Revealing vs. Concealing: More Simulation Games for Büchi Inclusion

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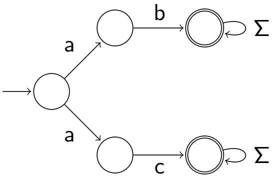
Büchi automata

- extends a finite automata to accept infinite sequences of letters
- represents a language over infinite words

Example

(Nondeterministic) Büchi automata $\mathcal{A}$ over $\Sigma = \{a, b, c\}$



abaaa... $\in L(\mathcal{A})$
aaaaa... $\notin L(\mathcal{A})$

The language $L(\mathcal{A}) = a \cdot (b \cup c) \cdot (a \cup b \cup c)^{\omega}$.

Büchi inclusion problem

- Given : two Büchi automata $\mathcal{A}$, $\mathcal{B}$
  Question : Is $L(\mathcal{A}) \subseteq L(\mathcal{B})$?

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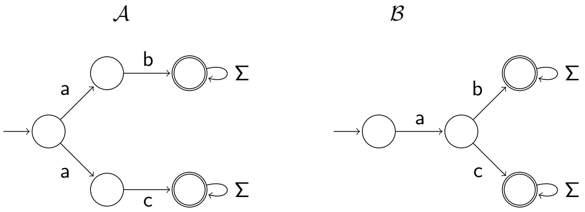

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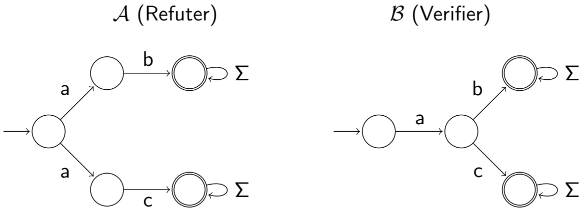

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  - "Refuter" plays against "Verifier"
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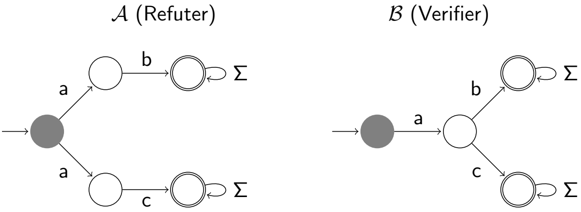

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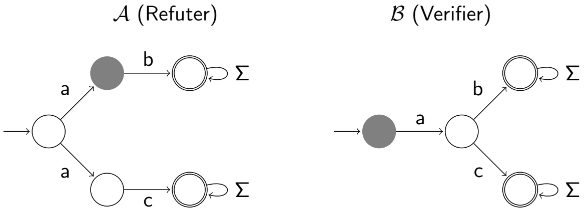

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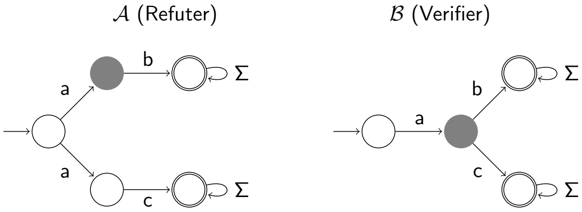

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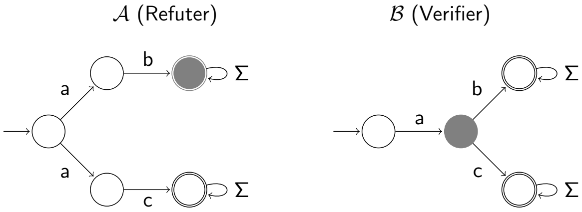

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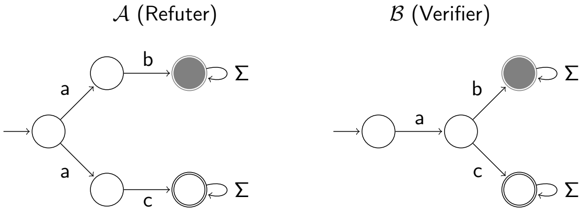

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- can be encoded to parity game with priorities: 0, 1, 2

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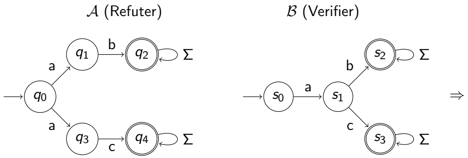

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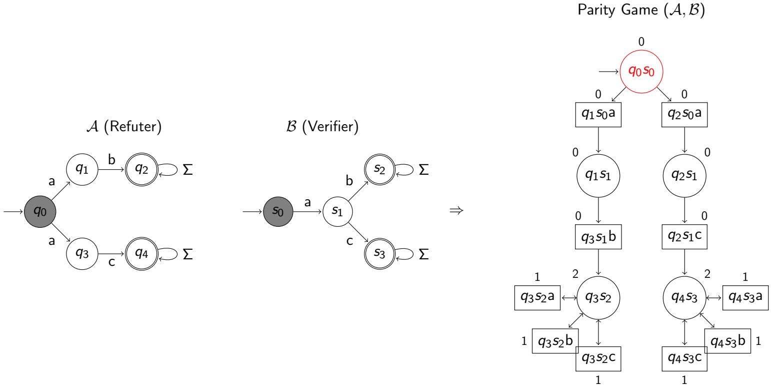

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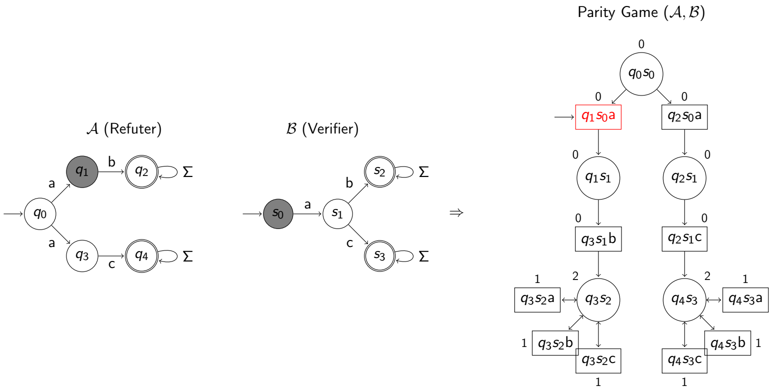

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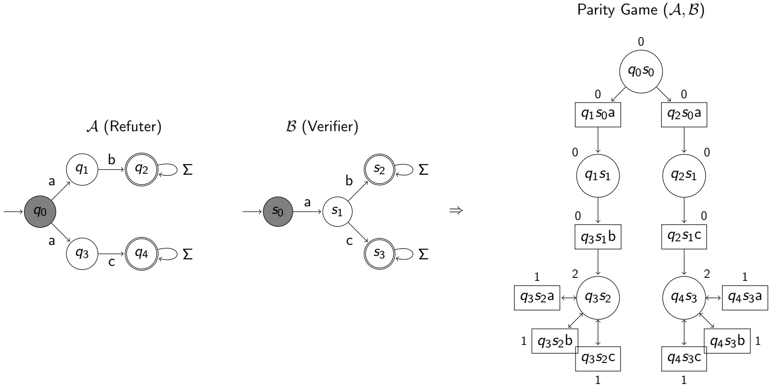

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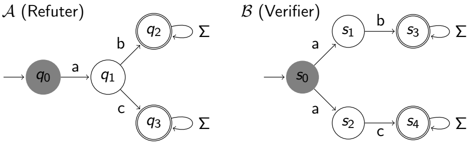

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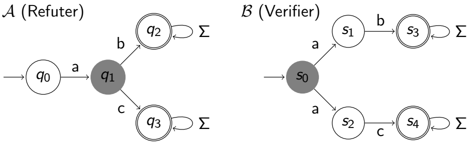

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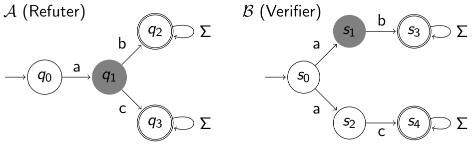

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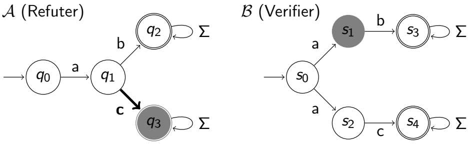

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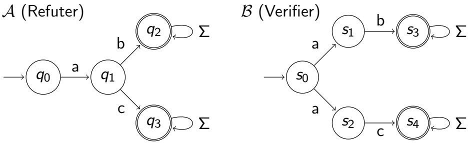

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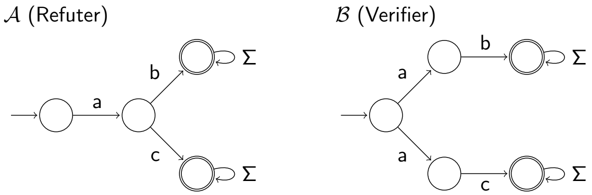
k-pebble game [Etessami02]



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- Verifier controls k pebbles
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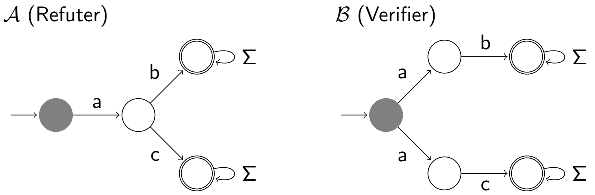

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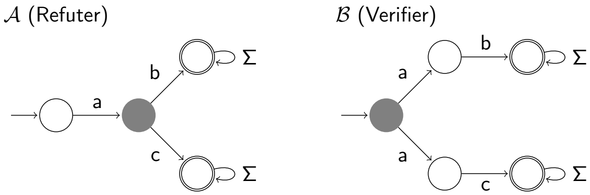

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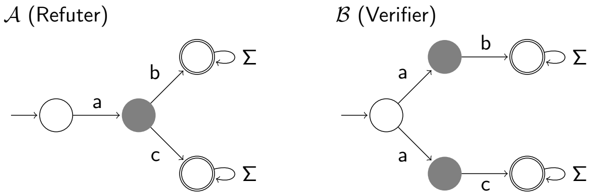

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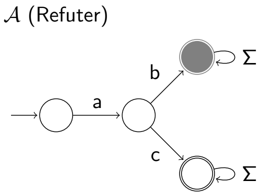

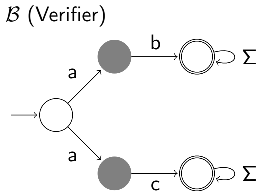

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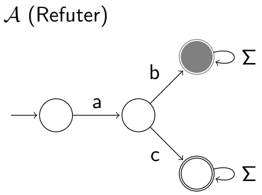

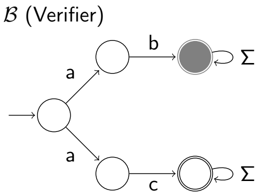

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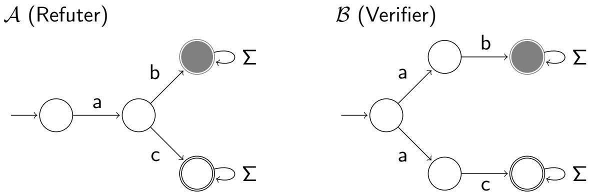

- forms a hierarchy:

$$\sqsubseteq^1_{\text{peb}} \subset \sqsubseteq^2_{\text{peb}} \subset \sqsubseteq^3_{\text{peb}} \subset \dots \subset \bigcup_{k\geq 1} \sqsubseteq^k_{\text{peb}} \subset \sqsubseteq_{\text{incl}}$$

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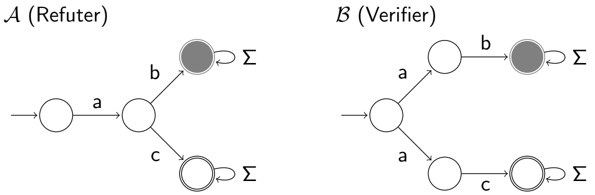

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- parity game size $< |\mathcal{A}| \cdot (2 \cdot |\mathcal{B}| + 1)^k \cdot (|\Sigma| + 1)$

Static k-letter game

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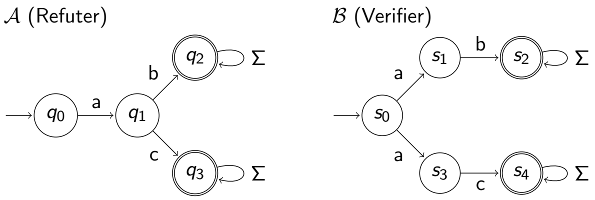
- both players control one pebble
- move k steps in each round

Example



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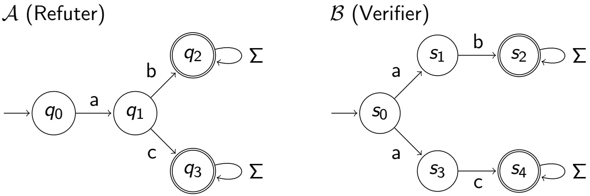

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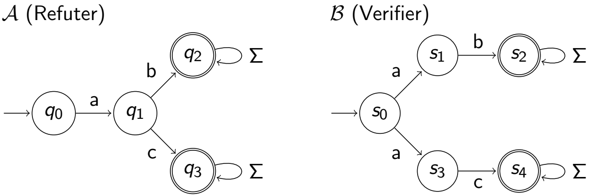

- parity game size $< |\mathcal{A}| \cdot |\mathcal{B}| \cdot (|\Sigma|^k + 1)$
- do not form a linear hierarchy

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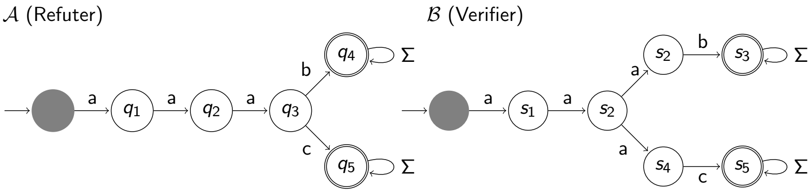

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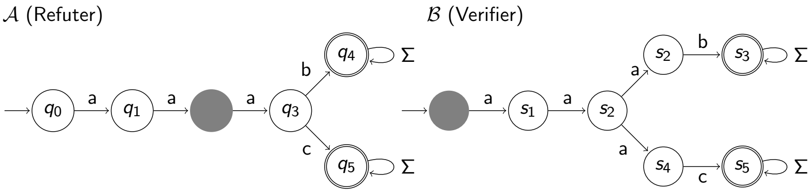

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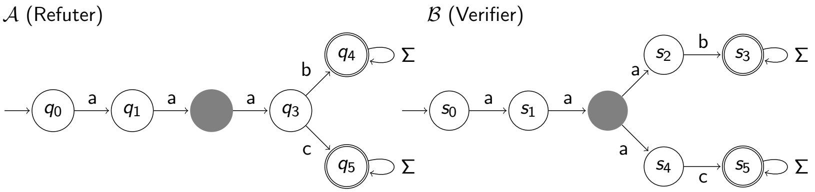

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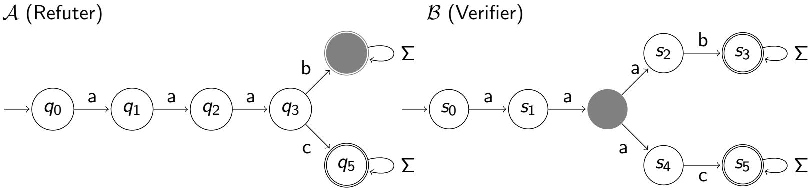

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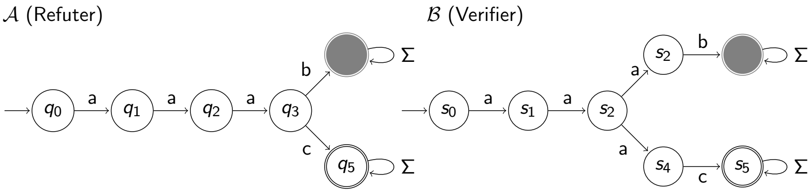

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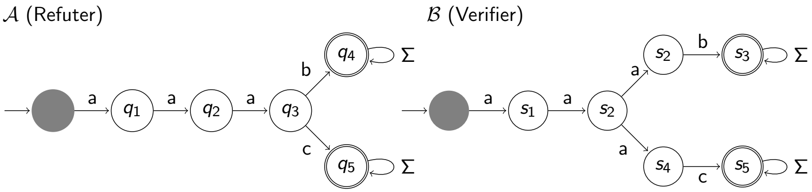

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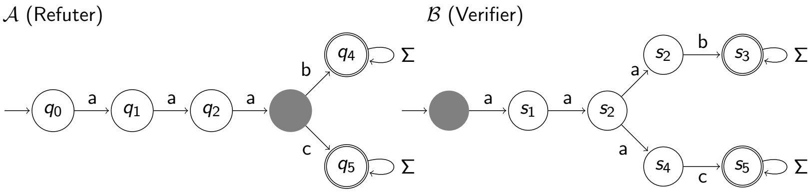

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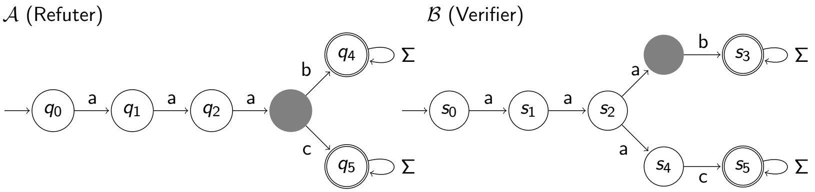

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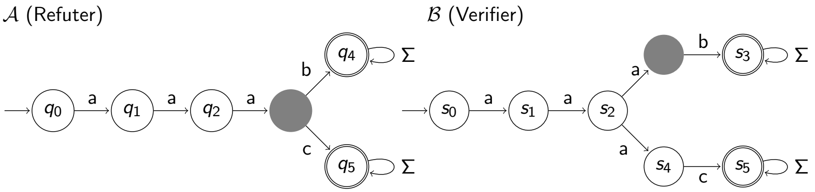

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$$\sqsubseteq^{\gcd(k,\ell)}_{\text{stat}} \subseteq \sqsubseteq^{k}_{\text{stat}} \;, \; \sqsubseteq^{\ell}_{\text{stat}} \subseteq \sqsubseteq^{\text{lcm}(k,\ell)}_{\text{stat}}$$

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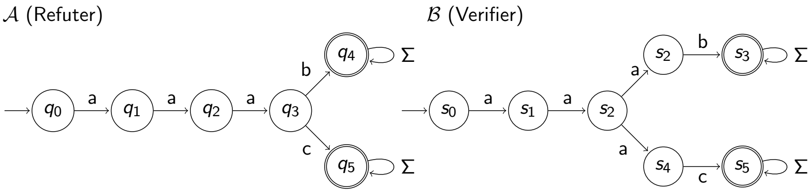

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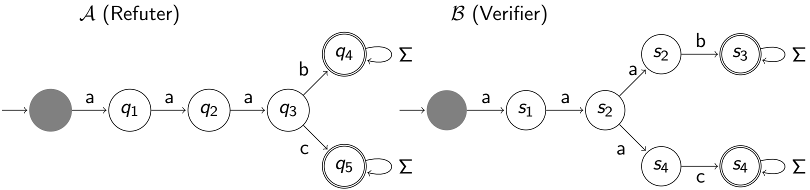
- hard to determine a search is hopeless already
- can we refine the game?

Dynamic k-letter game

- verifier may choose how far both players move.
- both players move h_i steps in round i, $h_i < k$.

Example ($\mathcal{A} \sqsubseteq^3_{\mathrm{dyn}} \mathcal{B}$)



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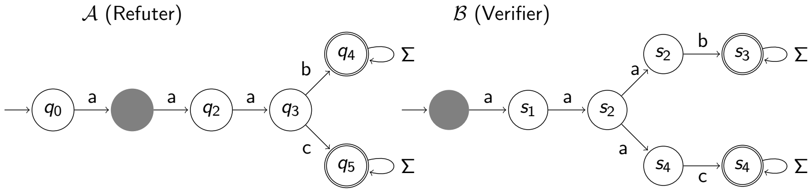

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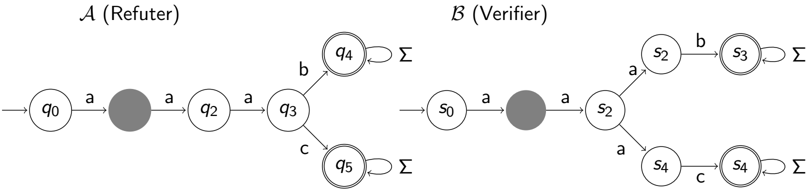

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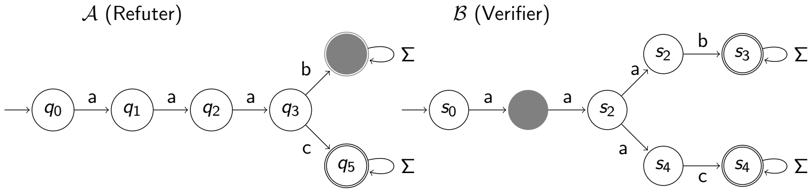

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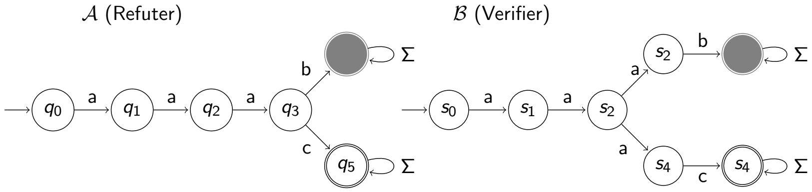

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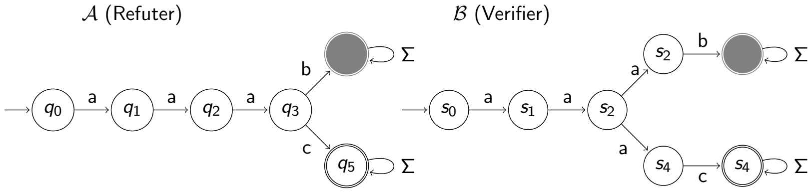

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Dynamic k-letter game

- more powerful than static game
- less powerful than pebble game

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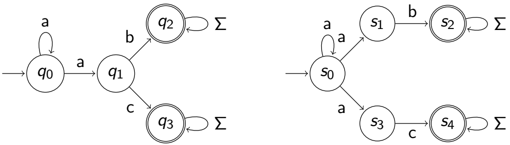

- upper bound:

$$\mathcal{A} \sqsubseteq^{k}_{\text{dyn}} \mathcal{B} \quad \Rightarrow \quad \mathcal{A} \sqsubseteq^{k_0}_{\text{dyn}} \mathcal{B}, \quad k_0 := 2^{(|\mathcal{A}|+|\mathcal{B}|)^3}$$

Experiments

Experiments

mutual exclusion protocols							
	multi-letter				multi-pebble		Rabit
	k	dynamic	k	static	k	pebble	
Mcs	1	22.94s	1	23.48s	1	25.41s	39.00s
FischerV2	1	0.07s	1	0.07s	1	0.07s	0.09s
Peterson	1	0.01s	1	0.01s	1	0.01s	0.03s
Bakery	1	7.22s	1	7.16s	1	7.23s	4.43s
Phils	1	0.02s	1	0.02s	1	0.02s	0.11s
Fischer	1	40.80s	1	47.30s	1	47.37s	3.41s
FischerV3	-	>1h	-	>1h	-	>1h	7.63s
FischerV4	-	>1h	-	>1h	-	>1h	2136.70s
BakeryV2	2	36.01s	2	7.06s	-	>1h	>1h

random NBA								
	size $= 30$, $d_{\mathrm{acc}} = 0.1$, $d_{\mathrm{tr}} = 2$				size $= 50$, $d_{\mathrm{acc}} = 0.6$, $d_{\mathrm{tr}} = 3$			
	dynamic	static	pebble	Rabit	dynamic	static	pebble	Rabit
time	59.12s	52.78s	18.22s	1655.84s	0.55s	0.52s	2.09s	127.53s
success %	80%	82%	86%	58%	100%	100%	100%	100%
average k	3.66	4.33	1.93	-	1.01	1.01	1.01	-

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- incremental testing
  - reasonable approach for inclusion problem
  - when succeed, comparable to the complete method

Literatures

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