Infinite Behaviour of Deterministic Petri Nets

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1 Introduction

Petri nets have turned out to be an adequate tool for modelling, designing, and analysing concurrent systems. Many concurrent systems are designed to run without explicit termination, e. g. operating systems. This leads to the investigation of infinite behaviour, especially of Petri nets. For the infinite behaviour we consider definitions which were given by [Landweber 69] for finite automata.

The cooperation of distributed actions can be sufficiently modelled by Petri nets, because concurrency is a fundamental element of the net theory. The actions will be represented by the transitions of the net. In Petri nets a transition always causes the same changing in any situation it appears. But often this is too strict, because it is not possible to model actions which may appear in different situations and cause different changes. On behalf of this labelled Petri nets were introduced, in which actions are represented by the label of transitions, i. e. several transitions may have the same label.

The problem in a labelled Petri net is that two or more transitions with the same label might be activated. There is no way to determine which transition will follow. This phenomenon causes severe problems, e. g. for labelled Petri nets it is undecidable if two given nets have the same behaviour.

Real systems mostly have the property that the changes resulting from an action are determined by the action and the situation in which it appears. This leads us to deterministic Petri nets. Here in any possible situation all activated transitions have different labels, hence every sequence of actions determines the corresponding sequence of transitions.

In the following section we will give necessary definitions and results concerning languages of labelled Petri nets and infinite behaviour of finite automata and nets. Then deterministic Petri nets will be compared with nondeterministic nets for finite behaviour. The fourth section investigates the infinite behaviour of deterministic nets with specifications on the markings. Whereas in the fifth section the infinite behaviour is defined on transitions which must appear. Finally the power of several classes of infinite behaviour of deterministic Petri nets is compared and an overall hierarchy is established.

2 Basic definitions and results

In this section we want to give the formal definition of Petri nets and some notations which will be used later. Finally we will define the infinite behaviour of Petri nets.

First some mathematical notations will be needed.

Definition 1 The positive integers are denoted by $\mathbb{N}^+$ and $\mathbb{N} = \{0\} \cup \mathbb{N}^+$. The cardinality of $\mathbb{N}$ is ω, i. e. $|\mathbb{N}| = \omega$. The quantors $\exists$ and $\forall$ will be extended to $\underset{\infty}{\exists}$ and $\underset{\infty}{\forall}$, there are infinitely many and for all but infinitely many. Let A be a set and p a predicate, then $\underset{\infty}{\exists}\, a \in A : p(a)$ means $|\{a \in A \mid p(a)\}| = \omega$. And $\underset{\infty}{\forall}$ is defined by $\left(\underset{\infty}{\forall}\, a \in A : \neg p(a)\right) \Leftrightarrow \left(\neg \underset{\infty}{\exists}\, a \in A : p(a)\right)$.

With the lemma of Dickson [Dickson 13], i. e., in every infinite sequence of vectors over positive integers there is an increasing infinite subsequence, it is easy to show the following technical lemma.

Lemma 1 *Let $A \subseteq (\mathbb{N}^k)^*$, $k \in \mathbb{N}^+$, be an infinite set of sequences of vectors, if*
a) there is $q \in \mathbb{N}$ with $\forall w \in A : \forall i, j \in \mathbb{N}, i < |w|, j \leq k : |w(i)(j) - w(i+1)(j)| \leq q$ and
b) there is a vector $x \in \mathbb{N}^k$ with $\forall w \in A/\{\lambda\} : w(1) = x$,
then it exists an $n \in \mathbb{N}$, so that for every sequence $v \in A$ with $|v| \geq n$: $\exists i, j \in \mathbb{N}^+, i < j \leq n : v(i) \leq v(j)$.

Definition 2 A *labelled Petri net* $N = (S, T; F, W, h, m_0)$ is given by two finite and disjunct sets, the set of *places* S, and the set of *transitions* T, a *flow relation* $F \subseteq (S \times T) \cup (T \times S)$, a *weight function* $W : F \to \mathbb{N}^+$, a *labelling function* $h : T \to (X \cup \{\lambda\})$, where X is a given finite alphabet and λ the empty word, and an *initial marking* $m_0 : S \to \mathbb{N}$. A net N is called λ-free if $h : T \to X$. A *marking* m of N is a mapping $m : S \to \mathbb{N}$, and it is given by a vector over $\mathbb{N}^S$.

The dynamic behaviour of a Petri net is described by the firing rule.

Definition 3 Let N be a Petri net. A transition $t \in T$ is *activated* by a marking $m \in \mathbb{N}^S$, written $m\ \langle t \rangle$, if on every input place s of t, $(s,t) \in F$, there are at least $W(s,t)$ tokens, i. e. $\forall s \in S : (s,t) \in F \Rightarrow W(s,t) \leq m(s)$. An activated transition may fire and it produces a *follower marking* m', $m\ \langle t \rangle\ m'$, by subtracting $W(s,t)$ tokens from every input place and adding $W(s',t)$ tokens on every output place s' of t, $(t,s') \in F$.
A sequence $v = t_1 t_2 \ldots t_n$ of transitions is called *firing sequence* from m, if it exists a sequence of markings $\delta(v,m) = m_1 m_2 \ldots m_{n+1}$ with $m = m_1\ \langle t_1 \rangle\ m_2\ \langle t_2 \rangle\ \ldots\ \langle t_n \rangle\ m_{n+1} = m'$, written $m\ \langle v \rangle\ m'$. The sequence $\delta(v,m)$ is the *corresponding marking sequence* of v starting from m, if it starts from the initial marking, it is defined: $\delta_0(v) = \delta(v, m_0)$. For an arbitrary sequence α, $\alpha(i)$ denotes the i^{th} element of α and $\alpha[i]$ the prefix of length i of α, i. e. $\alpha[i] = \alpha(1) \ldots \alpha(i)$.

Definition 4 Let N be a Petri net, then $\mathrm{F}(N) = \{v \in T^* \mid m_0\ \langle v \rangle\}$ is the *set of firing sequences* and $\mathrm{L}(N) = \{h(v) \in X^* \mid v \in \mathrm{F}(N)\}$ the *set of words* or the *language* of N. For the infinite behaviour we define: $\mathrm{F}_\omega(N) = \{v \in T^\omega \mid \forall i \in \mathbb{N}^+ : v[i] \in \mathrm{F}(N)\}$ is the *set of infinite firing sequences* and $\mathrm{L}_\omega(N) = \{h(v) \in X^\omega \mid v \in \mathrm{F}_\omega(N)\}$ the *set of infinite words* or *ω-language* of N. The *class* of all ω-languages of λ-free Petri nets will be denoted by $\mathcal{L}_\omega$, i. e. $\mathcal{L}_\omega = \{\mathrm{L}_\omega(N) \mid N \text{ is a } \lambda\text{-free Petri net}\}$, and $\mathcal{L}_{\omega\lambda}$ if arbitrary Petri nets are assumed.

To define infinite behaviours (languages) of finite automata [Landweber 69] introduces five levels of successful ω-sequences with respect to sets of states of the automaton. A word is called successful if the corresponding sequence of states fulfills certain properties. The individual levels were called i-successful, for $i \in \{1, 1', 2, 2', 3\}$. We will introduce an additional definition for 4-successful.

For the present we will define i-successful sequences independently of automata or Petri nets.

Definition 5 Let Y be a finite or an infinite set and $\mathcal{A} = \{A_1, A_2, \ldots, A_k\} \subseteq \wp(Y)$, with $A_i \neq \emptyset$, $1 \leq i \leq k$, a finite nonempty set of *anchorsets*, then an infinite sequence $\alpha \in Y^\omega$ is called
1-successful for $\mathcal{A}$, if $\exists A \in \mathcal{A}$: $\exists\, i \in \mathbb{N}^+$: $\alpha(i) \in A$, *1′-successful* for $\mathcal{A}$, if $\exists A \in \mathcal{A}$: $\forall\, i \in \mathbb{N}^+$: $\alpha(i) \in A$,
2-successful for $\mathcal{A}$, if $\exists A \in \mathcal{A}$: $\underset{\infty}{\exists}\, i \in \mathbb{N}^+$: $\alpha(i) \in A$, *2′-successful* for $\mathcal{A}$, if $\exists A \in \mathcal{A}$: $\underset{\infty}{\forall}\, i \in \mathbb{N}^+$: $\alpha(i) \in A$,
3-successful for $\mathcal{A}$, if $\exists A \in \mathcal{A}$: $\underset{\infty}{\forall}\, i \in \mathbb{N}^+$: $\alpha(i) \in A$ and $\forall a \in A$: $\underset{\infty}{\exists}\, i \in \mathbb{N}^+$: $\alpha(i) = a$.
4-successful for $\mathcal{A}$, if $\exists A \in \mathcal{A}$: $\forall a \in A$: $\underset{\infty}{\exists}\, i \in \mathbb{N}^+$: $\alpha(i) = a$.

Definition 6 Let N be a Petri net and $\mathcal{D} = \{D_1, D_2, \ldots, D_k\}$, $k \in \mathbb{N}^+$, a set of finite anchorsets of markings, $D_i \subseteq \mathbb{N}^S$. The *terminal language* of the net N and the anchorset D_1 is $\mathrm{L}_0(N, D_1) = \{h(v) \in X^* \mid v \in \mathrm{F}(N) \text{ and } \delta_0(v)(|v|) \in D_1\}$. For $i \in \{1, 1', 2, 2', 3, 4\}$ the *i-successful language* or the *i-behaviour* of $(N, \mathcal{D})$ is a language over infinite words: $\mathrm{L}^i_\omega(N, \mathcal{D}) = \{h(v) \in X^\omega \mid v \in \mathrm{F}_\omega(N) \text{ and } \delta_0(v) \text{ is } i\text{-successful for } \mathcal{D}\}$. The corresponding *classes of languages* are $\mathcal{L}_0$ or $\mathcal{L}^i_\omega$. Let $\mathcal{L}_1$ be a class of languages of finite words then we define $\mathcal{KC}_\omega(\mathcal{L}_1) := \left\{ \bigcup_{i=1}^{k} A_i B_i^\omega \mid A_i, B_i \in \mathcal{L}_1, k \in \mathbb{N}^+ \right\}$ to be the *ω-Kleene-Closure* of $\mathcal{L}_1$.

For the finite automaton it exists a similar definition of i-successful behaviour, but instead of markings we have to consider states. The classes of i-successful regular languages will be written as $\mathcal{R}^i_\omega$, they were introduced and investigated in [McNaughton 66], [Landweber 69], and [Hossley 72]. The following inclusions hold for nondeterministic noncomplete finite automata. Similar results were shown for push-down automata and Turing machines, where the anchorsets are defined for states of the finite control [Cohen, Gold 77].

Theorem 1

$$\mathcal{R}^{1'}_\omega \subset \mathcal{R}^1_\omega = \mathcal{R}^{2'}_\omega \subset \mathcal{R}^2_\omega = \mathcal{R}^3_\omega = \mathcal{KC}_\omega(\mathcal{R})$$

The integration of the 4-successful class causes no problems, it is easy to show that $\mathcal{R}^2_\omega = \mathcal{R}^4_\omega$.

For Petri nets we get quite different inclusions [Valk 83], [Carstensen, Valk 85].

Lemma 2

$$\begin{array}{ccccccccccc} \mathcal{L}_\omega & \subset & \mathcal{L}^1_\omega & & & & & & & & \\ \cup & & \cup & & & & & & & & \\ \mathcal{L}^{1'}_\omega & \subset & \mathcal{L}^{2'}_\omega & \subset & \mathcal{L}^3_\omega & \subset & \mathcal{L}^2_\omega & \subset & \mathcal{L}^4_\omega & \subset & \mathcal{KC}_\omega(\mathcal{L}_0) \end{array}$$

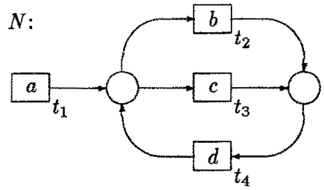

Figure 1: Petri net for the proof of lemma 5

Definition 7 Let N be a Petri net and $\mathcal{E} = \{E_1, E_2, \dots, E_k\}$, $k \in \mathbb{N}^+$, a set of anchorsets of transitions, $E_i \subseteq T$. For $i \in \{1, 1', 2, 2', 3, 4\}$ the *transitional i-behaviour* of $(N, \mathcal{E})$ is given by
$\mathrm{K}^i_\omega(N, \mathcal{E}) = \{h(v) \in X^\omega \mid v \in \mathrm{F}_\omega(N)$ and v is i-successful for $\mathcal{E}\}$.
The corresponding *classes of languages* are $\mathcal{K}^i_\omega$.

In [Carstensen, Valk 85] it was shown:

Lemma 3

$$\mathcal{K}^{1'}_\omega \subset \mathcal{K}^1_\omega = \mathcal{K}^{2'}_\omega \subset \mathcal{K}^2_\omega = \mathcal{K}^3_\omega = \mathcal{K}^4_\omega$$

Lemma 4

$$\mathcal{K}^1_\omega = \mathcal{L}_\omega$$

There are no further inclusions between the classes $\mathcal{K}^i_\omega$ and $\mathcal{L}^i_\omega$.

The proofs are given in [Valk 83], but one proof is not correct. The language which should be a counter example for $\mathcal{K}^1_\omega \not\subseteq \mathcal{L}^1_\omega$ is no counter example, but the mentioned idea leads to another example. We will give the proof here, since in the deterministic case we will refer to it.

Lemma 5

$$\mathcal{K}^1_\omega \not\subseteq \mathcal{L}^1_\omega$$

Proof: Take the net N in figure 1 with the language $L = \mathrm{K}^1_\omega(N, \{\{t_3\}\})$.

For every $n \in \mathbb{N}^+$ the word $a^n c b^{n-1} d^n (b^n d^n)^\omega$ is element of L. The language L cannot be described by a net N' without anchorsets, otherwise also a^ω is in the language of N', i. e. $L \notin \mathcal{L}_\omega$.

Suppose there is a net N' and a set of anchorsets $\mathcal{D}'$ with $L = \mathrm{L}^1_\omega(N', \mathcal{D}')$. Then $L = \mathrm{L}^1_\omega(N', \{D'\})$ for $D' = \{m \mid \exists D \in \mathcal{D}' : m \in D\}$, since only an arbitrary marking of the anchorsets must be reached.

For all $d' \in D'$ we can determine the length, so that in a sequence consisting only of transitions labelled with a, starting from d', two markings must be in the relation greater or equal, i. e. $len(d') = \min\left\{ l \in \mathbb{N}^+ \;\middle|\; \begin{array}{l} \forall v \in T^*, h(v) \in a^*, |v| \geq l: \\ \exists i, j \in \mathbb{N}^+, i < j \leq l : \delta(v, d')(i) \leq \delta(v, d')(j) \end{array} \right\}$. Let $len = \max\{len(d') \mid d' \in D'\}$.

Since the anchorset is finite, two different words of the described form must reach the same marking in D'. Let this be the words $v = a^i c b^{i-1} d^i (b^i d^i)^\omega$ and $w = a^j c b^{j-1} d^j (b^j d^j)^\omega$, with $i, j \in \mathbb{N}^+$ and $j - i > len$. Then for a marking $d' \in D'$ partitions $v_1 v_2 = v$ and $w_1 w_2 = w$ exist with $m_0 \langle v_1 \rangle\, d' \,\langle v_2 \rangle$ and $m_0 \langle w_1 \rangle\, d' \,\langle w_2 \rangle$. If $h(w_1) \in a^*$ and w_2 starts with at least len transitions labelled with a, then also a^ω would be 1-successful. Otherwise the word $h(v_1 w_2)$ is 1-successful. Both words are not in the language L, hence we have a contradiction.

Theorem 2 *The figure 2 shows all inclusions between the mentioned classes. There are no more inclusions between these classes with the exception of those resulting from transitivity.*

In [Carstensen 82] it was shown that every transitional 2-behaviour can be described as the ω-language of a λ-net. Note it was settled that an infinite sequence of λ-transitions is mapped on the empty word, i. e. it is no word of an ω-language. The proof uses the fact that only finitely many λ-transitions may follow each other, hence at least one other transition must appear infinitely often. With a transformation of the net one can ensure that this also holds for a certain transition.

Theorem 3

$$\mathcal{K}^2_\omega \subset \mathcal{L}_{\omega\lambda}$$

With the same ideas as for Petri net languages of finite words it could be shown that this inclusion is proper.

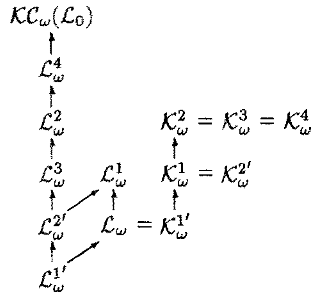

Figure 2: Hierarchy of the i-behaviours of Petri nets

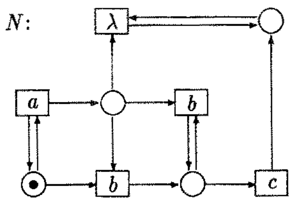

Figure 3: A deterministic net for the proof of lemma 6

3 Comparing deterministic and nondeterministic Petri nets

The deterministic Petri nets were introduced in [Vidal–Naquet 81] and the classes of languages of these nets were compared with the known classes.

Definition 8 Let N be a Petri net, N is called *deterministic*, if
$\forall t, t' \in T : \forall m \in R(N, m_0) : [(\ m\ (t\rangle \text{ and } m\ (t'\rangle\ ) \Rightarrow (h(t) \neq h(t') \text{ or } t = t')]$.
A λ-net N is called *deterministic*, if
$\forall t, t' \in T : \forall m \in R(N, m_0) : [(m\ (t\rangle \text{ and } m\ (t'\rangle\ ) \Rightarrow [(h(t) \neq h(t') \text{ or } t = t') \text{ and } (h(t) = \lambda \Rightarrow t = t')]]$.
The classes of languages of deterministic Petri nets will be denoted by $\mathcal{L}_{\mathrm{dt}}$, $\mathcal{L}_{0\mathrm{dt}}$, $\mathcal{L}_{\lambda\mathrm{dt}}$, $\mathcal{L}_{0\lambda\mathrm{dt}}$, $\mathcal{L}_{\omega\mathrm{dt}}$, $\mathcal{L}_{\omega\lambda\mathrm{dt}}$, $\mathcal{L}^i_{\omega\mathrm{dt}}$, $\mathcal{L}^i_{\omega\lambda\mathrm{dt}}$, $\mathcal{K}^i_{\omega\mathrm{dt}}$, and $\mathcal{K}^i_{\omega\lambda\mathrm{dt}}$, .

In deterministic nets between two transitions not labelled by λ only a certain number of λ-transitions may appear. Hence it is not difficult to transform an arbitrary net into a λ-free net by introducing new transitions for every possibility of sequences of λ-transitions ending with a labelled transition.

Theorem 4

$$\mathcal{L}_{\mathrm{dt}} = \mathcal{L}_{\lambda\mathrm{dt}}$$

In [Vidal–Naquet 82] this equality was also claimed for deterministic Petri nets with terminal markings, i. e. $\mathcal{L}_{0\mathrm{dt}} = \mathcal{L}_{0\lambda\mathrm{dt}}$. But this is not correct, since in a λ-net unbounded many λ-transitions may appear at the end of a firing sequence.

Lemma 6

$$\mathcal{L}_{0\mathrm{dt}} \subset \mathcal{L}_{0\lambda\mathrm{dt}}$$

Proof: We only have to show the inequality of the classes. Consider the language $L = \{a^n b^m c \mid n, m \in \mathbb{N}^+ : n \geq m\}$. The net in figure 3 shows that $L = \mathrm{L}_0(N, \{(0,0,0,1)\})$. N is deterministic, since the λ-transition may appear only at the end of firing sequences, hence $L \in \mathcal{L}_{0\lambda\mathrm{dt}}$.
Suppose $L \in \mathcal{L}_{0\mathrm{dt}}$, i. e., there is a λ-free Petri net N' and a finite anchorset D', with $L = \mathrm{L}_0(N', D')$. Every word $a^n b^n c$, $n \in \mathbb{N}^+$, is a word of L with a unique firing sequence in N'. Lemma 1 ensures that there is a $n \in \mathbb{N}^+$ with two markings m_1 and m_2 so that

m_0 (a^q) m_1 (a^r) m_2 $(b^n c)$ d, for $d \in D'$, $q, r \in \mathbb{N}^+ : q + r = n$ and $m_1 \leq m_2$. (We only consider names of firing sequences since in deterministic nets the corresponding firing sequence is determined.)

For all $i \in \mathbb{N}^+$ the sequence $a^q (a^r)^i b^n c$ must lead to a terminal marking. There are only finitely many terminal markings, hence for all i it must be the same marking, i. e., the sequence a^r may not change the marking in the net N'. Hence it holds for a marking m, m_0 (a^q) m (a^r) m. But now the subsequence a^r may be omitted and it follows m_0 $(a^q b^n c)$ d, $d \in D$, i. e. $a^q b^n c \in \mathrm{L_0}(N', D')$. This is a contradiction to the definition of L since $q < n$.

The next lemma was originally proved in [Vidal–Naquet 82]:

Lemma 7 *$\mathcal{L}_{\mathrm{dt}}$, $\mathcal{L}_{0\lambda\mathrm{dt}}$, and $\mathcal{L}_{0\lambda\mathrm{dt}}$ are not closed under union.*

Proof: In [Carstensen 87] it was shown for some $L1, L2 \in \mathcal{L}_{\mathrm{dt}}$ that $L1 \cup L2 =$
$L = \{w_1 w_2 \mid w_1 \in \{a,b\}^*, w_2 \in c^* \text{ and } [(|w_2|_c \leq |w_1|_a) \vee (|w_2|_c \leq |w_1|_b)\} \notin \mathcal{L}_{\mathrm{dt}}$. The same was shown for some $L1', L2' \in \mathcal{L}_{0\mathrm{dt}}$ that $L1' \cup L2' =$
$L' = \{w_1 w_2 \mid w_1 \in \{a,b\}^*, w_2 \in c^* \text{ and } (|w_2|_c = |w_1|_a) \vee (|w_2|_c = |w_1|_b)\} \notin \mathcal{L}_{0\mathrm{dt}}$.

Other closure properties of deterministic nets were studied in [Pelz 87]. These classes are not closed under most operations, with exception of the intersection.

4 i-behaviour of deterministic nets

Before the infinite behaviour, i. e. the (transitional) i-behaviour, of deterministic nets will be investigated, we will have a look on the corresponding classes for deterministic finite automata ([Hossley 72] and [Cohen, Gold 78]).

Theorem 5 *For $i \in \{1, 1', 2', 3\}$: $\mathcal{R}^i_{\omega\mathrm{dt}} = \mathcal{R}^i_\omega$ and $\mathcal{R}^2_{\omega\mathrm{dt}} \subset \mathcal{R}^2_\omega$*

In the case of noncomplete automata the following results hold:

Theorem 6

$$\begin{array}{ccccc} & & \mathcal{R}^2_{\omega\mathrm{dt}} & \subset & \mathcal{R}^3_{\omega\mathrm{dt}} \\ & & \cup & & \cup \\ \mathcal{R}^{1'}_{\omega\mathrm{dt}} & \subset & \mathcal{R}^1_{\omega\mathrm{dt}} & \subset & \mathcal{R}^{2'}_{\omega\mathrm{dt}} \end{array}$$

The classes of the 1- and of the 1′-behaviour are again incomparable if the automata must not only be deterministic but also complete, the other relationships will not change.

Corresponding investigations were made in [Cohen, Gold 78] for deterministic push-down automata. Here nondeterminism is of course more powerful than determinism, but there were the same relationships as shown in theorem 6. The classes of the 1- and of the 1′-behaviour are incomparable if only "continuable" push-down automata are allowed. This means that for every infinite input there is a computation in the push-down automata.

For the infinite behaviour of Petri nets we can show that λ-transitions can be eliminated. The same ideas with which $\mathcal{L}_{\lambda\mathrm{dt}} = \mathcal{L}_{\mathrm{dt}}$ was proved can also be applied here. The phenomenon that arbitrarily many λ-transitions may follow each other can only happen at the end of a firing sequence, but an infinite sequence never ends.

Theorem 7

$$\mathcal{L}_{\omega\lambda\mathrm{dt}} = \mathcal{L}_{\omega\mathrm{dt}}, \quad \mathcal{L}^i_{\omega\lambda\mathrm{dt}} = \mathcal{L}^i_{\omega\mathrm{dt}}, \quad \mathcal{K}^i_{\omega\lambda\mathrm{dt}} = \mathcal{K}^i_{\omega\mathrm{dt}}$$

This theorem allows us to restrict our investigations of ω-behaviour of deterministic Petri nets on λ-free nets.

Lemma 8 *For $i \in \{1, 2, 2', 3, 4\}$: $\mathcal{L}^i_{\omega\mathrm{dt}}$ is not closed under union.*

Proof: Consider the language L' of the proof of lemma 7. For all $i \in \{1, 2, 2', 3, 4\}$ the language $L'.d^\omega$ can be described as the union of two deterministic i-behaviours. If the classes $\mathcal{L}^i_{\omega\mathrm{dt}}$ would be closed under union, one could show that L' is a deterministic language of a net with terminal markings.

This shows that nondeterminism is more powerful, with the exception of the 1′-behaviour.

Theorem 8

a) $\mathcal{L}^{1'}_{\omega\mathrm{dt}} = \mathcal{L}^{1'}_\omega$, b) *For $i \in \{1, 2, 2', 3, 4\}$: $\mathcal{L}^i_{\omega\mathrm{dt}} \subset \mathcal{L}^i_\omega$*

Proof: a) The classes of 1′-behaviours of nets and finite automata are equivalent. It is easy to see that a deterministic automaton corresponds to a deterministic Petri net.
b) The classes $\mathcal{L}^i_\omega$ are closed under finite union, not so the classes $\mathcal{L}^i_{\omega\mathrm{dt}}$.

The following inclusions are valid between the classes of deterministic behaviours of nets:

Lemma 9

$$\text{a) } \mathcal{L}^{1'}_{\omega\mathrm{dt}} \subseteq \mathcal{L}_{\omega\mathrm{dt}}, \quad \text{b) } \mathcal{L}^{1'}_{\omega\mathrm{dt}} \subseteq \mathcal{L}^{2}_{\omega\mathrm{dt}}, \quad \text{c) } \mathcal{L}^{1'}_{\omega\mathrm{dt}} \subseteq \mathcal{L}^{2'}_{\omega\mathrm{dt}}, \quad \text{d) } \mathcal{L}_{\omega\mathrm{dt}} \subseteq \mathcal{L}^{1}_{\omega\mathrm{dt}}, \quad \text{e) } \mathcal{L}^{2'}_{\omega\mathrm{dt}} \subseteq \mathcal{L}^{3}_{\omega\mathrm{dt}}$$

$$\text{f) } \mathcal{L}^{1'}_{\omega\mathrm{dt}} \subseteq \mathcal{L}_{\omega\mathrm{dt}}, \quad \text{g) } \mathcal{L}^{2}_{\omega\mathrm{dt}} \subseteq \mathcal{KC}_\omega(\mathcal{L}_{0\mathrm{dt}}), \quad \text{h) } \mathcal{L}^{3}_{\omega\mathrm{dt}} \subseteq \mathcal{KC}_\omega(\mathcal{L}_{0\mathrm{dt}})$$

Proof: a) – c) Obviously the 1′-behaviour of a deterministic finite automaton is also the ω-language or the i-behaviour of a deterministic Petri net.
d) Choose the set with the initial marking as the anchorset, then $\mathrm{L}_\omega(N) = \mathrm{L}^1_\omega(N, m_0)$.
e) Let $L = \mathrm{L}^{2'}_\omega(N, \mathcal{D})$, then $L = \mathrm{L}^3_\omega(N, \{D' \mid \exists D \in \mathcal{D}: \emptyset \neq D' \subseteq D\})$.
f) Let $L = \mathrm{L}^2_\omega(N, \mathcal{D})$, then $L = \mathrm{L}^4_\omega(N, \{\{d\} \mid \exists D \in \mathcal{D}: d \in D\})$.
g) $\mathcal{KC}_\omega(\mathcal{L}_{0\mathrm{dt}})$ is closed under finite union by its definition. Hence it is sufficient to consider a language of the form $L = \mathrm{L}^2_\omega(N, \{\{d\}\})$. Then $L = A.B^\omega$ with $A = \mathrm{L}_0(N, \{d\})$ and $B = \mathrm{L}_0(N_d, \{d\})$, where N_d is the net N with the new initial marking d. If d can be reached in N then N_d is deterministic, too. Otherwise the language L is empty and can easily be represented. Hence $L \in \mathcal{KC}_\omega(\mathcal{L}_{0\mathrm{dt}})$.
h) Again we only have to consider a language of the form $L = \mathrm{L}^3_\omega(N, \{D\})$. Let $d \in D$, the language L can be written as $L = A.B^\omega$, where $A = \mathrm{L}_0(N, \{d\})$ and B is the set of all words of firing sequences starting and ending in d, visiting every marking of D. The language B is the behaviour of a finite automaton, hence it is also the terminal language of a deterministic net. Thus $L \in \mathcal{KC}_\omega(\mathcal{L}_{0\mathrm{dt}})$.

In the following we will show that there are no more inclusions between these classes. The question if the class $\mathcal{L}^4_{\omega\mathrm{dt}}$ is not in $\mathcal{KC}_\omega(\mathcal{L}_{0\mathrm{dt}})$ will remain a conjecture. First we will look at cases which are similar to the nondeterministic case. In several proofs it will be shown that a behaviour cannot even be represented by a nondeterministic net.

The next lemmata will show that the hierarchy is splitted in more branches than in the nondeterministic case.

Lemma 10

$$\text{a) } \mathcal{L}_{\omega\mathrm{dt}} \not\subseteq \mathcal{KC}_\omega(\mathcal{L}_{0\mathrm{dt}}), \quad \text{b) } \mathcal{L}_{\omega\mathrm{dt}} \not\subseteq \mathcal{L}^4_{\omega\mathrm{dt}}, \quad \text{c) } \mathcal{L}^1_{\omega\mathrm{dt}} \not\subseteq \mathcal{L}_{\omega\mathrm{dt}}, \quad \text{d) } \mathcal{L}^2_{\omega\mathrm{dt}} \not\subseteq \mathcal{L}^1_{\omega\mathrm{dt}}, \quad \text{e) } \mathcal{L}^2_{\omega\mathrm{dt}} \not\subseteq \mathcal{L}^3_{\omega\mathrm{dt}},$$

$$\text{f) } \mathcal{L}^3_{\omega\mathrm{dt}} \not\subseteq \mathcal{L}^{2'}_{\omega\mathrm{dt}}, \quad \text{g) } \mathcal{L}^4_{\omega\mathrm{dt}} \not\subseteq \mathcal{L}^2_{\omega\mathrm{dt}}$$

Proof: a), b) By theorem 2 it holds $\mathcal{L}^4_\omega \subseteq \mathcal{KC}_\omega(\mathcal{L}_{0\mathrm{dt}})$. Additionally [Valk 83] proved that $\mathcal{L}_\omega \not\subseteq \mathcal{KC}_\omega(\mathcal{L}_0)$ with a language having a not semilinear Parikh image. He used a net for this language which was deterministic.
c) It is easy to see that the language $L = a^*bc^\omega$ can be described as the 1-behaviour of a deterministic finite automaton, hence $L \in \mathcal{L}^1_{\omega\mathrm{dt}}$. If there would be a net N' with $L = \mathrm{L}_\omega(N')$, then also a^ω would be a possible word in N'.
d), e) We may use the corresponding proof in [Valk 83], since the considered language
$L = \{w \in \{a,b\}^\omega \mid \underset{\infty}{\exists}\, i \in \mathbb{N}^+: |w[i]|_a = |w[i]|_b \text{ and } \forall j \in \mathbb{N}^+ |w[j]|_a \geq |w[j]|_b\}$ has a deterministic net. Thus it holds $L \in \mathcal{L}^2_{\omega\mathrm{dt}}$, $L \notin \mathcal{L}^1_\omega$, and $L \notin \mathcal{L}^3_\omega$.
f) In [Valk 83] it was shown that for the language $L = (a^*b)^\omega$ that $L \notin \mathcal{L}^1_\omega$. With $\mathcal{L}^{2'}_\omega \subseteq \mathcal{L}^1_\omega$, L cannot be element of $\mathcal{L}^{2'}_\omega$. But L is the 3-behaviour of a finite automaton, thus $L \in \mathcal{L}^3_\omega$.
g) In [Carstensen 87] the inequality was shown for the nondeterministic case with a deterministic net.

The following lemmata show differences to the nondeterministic case, where we had inclusions.

Lemma 11

$$\text{a) } \mathcal{L}^{2'}_{\omega\mathrm{dt}} \not\subseteq \mathcal{L}^1_{\omega\mathrm{dt}}, \quad \text{b) } \mathcal{L}^{2'}_{\omega\mathrm{dt}} \not\subseteq \mathcal{L}^4_{\omega\mathrm{dt}}$$

Proof: Take net N from figure 4 and the anchorset $D = \{(0),(1)\}$. Let $L = \mathrm{L}^{2'}_{\omega\mathrm{dt}}(N, \{D\})$, i. e. $L = \{w \in \{a,b\}^* \mid \forall i \in \mathbb{N}^+: |w[i]|_a \geq |w[i]|_b \text{ and } |w|_a = |w|_b\}.(ab)^\omega$. The word $w = (ab)^\omega$ is element of L.
a) Suppose $L \in \mathcal{L}^1_{\omega\mathrm{dt}}$, i. e. $L = \mathrm{L}^1_\omega(N', \mathcal{D}')$. For w a partition $w = w_1\, w_2$ must exist with $m_0\ \langle w_1 \rangle\ d\ \langle w_2 \rangle$, where $d \in D'$ for a $D' \in \mathcal{D}'$. But also the word $w_1\, a^n b^n\, w_2$ is element of L for all $n \in \mathbb{N}$. Hence the word $w_1 a^\omega$ must be 1-successful, too, since a corresponding firing sequence is possible and a marking of $\mathcal{D}$ will be reached. This is a contradiction to the definition of L.

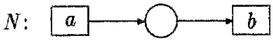

Figure 4: Petri net for the proof of lemma 11

b) Suppose $L \in \mathcal{L}^4_{\omega\mathrm{dt}}$, i. e. $L = \mathrm{L}^4_\omega(N', \mathcal{D}')$. For $w \in L$ there must be a partition $w = w_1 w_2 w_3$ and an anchorset $D \in \mathcal{D}'$, $D = \{d_1, d_2, \ldots, d_k\}$, so that $m_0\ \langle w_1 \rangle\ d_1\ \langle w_2 \rangle\ d_k\ \langle w_3 \rangle$ and in w_2 all markings of D will be reached, thus $w_2 = v_1 v_2 \ldots v_{k-1}$ with $d_1\ \langle v_1 \rangle\ d_2\ \langle v_2 \rangle \ldots \langle v_{k-1} \rangle\ d_k$. Also the sequence $w' = w_1 w_2 aabbw_3$ is element of L, thus also for this sequence there must be a partition and an anchorset, as described above, i. e. $m_0\ \langle w'_1 \rangle\ d'_1\ \langle w'_2 \rangle\ d'_{k'}\ \langle w'_3 \rangle$, where in w'_2 all markings of some $D' \in \mathcal{D}'$ will be reached at least once. Without loss of generality we may demand that $w'_1 > w_1 w_2$.

If $D \neq D'$, we start this consideration again with the sequence $w'' = w'_1 w'_2 aabbw'_3$. There are only finitely many anchorsets, hence some time two anchorsets must be the same. Let this be the set D''. Then $v = \underbrace{abab\ldots}_{v_1} v_2\, aabb \underbrace{\ldots}_{v_3} v_4 \underbrace{\ldots}_{v_5}$ with $m_0\ \langle v_1 \rangle\ d_1\ \langle v_2 \rangle\ d_k\ \langle aabbv_3 \rangle\ d_1\ \langle v_4 \rangle\ d_k\ \langle v_5 \rangle$, in v_2 and v_4 all markings of D'' will be reached. But then also $v_1(v_2 aabbv_3)^\omega$ would be 4-successful. This is a contradiction to the definition of L.

5 Transitional i-behaviour of deterministic nets

Just as for the i-behaviour the classes of the transitional i-behaviour of deterministic nets will be compared. We will see that fewer inclusions are valid than in the nondeterministic case. But the relationships among the classes will be the same as for deterministic noncomplete automata or deterministic push-down automata.

First we will compare the deterministic case with the unrestricted case.

Lemma 12 *For all $i \in \{1, 1', 2, 2', 3, 4\}$: $\mathcal{K}^i_{\omega\mathrm{dt}}$ is not closed under union.*

Proof: Similar as in the proof of lemma 8 this problem can be reduced to the proof for the languages of finite words. Take the language L from the proof of lemma 7. For all $i \in \{1, 1', 2, 2', 3, 4\}$ the language $L.d^\omega$ can be described as the union of two transitional i-behaviours. Remember that L includes all prefixes of itself. If there would be a net N' with $L.d^\omega$ as its transitional i-behaviour, L could be described as the finite behaviour of a deterministic net.

With this lemma it is obvious, that the transitional i-behaviour is more powerful in the nondeterministic case.

Lemma 13 *For all $i \in \{1, 1', 2, 2', 3, 4\} : \mathcal{K}^i_{\omega\mathrm{dt}} \subset \mathcal{K}^i_\omega$*

Comparing the classes of transitional i-behaviour we start with inclusions which were already shown in the nondeterministic case. The proofs are often more complicate, since the deterministic transitional i-behaviours are not closed under union. In general it is not sufficient to look at only one anchorset.

Lemma 14

$$\mathcal{L}_{\omega\mathrm{dt}} = \mathcal{K}^{1'}_{\omega\mathrm{dt}}$$

Proof: 1) For $L \in \mathcal{L}_{\omega\mathrm{dt}}$: Let $L = \mathrm{L}_\omega(N)$, then $L = \mathrm{K}^{1'}_\omega(N, \{T\})$.

2) For $L \in \mathcal{K}^{1'}_{\omega\mathrm{dt}}$: Let $L = \mathrm{K}^{1'}_\omega(N, \mathcal{E})$. Build a new net N' in the following way: The places consist of the old places S, and a place $s_{\tilde{\mathcal{E}}}$ for each set $\tilde{\mathcal{E}}$ with $\emptyset \neq \tilde{\mathcal{E}} \subseteq \mathcal{E}$. These new places will act as some kind of a finite control of the N', showing from which anchorsets the transitions of a firing sequence may be chosen, so that a 1′-successful continuation is still possible. At the initial marking on the places of S there lies the old marking and on $s_{\mathcal{E}}$ one token. The transitions consist of copies of the old transitions, which only differ with respect to the new places. For each place $s_{\tilde{\mathcal{E}}}$ take a set of transitions $T_{\tilde{\mathcal{E}}} = \{t_{\tilde{\mathcal{E}}} \mid \exists E \in \tilde{\mathcal{E}} : t \in E\}$. For each transition $t_{\tilde{\mathcal{E}}} \in T_{\tilde{\mathcal{E}}}$ construct the set $\tilde{\mathcal{E}}'(t) = \{E \mid E \in \tilde{\mathcal{E}} \wedge t \in E\}$. If this set is empty, eliminate the transition from N'. Every transition $t_{\tilde{\mathcal{E}}} \in T_{\tilde{\mathcal{E}}}$ has the place $s_{\tilde{\mathcal{E}}}$ as a precondition and it puts a token on $s_{\tilde{\mathcal{E}}'(t)}$.

It is easy to see that a firing sequence v is possible, iff there is at least one anchorset, so that v consists of copies of transitions from this anchorset. The net N' is deterministic, since at any time exactly one place $s_{\tilde{\mathcal{E}}}$ is marked and hence always only one set of copies of the original transitions is activated.

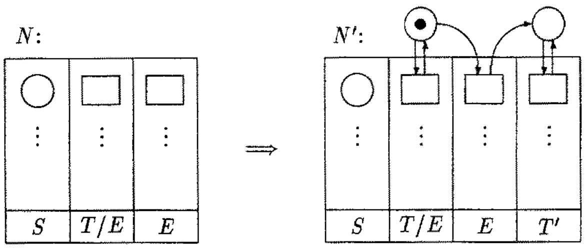

Figure 5: Transformation for the proof of lemma 15 e), f)

Lemma 15

$$\text{a) } \mathcal{L}_{\omega\mathrm{dt}} \subseteq \mathcal{K}^1_{\omega\mathrm{dt}}, \quad \text{b) } \mathcal{K}^{2'}_{\omega\mathrm{dt}} \subseteq \mathcal{K}^3_{\omega\mathrm{dt}}, \quad \text{c) } \mathcal{K}^4_{\omega\mathrm{dt}} \subseteq \mathcal{K}^3_{\omega\mathrm{dt}}, \quad \text{d) } \mathcal{K}^2_{\omega\mathrm{dt}} \subseteq \mathcal{K}^4_{\omega\mathrm{dt}},$$

$$\text{e) } \mathcal{K}^1_{\omega\mathrm{dt}} \subseteq \mathcal{K}^{2'}_{\omega\mathrm{dt}}, \quad \text{f) } \mathcal{K}^1_{\omega\mathrm{dt}} \subseteq \mathcal{K}^2_{\omega\mathrm{dt}}$$

Proof: a) Let $L = \mathrm{L}_\omega(N)$, then $L = \mathrm{K}^1_\omega(N, \{T\})$.
b) Let $L = \mathrm{K}^{2'}_\omega(N, \mathcal{E})$, then $L = \mathrm{K}^3_\omega(N, \{E' \mid \exists E \in \mathcal{E}: \emptyset \neq E' \subseteq E\})$, i. e., the set of the transitions appearing infinitely often must be a subset of an original anchorset.
c) Let $L = \mathrm{K}^4_\omega(N, \mathcal{E})$, then $L = \mathrm{K}^3_\omega(N, \{E' \mid \exists E \in \mathcal{E}: E \subseteq E'\})$, i. e., an original anchorset is subset of the transitions appearing infinitely often.
d) Let $L = \mathrm{K}^2_\omega(N, \mathcal{E})$, it holds $L = \mathrm{K}^2_\omega(N, \{\{t \mid \exists E' \in \mathcal{E}: t \in E\}\})$. Then $L = \mathrm{K}^4(N, \{\{t\} \mid t \in E\})$, i. e., at least one transition of E appears infinitely often.
e), f) Let $L = \mathrm{K}^1_\omega(N, \mathcal{E})$, then $L = \mathrm{K}^1_\omega(N, \{\{t \mid \exists E' \in \mathcal{E}: t \in E'\}\})$, since for the transitional i-behaviour it is sufficient that only some transition of some anchorset will be fired once. Construct a net N' according to the figure 5, where T' consists of copies of the original transitions, which have the same behaviour with respect to the places of S. Then $L = \mathrm{L}^i_\omega(N, \{T'\})$ for $i \in \{2, 2'\}$.

Lemma 16

$$\mathcal{K}^4_{\omega\mathrm{dt}} \subseteq \mathcal{K}^2_{\omega\mathrm{dt}}$$

Proof: Let $L \in \mathcal{K}^4_{\omega\mathrm{dt}}$, i. e. $L = \mathrm{K}^4_\omega(N, \{E_1, E_2, \ldots, E_k\})$. Construct the net N' from N in the following way. We use similar ideas as in the proof of lemma 14 ($\mathcal{K}^{1'}_{\omega\mathrm{dt}} \subseteq \mathcal{L}_{\omega\mathrm{dt}}$).
For the places in N' take all places of N and additionally for each subset of transitions, which is no anchorset, a new place $s_{\hat{T}}, \hat{T} \in \wp(T)/\mathcal{E}$. As the transitions take for every place $s_{\hat{T}}$ copies of the transitions T, these sets will be denoted by $T_{\hat{T}} = \{t_1^{\hat{T}}, t_2^{\hat{T}}, \ldots, t_n^{\hat{T}}\}$. Transition $t_i^{\hat{T}}$ acts like the original transition t_i with respect to the places of S. Additionally it takes a token from place $s_{\hat{T}}$ and puts one on $s_{\hat{T}'}$, where $\hat{T}' = \hat{T} \cup \{t_i\}$, if $\hat{T} \cup \{t_i\} \notin \mathcal{E}$ and $\hat{T}' = \emptyset$ otherwise. The initial marking remains unchanged on the places of S and a token will be on $s_\emptyset$.
In N' and N the same words are possible, since always exactly one of the places $s_{\hat{T}}$ contains a token, hence exactly the transitions of the set $T_{\hat{T}}$ are activated. These are copies of transitions from N. This also shows that N' is deterministic.
A token on place $s_{\hat{T}}$ means that copies of all transitions from $\hat{T}$ have fired after the last time $s_\emptyset$ was marked. Between two markings, in which $s_\emptyset$ has a token, the copies of all transitions of an anchorset must have fired. Since there are only finitely many different anchorsets, we only have to ensure that infinitely often on $s_\emptyset$ lies a token, to get the language L. This will be indicated by the firing of transitions of the set $T_\emptyset$, hence $L = \mathrm{K}^2_\omega(N', \{T_\emptyset\})$.

The following lemmata show nonexisting inclusions. As far as possible we will use examples of [Valk 83] and [Carstensen, Valk 85].

Lemma 17

$$\text{a) } \mathcal{K}^1_{\omega\mathrm{dt}} \not\subseteq \mathcal{L}_{\omega\mathrm{dt}}, \quad \text{b) } \mathcal{K}^2_{\omega\mathrm{dt}} \not\subseteq \mathcal{K}^{2'}_{\omega\mathrm{dt}}, \quad \text{c) } \mathcal{K}^{2'}_{\omega\mathrm{dt}} \not\subseteq \mathcal{K}^2_{\omega\mathrm{dt}}$$

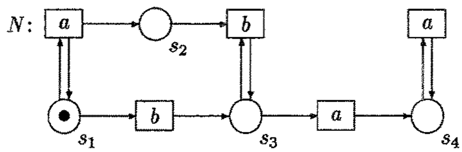

Figure 6: Petri net for the proof of lemma 18 a), b), c)

Proof: a) The language $L = a^*b^\omega$ is the 1-behaviour of a deterministic finite automaton, hence it is also the transitional 1-behaviour of a deterministic net. But $a^*b^\omega \notin \mathcal{L}_\omega$, because otherwise also a^ω would be possible.
b) The language $L = (a^*b)^\omega$ is the 2-behaviour of a deterministic finite automaton, hence it is also the transitional 2-behaviour of a deterministic net. But in [Valk 83] it was shown that $a^*b^\omega \notin \mathcal{L}_\omega$.
c) Consider the language $L = \{a,b\}^*.b^\omega$. For a net N with two transitions t_1 and t_2, $h(t_1) = a$ and $h(t_2) = b$, without places it holds: N is deterministic and $L = \mathrm{K}^{2'}_\omega(N, \{\{t_2\}\})$.
Suppose $L \in \mathcal{K}^2_{\omega\mathrm{dt}}$, then there is a net N', so that $L = \mathrm{K}^2_\omega(N', \mathcal{E}')$ and hence also $L = \mathrm{K}^2_\omega(N', E')$ for $E' = \{t \mid \exists E \in \mathcal{E}' : t \in E\}$. Starting from the initial marking m_0 we build a sequence of markings $\mu = m_0, m_1, m_2, \ldots \infty$ in the following way:
The sequence $w = b^\omega$ is 2-successful, i. e., there is a partition of $w = w_1 w_2$, so that some transition of E' appears in $w_1 \in b^+$. Then let m_1 be defined by $m_0\ (w_1\rangle\ m_1$. Now consider the sequence $w_1 ab^\omega \in L$. The marking m_2 is defined by $m_0\ (w_1\rangle\ m_1\ (aw_2\rangle\ m_2$, where some transition of E' appears in $w_2 \in b^+$. For each m_i consider the sequence $m_0\ (w_1\rangle\ m_1\ (aw_2\rangle\ \ldots\ m_{i-1}\ (aw_i\rangle\ m_i$, where some transition of E' appears in $w_i \in b^+$.
By the lemma of Dickson [Dickson 13] there must be two markings m_i and m_j in μ, with $i, j \in \mathbb{N}^+$, $i < j$ and $m_i \leq m_j$, i. e. $m_0 \underbrace{(w_1\rangle\ \ldots m_i}_{v_1} \underbrace{(aw_{i+1}\rangle\ \ldots m_j}_{v_2}$, $m_i \leq m_j$. Then also the sequence $v = v_1(v_2)^\omega$ is possible and 2-successful, since a transition of E' appears in w_{i+1}. But $h(v) \notin L$, because a appears infinitely often.

6 The hierarchy of deterministic (transitional) i-behaviour

In this section the relationship between the classes of the transitional i-behaviour and the i-behaviour, $\mathcal{L}^i_{\omega\mathrm{dt}}$, will be investigated and a hierarchy for all deterministic classes will be established. We will see that there are no further inclusions, just like in the nondeterministic case. Again we will use ideas of proof for the nondeterministic cases, but we have to inspect more cases.

Lemma 18

$$\text{a) } \mathcal{L}^1_{\omega\mathrm{dt}} \not\subseteq \mathcal{K}^3_{\omega\mathrm{dt}}, \quad \text{b) } \mathcal{L}^{2'}_{\omega\mathrm{dt}} \not\subseteq \mathcal{K}^3_{\omega\mathrm{dt}}, \quad \text{c) } \mathcal{L}^2_{\omega\mathrm{dt}} \not\subseteq \mathcal{K}^3_{\omega\mathrm{dt}}, \quad \text{d) } \mathcal{K}^1_{\omega\mathrm{dt}} \not\subseteq \mathcal{L}^1_{\omega\mathrm{dt}}$$

Proof: a), b), c) In [Valk 83] it was shown that the language $L = \{a^ib^ia^\omega \mid i \in \mathbb{N}^+\}$ is not a transitional 2-behaviour, i. e. $L \notin \mathcal{K}^2_\omega = \mathcal{K}^3_\omega$, hence more than ever $L \notin \mathcal{K}^3_{\omega\mathrm{dt}}$. In figure 6 a deterministic net N is given so that for $D = \{0,0,0,1\}$ it holds: $L = \mathrm{L}^1_\omega(N, \{D\}) = \mathrm{L}^{2'}_\omega(N, \{D\}) = \mathrm{L}^2_\omega(N, \{D\})$.
d) The net for the nondeterministic case in the proof of lemma 5 is deterministic.

Now we are able to establish the hierarchy.

Theorem 9 *The figure 7 shows all inclusions between the deterministic i-behaviour. The nonexistence of the inclusion between* $\mathcal{L}^4_{\omega\mathrm{dt}}$ *and* $\mathcal{KC}_\omega(\mathcal{L}_{0\mathrm{dt}})$ *can only be suggested.*

7 Conclusions

The infinite behaviour of deterministic Petri nets was investigated. The power of several definitions of infinite behaviour (i-behaviour and transitional i-behaviour) was compared with each other and with the nondeterministic case. It not only turned out that deterministic nets are less powerful than nondeterministic nets, but also that between the several classes of behaviours of nets there are less inclusions than in the

$$\begin{tikzcd}
 & \mathcal{KC}_\omega(\mathcal{L}_{0\mathrm{dt}}) & & \mathcal{K}^3_{\omega\mathrm{dt}} & \\
 & & & \mathcal{K}^{2'}_{\omega\mathrm{dt}} \arrow[u] & \mathcal{K}^2_{\omega\mathrm{dt}}=\mathcal{K}^4_{\omega\mathrm{dt}} \arrow[ul] \\
\mathcal{L}^4_{\omega\mathrm{dt}} & \mathcal{L}^3_{\omega\mathrm{dt}} \arrow[uu] & \mathcal{L}^1_{\omega\mathrm{dt}} & \mathcal{K}^1_{\omega\mathrm{dt}} \arrow[u] \arrow[ur] & \\
\mathcal{L}^2_{\omega\mathrm{dt}} \arrow[u] \arrow[uuur] & \mathcal{L}^{2'}_{\omega\mathrm{dt}} \arrow[u] & \mathcal{L}_{\omega\mathrm{dt}}=\mathcal{K}^{1'}_{\omega\mathrm{dt}} \arrow[u] \arrow[ur] & & \\
 & \mathcal{L}^{1'}_{\omega\mathrm{dt}} \arrow[ul] \arrow[u] \arrow[ur] & & &
\end{tikzcd}$$

Figure 7: Hierarchy of the deterministic i-behaviour

nondeterministic case. Also the influence of λ-transitions was investigated. The power of infinite behaviours of deterministic nets is not increased if λ-transitions are allowed. In contrast it was shown that nets with these transitions are more powerful for finite behaviour with terminal markings, as well as for infinite behaviour of nondeterministic nets. An open problem for the hierarchy is the problem if $\mathcal{L}^4_{\omega\mathrm{dt}} \subseteq \mathcal{KC}_\omega(\mathcal{L}_{0\mathrm{dt}})$. I suggest that this inclusion does not hold.

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