Reasoning about data repetitions with counter systems

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in LICS'13

reasoning on data words

data word

a	*b*	*a*	*b*	*a*	*b*	*b*	*b*	*b*	*a*	*b*	*b*
2	4	7	6	9	3	9	7	6	2	2	9

reasoning on data words

data word	a	b	a	b	a	b	b	b	b	a	b	b	$\in (\{a,b\} \times \mathbb{N})^*$
	2	4	7	6	9	3	9	7	6	2	2	9	

reasoning on data words

data word

$$\begin{array}{cccccccccccc} a & b & a & b & a & b & b & b & b & a & b & b \\ 2 & 4 & 7 & 6 & 9 & 3 & 9 & 7 & 6 & 2 & 2 & 9 \end{array} \in (\{a,b\} \times \mathbb{N})^*$$

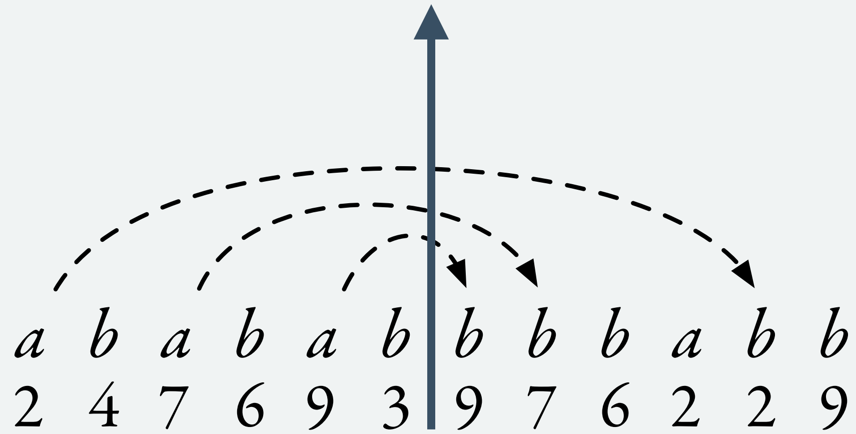
*"for every **a** there is a **b** with same data value to its right"*

reasoning with *infinite* alphabets = counting

a	*b*	*a*	*b*	*a*	*b*	*b*	*b*	*b*	*a*	*b*	*b*
2	4	7	6	9	3	9	7	6	2	2	9

*"for every **a** there is a **b** with same data value to its right"*

reasoning with *infinite* alphabets = counting



*"for every **a** there is a **b** with same data value to its right"*

reasoning with *infinite* alphabets = counting

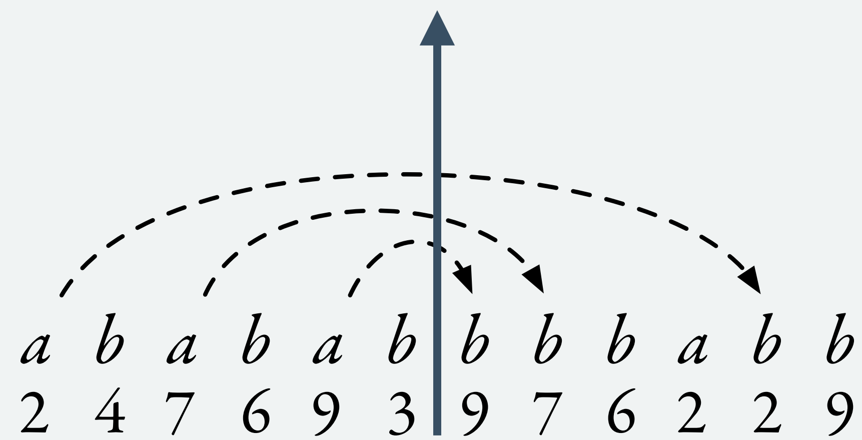
there must be 3 distinct data values to the right

a	*b*	*a*	*b*	*a*	*b*	*b*	*b*	*b*	*a*	*b*	*b*
2	4	7	6	9	3	9	7	6	2	2	9

“for every ***a*** *there is a* ***b*** *with same data value to its right”*

reasoning with *infinite* alphabets = counting

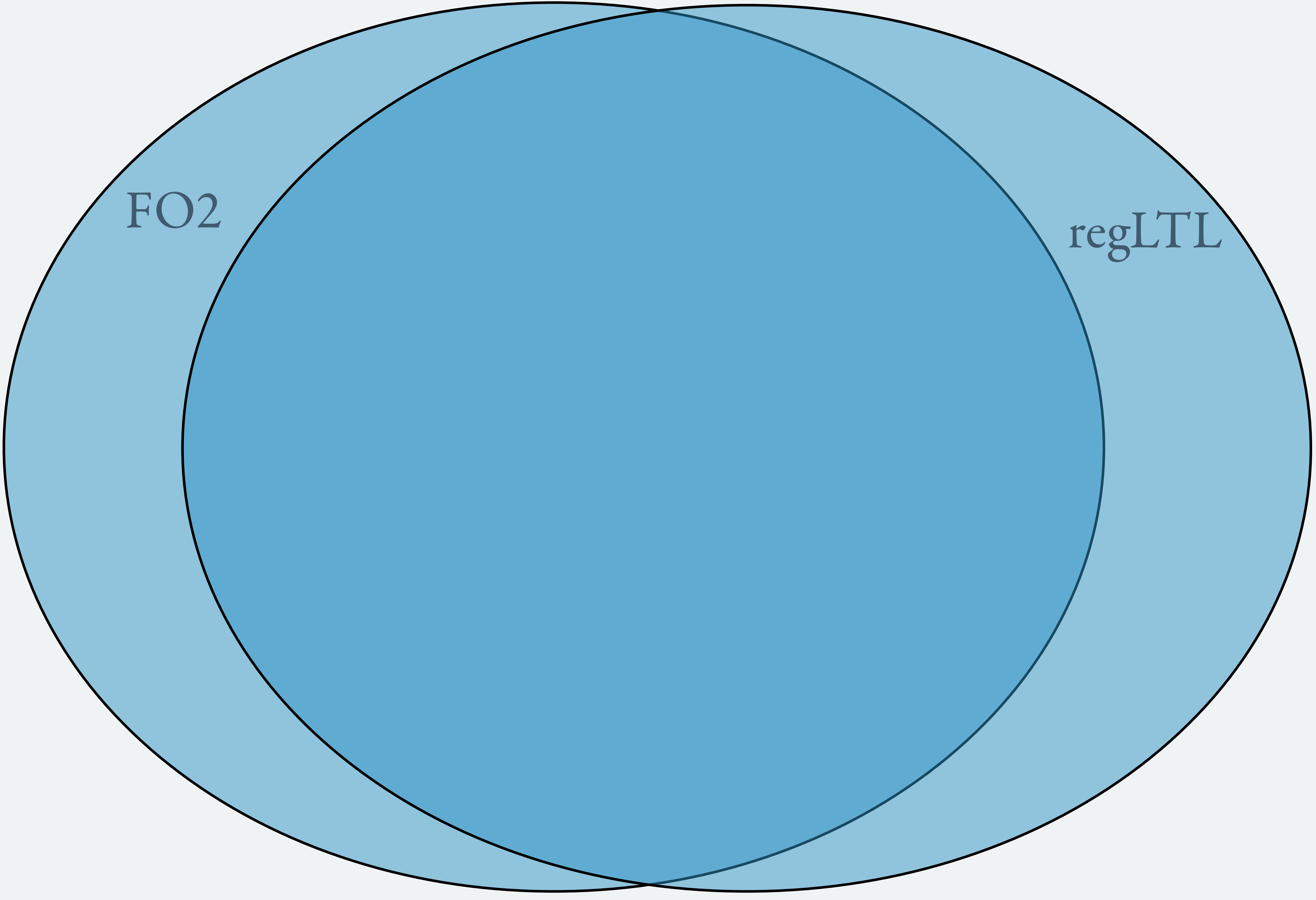
there must be 3 distinct data values to the right

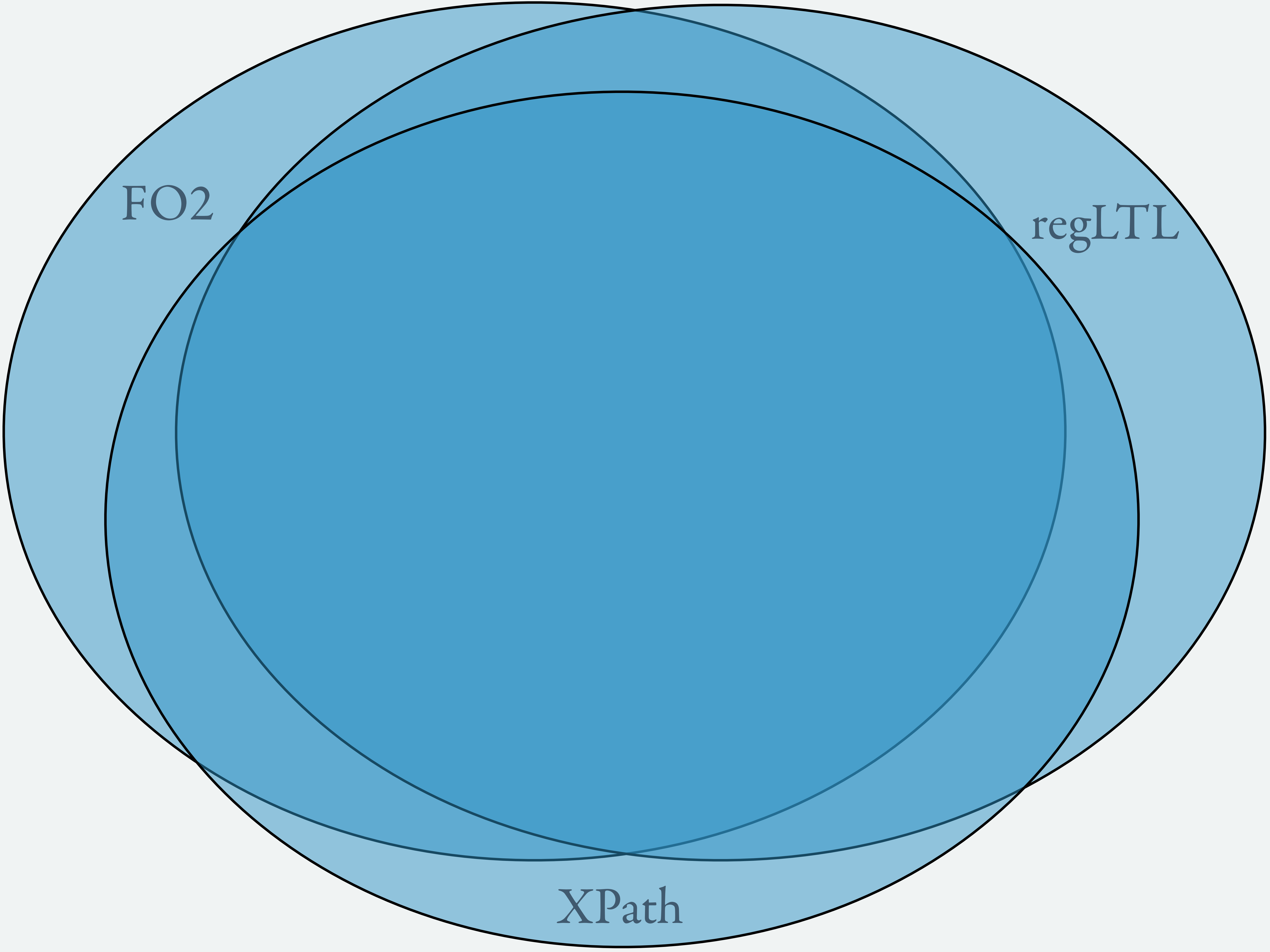


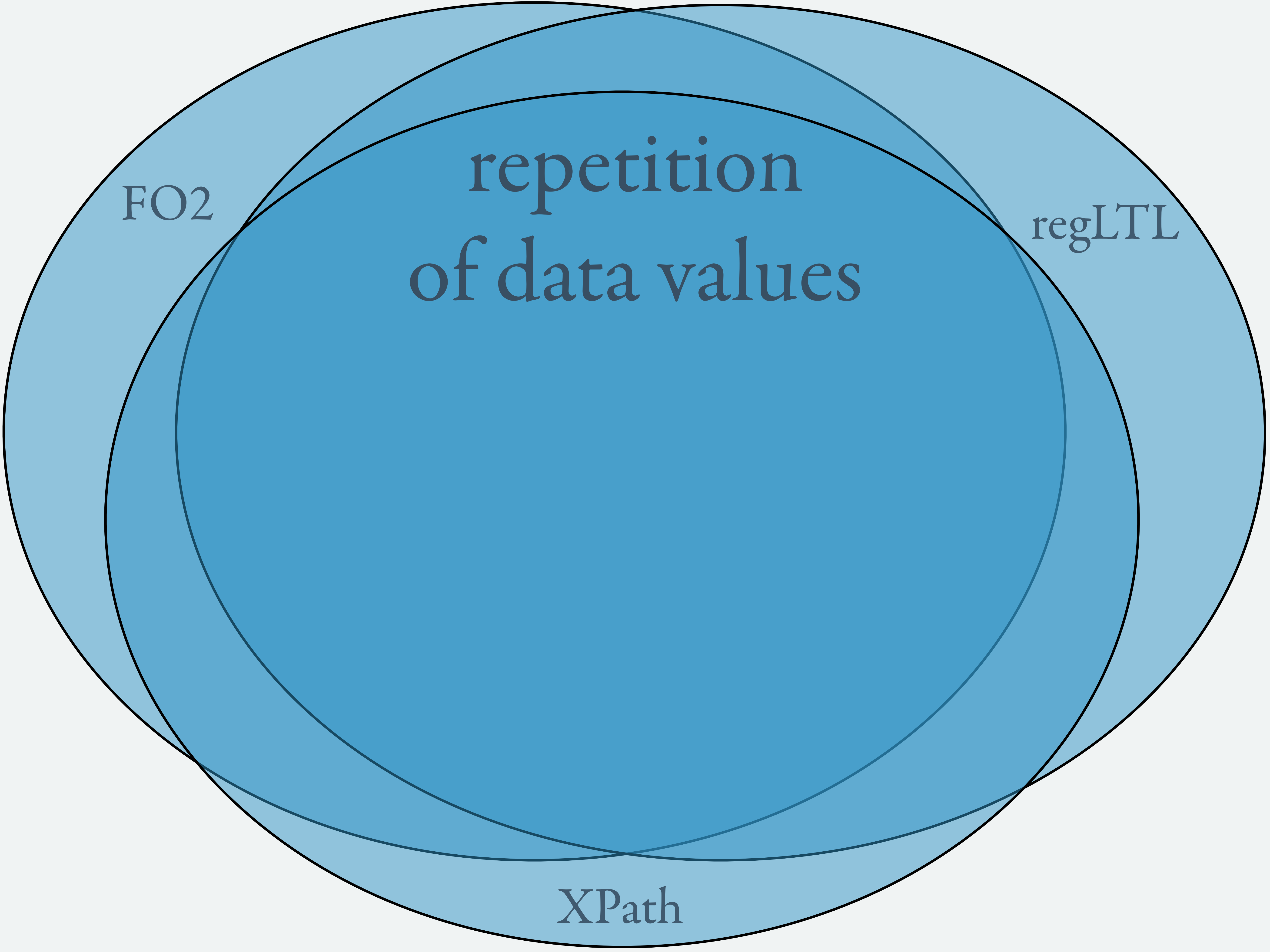
*"for every **a** there is a **b** with same data value to its right"*

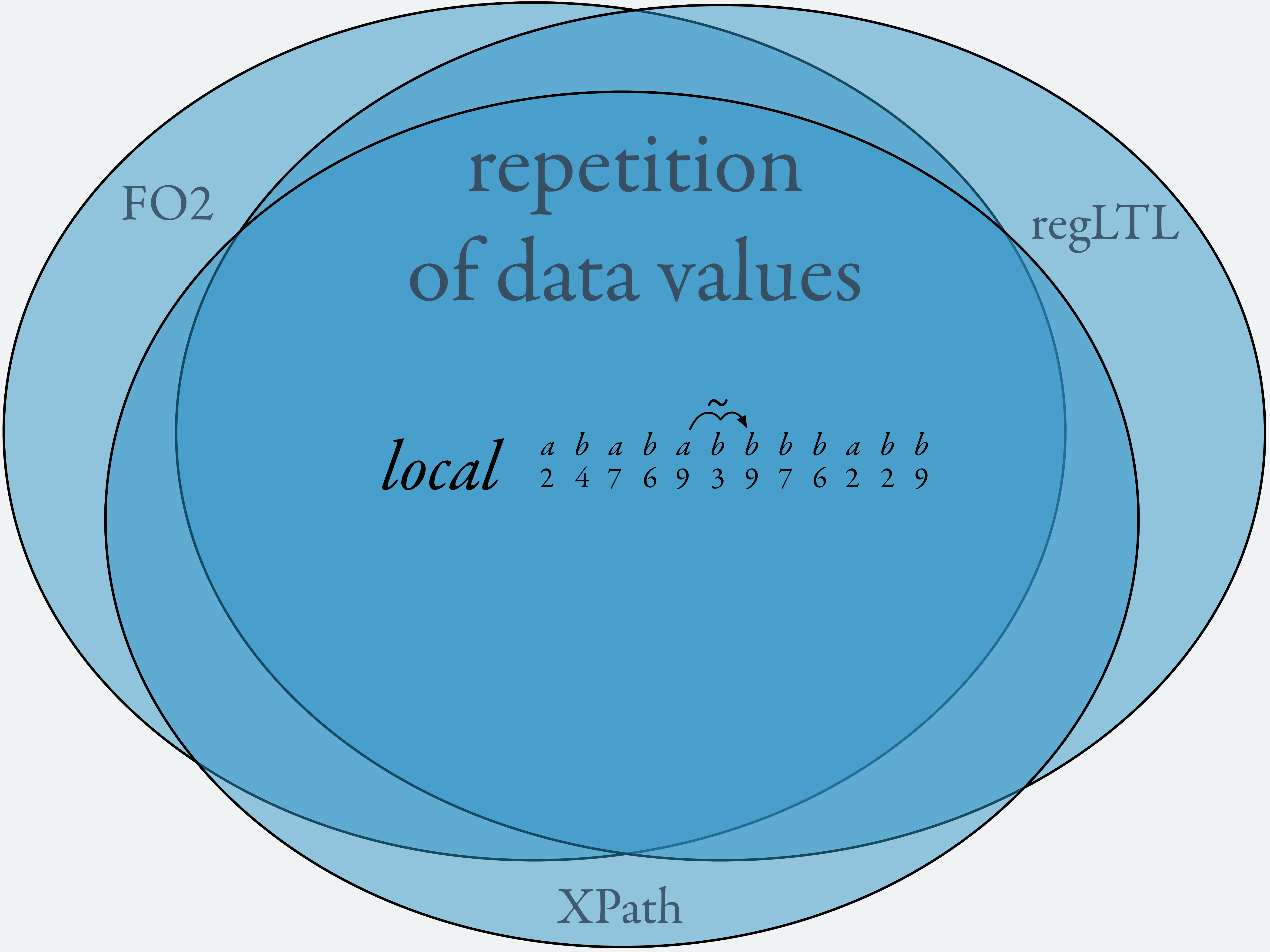
	DA / FO2 SAT	⟷	reachability of VASS	[Bojańczyk & al.]
	ARA-1 / 1-regLTL	⟷	reachability of ICA	[Demri & Lazić]
*on data **trees***	XPath SAT	⟷	reachability of ITCA	[Jurdziński & Lazić]
	FO2 SAT	⟷	reachability of BVASS+	[Dimino & al.]

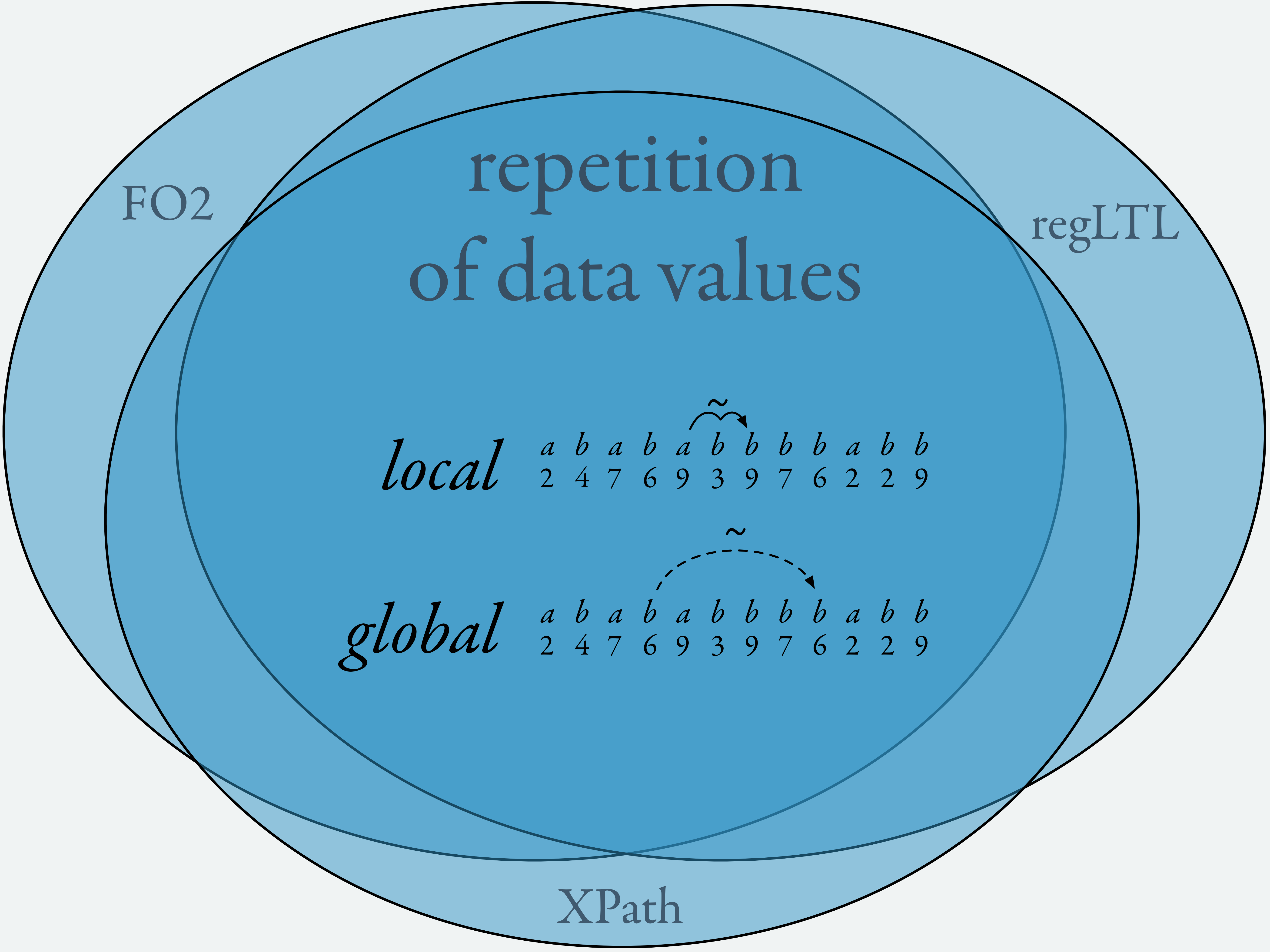
FO2


FO2
regLTL


FO2
regLTL
XPath


repetition
of data values
FO2
regLTL
XPath


FO2
regLTL
repetition
of data values
local
a b a b a b b b b a b b
2 4 7 6 9 3 9 7 6 2 2 9
~
XPath


repetition
of data values
FO2
regLTL
XPath
local
a b a b a b b b b a b b
2 4 7 6 9 3 9 7 6 2 2 9
global
a b a b a b b b b a b b
2 4 7 6 9 3 9 7 6 2 2 9

logic of repeating values

repeating values
on data words

logic of repeating values

repeating values
on data words

⟷

reachability of
counter systems

logic of repeating values

repeating values on data words	$\longleftrightarrow$	reachability of counter systems
different **flavours** of repeating values	$\longleftrightarrow$	different **precisions** of counting

a	*b*	*a*	*b*	*a*	*b*	*b*	*b*	*b*	*a*	*b*	*b*
2	4	7	6	9	3	9	7	6	2	2	9

data
word

	a	b	a	b	a	b	b	b	b	a	b	b
x_1	2	4	7	6	9	3	9	7	6	2	2	9
x_2	5	2	7	5	1	2	2	8	5	2	8	9
x_3	2	3	5	6	9	8	9	7	3	1	2	2
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

data word

	a	b	a	b	a	b	b	b	b	a	b	b
x_1	2	4	7	6	9	3	9	7	6	2	2	9
x_2	5	2	7	5	1	2	2	8	5	2	8	9
x_3	2	3	5	6	9	8	9	7	3	1	2	2
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

$\in (\{a, b\} \times \mathbb{N}^{\{x_1, x_2, \ldots\}})^*$

data word

LOGIC OF REPEATING VALUES

$$\text{LTL} + \begin{cases} \textit{global} \\ \textit{local} \end{cases}$$

	a	b	a	b	a	b	b	b	b	a	b	b
x_1	2	4	7	6	9	3	9	7	6	2	2	9
x_2	5	2	7	5	1	2	2	8	5	2	8	9
x_3	2	3	5	6	9	8	9	7	3	1	2	2
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

$\in (\{a, b\} \times \mathbb{N}^{\{x_1, x_2, \ldots\}})^*$

data word

LOGIC OF REPEATING VALUES

LTL + $\left\{ \begin{array}{l} \textit{global} \\ \textit{local} \end{array} \right.$ $\quad x_1 \sim \Diamond x_3$

$$\begin{array}{ccccccccccccc}
 & a & b & a & b & a & b & b & b & b & a & b & b \\
x_1 & 2 & 4 & 7 & 6 & 9 & 3 & 9 & 7 & 6 & 2 & 2 & 9 \\
x_2 & 5 & 2 & 7 & 5 & 1 & 2 & 2 & 8 & 5 & 2 & 8 & 9 \\
x_3 & 2 & 3 & 5 & 6 & 9 & 8 & 9 & 7 & 3 & 1 & 2 & 2 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots
\end{array} \in (\{a,b\} \times \mathbb{N}^{\{x_1,x_2,\dots\}})^*$$

data word

LOGIC OF REPEATING VALUES

$$\text{LTL} + \begin{cases} \textit{global} & x_1 \sim \Diamond x_3 \qquad x_1 \sim \langle \phi ? \rangle x_3 \\ \textit{local} & \end{cases}$$

$$\phi$$

$$\begin{array}{c|cccccccccccc}
 & a & b & a & b & a & b & b & b & b & a & b & b \\
x_1 & 2 & 4 & 7 & 6 & 9 & 3 & 9 & 7 & 6 & 2 & 2 & 9 \\
x_2 & 5 & 2 & 7 & 5 & 1 & 2 & 2 & 8 & 5 & 2 & 8 & 9 \\
x_3 & 2 & 3 & 5 & 6 & 9 & 8 & 9 & 7 & 3 & 1 & 2 & 2 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots
\end{array} \in (\{a, b\} \times \mathbb{N}^{\{x_1, x_2, \ldots\}})^*$$

data word

LOGIC OF REPEATING VALUES

$$\text{LTL} + \begin{cases} \textit{global} & x_1 \sim \Diamond x_3 \qquad x_1 \sim \langle \phi? \rangle x_3 \\ \textit{local} & x_3 \not\sim \mathrm{X}^{-3} x_2 \end{cases}$$

$$\begin{array}{c|cccccccccccc} & a & b & a & b & a & b & b & b & b & a & b & b \\ x_1 & 2 & 4 & 7 & 6 & 9 & 3 & 9 & 7 & 6 & 2 & 2 & 9 \\ x_2 & 5 & 2 & 7 & 5 & 1 & 2 & 2 & 8 & 5 & 2 & 8 & 9 \\ x_3 & 2 & 3 & 5 & 6 & 9 & 8 & 9 & 7 & 3 & 1 & 2 & 2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{array} \in (\{a,b\} \times \mathbb{N}^{\{x_1, x_2, \ldots\}})^*$$

data word

LOGIC OF REPEATING VALUES

LTL + $\begin{cases} \textit{global} \\ \textit{local} \end{cases}$

$\not\sim$ $\not\sim$

$x_1 \sim \Diamond x_3$ $x_1 \sim \langle \phi? \rangle x_3$

$\sim$

$x_3 \not\sim \mathrm{X}^{-3} x_2$

	a	*b*	*a*	*b*	*a*	*b*	*b*	*b*	*b*	*a*	*b*	*b*
x_1	2	4	7	6	9	3	9	7	6	2	2	9
x_2	5	2	7	5	1	2	2	8	5	2	8	9
x_3	2	3	5	6	9	8	9	7	3	1	2	2
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

$\in (\{a, b\} \times \mathbb{N}^{\{x_1, x_2, \ldots\}})^*$

data word

LOGIC OF REPEATING VALUES

LTL + $\begin{cases} \textit{global} \\ \textit{local} \end{cases}$

$x_1 \sim \Diamond x_3$ $\quad \not\sim \quad \Diamond^{-1}$

$x_1 \sim \langle \phi ? \rangle x_3$ $\quad \not\sim \quad \langle \phi ? \rangle^{-1}$

$x_3 \not\sim \mathrm{X}^{-3} x_2$ $\quad \sim \quad \mathrm{X}^3$

	a	b	a	b	a	b	b	b	b	a	b	b
x_1	2	4	7	6	9	3	9	7	6	2	2	9
x_2	5	2	7	5	1	2	2	8	5	2	8	9
x_3	2	3	5	6	9	8	9	7	3	1	2	2
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

$\in (\{a, b\} \times \mathbb{N}^{\{x_1, x_2, \dots\}})^*$

data word

VASS

Vector
Addition
Systems with
States

VASS

Vector
Addition
Systems with
States

finite state automaton A with n counters containing natural numbers

transitions can $\left\{\begin{matrix}\text{increment} \\ \text{decrement (if >0)}\end{matrix}\right\}$ a counter

run starts with $\left\{\begin{matrix}\text{initial state} \\ \text{all counters in zero.}\end{matrix}\right.$

VASS

Vector
Addition
Systems with
States

finite state automaton A with n counters containing natural numbers

transitions can $\left\{\begin{matrix}\text{increment} \\ \text{decrement (if} > 0)\end{matrix}\right\}$ a counter

run starts with $\begin{cases}\text{initial state} \\ \text{all counters in zero.}\end{cases}$

control-state reachability problem

whether A can reach a given state

VASS

Vector
Addition
Systems with
States

finite state automaton A with n counters containing natural numbers

transitions can $\left\{\begin{array}{l}\text{increment} \\ \text{decrement (if} > 0)\end{array}\right\}$ a counter

run starts with $\left\{\begin{array}{l}\text{initial state} \\ \text{all counters in zero.}\end{array}\right.$

- control-state reachability problem

  whether A can reach a given state

- reachability problem

  whether A can reach a given state <u>and counter values</u>

results

results

having or not $\langle \phi ? \rangle$ is the same

results

having or not $\langle \phi? \rangle$ is the same

with $\Diamond^{-1}$: SAT $\xleftrightarrow{\text{ExpTime}}$ reachability of VASS

results

having or not $\langle \phi ? \rangle$ is the same

with $\Diamond^{-1}$: SAT $\xleftrightarrow{\text{ExpTime}}$ reachability of VASS

without $\Diamond^{-1}$: SAT $\xleftrightarrow{\text{ExpTime}}$ control state reach. of VASS

results

having or not $\langle \phi ? \rangle$ is the same

with $\Diamond^{-1}$: SAT $\xleftrightarrow{\text{ExpTime}}$ reachability of VASS

without $\Diamond^{-1}$: SAT $\xleftrightarrow{\text{ExpTime}}$ control state reach. of VASS

SAT $\xleftrightarrow{\text{PTime}}$ control state reach. of $\underline{2^n\text{-VASS}}$

results

having or not $\langle \phi? \rangle$ is the same

with $\Diamond^{-1}$: SAT $\xleftrightarrow{\text{ExpTime}}$ reachability of VASS

without $\Diamond^{-1}$: SAT $\xleftrightarrow{\text{ExpTime}}$ control state reach. of VASS

SAT $\xleftrightarrow{\text{PTime}}$ $\underbrace{\text{control state reach. of } \underline{2^n\text{-VASS}}}_{\text{2ExpSpace-complete}}$

results

having or not $\langle \phi ? \rangle$ is the same

with $\Diamond^{-1}$: SAT $\xleftrightarrow{\text{ExpTime}}$ reachability of VASS [Demri, D'Souza & Gascon]

without $\Diamond^{-1}$: SAT $\xleftrightarrow{\text{ExpTime}}$ control state reach. of VASS

SAT $\xleftrightarrow{\text{PTime}}$ $\underbrace{\text{control state reach. of } \underline{2^n\text{-VASS}}}_{\text{2ExpSpace-complete}}$

the power of $\langle \phi ? \rangle$:

if we bound the number of variables: $\begin{cases} \text{SAT without } \langle \phi ? \rangle : \text{PSpace-complete} \\ \text{SAT with } \langle \phi ? \rangle \sim \text{reachability of VASS} \end{cases}$