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MORDCHAJ WAJSBERG. LIVE AND WORKS

Mordchaj Wajsberg is one of the most prominent representatives of the Polish school of mathematical logic of the mid-war period. His works concerning propositional calculus contain many results which evidently influenced further development of logic. Particularly Wajsberg:
(1) was a opneer in axiomatization of Łukasiewiczian logics and, more generally, of many valued logics,
(2) was the first to give adequate semantic characterization of Lewis' system $S5$, beginning the same research on algebraic semantic of modal logics,
(3) he worked out an original syntactic method for separable axiomatization of propositional calculus and was the first to prove the theorem about separable axiomatization of the intuitionistic logic,
(4) gave the criterion of axiomatization for a broad class of finitely many-valued logics including finite Łukasiewiczian logics, finite intermediate logics and others.

Mordchaj Wajsberg was born on May 10th 1902 in Łomża, Białostocki district. From 1909 to 1912 he was at the primary school in Łomża and next began studying in the grammar school but after two years had to stop because of the hostilities during the 1st World War. In 1919 he started preparations for the final external examinations. In 1920 his preparations were interrupted by the year of military service in the Polish Army. In 1921 he passed the entrance exam to the seventh form of the eight-form coeducational gymnasium owned by Szymon Goldlust in Łomża from which he graduated in June 1923.

In October 1923 he began mathematical studies at the Department of Philosophy of the University of Warsaw. As a specialization he chose mathematical logic studying under the direction of Professor Jan Łukasiewicz. Apart from Łukasiewicz's lectures he listened also to the lectures on logic

given by Professors StanisŁaw Leśniewski and Tadeusz Kotarbiński. On the second year of studies he gave two reports for the Section of the Philosophy of Mathematics (which worked in Philosophical Circle and in Math-Physics Circle of Warsaw University Students) on "B. Russel's theory of functions of apparent variable" and "In invariants of logistic transformations". On the third year Wajsberg obtained some original results, namely, he gave some new systems of axioms of axioms for the implicational propositional calculus and for the equivalential propositional calculus and among them one single implicational axiom built of 25 symbols, and two single equivalential axioms built of 15 symbols each (as it was later pointed out by Łukasiewicz the shortest single implicational axiom has 13 symbols and the shortest single equivalential axiom has 11 symbols). On the fourth year Wajsberg formulated a single axiom for Sheffer function which in contrary to Nicod's axiom was an organic formula. These results were collected in his graduate work entitled "Contributions to the research on mathematical logic" written under the guidance of Professor Łukasiewicz and defended with the best grade on the 2nd of October 1928. In 1927 Wajsberg started correspondence with C. I. Lewis. According to C. I. Lewis' and W. T. Parry's evidence in his letters from that period Wajsberg, among other things:
(1) proved that no Lewis' system is equivalent with the classical propositional calculus,
(2) outlined adequate semantic characterization of Lewis' system $S5$, and
(3) proved that if x is a classical tautology, then $NMNx$ (what is read: necessary that possible that necessary that x) is a theorem of Lewis' system $S1$.

From August 1929 till September 1930 Wajsberg served in the army – first at the officer cadet training unit and then in the 4th Regiment of the Tatra Highlands Gunners.

In September 1930 he enlisted for the Ph.D. studies in the University of Warsaw. His Ph.D. work entitled "Axiomatization of the three-valued propositional calculus" he prepared under the direction of Professor Łukasiewicz. He gave in it independent axiomatics for the three-valued implicational-negational Łukasiewiczian logic: $CpCqp$, $CCpqCCqrCpr$, $CCNpNqCqp$, $CCCpNppp$ and proved the theorem about non-axiomatizability of any sub-system of classical logic with help of axioms built at most of 2 different variables. These Wajsberg's results were presented by Łukasiewicz for the Warsaw Scientific Society on the 19th of February 1931 and were published in work [1]. His Ph.D. degree Wajsberg received during the festive promotion on May 29th 1931.

In that work Wajsberg did not insert all results concerning Łukasiewiczian logics. It is known that in that time he proved axiomatizability of all n-valued Łukasiewiczian logics, where $(n-1)$ is prime (for all natural n it was proved by A. Lindenbaum), and also confirmed the well-known Łukasiewicz's hypothesis about axiomatizability of infinite Łukasiewiczian logic with the help of axioms: $CpCqp$, $CCpqCCqrCpr$, $CCCpqqCCqpp$, $CCCpqCqpCqp$, $CCNpNqCqp$. This last result Wajsberg announced later in work [8] on page 240 but never published. The first published proof of Łukasiewicz's hypothesis gave A. Rose and J. B. Rosser only in 1958 in the work "Fragments of many-valued statement calculi". Let us notice that in the same year C. A. Meredith and C. C. Chang published independently that axiom $CCCpqCqpCqp$ is dependent.

1932 Wajsberg spent in Warsaw. In February and March 1932 during the sitting of the Section of Logic of the Warsaw Philosophical Society he delivered two reports: "From research on theory of deduction" and "Axiomatization of predicate calculus". During his second stay in Warsaw Wajsberg prepared for print, apart from his doctor's work [1], works [2] and [3]. In work [2] he proved the theorem according to which the set of formulas X, consisting of such implicational-negational tautologies in which negation may appear only with propositional variables, after adjusting it to the axioms for implication, axiomatises the implicational-negational logic if and only if every unary Boolean function different from negation does not satisfy at least one formula from X. In work [3] Wajsberg published this organic axiom for Sheffer function that he found as a student and inserted in his unpublished graduate work.

From Warsaw Wajsberg moved to Kowl in Volhynia, where he stayed until the end of June 1933 making his living probably of teacher's job. In that period were created his works [4], [5] and [6]. Work [4] concerns the first order predicate calculus. Wajsberg proved there the theorem according to which from an arbitrary formula true exclusively in k-element models results any formula true in k-element models. In work [5] he carried out an adequate semantic characterization of Lewis' system $S5$. It is the first example of an adequate semantics in the history of modal logic. Work [6], as [4], concerns also the first order predicate calculus and includes a detailed proof of the theorem according to which the degree of completeness (that is the number of super-systems) of the first order predicate calculus equals continuum.

From Kowl Wajsberg moved back to Łomża where he exercised teaching and where his remaining works [7], [8], [9], [10], and [11] were created. Work [7] includes one of the more important Wajsberg's results and concerns the conditions of axiomatizability of finite logical matrices including finite Łukasiewiczian matrices, finite intermediate logics and others. According to this theorem if the formulas: $CCpqCCqrCpr$, $CCqrCCpqCpr$, $CCpqCNqNp$, $CNqCCpqNp$, $CCqqCpp$ are satisfied in any finite logical matrix, then that matrix is axiomatizable. Work [7] contains also some other interesting results. In work [7] contains also some other interesting results. In work [8] Wajsberg made an attempt to classify logical matrices into types and described the methods for deciding about particular formulas in what matrices of a given type those formulas are satisfied. In work [9] Wajsberg established separable axiomatizability of the intuitionistic propositional calculus and also gave many interesting results concerning definability of propositional functors. In works [10] and [11] Wajsberg inserted many smaller information on different axiom systems of the classical propositional calculus and its different fragments. Some of them belonged to his unpublished graduate's work. Works [10] and [11] include also some other results as, among others, schematic description of, so called today, Wajsberg's method of proof the completeness for the propositional calculus with implication as the only primitive term.

Since the date of the IInd Word War outbreak we are short of reliable information concerning Wajsberg. What we know for sure is the only fact that he perished during the war.

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