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On the ω-language expressive power of extended Petri nets

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Abstract

In this paper, we study the expressive power of several monotonic extensions of Petri nets. We compare the expressive power of Petri nets, Petri nets extended with *non-blocking arcs* and Petri nets extended with *transfer arcs*, in terms of ω-languages. We show that the hierarchy of expressive powers of those models is strict. To prove these results, we propose *original techniques* that rely on well-quasi orderings and monotonicity properties.

Keywords: (extended) Petri nets; ω-languages; Expressiveness

1. Introduction

Reactive systems are non-terminating systems that interact with an environment. Those systems are often embedded in environments which are *safety critical*, making their correctness a crucial issue.

To formally reason about the correctness of such systems, we need formal models of their behaviours. At some abstract level, the behaviour of a non-terminating reactive system within its environment can be seen as an *infinite sequence of events* (usually taken within a finite set of events). The semantics of those systems is thus a (usually infinite) set of those infinite behaviours. Sets of infinite sequences of events have been studied intensively in automata theory where infinite sequences of events are called *infinite words*, and sets of such sequences are called *omega languages* (ω-languages).

If the global system (the reactive system and its environment) has a meaningful *finite state abstraction* then there are well-studied formalisms that can be used. For example, *finite state automata* allow us to specify any *omega regular language* [18]. Furthermore, as the global system is naturally composed of several components (at least two: the reactive system and its environment), it is convenient to model the reactive system and its environment compositionally by several (at least two) automata. This is possible using simple synchronization mechanisms. In the case of finite state machines, synchronizations on *common events* allow to model naturally most of the interesting communication mechanisms between processes.

Recently, many research works have tried to generalize the *computer aided verification methods* that have been proposed for finite state systems toward infinite state systems. In particular, interesting positive (decidability)

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results have been obtained for a class of *parametric systems*. New methods have been proposed for automatically verifying temporal properties of concurrent systems containing an *arbitrary number* (parametric number) of finite-state processes that communicate. Contrary to the finite state case, three primitives of communication have been proposed:

- in [13], German and Sistla introduce a model where an arbitrary number of processes communicate via *rendez-vous* (synchronization on common events);
- in [8,9], Emerson and Namjoshi and Esparza et al. study the automatic analysis of models where an arbitrary number of processes communicate through rendez-vous and *broadcasts*. A broadcast is a *non-blocking* synchronization mechanism where the emitter sends a signal to all the possibly awaiting processes, and continue its execution without waiting (whether there are receivers or not). In [4], Delzanno uses broadcast protocols to model and verify cache coherency protocols [14];
- In the model introduced in [5,16] by Delzanno et al., an arbitrary number of processes can communicate thanks to *non-blocking rendez-vous* (in addition to rendez-vous and broadcasts). In a non-blocking rendez-vous synchronization, the sender emits an event, and if there are automata waiting for that event, one of those automata is chosen non-deterministically and synchronizes with the sender. As for broadcast, this synchronization mechanism is non-blocking. This model is useful to model multi-threaded programs written in JAVA where instructions like `NotifyAll` are modeled by using broadcasts and `Notify` are modeled by using non-blocking rendez-vous.

In all those works, the identity of individual processes is irrelevant. Hence, we can apply to all those models the so-called *counting abstraction* [13,19] and equivalently see all those models as extended Petri nets. It has been shown in previous works that rendez-vous can be modeled by *Petri nets* [15], broadcasts can be modeled by *Petri nets extended with transfer arcs*, and non-blocking rendez-vous can be modeled by *Petri nets extended with non-blocking arcs* [5,19].

These two Petri nets extensions (and others like reset Petri nets, lossy Petri nets,...) are monotonic and well-structured [16]. Those models have attracted a lot of attention recently [6,7,17,11,12,9,5,16]. These papers study the main decidability problems for these models: even if the *general reachability problem* is undecidable, interesting sub-problems, like *control state reachability* and *termination*, are decidable for all those models, and the *boundedness* problem is decidable for Petri nets, Petri nets with non-blocking arcs and transfer nets. However, the expressiveness of those formalisms have not been studied carefully presumably because the finite word languages definable in those extensions of Petri nets are all equal to the recursively enumerable languages. Nevertheless, as recalled above, those formalisms are usually used to model non-terminating systems and so their expressive power should be measured in terms of definable omega languages.

There is currently no proof that the *expressive power* of Petri nets with transfer arcs or Petri nets with non-blocking arcs, measured in terms of *definable omega languages*, are strictly greater than the expressive power of Petri nets. In this paper, we solve this open problem. Our results are as follows. First (Section 3), we show that all the omega-languages definable by Petri nets with non-blocking arcs can be recognized by Petri nets with transfer arcs, but that some languages which are definable by Petri nets with transfer arcs are not recognizable by Petri nets with non-blocking arcs (even if we allow τ-transitions). Second (Section 4), we show that there exist omega languages that can be defined with Petri nets extended with non-blocking arcs and cannot be defined with Petri nets (even if we allow τ-transitions). The separation of expressive power over *definable omega languages* is surprising as the expressive power of those two extended Petri net models equals, as mentioned above, the expressive power of Turing Machines when measured on *finite word languages* defined with the help of a finite accepting set of markings. We also study the expressiveness of Petri nets with reset arcs in Section 5.

The techniques that we use to separate the expressive power of extended Petri nets on omega languages are based on properties of *well-quasi orderings* and *monotonicity*. They are, to the best of our knowledge, original in the context of (extended) Petri nets.

2. Preliminaries

In this section, we introduce the preliminaries of the discussion. In Section 2.1, we introduce two Petri nets extensions (non-blocking arcs and transfer arcs) and define the notion of ω-language *accepted* by these models. In Section 2.2 we recall and prove a basic result on well-quasi orderings, which is the cornerstone of the proofs of Sections 3 and 4.

2.1. Extended Petri nets

Definition 1. An *Extended Petri net* (EPN for short) $\mathcal{N}$ is a tuple $\langle \mathcal{P}, \mathcal{T}, \Sigma, \mathbf{m}_0 \rangle$, where $\mathcal{P} = \{p_1, p_2, \ldots, p_n\}$ is a finite set of places, $\mathcal{T}$ is finite set of transitions and Σ is a finite alphabet containing a special silent symbol τ. A *marking* of the places is a function $\mathbf{m} : \mathcal{P} \mapsto \mathbb{N}$. A marking $\mathbf{m}$ can also be seen as a vector v such that $v^T = [\mathbf{m}(p_1), \mathbf{m}(p_2), \ldots, \mathbf{m}(p_n)]$. $\mathbf{m}_0$ is the initial marking. Each transition is of the form $\langle I, O, s, d, b, \lambda \rangle$, where I and $O : \mathcal{P} \mapsto \mathbb{N}$ are multi-sets of input and output places, respectively. By convention, $O(p)$ (resp. $I(p)$) denotes the number of occurrences of p in O (resp. I). $s, d \in \mathcal{P} \cup \{\perp\}$ are respectively the source and the destination places of the extended arc, $b \in \mathbb{N} \cup \{+\infty\}$ is the bound and $\lambda \in \Sigma$ is the label of the transition.

Let us divide $\mathcal{T}$ into $\mathcal{T}_r$ and $\mathcal{T}_e$ such that $\mathcal{T} = \mathcal{T}_r \cup \mathcal{T}_e$ and $\mathcal{T}_e \cap \mathcal{T}_r = \emptyset$. Without loss of generality, we assume that for each transition $\langle I, O, s, d, b, \lambda \rangle \in \mathcal{T}$, either $b = 0$ and $s = \perp = d$ (regular Petri transitions, grouped into $\mathcal{T}_r$); or $b > 0$, $s \neq d$, $s \neq \perp$ and $d \neq \perp$ (extended transitions, grouped into $\mathcal{T}_e$). We identify several non-disjoint classes of EPN, depending on $\mathcal{T}_e$:

- *Petri net* (PN for short): $\mathcal{T}_e = \emptyset$.
- *Petri net with non-blocking arcs* (PN+NBA): $\forall t = \langle I, O, s, d, b, \lambda \rangle \in \mathcal{T}_e : b = 1$.
- *Petri net with transfer arcs* (PN+T): $\forall t = \langle I, O, s, d, b, \lambda \rangle \in \mathcal{T}_e : b = +\infty$.

As usual, places are graphically depicted by circles; transitions by filled rectangles. For any transition $t = \langle I, O, s, d, b, \lambda \rangle$, we draw an arrow from any place $p \in I$ to transition t and from t to any place $p \in O$. For a PN+NBA (resp. PN+T), we draw a dotted (grey) arrow from s to t and from t to d (provided that $s, d \neq \perp$).

Definition 2. Given an EPN $\mathcal{N} = \langle \mathcal{P}, \mathcal{T}, \Sigma, \mathbf{m}_0 \rangle$, and a marking $\mathbf{m}$ of $\mathcal{N}$, a transition $t = \langle I, O, s, d, b, \lambda \rangle$ is said to be *enabled in* $\mathbf{m}$ (notation: $\mathbf{m} \xrightarrow{t}$) iff $\forall p \in \mathcal{P} : \mathbf{m}(p) \geqslant I(p)$. An enabled transition $t = \langle I, O, s, d, b, \lambda \rangle$ can *occur*, which deterministically transforms the marking $\mathbf{m}$ into a new marking $\mathbf{m}'$ (we denote this by $\mathbf{m} \xrightarrow{t} \mathbf{m}'$). $\mathbf{m}'$ is computed as follows:

(1) First compute $\mathbf{m}_1$ such that: $\forall p \in \mathcal{P} : \mathbf{m}_1(p) = \mathbf{m}(p) - I(p)$.

(2) Then compute $\mathbf{m}_2$ as follows. If $s = d = \perp$, then $\mathbf{m}_2 = \mathbf{m}_1$. Otherwise

$$\mathbf{m}_2(s) = \begin{cases} 0 & \text{if } \mathbf{m}_1(s) \leqslant b, \\ \mathbf{m}_1(s) - b & \text{otherwise,} \end{cases} \qquad \mathbf{m}_2(d) = \begin{cases} \mathbf{m}_1(d) + \mathbf{m}_1(s) & \text{if } \mathbf{m}_1(s) \leqslant b, \\ \mathbf{m}_1(d) + b & \text{otherwise,} \end{cases}$$

$$\forall p \in \mathcal{P} \setminus \{d, s\} : \mathbf{m}_2(p) = \mathbf{m}_1(p).$$

(3) Finally, compute $\mathbf{m}'$, such that $\forall p \in O : \mathbf{m}'(p) = \mathbf{m}_2(p) + O(p)$.

Let $\sigma = t_1 t_2 \ldots t_n$ be a (possibly infinite) sequence of transitions. We write $\mathbf{m} \xrightarrow{\sigma} \mathbf{m}'$ to mean that there exist $\mathbf{m}_1, \ldots, \mathbf{m}_{n-1}$ such that $\mathbf{m} \xrightarrow{t_1} \mathbf{m}_1 \xrightarrow{t_2} \cdots \xrightarrow{t_{n-1}} \mathbf{m}_{n-1} \xrightarrow{t_n} \mathbf{m}'$.

Example 3. Fig. 1 presents a transition $t = \langle I, O, s, d, +\infty, \mathsf{a} \rangle$ equipped with a transfer arc. I and O are such that: $I(p_1) = I(s) = 1$, $I(p_2) = I(d) = 0$, $O(p_2) = 1$ and $O(p_1) = O(s) = O(d) = 0$.

The successive steps (according to Definition 2) to compute the effect of the firing of t are shown. Namely, (a) presents the marking $\mathbf{m}$ before the firing of t; (b) presents the marking $\mathbf{m}_1$ obtained by removing $I(p)$ tokens in every place p; (c) presents $\mathbf{m}_2$ obtained from $\mathbf{m}_1$ by transferring to d the two tokens present in s; and (d) presents the resulting marking $\mathbf{m}'$ obtained after producing $O(p)$ tokens in every place p.

If t had been equipped with a non-blocking arc (hence $t = \langle I, O, s, d, 1, \mathsf{a} \rangle$), only one token would have been transferred from s to d at step (c). In both cases, t would have been firable even if $\mathbf{m}(s) = 0$.

Given a EPN $\mathcal{N}$ with initial marking $\mathbf{m}_0$, $\mathbf{m}_0 \xrightarrow{t_1} \mathbf{m}_1 \xrightarrow{t_2} \cdots \xrightarrow{t_{n-1}} \mathbf{m}_{n-1} \xrightarrow{t_n} \cdots$ is called a *computation* of $\mathcal{N}$.

Definition 4. Let σ be a sequence of transitions. $\Lambda(\sigma)$ is defined inductively as follows (where λ_i denotes the label of t_i). If $\sigma = t_1$, then $\Lambda(\sigma) = \varepsilon$ if $\lambda_1 = \tau$; $\Lambda(\sigma) = \lambda_1$ otherwise. In the case where $\sigma = t_1 t_2 \ldots$, then $\Lambda(\sigma) = \Lambda(t_2 \ldots)$ if $\lambda_1 = \tau$; $\Lambda(\sigma) = \lambda_1 \cdot \Lambda(t_2 \ldots)$ otherwise.

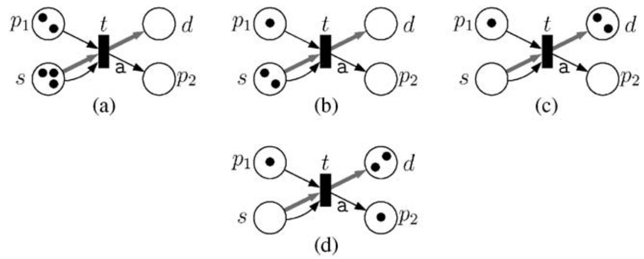

Fig. 1. The four steps to compute the effect of a transfer arc.

Remark that this definition is sound even in the case where σ is infinite since Λ associates one and only one word to any infinite sequence of transitions.

Definition 5. Let $\mathcal{N} = \langle \mathcal{P}, \mathcal{T}, \Sigma, \mathbf{m}_0 \rangle$ be an EPN. An infinite word x on Σ is said to be *accepted* by $\mathcal{N}$ if there exists an infinite sequence of transitions $\sigma = t_1 t_2 \dots$ and an infinite set of markings $\{\mathbf{m}_1, \mathbf{m}_2 \dots\}$ such that $\mathbf{m}_0 \xrightarrow{t_1} \mathbf{m}_1 \xrightarrow{t_2} \mathbf{m}_2 \dots$ and $\Lambda(\sigma) = x$. The *language* $L^\omega(\mathcal{N})$ is defined as the set of all the infinite words accepted by $\mathcal{N}$. The language $L^\omega_{\not\tau}(\mathcal{N})$ is the set of infinite words accepted by sequences of transitions of $\mathcal{N}$ that *do not contain* τ-transitions.

By abuse of notation we also write $\mathbf{m} \xrightarrow{x} \mathbf{m}'$ to mean that there exists a *finite* sequence of transitions σ such that $\Lambda(\sigma) = x$ and $\mathbf{m} \xrightarrow{\sigma} \mathbf{m}'$, and $\mathbf{m} \xrightarrow{x'}$ to mean that we can fire the *infinite* sequence of transitions σ' (with $\Lambda(\sigma') = x'$) from $\mathbf{m}$.

In the following, $L^\omega(PN)$ (respectively, L^ω(PN+T), L^ω(PN+NBA)) denotes the set of all the ω-languages that can be recognized by a PN (respectively, PN+T, and PN+NBA). $L^\omega_{\not\tau}(PN)$, $L^\omega_{\not\tau}$(PN+NBA) and $L^\omega_{\not\tau}$(PN+T) are defined similarly in the case where we disallow τ-transitions.

In the sequel a notion of ordering on the markings will appear to be useful. Let $\preccurlyeq$ denote the quasi ordering on markings, defined as follows: let $\mathbf{m}$ and $\mathbf{m}'$ be two markings on the set of places $\mathcal{P}$, then $\mathbf{m} \preccurlyeq \mathbf{m}'$ if $\forall p \in \mathcal{P}$: $\mathbf{m}(p) \leqslant \mathbf{m}'(p)$. We come back on important properties of $\preccurlyeq$ in Section 2.2.

An important property of sequences of transitions of PN is their *constant effect* (it is well-known that the effect of such a sequence, when it is enabled, can be expressed by a vector of integers—often called *Parikh vector*—stating how many tokens are removed and put in each place). In the case of PN+NBA or PN+T, the effect is not constant anymore, since it is dependant on the marking at the time of the firing. However, the effect of a sequence of transitions with non-blocking arcs can be *bounded*, as stated by the following Lemma.

Lemma 6. *Let $\mathcal{N} = \langle \mathcal{P}, \mathcal{T}, \Sigma, \mathbf{m}_0 \rangle$ be a* PN+NBA, *and let σ be a finite sequence of transitions of $\mathcal{N}$ that contains n occurrences of transitions in $\mathcal{T}_e$. Let $\mathbf{m}_1, \mathbf{m}'_1, \mathbf{m}_2$ and $\mathbf{m}'_2$ be four makings such that* (i) $\mathbf{m}_1 \xrightarrow{\sigma} \mathbf{m}'_1$, (ii) $\mathbf{m}_2 \xrightarrow{\sigma} \mathbf{m}'_2$ *and* (iii) $\mathbf{m}_2 \succcurlyeq \mathbf{m}_1$. *Then, for every place $p \in \mathcal{P}$: $\mathbf{m}'_2(p) - \mathbf{m}'_1(p) \geqslant \mathbf{m}_2(p) - \mathbf{m}_1(p) - n$.*

Proof. Let us consider a place $p \in \mathcal{P}$. First, we remark that when we fire σ from $\mathbf{m}_2$ instead of $\mathbf{m}_1$, its Petri net arcs will have the same effect on p. On the other hand, since we want to find a lower bound on $\mathbf{m}'_2(p) - \mathbf{m}'_1(p)$, we consider the situation where no non-blocking arcs affect p when σ is fired from $\mathbf{m}_1$, but they all remove one token from p when σ is fired from $\mathbf{m}_2$. In the latter case, the effect of σ on p is $\mathbf{m}'_1(p) - \mathbf{m}_1(p) - n$. We obtain thus: $\mathbf{m}'_2(p) \geqslant \max\{\mathbf{m}_2(p) + \mathbf{m}'_1(p) - \mathbf{m}_1(p) - n, 0\}$. Hence $\mathbf{m}'_2(p) \geqslant \mathbf{m}_2(p) + \mathbf{m}'_1(p) - \mathbf{m}_1(p) - n$, and thus: $\mathbf{m}'_2(p) - \mathbf{m}'_1(p) \geqslant \mathbf{m}_2(p) - \mathbf{m}_1(p) - n$. $\square$

2.2. Properties of infinite sequences on well-quasi ordered elements

Following [10,1], $\preccurlyeq$ is a *well-quasi ordering* (wqo for short). This means that $\preccurlyeq$ is a reflexive and transitive relation such that for any infinite sequence $\mathbf{m}_1, \mathbf{m}_2, \dots$ there exist $i < j$ such that $\mathbf{m}_i \preccurlyeq \mathbf{m}_j$. Hence, we get this property on $\preccurlyeq$:

Lemma 7. *Given an infinite sequence of markings* $\mathbf{m}_1, \mathbf{m}_2, \ldots$ *we can always extract an infinite sub-sequence* $\mathbf{m}_{i_1}, \mathbf{m}_{i_2}, \ldots (\forall j : i_j < i_{j+1})$ *such that for each place p, either* $\mathbf{m}_{i_j}(p) < \mathbf{m}_{i_{j+1}}(p)$ *for all* $j \geqslant 1$ *or* $\mathbf{m}_{i_j}(p) = \mathbf{m}_{i_{j+1}}(p)$ *for all* $j \geqslant 1$.

Proof. We exhibit an inductive reasoning on the dimension n (number of places) of the net.

Base case: Let $n = 1$. Then $\mathbf{m}_1\mathbf{m}_2 \ldots$ is a sequence of elements in $\mathbb{N}$. If one of the elements of the sequence occurs infinitely often, the property is trivially true. Otherwise, let min be the minimal value in this sequence and i_1 be its last index, then we take $\mathbf{m}_{i_1}$ as the first element and we iterate the construction on $\mathbf{m}_{i_1+1}\mathbf{m}_{i_1+2} \ldots$ We then obtain the desired sequence $\mathbf{m}_{i_1}\mathbf{m}_{i_2} \ldots$

Induction step: Let $\mathbf{m}_1\mathbf{m}_2 \ldots$ be a sequence of markings and $\mathbf{m}_{i_1}\mathbf{m}_{i_2} \ldots$ be an infinite sub-sequence such that each of the $n-1$ first components are constant or always increase. Such a sub-sequence exists by induction hypothesis. By applying the procedure of the base case, we can extract a sub-sequence of it which is either constant or strictly increasing for the nth component. □

The following Lemma is easy to prove [16].

Lemma 8 (*Monotonicity*). *Let* $\mathbf{m}_1$, $\mathbf{m}_2$ *and* $\mathbf{m}_1'$ *be three markings of an EPN, such that* $\mathbf{m}_1 \preccurlyeq \mathbf{m}_2$ *and* $\mathbf{m}_1 \xrightarrow{t} \mathbf{m}_1'$ *for some transition t of the EPN. Then, there exists* $\mathbf{m}_2'$ *such that* $\mathbf{m}_2 \xrightarrow{t} \mathbf{m}_2'$ *and* $\mathbf{m}_1' \preccurlyeq \mathbf{m}_2'$.

3. PN+T are more expressive than PN+NBA

In this section, one will find the first important result of the paper (as stated by Theorem 14): PN+T are strictly more expressive, on ω-languages, than PN+NBA. We prove this in two steps. First, we show that any ω-language accepted by a PN+NBA can be accepted by a PN+T (this is the purpose of Lemma 9 and Theorem 10). Then, we prove the strictness of the inclusion thanks to the PN+T $\mathcal{N}_1$ of Fig. 3(a). Namely, we show that $L^\omega(\mathcal{N}_1)$ contains at least the words $(\mathtt{a}^k\mathtt{b}^k)^\omega$, for any $k \geqslant 1$ (Lemma 11). On the other hand we show that $\mathcal{N}_1$ rejects the words whose prefix belongs to $(\mathtt{a}^{n_3}\mathtt{b}^{n_3})^*\mathtt{a}^{n_3}(\mathtt{b}^{n_1}\mathtt{a}^{n_1})^+\mathtt{b}^{n_2}$ with $n_1 < n_2 < n_3$ (Lemma 12). We finally show that any PN+NBA accepting words of the form $(\mathtt{a}^k\mathtt{b}^k)^\omega$ also has to accept words whose prefix belongs to $(\mathtt{a}^{n_3}\mathtt{b}^{n_3})^*\mathtt{a}^{n_3}(\mathtt{b}^{n_1}\mathtt{a}^{n_1})^+\mathtt{b}^{n_2}$ with $n_1 < n_2 < n_3$. Since $\mathcal{N}_1$ rejects the latter, we conclude that no PN+NBA can accept $L^\omega(\mathcal{N}_1)$.

3.1. PN+NBA *are not more expressive than* PN+T.

Let us consider a PN+NBA $\mathcal{N} = \langle \mathcal{P}, \mathcal{T}, \Sigma, \mathbf{m}_0 \rangle$, and let us show how to transform it into a PN+T $\mathcal{N}'$ such that $L^\omega(\mathcal{N}) = L^\omega(\mathcal{N}')$.

Let us consider the partition of $\mathcal{T}$ into $\mathcal{T}_e$ and $\mathcal{T}_r$ as defined in Definition 1, and a new place p_{Tr} (the trash place). We now show how to build $\mathcal{N}' = \langle \mathcal{P}', \mathcal{T}', \Sigma, \mathbf{m}_0' \rangle$. First, $\mathcal{P}' = \mathcal{P} \cup \{p_{\mathrm{Tr}}\}$. For each transition $t = \langle I, O, s, d, 1, \lambda \rangle$ in $\mathcal{T}_e$, we put in $\mathcal{T}'$ two new transitions $t_l = \langle I, O, s, p_{\mathrm{Tr}}, +\infty, \lambda \rangle$ and $t_e = \langle I_e, O_e, \bot, \bot, 0, \lambda \rangle$, such that: $\forall p \in \mathcal{P} : \big(p \neq s \Rightarrow I_e(p) = I(p) \wedge p \neq d \Rightarrow O_e(p) = O(p)\big)$, $I_e(s) = I(s) + 1$ and $O_e(d) = O(d) + 1$. We also add into $\mathcal{T}'$ all the transitions of $\mathcal{T}_r$. Finally, $\forall p \in \mathcal{P} = \mathbf{m}_0'(p) = \mathbf{m}_0(p)$ and $\mathbf{m}_0'(p_{\mathrm{Tr}}) = 0$. Fig. 2 illustrates the construction. Lemma 9 shows that this construction is correct.

Lemma 9. *For any* PN+NBA $\mathcal{N}$, *it is possible to construct a* PN+T $\mathcal{N}'$ *such that* $L^\omega(\mathcal{N}) = L^\omega(\mathcal{N}')$.

Proof. Let us consider a PN+NBA $\mathcal{N}$, and let $\mathcal{N}'$ be the PN+T obtained by applying to $\mathcal{N}$ the aforementioned construction. Let us show that $L^\omega(\mathcal{N}) = L^\omega(\mathcal{N}')$

[$L^\omega(\mathcal{N}) \subseteq L^\omega(\mathcal{N}')$] We show that, for every infinite sequence of transitions σ of $\mathcal{N}$, we can find a sequence of transitions σ' of $\mathcal{N}'$ such that $\Lambda(\sigma) = \Lambda(\sigma')$.

Let us define the function $f : \mathcal{T} \times \mathbb{N}^{|\mathcal{P}|} \to \mathcal{T}'$ such that $\forall t \in \mathcal{T}_r : f(t, \mathbf{m}) = t$ and $\forall t = \langle O, I, s, d, 1, \lambda \rangle \in \mathcal{T}_e : f(t, \mathbf{m}) = t_e$, if $\mathbf{m}(s) > I(s)$ (i.e., s will not be empty after $I(s)$ tokens have been removed from it and the non-blocking arc will actually remove tokens from s.); and $f(t, \mathbf{m}) = t_l$, otherwise.

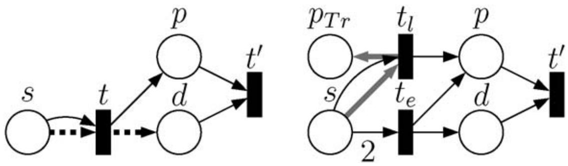

Fig. 2. A PN+NBA $\mathcal{N}$ (a) and the corresponding PN+T $\mathcal{N}'$ (b).

Let $\sigma = \mathbf{m}_0 \xrightarrow{t_1} \mathbf{m}_1 \xrightarrow{t_2} \cdots \xrightarrow{t_n} \mathbf{m}_n \xrightarrow{t_{n+1}} \cdots$ be a computation of $\mathcal{N}$. Then we may see that $\sigma' = \mathbf{m}'_0 \xrightarrow{f(t_1,\mathbf{m}_0)} \mathbf{m}'_1 \xrightarrow{f(t_2,\mathbf{m}_1)} \ldots \xrightarrow{f(t_n,\mathbf{m}_{n-1})} \mathbf{m}'_n \xrightarrow{f(t_{n+1},\mathbf{m}_n)} \ldots$ is a computation of $\mathcal{N}'$, where $\mathbf{m}'_i$ is such that $\mathbf{m}'_i(p) = \mathbf{m}_i(p)$ for all $p \in \mathcal{P}$ and $\mathbf{m}'_i(p_{\mathrm{Tr}}) = 0$ for all $i \geqslant 1$ (remark that, since f allows us to choose, for any $i \geqslant 1$, the *right transition* to simulate t_i, no tokens are put into p_{Tr} along the execution of σ'). Since we have $\forall i \geqslant 1 : \Lambda(t_i) = \Lambda(f(t_i, \mathbf{m}_{i-1}))$, we conclude that $\Lambda(\sigma) = \Lambda(\sigma')$, hence $L^\omega(\mathcal{N}) \subseteq L^\omega(\mathcal{N}')$.

$[L^\omega(\mathcal{N}') \subseteq L^\omega(\mathcal{N})]$ We show that, for every infinite sequence of transitions σ' of $\mathcal{N}'$, we can find a sequence of transitions σ of $\mathcal{N}$ such that $\Lambda(\sigma') = \Lambda(\sigma)$.

We define the function $g : \mathcal{T}' \to \mathcal{T}$ such that for all $t \in \mathcal{T}_r$: $g(t) = t$ and for all $t \in \mathcal{T}_e : g(t_e) = g(t_l) = t$. Moreover, we define the relation $\preccurlyeq_P$ that compares two markings only on the places that are in P. Thus, if $\mathbf{m}$ is defined on a set of places P and $\mathbf{m}'$ on a set of places P' with $P' \subseteq P$, we have $\mathbf{m}' \preccurlyeq_{P'} \mathbf{m}$ iff $\forall p \in \mathcal{P}' : \mathbf{m}'(p) \leqslant \mathbf{m}(p)$.

Let $\sigma' = \mathbf{m}'_0 \xrightarrow{t_1} \mathbf{m}'_1 \xrightarrow{t_2} \cdots \xrightarrow{t_n} \mathbf{m}'_n \xrightarrow{t_{n+1}} \cdots$ be a computation of $\mathcal{N}'$. Then, there exist $\mathbf{m}_1, \mathbf{m}_2, \ldots$ in $\mathcal{N}$ such that we have $\mathbf{m}_0 \xrightarrow{g(t_1)} \mathbf{m}_1 \xrightarrow{g(t_2)} \cdots \xrightarrow{g(t_n)} \mathbf{m}_n \xrightarrow{g(t_{n+1})} \cdots$ We prove that this computation exists by contradiction. Suppose that it is not the case. Let $i \geqslant 0$ be the smallest value such that $g(t_{i+1})$ is not firable from $\mathbf{m}_i$. Let us show by induction on the indexes, that $\mathbf{m}'_j \preccurlyeq_{\mathcal{P}} \mathbf{m}_j$ for all j such that $0 \leqslant j \leqslant i$.

Base case: $j = 0$. The base case is trivially verified.

Induction step: $j = k$. By induction hypothesis, we have: $\forall 0 \leqslant j \leqslant k-1 : \mathbf{m}'_j \preccurlyeq_{\mathcal{P}} \mathbf{m}_j$. In the case where $t_k = \langle I, O, s, d, b, \lambda\rangle$ (from $\mathbf{m}'_{k-1}$) has the same effect on $\mathcal{P}$ as $g(t_k)$ (from $\mathbf{m}_{k-1}$), we directly have that $\mathbf{m}'_k \preccurlyeq_{\mathcal{P}} \mathbf{m}_k$. This happens if t_k is a regular Petri transition or if $\mathbf{m}_{k-1}(s) = \mathbf{m}'_{k-1}(s) = I(s)$.

Otherwise t_k has a transfer arc and we must consider two cases:

- The transfer arc of t_k has no effect and the non-blocking arc of $g(t_k)$ moves one token from the source s to the target d, hence $I(s) = \mathbf{m}'_{k-1}(s) < \mathbf{m}_{k-1}(s)$. Since t_k and $g(t_k)$ have the same effect except that $g(t_k)$ removes one more token from s and adds one more token in d, and since $\mathbf{m}'_{k-1} \preccurlyeq_{\mathcal{P}} \mathbf{m}_{k-1}$ with $\mathbf{m}'_{k-1}(s) < \mathbf{m}_{k-1}(s)$, we conclude that $\mathbf{m}'_k \preccurlyeq_{\mathcal{P}} \mathbf{m}_k$.
- The transfer of t_k moves at least one token from the source s to p_{Tr} and the non-blocking arc of $g(t_k)$ moves one token from s to d. Since t_k and $g(t_k)$ have the same effect on the places in $\mathcal{P}$ except that $g(t_k)$ adds one more token in d and t_k may remove more tokens from s, and since $\mathbf{m}'_{k-1} \preccurlyeq_{\mathcal{P}} \mathbf{m}_{k-1}$, we conclude that $\mathbf{m}'_k \preccurlyeq_{\mathcal{P}} \mathbf{m}_k$.

Thus, for any $0 \leqslant j \leqslant i$: $\mathbf{m}'_j \preccurlyeq_{\mathcal{P}} \mathbf{m}_j$. Since t_{i+1} is firable from $\mathbf{m}'_i$, we conclude that $g(t_{i+1})$ is firable from $\mathbf{m}_i$ because $g(t_{i+1})$ consumes no more tokens in any place than t_{i+1} does. Hence the contradiction.

Thus, there exists $\mathbf{m}_1, \mathbf{m}_2 \ldots$ such that we have $\mathbf{m}_0 \xrightarrow{g(t_1)} \mathbf{m}_1 \xrightarrow{g(t_2)} \cdots \xrightarrow{g(t_n)} \mathbf{m}_n \xrightarrow{g(t_{n+1})} \cdots$ in $\mathcal{N}$. Since $\Lambda(t_k) = \Lambda(g(t_k))$ for all $k \geqslant 1$, we conclude that $\Lambda(\sigma') = \Lambda(\sigma)$, hence $L^\omega(\mathcal{N}') \subseteq L^\omega(\mathcal{N})$. $\square$

Theorem 10. *For every ω-language L that is accepted by a* PN+NBA, *there exists a* PN+T *that accepts L.*

Proof. The Theorem stems directly from Lemma 9. $\square$

3.2. PN+T *are more expressive than* PN+NBA

Let us now prove that L^ω(PN+NBA) is *strictly* included in L^ω(PN+T). We consider the PN+T $\mathcal{N}_1$ presented in Fig. 3(a) with the initial marking $\mathbf{m}_0(p_1) = 1$ and $\mathbf{m}_0(p) = 0$ for $p \in \{p_2, p_3, p_4\}$. The two following Lemmata allow us to better understand the behaviour of $\mathcal{N}_1$.

Lemma 11. *For any $k \geqslant 1$, the word $\left(\mathtt{a}^k\mathtt{b}^k\right)^\omega$ is accepted by $\mathcal{N}_1$.*

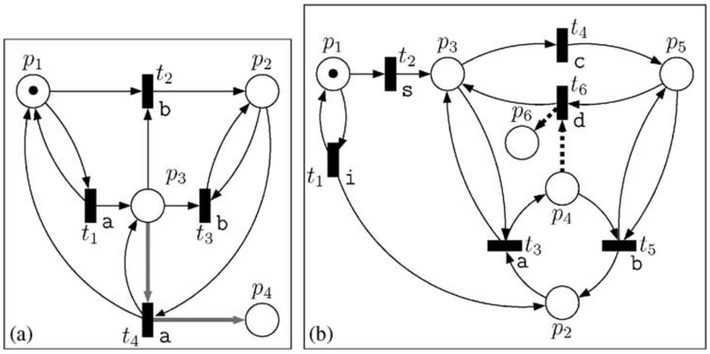

Fig. 3. (a) The PN+T $\mathcal{N}_1$ and (b) the PN+NBA $\mathcal{N}_2$.

Proof. The following holds for any $k \geqslant 1$. From the initial marking of $\mathcal{N}_1$, we can fire $t_1^k t_2 t_3^{k-1}$ (which accepts $\mathtt{a}^k\mathtt{b}^k$), and reach the marking $\mathbf{m}_1$ such that $\mathbf{m}_1(p_2) = 1$ and $\forall i \in \{1, 3, 4\} : \mathbf{m}_1(p_i) = 0$. Thus, t_4 is firable from $\mathbf{m}_1$ and does not transfer any token, but produces a token in p_3 and moves the token from p_2 to p_1. The marking that is obtained is $\mathbf{m}_2$ such that $\mathbf{m}_2(p_1) = \mathbf{m}_2(p_3) = 1$ and $\mathbf{m}_2(p_2) = \mathbf{m}_2(p_4) = 0$. It is not difficult to see now that $\mathbf{m}_2 \xrightarrow{t_1^{k-1}t_2t_3^{k-1}} \mathbf{m}_1 \xrightarrow{t_4} \mathbf{m}_2$. We can thus fire the sequence $t_1^{k-1}t_2t_3^{k-1}t_4$ arbitrarily often from $\mathbf{m}_2$. Hence $\left(\mathtt{a}^k\mathtt{b}^k\right)^\omega$ is accepted by $\mathcal{N}_1$. $\square$

Lemma 12. *Let n_1, n_2, n_3 and m be four natural numbers such that $0 < n_1 < n_2 < n_3$ and $m > 0$. Then, for any $k \geqslant 0$ the words $(\mathtt{a}^{n_3}\mathtt{b}^{n_3})^k\mathtt{a}^{n_3}(\mathtt{b}^{n_1}\mathtt{a}^{n_1})^m(\mathtt{b}^{n_2}\mathtt{a}^{n_2})^\omega$ are not accepted by $\mathcal{N}_1$.*

Proof. The following holds for any n_1, n_2, n_3 with $0 < n_1 < n_2 < n_3$ and for any $m > 0$ and $k \in \mathbb{N}$. From the initial marking of $\mathcal{N}_1$, the only sequence of transitions labelled by $\mathtt{a}^{n_3}$ is $t_1^{n_3}$. Firing this sequence leads to the marking $\mathbf{m}_1$ such that $\mathbf{m}_1(p_1) = 1, \mathbf{m}_1(p_3) = n_3$ and $\mathbf{m}_1(p) = 0$ if $p \in \{p_2, p_4\}$. From $\mathbf{m}_1$ the only firable sequence of transitions labelled by $\mathtt{b}^{n_3}$ is $t_2t_3^{n_3-1}$. This leads to the marking $\mathbf{m}_2$ such that $\mathbf{m}_2(p_2) = 1$ and $\mathbf{m}_2(p) = 0$ if $p \neq p_2$. The only sequence of transitions firable from $\mathbf{m}_2$ and labelled by $\mathtt{a}^{n_3}$ is $t_4t_1^{n_3-1}$. Since $\mathbf{m}_2(p_3) = 0$, the transfer of t_4 has no effect when fired from $\mathbf{m}_2$. Hence, we reach $\mathbf{m}_1$ again after firing $t_4t_1^{n_3-1}$. By repeating the reasoning, we conclude that the only sequence of transitions firable from the initial marking and labelled by $(\mathtt{a}^{n_3}\mathtt{b}^{n_3})^k\mathtt{a}^{n_3}$ (when $k > 0$) is $t_1^{n_3}t_2t_3^{n_3-1}(t_4t_1^{n_3-1}t_2t_3^{n_3-1})^{k-1}t_4t_1^{n_3-1}$ and it leads to $\mathbf{m}_1$. In the case where $k = 0$, the sequence $t_1^{n_3}$ is firable and leads to $\mathbf{m}_1$ too. From $\mathbf{m}_1$, the only firable sequence of transitions labelled by $\mathtt{b}^{n_1}$ is $t_2t_3^{n_1-1}$. This leads to a marking similar to $\mathbf{m}_2$, noted $\mathbf{m}_2'$, except that p_3 contains $n_3 - n_1$ tokens. Then, the only firable sequence of transitions labelled by $\mathtt{a}^{n_1}$ is $t_4t_1^{n_1-1}$. In this case, the transfer of t_4 moves the $n_3 - n_1$ tokens from p_3 to p_4 and we reach a marking similar to $\mathbf{m}_1$, noted $\mathbf{m}_1'$, except that p_4 contains $n_3 - n_1$ tokens and p_3 contains n_1 tokens. From $\mathbf{m}_1'$, the only firable sequence of transitions labelled by $\mathtt{b}^{n_1}\mathtt{a}^{n_1}$ is $t_2t_3^{n_1-1}t_4t_1^{n_1-1}$ and it leads to $\mathbf{m}_1'$.

However, after firing $t_2t_3^{n_1-1}$ from $\mathbf{m}_1'$, we reach a marking $\mathbf{m}_2''$ similar to $\mathbf{m}_2$ except that p_4 contains $n_3 - n_1$ tokens and from which no transition labelled by $\mathtt{b}$ is firable. Since $n_2 > n_1$, we conclude that there is no sequence of transitions labelled by $\mathtt{b}^{n_2}$ that is firable from $\mathbf{m}_1'$, hence $(\mathtt{a}^{n_3}\mathtt{b}^{n_3})^k\mathtt{a}^{n_3}(\mathtt{b}^{n_1}\mathtt{a}^{n_1})^m(\mathtt{b}^{n_2}\mathtt{a}^{n_2})^\omega$ is not accepted by $\mathcal{N}_1$. $\square$

We can now show that no PN+NBA can accept $L^\omega(\mathcal{N}_1)$. Remark that the proof technique used hereafter relies on Lemmata 7 and 8, and is somewhat similar to a *pumping Lemma*. To the best of our knowledge, it is the first time such a technique is applied in the context of Petri nets (and their extensions).

Lemma 13. *No PN+NBA accepts $L^\omega(\mathcal{N}_1)$.*

Proof. Let $\mathcal{N}$ be a PN+NBA such that $L^\omega(\mathcal{N}_1) \subseteq L^\omega(\mathcal{N})$. We will show that this implies that $L^\omega(\mathcal{N}_1) \subsetneq L^\omega(\mathcal{N})$. As $L^\omega(\mathcal{N}_1) \subseteq L^\omega(\mathcal{N})$, by Lemma 11 we know that, for all $k \geqslant 1$, the word $(\mathtt{a}^k\mathtt{b}^k)^\omega$ belongs to $L^\omega(\mathcal{N})$. Suppose that

$\mathbf{m}_{\text{init}}$ is the initial marking of $\mathcal{N}$. Thus, for all $k \geqslant 1$, there exists a marking $\widehat{\mathbf{m}}_k$, a finite sequence of transitions σ_k and a natural number ℓ_k such that

$$\mathbf{m}_{\text{init}} \xrightarrow{(\mathtt{a}^k\mathtt{b}^k)^{\ell_k}\mathtt{a}^k} \widehat{\mathbf{m}}_k \xrightarrow{\Lambda(\sigma_k)} \widehat{\mathbf{m}}'_k, \widehat{\mathbf{m}}'_k \succcurlyeq \widehat{\mathbf{m}}_k \quad \text{and} \quad \Lambda(\sigma_k) = (\mathtt{b}^k\mathtt{a}^k)^{n_k} \text{ with } n_k \geqslant 1.$$

Indeed, if it was not the case, we would have $\mathbf{m}_{\text{init}} \xrightarrow{\mathtt{a}^k} \mathbf{m}_1 \xrightarrow{\mathtt{b}^k\mathtt{a}^k} \mathbf{m}_2 \cdots \xrightarrow{\mathtt{b}^k\mathtt{a}^k} \mathbf{m}_i \xrightarrow{\mathtt{b}^k\mathtt{a}^k} \cdots$ such that there does not exist $1 \leqslant i < j$ with $\mathbf{m}_i \preccurlyeq \mathbf{m}_j$. But from Lemma 7, this never occurs.

Let us consider the infinite sequence $\widehat{\mathbf{m}}_1, \widehat{\mathbf{m}}_2, \ldots, \widehat{\mathbf{m}}_i, \ldots$ Following Lemma 7 again, we extract from it a subsequence $\widehat{\mathbf{m}}_{\rho(1)}, \widehat{\mathbf{m}}_{\rho(2)}, \ldots, \widehat{\mathbf{m}}_{\rho(n)}, \ldots$ such that: $\forall p \in \mathcal{P}$: either $\forall i \geqslant 1 : \widehat{\mathbf{m}}_{\rho(i)}(p) = \widehat{\mathbf{m}}_{\rho(i+1)}(p)$ or $\forall i \geqslant 1 : \widehat{\mathbf{m}}_{\rho(i)}(p) < \widehat{\mathbf{m}}_{\rho(i+1)}(p)$. Let us denote by $\mathcal{P}'$ the set of places that strictly increase in that sequence.

Let n be the number of occurrences of transitions of $\mathcal{T}_e$ in $\sigma_{\rho(1)}$ and let us consider $\widehat{\mathbf{m}}_{\rho(1)}, \widehat{\mathbf{m}}_{\rho(2)}, \widehat{\mathbf{m}}_{\rho(n+3)}$, and $\mathbf{m}$ such that: $\widehat{\mathbf{m}}_{\rho(n+3)} \xrightarrow{\sigma_{\rho(1)}} \mathbf{m}$ (from Lemma 8, the sequence $\sigma_{\rho(1)}$ is firable from $\widehat{\mathbf{m}}_{\rho(n+3)}$ since $\widehat{\mathbf{m}}_{\rho(1)} \preccurlyeq \widehat{\mathbf{m}}_{\rho(n+3)}$ and $\sigma_{\rho(1)}$ is firable from $\widehat{\mathbf{m}}_{\rho(1)}$). We first prove that $\mathbf{m} \succcurlyeq \widehat{\mathbf{m}}_{\rho(2)}$.

We know that

$$\widehat{\mathbf{m}}_{\rho(1)} \xrightarrow{\sigma_{\rho(1)}} \widehat{\mathbf{m}}'_{\rho(1)} \wedge \widehat{\mathbf{m}}'_{\rho(1)} \succcurlyeq \widehat{\mathbf{m}}_{\rho(1)} \tag{1}$$

$$\forall\, p \in \mathcal{P}' : \widehat{\mathbf{m}}_{\rho(n+3)}(p) \geqslant \widehat{\mathbf{m}}_{\rho(2)}(p) + n + 1 \tag{2}$$

$$\forall\, p \in \mathcal{P} \setminus \mathcal{P}' : \widehat{\mathbf{m}}_{\rho(1)}(p) = \widehat{\mathbf{m}}_{\rho(2)}(p) = \widehat{\mathbf{m}}_{\rho(n+3)}(p). \tag{3}$$

Thus

$$\begin{array}{lll}
\text{(a)} & \forall\, p \in \mathcal{P}' : \mathbf{m}(p) \geqslant \widehat{\mathbf{m}}'_{\rho(1)}(p) + \big(\widehat{\mathbf{m}}_{\rho(n+3)}(p) - \widehat{\mathbf{m}}_{\rho(1)}(p)\big) - n & \text{by Lemma 6} \\
\Rightarrow & \forall\, p \in \mathcal{P}' : \mathbf{m}(p) \geqslant \widehat{\mathbf{m}}_{\rho(1)}(p) + \big(\widehat{\mathbf{m}}_{\rho(n+3)}(p) - \widehat{\mathbf{m}}_{\rho(1)}(p)\big) - n & \text{by (1)} \\
\Rightarrow & \forall\, p \in \mathcal{P}' : \mathbf{m}(p) \geqslant \widehat{\mathbf{m}}_{\rho(n+3)}(p) - n & \\
\Rightarrow & \forall\, p \in \mathcal{P}' : \mathbf{m}(p) \geqslant \widehat{\mathbf{m}}_{\rho(2)}(p) + 1 & \text{by (2)} \\
\Rightarrow & \forall\, p \in \mathcal{P}' : \mathbf{m}(p) > \widehat{\mathbf{m}}_{\rho(2)}(p). &
\end{array}$$

(b) By monotonicity of PN+NBA (Lemma 8), we have that $\mathbf{m} \succcurlyeq \widehat{\mathbf{m}}'_{\rho(1)}$. Moreover, by (1), we have that $\widehat{\mathbf{m}}'_{\rho(1)} \succcurlyeq \widehat{\mathbf{m}}_{\rho(1)}$. Hence, $\forall p \in \mathcal{P} : \mathbf{m}(p) \geqslant \widehat{\mathbf{m}}_{\rho(1)}(p)$. As a consequence, $\forall p \in \mathcal{P} \setminus \mathcal{P}' : \mathbf{m}(p) \geqslant \widehat{\mathbf{m}}_{\rho(2)}(p)$ from (3).

From (a) and (b), we obtain $\mathbf{m} \succcurlyeq \widehat{\mathbf{m}}_{\rho(2)}$, hence $\sigma_{\rho(2)}$ is firable from $\mathbf{m}$. And so

$$\mathbf{m}_{\text{init}} \xrightarrow{\left(\mathtt{a}^{\rho(3+n)}\mathtt{b}^{\rho(3+n)}\right)^{\ell_{\rho(3+n)}}\mathtt{a}^{\rho(3+n)}} \widehat{\mathbf{m}}_{\rho(3+n)} \xrightarrow{\left(\mathtt{b}^{\rho(1)}\mathtt{a}^{\rho(1)}\right)^{n_{\rho(1)}}} \mathbf{m} \xrightarrow{\left(\mathtt{b}^{\rho(2)}\mathtt{a}^{\rho(2)}\right)^{n_{\rho(2)}}} \mathbf{m}'.$$

Finally, let us prove that we can fire $\sigma_{\rho(2)}$ infinitely often from $\mathbf{m}'$. Since $\mathbf{m} \succcurlyeq \widehat{\mathbf{m}}_{\rho(2)}$ and $\widehat{\mathbf{m}}_{\rho(2)} \xrightarrow{\sigma_{\rho(2)}} \widehat{\mathbf{m}}'_{\rho(2)}$, we have by Lemma 8 that $\mathbf{m}' \succcurlyeq \widehat{\mathbf{m}}'_{\rho(2)} \succcurlyeq \widehat{\mathbf{m}}_{\rho(2)}$, hence $\mathbf{m}' \xrightarrow{\sigma_{\rho(2)}} \mathbf{m}''$ for some marking $\mathbf{m}'' \succcurlyeq \widehat{\mathbf{m}}'_{\rho(2)} \succcurlyeq \widehat{\mathbf{m}}_{\rho(2)}$. Since we can repeat the reasoning infinitely often from $\mathbf{m}''$, we conclude that $\sigma_{\rho(2)}$ can be fired infinitely often from $\mathbf{m}''$ and $(\mathtt{a}^{\rho(n+3)}\mathtt{b}^{\rho(n+3)})^{\ell_{\rho(n+3)}}\mathtt{a}^{\rho(n+3)}\big(\mathtt{b}^{\rho(1)}\mathtt{a}^{\rho(1)}\big)^{n_{\rho(1)}}\big(\mathtt{b}^{\rho(2)}\mathtt{a}^{\rho(2)}\big)^{\omega}$ is a word of $L^{\omega}(\mathcal{N})$ (with $\rho(n+3) > \rho(2) > \rho(1) > 0$ and $n_{\rho(1)} > 0$). But, following Lemma 12, this word is not in $L^{\omega}(\mathcal{N}_1)$. We conclude that $L^{\omega}(\mathcal{N}_1) \subsetneq L^{\omega}(\mathcal{N})$. □

We can now state the main Theorem of this section.

Theorem 14. PN+T *are more expressive*, *on infinite words*, *than* PN+NBA, *i.e.*, $L^{\omega}(\text{PN+NBA}) \subsetneq L^{\omega}(\text{PN+T})$.

Proof. From Theorem 10, we have that $L^{\omega}(\text{PN+NBA}) \subseteq L^{\omega}(\text{PN+T})$. However, there exist languages that can be recognized by a PN+T but not by PN+NBA, following Lemma 13. Hence, we conclude that $L^{\omega}(\text{PN+NBA}) \subsetneq L^{\omega}(\text{PN+T})$. □

Remark that Theorem 10 still holds in the case where we disallow τ-transitions, since the construction used in Lemma 9 does not require the use of τ-transitions. Moreover, since $\mathcal{N}_1$ contains no τ-transitions and since we have made no assumptions regarding the τ-transitions in the previous proofs, we obtain:

Corollary 15. PN+T *without τ-transitions are more expressive on infinite words than* PN+NBA, *i.e.*, $L^{\omega}_{\not\tau}$(PN+NBA) $\subsetneq L^{\omega}_{\not\tau}$(PN+T).

4. PN+NBA are more expressive than PN

In this section we prove that the class of ω-languages accepted by any PN+NBA strictly contains the class of ω-languages accepted by any PN.

The strategy adopted in the proof is similar to the one we have used in Section 3. We look into the PN+NBA $\mathcal{N}_2$ of Fig. 3(b), and prove it accepts every word of the form $\mathtt{i}^k\mathtt{s}(\mathtt{a}^k\mathtt{c}\mathtt{b}^k\mathtt{d})^{\omega}$, for $k \geqslant 1$ (Lemma 16), but rejects words of the form $\mathtt{i}^{n_3}\mathtt{s}(\mathtt{a}^{n_3}\mathtt{c}\mathtt{b}^{n_3}\mathtt{d})^m\mathtt{a}^{n_3}\mathtt{c}(\mathtt{b}^{n_1}\mathtt{d}\mathtt{a}^{n_1}\mathtt{c})^k(\mathtt{b}^{n_2}\mathtt{d}\mathtt{a}^{n_2}\mathtt{c})^{\omega}$, for k big enough, and $0 < n_1 < n_2 < n_3$ (Lemma 17). Then, we prove Lemma 18, stating that any PN accepting at least the words of the first form must also accept the words of the latter form. We conclude that no PN can accept $L^{\omega}(\mathcal{N}_2)$. Since any PN is also a PN+NBA, the inclusion is immediate, and we obtain Theorem 19, that states the strictness of the inclusion $L^{\omega}(PN) \subsetneq L^{\omega}$(PN+NBA).

Let us consider the PN+NBA $\mathcal{N}_2$ in Fig. 3(b), with the initial marking $\mathbf{m}_0$ such that $\mathbf{m}_0(p_1) = 1$ and $\mathbf{m}_0(p) = 0$ for $p \in \{p_2, p_3, p_4, p_5, p_6\}$.

Lemma 16. *For any* $k \geqslant 0$, *the word* $\mathtt{i}^k\mathtt{s}(\mathtt{a}^k\mathtt{c}\mathtt{b}^k\mathtt{d})^{\omega}$ *is accepted by* $\mathcal{N}_2$.

Proof. The following holds for any $k \geqslant 0$. After firing the transitions $t_1^k t_2$ from the initial marking of $\mathcal{N}_2$, we reach the marking $\mathbf{m}_1$ such that $\mathbf{m}_1(p_2) = k$, $\mathbf{m}_1(p_3) = 1$, and $\mathbf{m}_1(p_j) = 0$ for $j \in \{1, 4, 5, 6\}$. Then, we can fire $t_3^k t_4$ from $\mathbf{m}_1$. This leads to the marking $\mathbf{m}_2$ such that $\mathbf{m}_2(p_4) = k$, $\mathbf{m}_2(p_5) = 1$, and $\mathbf{m}_2(p_j) = 0$ for $j \in \{1, 2, 3, 6\}$. From $\mathbf{m}_2$, t_5^k can be fired. This sequence of transitions moves the k tokens from p_4 to p_2. Then, from the resulting marking, t_6 can be fired. Since, p_4 is now empty, the effect of t_6 only consists in moving the token from p_5 to p_3 (its non-blocking arc has no effect) and we reach $\mathbf{m}_1$ again. Then, by applying the same reasoning, we fire infinitely often $t_3^k t_4 t_5^k t_6$. The word corresponding to such a sequence is $\mathtt{i}^k\mathtt{s}(\mathtt{a}^k\mathtt{c}\mathtt{b}^k\mathtt{d})^{\omega}$. □

Lemma 17. *Let* n_1, n_2 *and* n_3 *be three natural numbers such that* $0 < n_1 < n_2 < n_3$. *Then, for all* $m > 0$, *for all* $k \geqslant n_3 - n_1 - 1$: *the words*

$$\mathtt{i}^{n_3}\mathtt{s}(\mathtt{a}^{n_3}\mathtt{c}\mathtt{b}^{n_3}\mathtt{d})^m\mathtt{a}^{n_3}\mathtt{c}(\mathtt{b}^{n_1}\mathtt{d}\mathtt{a}^{n_1}\mathtt{c})^k(\mathtt{b}^{n_2}\mathtt{d}\mathtt{a}^{n_2}\mathtt{c})^{\omega}$$

are not accepted by $\mathcal{N}_2$.

Proof. In this proof, we will identify a sequence of transitions with the word it accepts (all the transitions have different labels). Clearly (see the proof of Lemma 16), the firing of $\mathtt{i}^{n_3}\mathtt{s}(\mathtt{a}^{n_3}\mathtt{c}\mathtt{b}^{n_3}\mathtt{d})^m$ from $\mathbf{m}_0$ leads to a marking $\mathbf{m}_1$ such that $\mathbf{m}_1(p_2) = n_3$, $\mathbf{m}_1(p_3) = 1$, and $\forall i \in \{1, 4, 5, 6\} : \mathbf{m}_1(p_i) = 0$ (the non-blocking arc of t_6 has not consumed any token in p_4). By firing $\mathtt{a}^{n_3}\mathtt{c}\mathtt{b}^{n_1}\mathtt{d}$ from $\mathbf{m}_1$, we now have n_1 tokens in p_2, $n_3 - n_1 - 1$ tokens in p_4 and one token in p_6 (this time the non-blocking arc has moved one token since $n_1 < n_3$). Clearly, at each subsequent firing of $\mathtt{a}^{n_1}\mathtt{c}\mathtt{b}^{n_1}\mathtt{d}$, the non-blocking arc of t_6 will remove one token from p_4 and its marking will strictly decrease until it becomes empty. Let $\ell = n_3 - n_1 - 1$. It is easy to see that $(\mathtt{a}^{n_1}\mathtt{c}\mathtt{b}^{n_1}\mathtt{d})^{\ell}$ leads to a marking $\mathbf{m}_2$ with $\mathbf{m}_2(p_2) = n_1$ $\mathbf{m}_2(p_3) = 1$ and $\forall j \in \{1, 4, 5\} : \mathbf{m}_2(p_j) = 0$. This characterization also implies that we can fire $\mathtt{a}^{n_1}\mathtt{c}\mathtt{b}^{n_1}\mathtt{d}$ an arbitrary number of times from $\mathbf{m}_2$ because $\mathbf{m}_2 \xrightarrow{\mathtt{a}^{n_1}\mathtt{c}\mathtt{b}^{n_1}\mathtt{d}} \mathbf{m}_2$. On the other hand, it is not possible to fire $\mathtt{a}^{n_1}\mathtt{c}\mathtt{b}^{n_2}\mathtt{d}$, with $n_2 > n_1$, from $\mathbf{m}_2$. Indeed $\mathbf{m}_2 \xrightarrow{\mathtt{a}^{n_1}\mathtt{c}\mathtt{b}^{n_1}} \mathbf{m}_3$, with $\mathbf{m}_3(p_5) = 1$, $\mathbf{m}_3(p_2) = n_1$ and $\forall j \in \{1, 3, 4\} : \mathbf{m}_3(p_j) = 0$, which does not allow to fire the b-labelled transition t_5 anymore. We conclude that, $\forall k \geqslant \ell = n_3 - n_1 - 1$, a sequence labelled by $\mathtt{i}^{n_3}\mathtt{s}(\mathtt{a}^{n_3}\mathtt{c}\mathtt{b}^{n_3}\mathtt{d})^m\mathtt{a}^{n_3}\mathtt{c}(\mathtt{b}^{n_1}\mathtt{d}\mathtt{a}^{n_1}\mathtt{c})^k\mathtt{b}^{n_2}\mathtt{d}\mathtt{a}^{n_2}\mathtt{c}$, is not firable in $\mathcal{N}_2$. Thus, we will not find in $L^{\omega}(\mathcal{N}_2)$ any word with this prefix, hence the Lemma. □

We are now ready to prove that no PN accepts exactly the ω-language of the PN+NBA $\mathcal{N}_2$.

Lemma 18. *No* PN *accepts* $L^{\omega}(\mathcal{N}_2)$.

Proof. Let $\mathcal{N}$ be a PN such that $L^\omega(\mathcal{N}_2) \subseteq L^\omega(\mathcal{N})$. We will show that this implies that $L^\omega(\mathcal{N}_2) \subsetneq L^\omega(\mathcal{N})$.

Suppose that $\mathbf{m}_{\text{init}}$ is the initial marking of $\mathcal{N}$. Following Lemma 16, since $L^\omega(\mathcal{N}_2) \subseteq L^\omega(\mathcal{N})$, we have $\forall k \geqslant 1$: $\mathtt{i}^k\mathtt{s}\left(\mathtt{a}^k\mathtt{cb}^k\mathtt{d}\right)^\omega \in L(\mathcal{N})$. Thus, for all $k \geqslant 1$, there exists a marking $\widehat{\mathbf{m}}_k$, a sequence of transitions σ_k and a natural ℓ_k such that

$$\mathbf{m}_{\text{init}} \xrightarrow{\mathtt{i}^k\mathtt{s}\left(\mathtt{a}^k\mathtt{cb}^k\mathtt{d}\right)^{\ell_k}\mathtt{a}^k\mathtt{c}} \widehat{\mathbf{m}}_k \xrightarrow{\Lambda(\sigma_k)} \widehat{\mathbf{m}}'_k \quad \text{with } \widehat{\mathbf{m}}_k \preccurlyeq \widehat{\mathbf{m}}'_k \quad \text{and} \quad \Lambda(\sigma_k) \in \left(\mathtt{b}^k\mathtt{da}^k\mathtt{c}\right)^+.$$

Indeed, if it is not the case, we would have $\mathbf{m}_{\text{init}} \xrightarrow{\mathtt{i}^k\mathtt{sa}^k\mathtt{c}} \mathbf{m}_1 \xrightarrow{\mathtt{b}^k\mathtt{da}^k\mathtt{c}} \cdots \xrightarrow{\mathtt{b}^k\mathtt{da}^k\mathtt{c}} \mathbf{m}_i \xrightarrow{\mathtt{b}^k\mathtt{da}^k\mathtt{c}} \ldots$ such that there do not exist $1 \leqslant i < j$ with $\mathbf{m}_i \preccurlyeq \mathbf{m}_j$. But, from Lemma 7, this never occurs.

Let us consider the sequence $\widehat{\mathbf{m}}_1, \widehat{\mathbf{m}}_2, \widehat{\mathbf{m}}_3, \ldots$ Following Lemma 7, we extract an infinite sub-sequence $\widehat{\mathbf{m}}_{\rho(1)}, \widehat{\mathbf{m}}_{\rho(2)}, \widehat{\mathbf{m}}_{\rho(3)}, \ldots$ such that $\forall p \in P$: either $\forall i \geqslant 1 : \widehat{\mathbf{m}}_{\rho(i)}(p) = \widehat{\mathbf{m}}_{\rho(i+1)}(p)$ or $\forall i \geqslant 1 : \widehat{\mathbf{m}}_{\rho(i)}(p) < \widehat{\mathbf{m}}_{\rho(i+1)}(p)$.

Since $\widehat{\mathbf{m}}_{\rho(3)} \succcurlyeq \widehat{\mathbf{m}}_{\rho(1)}$ and $\sigma_{\rho(1)}$ has a non-negative and constant effect on each place (its effect is characterized by its Parikh vector, which is a tuple of naturals), we can fire $\sigma_{\rho(1)}$ any number of times from $\widehat{\mathbf{m}}_{\rho(3)}$: for all $k' \geqslant 0$ we have $\widehat{\mathbf{m}}_{\rho(3)} \xrightarrow{(\sigma_{\rho(1)})^{k'}} \mathbf{m}^{k'}$ with $\mathbf{m}^{k'} \succcurlyeq \widehat{\mathbf{m}}_{\rho(3)}$. Since $\widehat{\mathbf{m}}_{\rho(3)} \succcurlyeq \widehat{\mathbf{m}}_{\rho(2)}$ and $\sigma_{\rho(2)}$ has a constant non-negative effect on each place, $\sigma_{\rho(2)}$ can be fired infinitely often from $\mathbf{m}^{k'}$ for any $k' \geqslant 1$. Thus:

$$\mathbf{m}_{\text{init}} \xrightarrow{\mathtt{i}^{\rho(3)}\mathtt{s}\left(\mathtt{a}^{\rho(3)}\mathtt{cb}^{\rho(3)}\mathtt{d}\right)^{\ell_{\rho(3)}}\mathtt{a}^{\rho(3)}\mathtt{c}} \widehat{\mathbf{m}}_{\rho(3)} \xrightarrow{(\sigma_{\rho(1)})^{k'}} \mathbf{m}^{k'} \xrightarrow{(\sigma_{\rho(2)})^{\omega}} .$$

Following Lemma 17, if we choose k' large enough (that is, $k' \geqslant \rho(3) - \rho(1) - 1$), the word accepted by the previous sequence is not in $L^\omega(\mathcal{N}_2)$. Hence, $L^\omega(\mathcal{N}_2) \subsetneq L^\omega(\mathcal{N})$. □

Theorem 19. PN+NBA *are more expressive*, *on infinite words*, *than* PN, *i.e.*, $L^\omega(PN) \subsetneq L^\omega(\text{PN+NBA})$.

Proof. As the PN class is a syntactic subclass of the PN+NBA, each PN-language is also a PN+NBA-language. On the other hand, some PN+NBA-languages are not PN-languages, by Lemma 18. Hence the Theorem. □

Again, since PN is a syntactic subclass of PN+NBA and we have made no assumptions about the τ-transitions in the previous proofs, and since $\mathcal{N}_2$ contains no τ-transition, we obtain:

Corollary 20. PN+NBA *are more expressive*, *on infinite words and without τ-transitions than* PN, *i.e.*, $L^\omega_{\not\tau}(PN) \subsetneq L^\omega_{\not\tau}(\text{PN+NBA})$.

5. Reset nets

In this section we show how Petri nets with reset arcs—another widely studied class of Petri nets [15,6]—fit into our classification. We first recall the definition of this class, then show that it is as expressive, on ω-languages, as PN+T. It is important to remark here that our construction requires τ-transitions.

An EPN $\mathcal{N} = \langle \mathcal{P}, \mathcal{T}, \Sigma, \mathbf{m}_0 \rangle$ is a *Petri net with reset arcs* (PN+R for short) if it is a PN+T, with the following additional restrictions: (i) there exists a place $p_{\text{Tr}} \in \mathcal{P}$ that is not an input or output place of any transition of $\mathcal{T}$ and (ii) for any extended transition $t = \langle I, O, s, d, +\infty, \lambda \rangle \in \mathcal{T}_e$, $d = p_{\text{Tr}}$. The special place p_{Tr} is called the *trashcan*. Intuitively, we see the reset of a place as a transfer where the consumed tokens are sent to the trashcan, from which they can never escape.

Let us now exhibit a construction to prove that any ω-language accepted by a PN+T can also be accepted by a PN+R. We consider the PN+T $\mathcal{N}_t = \langle \mathcal{P}, \mathcal{T}, \Sigma, \mathbf{m}_0 \rangle$, and build the reset $\mathcal{N}_r = \langle \mathcal{P}', \mathcal{T}', \Sigma, \mathbf{m}'_0 \rangle$ as follows. Let $\mathcal{P}' = \mathcal{P} \uplus \{p_b, p_{\text{Tr}}\} \uplus \{p_t | t \in \mathcal{T}_e\}$. Then for each transition $t = \langle I, O, s, d, +\infty, \lambda \rangle \in \mathcal{T}_e$, we put three transitions in $\mathcal{T}'$: $t^s = \langle I \uplus \{p_b\}, \{p_t\}, \bot, \bot, 0, \lambda \rangle$; $t^c = \langle \{p_t, s\}, \{p_t, d\}, \bot, \bot, 0, \tau \rangle$; and $t^e = \langle \{p_t\}, O \uplus \{p_b\}, s, p_{\text{Tr}}, +\infty, \tau \rangle$. For any $t = \langle I, O, \bot, \bot, 0, \lambda \rangle \in \mathcal{T}_r$, we add $t' = \langle I \uplus \{p_b\}, O \uplus \{p_b\}, \bot, \bot, 0, \lambda \rangle$ in $\mathcal{T}'$. Finally, $\forall p \in \mathcal{P} : \mathbf{m}'_0(p) = \mathbf{m}_0(p)$, $\mathbf{m}'_0(p_b) = 1$, $\mathbf{m}'_0(p_{\text{Tr}}) = 0$ and $\forall t \in \mathcal{T}_e : \mathbf{m}'_0(p_t) = 0$. Fig. 4 shows the construction.

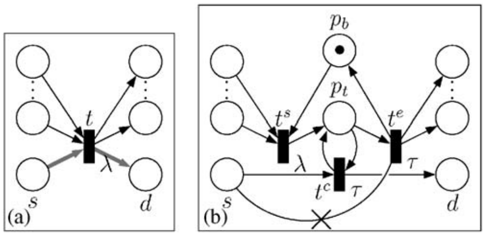

Fig. 4. How to transform a PN+T (a) into a PN+R (b). On this figure, we have adapted a conventional graphical convention to represent reset arcs: the arc bearing a $\times$ links the (source) place to be reset to the extended transition and p_{Tr} (irrelevant in this case) is not shown.

Let us now prove that the PN+R obtained thanks to this construction has the same ω-language as the PN+T it corresponds to.

Lemma 21. $L^\omega(\mathcal{N}_r) = L^\omega(\mathcal{N}_t)$.

Proof. $[L^\omega(\mathcal{N}_t) \subseteq L^\omega(\mathcal{N}_r)]$ Let $\sigma = t_1 t_2 \ldots$ be an infinite sequence of transitions of $\mathcal{N}_t$. Then, $\mathcal{N}_r$ accepts $\Lambda(\sigma)$ thanks to σ' built as follows. We simply replace in σ each regular Petri transition t by t' and each extended transition $t = \langle I, O, s, d, b, \lambda\rangle$ by $\sigma_t = t^s (t^c)^{(k-I(s))} t^e$, where k is the marking of place s that is reached in $\mathcal{N}_r$ before the firing of σ_t. Clearly $\Lambda(\sigma_t) = \Lambda(t)$ an their respective effects are equal on the places in $\mathcal{P}$.

$[L^\omega(\mathcal{N}_r) \subseteq L^\omega(\mathcal{N}_t)]$ Let $\sigma' = t'_1 t'_2 \ldots$ be an infinite sequence of transitions of $\mathcal{N}_r$ such that $\mathbf{m}'_0 \xrightarrow{t'_1} \mathbf{m}'_1 \xrightarrow{t'_2} \mathbf{m}'_2 \ldots$. We first extract from σ' the subsequences $t^s(t^c)^n t^e (n \in \mathbb{N})$ that correspond to a given extended transition t in $\mathcal{N}_t$. Thus, we obtain $\mathbf{m}'_0 = \mathbf{m}'_{k_0} \xrightarrow{\sigma_1} \mathbf{m}'_{k_1} \xrightarrow{\sigma_2} \mathbf{m}'_{k_2} \ldots$, where σ_i is either a single regular Petri transition t'_{k_i} corresponding to the simple regular Petri transition t_{k_i} or a sequence σ_t corresponding to the extended transition t. This is possible since the firing of t^s will remove the token from p_b and block the whole net. Hence no transitions can interleave with $t^s(t^c)^* t^e$. Moreover, σ' cannot have a suffix of the form $t^s(t^c)^\omega$ since t^c decreases the marking of the source place of the transfer of the corresponding transition t.

Then, we replace each σ_i of length > 1 by the transition t it corresponds to in $\mathcal{N}_t$. Hence, we obtain a new sequence $\sigma = t_1 t_2 \ldots$ of $\mathcal{N}_t$. Clearly, $\Lambda(\sigma) = \Lambda(\sigma')$. Let us now prove that σ is firable, i.e., $\mathbf{m}_0 \xrightarrow{t_1} \mathbf{m}_1 \xrightarrow{t_2} \mathbf{m}_2 \xrightarrow{t_3} \ldots$, by showing that $\forall i \geqslant 0 : \mathbf{m}'_{k_i} \preccurlyeq_{\mathcal{P}} \mathbf{m}_i$.

Base case: $i = 0$. The base case is trivially verified.

Induction Step: $i = \ell$. By induction hypothesis, we have that $\forall 0 \leqslant i \leqslant \ell - 1 : \mathbf{m}'_{k_i} \preccurlyeq_{\mathcal{P}} \mathbf{m}_i$. In the case where t_ℓ is a regular transition, it has the same effect on the places in $\mathcal{P}$ as $\sigma_\ell = t'_{k_\ell}$ and it can occur since $\mathbf{m}'_{k_{\ell-1}} \preccurlyeq_{\mathcal{P}} \mathbf{m}_{\ell-1}$. Hence $\mathbf{m}'_{k_\ell} \preccurlyeq_{\mathcal{P}} \mathbf{m}_\ell$, by monotonicity. Otherwise t_ℓ is an extended transition and its effect corresponds to the effect of σ_ℓ. Let us observe the effect of σ_ℓ: some tokens will be taken from s (the source place of the transfer) and put into d (the destination) by t^c_ℓ. Finally, the tokens remaining in s will be removed by the reset arc of t^e_ℓ. Hence, σ_ℓ removes the same number of tokens from s than t_ℓ, and cannot put more tokens in d than t_ℓ does. Moreover, the effect of σ_ℓ on the other places is the same than t_ℓ. Thus, $\mathbf{m}'_{k_\ell} \preccurlyeq_{\mathcal{P}} \mathbf{m}_\ell$. □

Theorem 22. PN+R *are as expressive as* PN+T *on infinite words, i.e.*, $L^\omega(\text{PN+R}) = L^\omega(\text{PN+T})$.

Proof. As any PN+R is a special case of PN+T, we have that $L^\omega(\text{PN+R}) \subseteq L^\omega(\text{PN+T})$. The other direction stems from Lemma 21. □

In the case where we disallow τ-transitions, the previous construction does not allow to prove whether $L^\omega_{\not\tau}(\text{PN+T}) \subseteq L^\omega_{\not\tau}(\text{PN+R})$ or not. Indeed, this construction requests the use of several τ-transitions (namely, the t_c and t_e transitions used in the widget that replaces any extended transition t. See Fig. 4). However, we have that $L^\omega(\text{PN+NBA}) \subsetneq L^\omega(\text{PN+R})$ and $L^\omega_{\not\tau}(\text{PN+NBA}) \subsetneq L^\omega_{\not\tau}(\text{PN+R})$, since the PN+T $\mathcal{N}_1$ we have used in the proof of Lemma 13 satisfies our definition of PN+R (in this case, the place p_4 is the trashcan) and has no τ-transitions.

With τ-transitions:

$$L^\omega(\mathsf{PN}) \subsetneq L^\omega(\mathsf{PN{+}NBA}) \subsetneq \begin{cases} L^\omega(\mathsf{PN{+}T}) \\ \quad \| \\ L^\omega(\mathsf{PN{+}R}) \end{cases}$$

Without τ-transitions:

$$L^\omega_{\not\tau}(\mathsf{PN}) \subsetneq L^\omega_{\not\tau}(\mathsf{PN{+}NBA}) \subsetneq \begin{cases} L^\omega_{\not\tau}(\mathsf{PN{+}T}) \\ \quad \text{I}\cup \\ L^\omega_{\not\tau}(\mathsf{PN{+}R}) \end{cases}$$

Fig. 5. Summary of the results.

6. Conclusion

In the introduction of this paper, we have recalled how important EPN are to study the *non-terminating* behaviour of concurrent systems made up of an arbitrary number of communicating processes (once abstracted thanks to predicate- and counting- abstraction techniques [2]). Our aim was thus to study and classify the expressive powers of these models, as far as ω-languages are concerned. This goal has been thoroughly fulfilled, as shown in the summary of our results at Fig. 5. Indeed, we have proved in Section 3 that any ω-language accepted by a PN+NBA can be accepted by a PN+T, but that there exist ω-languages that are recognized by a PN+T but not by a PN+NBA. A similar result has been demonstrated for PN+NBA and PN in Section 4. These results hold with or without τ-transitions. Finally, in Section 5 we have drawn a link between these results and the class of PN+R.

Future works: In [15], Peterson studies different classes of *finite words* languages of PN and Ciardo [3] extends the study to Petri nets *with marking-dependant arc multiplicity*, which subsume the four classes of nets we have studied here. The latter paper states some relations between the languages accepted by these classes of nets, but keeps several questions open. To the best of out knowledge, most of them are still open, and we strive for applying the new proof techniques developed in this paper to solve those open problems.

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