On Mathematicians Who Liked Logic
The Case of Max Newman

Ivor Grattan-Guinness

Middlesex University Business School, The Burroughs,
Hendon, London NW4 4BT, England

Abstract. The interaction between mathematicians and (formal) logicians has always been much slighter than one might imagine. After a brief review of examples of very partial contact in the period 1850-1930, the case of Max Newman is reviewed in some detail. The rather surprising origins and development of his interest in logic are recorded; they included a lecture course at Cambridge University, which was attended in 1935 by Alan Turing.

1 Cleft

One might expect that the importance to many mathematicians of means of proving theorems, and their desire in many contexts to improve the level of rigour of proof, would motivate them to examine and refine the logic that they were using. However, inattention has long been common.

A very important source of maintaining the cleft during the 19th century is the founding from the late 1810s onwards of the 'mathematical analysis' of real variables, grounded upon an articulated theory of limits, by the French mathematician A.-L. Cauchy. He and his followers extolled rigour, especially careful nominal definitions of major concepts and detailed proofs of theorems. From the 1850s onwards this aim was enriched by the German mathematician Karl Weierstrass and his many followers, who brought in, for example, multiple limit theory, definitions of irrational numbers, and an increasing use of symbols – and from the early 1870s, Georg Cantor and his set theory. However, absent from all these developments was explicit attention to any kind of logic.

This silence continued among the many set theorists who participated in the inauguration of measure theory, functional analysis and integral equations[1]. Artur Schoenflies and Felix Hausdorff were particularly hostile to logic, targeting Bertrand Russell. Even the extensive dispute over the axiom of choice focussed mostly on its legitimacy as an assumption in set theory and mathematics and use of higher-order quantification [Moore 1982]: its ability to state an infinitude of independent choices within *finitary* logic constituted a special difficulty for logicists such as Russell.

[1] The history of mathematical analysis is well covered; cf. especially [Rosenthal 1923; Bottazzini 1986; Medvedev 1991; Jahnke 2003]. A similar story obtains for complex-variable analysis.

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The creators of symbolic logics were exceptional among mathematicians in attending to logic, but also they made little impact on their colleagues. The algebraic tradition with George Boole, C. S. Peirce, Ernst Schröder and others from the mid 19th century was just a curiosity to most of their contemporaries. Similarly, when mathematical logic developed from the late 1870s, especially with Giuseppe Peano's 'logistic' programme at Turin from around 1890, he gained many followers there [Roero and Luciano 2010] but few elsewhere. However, followers in the 1900s included the Britons Russell and A. N. Whitehead, who adopted logistic (include Cantor's set theory) and converted it into their 'logicistic' thesis that all the "objects" of mathematics could be obtained; G. H. Hardy but not many other mathematicians responded [Grattan-Guinness 2000; chs. 8-9]. From 1903 onwards Russell publicised the mathematical logic and arithmetic logicism put forward from the late 1870s by Gottlob Frege, which had gained little attention hitherto even from students of foundations and did not gain much more in the following decades. In the late 1910s David Hilbert started the definitive phase of his programme of metamathematics and attracted several followers at Göttingen University and a few elsewhere; however, its impact among mathematicians was limited even in Germany[2].

The next generations of mathematicians include a few distinguished students of foundations. For example, in the USA from around 1900 E. H. Moore studied Peano and Hilbert and passed on an interest in logic and model theory to his student Oswald Veblen, then to Veblen's student Alonzo Church, and then to his students Stephen Kleene and Barkley Rosser [Aspray 1991]. At Harvard Peirce showed multiset theory to the Harvard philosopher Josiah Royce, who was led on to study logic, and especially to supervise around 1910 C. I. Lewis, Henry Sheffer, Norbert Wiener, Morris Cohen and C. J. Ducasse, the last the main founder of the Association of Symbolic Logic in the mid 1930s [Grattan-Guinness 2002]. In central Europe Johann von Neumann included metamathematics and axiomatic set theory among his concerns [Hallett 1984; ch. 8], while in Poland a distinguished group of logicians did not mesh with a distinguished group of mathematicians even though made both made much use of set theory [Kuratowski 1980]; for example, their joint journal *Fundamenta mathematicae* (1920+) rarely carried logical articles.

But the normal attitude of mathematicians remained indifference. For instance, around 1930 Alfred Tarski and others proved the 'deduction theorem' [Tarski 1941; 125-130]; [Kleene 1952; 90-98]; it gained the apathy of the mathematical community, although it came to be noted by the Bourbaki group, who normally were hostile to logics. (Maybe the reason was that their compatriot Jacques Herbrand had proved versions of it; if so, it was his sole impact on

[2] There does not seem to be an integrated social history of metamathematics, but one can be pieced together from the temporally ordered trio [Peckhaus 1992; Sieg 1999; Menzler-Trott 2001]. In the early stages Hilbert based logic on the existence of a thought-source, a rather peculiar notion already found in Dedekind; Zermelo worked with truth-functions, but the place of logic in his set theory is modest. Compare [Peckhaus 1994].

French mathematics.) Also, Kurt Gödel's theorems [Gödel 1931] on the incompletability of first-order arithmetic were appreciated fairly quickly by students of foundations, but the community did not become widely aware of them until the mid 1950s [Grattan-Guinness 2011].

2 Newman's Course at Cambridge

Turing's own career provides a good example of the cleft; for when he submitted his paper [Turing 1936] on computability to the London Mathematical Society they could not referee it properly, because Max Newman was the only other expert in Britain and he had been involved in its preparation [Hodges 1983, 109-114]; they seem to have accepted it on Newman's word. But this detail prompts an historical question that has not been examined: why was the mathematician Newman also a logician? An answer is given in the rest of this article; more details are given in [Grattan-Guinness 2012a]

Although he did not publish much on logic, it is clear that Newman was familiar with the technicalities of several parts of it. In particular, during his wartime period at Bletchley Park he published three technical papers, one written jointly with Turing [Newman 1942, 1943; Newman and Turing 1943]. At Cambridge in the 1930s he had taught a course on 'Foundations of mathematics', which, as is well known, was crucial for Turing when he attend it in 1935; for from it learnt about recursion theory and Gödel numbering from the only Briton who was familiar with it.

Ready for the academic year 1933-1934, Newman ran the course only for the two succeeding years before it was closed down, perhaps because of disaffection among staff as well as among students. In particular, Hardy, despite his familiarity with foundations, opined to Newman in 1937: 'though "Foundations" is now a highly respectable subject, and everybody ought to know <u>something</u> about it, it is, (like dancing or "groups") slightly dangerous for a bright young mathematician!'[3]. Somehow Newman continued to set questions for five of the six years that he was to remain at Cambridge before moving to Bletchley Park in 1942;[4] The questions for 1939 may have been set by Turing, who, presumably in resistance against Hardy's coolness, was invited to give a lecture course on foundations in the Lent term of 1939. He was asked to repeat it in 1940; but by then he also was at Bletchley Park[5]. How had Newman got involved in logic in the first place?

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[3] Newman's archive, Saint John's College Cambridge; thanks to David Anderson much of it is available in digital form at http://www.cdpa.co.uk/Newman/. Individual items are cited in the style 'NA, [box] a- [folder] b- [document] c'; here 2-12-3.

[4] A Mathematics Tripos course in 'logic' was launched in 1944 by S. W. P. Steen. The Moral Sciences Tripos continued to offer its long-running course on the more traditional parts of 'Logic'.

[5] On Turing's teaching, cf. [Hodges 1983; 153,177] and the Faculty Board minutes for 29 May 1939.

3 Newman's Way in to Logic

Maxwell Hermann Alexander Neumann (1897-1984) was born in London to a German father and an English mother. He gained a scholarship to St John's College Cambridge in 1915 and took Part I of the Mathematics Tripos in the following year. But carrying the surname 'Neumann' in Britain in the Great War was not a good idea; the family changed its surname to 'Newman', and Max had to leave his college until 1919, when he returned and completed Part II of the Tripos very well in 1921.

Then, very unusually, he spent much of the academic year 1922-1923 at Vienna University. He went with two other members of his college.

One was Lionel Penrose; as a schoolboy he had been interested in Russell's mathematical logic, and he specialised in traditional versions of logic as taught in the Moral Sciences Tripos. But he also examined mathematical logic, and may well have at least have alerted his friend Newman to the subject, which was absent from the Mathematics Tripos. He became interested also in psychology (well represented in his own Tripos syllabus), especially its bearing upon logic, and he wanted to meet Sigmund Freud and Karl Bühler and other psychologists in Vienna. He seems to have initiated the visit to Vienna; his family was wealthy enough to sustain it, especially as at that inflationary time British money would last a long time in Vienna. His friendship with Newman was multi-faceted and deep.

The other was Rolf Gardiner, who was later active in organic farming and folk dancing, enthused for the Nazis [Moore-Colyer 2001], and was to be the father of the conductor Sir John Eliot Gardiner. His younger sister Margaret came along; she became an artist and a companion to the biologist Desmond Bernal. She recalled 'the still deeply impoverished town' of Vienna, where Penrose and Newman would walk side-by-side down the street playing a chess game in their heads [Gardiner 1988; 61-68].

Of Newman's contacts with the mathematicians in Vienna we have only a welcoming letter of July 1922 from ordinary professor Wilhelm Wirtinger;[6] but it seems clear that his experience of Viennese mathematics was decisive in changing the direction of his researches. His principal research interest was to become topology, which was *not* a speciality of British mathematics. By contrast, in Vienna some of Wirtinger's own work related to the topology of surfaces; in 1922 the University recruited Kurt Reidemeister, who was to become a specialist in combinatorial topology, like Newman himself; Leopold Vietoris was a junior staff member; and a student was Karl Menger (though rather ill at the time and away from Vienna).

Most notably, ordinary professor Hans Hahn was not only a specialist in the topology of curves, and in real-variable mathematical analysis; he also regarded formal logic as both a research and as a teaching topic. In particular, while Newman was there he ran a preparatory seminar on 'algebra and logic', and in later years held two full seminars on *Principia mathematica*. In addition,

[6] The Wirtinger letter is at NA, 2-1-2.

he supervised doctoral student Kurt Gödel working on the completeness of the first-order functional calculus with identity, and as editor of the *Monatshefte für Mathematik und Physik* published both that thesis and the sequel paper [Gödel 1931] on the incompletability of first-order arithmetic (which was to be registered as Gödel's higher doctorate).

Hahn also engaged in philosophical debates. When he had studied at Vienna University from the mid 1890s to his higher doctorate in 1905 he had participated in some of the discussion groups that surrounded certain chairs in the university. After teaching elsewhere for several years, he returned to Vienna University as a full professor of mathematics in 1921. During 1922 he led the move to appoint to the chair of natural philosophy the German physicist and philosopher Moritz Schlick; after arriving in 1923 Schlick created what was to be known as the 'Vienna Circle', with Hahn as a leading member[7]. Further, while the Circle had no agreed philosophy among all its members, Schlick, Hahn and later Carnap strongly advocated positivism and empiricism, acknowledging major influences from Ernst Mach (who had held that chair in the 1890s) and Russell.

4 After Vienna

After his return Newman developed as a (pioneer) British topologist, with a serious interest in logic and logic education and (as we shall soon see) a readiness to engage with Russell's philosophy; surely one sees heavy Viennese influences here, especially from Hahn.

In 1923 Newman applied for a college fellowship. He submitted a paper [Newman 1923a] on the avoiding the axioms of choice in developing the theory of functions of a real variable that was published that year[8], some unidentifiable discussion of solutions of Laplace's equation, and a long unpublished essay [Newman 1923b] in the philosophy of science that was completed in August. Its title, 'The foundations of mathematics from the standpoint of physics', could well have originated in a Viennese chat. Maybe he wrote some of it there; unfortunately the 161 folios do not contain any watermarks.

In this essay Newman took the world of idealised objects that was customary adopted in applied mathematics (smooth bodies, light strings, and so on) as 'certain ideals, or abstractions [...] not applicable to those of real physical objects', and contrasted it with the world of real physical that one encounters and on which he wished to philosophise. He distinguished between these two kinds of philosophising by *the different logics that they used*. The idealisers would draw on the two-valued logic, for which he cited a recent metamathematical paper by Hilbert [1922] as a source; but those interested in real life would go to constructive logic, on which he cited papers by Brouwer [1918-1919] and Hermann Weyl [1921].

We see here Newman's notable readiness to admit logical pluralism, and to put logics at the centre of the analysis of a philosophical problem; most unusual for

[7] On the Vienna Circle, cf. [Stadler 2001]; on Hahn, cf. [Sigmund 1995].
[8] On the context, cf. *passim* in [Rosenthal 1923] and [Medvedev 1991].

a mathematician, and far more Viennese than Cantabrigian. His college referees, Ebenezer Cunningham and H. F. Baker, were not impressed by the essay but recommended the award of the Fellowship. He neither revised the essay nor seemed to seek its publication, although occasionally he alluded to its concerns; and it must be at least a major source of his recognition of the importance of logics.

This essay built upon the awareness of logic that he must have gained at Cambridge from Penrose. That contact will have continued, for after Vienna Penrose wrote several manuscripts on mathematical logic, especially the psychological aspects, in which he was influenced by Russell and also by Ludwig Wittgenstein's notion of tautology given in the recent *Tractatus logico-philosophicus* (1922). He worked on a doctoral dissertation on the psychology of mathematics, but then abandoned it[9]. From 1925 he studied for a degree in medicine at Cambridge and London, and became a distinguished geneticist, psychiatrist and statistician, and also father of the mathematicians Oliver and Sir Roger Penrose [Harris 1973].

An occasion for Newman to exercise his logical and philosophical talents arose when he attended a set of philosophical lectures that Russell gave in Trinity College Cambridge in 1926. They went into a book on 'the analysis of matter' [Russell 1927]. Newman helped Russell to write two chapters, and when the book appeared he criticised its philosophical basis most acutely in [Newman 1928]; Russell accepted the criticisms, which stimulated Newman to write Russell two long letters on logic and on topology [Grattan-Guinness 2012b].

Newman continued to pioneer both topology and logic at Cambridge. Doubtless with topology in mind, in 1936, a year before sinking the foundations course, Hardy had proposed Newman as a Fellow of the Royal Society, with J. E. Littlewood as seconder, although Newman was no Hardy-Littlewood analyst; the election was made in 1939. Newman used the Society to support his logical cause. In 1950 he proposed Turing as a Fellow, seconded by Russell, the election being accepted in 1951; five years later he wrote the obituary [Newman 1955] of Turing. In 1966 Newman proposed and Russell seconded Gödel as Foreign Member, duly gained two years later[10]. In 1970 he agreed to be the obituarist of Russell, to be helped by the philosopher A. J. Ayer, but he was not well enough to oblige. He died in 1984.

Among mathematicians who came to like logic, Newman is a very unusual case. The (sparse) evidence suggests two sources: Penrose's early interest; and the unusual mixture of mathematics, logic and philosophy in Vienna, which drew him also to topology. Thus he changed directions; had he stayed in Cambridge in 1922-1923, he would have surely continued in the direction indicated by the paper on avoiding the axioms of choice, namely, Hardy-Littlewood mathematical analysis. But then his interest in logic could have waned (and in topology never have flowered), so that maybe no foundations course would have existed for budding Hardy-Littlewood mathematical analyst Turing to take and thereby to

[9] In the Penrose Papers, University College London Archives, cf. especially boxes 20-21 and 26-28.

[10] Information comes from Royal Society Archives, and NA, 2-15-10 to -13.

learn of the subjects of recursive functions and undecidability. Then the story of Bletchley Park and afterwards could have been very different; neither he nor this alternative Newman would have been the obvious choices to go there, nor would they have been as effective as they actually were. The way that things turned out for Newman and Turing contained some strokes of luck!

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