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Exponential Space Complete Problems for Petri Nets and Commutative Semigroups: Preliminary Report*

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1. Introduction

The uniform word problem for commutative semigroups (UWCS) is the problem of determining from any given finite set of defining relations and any pair of words, whether the words describe the same element in the commutative semigroup defined by the relations. The effective decidability of this classical algebraic problem was first explicitly noted by Malcev [1958] and Emilichev [1958], though in retrospect this result can be seen to be contained in the earlier work of König [1903] and Hermann [1926] on polynomial ideals.

Our main result is:

Theorem 1. The UWCS is complete in exponential space under log-space transformability.

As a corollary of the proof of Theorem 1 we conclude

Corollary 1a. There is a constant $c > 0$ and an algorithm (Turing machine) which decides the UWCS and requires space at most 2^{cn} on any instance of the UWCS of size n.

1b. There is a constant $\varepsilon > 0$ such that any algorithm which decides the UWCS requires space exceeding $2^{\varepsilon n}$ on an instance of the UWCS of size n for infinitely many n.

We also observe that the word problem for commutative semigroups is a notational variant of the uniform reachability problem for reversible Petri nets, that is, Petri nets in which for each transition T there is a reverse transition T' whose firing undoes the effect of firing T (cf. [Karp and Miller, 1969], [Hack, 1975]). This yields

Corollary 2. The uniform reachability problem for reversible Petri nets is complete in exponential space under log-space transformability.

Corollary 2 strengthens a recent result of Lipton [1976], who established an exponential space complexity lower bound on the uniform reachability problem for unrestricted Petri nets. This latter problem is still not known to be decidable.

Our second main result concerns the containment and equivalence problems for reachability sets of Petri nets. Rabin has shown that the containment problem is undecidable. Hack has shown that the containment problem is reducible to the equivalence problem, so that equivalence is also undecidable (cf. [Hack 1975]). Karp and Miller [1969] proved that the finiteness problem for reachability sets is decidable, and it follows directly from their proof that the containment and equivalence problems for finite reachability sets is decidable by exhaustion.

Theorem 2. The containment problem and the equivalence problem for finite reachability sets are decidable, but neither problem has a primitive recursive decision procedure.

Theorem 2 provides the first examples of uncontrived decidable problems which are provably not primitive recursive.

2. Semi-Thue Systems, Semigroups, and Petri Nets

A semi-Thue system consists of a finite set $\mathcal{P}$ of productions of the form $\alpha_i \to \beta_i$ where $\alpha_i, \beta_i \in \Sigma^*$ for some finite alphabet Σ. A word $\alpha \in \Sigma^*$ derives in one step (in $\mathcal{P}$) a word $\beta \in \Sigma^*$, written $\alpha \Rightarrow \beta(\mathcal{P})$, iff there are words $\gamma, \delta \in \Sigma^*$ and a production $(\alpha_i \to \beta_i) \in \mathcal{P}$ such that $\alpha = \gamma\alpha_i\delta$ and $\beta = \gamma\beta_i\delta$. The word α derives β (in $\mathcal{P}$), written $\alpha \overset{*}{\Rightarrow} \beta(\mathcal{P})$, iff $\alpha = \beta$ or there is a sequence of $n > 1$ words $\gamma_1, \gamma_2, \ldots, \gamma_n$ such that $\gamma_i \Rightarrow \gamma_{i+1}(\mathcal{P})$ for $1 \le i < n$, $\alpha = \gamma_1$, and $\beta = \gamma_n$. Such a sequence $\gamma_1, \ldots, \gamma_n$ is called a derivation of β from α in $\mathcal{P}$.

The uniform derivability problem for semi-Thue systems (UDT) is

$$\mathrm{UDT} \overset{\mathrm{df}}{=} \{< \alpha, \beta, \mathcal{P} > \mid \alpha \overset{*}{\Rightarrow} \beta(\mathcal{P})\}.$$

A semi-Thue system $\mathcal{P}$ is said to be a semigroup presentation (or Thue system) iff for every production $(\alpha_i \to \beta_i) \in \mathcal{P}$, the reverse production $(\beta_i \to \alpha_i) \in \mathcal{P}$ also. For semigroup presentations it is customary to represent a matched pair of productions $\alpha_i \to \beta_i$ and $\beta_i \to \alpha_i$ by writing $\alpha_i \equiv \beta_i$. In this case derivability is an equivalence relation and by an abuse of notation we write $\alpha \equiv \beta(\mathcal{P})$ to indicate that $\alpha \overset{*}{\Rightarrow} \beta(\mathcal{P})$. The uniform word problem for semigroups (UWS) is

$$\mathrm{UWS} \overset{\mathrm{df}}{=} \{< \alpha, \beta, \mathcal{S} > \mid \mathcal{S} \text{ is a semigroup presentation and } \alpha \equiv \beta(\mathcal{S})\}.$$

Post [1947] and Markov [1947] proved that there is a particular semigroup presentation $\mathcal{S}_0$ for which $\{< \alpha, \beta > \mid \alpha \equiv \beta(\mathcal{S}_0)\}$ is not a recursive set, so that *a fortiori* UDT and UWS are not decidable.

A semi-Thue system or semigroup presentation is said to be commutative if for every pair of symbols $a, b \in \Sigma$, the "commutative" production $ab \to ba$ appears. All the systems considered in this paper

* Research for this paper was supported in part by the National Science Foundation under research grant no. GJ-43634X. A portion of the research was done while the latter two authors were visiting at the IBM Watson Research Center, Yorktown Heights, N.Y.

will be commutative. The uniform derivability problem for commutative semi-Thue systems (UDCT) and the uniform word problem for commutative semi-groups (UWCS) are respectively the special cases, i.e., subsets, of UDT and UWS for commutative semi-Thue systems $\mathcal{P}$ and commutative semigroup presentations $\mathcal{S}$.

Any commutative semi-Thue system $\mathcal{P}$ with alphabet Σ may be represented as a Petri net as follows: the net has one place for each symbol $b \in \Sigma$ and one transition for each production in $\mathcal{P}$. For $b \in \Sigma$, $\alpha \in \Sigma^*$, let $n(b,\alpha)$ equal the number of occurrences of b in α. The transition corresponding to the production $(\delta \to \gamma) \in \mathcal{P}$ has for each $b \in \Sigma$, $n(b,\delta)$ incoming arcs from the place corresponding to b and $n(b,\gamma)$ outgoing arcs to this b-place. For example, the system $\mathcal{P} = \{sa \to scc,\ sb \to sc,\ s \to f\}$ corresponds to the Petri net of Figure 1.

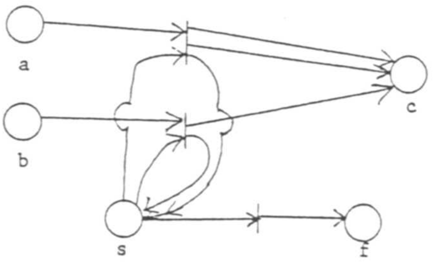

Figure 1. A Petri Net.

Any word $\alpha \in \Sigma^*$ corresponds to the marking of the Petri net in which the b-place contains $n(b,\alpha)$ tokens for each $b \in \Sigma$. Thus $\alpha \overset{*}{\Rightarrow} \beta(\mathcal{P})$ iff the marking corresponding to β is in the reachability set of the net corresponding to $\mathcal{P}$ initialized with the marking corresponding to α. By this correspondence, the UDCT and the uniform reachability problem for Petri nets are seen to be the same.

We define the size of a commutative semi-Thue system to be the sum of the lengths of left and righthand sides of all productions, ignoring the commutative production rules $ab \to ba$. That is, the size of the system equals the total number of arcs in the corresponding Petri net. The size of a triple $< \alpha, \beta, \mathcal{P} >$ is defined to be length(α) + length(β) + size($\mathcal{P}$). We remark that our results do not depend heavily on this definition of size. Thus, Theorems 1 and 2 and their corollaries remain true if we allow exponents in decimal notation in representing words and productions, so that, for example, the word consisting of ninety-nine b's could be written in length three, namely as "b99", instead of length ninety-nine.

It is not known if the UDCT is decidable. It is not even known if there is a nonrecursive reachability set, that is, a particular commutative semi-Thue system $\mathcal{P}_0$ and particular word $\alpha_0 \in \Sigma^*$, such that $\{\beta \in \Sigma^* \mid \alpha_0 \overset{*}{\Rightarrow} \beta(\mathcal{P}_0)\}$ is not recursive. On the other hand, Taiclin [1968] has shown that for any fixed commutative semigroup presentation $\mathcal{S}_0$ over alphabet $\Sigma = \{b_1,\ldots,b_m\}$, the set of nonnegative integer vectors

$$\{< n(b_1,\alpha),\ldots,n(b_m,\alpha),n(b_1,\beta),\ldots,n(b_m,\beta) > \in \mathbb{N}^{2m} \mid \alpha \equiv \beta(\mathcal{S}_0)\}$$

is semilinear (cf. Ginsburg and Spanier [1965]); consequently the word problem for any fixed $\mathcal{S}_0$ is computationally trivial, that is, real-time recognizable on a Turing machine even using decimal exponent notation.

3. Commutative Semigroups and Polynomial Ideals

For any finite alphabet Σ, let $Q[\Sigma]$ denote the ring of formal polynomials with rational coefficients over the indeterminates Σ. We identify any word $\alpha \in \Sigma^+$ with the monomial $\alpha \in Q[\Sigma]$ and identify the empty word in Σ^* with the constant $1 \in Q$. For $p_1,\ldots,p_k \in Q[\Sigma]$, let $(p_1,\ldots,p_k)$ denote the ideal generated by $p_1,\ldots,p_k$, that is

$$(p_1,\ldots,p_k) \overset{df.}{=} \{\sum_1^k g_i p_i \mid g_i \in Q[\Sigma]\}.$$

Now for any commutative semigroup presentation $\mathcal{S} = \{\alpha_i \equiv \beta_i\}_1^k$, define the ideal

$$I(\mathcal{S}) \overset{df.}{=} (\alpha_1-\beta_1,\ldots,\alpha_k-\beta_k).$$

The next two lemmas show that the UWCS is reducible to the membership problem for polynomial ideals.

Lemma 1. If $\alpha \equiv \beta(\mathcal{S})$, then $(\alpha-\beta) \in I(\mathcal{S})$.

Proof: Suppose $\gamma_1 \Rightarrow \gamma_2$ ($\mathcal{S}$) by one application of the production $(\alpha_i \to \beta_i) \in \mathcal{S}$. That is, $\gamma_1 = \gamma\alpha_i\delta$ and $\gamma_2 = \gamma\beta_i\delta$ for some $\gamma,\delta \in \Sigma^*$. Then in $Q[\Sigma]$,

$$\gamma_1-\gamma_2 = \gamma\delta(\alpha_i-\beta_i) \in I(\mathcal{S}).$$

Hence, if $\alpha = \gamma_1 \Rightarrow \gamma_2 \Rightarrow \ldots \Rightarrow \gamma_n = \beta(\mathcal{S})$, then $\gamma_i-\gamma_{i+1} \in I(\mathcal{S})$ and

$$\alpha-\beta = \sum_1^{n-1}(\gamma_i-\gamma_{i+1}) \in I(\mathcal{S}). \qquad \boxtimes$$

The degree $\deg(\alpha)$ of a word $\alpha \in \Sigma^*$ is its length. A polynomial $p \in Q[\Sigma]$ is uniquely expressible as a sum of distinct monomials with nonzero coefficients; the degree $\deg(p)$ is defined to be the maximum of the degrees of the monomials in p.

Lemma 2. If $(\alpha-\beta) \in I(\mathcal{S})$, then $\alpha \equiv \beta(\mathcal{S})$. In particular, if $\alpha-\beta = \sum_1^k g_i\cdot(\alpha_i-\beta_i)$ for $g_i \in Q[\Sigma]$, then there is a derivation $\gamma_1,\ldots,\gamma_n$ of β from α in $\mathcal{S}$ such that for $1 \le j \le n$,

$$\text{length}(\gamma_j) \le \max\ (\bigcup_1^k\{\deg(g_i\alpha_i),\deg(g_i\beta_i)\}).$$

We omit the proof.

Hermann [1926] used the following lemma to establish the decidability of the membership problem in polynomial ideals (cf. [Seidenberg 1974] for an improved proof).

Lemma 3. Let Σ be an alphabet of n symbols and let $p,p_1,\ldots,p_k \in Q[\Sigma]$. Let $m = \max\{\deg(p_i) \mid 1 \le i \le k\}$. If $p \in (p_1,\ldots,p_k)$, then there exist $g_1,\ldots,g_k \in Q[\Sigma]$ such that

1) $p = \sum_1^k g_i p_i$, and

2) $\deg(g_i) \le \deg(p) + 2n(mk)^{2^{n-1}}$ for $1 \le i \le k$.

As an immediate consequence of Lemmas 1,2, and 3 we obtain

Lemma 4. $\alpha \equiv \beta(\mathcal{S})$ iff there is a derivation of β from α in $\mathcal{S}$ such that each word in the derivation is of length at most $2^{2^{c\cdot size(<\alpha,\beta,\mathcal{S}>)}}$, for some universal constant $c > 0$ independent of $<\alpha,\beta,\mathcal{S}>$.

Now note that, since the exact order in which symbols occur in a word in a commutative system is irrelevant, any word $\gamma \in \Sigma^*$ may be represented on a Turing machine tape by storing the decimal representations of the numbers $n(b,\gamma)$ for $b \in \Sigma$. It follows from Lemma 4 that to decide whether $\alpha \equiv \beta(\mathcal{S})$, a nondeterministic Turing machine need only "guess" successive words in a derivation of β from α, storing the representations of two words from the derivation on its tape at any time and allocating only $2^{d\cdot size(<\alpha,\beta,\mathcal{S}>)}$ tape squares for the representations of these words, where $d > 0$ is some universal constant. Of course this nondeterministic Turing machine may be simulated by an ordinary deterministic Turing machine which also requires only exponential space [Savitch 1970], thus establishing Corollary 1a.

4. Space Bounded Counter Machines

Let L be a language recognizable by a Turing machine using space at most 2^n on inputs of length n. Then there is a counter machine C recognizing L whose counters only contain nonnegative integers less than 2^{2^n} during computations on inputs of length n [Fischer, Meyer, Rosenberg, 1968]. We may construct, from C and an input of length n, a corresponding instance of the UWCS.

Suppose, for example, C has two counters and at some step of its computation on an input of length n, C is in state q and its counters respectively contain nonnegative integers n_1,n_2. We may represent this situation by a word

$w(q,n_1,n_2) \in \{q,b_1,b_2,c_1,c_2\}^*$:

$$w(q,n_1,n_2) \overset{df.}{=} qb_1^{k-n_1}c_1^{n_1}b_2^{k-n_2}c_2^{n_2} \text{ where } k = 2^{2^n}.$$

Now corresponding to the defining rules of C, we may define a semi-Thue system $\mathcal{S}$ such that $w(q,n_1,n_2) \overset{*}{\Rightarrow} w'(\mathcal{S})$ iff w' represents the state and counter contents of C at some step of C's computation after C is in the situation (q,n_1,n_2).

For example, if, when C is in state q and counter 1 is nonzero, C enters state q_1 and decrements counter 1, then we introduce a production

$$(qc_1 \to q_1b_1) \in \mathcal{S},$$

which ensures that $w(q,n_1,n_2) \overset{*}{\Rightarrow} w(q_1,n_1-1,n_2)(\mathcal{S})$.

In order to simulate by rules in $\mathcal{S}$ a zero test by C, we suppose there are productions in $\mathcal{S}$ which ensure precisely that $(b_i)^k \equiv a_i(\mathcal{S})$ for $i = 1,2$.

Thus, $w(q,0,n_2) = qb_1^k b_2^{k-n_2} c_2^{n_2} \equiv qa_1b_2^{k-n_2}c_2^{n_2}(\mathcal{S})$. If, when C is in state q and counter 1 contains zero, C enters state q_2 and increments counter 1, then we introduce productions in $\mathcal{S}$:

$$(qa_1 \to q'a_1) \quad (q'b_1 \to q_2c_1)$$

which ensure that $w(q,0,n_2) \overset{*}{\Rightarrow} w(q_2,1,n_2)(\mathcal{S})$.

Now it turns out that the fact that C is a deterministic counter machine implies that the correspondence outlined above between derivations in $\mathcal{S}$ and computations of C is preserved even if the productions of $\mathcal{S}$ are reversible, viz., if $\mathcal{S}$ is a semigroup presentation (cf. [Cardoza, 1975]). Moreover, the construction of $\mathcal{S}$ from C and an input of length n to C may be carried out efficiently (by a Turing machine using space log n, not counting the space occupied by input and output), once we have determined how to express the property that $a_i \equiv b_i^k(\mathcal{S})$ succinctly. Thus, to test a word w for membership in L, we need merely test whether a word α, coding the initial configuration of C on w, derives in $\mathcal{S}$ the word β corresponding to the accepting configuration of C.

In the next section we indicate how to express the relation $a_i \equiv b_i^k$ where $k = 2^{2^n}$ by an $\mathcal{S}$ of size proportional to n. Combining this with the argument outlined above we obtain

Lemma 5. Let $L \subset \Delta^+$ be recognizable by a Counter machine C whose counters are bounded by 2^{2^n} on inputs of length n. Then there is a logspace computable function f such that

1) for any $w \in \Delta^+$, $f(w) = <\alpha,\beta,\mathcal{S}>$ where $\mathcal{S}$ is a semigroup presentation over some alphabet Σ and $\alpha,\beta \in \Sigma$,
2) $w \in L$ iff $f(w) \in$ UWCS, and
3) there is a constant $c > 0$ such that $size(f(w)) \le c\cdot length(w)$ for all $w \in \Delta^+$.

Thus, if L is recognizable in exponential space on a Turing machine, L is log-space transformable into the UWCS. This proves Theorem 1. Corollary 1b follows from the additional condition (3) of Lemma 5 and the usual properties of log-space reducibility ([Stockmeyer and Meyer, 1973], [Stockmeyer, 1974]).

5. Succinct Semigroup Presentations

Our aim is to show how to construct a semigroup presentation $\mathcal{S}$ such that size($\mathcal{S}$) is proportional to n and such that $\mathcal{S}$ has precisely the same "meaning" as the single relation $a \equiv b^{2^{2^n}}$. Let $N = 2^{2^n}$ and note that $N^2 = 2^{2^{n+1}}$. We proceed by induction, constructing from $\mathcal{S}_n$, which represents relations of approximately the form $a \equiv b^N$, a presentation $\mathcal{S}_{n+1}$ representing relations of the form $a' \equiv (b')^{N^2}$. $\mathcal{S}_{n+1}$ is obtained by adding only a fixed (independent of n) number of symbols and relations to $\mathcal{S}_n$, so that size($\mathcal{S}_n$) will be proportional to n by induction.

Specifically, suppose we have a semigroup presentation $\mathcal{S}_n$ over an alphabet $\Sigma = S \cup F \cup C \cup \Delta$ where $S = \{s_i\}_1^5$, $F = \{f_i\}_1^5$, $C = \{a_i,b_i\}_1^5$ and Δ is disjoint from $S \cup F \cup C$. Let $\mathcal{C}$ denote the commutative relations on Σ; $\mathcal{C} \overset{df.}{=} \{(ab \equiv ba) \mid a,b \in \Sigma\}$. Assume $\mathcal{S}_n$ "means" $s_ia_i \equiv f_i(b_i)^N$ for $i=1,\ldots,5$ in the following precise sense:

1) $s_i a_i \equiv f_i(b_i)^N(\mathcal{S}_n)$ for $i = 1,\ldots,5$, and

2) if $\alpha \in SC^*$ and $\alpha \equiv \beta(\mathcal{S}_n)$ for some $\beta \in F\cdot\Sigma^*$, then there is a word $\beta' \equiv \beta(\mathcal{G})$ such that $\alpha \Rightarrow \beta'(\{s_i a_i \rightarrow f_i(b_i)^N\})$ for some $i = 1,\ldots,5$.

We shall add to $\mathcal{S}_n$ five groups of four new symbols not in Σ which will play the role in $\mathcal{S}_{n+1}$ that the five given groups of four symbols $\{s_i, a_i, f_i, b_i\}$ play in $\mathcal{S}_n$. Let $s,f,a,b \notin \Sigma$ be one of these five new groups of symbols. We introduce eight more symbols $q_1,\ldots,q_8$ distinct from all others and add the following twelve relations (plus new commutative relations) to $\mathcal{S}_n$:

New relation	Some words derivable from sa
$sa \equiv q_1 s_1 a_1$	$q_2 b_1^{N}$
$q_1 f_1 \equiv q_2$	
$q_2 b_1 \equiv q_2 a_2 b_3$	$q_2 b_1^{N-\ell}(a_2 b_3)^{\ell}$
$q_2 \equiv q_3 f_3$	$q_4 a_2^{N}$
$q_3 s_3 a_3 \equiv q_4$	
$q_4 \equiv q_5 s_2$	$q_5 s_2 a_2^{N-\ell} b_4^{\ell} b^{N\ell}$
$q_5 f_2 \equiv q_6 b_4$	$q_6 a_2^{N-(\ell+1)} b_4^{\ell+1} b^{N\ell} b_2^{N}$
$q_6 b_2 \equiv q_6 b b_5$	$q_6 a_2^{N-(\ell+1)} b_4^{\ell+1} b^{N\ell+j} b_2^{N-j} b_5^{j}$
$q_6 \equiv q_7 f_5$	$q_4 a_2^{N-(\ell+1)} b_4^{\ell+1} b^{N(\ell+1)}$
$q_7 s_5 a_5 \equiv q_4$	
$q_4 \equiv q_8 f_4$	fb^{N^2}
$q_8 s_4 a_4 \equiv f$	

We claim that by adding these $5(4+8) = 60$ new symbols and $5\cdot 12 = 60$ new relations to $\mathcal{S}_n$ (plus new commutative relations) we obtain a presentation $\mathcal{S}_{n+1}$ with the correct meaning in the sense of (1) and (2) above with N replaced by N^2 (and symbols in Σ replaced by the corresponding new symbols). Condition (1) is easily verified by treating the new relations as productions directed from left to right and considering the accompanying list of words derivable from sa. The verification of condition (2) is more difficult since "side effects" caused by using the relations as productions from right to left must be analyzed; we omit the proof.

We remark that another consequence of this construction is that the double exponential growth bound obtained by Hermann and Seidenberg on the degrees of the polynomials in Lemma 3 is roughly achievable.

6. The Equivalence Problem for Finite Reachability Sets

Let $\mathcal{P}$ be a semi-Thue system over an alphabet Σ and let $\alpha \in \Sigma^*$. Regarding $< \mathcal{P},\alpha >$ as an initialized Petri net as in section 2, we may define the reachability set

$$R(\mathcal{P},\alpha) \overset{\text{df.}}{=} \{\beta \in \Sigma^* \mid \alpha \overset{*}{\Rightarrow} \beta(\mathcal{P})\}.$$

The containment problem is to determine given $< \mathcal{P},\alpha >$ and $< \mathcal{P}',\alpha' >$ whether $R(\mathcal{P},\alpha) \supset R(\mathcal{P}',\alpha')$ and for the equivalence problem whether $R(\mathcal{P},\alpha) = R(\mathcal{P}',\alpha')$. Rabin's proof (given in [Hack, 1975]) of the undecidability of the containment problem proceeds by a reduction of Hilbert's Tenth Problem on Diophantine polynomials to the containment problem. Recently, Adleman and Manders [1975] have considered bounded versions of Hilbert's Tenth Problem and established their inherent complexity. In particular, let

$$A_0(x) = 2x,$$

$$A_{n+1}(0) = 2,$$

$$A_{n+1}(x+1) = A_n(A_{n+1}(x)),$$

$$A(n) = A_n(2).$$

It is not hard to show that the function A, which is a variant of Ackermann's function, majorizes the primitive recursive functions. Let $\mathbb{N}[X]$ denote the ring of polynomials in the finite set of indeterminates X with nonnegative integer coefficients. Define the bounded polynomial inequality problem (BPI) as follows:

$$\text{BPI} \overset{\text{df.}}{=} \{< p,q,n > \mid p,q \in \mathbb{N}[X] \text{ for some } X \text{ and } (\forall \vec{y} \in \{0,1,\ldots,A(n)\}^{\text{card}(X)})[p(\vec{y}) \geq q(\vec{y})]\}.$$

Adleman and Mander's results easily yield

Lemma 6. BPI is not primitive recursive.

A relatively straightforward adaptation of Rabin's proof may be carried through to reduce BPI to the containment problem. Moreover, the reachability sets to which BPI reduces will be finite. To make the reduction efficient, the difficulty is that Petri nets of size proportional to n must be constructed with finite reachability sets of size at least A(n).

Suppose $f: \mathbb{N} \rightarrow \mathbb{N}$ is a nondecreasing function such that $f(0) = 2$ and there is a Petri net $\mathcal{P}_f$ which "weakly computes" f in the sense that $f(x) = \max\{k \in \mathbb{N} \mid sa^x \equiv b^k(\mathcal{P}_f)\}$, for some symbols s,a,b and all $x \in \mathbb{N}$. Define $g: \mathbb{N} \rightarrow \mathbb{N}$ by letting $g(0) = 2$ and $g(x+1) = f(g(x))$; we may define a similar weak computer $\mathcal{P}_g$ for g as in Figure 2. Moreover, if $R(\mathcal{P}_f, sa^x)$ is finite for all $x \in \mathbb{N}$, so is $R(\mathcal{P}_g, s'(a')^x)$ for all $x \in \mathbb{N}$. Repeating this construction n times we may construct an initialized net of size proportional to n whose reachability set is finite and in which the maximum number of tokens which may appear in a designated place is precisely A(n). (This observation is due to M. Hack.)

Definte the containment problem for finite reachability sets (CFR) to be

$$\text{CFR} \overset{\text{df.}}{=} \{< \mathcal{P},\alpha,\mathcal{P}',\alpha' > \mid R(\mathcal{P},\alpha) \text{ is finite and } R(\mathcal{P},\alpha) \supset R(\mathcal{P}',\alpha')\}.$$

We have now outlined the proof of

Lemma 7. BPI is log-space transformable to CFR.

It follows from Lemmas 6 and 7 that CFR is not primitive recursive.

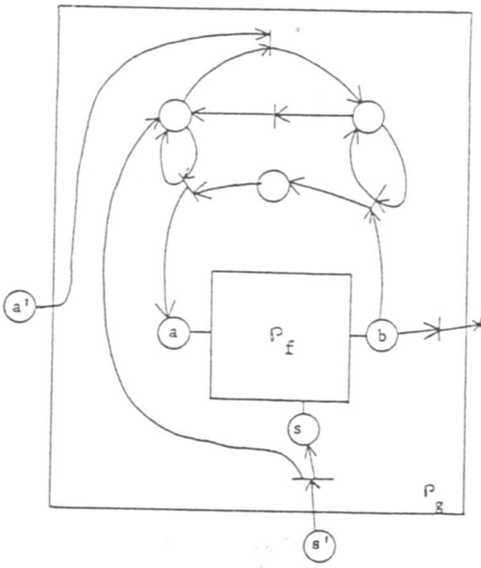

Figure 2. Weak computer for g(x+1) = f(g(x)).

It is also straightforward to modify Hack's [1975] reduction of the containment problem to the equivalence problem to show that CFR is log-space transformable to the equivalence problem for finite reachability sets, thus proving Theorem 2.

7. An Open Problem

The results of Biryukov [1967] and Taiclin [1968] imply that the reachability set of any initialized reversible Petri net, i.e., semigroup presentation, is a semilinear set of vectors which is constructible uniformly. It follows that the containment and equivalence problems for reversible nets are decidable. It is not known if these problems are primitive recursive.

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