A Tighter Bound for the Determinization of Visibly Pushdown Automata*

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Visibly pushdown automata (VPA), introduced by Alur and Madhusuan in 2004, is a subclass of pushdown automata whose stack behavior is completely determined by the input symbol according to a fixed partition of the input alphabet. Since its introduce, VPAs have been shown to be useful in various context, *e.g.,* as specification formalism for verification and as automaton model for processing XML streams. Due to high complexity, however, implementation of formal verification based on VPA framework is a challenge. In this paper we consider the problem of implementing VPA-based model checking algorithms. For doing so, we first present an improvement on upper bound for determinization of VPA. Next, we propose simple on-the-fly algorithms to check universality and inclusion problems of this automata class. Then, we implement the proposed algorithms in a prototype tool. Finally, we conduct experiments on randomly generated VPAs. The experimental results show that the proposed algorithms are considerably faster than the standard ones.

1 Introduction

Visibly pushdown automata [1] are pushdown automata whose stack behavior (i.e. whether to execute a push, a pop, or no stack operation) is completely determined by the input symbol according to a fixed partition of the input alphabet. As shown in [1], this class of visibly pushdown automata enjoys many good properties similar to those of the class of finite automata. The main reason for this being that is, each nondeterministic VPA can be transformed into an equivalent deterministic one. Therefore, checking context-free properties of pushdown models is feasible as long as the calls and returns are made visible. As a result, visibly pushdown automata have turned out to be useful in various context, *e.g.* as specification formalism for verification and synthesis problem for pushdown systems [3, 4, 11], as automaton model for processing XML streams [10, 9], and as AOP protocols for component-based systems [12, 13].

As each nondeterministic VPA can be determinized, all problems that concern the accepted languages such as universality and inclusion problems are decidable. To check universality for a nondeterministic VPA M over its alphabet Σ (that is, to check if $L(M) = \Sigma^*$), the standard method is first to make it complete, determinize it, complement it, and then checks for emptiness. To check the inclusion problem $L(M) \subseteq L(N)$, the standard method computes the complement of N, takes its intersection with M and then, check for emptiness. This is costly as computing the complement necessitates a full determinization. This explosion is in some sense unavoidable, because determinization for VPAs requires exponential time blowup [1]. Therefore, one of the questions raised is that whether one can implement

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efficiently operations like determinization as well as decision procedures like universality (or. inclusion) checking for VPAs.

During the recent years, a new approach called *antichain method* has been proposed to implement efficiently operations like universality or inclusion checking on nondeterministic word or tree automata [14, 6]. Unfortunately, the antichain technique cannot be directly used for checking universality and inclusion of VPA. This is because the set of configurations of a VPA is infinite and thus, computing the set of antichains may not terminate. In this paper, we focus on the problem of checking universality and inclusion for VPAs. We make the following contributions towards to this overall goal.

- First, we present an improvement on upper bound for determinization of VPA. In [1], Alur and Madhusudan showed that any nondeterministic VPA with n states can be translated into a deterministic one with at most 2^{n^2+n} states. Here, we show that this upper bound can be made tighter. More precisely, we optimize Alur-Madhusudan's determinization procedure, and show that any nondeterministic VPA with n states can be transformed into a deterministic one with at most 2^{n^2} states.

- Second, we apply the standard method to check universality and inclusion problems for nondeterministic VPA. This method includes two main steps: determinization and reachability checking for non-accepting configurations. For determinization, we use the Alur-Madhusudan's procedure [1]. For reachability checking, we apply the symbolic technique $\mathscr{P}$-automata [7, 8] to compute the sets of all reachable configurations of a VPA.

- Third, we present an on-the-fly method to check universality of VPA. The idea is very simple that we perform determinization and reachability checking by $\mathscr{P}$-automaton simultaneously. For checking universality of nondeterministic VPA M, we first create the initial state of the determinized VPA M^d and, initiate a $\mathscr{P}$-automaton A to represent the initial configuration of M^d. Second, construct new transitions departing from the initial states, and update the $\mathscr{P}$-automaton A. Then, the determinized VPA M^d is updated using new states and transitions of A (which correspond to pairs of the states and topmost stack symbols of M^d), and so on. When a non-accepting state is added to A, we stop and report that M is not universal.

- Fourth, we also propose a new algorithmic solution to inclusion checking for VPAs using on-the-fly manner. Again, no explicit determinization is performed. To solve the language-inclusion problem for nondeterministic VPAs, $L(M) \subseteq L(N)$, the main idea is to find at least one word w accepted by M but not accepted by N, *i.e.,* $w \in L(M) \setminus L(N)$.

- Finally, we have implemented all algorithms in a prototype tool (written in Java 1.5) and tested them in a series of experiments. Although the standard methods (as well as on-the-fly ones) have the same worst case complexity, our preliminary experiments on randomly generated visibly pushdown automata show a significant improvement of on-the-fly methods compared to the standard ones.

The remainder of this paper is organized as follows. In Section 2 we recall notions and properties of VPAs, and then we give an improvement on determinization of VPAs. Section 3 presents new algorithms for checking universality and inclusion of VPAs. Implementation as well as experimental results are presented and analyzed in Section 4. Section 5 discusses about related works. Finally, we conclude the paper in Section 6.

2 Visibly Pushdown Automata

2.1 Definitions

In this section we briefly recall the notions and properties of visibly pushdown automata. Readers are referred to the seminal paper [1] for their more details.

Let Σ be the finite input alphabet, and let $\Sigma = \Sigma_c \cup \Sigma_r \cup \Sigma_i$ be a partition of Σ. The intuition behind the partition is: Σ_c is the finite set of *call* (push) symbols, Σ_r is the finite set of *return* (pop) symbols, and Σ_i is the finite set of *internal* symbols. Visibly pushdown automata are formally defined as follows:

Definition 1 *A* visibly pushdown automaton *(VPA) M over Σ is a tuple $(Q, Q_0, \Gamma, \Delta, F)$ where Q is a finite set of states, $Q_0 \subseteq Q$ is a set of initial states, $F \subseteq Q$ is a set of final states, Γ is a finite stack alphabet with a special symbol $\perp$ (representing the* bottom-of-stack*), and $\Delta = \Delta_c \cup \Delta_r \cup \Delta_i$ is the transition relation, where $\Delta_c \subseteq Q \times \Sigma_c \times Q \times (\Gamma \setminus \{\perp\})$, $\Delta_r \subseteq Q \times \Sigma_r \times \Gamma \times Q$, and $\Delta_i \subseteq Q \times \Sigma_i \times Q$.*

If $(q, c, q', \gamma) \in \Delta_c$, where $c \in \Sigma_c$ and $\gamma \neq \perp$, there is a *push-transition* from q on input c where on reading c, γ is pushed onto the stack and the control changes from state q to q'; we denote such a transition by $q \xrightarrow{c/+\gamma} q'$. Similarly, if (q, r, γ, q'), there is a *pop-transition* from q on input r where γ is read from the top of the stack and popped (if the top of the stack is $\perp$, then it is read but not popped), and the control state changes from q to q'; we denote such a transition $q \xrightarrow{r/-\gamma} q'$. If $(q, i, q') \in \Delta_i$, there is an *internal-transition* from q on input i where on reading i, the state changes from q to q'; we denote such a transition by $q \xrightarrow{i} q'$. Note that there are no stack operations on internal transitions.

We write *St* for the set of *stacks* $\{w\perp \mid w \in (\Gamma \setminus \{\perp\})^*\}$. A *configuration* is a pair (q, σ) of $q \in Q$ and $\sigma \in St$. The transition function of a VPA can be used to define how the configuration of the machine changes in a single step: we say $(q, \sigma) \xrightarrow{a} (q', \sigma')$ if one of the following conditions holds:

- If $a \in \Sigma_c$ then there exists $\gamma \in \Gamma$ such that $q \xrightarrow{a/+\gamma} q'$ and $\sigma' = \gamma \cdot \sigma$
- If $a \in \Sigma_r$, then there exists $\gamma \in \Gamma$ such that $q \xrightarrow{a/-\gamma} q'$ and either $\sigma = \gamma \cdot \sigma'$, or $\gamma = \perp$ and $\sigma = \sigma' = \perp$
- If $a \in \Sigma_i$, then $q \xrightarrow{a} q'$ and $\sigma = \sigma'$.

A (q_0, w_0)-*run* on a word $u = a_1 \cdots a_n$ is a sequence of configurations $(q_0, w_0) \xrightarrow{a_1} (q_1, w_1) \cdots \xrightarrow{a_n} (q_n, w_n)$, and is denoted by $(q_0, w_0) \xrightarrow{u} (q_n, w_n)$. A word u is accepted by M if there is a run $(q_0, w_0) \xrightarrow{u} (q_n, w_n)$ with $q_0 \in Q_0$, $w_0 = \perp$, and $q_n \in Q_F$. The language $L(M)$ is the set of words accepted by M. The language $L \subseteq \Sigma^*$ is a *visibly pushdown language* (VPL) if there exists a VPA M with $L = L(M)$.

Definition 2 *A VPA M is* deterministic *if $|Q_0| = 1$ and for every configuration (q, σ) and $a \in \Sigma$, there are at most one transition from (q, σ) by a. For deterministic VPAs (DVPAs) we denote the transition relation by δ instead of Δ, and write:*

1. *$\delta(q, a) = (q', \gamma)$ instead of $(q, a, q', \gamma) \in \Delta$ if $a \in \Sigma_c$,*
2. *$\delta(q, a, \gamma) = q'$ instead of $(q, a, \gamma, q') \in \Delta$ if $a \in \Sigma_r$, and*
3. *$\delta(q, a) = q'$ instead of $(q, a, q') \in \Delta$ if $a \in \Sigma_i$.*

2.2 Determinization

As shown in [1], any nondeterministic VPA can be transformed into an equivalent deterministic one. The key idea of the determinization procedure is to do subset construction, but postponing handling push transitions. The push transitions are stored into the stack and simulated at the time of matching pop transitions. The construction has two components: a set of *summary edges* S, that keeps track of what state transitions are possible from a push transition to the corresponding pop transition, and a set of *path edges* R, that keeps track of all possible state reached from initial states. For completeness, let us briefly recall the original determinization procedure [1] as below.

Let $M = (Q, \Gamma, Q_0, \Delta, F)$ be a nondeterministic VPA. We construct an equivalent deterministic VPA $M' = (Q', \Gamma', Q'_0, \Delta', F')$ as follows: $Q' = 2^{Q \times Q} \times 2^Q$, $Q'_0 = \{(Id_Q, Q_0)\}$ where $Id_Q = \{(q,q) \mid q \in Q\}$, $F' = \{(S,R) \mid R \cap F \neq \varnothing\}$, $\Gamma' = Q' \times \Sigma_c$, and the transition relation $\Delta' = \Delta'_i \cup \Delta'_c \cup \Delta'_r$ is given by:

- **Internal:** For every $a \in \Sigma_i$, $(S,R) \xrightarrow{a} (S',R') \in \Delta'_i$ where $S' = \{(q,q') \mid \exists q'' \in Q : (q,q'') \in S, q'' \xrightarrow{a} q' \in \Delta_i\}$, and $R' = \{q' \mid \exists q \in R : q \xrightarrow{a} q' \in \Delta_i\}$.
- **Push:** For every $a \in \Sigma_c$, $(S,R) \xrightarrow{a/+(S,R,a)} (Id_Q, R') \in \Delta'_c$ where $R' = \{q' \mid \exists q \in R : q \xrightarrow{a/+\gamma} q' \in \Delta_c\}$.
- **Pop:** For every $a \in \Sigma_r$,
  - if the stack is empty : $(S,R) \xrightarrow{a/-\perp} (S',R') \in \Delta'_r$ where $S' = \{(q,q') \mid \exists q'' \in Q : (q,q'') \in S, q'' \xrightarrow{a/-\perp} q' \in \Delta_r\}$ and $R' = \{q' \mid \exists q \in R : q \xrightarrow{a/-\perp} q' \in \Delta_r\}$.
  - otherwise: $(S,R) \xrightarrow{a/-(S',R',a')} (S'',R'') \in \Delta'_r$, where

$$\begin{cases} R'' & = & \{q' \mid \exists q \in R' : (q,q') \in Update\} \\ S'' & = & \{(q,q') \mid \exists q_3 \in Q : (q,q_3) \in S', (q_3,q') \in Update\} \\ Update & = & \left\{ (q,q') \;\middle|\; \begin{array}{l} \exists q_1 \in Q, q_2 \in R : (q_1,q_2) \in S, \\ q \xrightarrow{a'/+\gamma} q_1 \in \Delta_c, q_2 \xrightarrow{a/-\gamma} q' \in \Delta_r \end{array} \right\} \end{cases}$$

Theorem 1 ([1, Theorem 2]) *Let M be a VPA. The VPA M' is deterministic and $L(M') = L(M)$. Moreover, if M has n states, one can construct M' with at most 2^{n^2+n} states and with stack alphabet of size $|\Sigma_c| \cdot 2^{n^2+n}$.*

Example 1 *We illustrate the original determinization procedure by an example in Figure 1.*

2.3 An Improvement on Complexity for Determinization

During implementation of VPA's operations, we found that the set of summaries S in the determinization may contain unnecessary pairs in the sense that these pairs do not keep information of reachable states. In other words, for any state (S,R) of the determinized VPA, $\Pi_2(S)$ does not always equal to R in which Π_2 is the projection on the second component. In the following, we present an optimization for determinization by keeping the set of summaries as few as possible. This simple observation, however, leads to a tighter bound for determinization.

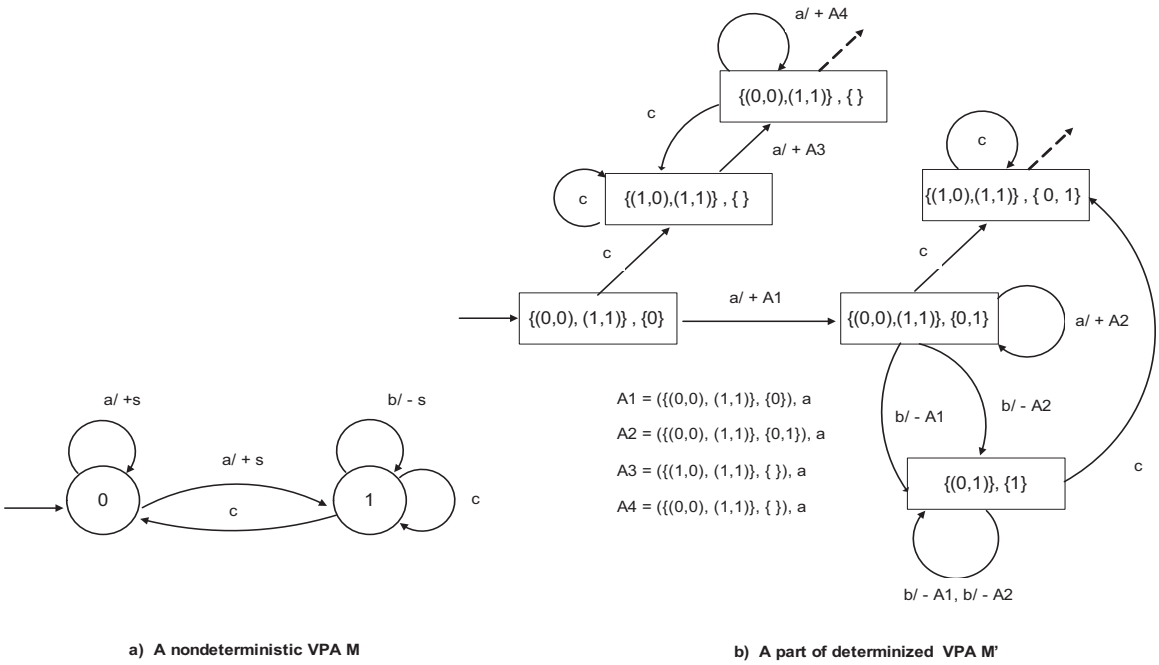

Figure 1: An example for determinization of VPA

2.3.1 Optimize S-Component

We first optimize Alur-Madhusudan's determinization of VPA by minimizing the set of summaries S. Given a finite set X, let us denote $Id_X = \{(q,q) \mid q \in X\}$.

Let $M = (Q, \Gamma, Q_0, \Delta, F)$ be a nondeterministic VPA. We construct an equivalent deterministic VPA $M^d = (Q', \Gamma', Q_0', \Delta', F')$ as follows: $Q' = 2^{Q \times Q} \times 2^Q$, $Q_0' = \{(Id_{Q_0}, Q_0)\}$ where $F' = \{(S,R) \mid R \cap F \neq \varnothing\}$, $\Gamma' = Q' \times \Sigma_c$, and the transition relation $\Delta' = \Delta_i' \cup \Delta_c' \cup \Delta_r'$ is given by:

- **Internal:** For every $a \in \Sigma_i$, $(S,R) \xrightarrow{a} (S',R') \in \Delta_i'$ where $S' = \{(q,q') \mid \exists q'' \in Q : (q,q'') \in S, q'' \xrightarrow{a} q' \in \Delta_i\}$ and $R' = \{q' \mid \exists q \in R : q \xrightarrow{a} q' \in \Delta_i\}$
- **Push:** For every $a \in \Sigma_c$, $(S,R) \xrightarrow{a/+(S,R,a)} (Id_{R'}, R') \in \Delta_c'$ where $R' = \{q' \mid \exists q \in R : q \xrightarrow{a/+\gamma} q' \in \Delta_c\}$
- **Pop:** For every $a \in \Sigma_r$,
  - if the stack is empty : $(S,R) \xrightarrow{a/-\perp} (S',R') \in \Delta_r'$ where $S' = \{(q,q') \mid \exists q'' \in Q : (q,q'') \in S, q'' \xrightarrow{a/-\perp} q' \in \Delta_r\}$ and $R' = \{q' \mid \exists q \in R : q \xrightarrow{a/-\perp} q' \in \Delta_r\}$.
  - otherwise: $(S,R) \xrightarrow{a/-(S',R',a')} (S'',R'') \in \Delta_r'$, where

$$\begin{cases} R'' & = & \{q' \mid \exists q \in R' : (q,q') \in Update\} \\ S'' & = & \{(q,q') \mid \exists q_3 \in Q : (q,q_3) \in S', (q_3,q') \in Update\} \\ Update & = & \left\{ (q,q') \;\middle|\; \begin{array}{l} \exists q_1, q_2 \in Q : (q_1,q_2) \in S, \\ q \xrightarrow{a'/+\gamma} q_1 \in \Delta_c, q_2 \xrightarrow{a/-\gamma} q' \in \Delta_r \end{array} \right\} \end{cases}$$

Remark 1 *The main differences of our construction with the original one are: (1) we initiate the initial state as (Id_{Q_0}, Q_0) instead of (Id_Q, Q_0); and (2) after reading a push symbol, the automaton will enter the state $(Id_{R'}, R')$ instead of (Id_Q, R').*

Lemma 1 *For a given nondeterministic VPA M, let M^d be the deterministic VPA constructed from M as above. Then, $\Pi_2(S) = R$ for any state (S,R) of M^d, where Π_2 is the projection on the second component.*

Proof 1 *Since states of M^d are generated on-the-fly manner, we prove the lemma by induction on the length of input words. Let w be an input word.*

1. *If $|w| = 0$, the lemma holds because $Q'_0 = (Id_{Q_0}, Q_0)$ and $\Pi_2(Id_{Q_0}) = Q_0$.*
2. *If $|w| = 1$, then $w = a \in \Sigma$. Consider three cases of a:*
   - *If $a \in \Sigma_i$: Based on the construction of transitions, we have $(Id_{Q_0}, Q_0) \xrightarrow{a} (S,R) \in \Delta'_i$ where $S = \{(q,q') \mid \exists q'' \in Q_0 : (q,q) \in Id_{Q_0}, q \xrightarrow{a} q' \in \Delta_i\}$ and $R = \{q' \mid \exists q \in Q_0 : q \xrightarrow{a} q' \in \Delta_i\}$. It is easy to verify that $\Pi_2(S) = R$.*
   - *If $a \in \Sigma_c$: The proof is trivial.*
   - *If $a \in \Sigma_r$: Since the stack now is empty, the proof is similar to the case of internal symbols.*
3. *If $|w| = 2$, assume that $w = a_1a_2$. The proof is trivial for the cases: $a_1 \in \Sigma_i \cup \Sigma_c \wedge a_2 \in \Sigma_i \cup \Sigma_c$; $a_1 \in \Sigma_i \wedge a_2 \in \Sigma_r$. We now check the last case: $a_1 \in \Sigma_c \wedge a_2 \in \Sigma_r$. After reading a_1, the current state of M^d is (S,R) (with $\Pi_2(S) = R$ by the induction assumption) and the stack content is $(Id_{Q_0}, Q_0, a_1)\perp$. On reading a_2, a new transition of M^d is created: $(S,R) \xrightarrow{a/-(Id_{Q_0},Q_0,a_1)} (S',R') \in \Delta'_r$ where $R' = \{q' \mid \exists q \in Q_0 : (q,q') \in Update\}$, $S' = \{(q,q') \mid \exists q \in Q : (q,q) \in Id_{Q_0}, (q,q') \in Update\}$, and $Update = \{(q,q') \mid \exists q_1, q_2 \in Q : (q_1,q_2) \in S, q \xrightarrow{a_1/+\gamma} q_1 \in \Delta_c, q_2 \xrightarrow{a_2/-\gamma} q' \in \Delta_r\}$. It is easy to see in this case that $\Pi_2(S') = R' = \Pi_2(Update)$.*
4. *Now, let us assume that the lemma holds with $|w| = n$. Without loss of generality, we suppose that $w = w_1a_1w_2a_2 \cdots w_k$ where in w_1 every call is matched by a return, but there may be unmatched returns; w_i ($i = 2 \cdots k$) are well-matched words, and a_i ($i = 1 \cdots k$) are calls. After reading w, M^d will have its stack $(S_{k-1}, R_{k-1}, a_{k-1}) \cdots (S_1, R_1, a_1)\perp$ and its control state will be (S_k, R_k). By the assumption, we have $\Pi_2(S_k) = R_k$. Assume that M^d read an input symbol a_k. There are three cases of a_k:*
   - *If $a_k \in \Sigma_i$: The automaton will go to the control state (S',R'). Similar to the proof for the case $|w| = 1$, we get $\Pi_2(S') = R'$.*
   - *If $a_k \in \Sigma_c$: The proof is trivial.*
   - *If $a_k \in \Sigma_r$: The automaton changes control state to (S',R') and pops the stack symbol $(S_{k-1}, R_{k-1}, a_{k-1})$. Namely, $(S_k, R_k) \xrightarrow{a_k/-(S_{k-1},R_{k-1},a_{k-1})} (S',R') \in \Delta'_r$,*

$$\begin{cases} R' & = & \{q' \mid \exists q \in R_{k-1} : (q,q') \in Update\} \\ S' & = & \{(q,q') \mid \exists q, q_3 \in Q : (q,q_3) \in S_{k-1}, (q_3,q') \in Update\} \\ Update & = & \left\{(q,q') \middle| \begin{array}{l} \exists q_1, q_2 \in Q : (q_1,q_2) \in S_k, \\ q \xrightarrow{a_1/+\gamma} q_1 \in \Delta_c, q_2 \xrightarrow{a_2/-\gamma} q' \in \Delta_r \end{array} \right\} \end{cases}$$

Since $\Pi_2(S_{k-1}) = R_{k-1}$, we obtain that $\Pi_2(S') = R'$. The lemma is proved.

2.3.2 Remove R-Component

As can be seen in the previous section, the component S in a state of M^{od} satisfies the condition $\Pi_2(S) = R$. Therefore, we can further optimize this determinization procedure by using the second component of the summary S as the set of reachable states.

Let $M = (Q, \Gamma, Q_0, \Delta, F)$ be a nondeterministic VPA. We construct an equivalent deterministic VPA $M^{od} = (Q', \Gamma', Q'_0, \Delta', F')$ as follows: $Q' = 2^{Q \times Q}$, $Q'_0 = Id_{Q_0}$ where $F' = \{S \mid \Pi_2(S) \cap F \neq \varnothing\}$, $\Gamma' = Q' \times \Sigma_c$, and the transition relation $\Delta' = \Delta'_i \cup \Delta'_c \cup \Delta'_r$ is given by:

- **Internal:** For every $a \in \Sigma_i$, $S \xrightarrow{a} S' \in \Delta'_i$ where $S' = \{(q,q') \mid \exists q'' \in Q : (q,q'') \in S, q'' \xrightarrow{a} q' \in \Delta_i\}$.
- **Push:** For every $a \in \Sigma_c$, $S \xrightarrow{a/+(S,a)} Id_{R'} \in \Delta'_c$ where $R' = \{q' \mid \exists q \in \Pi_2(S) : q \xrightarrow{a/+\gamma} q' \in \Delta_c\}$.
- **Pop:** For every $a \in \Sigma_r$,
  - if the stack is empty : $S \xrightarrow{a/-\perp} S' \in \Delta'_r$ where $S' = \{(q,q') \mid \exists q'' \in Q : (q,q'') \in S, q'' \xrightarrow{a/-\perp} q' \in \Delta_r\}$.
  - otherwise: $S \xrightarrow{a/-(S',a')} S'' \in \Delta'_r$, where

$$\begin{cases} S'' & = & \{(q,q') \mid \exists q_3 \in Q : (q,q_3) \in S', (q_3,q') \in Update\} \\ Update & = & \left\{ (q,q') \;\middle|\; \begin{array}{l} \exists q_1, q_2 \in Q : (q_1,q_2) \in S, \\ q \xrightarrow{a'/+\gamma} q_1 \in \Delta_c, q_2 \xrightarrow{a/-\gamma} q' \in \Delta_r \end{array} \right\} \end{cases}$$

The next theorem immediately follows from the above construction.

Theorem 2 *For a given nondeterministic VPA M of n states. One can construct a deterministic VPA M^{od} such that $L(M^{od}) = L(M)$. Moreover, the number of states and stack symbols of M^{od} in the worst case are 2^{n^2} and $|\Sigma_c| \cdot 2^{n^2}$, respectively.*

Example 2 *We illustrate the optimized procedure by determinizing non deterministic VPA M in Figure 1. The result of this optimized determinization is given in Figure 2. We can see that the size of the determinized VPA is reduced.*

Remark 2 *We should mention a fact that the model of nested words was proposed in [2] for representation of data with both a linear ordering and a hierarchically nested matching of items. Recall that the input word of VPA has an implicit nesting structure defined by matching occurrences of symbols in Σ_c with symbols in Σ_r. In nested words, this nesting is given explicitly, and thus they defined finite-state acceptors (with out stacks) for nested words, so-called nested word automata. One can interpret a nested word automaton as a visibly pushdown automaton over classical words. As shown in [2], a nondeterministic nested word automaton with n states can be translated into a deterministic nested word automaton with at most 2^{n^2} states. In this paper, we show that the direct determinization of VPAs can be made tighter. As stack-based implementation is the most natural way in modeling recursive programs, we hope that our simple improvement on determinization procedure of VPAs is still useful.*

3 Universality and Inclusion Checking

According to visibility and determinizablity, the class of VPAs is closed under union and intersection, and complementation. Moreover, it has been shown that the universality and inclusion problems are EXPTIME-complete [1].

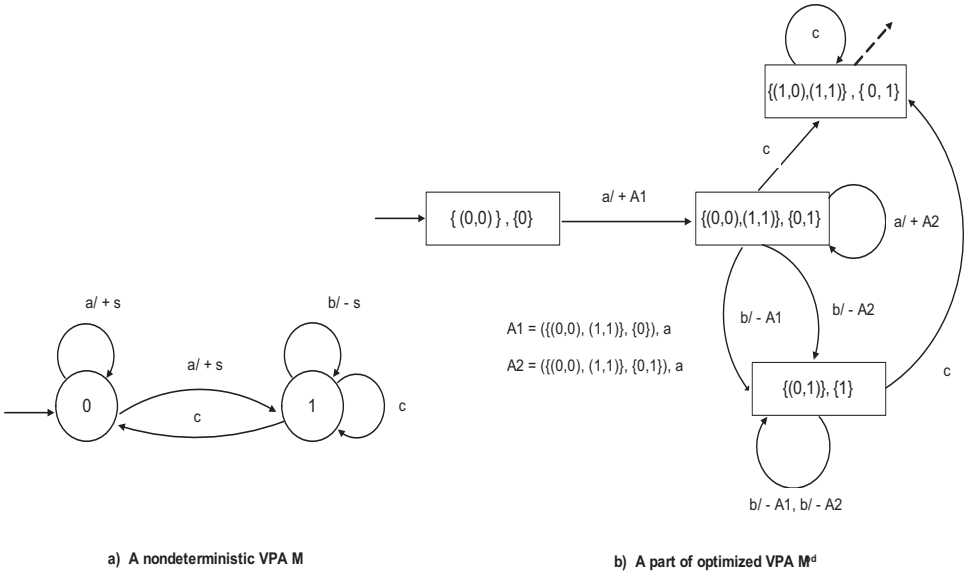

Figure 2: An example for optimized determinization of VPA

3.1 Emptiness Checking

A *pushdown system*(see [5, 7], for example) is pushdown automaton that is regardless of input symbols. Bouajjani *et al.* [5] have introduced an efficient symbolic method to compute reachable configurations of a pushdown system (This method was extended for model checking LTL properties of pushdown systems by Esparza *et al.* [7, 8]). The key of their technique is to use a finite automaton so-called $\mathscr{P}$*-automaton* to encode a set of infinite configurations of a pushdown system. It is easy to see that the $\mathscr{P}$-automaton technique can be used to solve emptiness problem for pushdown automata (or, visibly pushdown automata). To check the emptiness of a pushdown automaton M, the first step is to compute the set of its reachable configurations using $\mathscr{P}$-automata. Second, if there exists an accepting configurations, we conclude that the language of M is not empty.

In the following, we adapt $\mathscr{P}$-automata technique to checking emptiness of visibly pushdown automaton. Our definition, though in essence do not differ from the one in [5, 7, 8], has been tailored so that concepts discussed in this paper are easily related to the definition. Given a VPA $\mathscr{P} = (Q, \Gamma, Q_0, \Delta, F)$, a $\mathscr{P}$-automaton is used in order to represent sets of configurations C of $\mathscr{P}$. A $\mathscr{P}$-automaton uses Γ as the input alphabet, and Q as set of initial states. Formally,

Definition 3 ($\mathscr{P}$-automata) *1. A $\mathscr{P}$-automaton of a VPA $\mathscr{P}$ is a finite automaton $A = (P, \Gamma, \delta, Q, F_A)$ where P is the finite set of states, $\delta \subseteq P \times \Gamma \times P$ is the set of transitions, Q is the set of* initial states *and $F_A \subseteq P$ is the set of* final states.

2. A $\mathscr{P}$-automaton accepts *or* recognizes *a configuration (p, w) if $p \xrightarrow{w} q$, for some $p \in Q$, $q \in F_A$. The set of configurations recognized by $\mathscr{P}$-automaton A is denoted by $Conf(\mathscr{P})$.*

For a VPA $\mathscr{P} = (Q, \Gamma, Q_0, \Delta, F)$ and the set of configurations C, let A be a $\mathscr{P}$-automaton representing C. The $\mathscr{P}$-automaton A_{post^*} representing the set of configurations reachable from C ($Post^*(C)$) is constructed as follows: We compute $Post^*(C)$ as a language accepted by a $\mathscr{P}$-automaton A_{post^*} with

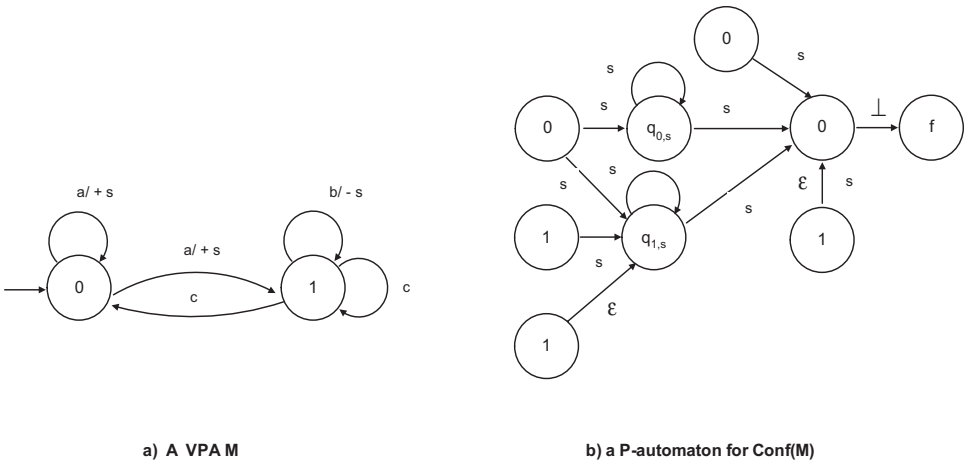

Figure 3: An example of $\mathscr{P}$-automata

ε-moves. We denote the relation $q(\xrightarrow{\varepsilon})^* \cdot \xrightarrow{\gamma} \cdot (\xrightarrow{\varepsilon})^* \cdot p$ by $\Longrightarrow^{\gamma}$. Formally, A_{post^*} is obtained from A in two stages:

- For each pair (q', γ') such that $\mathscr{P}$ contains at least one rule of the form $q \xrightarrow{a/+\gamma'} q' \in \Delta_c$, add a new state $p_{(q',\gamma')}$ to A.
- Add new transitions to A according to the following saturation rules:

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1. **Internal:** If $q \xrightarrow{a} q' \in \Delta_i$ and $q \Longrightarrow^{\gamma} p$ in the current automaton, add a transition (q', γ, p).
2. **Push:** If $q \xrightarrow{a/+\gamma'} q' \in \Delta_c$ and $q \Longrightarrow^{\gamma} p$ in the current automaton, first add $(q', \gamma', p_{(q',\gamma')})$, and then add $(p_{(q',\gamma')}, \gamma, p)$.
3. **Pop:** If $q \xrightarrow{a/-\gamma} q' \in \Delta_r$ and $q \Longrightarrow^{\gamma} p$ in the current automaton, add a transition (q', ε, p).

Example 3 *Let us revisit nondeterministic VPA M in Figure 1. A $\mathscr{P}$-automaton for the set of all reachable configurations of M is given in Figure 3.*

3.2 Universality Checking

In this section, we propose an *on-the-fly* method to solve the universality and inclusion problems for visibly pushdown automata. We first briefly recall the standard method in the next subsection.

3.2.1 Standard Methods

The standard algorithm for universality of VPA is to first determinize the automaton, and then check for the reachability of a non-accepting states. Reachable configurations of a determinized VPA can be computed by using $\mathscr{P}$-automata technique. A configuration $c = (q, w)$ is said a rejecting configuration if

q is not a final location. Whenever a rejecting configuration is found, we stop and report that the original VPA is not universal. Otherwise, if all reachable configurations of determinized VPA are accepting configurations, the original VPA is universal.

3.2.2 On-the-fly Methods

To improve efficiency of checking, we perform simultaneously on-the-fly determinization and $\mathscr{P}$-automata construction. There are two interleaving phases in this approach. First, we determinize VPA M step by step (iterations). After each step of determinization, we update the $\mathscr{P}$-automaton. Then, using the $\mathscr{P}$-automaton, we perform determinization again, and so on. It is crucial to note that this procedure terminates. This is because the size of the M^{od} is finite, and the $\mathscr{P}$-automaton construction is terminated. However, once a rejecting state is added to the $\mathscr{P}$-automaton, we stop and report that the VPA is not universal. Let $\mathsf{Conf}(M^{od})$ and $\mathsf{Rejecting\text{-}Conf}(M^{od})$ denote the sets of reachable and rejecting configurations of M^{od}, respectively. With the above observation, the following lemma holds:

Lemma 2 *Let M be a nondeterministic VPA. The automaton M is not universal iff there exists a rejecting reachable configuration of M^{od},* i.e., $\mathsf{Conf}(M^{od}) \cap \mathsf{Rejecting\text{-}Conf}(M^{od}) \neq \varnothing$.

Therefore checking universality of M amounts to finding a rejecting configuration of M^{od}. In Algorithm 1, we present an on-the-fly way to explore such rejecting configurations.

Algorithm 1 On-the-fly algorithm

Input: A nondeterministic VPA $M = (Q, Q_0, \Gamma, \Delta, F)$
Result: Universality of M

begin
Create the initial state of the determinized VPA M^{od};
Initiate $\mathscr{P}$-automaton A to present the initial configuration of M^{od};
$A_{post^*} \longleftarrow A$;
Create transitions of M^{od} departing from the initial state;

while (the set of new transitions of M^{od} is not empty) **do**
  Update the $\mathscr{P}$-automaton A_{post^*} using new transitions of M^{od};
  if a rejecting state is added to A_{post^*} **then**
    return False;
  end if
  Update M^{od} using new transitions of A_{post^*};
end while
return True;
end

Having said this, time complexity of the on-the-fly method is same as the complexity of the standard one. However, if the input VPA is not universal, the on-the-fly method is significantly faster. This is because the on-the-fly method does not need to perform full determinization, and thus it will immediately stop whenever a rejecting state is found.

Example 4 *We illustrate the on-thy-fly algorithm by an example given in Figure 4. We assume that $a \in \Sigma_c$, $b \in \Sigma_i$, and $c \in \Sigma_r$. The process of the algorithms is performed as below:*

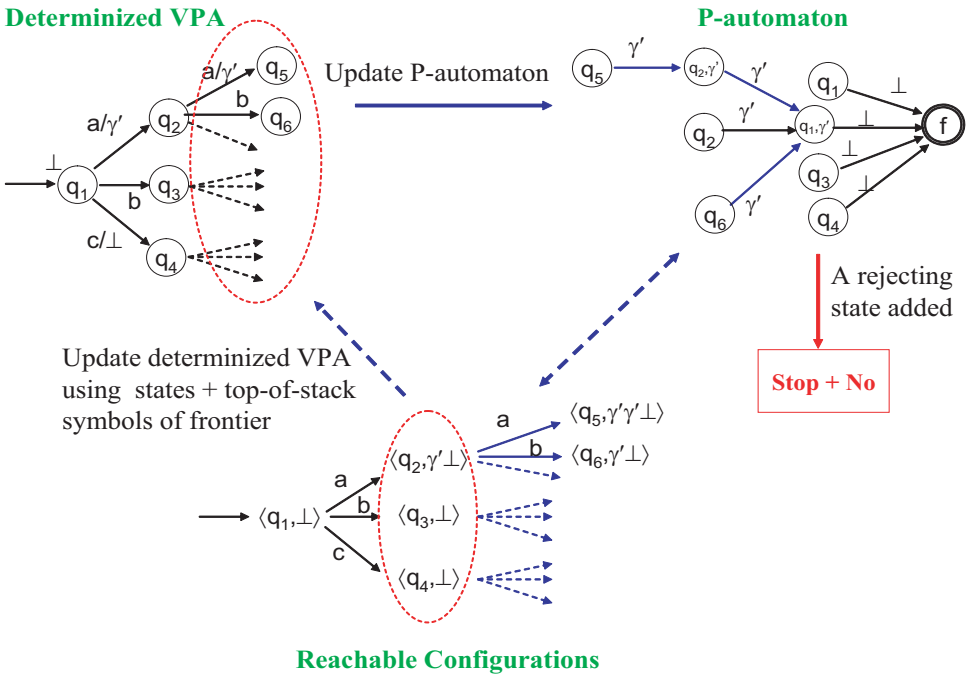

Figure 4: Simulation for On-the-fly Method

1. *At the first time, assume that the initial state q_1 of determinized VPA M^{od} is created.*
2. *Then, the $\mathscr{P}$-automaton A is constructed which includes two states $\{q_1, f\}$ and one transition $q_1 \xrightarrow{\perp} f$, where f is a unique final state. $\mathscr{P}$-automaton A represents a set of initial configurations $\{(q_1, \perp)\}$ of M^{od}.*
3. *Update M^{od} using A. Suppose that M^{od} has new states $\{q_2, q_3, q_4\}$; and new transitions $\{q_1 \xrightarrow{a/+\gamma'} q_2, q_1 \xrightarrow{b} q_3, q_1 \xrightarrow{c/-\perp} q_4\}$.*
4. *Update $\mathscr{P}$-automaton A using new transitions of M^{od}. A has new states $\{q_2, q_3, q_4, p_{(q_1,\gamma')}\}$ and transitions $\{q_2 \xrightarrow{\gamma'} p_{(q_1,\gamma')}, p_{(q_1,\gamma')} \xrightarrow{\perp} f, q_3 \xrightarrow{\perp} f, q_4 \xrightarrow{\perp} f\}$.*
5. *Again, update M^{od} using new transitions of A, and so on.*

3.3 Inclusion Checking

Let A and B be two VPAs. We want to check whether $L(A) \subseteq L(B)$. The standard method is to check whether $L(A \times \overline{B}) = \varnothing$, where $\overline{B}$ is the complement of B.

The on-the-fly approach tries to find if there exists at least a word $w \in L(A) \setminus L(B)$. If such a word w was found, we can conclude that $L(A) \nsubseteq L(B)$. Otherwise, $L(A)$ is a subset of $L(B)$. To do so, similar to the case of universality checking, we perform on-the-fly determinization for B and simultaneously $\mathscr{P}$-automata construction for the product VPA $A \times B^{od}$, where B^{od} is determinized counterpart of B. Once a state $(p,q) \in (F_A \times (Q_{B^{od}} \setminus F_{B^{od}})$ is added to the $\mathscr{P}$-automaton. There exists a word w such that, after

Table 1: Universality checking for VPA generated by random 1

	number of states										
ON-THE-FLY	5	10	20	30	40	50	60	70	80	90	100
success	50	50	50	50	50	50	50	50	50	50	46
total time	23	46	52	71	110	186	210	274	247	407	686
timeout number (*60* s)	0	0	0	0	0	0	0	0	0	0	4
	number of states										
STANDARD	5	10	20	30	40	50	60	70	80	90	100
success	21	1	0	0	0	0	0	0	0	0	0
total time	456	31	0	0	0	0	0	0	0	0	0
timeout number (*60* s)	29	49	50	50	50	50	50	50	50	50	50

reading w, A leads to an accepting configuration whereas B^{od} leads to a rejecting configuration. This means that there exists a word $w \in L(A) \setminus L(B)$. In this case, we stop and report that $L(A) \not\subseteq L(B)$.

It is crucial to note that, if $L(A) \subseteq L(B)$, the on-the-fly approach needs to fully determinize B, and this is similar to the standard approach. Therefore, in the worst case, the time complexity of the on-the-fly approach equals to that of the standard one.

4 Implementation and Experiments

We have implemented the above approaches for testing universality and inclusion of VPA in a prototype tool. The package is implemented in Java 1.5.0 on Windows XP. To compare the on-the-fly algorithm with the standard algorithm, we run our implementations on randomly generated VPAs. All tests are performed on a PC equipped with 1.50 GHz Intel® Core™ Duo Processor L2300 and 1.5 GB of memory.

During experiments, we fix the size of the input alphabet to $|\Sigma_c| = |\Sigma_r| = |\Sigma_i| = 2$, and the size of the stack alphabet to $|\Gamma| = 3$. We first set parameters of the tests as follows:

Definition 4 (random 1) *The* density of final states $f = \frac{|F|}{|Q|} = 1$ *and the* density of transitions $r = \frac{k_a}{|Q|} = 2$, *where k_a is the number of transitions for each input symbol a.*

We ran our tests on randomly VPA generated by the parameter random 1. We have tried VPAs sizes from 10 to 100. We generated 50 VPAs for each sample point, and setting timeout to 60 seconds. The experimental results are given in Table 1. We found that all successfully checked VPAs are not universal, and thus we omit the row for universal results in the table. The experiments shows that STANDARD can solve for generated VPA instances with 5 states only. It gets stuck when the number of states greater than or equal to 10. Meanwhile, ON-THE-FLY is significantly efficient than STANDARD, they can check for almost VPAs.

The parameter random 1 does not guarantee the completeness of VPAs. Therefore, the probability of being universal is very low. In order to increase the probability of being universal, we set a new parameter as below:

Definition 5 (random 2) *The* density of final states $f = \frac{|F|}{|Q|}$ *and the* density of transitions $r : Q \times \Sigma \to N$; *$r(q,a)$ depends on not only the input symbol a but also on the state q. In particular, we select $r(q,a) = 2$ for all $q \in Q$ and $a \in \Sigma_c$, $r(q,b) = 6$ for all $q \in Q$ and $b \in \Sigma_r$, and $r(q,c) = 2$ for all $q \in Q$ and $c \in \Sigma_i$.*

Table 2: Universality checking for VPA generated by random 2, f = 0.6

	number of states						
ON-THE-FLY	5	10	15	20	30	40	50
success	50	43	33	18	6	2	0
total time	68	1425	2310	1950	1024	345	0
timeout number (*180* s)	0	7	17	32	44	48	50
	number of states						
STANDARD	5	10	15	20	30	40	50
success	20	0	0	0	0	0	0
total time	3240	0	0	0	0	0	0
timeout number (*180* s)	0	50	50	50	50	50	50

Table 3: Checking inclusion with $r(q,a) = 2$, $f = 0.5$

	number states of A and B					
ON-THE-FLY	(10,5)	(100,5)	(200,5)	(500,5)	(1000,5)	(3000,5)
success	20	20	15	7	5	2
total time	27	910	830	336	1257	357
timeout number (*300* s)	0	0	13	15	15	18

As can be seen, with random 2, a VPA with 10 states has 200 transitions. We again test for various sizes of VPAs from 5 to 50. We ran with 50 samples for each point, setting timeout to 180 seconds. The results are reported in Table 2. For this parameter, results of STANDARD are almost timeout even with only 5 states. ON-THE-FLY behaves in significantly better ways than those of STANDARD.

We also performed experiments for inclusion checking $L(A) \subseteq L(B)$. For this, we selected parameter random 2 for $f = 0.5$. We generated various sizes of A (10, 100, 200, 500, 1000, and 3000 states) and B (5 and 10 states). We ran with 20 samples for each point, setting timeout to 300 seconds. For this test, STANDARD does not work well, it get all timeout for the smallest size $(10,5)$. Meanwhile, ON-THE-FLY behaves in a significant way. The detailed experimental results of ON-THE-FLY for inclusion checking are reported in Table 3.

5 Related Work

The model of *nested words* was proposed in [2] for representation of data with both a linear ordering and a hierarchically nested matching of items. Recall that the input word of VPA has an implicit nesting structure defined by matching occurrences of symbols in Σ_c with symbols in Σ_r. In nested words, this nesting is given explicitly, and thus they defined finite-state acceptors (with out stacks) for nested words, so-called nested word automata. One can interpret a *nested word* automaton as a visibly pushdown automaton over classical words. As shown in [2], a nondeterministic nested word automaton with n states can be translated into a deterministic nested word automaton with at most 2^{n^2} states. In this paper, we show that the direct determinization of VPAs can be made tighter. As stack-based implementation is the most natural way in modeling recursive programs, we hope that our simple improvement on

determinization procedure of VPAs is still useful.

The first implementation of VPA, named VPAlib [1], only works for basic operations such as union, intersection, and determinization. In their implementation, however, determinization was performed in an exhaustive way. Namely, unreachable states and redundant transitions were also generated. Therefore their determinization easily gets stuck with VPAs of small size. We implemented our prototype tool upon the top of VPAlib. In particular, we first reused and improved data structures as well as basic operations of VPAlib. Next, we implemented determinization on-the-fly manner, in which only reachable states and necessary transitions were created. Then, we used $\mathscr{P}$-automata technique to check emptiness (as well as computing reachable configurations) of VPAs. Finally, we implemented the standard and on-the-fly methods to check universality and inclusion of VPAs.

6 Conclusion

In this paper we have shown that the upper bound for determinization of VPA can be made tighter. Our improvement comes from a simple observation that, in Alur-Madhusudan determinization procedure, the set of summaries S may contain unnecessary pairs in the sense that these pairs do not keep information of reachable states. We exploit this observation to present a new algorithm for determinization by keeping the second component of S always equal to R. This leads to an optimization of the determinization algorithm by using the second component of the summary edge S as the set of reachable states R and this permits to construct a deterministic VPA with only 2^{n^2} states.

We also have presented on-the-fly algorithms for testing universality and inclusion of nondeterministic VPAs. In summary, to check universality of a nondeterministic VPA M, the intuition behind on-the-fly manner is try to find whether there exists a word w such that $w \notin L(M)$. Similarly, to check inclusion $L(M) \subseteq L(N)$, the ideas behind is to find whether there exists at least a word w such that $w \in L(M) \backslash L(N)$. All algorithms has been implemented in a prototype tool. Although the ideas of the on-the-fly methods are simple, the experimental results showed that the proposed algorithms are considerably faster than the standard ones, especially for the cases universality / inclusion do not hold.

Finally, we should emphasize that we need to improve our tool (as well as algorithms) to check larger examples. On the other hand, we also need to consider to apply the tool to case studies in practice. At the moment, the data structures for VPA are rather naive. That is why the running time of our tool is not fast. It would be interesting to explore a more compact data structure. For this, we plan to manipulate VPA using BDD-based representation. Despite these many limitations, however, we believe that this paper provides a first stepping-stone for developing a VPA-based model checker.

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[1] http://www.emn.fr/x-info/hnguyen/vpa/

References

[1] R. Alur and P. Madhusudan (2004): *Visibly pushdown languages*. In: *Proc. of the 36th ACM Symposium on Theory of Computing (STOC'04)*, pp. 202–211. ACM Press.

[2] R. Alur and P. Madhusudan (2009): *Adding Nesting Structure to Words*. In: *Journal of ACM*, Volume 56(3). ACM Press.

[3] R. Alur, K. Etessami, P. Madhusudan (2004): *A temporal logic of nested calls and returns*. In: *Proc. of the 10th International Conference on Tools and Algorithms for the Construction and Analysis of Systems (TACAS'04)*, LNCS 2988, pp. 467–481. Springer-Verlag.

[4] R. Alur, S. Chaudhuri, and P. Madhusudan (2006): *A fixpoint calculus for local and global program flows*. In: *Proc. of the 33rd Annual ACM SIGPLAN - SIGACT Symposium on Principles of Programming Languages (POPL'06)*, pp. 153–165. ACM Press.

[5] Ahmed Bouajjani, Javier Esparza and Oded Maler (1997): *Reachability Analysis of Pushdown Automata: Application to Model-Checking*. In: *Proc. of the 8th International on Conference Concurrency Theory (CONCUR'97)*, LNCS 1243, pp. 135–150. Springer Verlag.

[6] Ahmed Bouajjani, Peter Habermehl, Lukas Holik, Tayssir Touili, and Tomas Vojnar (2008): *Antichain-based Universality and Inclusion Testing over Nondeterministic Finite Tree Automata*. In: *Proc. of the 13th International Conference on Implementation and Applications of Automata (CIAA'08)*, LNCS 5148, pp. 57–67. Springer-Verlag.

[7] J. Esparza, D. Hansel, P. Rossmanith, and S. Schwoon (2000): *Efficient algorithms for model checking pushdown systems*. In: *Proc. of the 12th International Conference on Computer Aided Verification (CAV 2000)*, LNCS 1855, pp. 232–247. Springer-Verlag.

[8] J. Esparza, A. Kucera, and S. S. Schwoon (2003): *Model-checking LTL with regular valuations for pushdown systems*. In: *Information and Computation*, 186(2), pp. 355–376. Elsevier Publisher.

[9] V. Kumar, P. Madhusudan, and M. Viswanathan (2007): *Visibly pushdown automata for streaming XML*. In: *Proc. of 16th International World Wide Web Conference (WWW2007)*, pp. 1053–1062.

[10] C. Pitcher. C. Pitcher (2005): *Visibly pushdown expression effects for XML stream processing*. In: *Proc. of PLAN-X'05*, pp. 5–19.

[11] C. Löding, P. Madhusudan, and O. Serre (2004): *Visibly pushdown games*. In: *Proc. of the 24th International Conference on Foundations of Software Technology and Theoretical Computer Science (FSTTCS04)*, LNCS 3328, pp. 408–420. Springer-Verlag.

[12] D. H. Nguyen and M. Südholt (2006): *VPA-based aspects: better support for AOP over protocols*. In: *Proc. of the 4th International Conference on Software Engineering and Formal Methods (SEFM'06)*, pp. 167–176. IEEE Computer Society.

[13] D. H. Nguyen and M. Südholt (2007): *Property-preserving evolution of components using VPA-based aspects*. In: *Proc. of DOA'07*, LNCS 4803, pp. 613–629. Spinger-Verlag.

[14] M. De Wulf, L. Doyen, T. A. Henzinger, and J. F. Raskin (2006): *Antichains: A New Algorithm for Checking Universality of Finite Automata*. In: *Proc. of the 18th International Conference on Computer Aided Verification (CAV'06)*, LNCS 4144, pp. 17–30. Springer-Verlag.