Automaton semigroups: the two-state case

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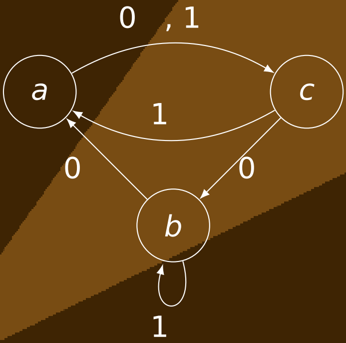
Automata

finite stateset

finite alphabet

$$\left(A, \Sigma, \delta = (\delta_i : A \to A)_{i \in \Sigma} \right)$$

transition functions

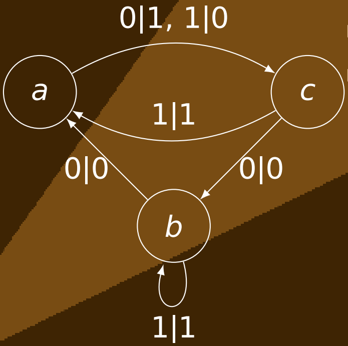


- no initial state
- no final state
- finite
- deterministic
- complete

Mealy automata

production functions

$$\mathcal{A} = \left(A, \Sigma, \delta = (\delta_i : A \to A)_{i\in\Sigma}, \rho = (\ \rho_x\ : \Sigma \to \Sigma)_{a\in A}\right)$$



- (A, Σ, δ) is an automaton
- (Σ, A, ρ) is an automaton

Mealy automata

production functions

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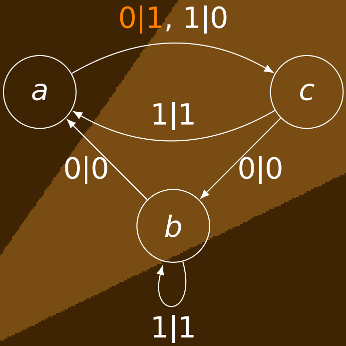

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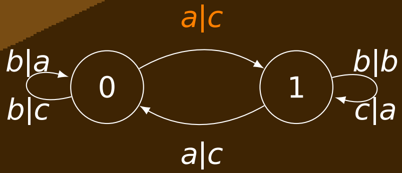
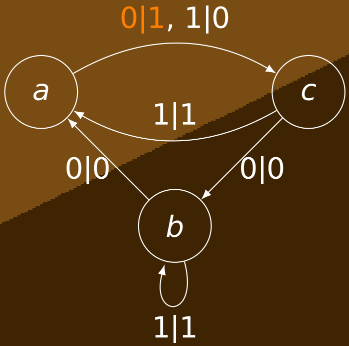


Generated semigroups

$\mathcal{A} = (A, \Sigma, \delta, \ \rho \ )$

$\rho_x : \Sigma \ \ \longrightarrow \Sigma$

$\text{ex} : \rho_a : 0 \qquad \mapsto 1$

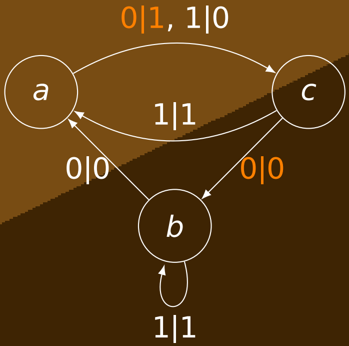


Generated semigroups

$\mathcal{A} = (A, \Sigma, \delta, \ \rho \ )$

$\rho_x : \Sigma^* \longrightarrow \Sigma^*$

ex : $\rho_a : 00\ldots \mapsto 10\ldots$



Generated semigroups

$$\mathcal{A} = (A, \Sigma, \delta, \rho)$$

$$\rho_x : \Sigma^* \longrightarrow \Sigma^*$$

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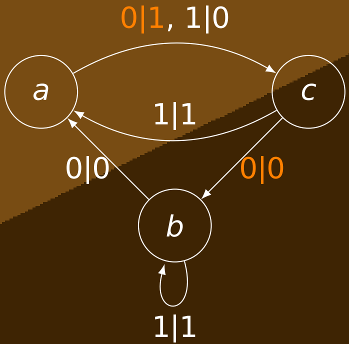


generated semigroup :

$$\langle \mathcal{A} \rangle_+ = \langle \rho_x, x \in A \rangle_+$$

why automaton (semi)groups ?

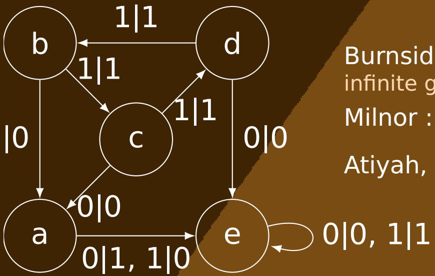
automaton (semi)groups

easy handling

Mealy automata
automata theory

Tree automorphisms
geometric (semi)group theory

Grigorchuk automaton



Burnside : infinite torsion group

infinite group with only finite order elements

Milnor : intermediate growth group

Atiyah, Day, Gromov, etc.

complex behaviour

automaton (semi)groups

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Burnside problem

Theorem 1

$$\mathcal{A} \text{ two-state and reversible}$$
$$\Downarrow$$
$$\langle \mathcal{A} \rangle_+ \text{ finite or free of rank 2}$$

Burnside problem

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Conjecture 1

$$\mathcal{A} \text{ reversible}$$
$$\Downarrow$$
$$\langle \mathcal{A} \rangle_+ \text{ cannot answer to the Burnside problem}$$

Finiteness problem

The finiteness problem for automaton semigroups is undecidable,
P. Gillibert.
arXiv :cs.FL/1304.2295 (2013)

Theorem 2
The finiteness of a semigroup generated by a two-state bireversible automaton is decidable.

$\rightsquigarrow$ decidability for two-letter bireversible automata

Finiteness problem

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⇝ decidability for two-letter bireversible automata

Conjecture 2
The finiteness of a semigroup generated by a bireversible automaton is decidable.

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