chapters are as follows: I. General remarks on processing of information; II. Abstract (= behavioristic) theory of automata; III. Structural theory of automata; IV. Minimization of Boolean functions; V. Design of combinational (= memoryless) circuits using a binary alphabet; VI. Some questions concerning the reliability of automata; VII. Algorithmic structure of universal digital computers. The connection of the contents of the book with logic is chiefly via Boolean functions. Chapter III contains an exposition of fundamental facts from Boolean algebra, including a proof of the theorem stating necessary and sufficient conditions for a set of Boolean functions to be complete. (Properly speaking, by "Boolean algebra" the author means a theory based on negation, conjunction, and disjunction, while a theory based on conjunction and addition modulo two is what he calls "Žégalkin's algebra.") In Chapter IV there is an exposition of the minimization methods due to Quine, McCluskey, Blake, and Nelson, as well as of graphical methods using Karnaugh's maps or Veitch's diagrams. In addition, in Chapter VII, there is an outline of the proof of a theorem to the effect that an arbitrary Markov algorithm can be realized by a certain type of universal computer, called "universal programming automaton" (the concept of Turing machine, however, is not introduced). JIŘÍ BEČVÁŘ

Ú. T. MÉDVÉDÉV. *O klassé sobytij, dopuskaúščih prédstavlénié v konéčnom avtomaté* (On a class of events representable in a finite automaton). ***Avtomaty,*** Izdatél'stvo Inostrannoj Litératury, Moscow 1956, pp. 385–401.

The intention of the author is to describe finite automata (in the sense of Kleene) in a more comprehensible form without the use of the notions of regular event and regular set of tables.

*Definition.* A finite automaton is a device with a finite set $M = \{a_1, \ldots, a_m\}$, $m \geqq 1$, of states and a finite set $J = \{I_1, \ldots, I_n\}$, $n \geqq 1$, of inputs. The inputs are treated here as in Kleene (XXIII 59), i.e., each one can be in two states, let us say 0 and 1, and at each moment only one input $I_k$ can be in the state 1. We suppose further that every $I_k$ defines a function $I_k : M \to M$, such that $I_k a_i = a_j$ means that if the automaton is in the state $a_i$ and the input $I_k$ is in state 1, then the next state of the automaton is $a_j$.

It is shown (Theorem 1) that this notion is equivalent to each of the five following notions: (1) McCulloch-Pitts nerve nets; (2) finite automata in the sense of Kleene; (3) finite automata described by semigroups; (4) Turing machines with finite tape; and (5) finite automata in the sense of Moore (with outputs).

We can consider our automaton as having only one input with $n$ possible states $S_1, \ldots, S_n$, where $S_k$ means that $I_k$ is in the state 1. We shall denote by $\Sigma$ the set of all finite words in the alphabet $S_1, \ldots, S_n$. An event will be equivalent to a subset $C \subset \Sigma$ in the sense that every subset $C$ of $\Sigma$ defines some event $E$. If $\sigma \in C$ then $\sigma$ is called a realization of the event $E$.

*Definition.* Let $A$ be an automaton defined by the set of states $M = \{a_1, \ldots, a_m\}$ and the set of inputs $J = \{I_1, \ldots, I_k\}$. Let $M^0 \subset M$ and $a^0 \in M$. We shall say that
Recognisable
an event $E$ is representable in $A$ by $M^0$ and $a^0$ if for every $\sigma \in \Sigma$, $\sigma = S_{i_1} \ldots S_{i_n}$, the following holds: $\sigma$ is a realization of $E$ if and only if $I_{i_n} I_{i_{n-1}} \ldots I_{i_1} a^0 \in M^0$. We denote this fact by $E \sim A\{a^0, M^0\}$. $E$ is said to be representable if there exist $A$, $a^0$, and $M^0$ such that $E \sim A\{a^0, M^0\}$.

The following four kinds of events will be called events of elementary type:

(1) $E_{p=1}$, an event realized by all words of length 1;

(2) $E_{p=2}$, an event realized by all words of length 2;

(3) $E_{S_i}^{(p)}$, an event realized by all words of length $p$ with the symbol $S_i$ at the end;

(4) $E_{S_i}^{(p-1)}$, either an event realized by all words of length $p$ with $S_i$ in the $(p-1)$th position if $p > 1$, or an event $E_{p=1}$ if $p = 1$.

Let $E$, $E'$ be events. We shall define new events $E \vee E'$, $\bar{E}$, $(t)_{t \leqq p}E$, $[S_i \to S]E$ as follows:

(1) $E \vee E'$ is realized by $\sigma$ if and only if either $E$ or $E'$ is realized by $\sigma$;

(2) $\bar{E}$ is realized by $\sigma$ if and only if $E$ is not realized by $\sigma$;

(3) $(t)_{t \leqq p}E$ is realized by $\sigma = S_{i_1} \ldots S_{i_p}$ if and only if for every $1 \leqq t \leqq p$, $E$ is realized by $\sigma = S_{i_1} \ldots S_{i_t}$;

some

(4) $[S_i \to S]E$ is realized by $\sigma$ if and only if $E$ is realized by $\sigma'$, where $\sigma =$ Sb $(S_i/S)\sigma'$.

substitution

The operations $\vee$, $^{-}$, $(t)_{t \leqq p}$, $[S_i \to S]$ are called the elementary operations.

representable? *Theorem* 2. The class of all realizable events is the least class containing all elementary events and closed under the elementary operations.

It is interesting to note that the definition of finite automaton given here is very similar to that given by Rabin and Scott in XXV 163. The notion of event corresponds to the notion of set of tapes and the notion of representability of events to that of acceptability of tapes. Médvédév's results in Theorem 2 can also be related to some results of Rabin and Scott. ANDRZEJ J. BLIKLE

HEINZ ZEMANEK. *Automaten und Denkprozesse.* German, with summaries in German, English, and French. ***Digitale Informationswandler — Digital information processors,*** edited by Walter Hoffmann, Friedr. Vieweg & Sohn, Braunschweig, and Interscience Publishers, New York, 1962, pp. 1–66.

This is an excellent paper, devoted mainly to exposition and discussion of matters outside the field of this JOURNAL, in particular to the question whether and in what sense it can properly be said that machines think.

In the one paragraph that does venture on the field of the JOURNAL there is, in the reviewer's opinion, a substantial correction necessary. The author writes (p. 7): "... was immer logisch exakt beschreibbar ist, läßt sich als Automat realisieren. Die Bedeutung dieses Satzes ist schwerwiegend; er macht die meisten, vielleicht sogar alle Argumentationen wertlos, die mit den Worten '... *aber man wird nie einen Automaten bauen können, der* ...' eingeleitet werden. Vermag man nur das, was der Automat können (oder nicht können) soll, exakt zu beschreiben, so kann man diesen Automaten auch bauen ..." This has failed to take the Gödel incompleteness theorem into account (and the context moreover suggests that the author is erroneously identifying logic with propositional calculus). The (theoretical) existence of a (potentially infinite) automaton is a criterion, not for the existence of an exact logical description, but for effective producibility, which is another matter. At least from the viewpoint of classical mathematics—as opposed to that of intuitionism or some form of finitism—the class of valid formulas of arithmetic in some specific formalized language, e.g. the standard first-order arithmetic, has an exact description: one knows what it would mean for an automaton to produce them all, but one also knows that no automaton can do this. ALONZO CHURCH

YEHOSHUA BAR-HILLEL. *Preface.* ***Language and information, Selected essays on their theory and application,*** by Yehoshua Bar-Hillel, Addison-Wesley Publishing Company, Inc., Reading, Mass., Palo Alto, London, and The Jerusalem Academic Press Ltd., Jerusalem, Israel, 1964, pp. vii-viii.

YEHOSHUA BAR-HILLEL. *Introduction.* Ibid., pp. 1–16.

YEHOSHUA BAR-HILLEL. *On syntactical categories.* A reprint of XV 220. Ibid., pp. 19–37.

YEHOSHUA BAR-HILLEL. *Logical syntax and semantics.* A reprint of XX 290. Ibid., pp. 38–46.

YEHOSHUA BAR-HILLEL. *Idioms.* A slightly revised reprint of XXIX 68. Ibid., pp. 47–55.