

ALONZO CHURCH. *Logic, arithmetic, and automata. Proceedings of the International Congress of Mathematicians, 15-22 August 1962*, Institut Mittag-Leffler, Djursholm, Sweden, 1963, pp. 23-35.

This is a rather complete summary of the main results obtained in the theory of finite automata, during the first decade of its existence. It will be useful to both the outsider (logician and algebraist on the one side and practitioner on the other) who desires a survey over a small but active field related to his principal interest, and the beginning student of finite automata. There does not seem to be much sense in trying to condense further what is already a well organized outline. Instead the reviewer would like to add the following comments.

*History*: The author presents a careful analysis of various ideas and suggestions, which may be viewed as forerunners of Kleene's definition of finite automata, the logical nets of Burks and Wright, and his own concept of restricted recursion. In a more superficial sense, the people mentioned in this connection, certainly deserve some credit for pioneering the study of finite automata. However, it seems more significant to stress various ideas and results of Skolem's, dating back to the twenties. The following are strikingly relevant to the logical study of automata: recursive arithmetic, replacing bounded number-quantifiers by recursions, Skolem-Behmann method for handling monadic predicate variables and quantifiers, Skolem-Presburger decision method.

*Algebra*: More recent presentations of Kleene's work on regular events show the usefulness of basic ideas of algebra for the understanding of finite automata.

*Sequential calculus* (S.C.): This is the restricted (to monadic predicate variables) second-order language of the successor function on natural numbers. S.C. occurs as a particularly natural formalism for expressing synthesis requirements on finite automata. To find a Synthesis Algorithm for S.C. seems one of the more important tasks in the author's program for work on design-algorithms; his synthesis method for a small fragment of S.C. and the reviewer's decision method for S.C. are the most basic results now available in this field.

*Corrections*: Page 31, in (3), for "*singularly predicate meaning 'is power of 2'.*" read "*binary predicate meaning 'x is power of 2 and occurs in the binary expansion of y'.*" (this error occurs in the original paper mentioned and was discovered by McNaughton); page 32, line 7 from below, for "the symbols =, <, 0, and ',' read "the symbols = and <" (noted by the author).

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A. W. BURKS and J. B. WRIGHT. *Sequence generators and digital computers. Recursive function theory*, Proceedings of symposia in pure mathematics, vol. 5, American Mathematical Society, Providence 1962, pp. 139-199.

ARTHUR W. BURKS and JESSE B. WRIGHT. *Sequence generators, graphs, and formal languages. Information and control*, vol. 5 (1962), pp. 204-212.

There is a need for a single concept to apply to many devices and activities in which there is a finite number of possible configurations or states of affairs, with a limited set of transitions from one configuration to another. The applications of such a concept (as pointed out on page 146 of the first article) would include many games and puzzles, digital computer circuits and other gadgets (from a certain aspect), certain portions of mathematical linguistics, and finite directed graphs; also, applications to the study of certain systems of symbolic logic would be forthcoming. The most significant application, however, would be in the theory of automata, where the concept of finite automaton, or sequential circuit, is already well developed. All of the conceptual development in both papers is motivated by applications to the theory of automata.

These papers present such a concept under the name "sequence generator." A