GAME OR NOT GAME?

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Index / Wadge problem for a class C

INPUT: a language L in C

OUTPUT:
- the minimal (non-det / alternating) index needed to recognize L
- the Wadge degree of L

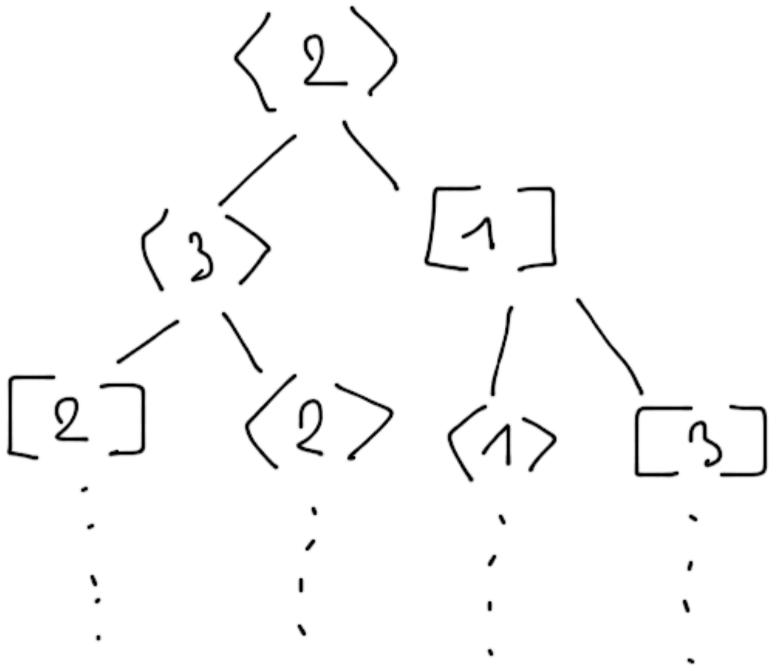
Game Automata

- a finite alphabet Σ,
- a finite set of states Q,
- an initial state $q_I \in Q$,
- a transition function $\delta : Q \times \Sigma \to \begin{cases} (0, q_0) \vee (1, q_1) \\ (0, q_0) \wedge (1, q_1) \end{cases}$,
- a rank function $\mathsf{rank} : Q \to \mathbb{N}$

$W_{(1,3)}$

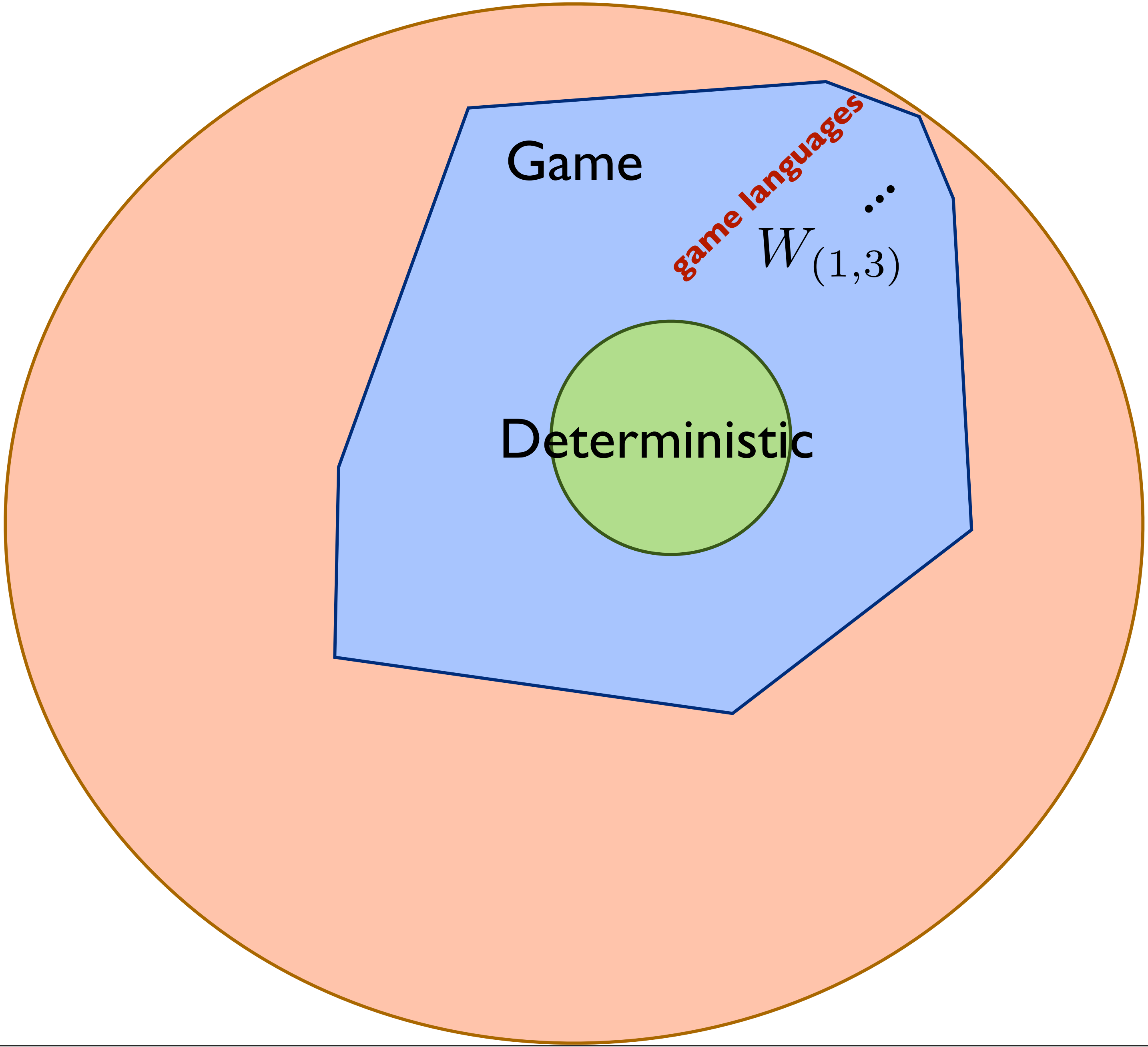
$(\Diamond, m) \mapsto \langle m \rangle$

$(\Box, m) \mapsto [m]$

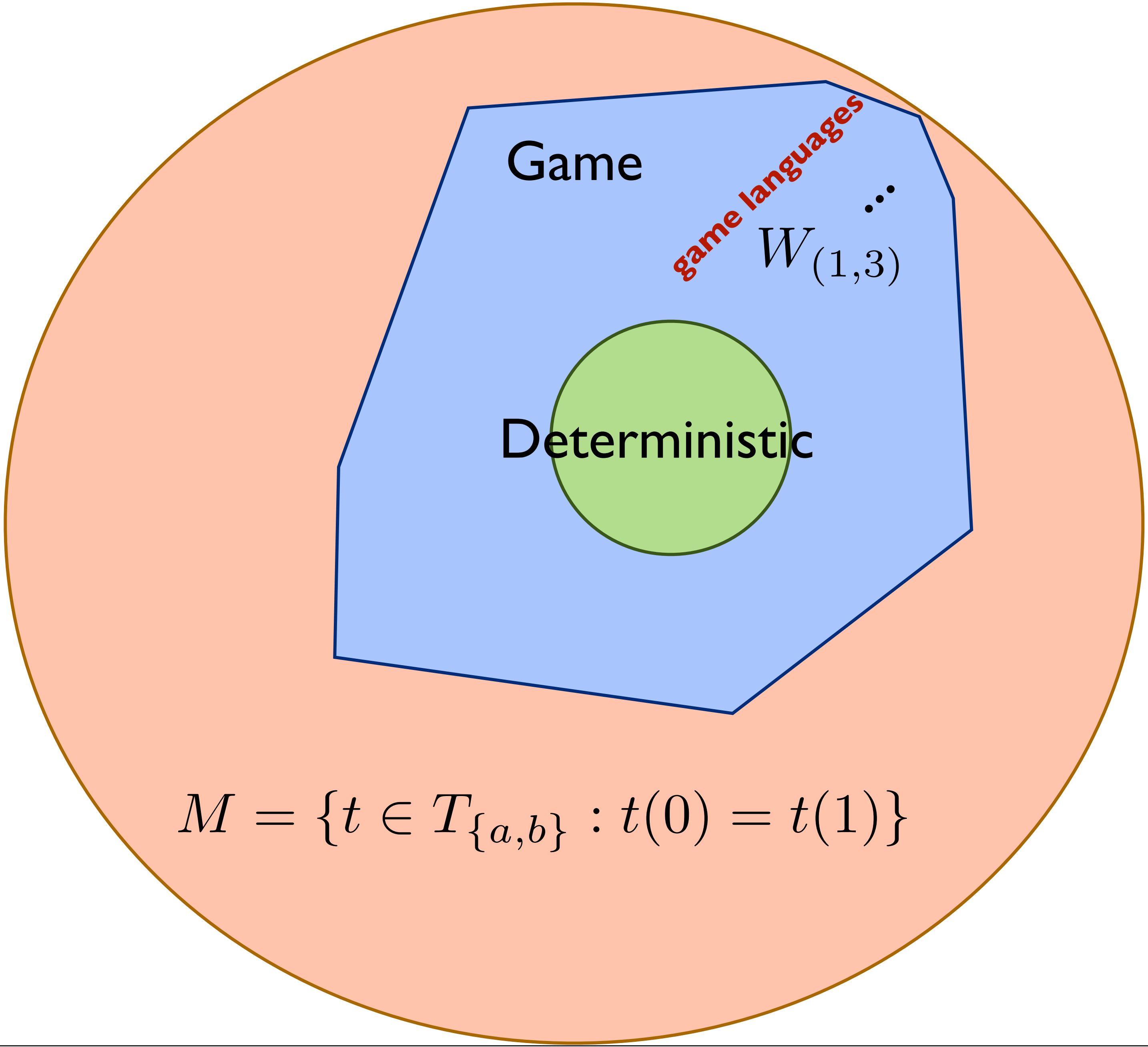


$t \in T_\Sigma$, where $\Sigma = \{\Diamond, \Box\} \times \{1, 2, 3\}$

Non-deterministic / Alternating



Non-deterministic / Alternating



Proposition (Duparc, F., M., 11): The class of game languages is the largest class of regular languages:

- extending the deterministic one,

- closed under complementation and substitution,

- and for which substitution preserves the equivalence relations of having the same index and having the same Wadge degree

Index problem for a class C

INPUT: a language L in C

OUTPUT: the minimal (non-det / alternating) index needed to recognize L

Index problem for a class C

INPUT: a language L in C

OUTP…

… to

Theorem (FMS, LICS 13): The non-deterministic and alternating index problems are decidable for game automata

Index problem for a class C

INPUT: a language L in C

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Deciding membership in C

Theorem (Niwinski-Walukiewicz 03): Given a regular language L, it is decidable whether L is recognizable by a **deterministic** automaton

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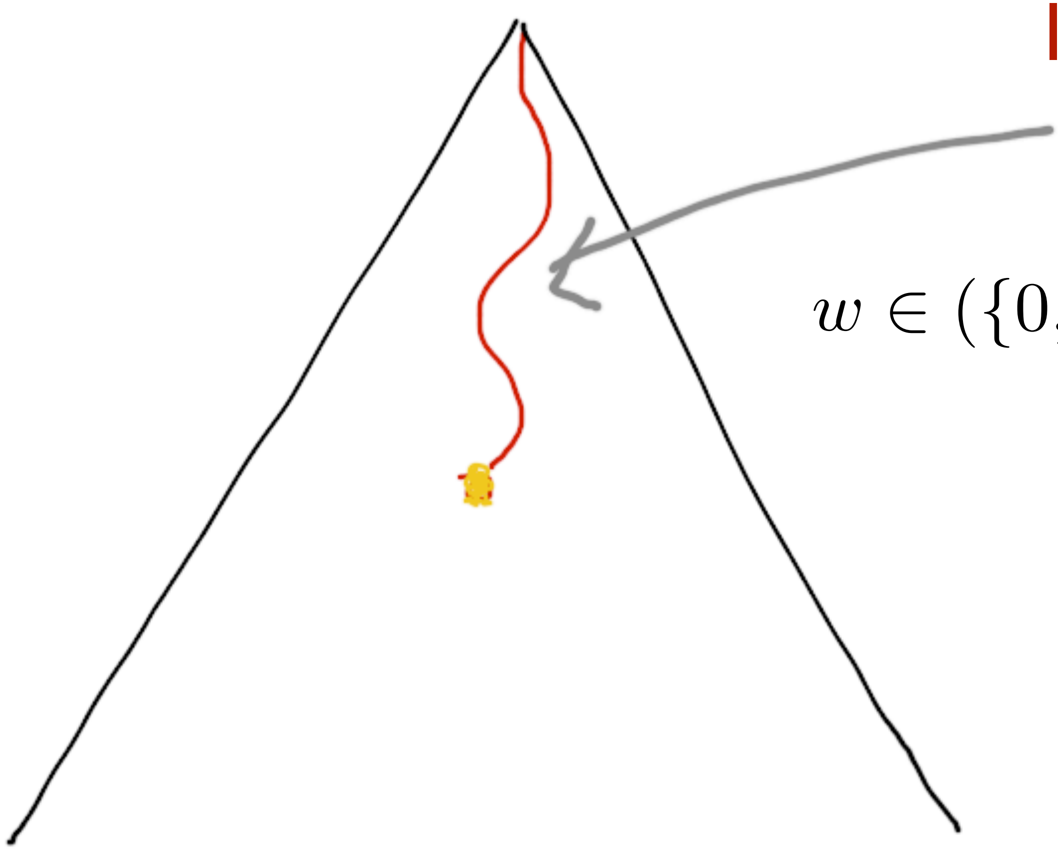
Theorem (FMS, LICS 13): Given a regular language L, it is decidable whether L is recognizable by a **game** automaton

Proof idea: Use of some basic tools from the composition methods.



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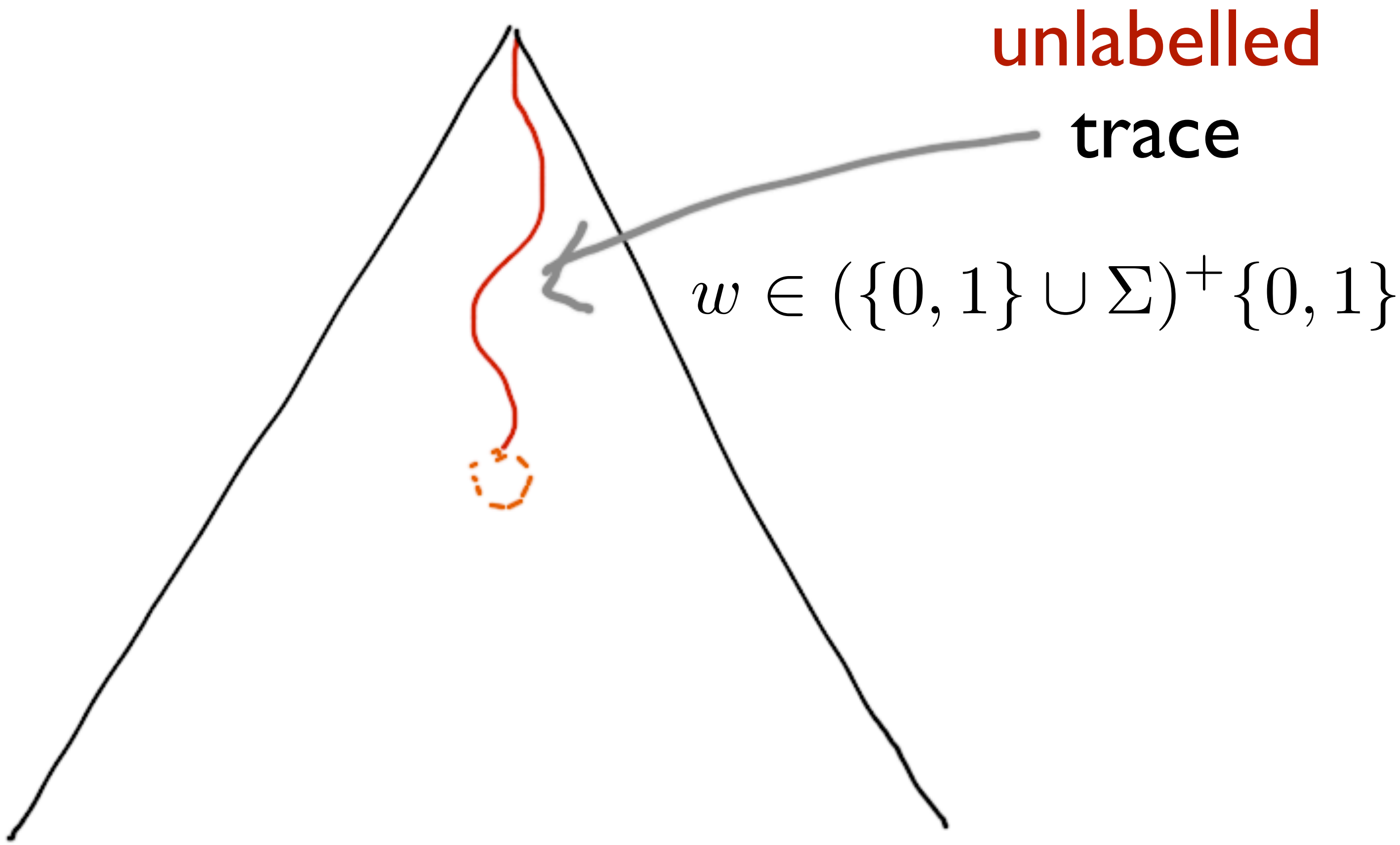
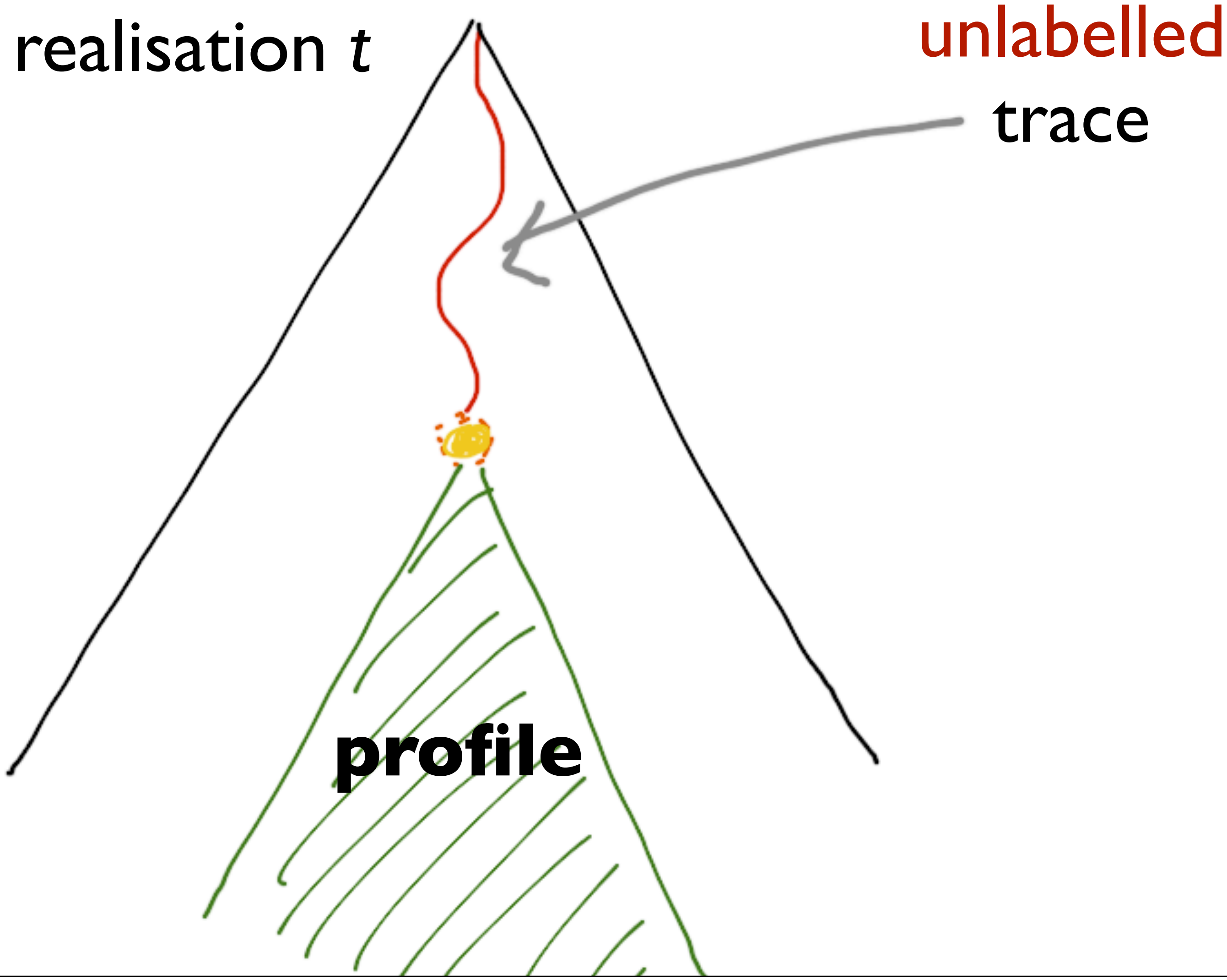


Proof idea: Given a regular language L,



binary profiles

unary profiles

- $Z_0 \times Tr_\Sigma$
- $Tr_\Sigma \times Z_1$
- $Z_0 \times Tr_\Sigma \cup Tr_\Sigma \times Z_1$
- $Z_0 \times Z_1$

- Z

non trivial

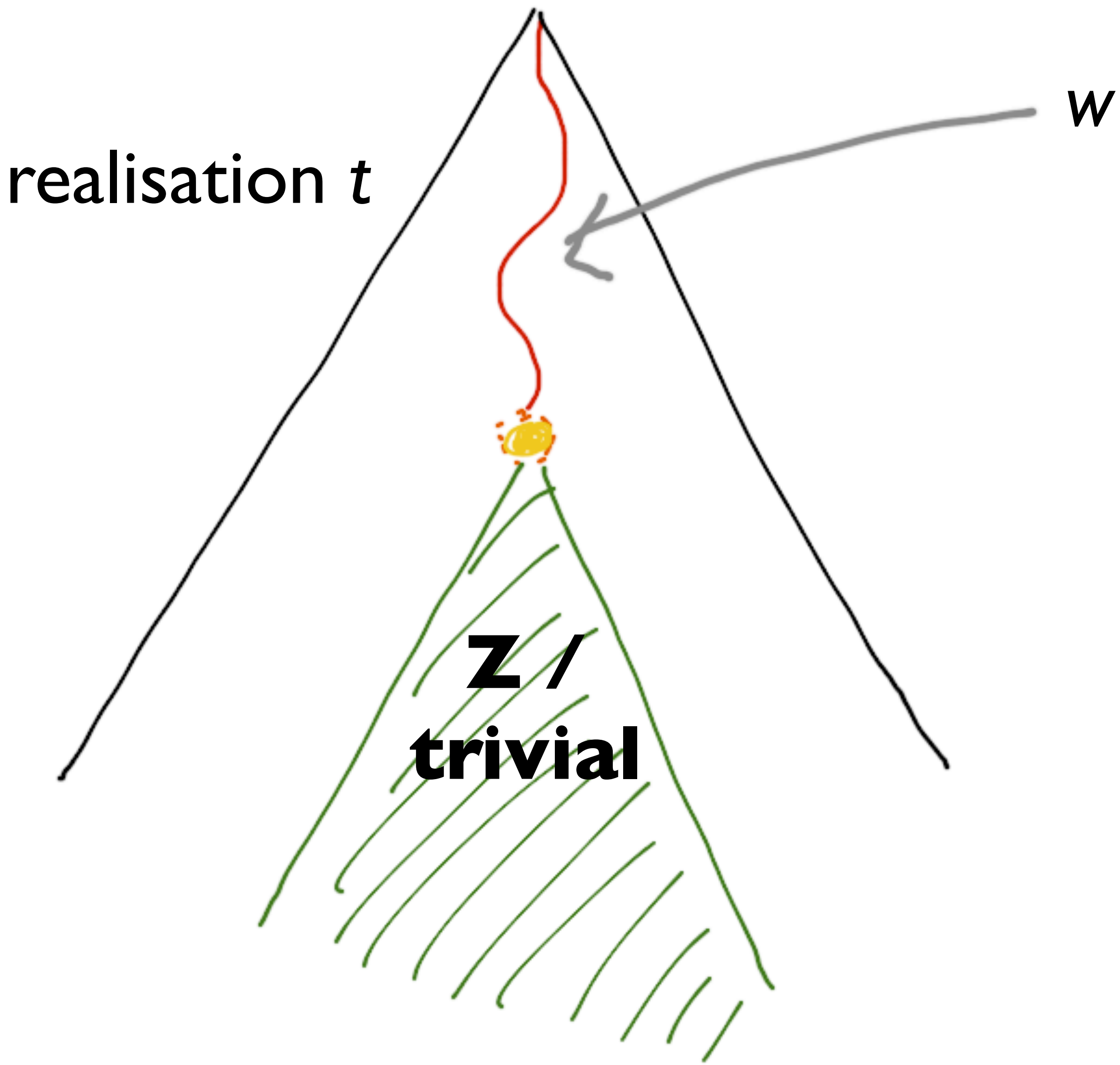
(labelled traces)

(unlabelled traces)

Definition :

- A trace w has non-trivial profile Z in a regular language M, if for each realisation t of w either $t^{-1}M$ is trivial or $t^{-1}M = Z$, and for some realisation t_0, $t_0^{-1}M = Z$.

Given L,



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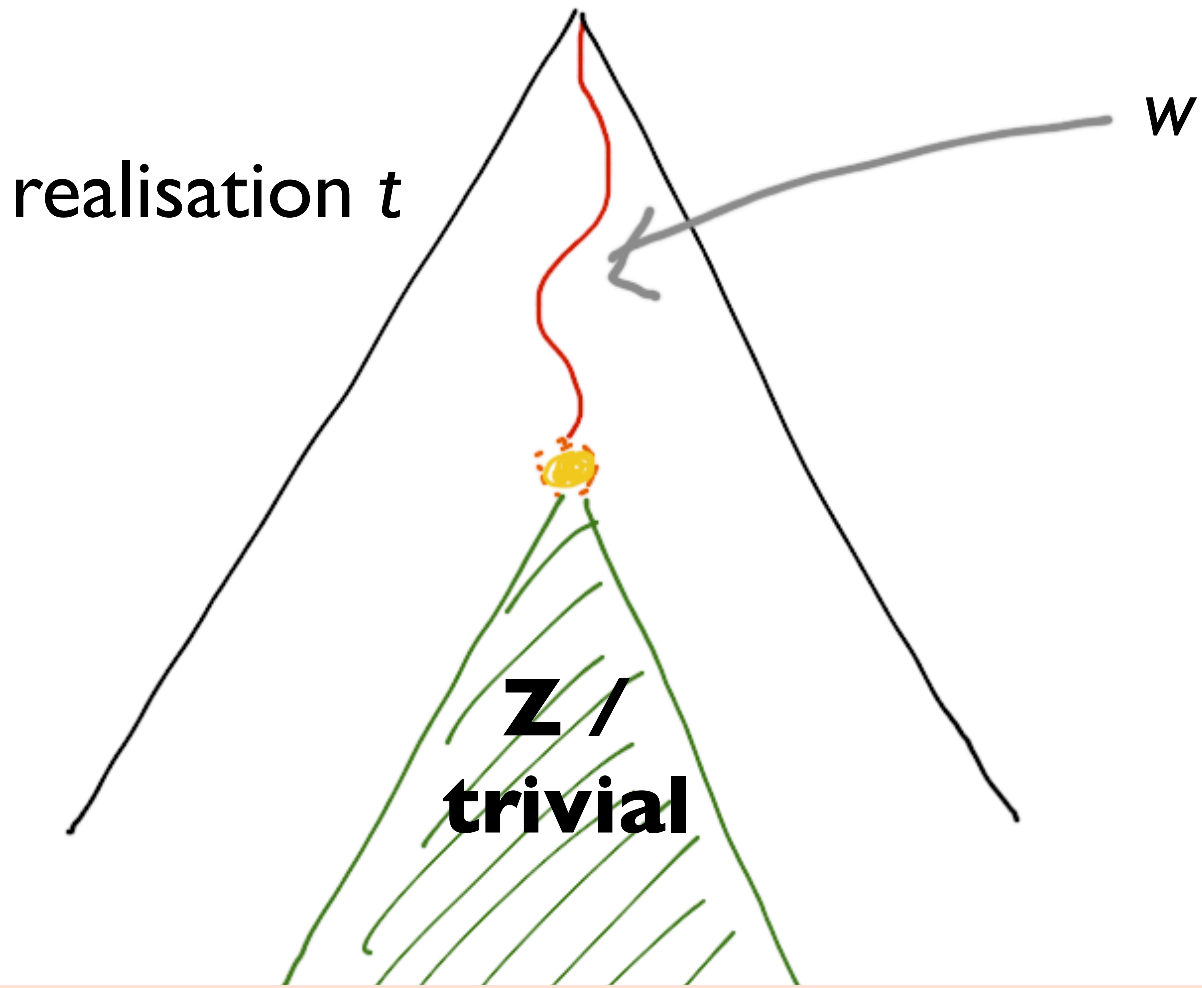

Note that: every trace has at most one profile in a regular language

$$M = \{t \in T_{\{a,b\}} : t(0) = t(1)\}$$

0 has no profile in M

A regular language is **locally game**
if every trace has a profile in it.

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regular property

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L is game

$\downarrow$

L is locally game

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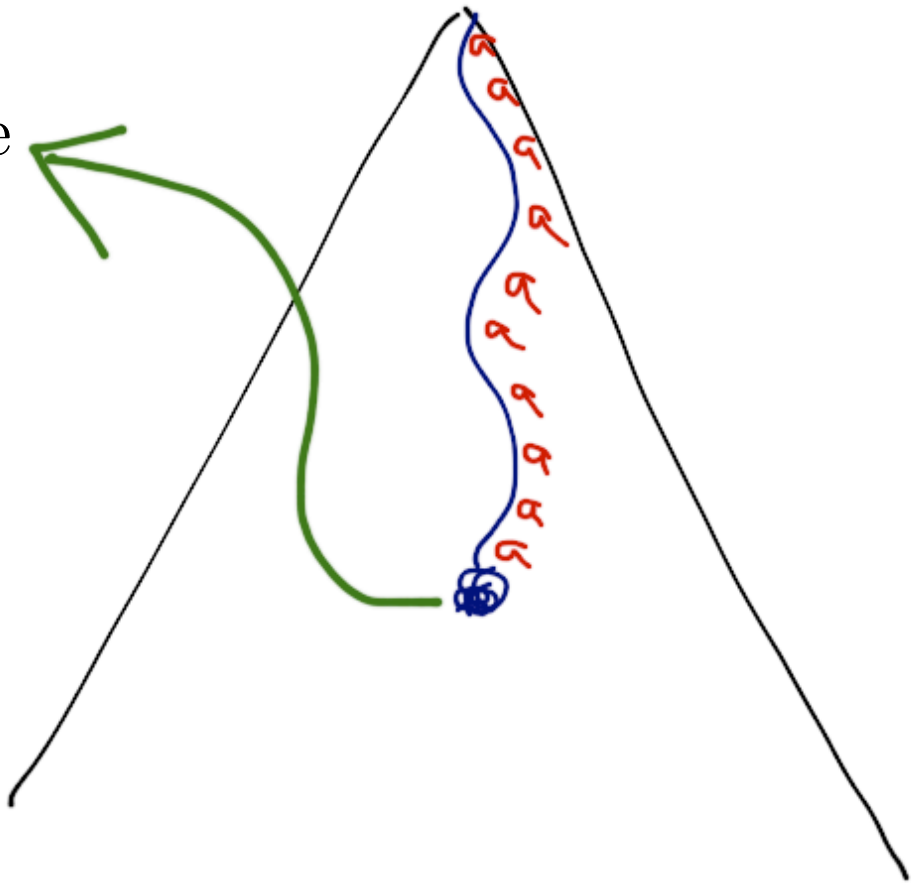
L is game

??

L is locally game

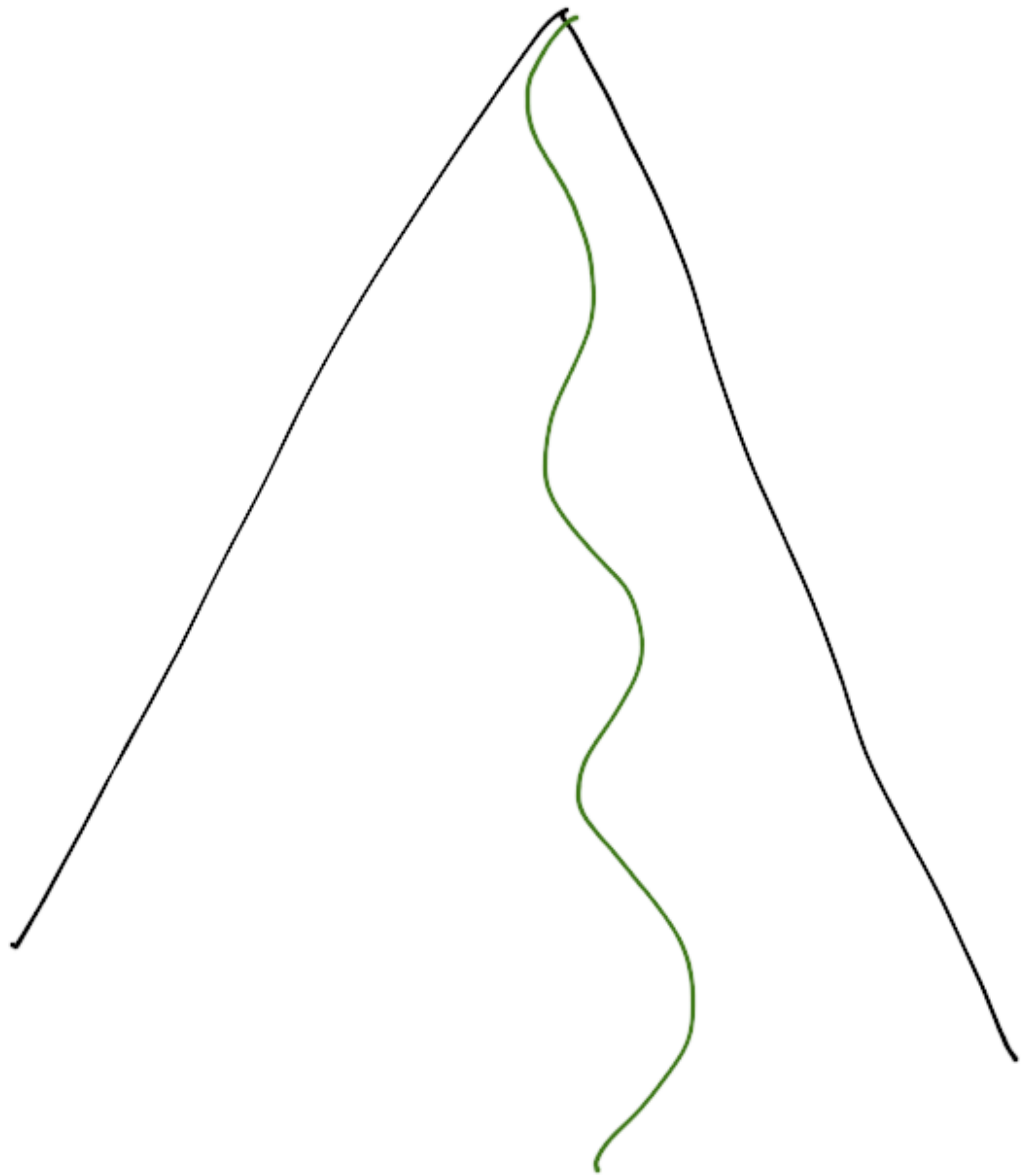
Counter-example:

a-reachable



$$Thin := \{t \in Tr_{\{a,b\}} \mid \|\{x \in dom(t) \mid x \text{ is } a\text{-reachable}\}\| \leq \aleph_0\}$$

M, t, π



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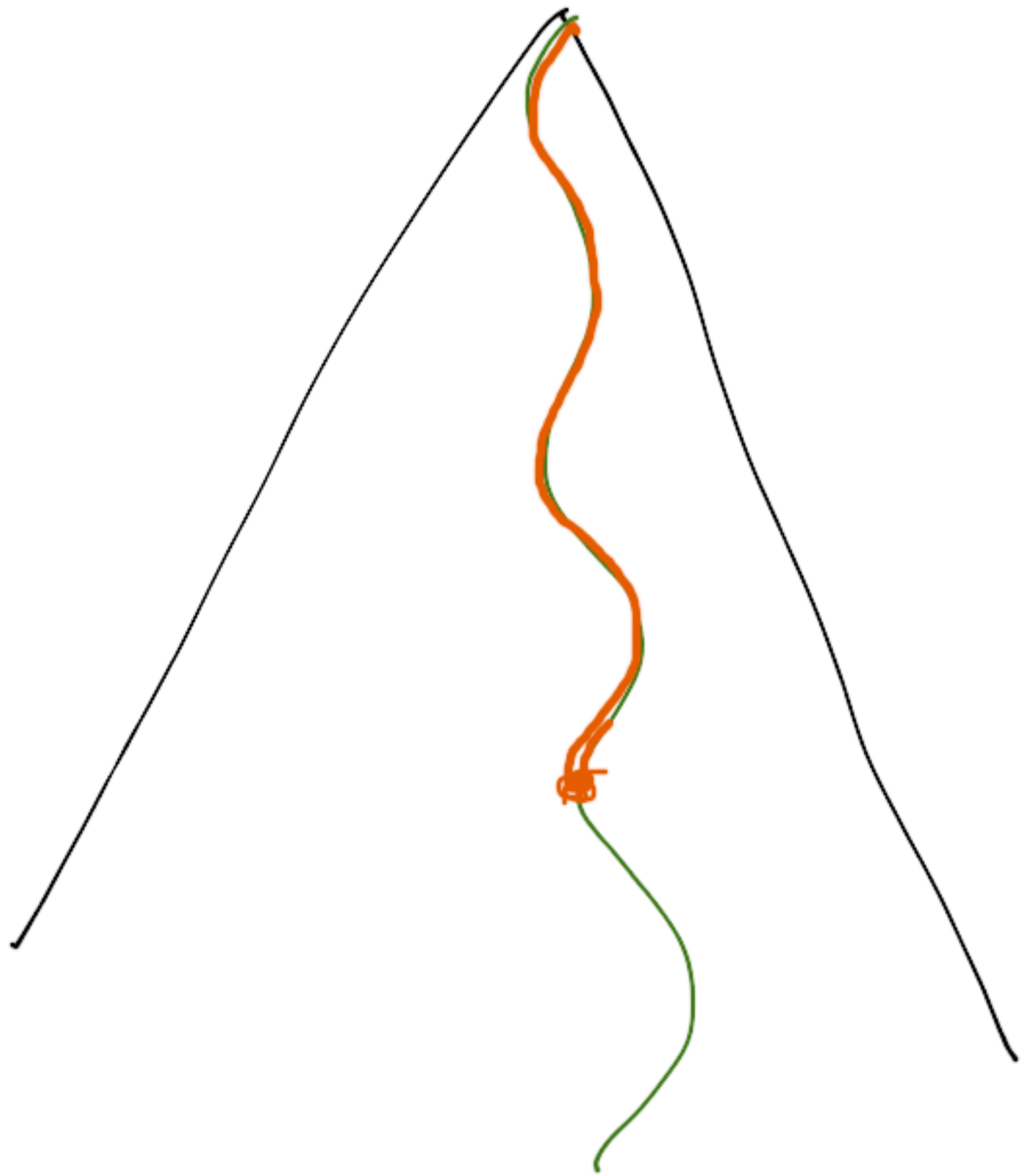

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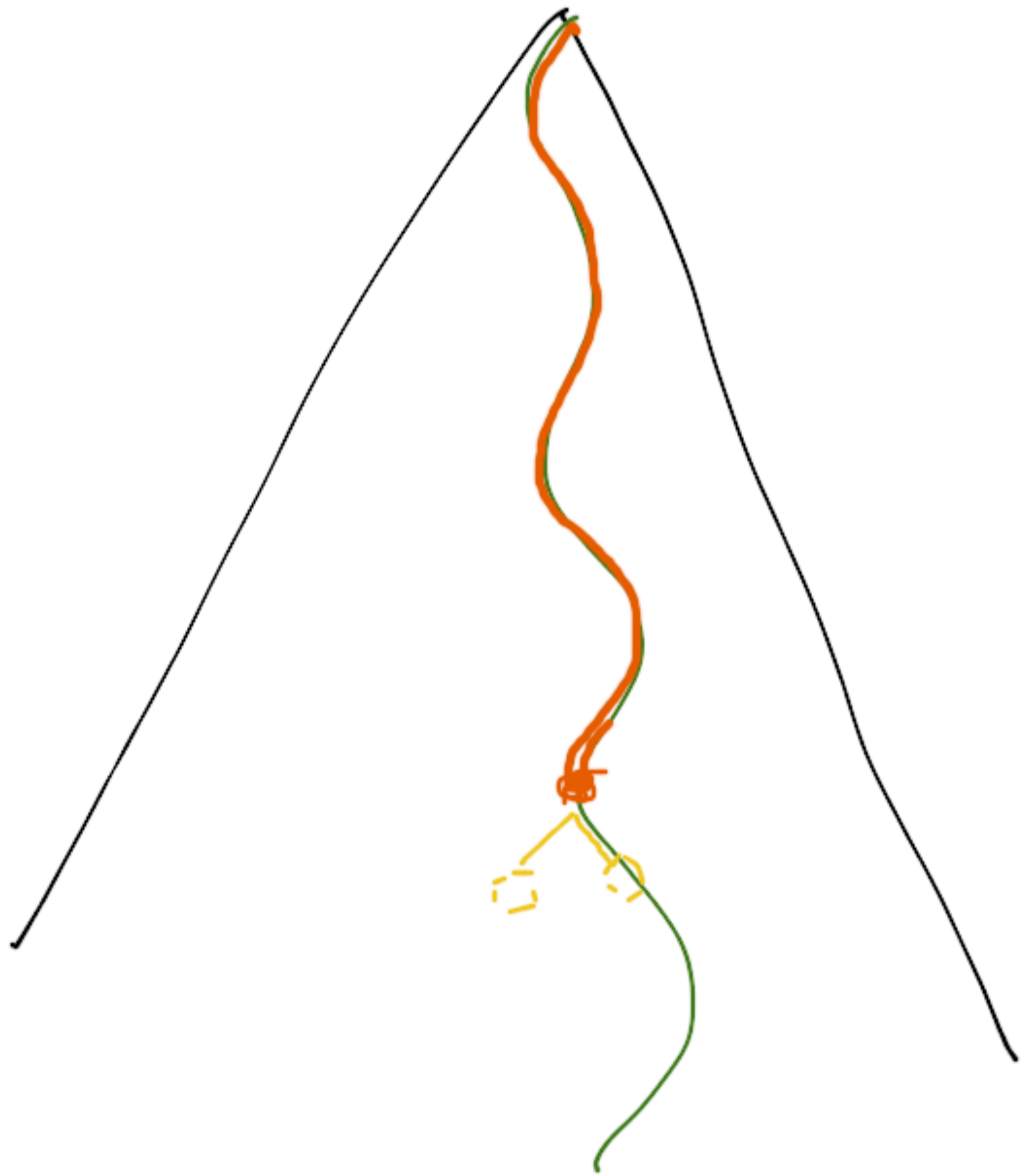

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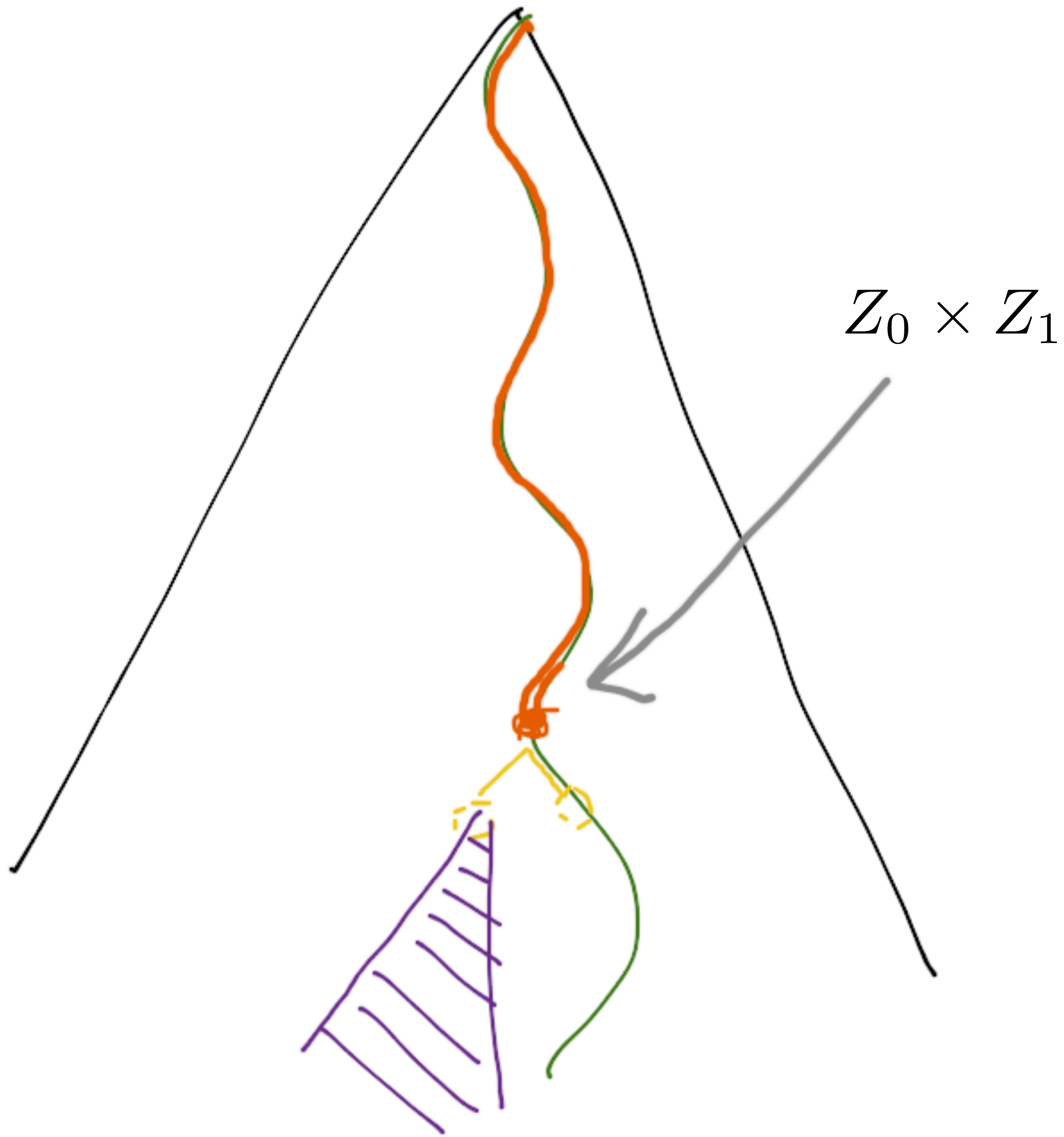

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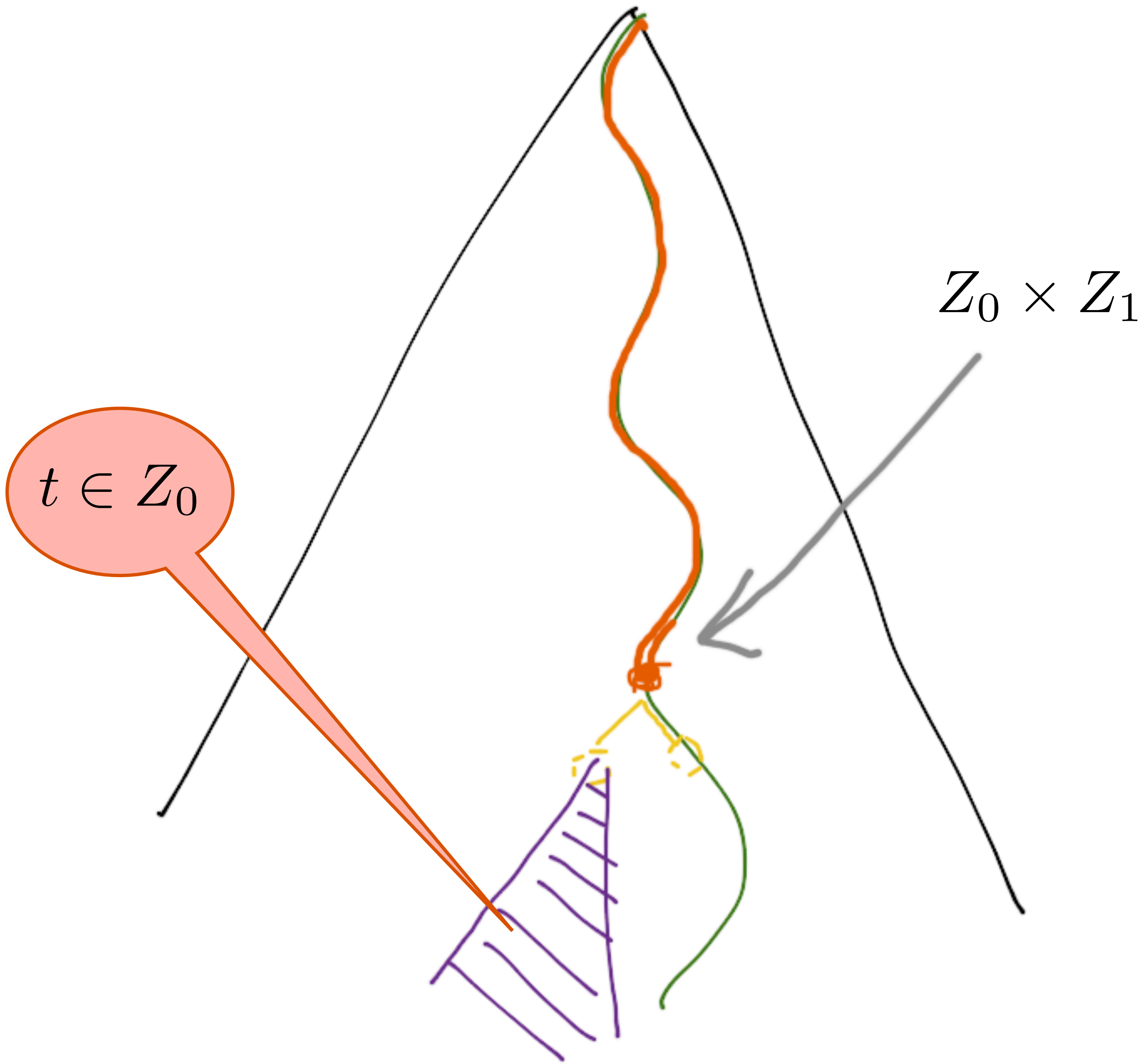

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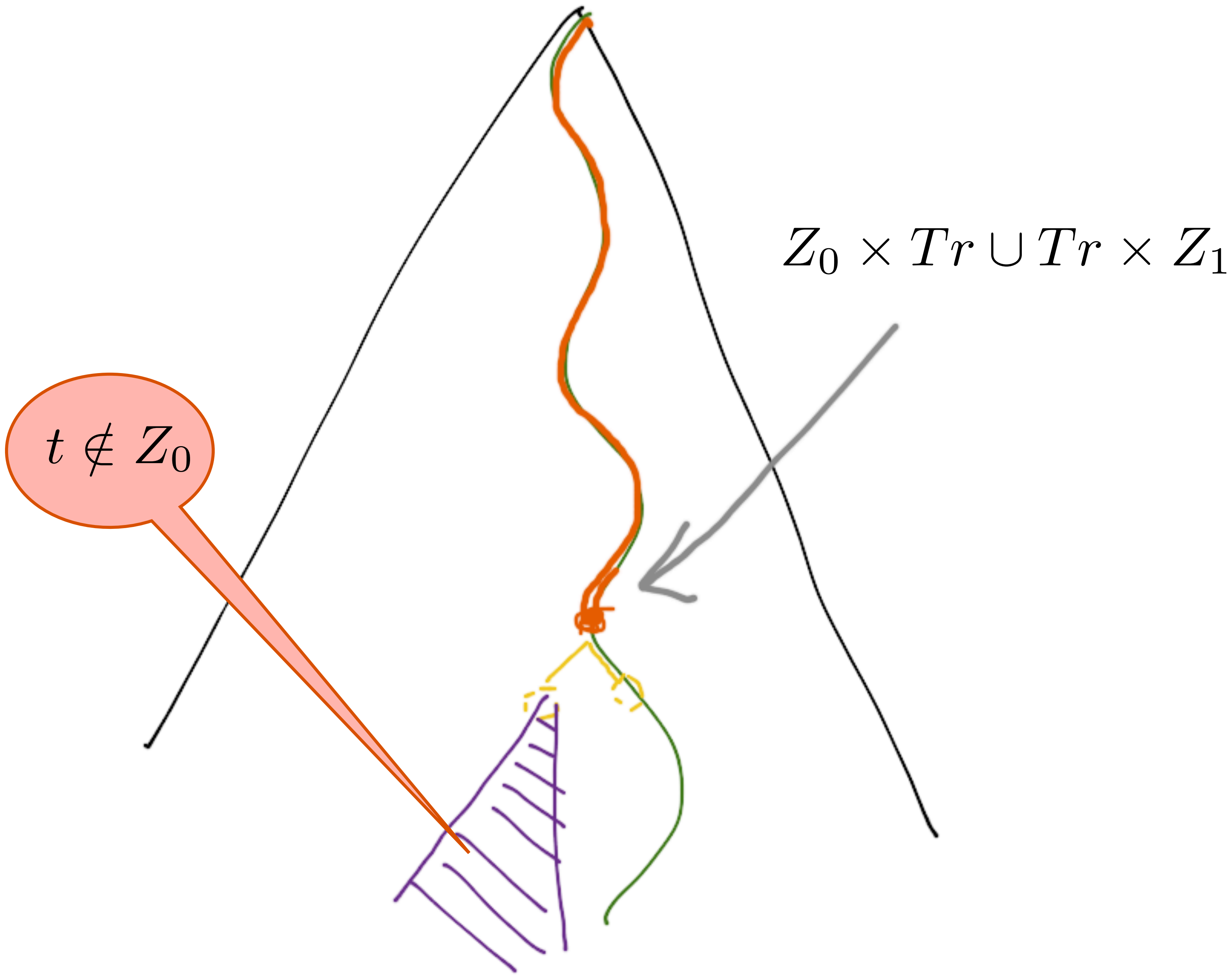


M, t, π

t resolves M up to π

if there is such a t, π is M-correct

M, t, π

t resolves M up to π

if there is such a t, π is M-correct

regular property

DFA

A

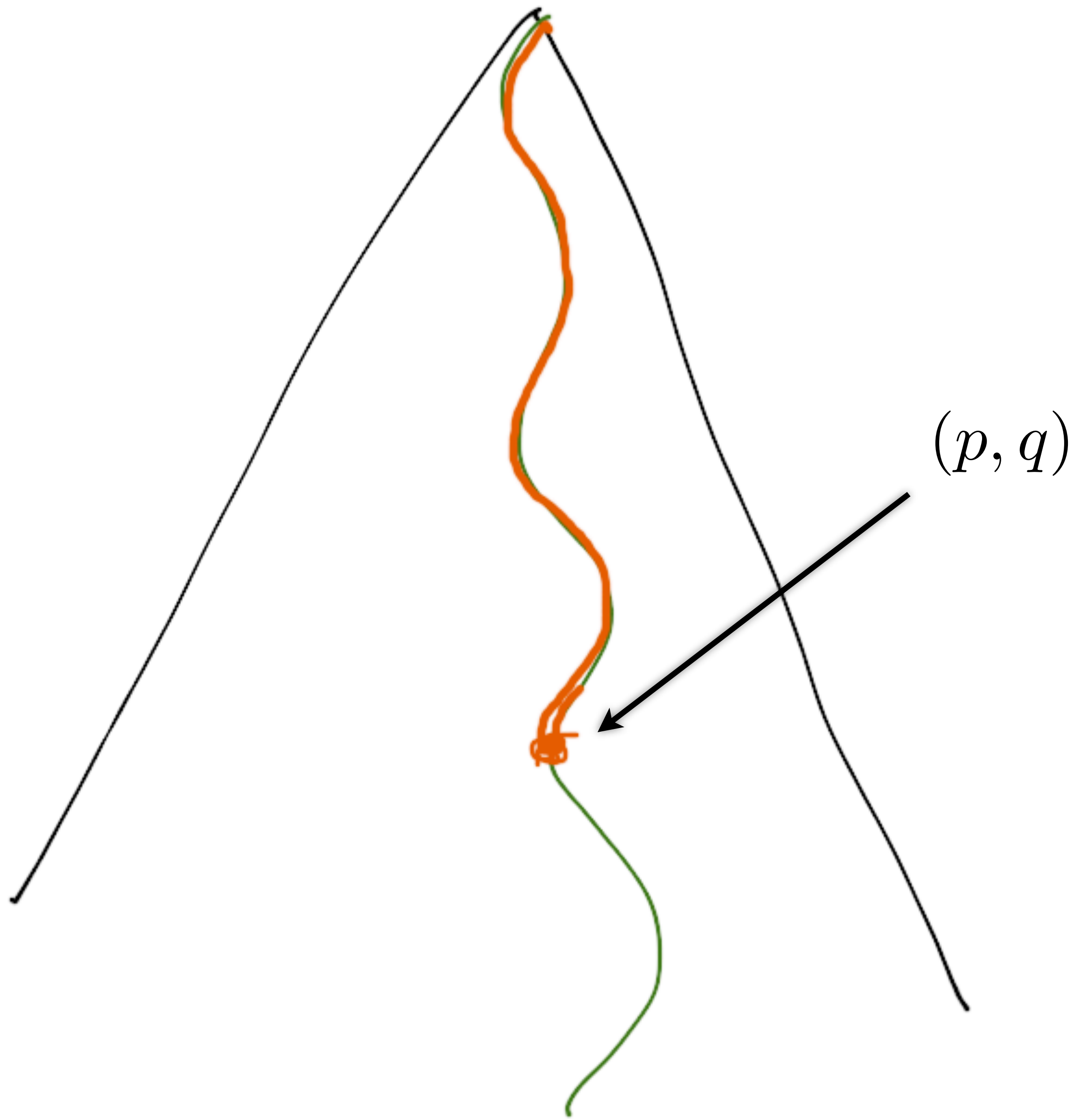
being locally game

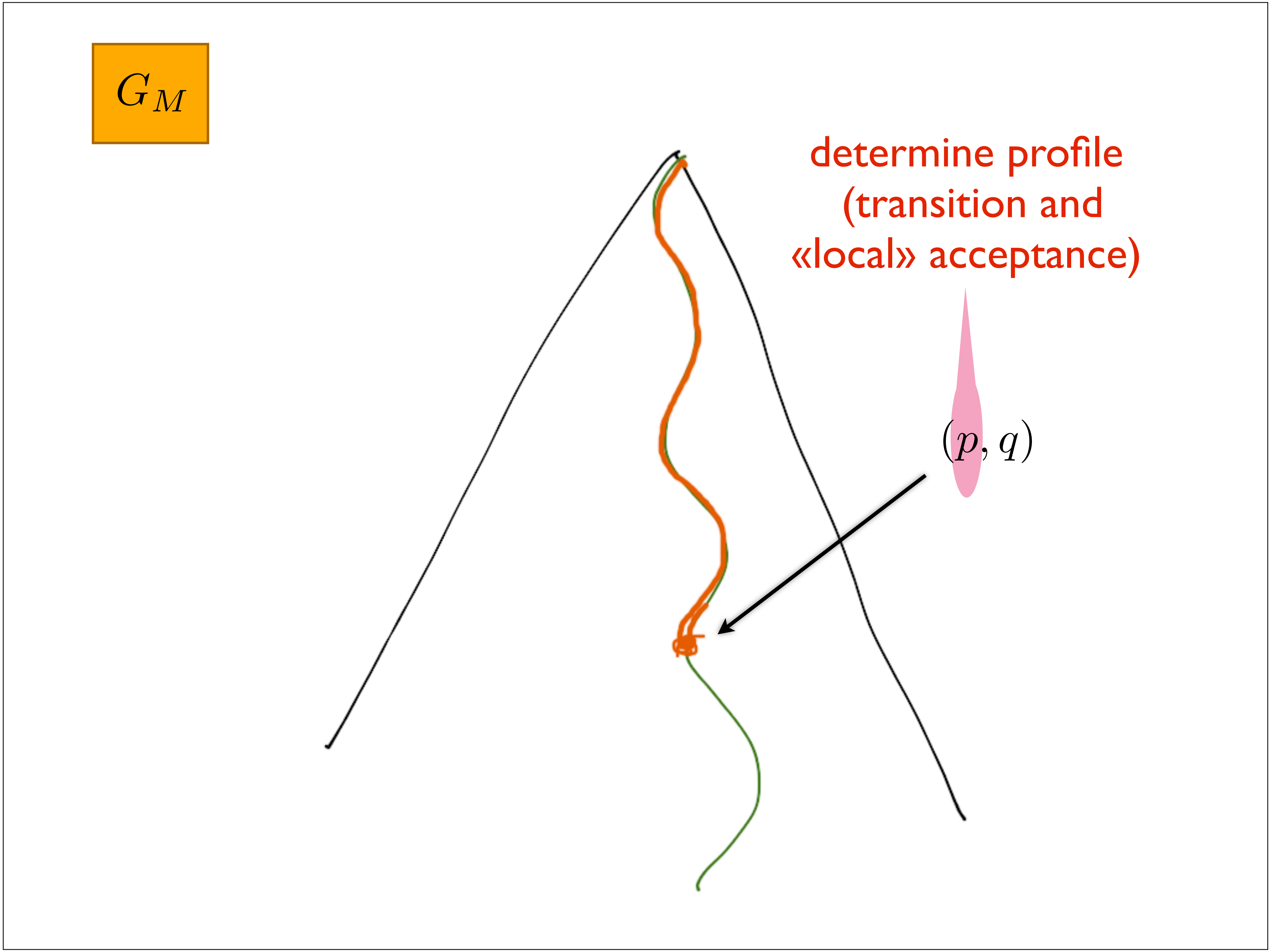
Deterministic parity aut.

B

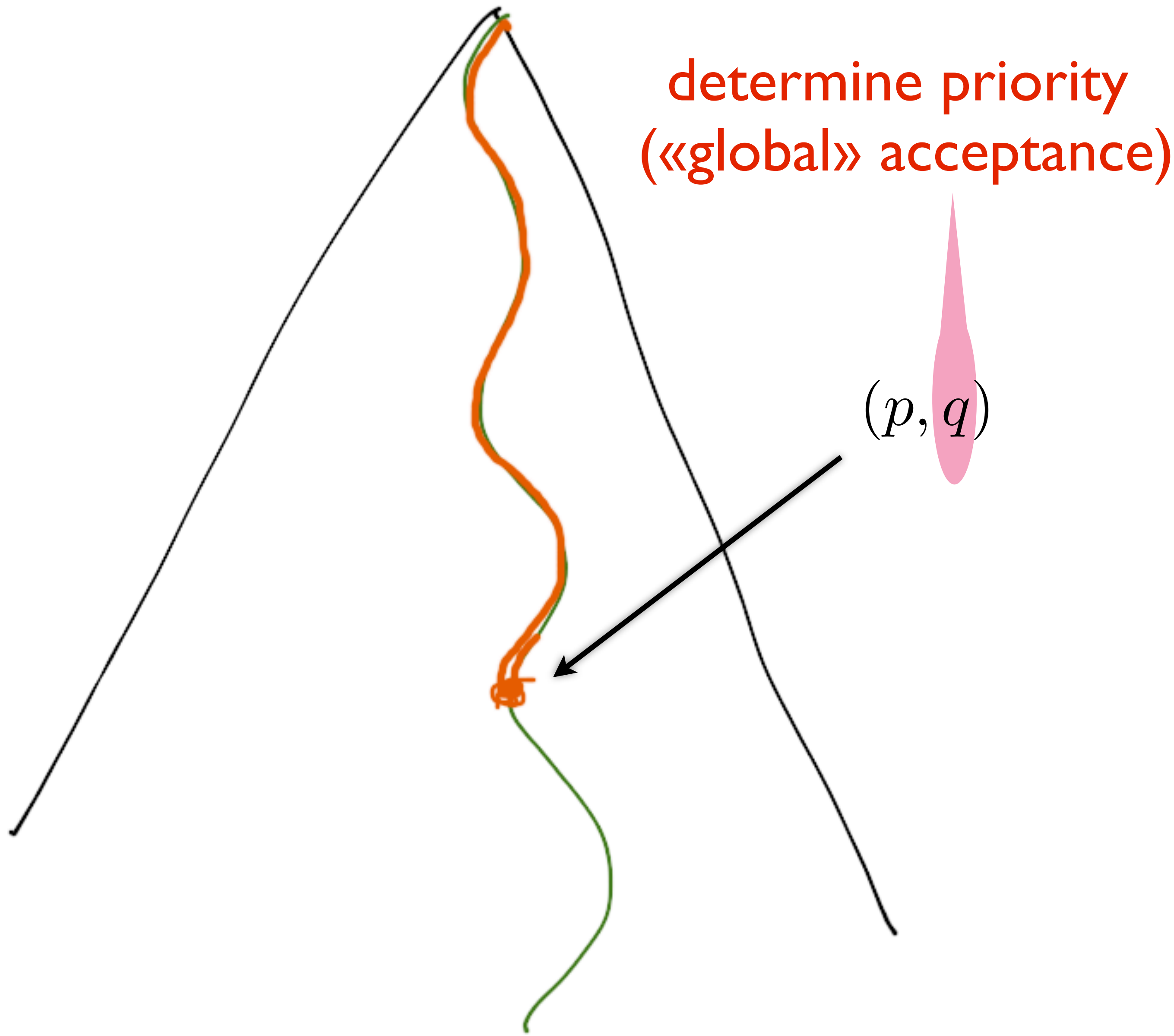
being M-correct

G_M




G_M
determine profile
(transition and
«local» acceptance)
(p, q)

G_M



Theorem : A regular language M is recognised by a game automaton iff M is locally game and

$$\mathcal{L}(G_M, q_M) = M.$$