symbols after "1°" and replace them by a horizontal line with "$\alpha = \alpha'$" above the line and "$\psi \equiv \psi'$" below the line; p. 566, line 8, replace "ψ" by "$\psi(\alpha)$".

STEVEN OREY

GAISI TAKEUTI. *An example on the fundamental conjecture of GLC.* **Journal of the Mathematical Society of Japan,** vol. 12 (1960), pp. 238–242.

The author's *GLC* is a system of sequents (in the manner of Gentzen) formalizing simple type theory. G^iLC is the restriction of *GLC* to formulas of type i. For some sub-systems of *GLC*, the author has proved in several previous papers his fundamental conjecture that every provable sequent can be proved without using Gentzen's cut rule. In this paper he proves the following two theorems: (1) If a sequent $\Gamma \to \Delta$ is provable in $G^{i+1}LC$, then $e(a)$, $\Gamma \to \Delta$ is provable without cut in $G^{i+1}LC$, where $e(a)$ is the formula which means that the term a satisfies the principle of mathematical induction. (2) If A and B are formulas, one of which is a prenex normal form of another, and $\Gamma \to \Delta$, A is provable without cut, then $\Gamma \to \Delta$, B is provable without cut in $G^{i+1}LC$. From these two theorems it follows that the fundamental conjecture holds for examples which were given by K. Gödel and G. Kreisel as possible counter-examples to the fundamental conjecture.

KURT SCHÜTTE

HAO WANG. *Eighty years of foundational studies.* **Dialectica,** vol. 12 (1958), pp. 466–497; also **Logica, Studia Paul Bernays dedicata,** Bibliothèque scientifique no. 34, Éditions du Griffon, Neuchâtel 1959, pp. 262–293.

The author presents reflections on some philosophical problems developed during the last eighty years of foundational studies. His main concern is with various degrees of constructivity.

Five shades of constructivism are distinguished. At one extreme is *anthropologism.* This viewpoint draws attention to what we can actually calculate, e.g. we can not actually write down the decimal expansion of $67^{257^{729}}$. A more important position is that of *finitism.* In his discussion the author includes attempts by Skolem and Kreisel to delineate this position formally. Next in the spectrum is *intuitionism*; this school of thought is discussed briefly, with emphasis on the possibilities for formalization. There follows a discussion of *predicativism,* including remarks on predicative set theory. Finally, at the opposite extreme from anthropologism the author finds *Platonism.*

The discussion is informal and rambling but stimulating. More detailed references to the literature would have been useful.

STEVEN OREY

A. NERODE. *Linear automaton transformations.* **Proceedings of the American Mathematical Society,** vol. 9 (1958), pp. 541–544.

Let both the input and output alphabet of a finite automaton be the elements of a finite ring with unit. (For example, 0 and 1, with addition mod 2 providing the ring sum and conventional multiplication providing the product.) If the automaton has a single input and single output then it transforms the class of infinite input sequences into the class of infinite output sequences. This transformation is linear if the following condition is satisfied for every pair of ring elements r and r': if (for each n) u_n, u_n', w_n are, respectively, the n^{th} terms of the output sequences that result, respectively, from the input sequences whose n^{th} terms are, respectively, i_n, i_n', $ri_n + r'i_n'$ then $w_n = ru_n + r'u_n'$.

The author gives two mathematical characterizations of such transformations. The first (Theorem 1 as restated by the reviewer) is that $u_n = a_{n0}i_0 + a_{n1}i_1 + \dots + a_{n(n-1)}i_{n-1}$, where, for some p and q and for all $n > q$, $a_{nj} = a_{(n+p)j} = a_{(n+p)(j+p)}$. In other words, the output at time n equals a linear polynomial in the inputs at time 0 through time $n - 1$, where, for a constant j, the coefficient of the input at time j

for the output at time n is ultimately periodic in n; and where, for a constant k $(= n - j)$, the coefficient of the input at time $n - k$ for the output at time n is ultimately periodic in n. (My a_{nj} is his $W_{nj} = u_{(n-j)j}$.)

The second characterization (Theorem 2) is that, for some k, the last k terms of the output and input sequences to time n are connected as in the linear difference equation $S_1(n-1)i_{n-1} + \ldots + S_k(n-k)i_{n-k} = u_n + T_1(n-1)u_{n-1} + \ldots + T_k(n-k)u_{n-k}$, where the functions $S_1, \ldots, S_k, T_1, \ldots, T_k$ are ultimately periodic and are 0 for negative arguments. Here u_{x+1} and i_x are 0 for x negative.

To the reviewer's knowledge, the transformations realized by all finite automata have not been characterized in any such mathematical manner. Finite linear automata have received considerable attention in the switching-theory literature since this paper appeared. It appears that the main reason for attention is their mathematical tractability, rather than any special utility as machines. ROBERT MCNAUGHTON

GR. C. MOISIL. *Rapport sur le développement dans la R.P.R. de la théorie algébrique des mécanismes automatiques.* **Analele Universității C. I. Parhon, seria Acta logica,** vol. 2 no. 1 (1959), pp. 145–199.

This survey gives a historical sketch of the Rumanian work on the application of algebra and logic to switching circuits. It is similar in scope to the paper reviewed in XXVIII 104, but it is to be preferred to it since it gives fuller explanations, has a few more recent bibliographic references, and is more accessible by being in French.

EDWARD F. MOORE

ERIC FOXLEY. *The determination of all Sheffer functions in 3-valued logic, using a logical computer.* **Notre Dame journal of formal logic,** vol. 3 (1962), pp. 41–50.

In XXI 199, Norman Martin gives four properties which a binary function must fail to have in order to satisfy what he calls "property P." He then shows that property P provides a necessary and sufficient condition for a binary function to be a three-valued Sheffer function.

The present paper describes how Martin's results were used to calculate the set of three-valued Sheffer functions on the Nottingham University Logical Computer, and indicates how the computer demonstrated that Martin's condition of co-substitution is superfluous.

To solve this problem of three-valued logic on a a two-state computer, the truth-values 1, 2, 3 were represented by the ordered assignments TT, TF, FT, FF with the ordered pair FT taken as meaningless. Thus, Foxley's method is related to but different from that given in Alan Rose's XXVII 250.

The complications encountered as a result of doing three-valued logic on a two-state computer suggests to this reviewer that there might be a distinct advantage in using an M-state computer to deal efficiently with problems in M-valued logic.

ATWELL R. TURQUETTE

ANTÓNIO MONTEIRO. *Matrices de Morgan caractéristiques pour le calcul propositionnel classique.* **Anais da Academia Brasileira de Ciências,** vol. 32 (1960), pp. 1–7.

A matrix is *characteristic for classical propositional logic* (ccpl) if each classical tautology assumes only designated values for any assignment of matrix elements to its variables, while no non-tautology has this property. In XIX 233(4) Church describes a class of ccpl matrices which are *irregular*, in the sense that there are designated elements x and $x \rightarrow y$ such that y is non-designated, and he raises the question what other such matrices exist. Here Monteiro describes an interesting class of ccpl matrices which properly contains Church's class.

A distributive lattice with an operation - satisfying the identities $--x = x$ and