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Editorial

Applications of Delay Differential Equations in Biological Systems

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Mathematical modeling with delay differential equations (DDEs) is widely used for analysis and predictions in various areas of life sciences, for example, population dynamics, epidemiology, immunology, physiology, and neural networks [1–5]. The time delays or time lags, in these models, can be related to the duration of certain hidden processes like the stages of the life cycle, the time between infection of a cell and the production of new viruses, the duration of the infectious period, the immune period, and so on [6]. In ordinary differential equations (ODEs), the unknown state and its derivatives are evaluated at the same time instant. In a DDE, however, the evolution of the system at a certain time instant depends on the past history/memory. Introduction of time delays in a differential model significantly increases the complexity of the model. Therefore, studying qualitative behaviors of such models, using stability or bifurcation analysis, is necessary [7].

There is no doubt that some of the recent developments in the theory of DDEs have enhanced our understanding of the qualitative behavior of their solutions and have many applications in mathematical biology and other related fields. Both theory and applications of DDEs require a bit more mathematical maturity than their ODEs counterparts. The mathematical description of delay dynamical systems will naturally involve the delay parameter in some specified way. Nonlinearity and sensitivity analysis of DDEs have been studied intensely in recent years in diverse areas of science and technology, particularly in the context of chaotic dynamics [8, 9].

This special issue aims at creating a multidisciplinary forum of discussion on recent advances in differential equations with memory such as DDEs or fractional-order differential equations (FODEs) in biological systems as well as new applications to economics, engineering, physics, and medicine. It provides an opportunity to study the new trends and analytical insights of the delay differential equations, existence and uniqueness of the solutions, boundedness and persistence, oscillatory behavior of the solutions, stability and bifurcation analysis, parameter estimations and sensitivity analysis, and numerical investigations of solutions.

In the paper "Oscillation Criteria for Delay and Advanced Differential Equations With Nonmonotone Arguments" by G. E. Chatzarakis and T. Li, the authors study the oscillatory behavior of differential equations with nonmonotone deviating arguments and nonnegative coefficients. New oscillation criteria, involving lim sup and lim inf, are obtained based on an iterative method. Some numerical examples are given to illustrate the applicability and strength of the obtained conditions over known ones.

In the paper "Bifurcations and Dynamics of the Rb-E2F Pathway Involving miR449" by L. Li and J. Shen, the authors focus on the gene regulative network involving Rb-E2F pathway and microRNAs (miR449) and studied the influence of time delay on the dynamical behaviors of Rb-E2F pathway by using Hopf bifurcation theory. It is shown that under certain assumptions the steady state of the delay model is asymptotically stable for all delay values; there is a critical value under another set of conditions; the steady state is stable when the time delay is less than the critical value, while the steady state is changed to be unstable when the time delay is greater than the critical value. Hopf bifurcation appears at the steady state when the delay passes through the critical value. Numerical simulations are presented to illustrate the theoretical results.

In the paper "Maximum Likelihood Inference for Univariate Delay Differential Equation Models with Multiple Delays" by A. A. Mahmoud et al., the authors study statistical inference methodology based on maximum likelihoods for DDE models in the univariate setting. Maximum likelihood inference is obtained for single and multiple unknown delay parameters as well as other parameters of interest that govern the trajectories of the DDE models. The maximum likelihood estimator is obtained based on adaptive grid and Newton-Raphson algorithms. The methodology estimates correctly the delay parameters as well as other unknown parameters (such as the initial starting values) of the dynamical system based on simulation data. They also develop methodology to compute the information matrix and confidence intervals for all unknown parameters based on the likelihood inferential framework. The authors present three illustrative examples related to biological systems.

In the paper "Impact of Time Delay in Perceptual Decision-Making: Neuronal Population Modeling Approach" by U. Foryś et al., the authors study the basis of novel time-delayed neuronal population model, if the delay in self-inhibition terms can explain those impairments. Analysis of proposed system reveals that there can be up to three positive steady states, with the one having the lowest neuronal activity being always locally stable in nondelayed case. They show, however, that this steady state becomes unstable above a critical delay value for which, in certain parameter ranges, a subcritical Hopf bifurcation occurs. They apply psychometric function to translate model-predicted ring rates into probabilities that a decision is being made. Using numerical simulations, they demonstrate that for small synaptic delays the decision-making process depends directly on the strength of supplied stimulus and the system correctly identifies to which population the stimulus was applied. For delays above the Hopf bifurcation threshold they observe complex impairments in the decision-making process; that is, increasing the strength of the stimulus may lead to the change in the neuronal decision into a wrong one. Furthermore, above critical delay threshold, the system exhibits ambiguity in the decision-making.

In the paper "On Coupled p-Laplacian Fractional Differential Equations with Nonlinear Boundary Conditions" by A. Khan et al., the authors investigate the existence and uniqueness of solutions to a coupled system of fractional differential equations (FDEs) with nonlinear p-Laplacian operator by using fractional integral boundary conditions with nonlinear term and also to checking the Hyers-Ulam stability for the proposed problem. The functions involved in the proposed coupled system are continuous and satisfy certain growth conditions. By using topological degree theory some conditions are established which ensure the existence and uniqueness of solution to the proposed problem. Further, certain conditions are developed corresponding to Hyers-Ulam type stability for the positive solution of the considered coupled system of FDEs. Also, from applications point of view, an example is given.

In the paper "Analytical Solution of the Fractional Fredholm Integrodifferential Equation Using the Fractional Residual Power Series Method" by M. I. Syam, the author provides a numerical method to find the solution of fractional Fredholm integrodifferential equation. A modified version of the fractional power series method (RPS) is presented to extract an approximate solution of the model. The RPS method is a combination of the generalized fractional Taylor series and the residual functions. Numerical results are also presented.

In the paper "Hybrid Adaptive Pinning Control for Function Projective Synchronization of Delayed Neural Networks with Mixed Uncertain Couplings" by T. Botmart et al., the authors study the function projective synchronization problem of neural networks with mixed time-varying delays and uncertainties asymmetric coupling. The function projective synchronization of this model via hybrid adaptive pinning controls and hybrid adaptive controls, composed of nonlinear and adaptive linear feedback control, is investigated. Based on Lyapunov stability theory combined with the method of the adaptive control and pinning control, some novel and simple sufficient conditions are derived for the function projective synchronization problem of neural networks with mixed time-varying delays and uncertainties asymmetric coupling, and the derived results are less conservative. Particularly, the control method focuses on how to determine a set of pinned nodes with xed coupling matrices and strength values and randomly select pinning nodes. Based on adaptive control technique, the parameter update law, and the technique of dealing with some integral terms, the control may be used to manipulate the scaling functions such that the drive system and response systems could be synchronized up to the desired scaling function. Some numerical examples are given to illustrate the effectiveness of the proposed theoretical results.

In paper "Numerical Study for Time Delay Multistrain Tuberculosis Model of Fractional Order" by N. H. Sweilam et al., the authors provide a novel mathematical fractional model of multistrain tuberculosis with time delay memory. The proposed model is governed by a system of fractional-order DDEs, where the fractional derivative is defined in the sense of the Grünwald–Letinkov definition. Modified parameters are introduced to account for the fractional order. The stability of the equilibrium points is investigated for any time delay. Nonstandard finite deference method is proposed to solve the resulting system of fractional-order DDEs. The numerical simulations show that nonstandard finite difference method can be applied to solve such fractional-order DDEs simply and effectively.

In paper "Extinction and Persistence in Mean of a Novel Delay Impulsive Stochastic Infected Predator-Prey System with Jumps" by G. Liu et al., the authors explore an impulsive stochastic infected predator-prey system with Levy jumps and delays. The main aim of this paper is to investigate the effects of time delays and impulse stochastic interference on dynamics of the predator-prey model. They prove some properties of the subsystem of the system. Second, in view of comparison theorem and limit superior theory, the authors obtain some sufficient conditions for the extinction of this system. Furthermore, persistence in mean of the system is investigated by using the theory of impulsive stochastic differential equations and DDEs. The authors carry out some simulations to verify our main results and explain the biological implications.

In the paper "A Novel Approach to Numerical Modeling of Metabolic System: Investigation of Chaotic Behavior in Diabetes Mellitus" by P. S. Shabestari et al., the authors consider that some mathematical models have been presented for glucose and insulin interaction. The dynamical behavior of a regulatory system of glucose insulin incorporating time delay is studied and a new property of the presented model is revealed. This property can describe the diabetes disease better and therefore may help one in deeper understanding of the diabetes, interactions between glucose and insulin, and possible cures for the widespread disease.

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References

[1] G. A. Bocharov and F. A. Rihan, "Numerical modelling in biosciences using delay differential equations," *Journal of Computational and Applied Mathematics*, vol. 125, no. 1-2, pp. 183–199, 2000.

[2] F. A. Rihan, D. H. Abdelrahman, F. Al-Maskari, F. Ibrahim, and M. A. Abdeen, "Delay differential model for tumour-immune response with chemoimmunotherapy and optimal control," *Computational and Mathematical Methods in Medicine*, vol. 2014, Article ID 982978, 15 pages, 2014.

[3] C. T. H. Baker, G. A. Bocharov, C. A. H. Paul, and F. A. Rihan, "Modelling and analysis of time-lags in some basic patterns of cell proliferation," *Journal of Mathematical Biology*, vol. 37, no. 4, pp. 341–371, 1998.

[4] S. Lakshmanan, F. A. Rihan, R. Rakkiyappan, and J. H. Park, "Stability analysis of the differential genetic regulatory networks model with time-varying delays and Markovian jumping parameters," *Nonlinear Analysis: Hybrid Systems*, vol. 14, pp. 1–15, 2014.

[5] R. Rakkiyappan, G. Velmurugan, F. . Rihan, and S. Lakshmanan, "Stability analysis of memristor-based complex-valued recurrent neural networks with time delays," *Complexity*, vol. 21, no. 4, pp. 14–39, 2015.

[6] F. A. Rihan, D. H. Abdel Rahman, S. Lakshmanan, and A. S. Alkhajeh, "A time delay model of tumour–immune system interactions: global dynamics, parameter estimation, sensitivity analysis," *Applied Mathematics and Computation*, vol. 232, pp. 606–623, 2014.

[7] M. Gozen and C. Tunc, "Stability in functional integro-differential equations of second order with variable delay," *Journal of Mathematical and Fundamental Sciences*, vol. 49, no. 1, pp. 66–89, 2017.

[8] F. A. Rihan, A. A. Azamov, and H. J. Al-Sakaji, "An Inverse problem for delay differential equations: parameter estimation, nonlinearity, sensitivity," *Applied Mathematics & Information Sciences*, vol. 12, no. 1, pp. 63–74, 2018.

[9] F. A. Rihan, "Sensitivity analysis for dynamic systems with time-lags," *Journal of Computational and Applied Mathematics*, vol. 151, no. 2, pp. 445–462, 2003.