T has property C of order $\chi(p, S, T)$, where, for given S, T, $\chi(p, S, T)$ is a certain computable function of p.

A function χ for which the weakened lemma holds is defined as follows: Let r be the number of restricted quantifiers within whose scopes the transformed expression (in the passage from S to T) lies; and let $n(i)$ $(1 \leqq i \leqq p - 1)$ be the number of elements of D_i^S, and $n(0) = 0$; then

$$\chi(p, S, T) = 2p - 1, \quad \text{for } r = 0,$$
$$\chi(p, S, T) = 1 + \sum_{k=1}^{p-1} \prod_{i=1}^{k} [1 + n(i)^r - n(i-1)^r], \quad \text{for } r > 0.$$

A proof of this is announced as forthcoming in an introduction by Burton Dreben to an edition of Herbrand's ***Écrits logiques*** by John van Heijenoort.

The authors have the great merit of having clarified a question which has been neglected for a long time.

As a point of detail it may be asked whether some simplification might not result by leaving out the added symbol "1" whenever free variables are present in $F(S)$, and ascribing then to these variables the height 1. PAUL BERNAYS

SHÔJI MAEHARA. *Another proof of Takeuti's theorems on Skolem's paradox.* ***Journal of the Faculty of Science, University of Tokyo,*** section I, vol. 7 part 5 (1958), pp. 541–556.

Choosing Gentzen's LK (442**2**) as a system for predicate calculus and Gentzen's system of IV 31 as a number-theoretic formal system, the author gives a finitary proof of the following: If Γ is an axiom system that does not involve the number-theoretic vocabulary and Γ is consistent in the restricted predicate calculus (in the predicate calculus with identity) then Γ (Γ^*) is consistent in the number-theoretic formal system. Γ^* results from Γ by introducing a new unary predicate $\mathscr{P}$ and transforming the formulas of Γ by restricting variables to $\mathscr{P}$.

In view of Gödel's completeness theorem this result may be viewed as a proof-theoretic version of the Löwenheim-Skolem theorem. Other such versions, due to Takeuti (XXIV 66(1), 66(2)) are shown to follow. ERWIN ENGELER

SHÔJI MAEHARA. *Remark on Skolem's theorem concerning the impossibility of characterization of the natural number sequence.* ***Proceedings of the Japan Academy,*** vol. 33 (1957), pp. 588–590.

The author shows (using the method of adjunction of a new individual constant and compactness) that no first-order axiom system can characterize the natural number sequence. He is apparently unaware of the previous wide applications of this method.

ERWIN ENGELER

RAPHAEL M. ROBINSON. *Restricted set-theoretical definitions in arithmetic.* ***Proceedings of the American Mathematical Society,*** vol. 9 (1958), pp. 238–242.

RAPHAEL M. ROBINSON. *Restricted set-theoretical definitions in arithmetic.* ***Summaries of talks presented at the Summer Institute for Symbolic Logic, Cornell University, 1957,*** 2nd edn., Communications Research Division, Institute for Defense Analyses, Princeton, N.J., 1960, pp. 139–140.

The second of these papers is a summary of the first. Two questions posed by Tarski are cited: "Problem 1. Is it possible to give a restricted set-theoretical definition of addition of natural numbers in terms of successor? Problem 2. Is there a decision method for the arithmetic of natural numbers based on the notion of successor and using restricted set theory?" Restricted set theory has two types of variable, one ranging over individuals (i.e., natural numbers), the other over sets of individuals (and thus no variables ranging over relations).

The author offers partial results. First, if α is a finite set, a natural number is associated with α equal to $\Sigma 2^i$, where i ranges over members of α. It is not difficult to define $\alpha = \beta \oplus \gamma$, which means that α, β, and γ are finite sets and the number associated with α equals the sum of the numbers associated with β and γ. The second result is that a restricted set-theoretical definition of addition is possible in terms of double and successor.

Several years after these papers appeared, Büchi managed to settle Tarski's two questions. See XXVIII 100. ROBERT MCNAUGHTON

K. JAAKKO HINTIKKA. *Reductions in the theory of types.* ***Two papers on symbolic logic,*** Acta philosophica Fennica no. 8, Helsinki 1955, pp. 57–115.

In this paper on the simple theory of types, Hintikka shows the power of his theory of model sets. After a clear brief exposition of the syntax and semantics of the simple theory of types, Hintikka presents a self-contained development of those apparatus of model sets appropriate to the simple theory of types. The bulk of the remainder of the paper consists in showing that the definition of satisfaction for formulas of arbitrary type can be characterized within a limited portion of the simple theory of types. In particular it is shown that there is a formula D of the second-order functional calculus and an effective rule which associates with each formula K of the theory of types a formula $\xi(K)$ of the first-order functional calculus such that if K is closed, then (1) K is valid if, and only if, $(D \rightarrow \xi(K))$ is valid and (2) K is logically false if, and only if, $(D \rightarrow \sim\xi(K))$ is valid. In addition D has only one quantified monadic predicate variable and $\xi(K)$ has only those free predicate variables which already occur in D. D is the conjunction of twenty sentences which are a line by line rendering of the definition of the notion of model set for the simple theory of types. All of these sentences are first-order except one which has a lone second-order quantifier. The constant predicates used represent such primitive notions as "a belongs to the type of individuals" and "the string c is obtained by concatenating the strings a and b (in this order)." $\xi(K)$ is a translation of the formula K into the first-order functional calculus in which the constant predicates like those just mentioned are so used that if the properties attributed to them by D hold, then the validity of K is equivalent to the validity of $\xi(K)$. Thus (1) and (2) hold. The proofs are presented in sufficient, but not unnecessary, dreary detail. It is also shown as a corollary that if we consider satisfaction in general models, that is to say models in which the domain of quantification of variables beyond the first order does not always have to encompass every possible appropriate subset or relation of the domain of next lower type, then the only formula in D with second-order quantifiers will be eliminated and the procedures in the paper reduce, in the sense explained above, every closed formula in the simple theory of types to a formula in the first-order functional calculus. W. B. PITT

SOLOMON FEFERMAN. *Systems of predicative analysis.* ***The journal of symbolic logic,*** Bd. 29 Heft 1 (1964), S. 1–30.

Die Arbeit enthält im wesentlichen den Inhalt eines Hauptvortrages, den der Verf. im Januar 1963 auf der Tagung der Association for Symbolic Logic in Berkeley gehalten hat.

Im I. Teil werden die Grundgedanken diskutiert: Die Imprädikativität der klassischen Analysis, die verzweigte Typenlogik von Russell und die prädikativ aufgebaute elementare Analysis von Hermann Weyl. Auf der Grundlage der verzweigten Typenlogik ergibt sich eine Schichtung der "prädikativ definierbaren" Mengen, die bis zu einer Mengengesamtheit $\mathscr{M}_{\omega_1}$ führt, wobei ω_1 die kleinste nicht rekursiv definierbare Ordinalzahl ist. Nach Kleene stimmt $\mathscr{M}_{\omega_1}$ mit der Gesamtheit der hyperarithmetischen Mengen überein. Dieselbe Mengengesamtheit ergibt sich mit einer Progression, die nicht auf der Schichtung der verzweigten Typenlogik, sondern auf einem semantisch