Unambiguous Büchi Automata

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Abstract. In this paper, we introduce a special class of Büchi automata called unambiguous. In these automata, any infinite word labels exactly one path going infinitely often through final states. The word is accepted by the automaton if this path starts at an initial state. The main result of the paper is that any rational set of infinite words is recognized by such an automaton. We also provide two characterizations of these automata. We finally show that they are well suitable for boolean operations.

1 Introduction

Automata on infinite words have been introduced by Büchi [3] in order to prove the decidability of the monadic second-order logic of the integers. Since then, automata on infinite objects have often been used to prove the decidability of numerous problems. From a more practical point of view, they also lead to efficient decision procedures as for temporal logic [12]. Therefore, automata of infinite words or infinite trees are one of the most important ingredients in model checking tools [14]. The complementation of automata is then an important issue since the systems are usually modeled by logical formulas which involve the negation operator.

There are several kinds of automata that recognize sets of infinite words. In 1962, Büchi [3] introduced automata on ω-words, now referred to as *Büchi automata*. These automata have initial and final states and a path is successful if it starts at an initial state and goes infinitely often through final states. However, not all rational sets of infinite words are recognized by a deterministic Büchi automaton [5]. Therefore, complementation is a rather difficult operation on Büchi automata [12].

In 1963, Muller [9] introduced automata, now referred to as *Muller automata*, whose accepting condition is a family of accepting subsets of states. A path is then successful if it starts at the unique initial state and if the set of states which occurs infinitely in the path is accepting. A deep result of McNaughton [6] shows that any rational set of infinite words is recognized by a deterministic Muller automaton. A deterministic automaton is unambiguous in the following sense. With each word is associated a canonical path which is the unique path starting at the initial state. A word is then accepted iff its canonical path is successful. In a deterministic Muller automaton, the unambiguity is due to the uniqueness

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of the initial state and to the determinism of the transitions. Independently, the acceptance condition determines if a path is successful or not. The unambiguity of a deterministic Muller automaton makes it easy to complement. It suffices to exchange accepting and non-accepting subsets of states. However, the main drawback of using deterministic Muller automata is that the acceptance condition is much more complicated. It is a family of subsets of states instead of a simple set of final states. There are other kinds of deterministic automata recognizing all rational sets of infinite words like Rabin automata [11], Street automata or parity automata [8]. In all these automata, the acceptance condition is more complicated than a simple set of final states.

In this paper, we introduce a class of Büchi automata in which any infinite word labels exactly one path going infinitely often through final states. A canonical path can then be associated with each infinite word and we call these automata unambiguous. In these automata, the unambiguity is due to the transitions and to the final states whereas the initial states determine if a path is successful. An infinite word is then accepted iff its canonical path starts at an initial state. The main result is that any rational set of infinite words is recognized by such an automaton. It turns out that these unambiguous Büchi automata are codeterministic, i.e., reverse deterministic. Our result is thus the counterpart of McNaughton's result for codeterministic automata. It has already been proved independently in [7] and [2] that any rational set of infinite words is recognized by a codeterministic automaton but the construction given in [2] does not provide unambiguous automata. We also show that unambiguous automata are well suited for boolean operations and especially complementation. In particular, our construction can be used to find a Büchi automaton which recognizes the complement of the set recognized by another Büchi automaton. For a Büchi automaton with n states, our construction provides an unambiguous automaton which has at most $(12n)^n$ states.

The unambiguous automata introduced in the paper recognize right-infinite words. However, the construction can be adapted to bi-infinite words. Two unambiguous automata on infinite words can be joined to make an unambiguous automaton on bi-infinite words. This leads to an extension of McNaughton's result to the realm of bi-infinite words.

The main result of this paper has been first obtained by the second author and his proof has circulated as a hand-written manuscript among a bunch of people. It was however never published. Later, the first author found a different proof of the same result based on algebraic constructions on semigroups. Both authors have decided to publish their whole work on this subject together.

The paper is organized as follows. Section 2 is devoted to basic definitions on words and automata. Unambiguous Büchi automata are defined in Sect. 3. The main result (Theorem 1) is stated there. The first properties of these automata are presented in Sect. 4. Boolean Operations are studied in Sect. 5.

2 Automata

We recall here some elements of the theory of rational sets of finite and infinite words. For further details on automata and rational sets of finite words, see [10] and for background on automata and rational sets of infinite words, see [13]. Let A be a set called an *alphabet* and usually assumed to be finite. We respectively denote by A^* and A^+ the set of finite words and the set of nonempty finite words. The set of right-infinite words, also called ω-words, is denoted by A^ω.

A *Büchi automaton* $\mathcal{A} = (Q, A, E, I, F)$ is a non-deterministic automaton with a set Q of states, subsets $I, F \subset Q$ of *initial* and *final* states and a set $E \subset Q \times A \times Q$ of *transitions*. A transition (p, a, q) of $\mathcal{A}$ is denoted by $p \xrightarrow{a} q$. A *path* in $\mathcal{A}$ is an infinite sequence

$$\gamma : q_0 \xrightarrow{a_0} q_1 \xrightarrow{a_1} q_2 \cdots$$

of consecutive transitions. The *starting* state of the path is q_0 and the ω-word $\lambda(\gamma) = a_0 a_1 \ldots$ is called the *label* of γ. A *final* path is a path γ such that at least one of the final states of the automaton is infinitely repeated in γ. A *successful* path is a final path which starts at an initial state.

As usual, an ω-word is accepted by the automaton if it is the label of a successful path. The set of accepted ω-words is said to be recognized by the automaton and is denoted by $\mathrm{L}(\mathcal{A})$. It is well known that a set of ω-words is rational iff it is recognized by some automaton.

A state of a Büchi automaton $\mathcal{A}$ is said to be *coaccessible* if it is the starting state of a final path. A Büchi automaton is said to be *trim* if all states are coaccessible. Any state which occurs in a final path is coaccessible and thus non-coaccessible states of an automaton can be removed. In the sequel, automata are usually assumed to be trim.

An automaton $\mathcal{A} = (Q, A, E, I, F)$ is said to be *codeterministic* if for any state q and any letter a, there is at most one incoming transition $p \xrightarrow{a} q$ for some state p. If this condition is met, for any state q and any finite word w, there is at most one path $p \xrightarrow{w} q$ ending in q.

3 Unambiguous Automata

In this section, we introduce the concept of unambiguous Büchi automata. We first give the definition and we state one basic property of these automata. We then establish a characterization of these automata. We give some examples and we state the main result.

Definition 1. *A Büchi automaton $\mathcal{A}$ is said to be* unambiguous *(respectively* complete*) iff any ω-word labels at most (respectively at least) one final path in $\mathcal{A}$.*

The set of final paths is only determined by the transitions and the final states of $\mathcal{A}$. Thus, the property of being unambiguous or complete does not depend on the set of initial states of $\mathcal{A}$. In the sequel, we will freely say that an automaton $\mathcal{A}$ is unambiguous or complete without specifying its set of initial states.

The definition of the word "complete" we use here is not the usual definition given in the literature. A deterministic automaton is usually said to be complete if for any state q and letter a there is at least an outgoing transition labeled by a. This definition implies that any finite or infinite word labels at least a path starting at the initial state. It will stated in Proposition 1 that the unambiguity implies that the automaton is codeterministic. Thus, we should reverse the definition and we should say that for any state q and letter a there is at least an incoming transition labeled by a. However, since the words are right-infinite, this condition does not imply anymore that any ω-word labels a path going infinitely often though final states as it is shown in Example 3. Thus, the definition chosen in this paper really insures that any ω-word is the label of a final path. It will be stated in Proposition 1 that this condition is actually stronger that the usual one.

In the sequel, we write UBA for Unambiguous Büchi Automaton and CUBA for Complete Unambiguous Büchi Automaton. The following example is the simplest CUBA.

Example 1. The automaton $(\{0\}, A, E, I, \{0\})$ with $E = \{0 \xrightarrow{a} 0 \mid a \in A\}$ is obviously a CUBA. It recognizes the set A^ω of all ω-words if the state 0 is initial and recognizes the empty set otherwise. It is called the *trivial* CUBA.

The following proposition states that an UBA must be codeterministic. Such an automaton can be seen as a deterministic automaton which reads infinite words from right to left. It starts at infinity and ends at the beginning of the word. Codeterministic automata on infinite words have already been considered in [2]. It is proved in that paper that any rational set of ω-words is recognized by a codeterministic automata. Our main theorem generalizes this results. It states that any rational set of ω-words is recognized by a CUBA.

Proposition 1. *Let $\mathcal{A} = (Q, A, E, I, F)$ be a trim Büchi automaton. If $\mathcal{A}$ is unambiguous, then $\mathcal{A}$ is codeterministic. If $\mathcal{A}$ is complete, then for any state q and any letter a, there is at least one incoming transition $p \xrightarrow{a} q$ for some state p.*

The second statement of the proposition says that our definition of completeness implies the usual one. Example 3 shows that the converse does not hold. However, Proposition 3 provides some additional condition on the automaton to ensure that it is unambiguous and complete.

Before giving some other examples of CUBA, we provide a simple characterization of CUBA which makes it easy to verify that an automaton is unambiguous and complete. This proposition also shows that it can be effectively checked if a given automaton is unambiguous or complete.

Let $\mathcal{A} = (Q, A, E, I, F)$ be a Büchi automaton and let q be a state of $\mathcal{A}$. We denote by $\mathcal{A}_q = (Q, A, E, \{q\}, F)$ the new automaton obtained by taking the singleton $\{q\}$ as set of initial states. The set $\mathrm{L}(\mathcal{A}_q)$ is then the set of ω-words labeling a final path starting at state q.

Proposition 2. *Let $\mathcal{A} = (Q, A, E, I, F)$ be a Büchi automaton. For $q \in Q$, let $\mathcal{A}_q$ the automaton $(Q, A, E, \{q\}, F)$. The automaton $\mathcal{A}$ is unambiguous iff the sets $\mathrm{L}(\mathcal{A}_q)$ are pairwise disjoint. The automaton $\mathcal{A}$ is complete iff $A^\omega \subset \bigcup_{q \in Q} \mathrm{L}(\mathcal{A}_q)$*

In particular, the automaton $\mathcal{A}$ is unambiguous and complete iff the family of sets $\mathrm{L}(\mathcal{A}_q)$ for $q \in Q$ is a partition of A^ω. It can be effectively verified that the two sets recognized by the automata $\mathcal{A}_q$ and $\mathcal{A}_{q'}$ are disjoint for $q \neq q'$. It can then be checked if the automaton is unambiguous. Furthermore, this test can be performed in polynomial time. The set $\bigcup_{q \in Q} \mathrm{L}(\mathcal{A}_q)$ is recognized by the automaton $\mathcal{A}_Q = (Q, A, E, Q, F)$ whose all states are initial. The inclusion $A^\omega \subset \bigcup_{q \in Q} \mathrm{L}(\mathcal{A}_q)$ holds iff this automaton recognizes A^ω. This can be checked but it does not seem it can be performed in polynomial time.

We now come to examples. We use Proposition 2 to verify that the following two automata are unambiguous and complete. In the figures, a transition $p \xrightarrow{a} q$ of an automaton is represented by an arrow labeled by a from p to q. Initial states have a small incoming arrow while final states are marked by a double circle. A Büchi automaton which is complete but ambiguous is given is Example 4.

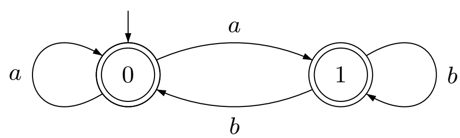

Fig. 1. CUBA of Example 2

Example 2. Let A be the alphabet $A = \{a, b\}$ and let $\mathcal{A}$ be the automaton pictured in Fig. 1. This automaton is unambiguous and complete since we have $\mathrm{L}(\mathcal{A}_0) = aA^\omega$ and $\mathrm{L}(\mathcal{A}_1) = bA^\omega$. It recognizes the set aA^ω of ω-words beginning with an a.

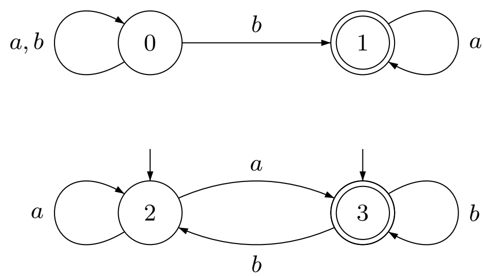

Fig. 2. CUBA of Example 3

The following example shows that a CUBA may have several connected components.

Example 3. Let A be the alphabet $A = \{a, b\}$ and let $\mathcal{A}$ be the automaton pictured in Fig. 2. It is unambiguous and complete since we have $\mathrm{L}(\mathcal{A}_0) = A^*ba^\omega$, $\mathrm{L}(\mathcal{A}_1) = a^\omega$, $\mathrm{L}(\mathcal{A}_2) = a(A^*b)^\omega$ and $\mathrm{L}(\mathcal{A}_3) = b(A^*b)^\omega$. It recognizes the set $(A^*b)^\omega$ of ω-words having an infinite number of b.

The automaton of the previous example has two connected components. Since it is unambiguous and complete any ω-word labels exactly one final path in this automaton. This final path is in the first component if the ω-word has finitely many b and it is the second component otherwise. This automaton shows that our definition of completeness for an unambiguous Büchi automaton is stronger than the usual one. Any connected component is complete in the usual sense if it is considered as a whole automaton. For any letter a and any state q in this component, there is exactly one incoming transition $p \xrightarrow{a} q$. However, each component is not complete according to our definition since not any ω-word labels a final path in this component.

In the realm of finite words, an automaton is usually made unambiguous by the usual subsets construction [4, p. 22]. This construction associates with an automaton $\mathcal{A}$ an equivalent deterministic automaton whose states are subsets of states of $\mathcal{A}$. Since left and right are symmetric for finite words, this construction can be reversed to get a codeterministic automaton which is also equivalent to $\mathcal{A}$. In the case of infinite words, the result of McNaughton [6] states that a Büchi automaton can be replaced by an equivalent Muller automaton which is deterministic. However, this construction cannot be reversed since ω-words are right-infinite. We have seen in Proposition 1 that a CUBA is codeterministic. The following theorem is the main result of the paper. It states that any rational set of ω-words is recognized by a CUBA. This theorem is thus the counterpart of McNaughton's result for codeterministic automata. Like Muller automata, CUBA make the complementation very easy to do. This will be shown in Sect. 5. The proof of Theorem 1 contains a new proof that the class of rational sets of ω-words is closed under complementation.

Theorem 1. *Any rational set of ω-words is recognized by a complete unambiguous Büchi automaton.*

There are two proofs of this result which are both rather long. Both proofs yield effective procedures which give a CUBA recognizing a given set of ω-words. The first proof is based on graphs and it directly constructs a CUBA from a Büchi automaton recognizing the set. The second proof is based on semigroups and it constructs a CUBA from a morphism from A^+ into a finite semigroup recognizing the set. An important ingredient of both proofs is the notion of a generalized Büchi automaton.

In a Büchi automaton, the set of final paths is the set of paths which go infinitely often through final states. In a generalized Büchi automaton, the set of final paths is given in a different way. A generalized Büchi automaton is equipped with an output function μ which maps any transition to a nonempty word over an alphabet B and with a fixed set K of ω-words over B. A path is final if the concatenation of the outputs of its transitions belongs to K. A generalized Büchi automaton can be seen as an automaton with an output function. We point out that usual Büchi automata are a particular case of generalized Büchi automata. Indeed, if the function μ maps any transition $p \xrightarrow{a} q$ to 1 if p or q is final and to 0 otherwise and if $K = (0^*1)^\omega$ is the set of ω-words over $\{0,1\}$ having an infinite number of 1, a path in $\mathcal{A}$ is final if some final state occurs infinitely often in it.

The notions of unambiguity and completeness are then extended to generalized Büchi automata. A generalized Büchi automaton is said to be unambiguous (respectively complete) if any ω-word labels at most (respectively at least) one final path.

The generalized Büchi automata can be composed. If a set X is recognized by an automaton $\mathcal{A}$ whose fixed set K is recognized by automaton $\mathcal{B}$ which has a fixed set K', then X is also recognized by an automaton having the fixed set K' which can be easily constructed from $\mathcal{A}$ and $\mathcal{B}$. Furthermore, this composition is compatible with unambiguity and completeness. This means that if both automata $\mathcal{A}$ and $\mathcal{B}$ are unambiguous (respectively complete), then the automaton obtained by composition is also unambiguous (respectively complete).

4 Properties and Characterizations

In this section, we present some additional properties of CUBA. We first give another characterization of CUBA which involves loops going through final states. We present some consequences of this characterization. The characterization of CUBA given in Proposition 2 uses sets of ω-words. The family of sets of ω-words labeling a final path starting in the different states must be a partition of the set A^ω of all ω-words. The following proposition only uses sets of finite words to characterize UBA and CUBA.

Proposition 3. *Let $\mathcal{A} = (Q, A, E, I, F)$ be a Büchi automaton such that for any state q and any letter a, there exists exactly one incoming transition $p \xrightarrow{a} q$. Let S_q be the set of nonempty finite words w such that there is a path $q \xrightarrow{w} q$ going through a final state. The automaton $\mathcal{A}$ is unambiguous iff the sets S_q are pairwise disjoint. The automaton $\mathcal{A}$ is unambiguous and complete iff the family of sets S_q for $q \in Q$ is a partition of A^+. In this case, the final path labeled by the periodic ω-word w^ω is the path*

$$q \xrightarrow{w} q \xrightarrow{w} q \cdots$$

where q is the unique state such that $w \in S_q$.

The second statement of the proposition says that if that if the automaton $\mathcal{A}$ is supposed to be unambiguous, it is complete iff the inclusion $A^+ \subset \bigcup_{q \in Q} S_q$ holds. The assumption that the automaton is unambiguous is necessary. As the following example shows, it is not true in general that the automaton is complete iff the inclusion holds.

Example 4. The automaton of Fig. 3 is ambiguous since the ω-word b^ω labels two final paths. Since this automaton is deterministic and all states are final, it

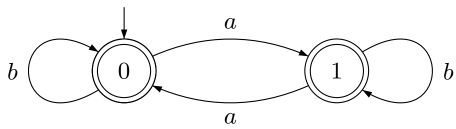

Fig. 3. CUBA of Example 4

is complete. However, it is not true that $A^+ \subset \bigcup_{q \in Q} S_q$. Indeed, no loop in this automaton is labeled by the finite word a.

Proposition 3 gives another method to check if a given Büchi automaton is unambiguous and complete. It must be first verified that for any state q and any letter a, there is exactly one incoming transition $p \xrightarrow{a} q$. Then, it must be checked if the family of sets S_q for $q \in Q$ forms a partition of A^+. The sets S_q are rational and a codeterministic automaton recognizing S_q can be easily deduced from the automaton $\mathcal{A}$. It is then straightforward to verify that the sets S_q form a partition of A^+.

The last statement of Proposition 3 says that the final path labeled by a periodic word is also periodic. It is worth mentioning that the same result does not hold for deterministic automata.

If follows from Proposition 3 that the trivial CUBA with one state (see Example 1) is the only CUBA which is deterministic.

5 Boolean Combinations

In this section, we show that CUBA have a fine behavior with the boolean operations. From CUBA recognizing two sets X and Y, CUBA recognizing the complement $A^\omega \setminus X$, the union $X \cup Y$ and the intersection $X \cap Y$ can be easily obtained. For usual Büchi automata or for Muller automata, automata recognizing the union and the intersection are easy to get. It is sufficient to consider the product of the two automata with some small additional memory. However, complementation is very difficult for general Büchi automata.

5.1 Complement

We begin with complementation which turns out to be a very easy operation for CUBA. Indeed, it suffices to change the initial states of the automaton to recognize the complement.

Proposition 4. *Let $\mathcal{A} = (Q, A, E, I, F)$ be a CUBA recognizing a set X of ω-words. The automaton $\mathcal{A}' = (Q, A, E, Q \setminus I, F)$ where $Q \setminus I$ is the set of non initial states, is unambiguous and complete and it recognizes the complement $A^\omega \setminus X$ of X.*

It must be pointed out that it is really necessary for the automaton $\mathcal{A}$ to be unambiguous and complete. Indeed, if $\mathcal{A}$ is ambiguous, it may happen that an ω-word x of X labels a final path starting at an initial state and another final path starting at a non initial state. In this case, the ω-word x is also recognized by the automaton $\mathcal{A}'$. If $\mathcal{A}$ is not complete, some ω-word x labels no final path. This ω-word which does not belong to X is not recognized by the automaton $\mathcal{A}'$.

By the previous result, the proof of Theorem 1 also provides a new proof of the fact that the family of rational sets of ω-words is closed under complementation.

5.2 Union and Intersection

In this section, we show how CUBA recognizing the union $X_1 \cup X_2$ and the intersection $X_1 \cap X_2$ can be obtained from CUBA recognizing X_1 and X_2.

We suppose that the sets X_1 and X_2 are respectively recognized by the CUBA $\mathcal{A}_1 = (Q_1, A, E_1, I_1, F_1)$ and $\mathcal{A}_2 = (Q_2, A, E_2, I_2, F_2)$. We will construct two CUBA $\mathcal{U} = (Q, A, E, I_{\mathcal{U}}, F)$ and $\mathcal{I} = (Q, A, E, I_{\mathcal{I}}, F)$ respectively recognizing the union $X_1 \cup X_2$ and the intersection $X_1 \cap X_2$. Both automata $\mathcal{U}$ and $\mathcal{I}$ share the same states set Q, the same transitions set E and the same set F of final states.

We first describe the states and the transitions of both automata $\mathcal{U}$ and $\mathcal{I}$. These automata are based on the product of the automata $\mathcal{A}_1$ and $\mathcal{A}_2$ but a third component is added. The final states may not appear at the same time in $\mathcal{A}_1$ and $\mathcal{A}_2$. The third component synchronizes the two automata by indicating in which of the two automata comes the first final state. The set Q of states is $Q = Q_1 \times Q_2 \times \{1, 2\}$. Each state is then a triple (q_1, q_2, ε) where q_1 is a state of $\mathcal{A}_1$, q_2 is a state of $\mathcal{A}_2$ and ε is 1 or 2. There is a transition $(q_1', q_2', \varepsilon') \xrightarrow{a} (q_1, q_2, \varepsilon)$ if $q_1' \xrightarrow{a} q_1$ and $q_2' \xrightarrow{a} q_2$ are transitions of $\mathcal{A}_1$ and $\mathcal{A}_2$ and if ε' is defined as follows.

$$\varepsilon' = \begin{cases} 1 & \text{if } q_1 \in F_1 \\ 2 & \text{if } q_1 \notin F_1 \text{ and } q_2 \in F_2 \\ \varepsilon & \text{otherwise} \end{cases}$$

This definition is not completely symmetric. When both q_1 and q_2 are final states, we choose to set $\varepsilon' = 1$. We now define the set F of final states as

$$F = \big\{(q_1, q_2, \varepsilon) \big| q_2 \in F_2 \text{ and } \varepsilon = 1\big\}.$$

This definition is also non symmetric.

It may be easily verified that any loop around a final state (q_1, q_2, ε) also contains a state $(q_1', q_2', \varepsilon')$ such that $q_2' \in F_2$. This implies that the function which maps a path γ to the pair (γ_1, γ_2) of paths in $\mathcal{A}_1$ and $\mathcal{A}_2$ is one to one from the set of final paths in $\mathcal{U}$ or $\mathcal{I}$ to the set of pairs of final paths in $\mathcal{A}_1$ and $\mathcal{A}_2$. Thus if both $\mathcal{A}_1$ and $\mathcal{A}_2$ are unambiguous and complete, then both automata $\mathcal{U}$ and $\mathcal{I}$ are also unambiguous and complete.

If q_1 and q_2 are the respective starting states of γ_1 and γ_2, the starting state of γ is then equal to (q_1, q_2, ε) with $\varepsilon \in \{1, 2\}$. We thus define the sets $I_{\mathcal{U}}$ and $I_{\mathcal{I}}$

of initial states of the automata $\mathcal{U}$ and $\mathcal{I}$ as follows.

$$I_{\mathcal{U}} = \{(q_1, q_2, \varepsilon) | (q_1 \in I_1 \text{ or } q_2 \in I_2) \text{ and } \varepsilon \in \{1, 2\}\}$$
$$I_{\mathcal{I}} = \{(q_1, q_2, \varepsilon) | q_1 \in I_1 \text{ and } q_2 \in I_2 \text{ and } \varepsilon \in \{1, 2\}\}$$

From these definitions, it is clear that both automata $\mathcal{U}$ and $\mathcal{I}$ are unambiguous and complete and that they respectively recognize $X_1 \cup X_2$ and $X_1 \cap X_2$.

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