A machine-independent characterization of timed languages

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University of Warsaw

joint work with Mikołaj Bojańczyk

HIGHLIGHTS 2013

deterministic timed automata

with uninitialized clocks

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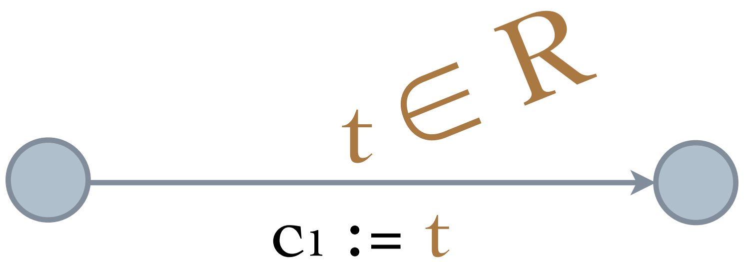

deterministic timed automata

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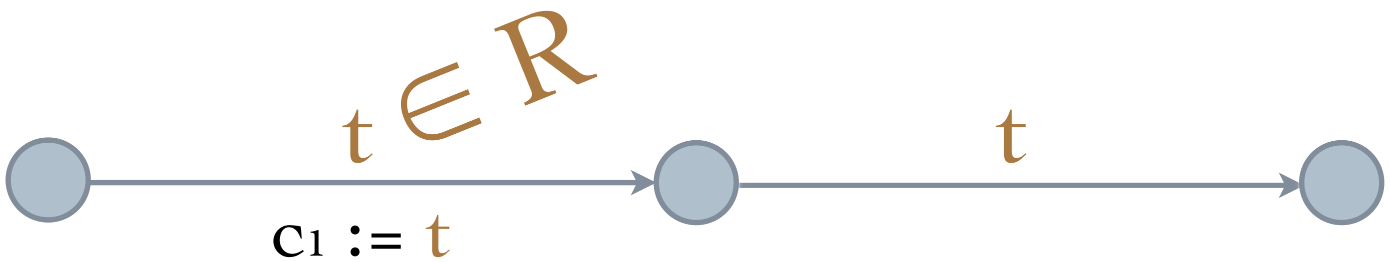

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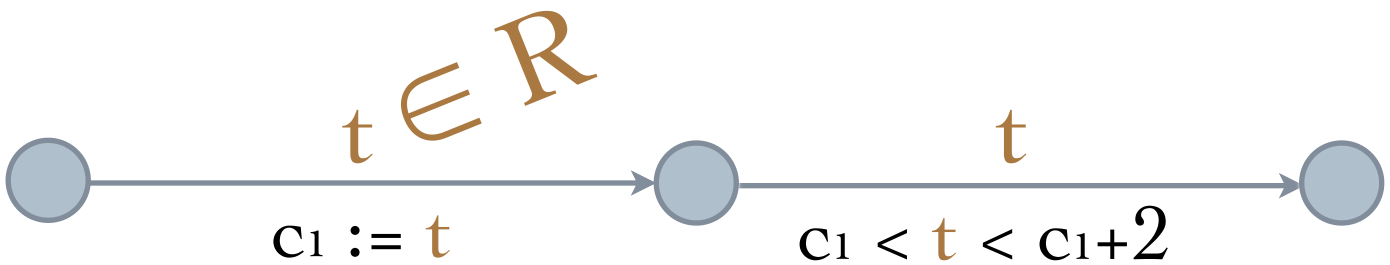

the guards use the structure $(\mathbb{R}, <, +1)$

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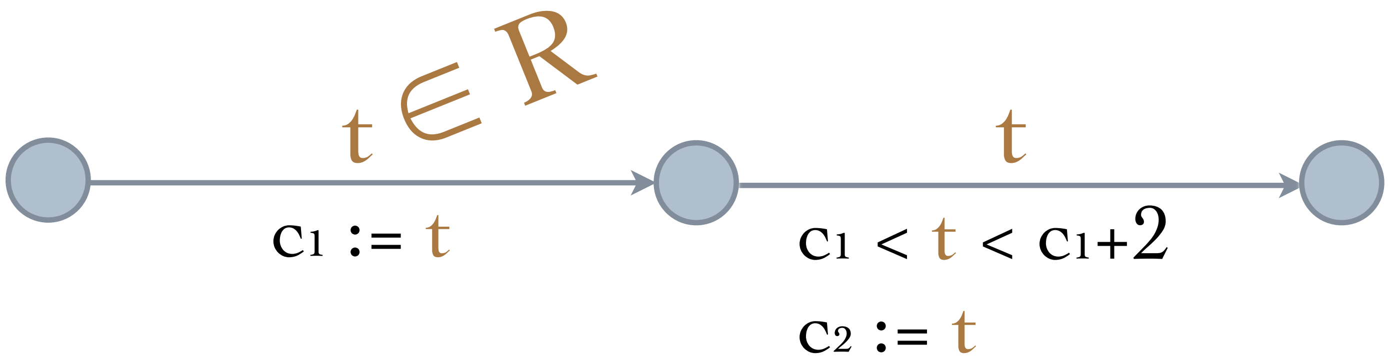

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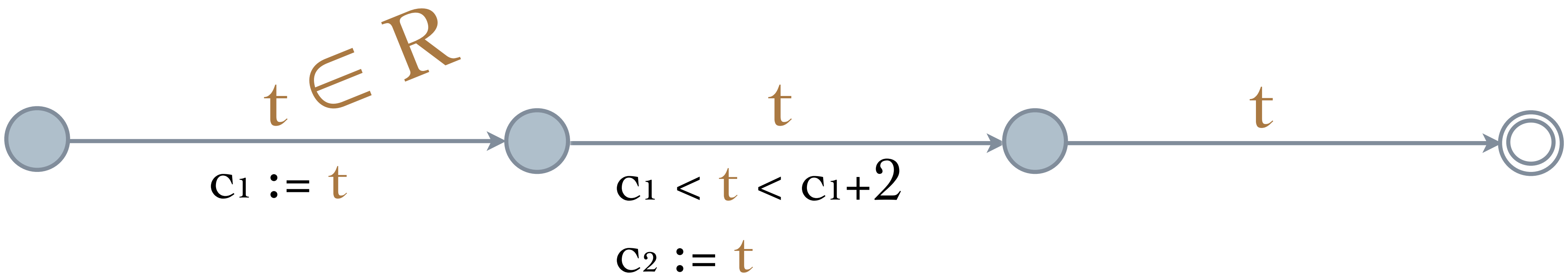

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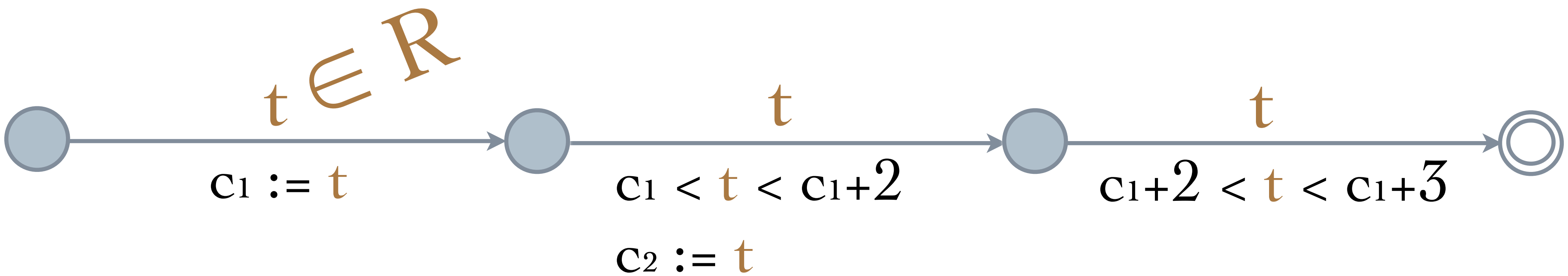

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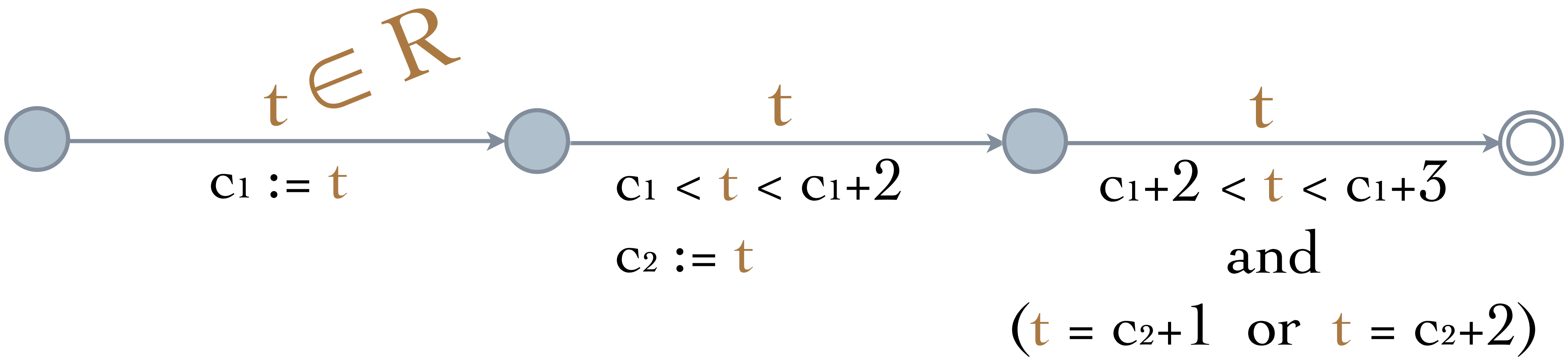

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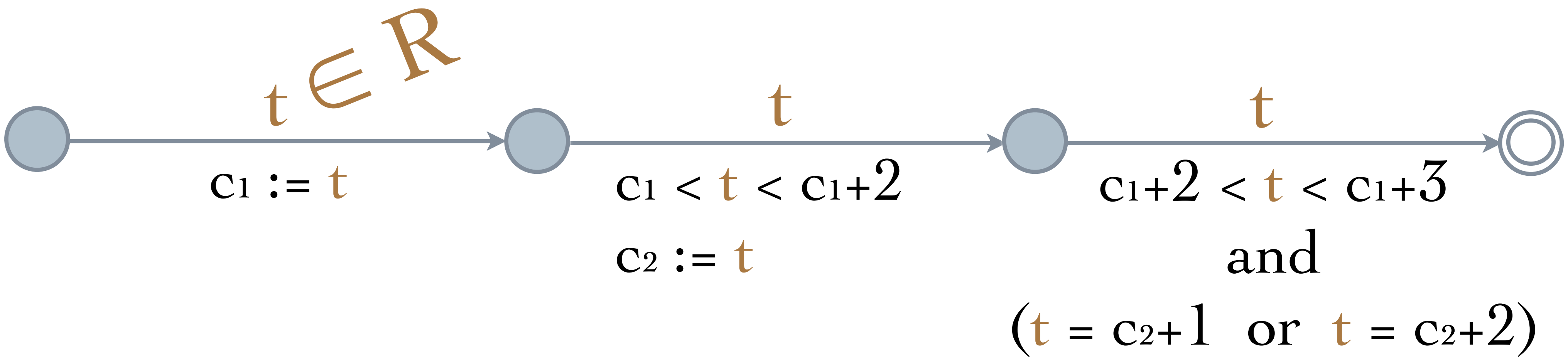

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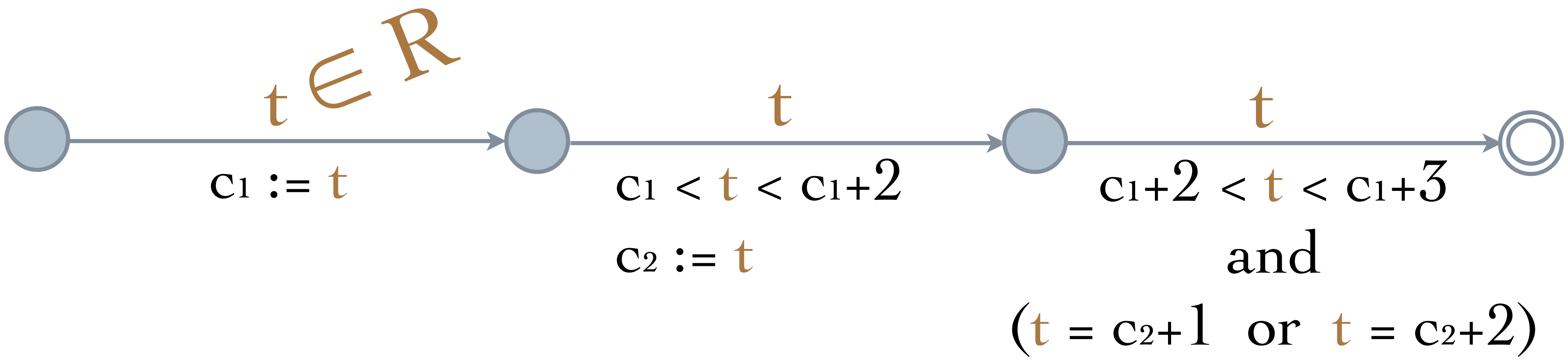

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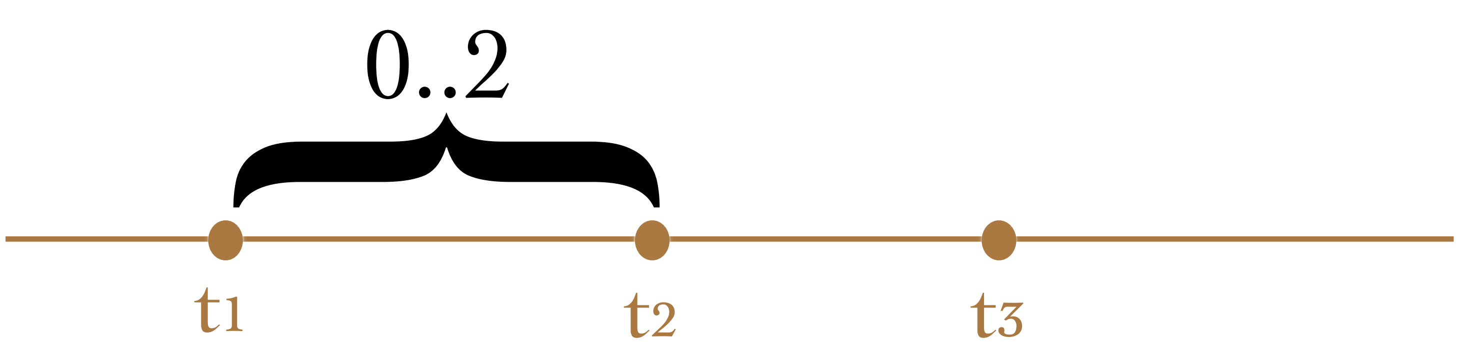

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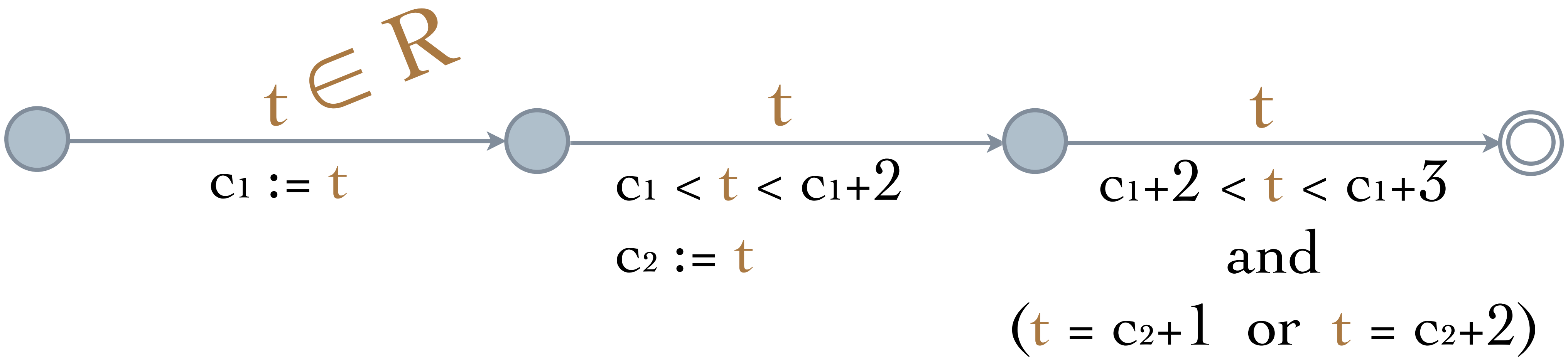

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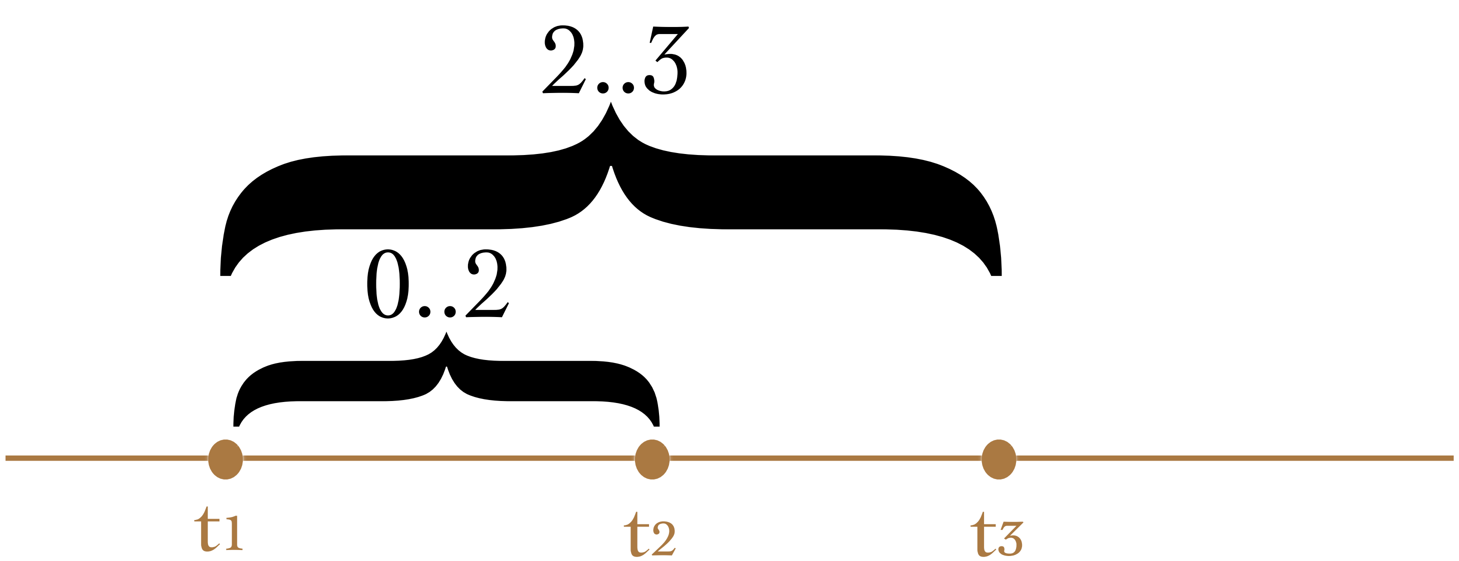

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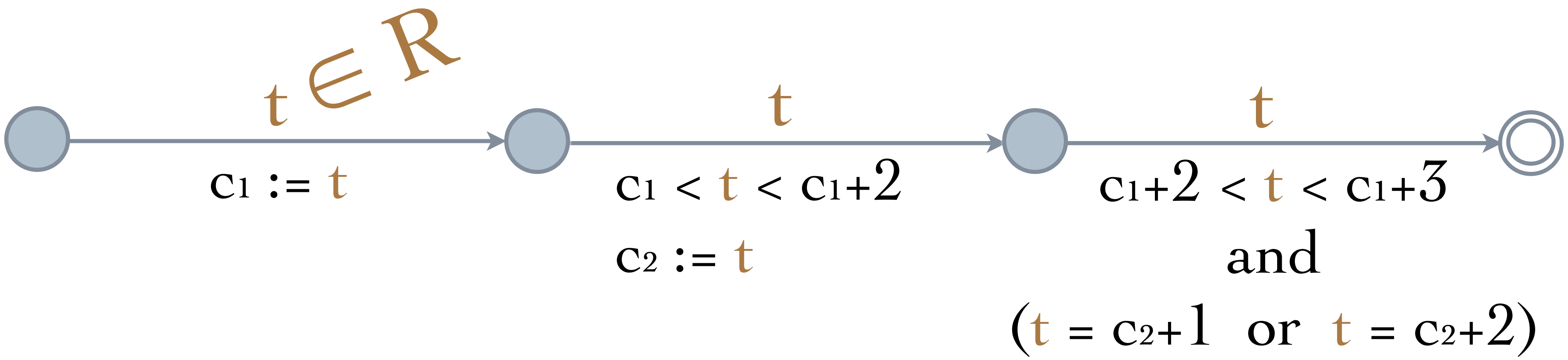

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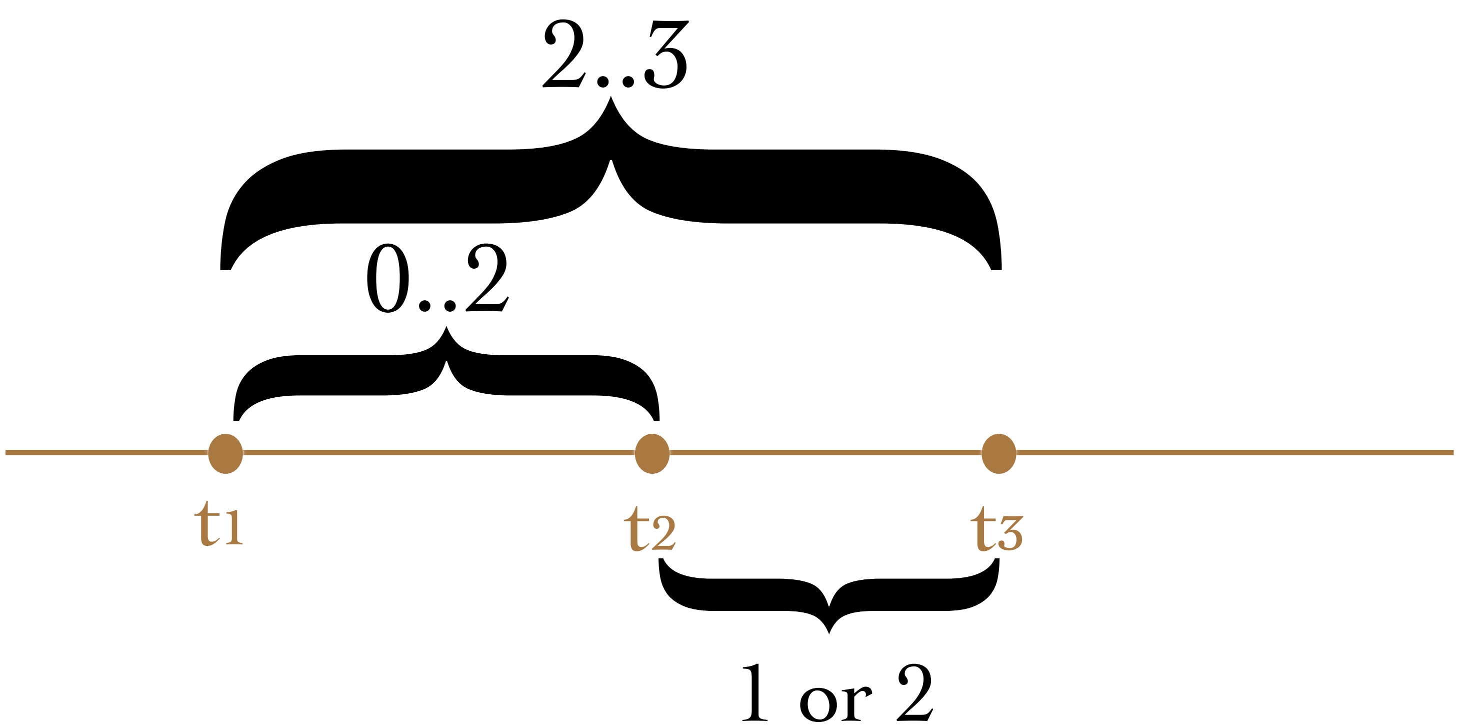

Myhill-Nerode theorem

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let L be a language over a finite alphabet A

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Problems:

- infinitely many equivalence classes
- no canonical minimal timed automaton

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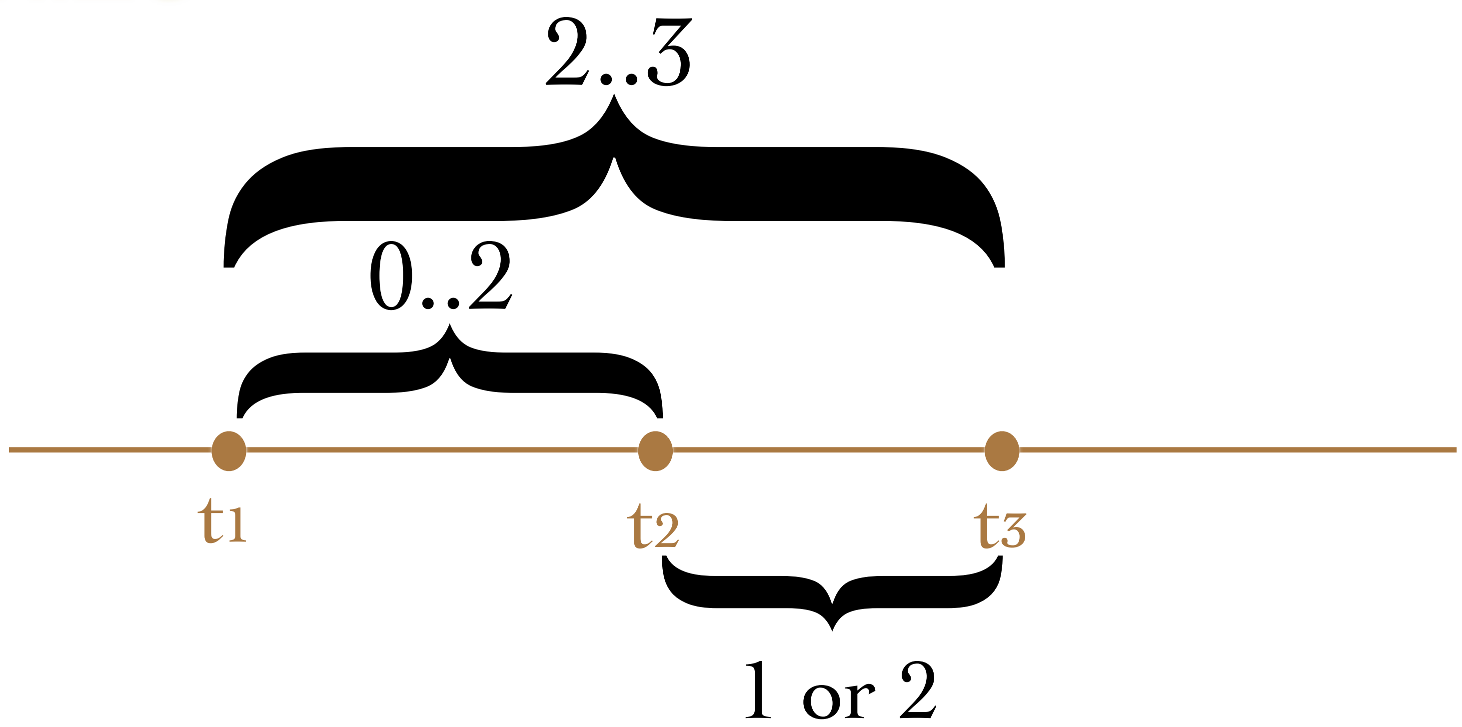

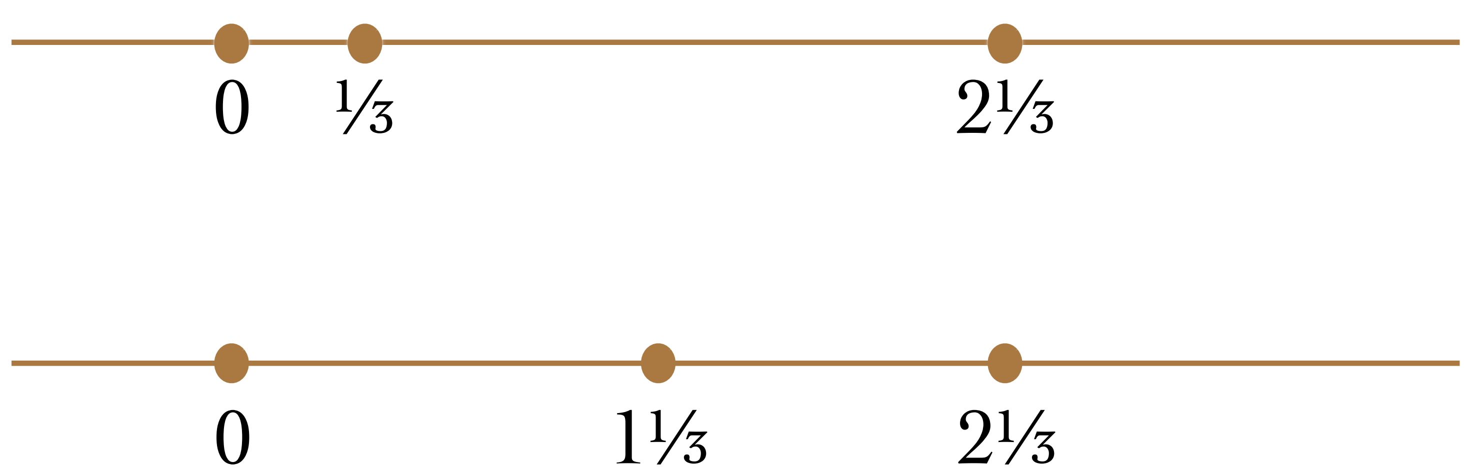

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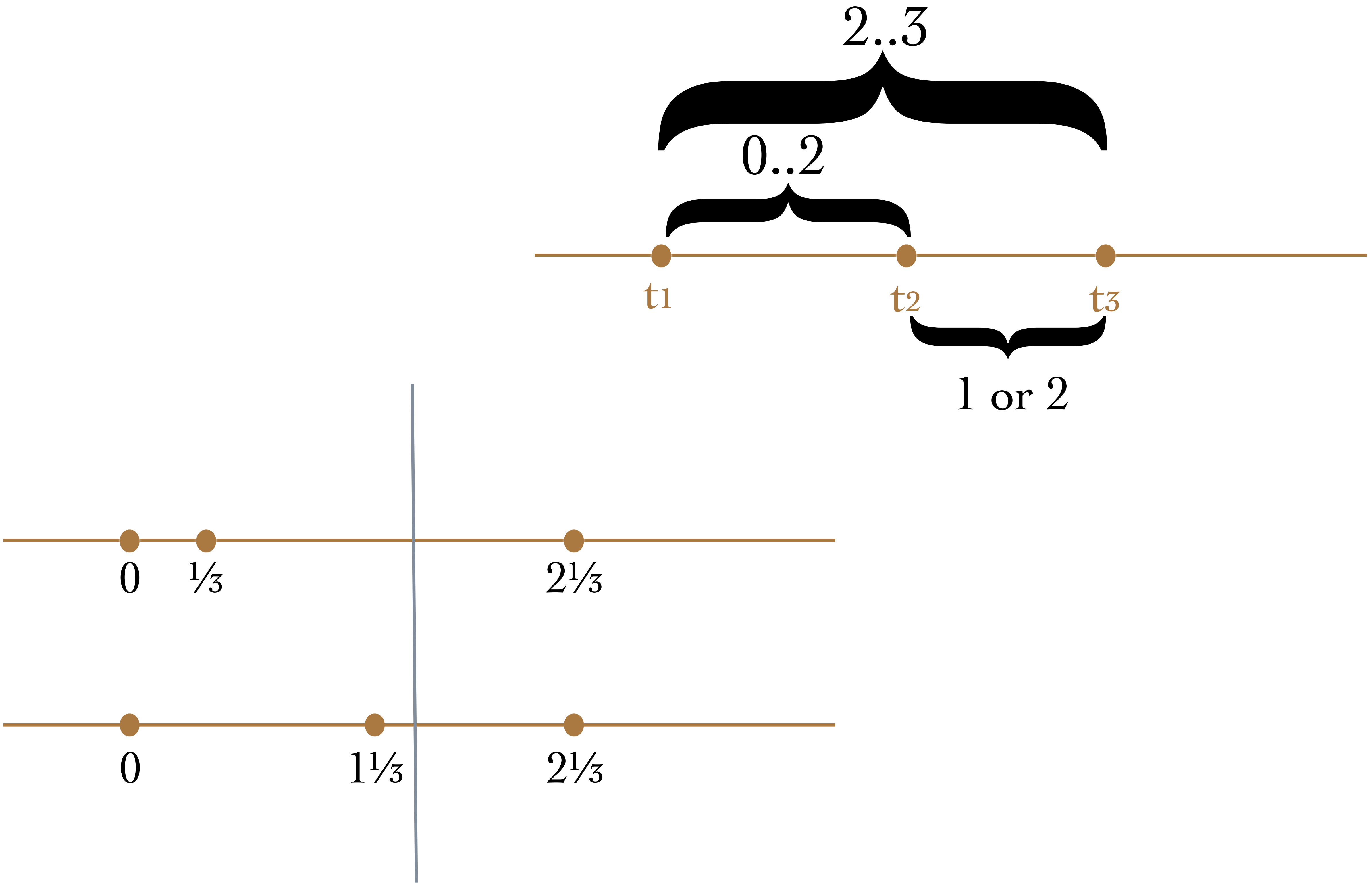

Solution: move to sets with atoms

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deterministic orbit-finite automata
in sets with atoms $(\mathbb{R}, <, +1)$

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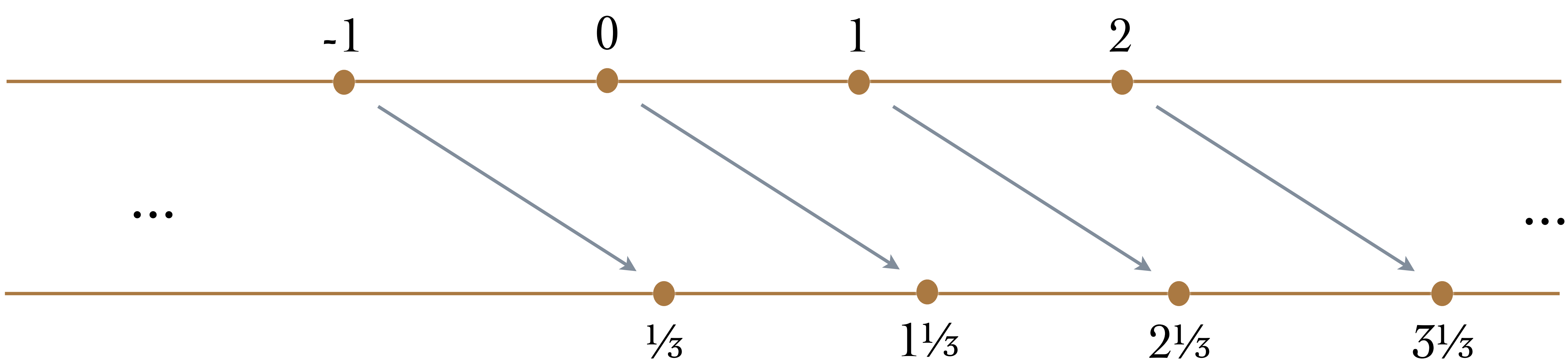

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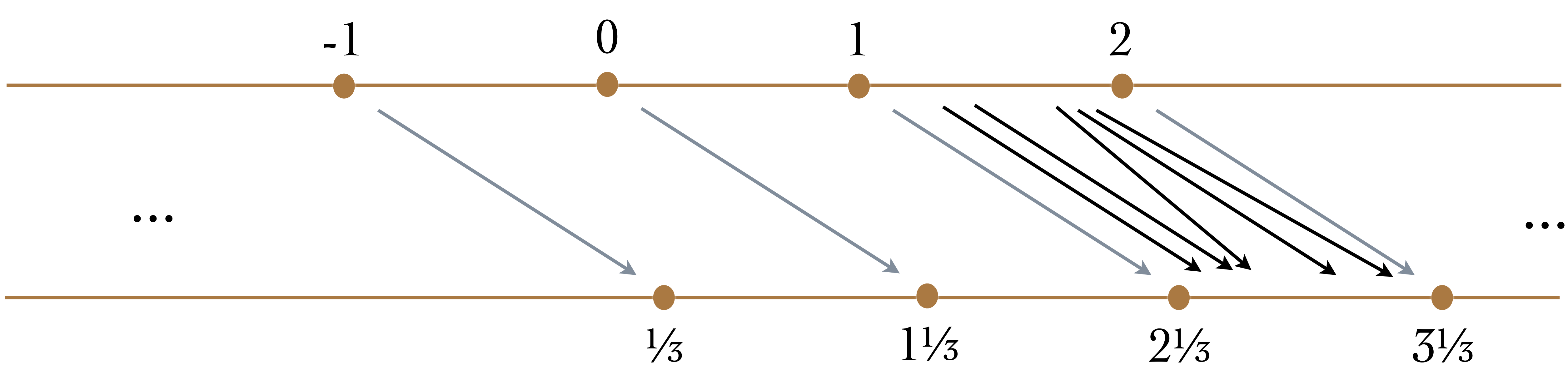

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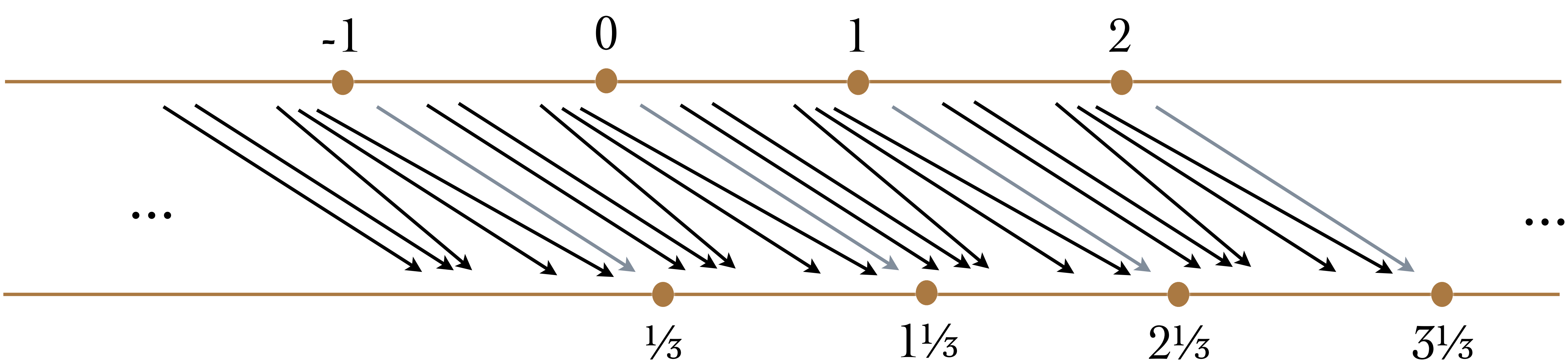

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minimal automata for languages
of deterministic timed automata
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Myhill-Nerode theorem for timed languages

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- $\approx_L$ has orbit-finite set of equivalence classes

deterministic orbit-finite automata in sets with atoms (R, <, +1)

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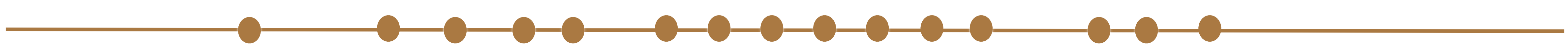

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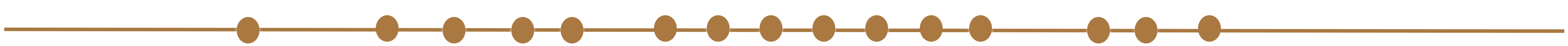

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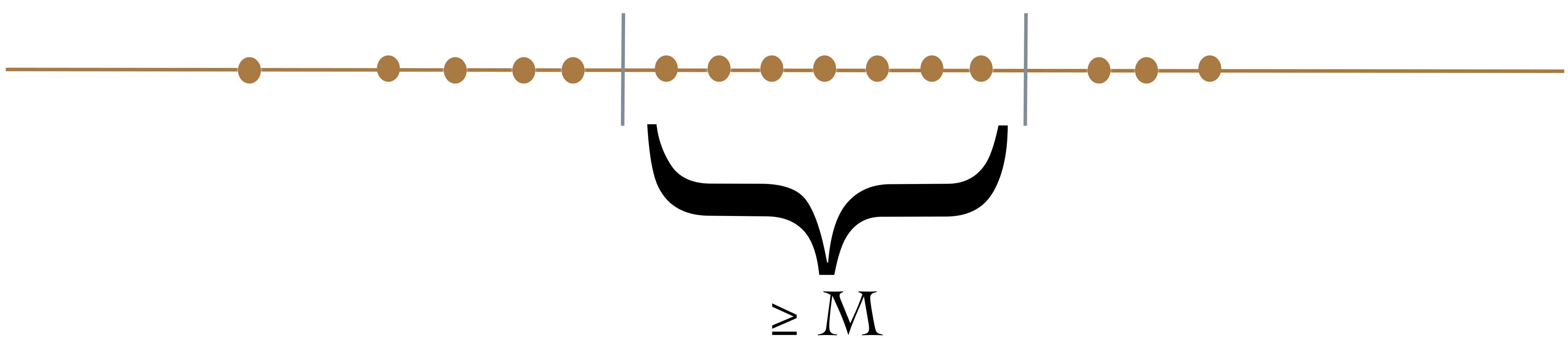

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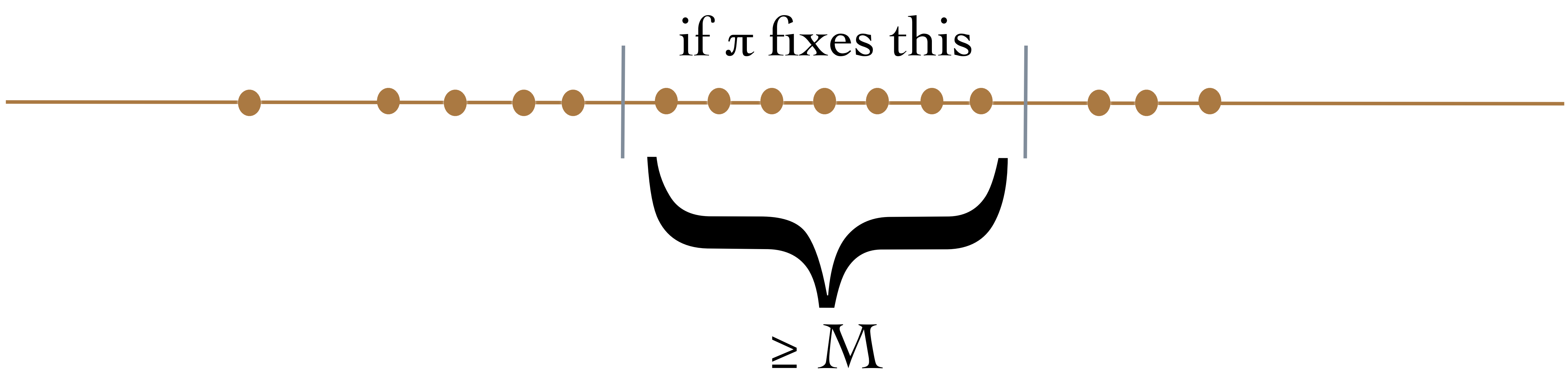

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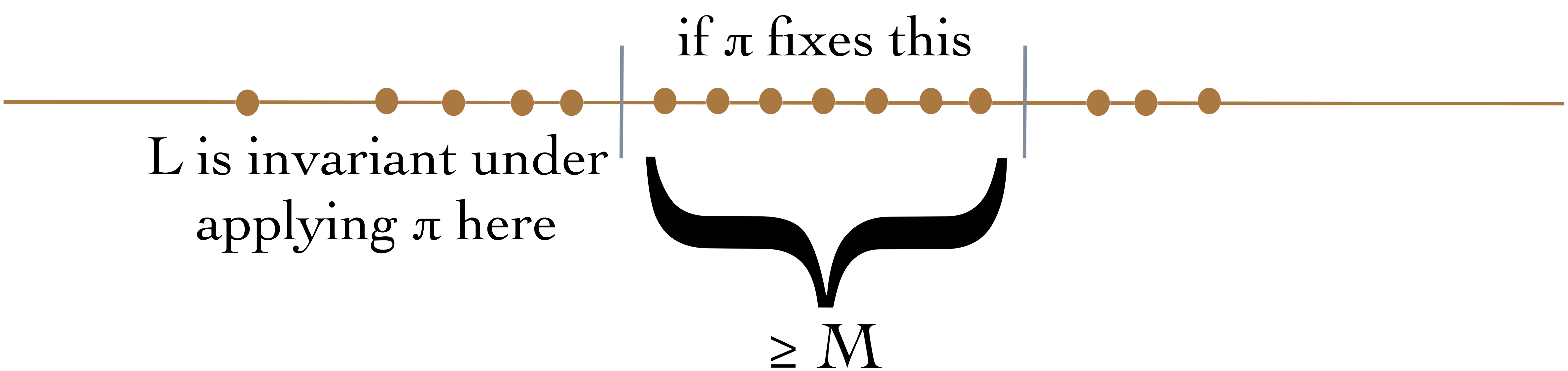

summary

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- superclass of deterministic timed automata closed under minimization

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Thank you!