Σ_C in Fig. 2 whose description is provided below (an example is provided in the Appendix).

1) To each clause C_j, $1 \leq j \leq m$, a place s_{C_j} is associated in Σ_C.
2) To each variable x_i, $1 \leq i \leq n$, four transitions t_{x_i}, $t_{\bar{x}_i}$, $\bar{t}_{x_i}$, and $\bar{t}_{\bar{x}_i}$ are associated in Σ_C. They are connected to places as follows (recall that, given a literal ℓ and a clause C_j, "$\ell \in C_j$" means that ℓ occurs in C_j).
   a) ${}^\bullet t_{x_1} = \{I_i\} = {}^\bullet t_{\bar{x}_i}$.
   b) ${}^\bullet \bar{t}_{x_1} = \{s_{x_i}, J_i\}$ and ${}^\bullet \bar{t}_{\bar{x}_i} = \{s_{\bar{x}_i}, J_i\}$.
   c) $t_{x_i}^\bullet = \{s_{x_i}\} \cup \{s_{C_j} \mid x_i \in C_j\}$.
   d) $t_{\bar{x}_i}^\bullet = \{s_{\bar{x}_i}\} \cup \{s_{C_j} \mid \bar{x}_i \in C_j\}$.
   e) $\bar{t}_{x_i}^\bullet = \{s_{C_j} \mid \bar{x}_i \in C_j\}$.
   f) $\bar{t}_{\bar{x}_i}^\bullet = \{s_{C_j} \mid x_i \in C_j\}$.
3) The transition t_0, when applied, marks the places $I_1, \ldots, I_n$. This allows exactly one of the transitions t_{x_i} and $t_{\bar{x}_i}$ to fire, for each $1 \leq i \leq n$. In this way, an assignment of the variables

x_i is simulated in Σ_C (remark that each place s_{C_j} will hold at most three tokens).

4) If the places s_{C_j}, $1 \le j \le m$, get marked (that is, the formula C is satisfiable) then the transition t can fire and a token in o is inserted. Clearly, in this case, at least one of the places s_{x_i} or $s_{\bar{x}_i}$ remains marked, which shows that the WF net Σ_C is not weakly sound.
5) If at least one place s_{C_j} cannot be marked, then t never fires (this corresponds to the fact that the formula C is not satisfiable); however, $\bar{t}$ can fire and it inserts one token in J_i, for all i. In this way, exactly one of $\bar{t}_{x_i}$ and $\bar{t}_{\bar{x}_i}$ can fire, for all i. When all these transitions have fired, each place s_{C_j} gets marked by exactly three tokens and the transitions t_1, t_2, and t_3 will empty the WF net, except for the place o which gets marked by exactly one token. Therefore, Σ_C is weakly sound.

A simple inspection of the WF net Σ_C shows that it is 3-bounded and acyclic. Moreover, the formula C is nonsatisfiable if and only if Σ_C is weakly sound.

The second step of our proof is to transform Σ_C into a 3-bounded acyclic ACWF net, equivalent to Σ_C with respect to soundness. The WF net Σ_C is not asymmetric choice because, for any $1 \le j \le m$, $s^\bullet \cap s^\bullet_{C_j} \neq \emptyset$ and neither $s^\bullet \subseteq s^\bullet_{C_j}$ nor $s^\bullet_{C_j} \subseteq s^\bullet$. However, if we insert in between s and t a new transition t' and a new place s' [and replace the arc (s, t) by the arcs (s, t'), (t', s'), and (s', t)], then the WF net Σ'_C such obtained is a 3-bounded acyclic ACWF net with the property that the formula C is nonsatisfiable if and only if Σ'_C is weakly sound.

As the complement of 3SAT is co-NP-complete, we conclude that the weak soundness of 3-bounded acyclic ACWF nets is co-NP-hard. ■

Corollary 2: The weak soundness problem of (3-bounded) acyclic ACWF nets is co-NP-complete.

Proof: The weak soundness problem of acyclic WF nets is in co-NP (Theorem 1) and it is co-NP-hard for 3-bounded acyclic ACWF nets (Theorem 2). ■

V. Conclusion

We have investigated in this paper the complexity of the soundness problem of acyclic WF nets, and we have proved two main results.

1) Soundness of 1-bounded acyclic WF nets is co-NP-complete.
2) Weak soundness of 3-bounded acyclic ACWF nets is co-NP-complete.

The results obtained in this paper are important from both theoretical and practical points of view. From a theoretical point of view, these results clearly position the soundness problem of acyclic WF nets into the hierarchy of complexity classes. They may also help to deeply understand the nature of the soundness problem of WF nets, as well as to show that other problems in Petri net or WF net theory are co-NP-hard. From a practical point of view, these results draw attention to the difficulty of checking the soundness of practical WFs modeled by acyclic (asymmetric-choice) WF nets.

Acyclicity and free-choiceness are two distinct structural constraints that can be imposed on Petri WF nets (a Petri net is free-choice if $s_1^\bullet = s_2^\bullet$, for any two places s_1 and s_2 with $s_1^\bullet \cap s_2^\bullet \neq \emptyset$): an acyclic WF net may be free-choice or not, and a free-choice WF net may be acyclic or not. Although acyclicity seems to be a very strong constraint, the soundness of acyclic WF nets has a quite high complexity (it is co-NP-complete) in comparison with the soundness of free-choice WF nets which can be solved in polynomial time. A deeper study of these two constraints, acyclicity and free-choiceness, in the context of soundness of WF nets, would be necessary.

The complexity of (weak) soundness of 1- or 2-bounded acyclic ACWF nets remains an interesting open problem.

Appendix

Fig. 3 illustrates the construction in Section III, applied to the Boolean formula

$$C = (x_1 \wedge \bar{x}_2 \wedge x_4) \vee (x_2 \wedge x_3 \wedge \bar{x}_4) \vee (\bar{x}_1 \wedge \bar{x}_3 \wedge x_5)$$

over the set $\{x_1, x_2, x_3, x_4, x_5\}$ of Boolean variables.

Fig. 4 illustrates the construction in Section IV, applied to the Boolean formula

$$C = (x_1 \vee x_2 \vee \bar{x}_3) \wedge (x_1 \vee \bar{x}_2 \vee x_3) \wedge (\bar{x}_1 \vee \bar{x}_2 \vee x_3)$$

over the set $\{x_1, x_2, x_3\}$ of Boolean variables.

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