SYMBOLIC NETWORK ANALYSIS WITH THE VALID TREES AND THE VALID TREE-PAIRS

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ABSTRACT

The valid tree and the valid tree-pair as well as their values are defined, Algorithms to look for all valid trees and valid tree-pairs and to obtain their values are given. A theorem points out that the determinant of a closed network is equal to the sum of all valid tree's values and all valid tree-pair's values of the network. A unified topological formula for general active circuits to solve various types of symbolic network functions is put forward.

1. INTRODUCTION

Solving symbolic network functions by the topological graph and element parameters of a circuit without establishing equations of any form is the goal of topological analysis.

Tree enumeration approach is classical and useful method for symbolic network analysis. There are two basic tree enumeration approaches [1,2], the directed tree enumeration [1,3] and the two-graph tree enumeration [1,2,4]. Because based on node equations of circuits, they are limited to application. They handle only, in principle, the elements of admittance and VCCS and give different formulas for various network functions. Many researchers have developed these methods for general networks including various active elements and used some techniques such as decomposition and simplification for large analog circuits [2].

The pair of conjugate trees is a wonderful idea that had been used for the solvability and the generic order of complexity of active networks. [5] But it does not be applied to solve symbolic network functions, because there is no algorithm to find all pairs of conjugate trees and obtain their values.

In this paper, a new idea is presented, which is similar to the pair of conjugate trees [5] but independent on it, that is the valid tree and the valid tree-pair. The fact that the determinant of a closed network is equal to the sum of all valid tree's values and all valid tree-pair's values and algorithms to find all valid trees and all valid tree-pairs and their values are given. Then a new approach to exact symbolic analysis is put forward, which is capable of analyzing general circuits including four types of dependent sources and nullors and giving a uniform formula for various network functions. It improves and develops existing tree enumeration methods and is more general for symbolic analysis of active networks.

2. PROBLEM AND NOTATIONS

2.1 Network functions

Consider a linear time-invariant circuit **N**. It may contain admittance, impedance, four types of dependent sources, nullors and independent sources. Resistors, inductors, capacitors, mutual inductors, ideal transformers, operation amplifiers, gyrators and other active elements can be simulated by these elements in frequency or complex frequency domain.

The output variables is $\boldsymbol{x}$ and the independent sources (input variables) is $\boldsymbol{w}$ of **N**. The network functions matrix is **H**. Then

$$\boldsymbol{x}=\mathbf{H}\boldsymbol{w} \tag{1}$$

Every content of H is

$$\mathrm{H_{ji}}=\frac{\mathrm{N}}{\mathrm{D}} \tag{2}$$

Here N and D are polynomials and both expressions are sums of element symbol products.

Then

$$x_j=\sum_i \mathrm{H_{ji}}\,w_i \tag{3}$$

For a network of one output port-one input port the $\mathrm{H_{ji}}$ is the network function

$$\mathrm{H_{ji}}=\frac{x_{\mathrm{out}}}{w_{\mathrm{in}}} \tag{4}$$

2.2 Closed Network

A closed network **N** is such circuit that may contain impedance, admittance, dependent sources and nullors but no independent source.

Equations of a closed network are as the following.

$$\mathrm{M}x=0 \tag{5}$$

We called determinant det(M) of matrix M as the determinant of closed network **N**. It is worth mentioning

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that although the determinant is defined with equations (1) it is relation to the graph and parameters of circuits but not to some form of equations, even the M.

Let **G** is undirected graph of a closed network **N**. Vertexes of **G** are nodes of **N**, edges of **G** are branches of **N**. According to element's types, edges of **G** are called respectively Y edge, Z edge, VS edge (controlled voltage branch), CS edge (controlled current branch), VC edge (controlling voltage branch), CC edge (controlling current branch), norator edge and nullator edge. **G** is an undirected graph, but the VS, CS, VC, CC, norator and nullator edges can be put the direction identical with the orientation of the branch voltages and currents. The symbol parameters of admittance, impedance and dependent source are separately Y_i , Z_i and X_{ij} , the subscripts i and j are the branch's numbers. Assume the voltage controlling branches all are the opens and the current controlling branches all are the shorts.

2.3 The Topological formula

Now we give the topological formula for network function H_{ji}.

Replace independent source w_i by depend source $-F_{ij}x_j$, That is

$$w_i = -F_{ij}x_j \tag{6}$$

Then replace other independent sources by shorts for voltage sources and opens for current sources. **N** becomes to closed network **N'**, whose equations are

$$M'x=0 \tag{7}$$

And its determinant is det(M'). According to F_{ij} included or not, separate det(M') into two parts as the following.

$$\det(M')=N+F_{ij}D \tag{8}$$

Since the right side of equations (7) are all zeros,

$$\det(M')=N+F_{ij}D=0 \tag{9}$$

So

$$H_{ji}=\frac{x_j}{w_i}=-\frac{1}{F_{ij}}=\frac{N}{D} \tag{10}$$

Let Δ_{ij} equals N and Δ_0 equals D, then

$$H_{ji}=\frac{\Delta_{ij}}{\Delta_0} \tag{11}$$

Here Δ_0 is the sum of all terms including no F_{ij} of the determinant of closed network **N'**. By later text Δ_0 also is the determinant of **N** opened all independent circuit sources and shorted all independent voltage sources. Δ_{ij} is the sum of all terms including F_{ij} divided by F_{ij} in the determinant of closed network **N'** which comes from **N** replaced independent source w_i by depend source $-F_{ij}x_j$.

Obviously, the key to solve network functions is to calculate the determinant of a closed network.

3. DEFINITIONS

Definition 1: When **N** does not contain any nullor, If T is a tree of **G** and includes all VS edges and CC edges, but no CS edge or VC edge, then T is called a valid tree of **N**.

Definition 2: Suppose T is a valid tree, let

$$P=\Pi Y_i \Pi Z_i \tag{12}$$

Here, Y_i is the admittance of Y edge included by T, Z_i is the impedance of Z edge not included by T. Then P is called value of the valid tree.

Definition 3: If Both of T_1 and T_2 are trees of **G** and satisfy the following conditions, then T_1 and T_2 are called a valid tree-pair of **N**.

- T_1 contains all norator edges and no nullator edge, T_2 contains all nullator edges and no norator edge, When **N** contains some nullors.
- The edges included by T_1 not by T_2 are some controlled edges, the edges included by T_2 not by T_1 are controlling edges of the same dependent sources.
- The edges included by both T_1 and T_2 may be Y, Z, VS and CC edges, not be CS or VC edges.
- The edges included by neither T_1 nor T_2 may be Y, Z, CS and VC edges, not be VS or CC edges.

Definition 4: Suppose T_1 and T_2 are a valid tree-pair, let

$$P=\Pi Y_i \Pi Z_i \Pi X_{ij} \tag{13}$$

Here, Y_i is the admittance of Y edge included by T_1 and T_2, Z_i is the impedance of Z edge not included by T_1 and T_2, X_{ij} is the parameter of dependent source, its controlled edge i in T_1 and its controlling branch j in T_2. Then P is called parameter of the valid tree-pair.

Definition 5: Suppose T_1 and T_2 are a valid tree-pair, execute the following steps, the result ε is called coefficient of the valid tree-pair.

- Short the edges in both T_1 and T_2, open the edges neither in T_1 nor in T_2, get a sub-graph $\mathbf{G_d}$ of **G**. The $\mathbf{G_d}$ contains the controlled edges and norator edges in T_1 and the same number controlling edges and nullator egdes in T_2.
- Find a loop in $\mathbf{G_d}$ of which only one edge belongs to T_2 and the others to T_1. Suppose the edge belonging to T_2 is j and one of the other edges is i. If the directions of edge i and edge j are same along the loop let $B_{ji}=1$, otherwise let $B_{ji}=-1$.
- Short j and open i in $\mathbf{G_d}$ and reduce $\mathbf{G_d}$.
- Repeat above two steps, till find all B_{ji} and reduce $\mathbf{G_d}$ to a node or some solitary nodes.
- Let

$$\varepsilon=(-1)^{m+c}\Pi B_{ji} \tag{14}$$

Here, m is the number of times to exchange the second subscripts i of B_{ji} so that the first subscript j is the controlling edge of a dependent source X_{ij} and the second subscript i is just the controlled edge of the same dependent source, for every B_{ji}. And c is the number of CS edges in G_d.

Definition 6: The value V of a valid tree-pair is equal to the product of its coefficient ε and parameter P.

4. THEOREM AND ALGORITHMS

Theorem 1: The determinant of a closed circuit **N** is equal to the sum of the values of all valid trees and all valid tree-pairs of **N**.

That is say

$$\triangle = \Sigma V_k \qquad (15)$$

Here V_k is the value of a valid tree or a valid tree-pair.

Due to limited space the proof are omitted.

Algorithm 1: If **N** contains no nullor, then find all valid trees by the following steps.

- Open all CS edges and all VC edges from **G**, then short all VS edges and all CC edges, get **G'**.
- A tree T' of **G'** and all shorted edges make up a valid tree of **N**.
- All valid trees of **N** come from all trees of T' of **G'**.

Algorithm 2: Find all valid tree-pairs by the following steps.

- Assume parameter P contains some X_{ij}, such as n X_{ij}.
- Short all nullator edges and open all norator edges, Short the n controlled edges and open the n controlling edges, then short other VS and CC edges and open other CS and VC edges, from **G**, get $\mathbf{G_1}$.
- Short all norator edges and open all nullator edges, Short the n controlling edges and open the n controlled edges, then short other VS and CC edges and open other CS and VC edges, from **G**, get $\mathbf{G_2}$.
- A common tree T' of $\mathbf{G_1}$ and $\mathbf{G_2}$ adding all shorted edges from **G** to get $\mathbf{G_1}$ make up a tree T_1 of **G**, While T' adding all shorted edges from **G** to get $\mathbf{G_2}$ make up a another tree T_2 of **G**. Then T_1 and T_2 make up a valid tree-pair of **N**.
- All valid tree-pairs containing the n X_{ij} come from all common trees of $\mathbf{G_1}$ and $\mathbf{G_2}$.
- Make all possible assumptions, all valid tree-pairs can be found.

5. EXAMPLE

Example 1:

Ask the function $Z_T=U_0/I_S$ for the circuit in Fig. 1.

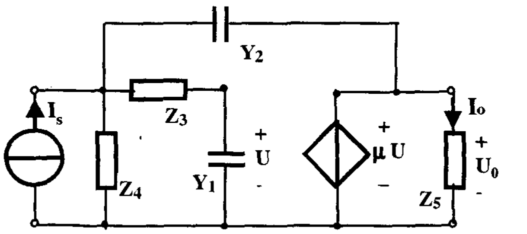
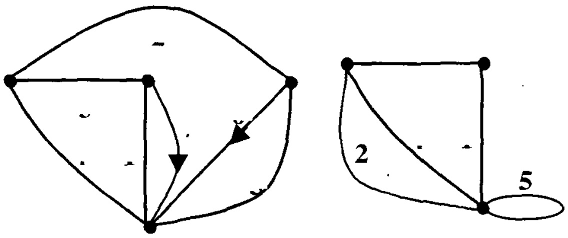


Fig.1 The circuit

Solution:

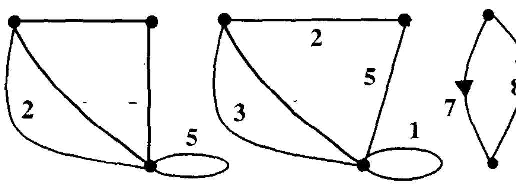
To calculate Δ_0 we open independent circuit source I_s, get graph $\mathbf{G_0}$ as Fig.2-a . There 1 and 2 are Y edges, their parameters are Y_1 and Y_2. 3, 4 and 5 are Z edges, their parameters are Z_3, Z_4 and Z_5. 6 is VS edge, 7 is VC edge, their parameters are $\mu_{67}=\mu$ and $Y_7=0$.



(a) the graph $\mathbf{G_0}$ (b) the graph $\mathbf{G_0'}$

Fig.2

To find all valid trees of $\mathbf{G_0}$, short VS edge 6 and open VC edge 7 from $\mathbf{G_0}$, get $\mathbf{G_0'}$ as Fig.2-b. The trees of $\mathbf{G_0'}$ are (1,2), (1,3), (1,4), (2,3) and (3,4). So the valid trees of $\mathbf{G_0}$ are (1,2,6), (1,3,6), (1,4,6), (2,3,6) and (3,4,6), their values are $Y_1Y_2Z_3Z_4Z_5$, $Y_1Z_4Z_5$, $Y_1Z_3Z_5$, $Y_2Z_4Z_5$, and Z_5.



(a) The graph $\mathbf{G_{0\text{-}1}}$ (b) the graph $\mathbf{G_{0\text{-}2}}$ (c) the graph $\mathbf{G_{0\text{-}d}}$

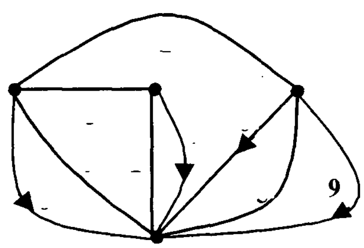
Fig.3

To find all valid tree-pairs including μ_{67} of $\mathbf{G_0}$, short edge 6 and open edge 7 from $\mathbf{G_0}$, get $\mathbf{G_{0\text{-}1}}$ as Fig.3-a; short edge 7 and open edge 6 from $\mathbf{G_0}$, get $\mathbf{G_{0\text{-}2}}$ as Fig.3-b. There is only one common tree (2,3) of $\mathbf{G_{0\text{-}1}}$ and $\mathbf{G_{0\text{-}2}}$, so the only valid tree-pair of $\mathbf{G_0}$ is {(6,2,3), (7,2,3)}, its parameter is $\mu_{67}Y_2Z_4Z_5$. Short edge 2 and 3 and open edge 1,4 and 5 from G_0, get $G_{0\text{-}d}$ as Fig.3-c. There B_{76}=-1, m=0, c=0, so $\varepsilon=(-1)^{0+0}(-1)=-1$. The value of valid tree-pair {(6,2,3), (7,2,3)} is $-\mu_{67}Y_2Z_4Z_5$.

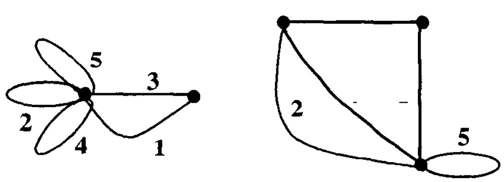
Thus

$$\Delta_0= Y_1Y_2Z_3Z_4Z_5+Y_1Z_4Z_5+Y_1Z_3Z_5+Y_2Z_4Z_5+Z_5 -\mu_{67}Y_2Z_4Z_5$$

To calculate Δ_{ij} we replace I_s breach by $g_{89}=-1/Z_T$, and construct closed network **N**, its graph **G** is Fig.4. There 8 is CS edge, 9 is VC edge, their parameters are $g_{89}=-1/Z_T$ and $Y_9=0$. Now we are finding all valid tree-pairs including g_{89} of **G**.



Assume P includes g_{89} but no μ_{67}. Short edge 8 and open edge 9, short VS edge 6 and open VC edge 7, from **G**, get G_1 as Fig.5-a. Short edge 9 and open edge 8, short VS edge 6 and open VC edge 7, from **G**, get G_2 as Fig.5-b. There is no common tree of G_1 and G_2, so no valid tree-pair.



(b) The graph G_1 **(b)** the gr··h G_2

Fig.5

Assume P includes g_{89} and μ_{67}. Short edge 8 and 6, open edge 9 and 7, from **G**, get G_1 as Fig.6-1, Short edge 9 and 7, open edge 8 and 6, from **G**, get G_2 as Fig.6-2. There is only one common tree is (3) of G_1 and G_2, so the only valid tree-pair of **G** is {(8,6,3), (9,7,3)}, its parameter is $g_{89}\mu_{67}Y_2Z_4Z_5$. Short edge 3 and open edge 1,2,4 and 5 from **G**, get G_d as Fig.6-3. There B_{78}=-1, B_{96}=-1, n=1,c=1, so $\varepsilon=(-1)^{1+1}(-1)(-1)=1$. The value of value tree-pair {(8,6,3), (9,7,3)} of G is $g_{89}\mu_{67}Z_4Z_5$.

Thus

$$\Delta_{ij}=\mu_{67}Z_4Z_5$$

Therefore the function

$$Z_T=\frac{U_0}{I_s}=\frac{\Delta_{ij}}{\Delta_0}=$$

$$\frac{\mu_{67}Z_4Z_5}{Y_1Y_2Z_3Z_4Z_5+Y_1Z_4Z_5+Y_1Z_3Z_5+Y_2Z_4Z_5+Z_5-\mu_{67}Y_2Z_4Z_5}$$

$$=\frac{\mu Z_4}{Y_1Y_2Z_3Z_4+Y_1Z_4+Y_1Z_3+Y_2Z_4+1-\mu_{67}Y_2Z_4}$$

There is not enough space to show an example containing nullors. We may look on a nullor as a dependent source which parameter is infinity and every valid terms of the determinant must include this parameter, then the parameter will be reduced from the numerator and denominator. The definitions, theorem and algorithms are available for the circuits containing nullor.

6. SUMMRA

A new symbolic analysis approach by topological operation is put forward. This approach offers such advantages. First it applies to general linear active circuits containing impedance, admittance, four types of dependent sources and nullors, and can generate various types of network functions by unified topological formula. Secondly it uses the circuit's topological graph and parameter symbols, and directly operates on them without establishing equations of any form Next only the valid trees and the valid tree-pairs need to be looked for, and only the coefficients of the valid tree-pairs need to be calculated. There is no term cancelled in the process. Therefor the approach is a more general and more efficient than the tree enumeration methods now available.

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