Hardness of Untimed Language Universality

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Thanks to Stefan Göller

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Timed Systems

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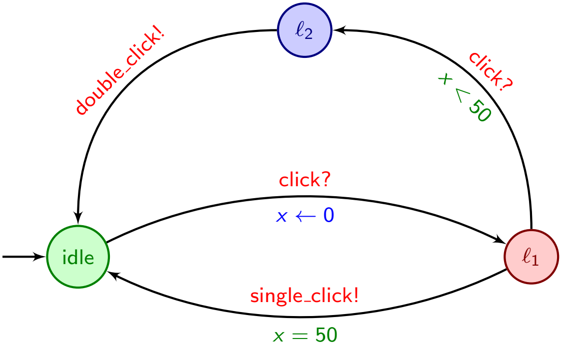
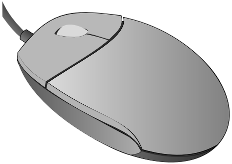
- Modeled by Timed Automata

Timed Automata (TA) [Alur and Dill 1994]

Finite automata + Analog clocks 





- Clocks cannot be stopped, all grow at the same rate.
- An edge is activated when its **clock constraint** holds.
- A clock can be **reset** by a transition.

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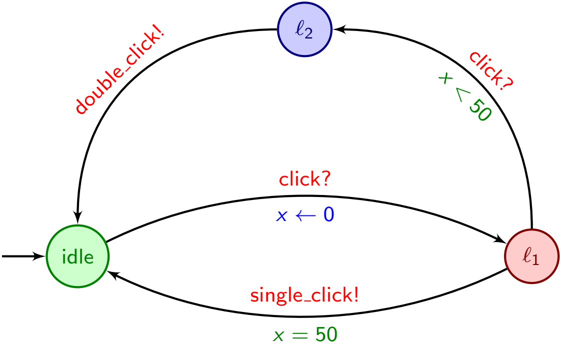

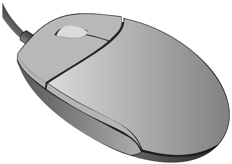

Runs of a timed automaton

$(\text{idle}, x = 0) \xrightarrow{23.7} (\text{idle}, x = 23.7) \xrightarrow{\textit{click?}} (\ell_1, x = 0) \xrightarrow{10} (\ell_1, x = 10)$
$\xrightarrow{\textit{click?}} (\ell_2, x = 10) \xrightarrow{\textit{double\_click!}} (\text{idle}, x = 10) \cdots$

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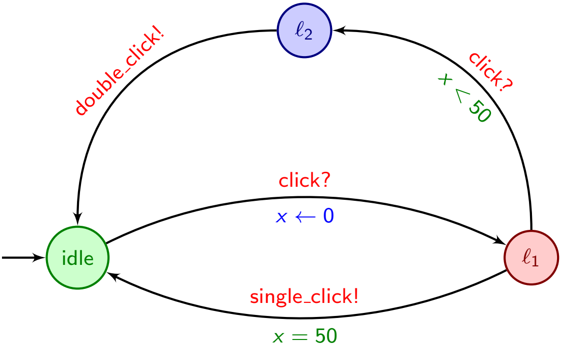

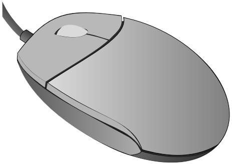

The **untimed language** of a timed automaton

Sequences of edge labels along which there is a run.

For instance $(\text{click?} \cdot \text{single\_click!})^* \subseteq L(\mathcal{A})$.

Known Results about TA

- Emptiness is PSPACE-complete; $L(\mathcal{A}) = \varnothing$? $L^t(\mathcal{A}) = \varnothing$?

- Timed language universality is undecidable;
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- Untimed language universality and inclusion are PSPACE-hard and in EXPSPACE. $L(\mathcal{A}) = \Sigma^*$? $L(\mathcal{A}) = L(\mathcal{B})$? $L(\mathcal{A}) \subseteq L(\mathcal{B})$?

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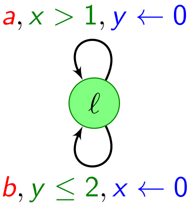
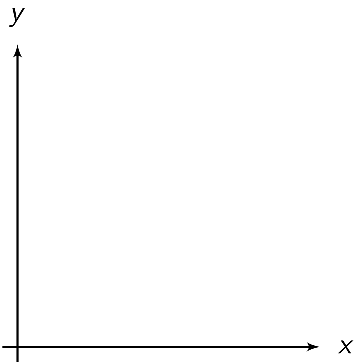
  $L(\mathcal{A}) = \Sigma^*$? $L(\mathcal{A}) = L(\mathcal{B})$? $L(\mathcal{A}) \subseteq L(\mathcal{B})$?
  - What is the exact complexity of these problems?

The Region Abstraction





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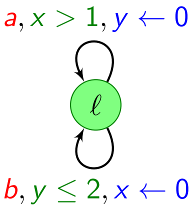

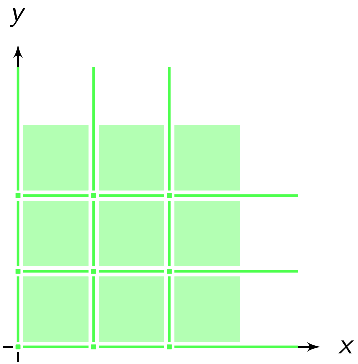

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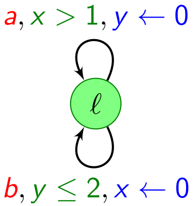

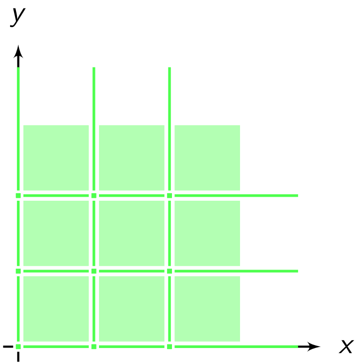

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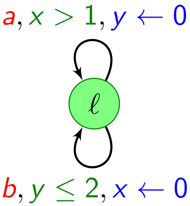

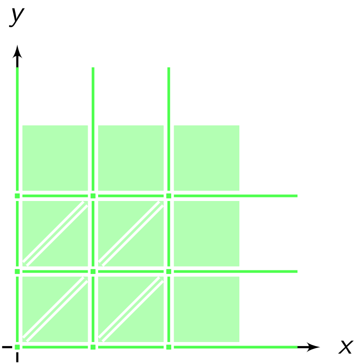

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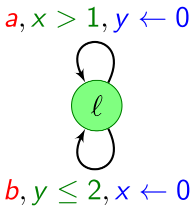

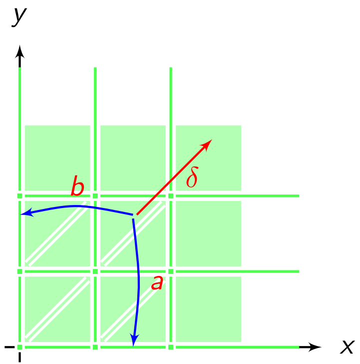

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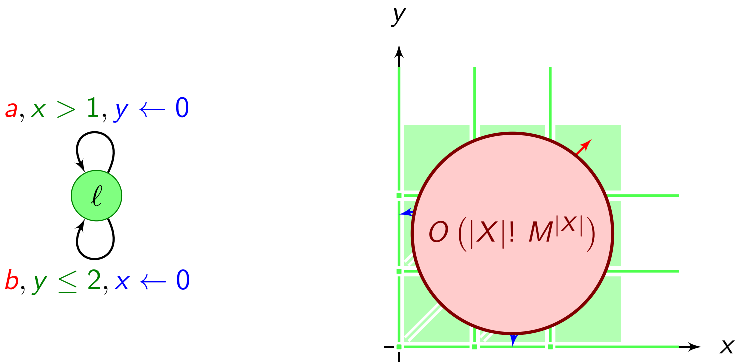

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Theorem

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Algorithm

Given timed automata $\mathcal{A}$ and $\mathcal{B}$

- construct the corresponding region automata, $R(\mathcal{A})$, $R(\mathcal{B})$;
- use a PSPACE algorithm to check language inclusion on the region automata, $R(\mathcal{A}) \subseteq R(\mathcal{B})$.

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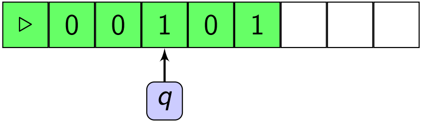
$\Rightarrow$ The automaton is **universal** if and only if the input x is **not accepted** by the Turing machine.

A configuration



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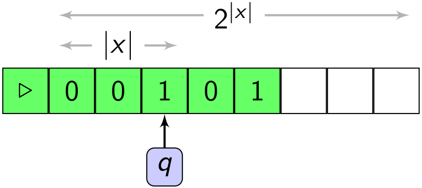

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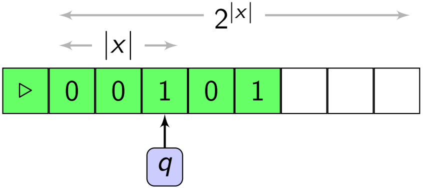

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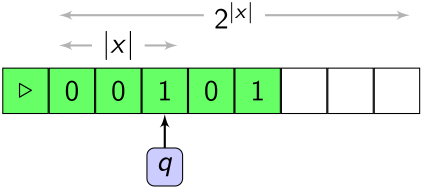

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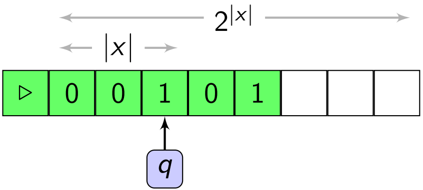

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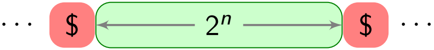
The alphabet of the timed automaton is $\Gamma = \Sigma \cup Q \cup \{\$\}$

Detecting Errors with an Automaton



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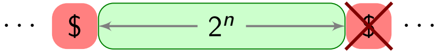

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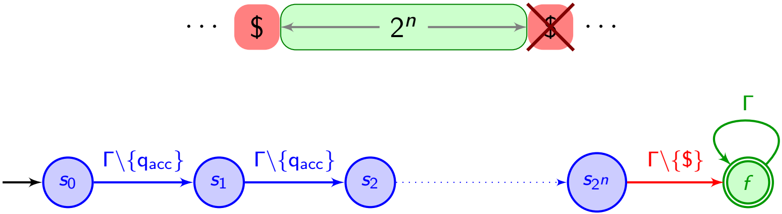

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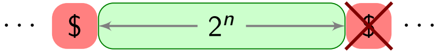

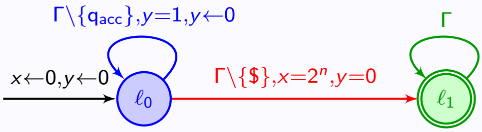

Module $D(\$, 2^n)$

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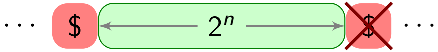

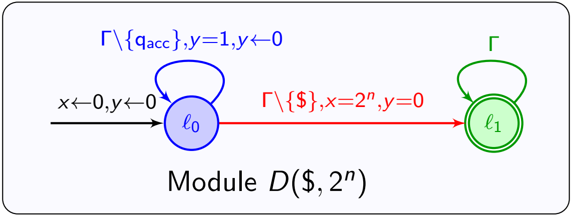

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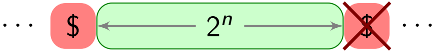

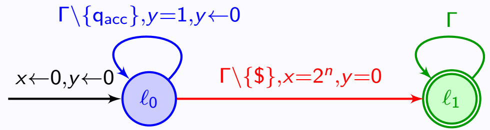

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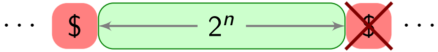

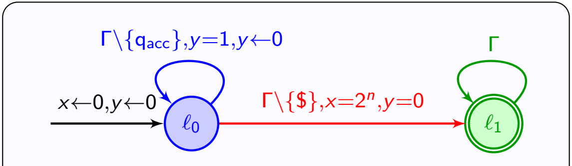

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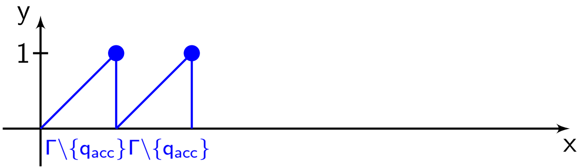

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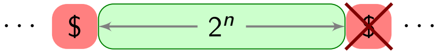

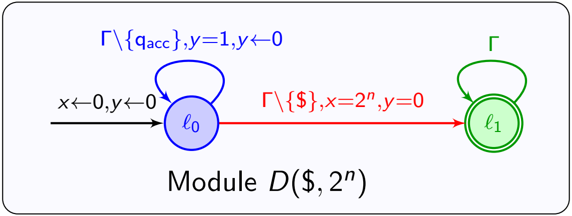

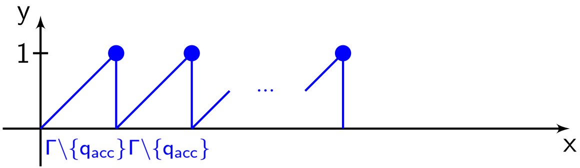

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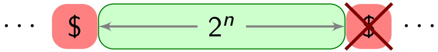

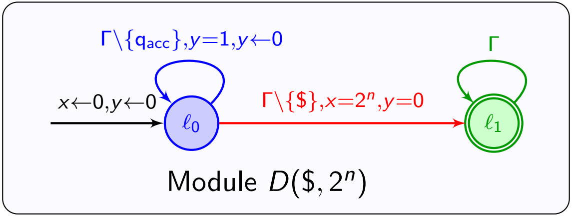

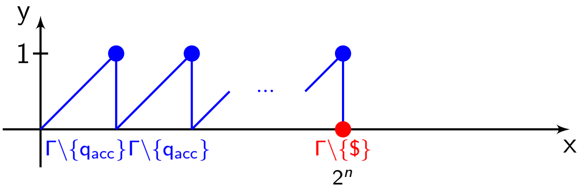

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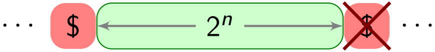

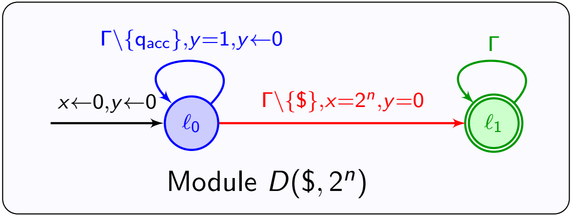

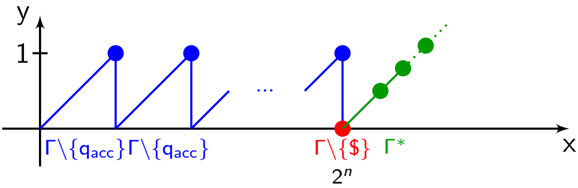

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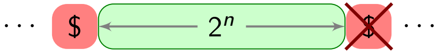
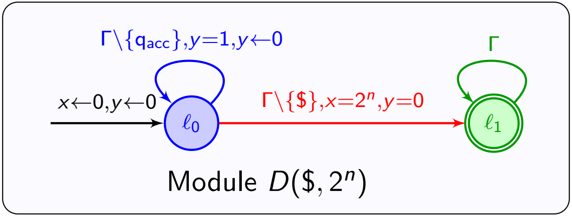
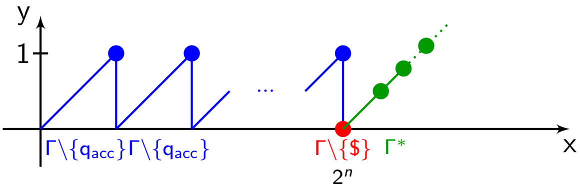
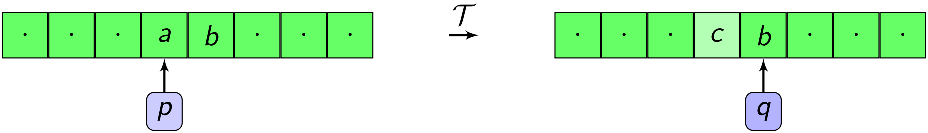






$$L(D(a,m)) = (\Gamma \setminus \{q_{acc}\})^m \cdot (\Gamma \setminus \{a\}) \cdot \Gamma^*$$

Encoding an Instruction



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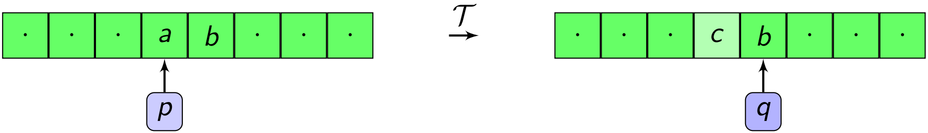

$$\cdots \quad a \quad p \quad b \quad \cdots \quad \$ \quad \cdots \quad c \quad b \quad q \quad \cdots$$

$$\longleftarrow 2^n - 1 \longrightarrow$$

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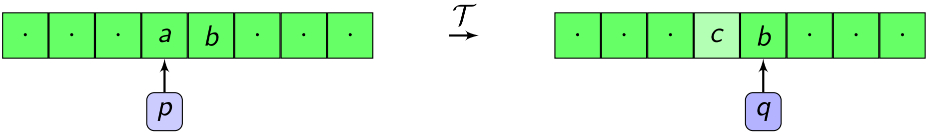

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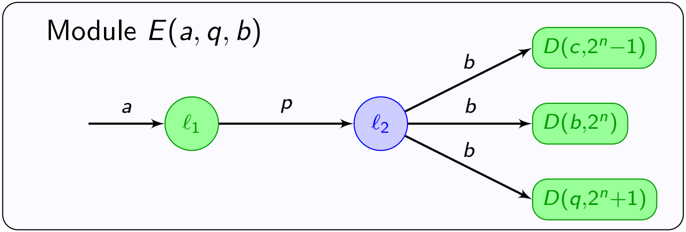
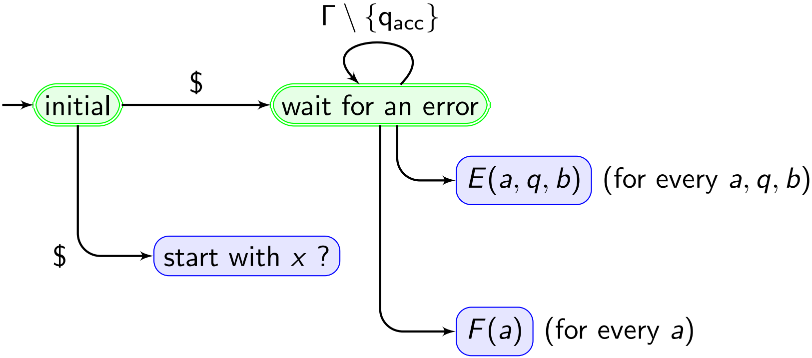


The Global Construction



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Thank you for your attention