On the decidability of priced timed games

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HIGHLIGHTS of Logic, Games and Automata – Paris

Outline of the talk

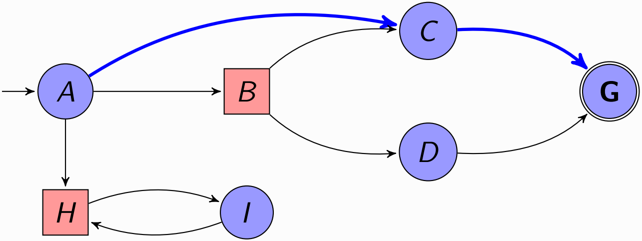
1. Priced Timed Games and Optimal Strategies

2. Existing results

3. Going further...

Classical game (qualitative, zero-sum, turn-based)

A game $\mathcal{G} = ((V, E), V_0, V_1, \mathbf{G})$. Pl. 0 aims at reaching $\mathbf{G}$.



A winning strategy for Pl. 0 from A

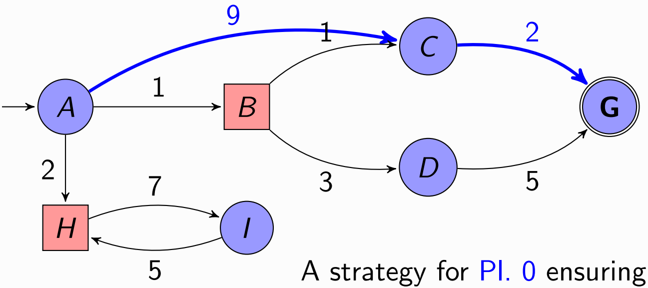
Winning strategy for Pl. 0

A strategy $\lambda_0 : V^* V_0 \to V$ is winning for Pl. 0 from v iff

$$\forall \lambda_1 : V^* V_1 \to V \quad \mathrm{Out}(v, \lambda_0, \lambda_1) \text{ visits } \mathbf{G}.$$

Priced Game

Pl. 0 aims at reaching **G** while minimising the cost.



A strategy for Pl. 0 ensuring 11 from A

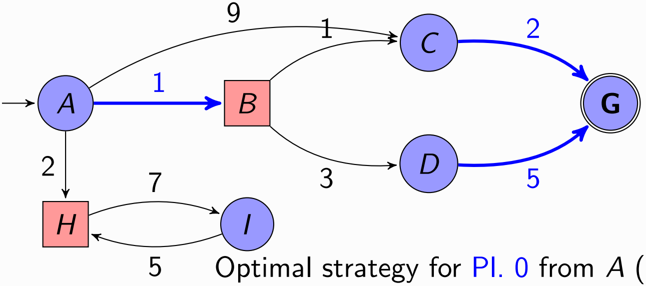
Strategy for Pl. 0 ensuring a cost K

A strategy λ_0 ensures a cost K for Pl. 0 from v iff

$$\sup_{\lambda_1} \mathrm{Cost}(\mathrm{Out}(v, \lambda_0, \lambda_1)) \leqslant K.$$

Priced game

Pl. 0 aims at reaching **G** while minimising the cost.



Optimal strategy for Pl. 0 from A (ensuring 9)

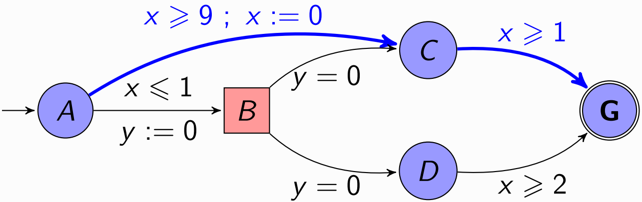
Optimal Strategy for Pl. 0

A strategy λ_0^* is optimal for Pl. 0 from v iff

$$\sup_{\lambda_1} \mathrm{Cost}(\mathrm{Out}(v, \lambda_0^*, \lambda_1)) = \inf_{\lambda_0} \sup_{\lambda_1} \mathrm{Cost}(\mathrm{Out}(v, \lambda_0, \lambda_1)).$$

Timed Game

Pl. 0 aims at reaching **G** while minimising the time.



A strategy for Pl. 0 ensuring 10 t.u. from $(A, 0, 0)$

Strategy for Pl. 0 ensuring a time T

A strategy λ_0 ensures a time T for Pl. 0 from $(\ell, 0)$ iff

$$\sup_{\lambda_1} \mathrm{Time}(\mathrm{Out}((\ell, 0), \lambda_0, \lambda_1)) \leqslant T.$$

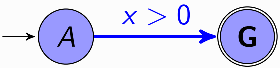
About optimal strategies in Timed Game

Optimal Strategy for Pl. 0

A strategy λ_0^* is optimal for Pl. 0 from $(\ell, 0)$ iff

$$\sup_{\lambda_1} \mathsf{Time}(\mathsf{Out}((\ell,0),\lambda_0^*,\lambda_1)) = \inf_{\lambda_0}\sup_{\lambda_1} \mathsf{Time}(\mathsf{Out}((\ell,0),\lambda_0,\lambda_1)).$$

Optimal strategies do not always exist in timed game ! ! !



Pl. 0 can choose to wait any $t > 0$ before reaching **G**.

$$\inf_{\lambda_0}\sup_{\lambda_1} \mathsf{Time}(\mathsf{Out}((A,0),\lambda_0,\lambda_1)) = \inf_{t>0} t = 0.$$

However, there is no strategy λ_0^* such that

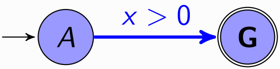
$$\sup_{\lambda_1} \mathsf{Time}(\mathsf{Out}((A,0),\lambda_0^*,\lambda_1)) = 0.$$

About optimal strategies in Timed Game (continued)

ϵ-optimal Strategy for Pl. 0

Given $\epsilon > 0$, a strategy λ_0^* is ϵ-optimal for Pl. 0 from $(\ell, 0)$ iff

$$\sup_{\lambda_1} \mathsf{Time}(\mathsf{Out}((\ell,0),\lambda_0^*,\lambda_1)) \leqslant \inf_{\lambda_0} \sup_{\lambda_1} \mathsf{Time}(\mathsf{Out}((\ell,0),\lambda_0,\lambda_1)) + \epsilon.$$



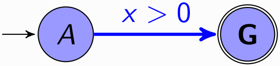
There is no optimal strategy for Pl. 0 from $(A, 0)$.
But for all $\epsilon > 0$, there is an ϵ-optimal strategy for Pl. 0 from $(A, 0)$.

About optimal strategies in Timed Game (continued)

ϵ-optimal Strategy for Pl. 0

Given $\epsilon > 0$, a strategy λ_0^* is ϵ-optimal for Pl. 0 from $(\ell, 0)$ iff

$$\sup_{\lambda_1} \mathsf{Time}(\mathsf{Out}((\ell,0), \lambda_0^*, \lambda_1)) \leqslant \inf_{\lambda_0} \sup_{\lambda_1} \mathsf{Time}(\mathsf{Out}((\ell,0), \lambda_0, \lambda_1)) + \epsilon.$$



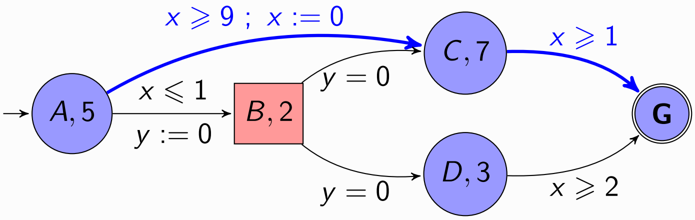
There is no optimal strategy for Pl. 0 from $(A, 0)$.
But for all $\epsilon > 0$, there is an ϵ-optimal strategy for Pl. 0 from $(A, 0)$.

Remark

The classical *region* of timed automata is a right tool to solve timed game.

Priced Timed Game

Pl. 0 aims at reaching **G** while minimising the cost.



A strategy for Pl. 0 ensuring 52 from $(A, 0, 0)$

Strategy for Pl. 0 ensuring a cost K

A strategy λ_0 ensures a cost K for Pl. 0 from $(\ell, 0)$ iff

$$\sup_{\lambda_1} \mathrm{Cost}(\mathrm{Out}((\ell, 0), \lambda_0, \lambda_1)) \leqslant K.$$

About optimal strategies in Priced Timed Game

Remark

Clearly optimal strategies do not always exists in Priced Timed Game

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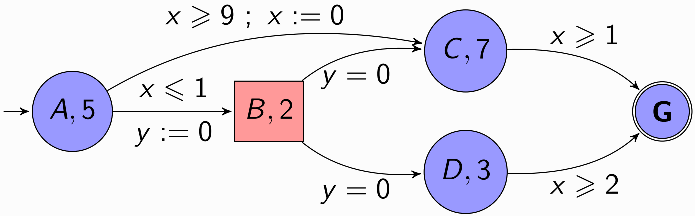
Clearly optimal strategies do not always exists in Priced Timed Game

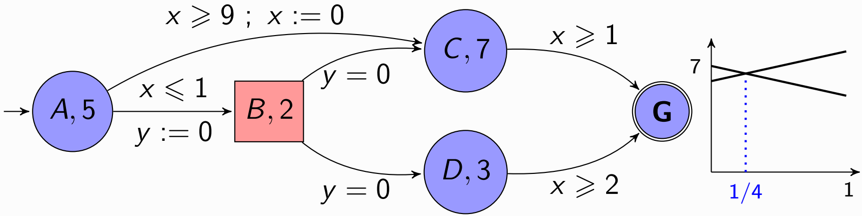


$$\inf_{\lambda_0} \sup_{\lambda_1} \mathrm{Cost}(\mathrm{Out}((A,0), \lambda_0, \lambda_1)) = \inf_{0 \leqslant t \leqslant 1} \max\{5t + 7(1-t), 5t + 3(2-t)\}$$

About optimal strategies in Priced Timed Game

Remark

Clearly optimal strategies do not always exists in Priced Timed Game



$$\inf_{\lambda_0} \sup_{\lambda_1} \mathrm{Cost}(\mathrm{Out}((A,0), \lambda_0, \lambda_1)) = \inf_{0 \leqslant t \leqslant 1} \max\{5t + 7(1-t), 5t + 3(2-t)\}$$

$$= \inf_{0 \leqslant t \leqslant 1} \max\{-2t + 7, 2t + 6\} = 6.5$$

The optimal strategy for Pl. 0 asks to take the transition after $\frac{1}{4}$ t.u.

Outline of the talk

1. Priced Timed Games and Optimal Strategies
2. Existing results
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The K-bounded problem

The K-bounded Problem

Given $\mathcal{A}$ a PTG and $K \in \mathbb{N}$, decide whether there exists λ_0^* such that

$$\sup_{\lambda_1} \mathsf{Cost}(\mathsf{Out}((\ell_0, 0), \lambda_0^*, \lambda_1)) \leqslant K.$$

Decidability results

The K-bounded problem is **decidable** on

- **Timed Games**

  E. Asarin and O. Maler. As soon as possible : Time optimal control for timed automata. 1999

- **Priced Timed Games** under strong non-Zenoness of the cost

  R. Alur, M. Bernadsky, and P. Madhusudan. Optimal reachability for weighted timed games. 2004

  P. Bouyer, F. Cassez, E. Fleury, and K. G. Larsen. Optimal strategies in priced timed game automata. 2004

- **Priced Timed Games** with one clock

  P. Bouyer, K. Larsen, N. Markey, and J. Rasmussen. Almost optimal strategies in one clock priced timed games. 2006

The value $\inf_{\lambda_0} \sup_{\lambda_1} \mathrm{Cost}(\mathrm{Out}((\ell_0, 0), \lambda_0, \lambda_1))$ can be computed.

Undecidability results

The K-bounded Problem is **undecidable** on

- **Priced Timed Games** with 6 clocks and non-negative prices.

  T. Brihaye, V. Bruyère, J.-F. Raskin. On optimal timed strategies. 2005.

- **Priced Timed Games** with 3 clocks and non-negative prices.

  P. Bouyer, T. Brihaye, N. Markey. Improved Undecidability Results on Weighted Timed Automata. 2006

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Questions still open...

- What about the variants of the K-bounded Problem ?

The K-bounded Problem (strict version)

Given $\mathcal{A}$ a PTG and $K \in \mathbb{N}$, decide whether there exists λ_0^* such that

$$\sup_{\lambda_1} \mathrm{Cost}(\mathrm{Out}((\ell_0, 0), \lambda_0^*, \lambda_1)) < K.$$

The K-bounded Problem (ϵ-version)

Given $\mathcal{A}$ a PTG and $K \in \mathbb{N}$, decide whether for all $\epsilon > 0$, there exists λ_0^ϵ such that

$$\sup_{\lambda_1} \mathrm{Cost}(\mathrm{Out}((\ell_0, 0), \lambda_0^*, \lambda_1)) \leqslant K + \epsilon.$$

- What happens if we consider concurrent games, positive costs,... ?

The time-bounded framework

Undecidable problems become **decidable** when considering their **time-bounded** version :

- Time-bounded language inclusion for Timed Automata

  J. Ouaknine, A. Rabinovich, and J. Worrell. Time-bounded verification. 2009

- Reachability for Hybrid Automata

  T. Brihaye, L. Doyen, G. Geeraerts, J. Ouaknine, J.-F. Raskin, J. Worrell : On Reachability for Hybrid Automata over Bounded Time. 2011

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Would this work for the K-bounded Problem on PTG ?

New (undecidability) results

- The **time-bounded**, K-bounded Pbm is **undecidable** on **PTG**.
- The **strict version of** the K-bounded Pbm is **undecidable** on **PTG**.
- The ϵ-**version of** the K-bounded Pbm is **undecidable** on **PTG**.
- The K-bounded Pbm is **undecidable** on **concurrent PTG with 2 clocks**.

New (undecidability) results

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- The **strict version of** the K-bounded Pbm is **undecidable** on **PTG**.
- The ϵ**-version of** the K-bounded Pbm is **undecidable** on **PTG**.
- The K-bounded Pbm is **undecidable** on **concurrent PTG with 2 clocks**.

We hope to obtain :

- a precise characterisation of the decidability border,
- a new **decidability result** (with positive cost, few clocks, and ? ? ?)

Thank you ! ! !