Nonlinear Systems Fault Diagnosis with Differential Elimination

Zhang Jiafan
Department of Mechanical Engineering
Wuhan Polytechnic University
Wuhan 430023, China
e-mail: jfz@whpu.edu.cn

Abstract—**The differential elimination algorithm is used to eliminate the non-observed variables of the nonlinear systems. By incorporating the algebraic observability and diagnosability concepts and using numerical differentiation algorithms, another approach to the certain classes of nonlinear systems fault diagnosis problem is presented.**

Keywords--nonlinear systems; fault diagnosis; differential elimination; algebraic observability and diagnosability

I. INTRODUCTION

Numerous control strategies and fault detection algorithms are based on the assumption that all internal states of the dynamic systems are available. In most cases, however, only a few of the states or some functions of the states can be measured. Different from the usual state reconstruction and estimation methods, in this paper, we try to present another approach to the some nonlinear systems diagnosis, the idea here consists in eliminating the non-observed variables of the model using the differential elimination algorithm. In other words, the idea consists in computing a differential polynomial which lies in the differential ideal generated by the model equations and which only involves the observed variables, their derivatives up to some finite orders, the unknown fault variables and the model parameters. The fault diagnosis problem of the some nonlinear systems thus could be solved based upon algebraic fault observability and diagnosability[1-4].

Commutative algebra, which is mainly concerned with the study of commutative rings and fields, provides the right tools for understanding algebraic equations. Differential algebra, which was mainly founded by Ritt[5] and Kolchin[6], extends to differential equations concepts and results from commutative algebra. The differential elimination is an algorithmic subtheory of the differential algebra[7,8]. It solves the membership problem for radical differential ideals. The differential elimination processes that are presented in this paper take as input two parameters: a system of polynomial (thus nonlinear) differential equations, ordinary or with partial derivatives and a ranking. They produce on the output an equivalent finite set of polynomial differential systems, which are simpler, in the sense that they involve some differential equations which are consequences of the input system but were somehow hidden.

The rest of this paper is organized as follows: in Section 2, we introduce some basic differential algebra definitions related to the differential elimination. The algebraic observability and diagnosability are briefly described in Section 3. In Section 4, an academic example is presented to show the differential elimination application in the nonlinear systems diagnosis, with the help of MAPLE computer algebra software. Finally, in Section 5 the paper is closed with some concluding remarks.

II. BASIC CONCEPTS ON DIFFERENTIAL ELIMINATION

Some basic concepts regarding differential algebra and the differential elimination are listed. Further details are obtained in [5-8] and references therein.

A *differential ring* (resp. field) is a ring (resp. field) R endowed with a derivation (this paper is restricted to the case of a single derivation but the theory is more general) $\frac{d}{dt}: R \to R$ such that, for any $a, b \in R$

1) $\frac{d}{dt}(a+b) = \dot{a} + \dot{b}$,

2) $\frac{d}{dt}(ab) = \dot{a}b + a\dot{b}$.

where $\frac{da}{dt} = \dot{a}$, $\frac{d^{\nu}a}{dt^{\nu}} = a^{(\nu)}$, $\nu \geq 0$.

Algorithmically, one is led to manipulate finite subsets of some *differential polynomial ring* $R = K\{U\}$ where K is the differential field of coefficients (in practice, $K = \mathbb{Q}$, $\mathbb{Q}(t)$ or $\mathbb{Q}(k_1, \ldots, k_r)$ where the k_i denote parameters that would be assumed to be algebraically independent) and U is a finite set of dependent variables $u_1,\ldots,u_m$. The elements of R, the differential polynomials are just polynomials in the usual sense, built over the infinite set, denoted ΘU, of all the derivatives of the dependent variables u_i.

Definition 2.1: A subset I of a differential ring R is a *differential ideal* of R if it is an algebraic ideal of R and moreover, it is closed under the action of the derivation.

If $p_1, \ldots, p_n$ are elements of R, the differential ideal $I = [p_1, .., p_n]$ generated by the p_i is the set of all the elements of R that are finite linear combinations (with elements of R for coefficients) of the p_i and their derivatives up to any order.

978-0-7695-3645-3/09 $25.00 © 2009 IEEE
DOI 10.1109/CINC.2009.38

The radical of a differential ideal I of R is the set of all the ring elements a power of which belongs to I, i.e. if an element r of R belongs to I whenever there exists some nonnegative integer p such that $r^p \in I$. The *radical differential ideal* $J = \{p_1, .., p_n\}$ generated by the p_i is the set of all the elements of R, a power of which belongs to the differential ideal $I = [p_1, .., p_n]$.

Theorem 2.1[5]: Let R be a differential polynomial ring and F be a finite subset of R. A differential polynomial p of R lies in the radical of the differential ideal generated by F if and only if it vanishes over every analytic solution of F.

The study of the radical of the differential ideal generated by a finite system of differential polynomials is strongly related to the study of the analytic solutions of this system. Consider a differential system given by equations of the form p_1=0,...., p_n=0, where the p_i's are differential polynomials in some unknown functions (i.e. dependent variables) $u_1,...,u_m$ of t and their derivatives. Actually, the radical differential ideal generated by p_1,...., p_n is the biggest set of equations with the same meromorphic solutions as the system p_1=0,...., p_n=0. Furthermore, the system p_1=0,...., p_n=0 has at least one meromorphic solution if and only if the radical differential ideal generated by p_1,...., p_n does not contain an element independent of the unknown functions. These two last facts are the fundamental results of differential algebra. Theorem 2.1 establishes a one-to-one correspondence between the zero sets of differential systems and radical differential ideals generated by the differential systems. Studying and decomposing the zero set of a differential system amounts to studying and decomposing the radical differential ideal it generates. The Rosenfeld-Gröebner algorithm computes a characteristic decomposition of a radical differential ideal generated by a finite set of differential polynomials, solves the membership problem to radical differential ideals [7]. It is implemented in the *diffalg* package of MAPLE. To present it, one needs to define the concept of ranking.

Definition 2.2: A *ranking* is a total order over the set ΘU of the derivatives of the dependent variables of R which satisfies:

$a < \dot{a}$ *and* $a < b \Rightarrow \dot{a} < \dot{b}$ *for all* $a, b \in \Theta U$.

Let U be a finite set of dependent variables. A ranking such that, for every $u, v \in U$, the ith derivative of u is greater than the jth derivative of v whenever $i > j$ is said to be *orderly*. An elimination ranking between the two dependent variables u, v (say $u > v$) satisfies that any derivative of u is greater than any derivative of v. Such rankings are said to *eliminate* u w.r.t. v.

The Rosenfeld-Gröebner algorithm gathers as input a finite system F of differential polynomials and a ranking. It returns a finite family (possibly empty) $C_1, . . . , C_r$ of finite subsets of $K\{U\}\backslash K$, called *regular differential chains*. Each system C_i defines a differential ideal G_i (it is a *characteristic set* of G_i). The relationship with the radical $\Im$ of the differential ideal generated by F is the following:

$$\Im = G_1 \cap \cdots \cap G_r$$

The computations and the differential characteristic sets obtained depend on the chosen ranking. Various properties of the zero set of a system of differential polynomials can be exhibited with an appropriate choice of the ranking.

III. DIFFERENTIAL AND ALGEBRAIC APPROACH OF NONLINEAR SYSTEMS DIAGNOSIS

A variety of approaches have been proposed to solve the diagnosis problem for nonlinear systems. Some model-based approaches can be found, as the approaches based upon differential geometric methods [9,10]. On the other hand there are approaches proposed in an algebraic and differential framework[1-4]. In their papers, some results based on the differential algebra are used to propose the algebraic observability concept of the variable which models the failure presence for the solvability of the nonlinear systems diagnosis problem, determine fault diagnosability with the minimum number of measurements from the system. In the following, an introduction to the algebraic and differential method is briefly described. Firstly, some related differential algebraic concepts are further listed.

Definition 3.1: Let L and K be differential fields. A differential field extension L/K is given by K and L such that: 1) K is a subfield of L and; 2) the derivation of K is the restriction to K of the derivation of L.

Definition 3.2: An element is said to be differentially algebraic with respect to the field K if it satisfies a differential algebraic equation with coefficients over K.

Definition 3.3: An element is said to be differentially transcendental over K, if and only if, it is not differentially algebraic over K.

Definition 3.4: Let G, $K\langle u\rangle$ be differential fields. A Dynamics consists in a finitely generated differential algebraic extension $G/K\langle u\rangle$, ($G = K\langle u, \xi\rangle$, $\xi \in G$). Any element of G satisfies an algebraic differential equation with coefficients being rational functions over K in the elements of u and a finite number of their time derivatives.

Definition 3.5: Let a subset $\{u,y\}$ of G in a dynamics $G/K\langle u\rangle$. An element in G is said to be algebraically observable with respect to $\{u,y\}$ if it is algebraic over $K\langle u,y\rangle$. Therefore, a state x is said to be algebraically observable if, and only if, it is algebraically observable with respect to $\{u,y\}$. A dynamics $G/K\langle u\rangle$, with output y in G is said to be algebraically observable if, and only if, the state has this property.

Definition 3.6: A fault is a not permitted deviation of at least one characteristic property or parameter of any process in relation to the development of the same parameter under normal conditions. Faults are defined as transcendent elements over $K\langle u\rangle$, therefore, a system with the presence of faults is a differential transcendental extension, denoted as $K\langle u,f,y\rangle/K\langle u\rangle$, where f is a fault vector and its time derivatives.

Let us consider the class of nonlinear systems with faults described by the following equation:

$$\begin{cases} \dot{x}(t) = A(x, f, u) \\ y(t) = h(x, u) \end{cases} \tag{1}$$

Where $x = (x_1, . . . , x_n)^T \in \mathbb{R}^n$ is a state vector, $u = (u_1, . . . , u_m) \in \mathbb{R}^m$ is a known input vector, $f = (f_1, . . . , f_\mu) \in \mathbb{R}^\mu$ is a fault vector, $y = (y_1, \ldots, y_p) \in \mathbb{R}^p$ is an output vector. A and h are assumed to be analytical vector functions.

Definition 3.7(Algebraic observability)[4]: An element $f\in K\langle u,f\rangle$ is said to be algebraically observable if f satisfies a differential algebraic equation with coefficients over $K\langle u,y\rangle$.

Definition 3.8 (Diagnosability)[4]: A nonlinear system described by Eq. (1) is said to be diagnosable if it is possible to estimate the fault f from the system equations and the time histories of the data u and y, i.e. it is diagnosable if f is algebraically observable with respect to u and y.

There is an immediate consequence of the diagnosability condition given by the definition 3.8, i.e., it is required for the solvability of the nonlinear systems diagnosis problem that each fault component be able to be written as a solution of a polynomial equation in f_i and finitely many time derivatives of u and y with coefficients in K

$$H_i(f_i, u, \dot{u}, \cdots, y, \dot{y}, \cdots) = 0. \qquad (2)$$

Obviously, when each fault component can be able to be written as the form of equations (2) in f_i and finitely many time derivatives of u and observed state x, the nonlinear systems is also diagnosable.

It should be noted that, based on aforementioned fault observability and diagnosability conditions, the nonlinear systems diagnosis problem boils down to numerical differentiation, i.e., to the derivatives estimations of noisy time signals (i.e., the data u and observed state x). This classic ill-posed mathematical problem has been already attacked by numerous means[11-14].

IV. AN EXAMPLE

Let consider the following system:

$$\begin{cases} \dot{x}_1 = -k_1 x_1 + k_2 x_2 - \dfrac{f\, x_1}{k_3 + x_1} \\ \dot{x}_2 = k_1 x_1 - k_2 x_2 \end{cases} \qquad (3)$$

x_1 is assumed to be an observed variable, i.e. the time histories of measures are assumed to be available for x_1, x_2 is the non-observed variable of the system, f the fault variable, and k_1, k_2, k_3 are non-negative known model parameters. This academic example is adapted from [8], used here to show the differential elimination application in the nonlinear systems diagnosis. the idea here consists in computing a differential polynomial which lies in the differential ideal generated by the model equations and which only involves the observed variable x_1, its derivatives up to some finite orders, the unknown fault variables and the model parameters. Let us show how to do this with the help of *diffalg*.

To compute this polynomial, the Rosenfeld-Gröebner algorithm is applied over the equations (3). The ranking eliminates x_2 w.r.t. x_1: $x_2 >> x_1$. In other words, the ranking means that we are looking for a polynomial free of x_2. The right-hand side of the first model equation is rewrited as a rational fraction form, its numerator is stored in the list of the equations (first parameter to the Rosenfeld-Gröebner command of *diffalg*). The denominator is stored in the list of the inequations (second parameter to the Rosenfeld-Gröebner command of *diffalg*). To avoid splitting cases on parameters values, one views them as (transcendental) elements of the base field of the differential polynomials, the fault variables are also defined as transcendental elements.

K := field_extension(transcendental_elements = [k_1, k_2, k_3,f]):

R := differential_ring(derivations = [t], notation = *diff*, field_of_constants = K, ranking = [x_2, x_1]):

ideal:=Rosenfeld-Groebner([*numer*(*eq1*), *eq2*], [*denom*(*eq1*)], R);

ideal := [*characterizable*]

The characteristic set *ideal* involves two polynomials. The one which does not involve x_2 is the second one, which is displayed below (as shown in [8]):

$$\ddot{x}_1(x_1+k_3)^2 + [k_1+k_2]\dot{x}_1(x_1+k_3)^2 + fk_3\dot{x}_1 + fk_2x_1(x_1+k_3) = 0$$

This differential polynomial equation is rewrited as follows:

$$f = -\frac{\ddot{x}_1(x_1+k_3)^2 + [k_1+k_2]\dot{x}_1(x_1+k_3)^2}{k_3\dot{x}_1 + k_2x_1(x_1+k_3)} \qquad (4)$$

With the measure data x_1, its derivatives $\dot{x}_1$ and $\ddot{x}_1$ can be estimated using numerical differentiation algorithms. So it is possible to estimate the fault f from the available measurements of the system.

V. CONCLUDING REMARHS

A subtheory of the differential algebra, the differential elimination, has proved to be useful in the diagnosis problem for nonlinear systems. It could eliminate the non-observed variables of the systems, compute an equivalent finite set of differential polynomial systems, which only involves the observed variables, their derivatives up to some finite orders, the unknown fault variables and the model parameters. By incorporating the algebraic fault observability and diagnosability conditions and using numerical differentiation algorithms, the differential elimination seems to be a very promising tool for the certain classes of nonlinear systems fault diagnosis problem.

REFERENCES

[1] R. Martinez-Guerra, J. Mendoza-Camargo, “Observers for a class of liouvillian and non-differentially flat systems,” IMA Journal of Mathematical Control and Information, no. 21, pp. 493–509, 2004.

[2] M. Fliess, C. Join, H. Mounier, “An introduction to nonlinear fault diagnosis with an application to a congested internet router,” Advances in Communication Control Networks, C.T. Abdallah, J. Chiasson (Eds), Lecture Notes, Conf. Inf. Sci, Springer, Berlin, vol. 308, pp. 327–343, 2005.

[3] R. Martinez-Guerra,A. Luviano-Juarez,J. J. Rincon-Pasaye,“Fault estimation using algebraic observers,” Proceedings of the 2007 American Control Conference,New York City, USA, July 11-13, 2007.

[4] J. C. Cruz-Victoria, R. Martinez-Guerra,J. J. Rincon-Pasaye,“On nonlinear systems diagnosis using differential and algebraic methods,” Journal of the Franklin Institute, vol. 345, pp.102–118, 2008.

[5] J.F. Ritt, Differential Algebra, Dover Publications Inc., New York, 1950.

[6] E.R. Kolchin, Differential Algebra and Algebraic Groups, Academic Press, 1973.

[7] F.Boulier, D.Lazard, F.Ollivier, and M.Petitot,"Representation for the radical of a finitely generated differential ideal," Proceedings of the 1995 international symposium on Symbolic and algebraic computation, ACM Press, NY, USA, pp.158–166, 1995.

[8] F. Boulier,"Differential Elimination and Biological Modelling," Johann Radon Institute for Computational and Applied Mathematics Book Series, Vol..2, pp.111-139. 2007.

[9] C. D. Persis, A. Isidori, "A geometric approach to nonlinear fault detection and isolation," IEEE Trans. Automat. Control, vol. 46, no. 6, pp. 853–865, 2001.

[10] M. Massoumnia, G. Verghese, A. Willsky, "Failure detection and identification," IEEE Transactions in Automatic Control, vol. 34, pp. 316–321,1989.

[11] T.E. Duncan, P. Mandl, B. Pasik-Duncan, "Numerical differentiation and parameter estimation in higher-order linear stochastic systems," IEEE Trans. Automat. Control, vol. 41, pp.522–532,1996.

[12] S.Ibrir, S. Diop,"A numerical procedure for filtering and efficient high-order signal differentiation," Int. J. Appl. Math. Comput. Sci., Vol. 14, pp. 201-208, 2004.

[13] S.Ibrir, "Online exact differentiation and notion of asymptotic algebraic observers," IEEE Trans. Automat. Control, vol. 48, pp.2055-2060, 2003.

[14] M.Braci,S.Diop, "On numerical differentiation algorithms for nonlinear estimation," Proc. 42nd IEEE Conf. Decision Control, Maui, Hawaii,2001.