Synthesis of Winning Strategies for Interaction under Partial Information

Oberseminar Informatik

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RWTH Aachen University

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Interaction: General Properties

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- single antagonist (player 0)

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- Partial Information about history of past events

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- finite set A of actions $a = (a_0, \dots, a_n)$

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- equivalence relations $\sim_i \subseteq V \times V$

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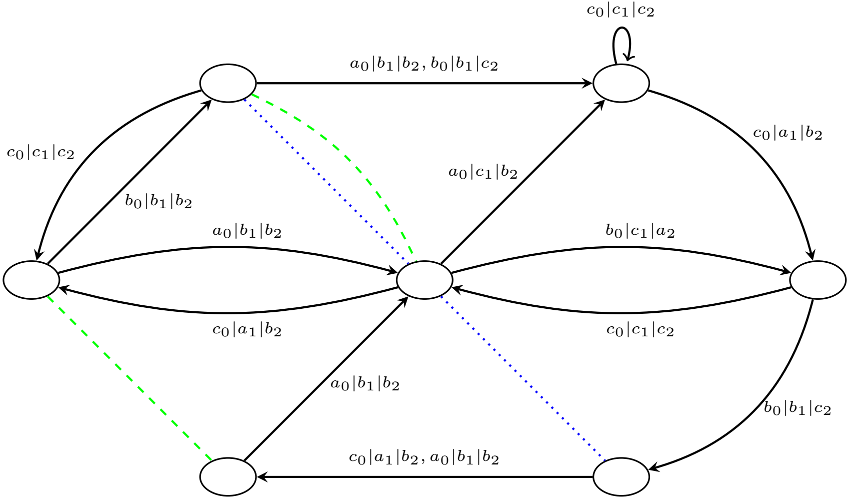

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Joint strategy for the grand coalition:

$$\sigma = (\sigma_1, \dots, \sigma_n)$$

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(strategy that guarantees winning for the grand coalition against **all** possible behaviors of player 0)

Strategy Synthesis: Problem

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Question: Does the grand coalition have a joint winning strategy?

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Implementation by

- Finite state machine
- Pushdown machine

Strategy Synthesis: Application in Computer Science

Controller-Synthesis for open reactive systems:

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Church's Circuit Synthesis Problem, 1957:
Two-player games with full information and regular winning conditions written in MSO

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Two-player games with full information and regular winning conditions written in MSO

Theorem (Büchi, Landweber 1969)

Given an MSO-formula ϕ, one can decide whether there exists a winning strategy σ for player 1. If so, a finite state winning strategy can be constructed effectively.

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Other Extensions: Asynchronous Systems, Stochastic Systems, Hybrid Systems, . . .

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Strategy problem is undecidable (not recursively enumerable) for three players and safety winning conditions.

- strategy problem for safety games with full information can be solved in linear time
- under full information, grand coalition reduces to single player

Strategy Synthesis: Refinements

What are the relevant parameters of strategy synthesis?

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- number of (cooperating) players
- complexity of winning condition:
  - expressive complexity of winning condition
  - information complexity of winning condition
    (extent to which winning conditions may involve facts that the player(s) cannot observe)
- complexity of information flow between the players
- structural complexity of game graph

Topics

- Parity Games on Simple Graphs
  Joint work with Roman Rabinovich
- Locally decomposable winning conditions
  Joint work with Wladimir Fridman
- Knowledge tracking for multiplayer games
  Joint work with Dietmar Berwanger and Łukasz Kaiser

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Parity Games on Simple Graphs

- **Two**-player games on finite graphs $\mathcal{G} = (V, \delta, \sim_1)$
- **Parity** conditions:
  col : $V \to \{0, \ldots, k\}$, $\min \mathrm{col} \inf v_0 v_1 v_2 \ldots$ even
- **Observable** coloring: $v \sim_1 w \quad \Rightarrow \quad \mathrm{col}(v) = \mathrm{col}(w)$

Parity Games on Simple Graphs: Full Information

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Theorem (Berwanger, Dawar, Hunter, Kreutzer 2006)

Strategy problem for parity games with full information can be solved in polynomial time on graphs of bounded DAG-width.

DAG-width is a measure of structural complexity for directed graphs:
How close is a graph to a directed acyclic graph (DAG)?

Parity Games on Simple Graphs: Partial Information

How about parity games on game graphs with partial information?

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Theorem (2010)

Strategy problem for safety games on finite game graphs with partial information information is

- EXPTIME*-hard on graphs of DAG-width at most* 3
- PSPACE*-hard on graphs of DAG-width at most* 1.

Parity Games on Simple Graphs: Bounded Partial Information

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Strategy problem for parity games on game graphs with bounded partial information and bounded DAG-width can be solved in polynomial time.

Locally Decomposable Winning Conditions

- Arbitrarily many players
- Regular and context-free winning conditions

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$\rightsquigarrow$ Decidability rather than complexity

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Strategy problem is undecidable for regular winning conditions **iff** *the communication graph contains incomparably informed players.*

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Different representation of interactive scenarios:
No predefined game graph!

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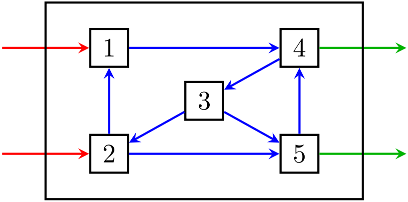

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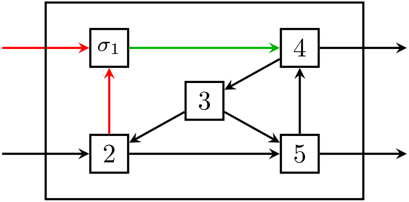

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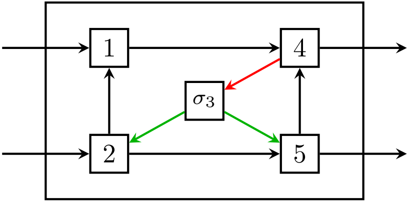

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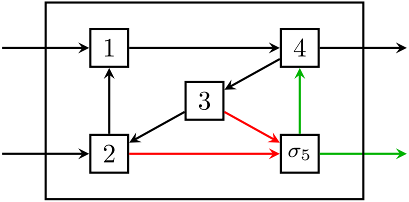

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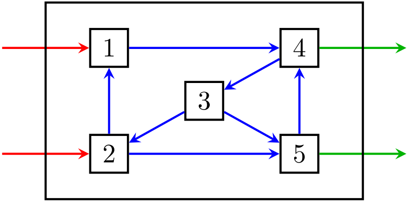

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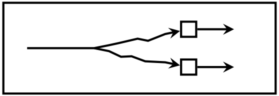

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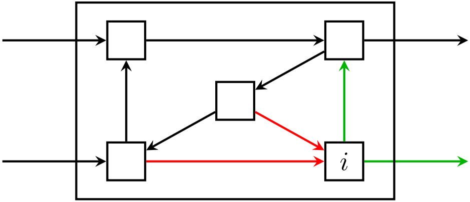

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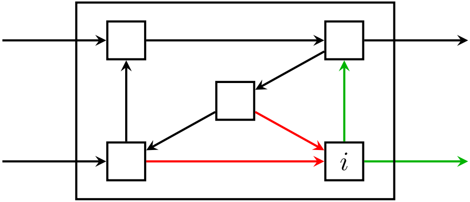

- Local Winning Conditions: W_i for player i
- Global Winning Condition $W = \bigcap W_i$

Locally Decomposable Winning Conditions: Regular Case

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Strategy problem is decidable for locally decomposable regular winning conditions **iff** *each connected component of the communication graph is a pipeline or a two-flanked pipeline.*

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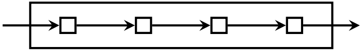

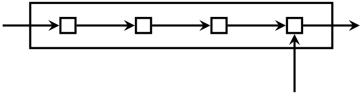

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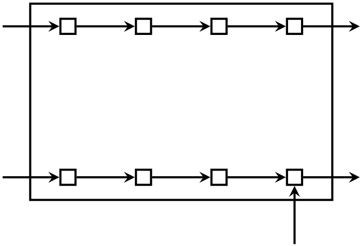

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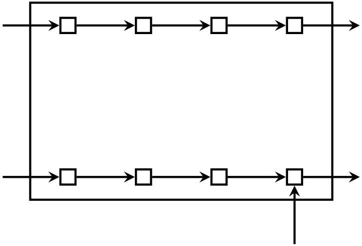

They considered only acyclic communication graphs!

Locally Decomposable Winning Conditions: Extensions

Extensions:

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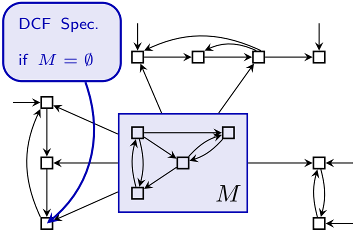

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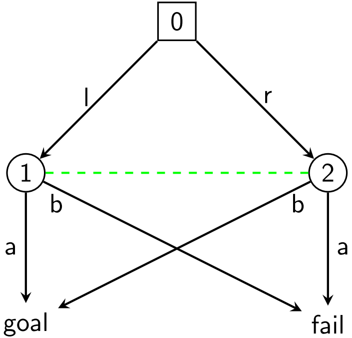
$\rightsquigarrow$ Represent all the possible states of mind of the players of the grand coalition and their dynamics explicitly

- Knowledge in multiplayer games:
  strategic dependencies and higher order knowledge
- Knowledge tracking:
  Epistemic Unfolding

Strategic Dependencies



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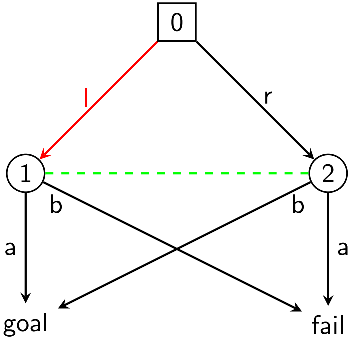

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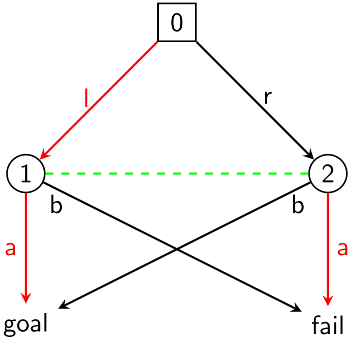

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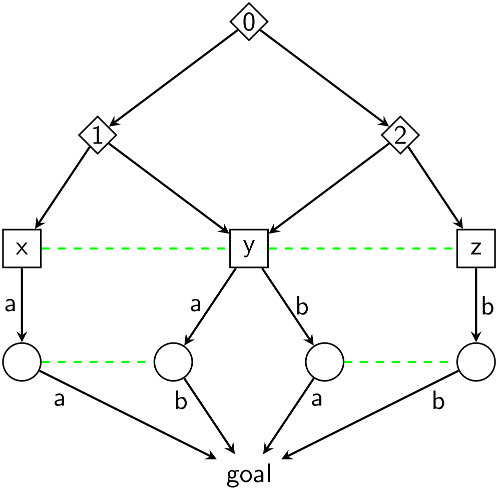

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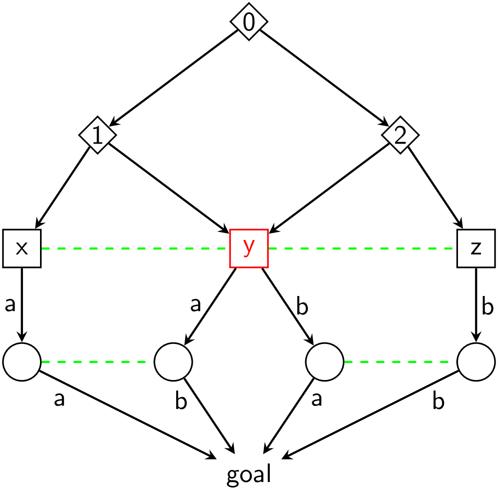

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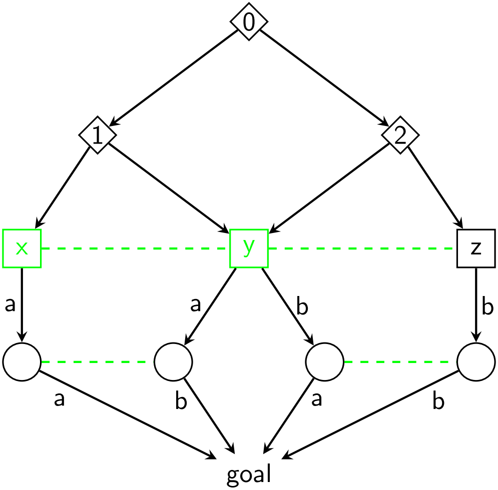

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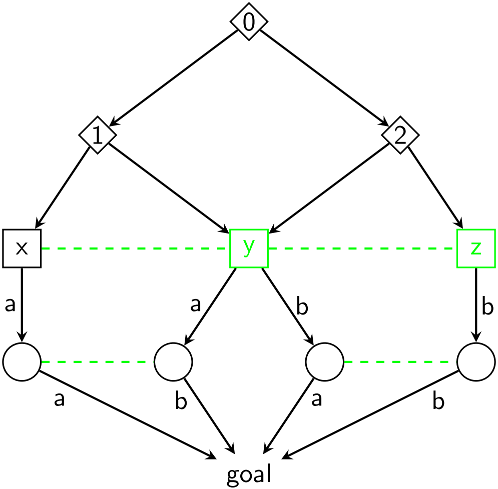

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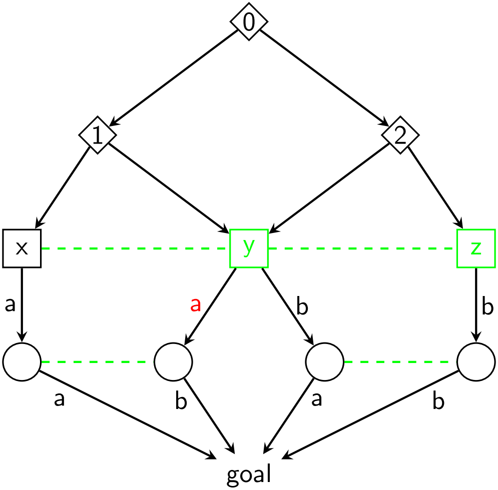

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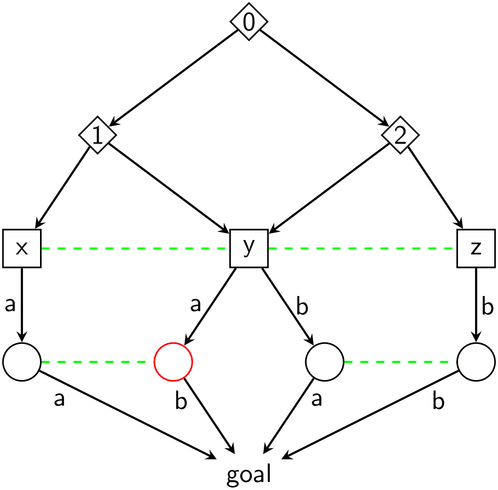

Epistemic Unfolding

Knowledge tracking construction has to take both these aspects into account!

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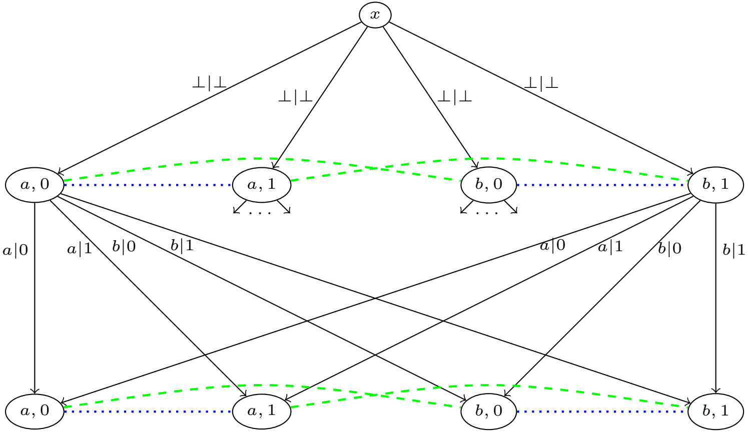

Epistemic Unfolding

x

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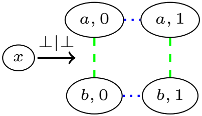
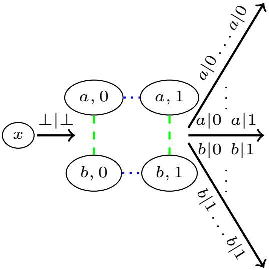
$$x \xrightarrow{\perp|\perp}$$

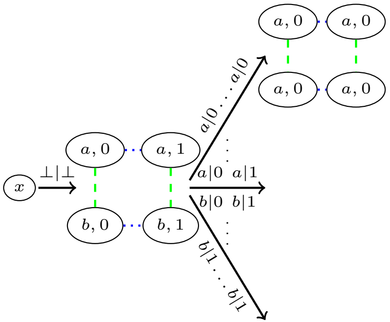
Epistemic Unfolding



Epistemic Unfolding



Epistemic Unfolding



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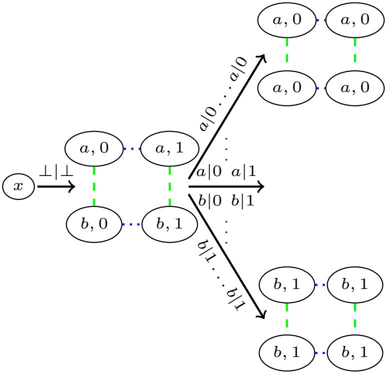

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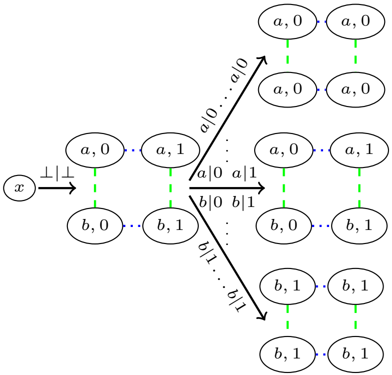

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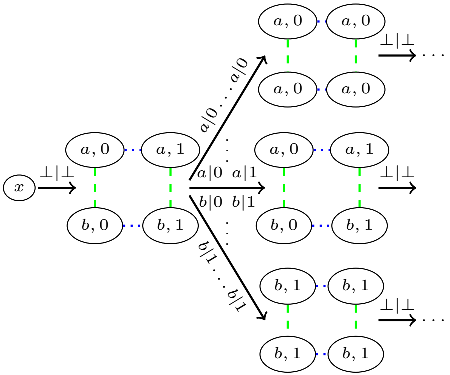

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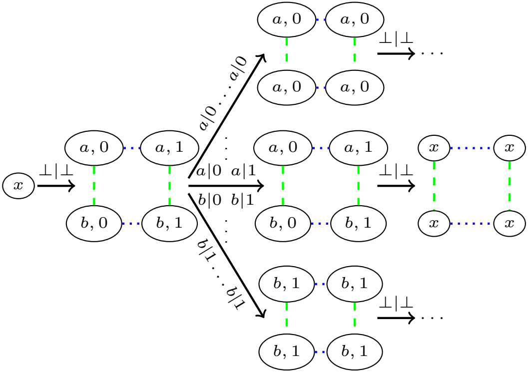

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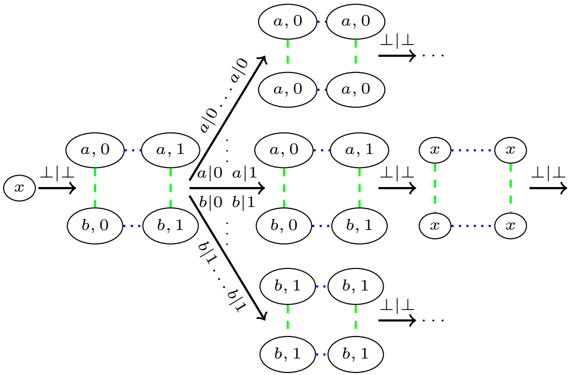

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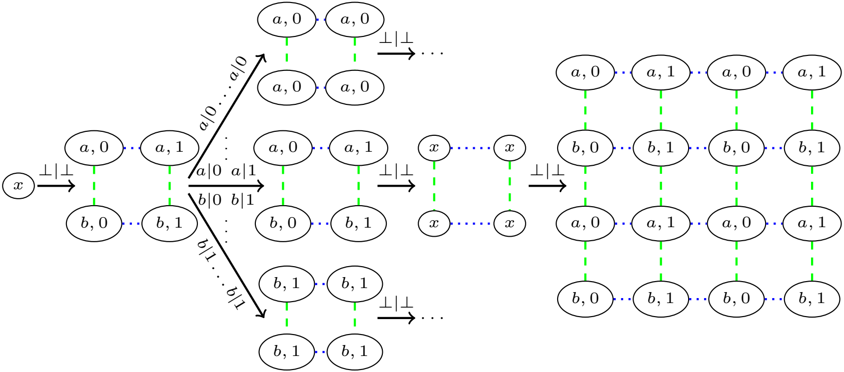

Epistemic Unfolding

Succinct Representation: Homomorphic Equivalence!

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Epistemic Unfolding

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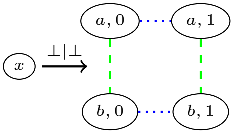

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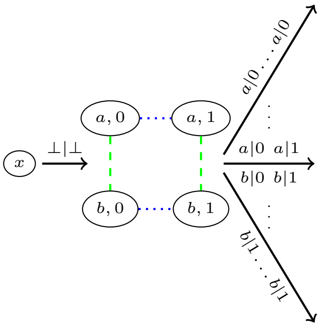

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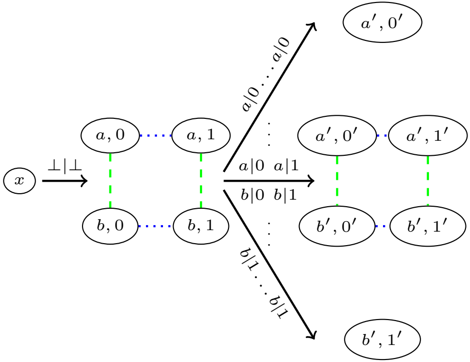

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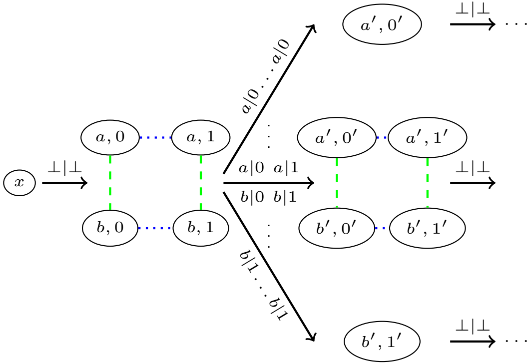

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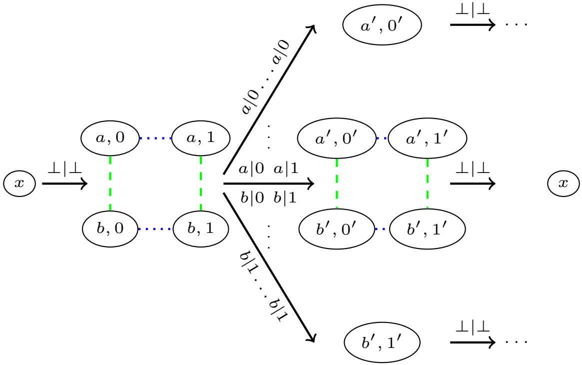

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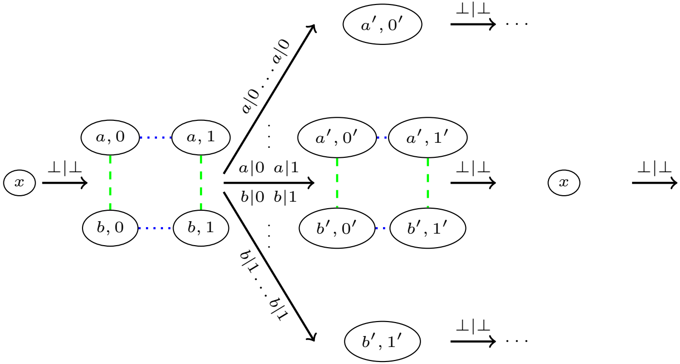

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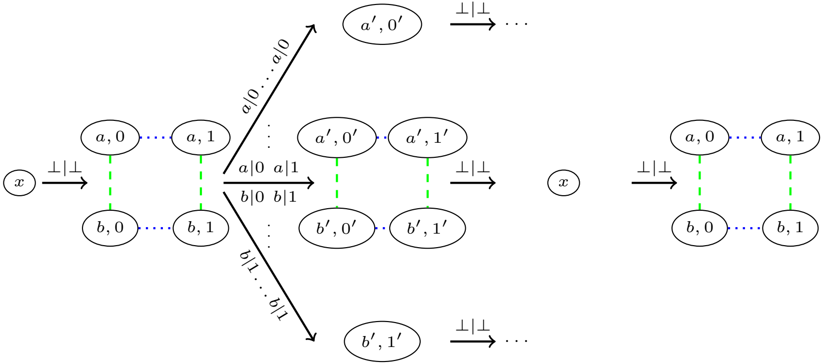

Epistemic Unfolding

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Theorem

For game graphs with hierarchical partial information, the quotient modulo homomorphic equivalence is finite.

Corollary

Srategy problem for hierarchical games with observable deterministic contextfree winning conditions is decidable.

Conclusion

Parity Games on Simple Graphs:

- Safety games on finite game graphs with partial information and DAG-width at most 3 are EXPTIME-hard
- Parity games on finite game graphs with bounded partial information and bounded DAG-width can be solved in polynomial time

Locally Decomposable Specifications:

- Most communication graphs have undecidable strategy problems, even for regular winning conditions
- There are relevant cases of communication graphs, where even (deterministic) contextfree winning conditions can be allowed

Knowledge in Multiplayer Games:

- Strategic Dependencies and higher order knowledge make reasoning about knowledge cumbersome in multiplayer games
- Knowledge tracking is possible and for observable winning conditions, homomorphic equivalence yields a sound quotient