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# AN ALGORITHM TO TEST IDENTIFIABILITY OF NON-LINEAR SYSTEMS

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Abstract: This paper considers an approach for analyzing the identifiability of nonlinear controlled or uncontrolled dynamical systems. The method is based on the computation of the ideal containing the differential algebraic relations between the input and the output of the model, a such ideal is called input-output ideal.
Our contribution consists in developing the corresponding algorithm in a symbolic computing language. This algorithm is based on differential algebra. *Copyright © 2001 IFAC*

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## 1. INTRODUCTION

We consider implicit controlled or uncontrolled ($g \equiv 0$) systems whose initial conditions are assumed to be unknown :

$$\Sigma^\theta \begin{cases} \dot{x}(t,\theta) = f(x(t,\theta),\theta) + u(t)g(x(t,\theta),\theta), \\ y(t,\theta) = h(x(t,\theta),\theta). \end{cases}$$

$x(t,\theta) \in \mathbf{R}^n$ and $y(t,\theta) \in \mathbf{R}^m$ denote the state variables and the measured outputs, respectively. It is assumed that $u \in \mathcal{U}_p$, a set of analytic functions, and $\theta \in \mathcal{U}_{ad}$ where $\mathcal{U}_{ad}$ is an open subset of $\mathbf{R}^p$. The single-input case is considered for notational simplicity; all the results can be readily generalized.

The functions $f(x,\theta)$, $g(x,\theta)$ and $h(x,\theta)$ are real and analytic on $M$ for every $\theta \in \mathcal{U}_{ad}$ ($M$ is a connected open subset of $\mathbf{R}^n$ such that $x(t,\theta) \in M$ for every $\theta \in \mathcal{U}_{ad}$ and every $t \in \mathbf{R}^+$).

As the initial conditions are assumed to be unknown, the solution corresponding to the pair $(\theta, u)$, may not be unique and some solutions may be degenerate. So $\bar{y}(\theta,u)$ is defined as a set of non-degenerate solutions (Ljung and Glad, 1994). Then a model $\Sigma^\theta$ is globally identifiable at $\theta \in \mathcal{U}_{ad}$ if for any $\bar{\theta} \in \mathcal{U}_{ad}$, $\bar{\theta} \neq \theta$ there exists a control $u \in \mathcal{U}_p$, such that $\bar{y}(\theta,u) \neq \emptyset$ and $\bar{y}(\theta,u) \cap \bar{y}(\bar{\theta},u) = \emptyset$.

When the functions $f, g$ and $h$ are rational functions, the techniques based on differential algebra can be used. Indeed, the controlled system is considered as a set of variables $u, x, y, \theta$ linked by a differential ideal.

The aim of this paper is to present an algorithm based on an approach given by F. Ollivier (Ollivier, 1997) who proposed to compute an input-output ideal of differential algebraic relations between $u$ and $y$. This approach is not the same as that of (Ljung and Glad, 1994) who express each parameter as a function of $u$, $y$ and their derivatives. More it is different of the procedure (Glad, 1998) in the sense that it does not only test the identifiability of the model but it also gives some identifiable combinations of parameters.

On the other hand, our algorithm uses another algorithm, named Rosenfeld-Groebner, written and efficiently implemented by (Boulier *et al.*, 1995).

The paper is organized as follows. In section 2, some notions of differential algebra are briefly given. Then section 3 presents the input-output method. More precisely, some theorical results are shown in order to justify the algorithm which is given in section 4 and illustrated by four examples.

## 2. DEFINITIONS

### 2.1 *Commutative algebra*

Let $K$ a field, $X$ an ordered alphabet and $K[X]$ a polynomial ring.

Let $p \in K[X]\backslash K$ be a polynomial. The leader of $p$ is the greatest indeterminate $x$ which occurs in $p$ and we denote by $d$ the highest power of this variable in $p$.

The *separant* $s_p$ of $p$ is the partial derivative of $p$ with respect to the leader $x$ while the *initial* $i_p$ is the coefficient of $x^d$.

Let $A \subset K[X]\backslash K$. $H_A$ denotes the set of initials and separants of its elements. $A$ is said triangular if its elements have distinct leaders. A polynomial $p$ is said *strongly normalized* with respect to (w.r.t) $A$ if no leader of $A$ occurs in the initial of $p$. The set $A$ is said to be *strongly normalized* if every $p$ in $A$ is strongly normalised w.r.t $A\backslash\{p\}$.

If $R$ is a unique factorization domain and $p \in R[X]$ then $p$ can be rewritten as $p = a_0t_0 + \ldots + a_kt_k$, where the $t_i$ are a power product of elements of $X$ and the $a_i \in R$. The *content* of $p$ over $R$ is the great common divisor of its coefficients, i-e $cont(p) = gcd(a_0, \ldots, a_k)$. The primitive part of $p$ over $R$ is the polynomial $pp(p) = \dfrac{p}{cont(p)}$. A polynomial is said *primitive* if it is equal to its primitive part.

Let $\mathcal{I}$ an ideal over $K[X]$. If $S = \{s_1, \ldots, s_t\}$ is a finite family of elements of $K[X]$ then the *saturation* of $\mathcal{I}$ by $S$, noted $\mathcal{I}:S^\infty$, is the ideal : $\{p \in K[X] \mid \exists\ (n_1, \ldots, n_t) \in \mathbb{N}^t\ ,\ s_1^{n_1} \ldots s_t^{n_t} p \in \mathcal{I}\}$.

### 2.2 *Differential algebra : reduction*

Let $V = \{v_1, \ldots, v_n\}$ be a set of *differential indeterminates* and $\Delta = \{\delta_1, \ldots, \delta_m\}$ a set of derivations which commute pairwise. We denote by $\Theta$ the commutative monoid generated by $\Delta$ and by $\Theta V$ the set of all the *derivatives* $\phi u, \phi \in \Theta, v \in V$. Let $K$ a differential field, we denote by $K\{V\}$ the differential ring of the differential polynomials built over the alphabet $\Theta V$ with coefficients in $K$.

A *ranking* $R$ is a well-ordering over the variables and their derivatives with the following properties for every derivation $\delta \in \Delta$ :

(1) $u \prec \delta u$,
(2) $u \prec v \Rightarrow \delta u \prec \delta v$.

Rankings such that $u \prec v \Rightarrow \delta u \prec \phi v$ (for every derivation operators $\delta \in \Delta$ and $\phi \in \Delta$, and every differential indeterminates $v, u$) are said *elimination* rankings and they are written $[u] \prec [v]$.

With a given ranking, the alphabet get ordered so the leader, separant, initial... are well defined. The highest ranking variable or derivative of a variable in a differential polynomial is called *the leader*.

Let $v$ the leader of $p$ and $d$ the degree of $p$ in this variable. A differential polynomial $q$ is said to be *partially reduced* w.r.t $p$ if no proper derivative of $v$ occurs in $q$. It is said to be *reduced* w.r.t $p$ if it is partially reduced w.r.t $p$ and $\deg(q, v) \leq d$.

A set $A$ of differential polynomials is said to be *auto-reduced* if its elements are pairwise reduced.

### 2.3 *Differential algebra : characteristic presentation*

A differential ideal of $K\{V\}$ is an ideal of $K\{V\}$ stable under derivation. Let $\mathcal{J}$ a differential ideal of $K\{V\}$ and a ranking $R$.

A set $\mathcal{D} \subset \mathcal{J}$ is a characteristic set if $\mathcal{D}$ is autoreduced and $\mathcal{J}$ contains no nonzero polynomial reduced w.r.t $\mathcal{D}$.

Let a set $\mathcal{C} \subset \mathcal{J}$, we introduce now the following notations :

- $L$ denotes the set of the leaders of the elements of $\mathcal{C}$.
- $N$ denotes the other derivatives occuring in $\mathcal{C}$.

We will say that a set $\mathcal{C} \subset \mathcal{J}$ is a characteristic presentation of $[\mathcal{C}] : H_{\mathcal{C}}^\infty$ if and only if $\mathcal{C}$ is a characteristic set of $[\mathcal{C}] : H_{\mathcal{C}}^\infty$ and $\mathcal{C}$ is a strongly normalized autoreduced set of $K[L, N]$ such that the elements of $\mathcal{C}$ are primitive over $K[N]$ (Boulier and Lemaire, 2000).

This notion supplies an effective way for testing the equality of two regular ideals : such ideals are equal if and only if, endowed with the same ranking, their characteristic presentation contains the same polynomials (we will say that the characteristic presentations are equal).

## 3. INPUT-OUTPUT APPROACH

As mentioned in the introduction, we have developed a method to check the identifiability of a large class of parametric models $\Sigma^\theta$.

Let us introduce some notations :

- we denote by $\mathcal{I}$, the radical of the differential ideal generated by the equations of $\Sigma^\theta$ completed with $\dot\theta_i = 0$, $i = 1, \dots, q$. In order to simplify the statement, we assume that $\mathcal{I}$, endowed by the following ranking which eliminates the state variables :

$$[\theta] \prec [y,u] \prec [x], \qquad (1)$$

admits, as a characteristic presentation, the set

$$\mathcal{C} = \{c_1, \dots, c_{m+n}, \dot\theta_1, \dots, \dot\theta_q\}.$$

Moreover, we note $\mathcal{C}(\theta)$ the evaluation of $\mathcal{C}$ in the particular value $\theta$.

- $\mathcal{I}_\theta$ is the radical of the differential ideal generated by the previous equations for the particular value of parameter $\theta$. We still assume that, for all $\theta \in \mathcal{U}_{ad}$, $\mathcal{I}_\theta$ is a regular ideal presented by a characteristic presentation $C_\theta$ if it is endowed by the ranking $[y,u] \prec [x]$.
- Finally, if we note $\mathcal{I}_\theta^{io}$ the ideal obtained after eliminating state variables, the set

$$C_\theta^{io} = C_\theta \cap \mathbf{Q}(\theta)\{U, Y\}$$

is a characteristic presentation of this ideal.

Evaluating the initial system in every parameter value and computing the associated characteristic presentation, are an unrealistic procedure. The following proposition simplifies the procedure. Indeed, it gives a sufficient condition of the equality $C_\theta = \mathcal{C}(\theta)$.

*Proposition 3.1.* If for every $\theta \in \mathcal{U}_{ad}$ and for every $i = 1, \dots, m+n$, the initial of $c_i(\theta) \in \mathcal{C}(\theta)$ is not equal to zero and none of its factors ($\neq 1$) is a divisor of all the other coefficients of $c_j(\theta)$ in $K[N]$, $\forall j = 1, \dots, m+n$ ($j \neq i$), then $C_\theta = \mathcal{C}(\theta)$ $\forall \theta \in \mathcal{U}_{ad}$.

*Proof -*
Notice that $\mathcal{C}$ is strongly normalized so each initial of $\mathcal{C}$ is in $K[N]$ and $\theta_i$ can not be the leader of any polynomial in $\mathcal{C}$ (see (Noiret, 2000)).

- $\mathcal{C}(\theta)$ is an auto-reduced set, indeed evaluating the polynomials keeps the leader ($\theta$ has been assumed not to cancel initials). Moreover, such an evaluation does not change the degree of elements of $L$ appearing in the polynomials.
- Suppose that there exists a polynomial $p$ reduced w.r.t $\mathcal{C}(\theta)$, a such polynomial is reduced w.r.t $\mathcal{C}$. As $\mathcal{C}$ is a characteristic set of $\mathcal{I}$, $p = 0$.

Therefore, $\mathcal{C}(\theta)$ is a characteristic set (section 2.3).

Next, evaluating in $\theta$ has not changed the leaders of all polynomials of $\mathcal{C}$. So since $\mathcal{C}$ is strongly normalized, $\mathcal{C}(\theta)$ is also strongly normalized. Afterwards, the elements of $\mathcal{C}(\theta)$ are primitive on $K[N]$ because, by hypothesis, the evaluated coefficients have no common divisors.

In that way, $\mathcal{C}(\theta)$ is a characteristic set of $\mathcal{I}_\theta$. By unicity of the characteristic set of an ideal endowed by a ranking, $\mathcal{C}_\theta = \mathcal{C}(\theta)$. □

We give here two propositions to test identifiability. This first one is due to Ollivier (Ollivier, 1997).

*Proposition 3.2.* If there exists some generic solution of $\Sigma^\theta$, then $\Sigma^\theta$ is globally identifiable at $\theta$ if and only if for every $\bar\theta \in \mathcal{U}_{ad}$ ($\bar\theta \neq \theta$), the two corresponding input-output characteristic presentations are distinct.

A solution is called generic if it verifies no equation out of the ideal's equations (Ollivier, 1997) (we notice that a generic solution is also non-degenerate). On the other hand, the genericity assumption is hard to test but the strong accessibility from all initial conditions in a dense open subset of $\mathbb{R}^n$ implies this property. And the strong accessibility is obtained by the strong accessibility rank criterion (SARC) (van der Schaft and Nijmeijer, 1996) at every initial condition of this dense open subset. But when we study an uncontrolled model, this test is not valid. So, we have investigated for another characterization of the identifiability. The polynomials $P_i, i = 1 \dots m$, of $C_\theta^{io}$ can be seen as polynomials in $y$ and their derivatives with coefficients in $K(\theta)$. We know that a polynomial whose indeterminates are independant, is null if and only if all its coefficients are zero. We note :

$$P_i(y,u) = m_0(y,u) + \sum_{k=1}^{n_i} p_k^i(\theta) m_i(y,u),$$
$$\Delta(P_i)(y,u) = \det\left(m_k(y,u), k = 1, \dots, n_i\right),$$
$$Det(P_i)(y,u) = \det\left(m_l(y,u), l = 0, \dots, n_i\right).$$

Then we prove the following proposition :

*Proposition 3.3.* If $\Delta(P_i)(y,u) \neq 0, \forall(y,u)$ solution of $\mathcal{I}_\theta^{io}$ and $\forall i = 1 \dots m$, then $\Sigma^\theta$ is globally identifiable at $\theta$ if and only if for every $\bar{\theta} \in \mathcal{U}_{ad}$ ($\bar{\theta} \neq \theta$), the two corresponding input-output characteristic presentations are distinct.

*Proof*-
The necessary condition is straightforward. Let us show the sufficient condition :

Let $\theta \in \mathcal{U}_{ad}$, suppose there exists an input $u^*$ such that $\bar{y}(\theta, u^*) \neq \emptyset$ and $y^* \in \bar{y}(\theta, u^*) \cap \bar{y}(\bar{\theta}, u^*)$ for a value $\bar{\theta} \in \mathcal{U}_{ad}$.

We have

$$P_j(y^*, u^*, \theta) = P_j(y^*, u^*, \bar{\theta}), \forall j \in \{1, \dots, m\}.$$

For the fixed index $j$, we denote by $Q_j(y,u)$ the polynomial $P_j(y,u,\theta) - P_j(y,u,\bar{\theta})$. Moreover $Det(Q_j)(y,u) = \Delta(P_j)(y,u)$. By hypothesis, this determinant evaluated in $(y^*, u^*)$ and their derivatives is not equal to zero. Therefore, we obtain the identifiability of the coefficients $p_k^j(\theta)$, for the indexes $j = 1, \dots, m$ and $k = 1, \dots, n_j$.

Since, by assumption on $\mathcal{U}_{ad}$, the two characteristic presentations $C_\theta^{io}$ and $C_{\bar{\theta}}^{io}$ contain the differential polynomials which are in $C^{io}$ evaluated respectively in $\theta$ et $\bar{\theta}$, the two characteristic presentations are equal, i-e $C_\theta^{io} = C_{\bar{\theta}}^{io}$. So $\theta = \bar{\theta}$ and $\Sigma^\theta$ is globally identifiable at $\theta$. $\square$

## 4. AN ALGORITHM TO TEST IDENTIFIABILITY

### 4.1 *Rosenfeld-Gröebner*

Technically, from a finite set of differential polynomials and a given ranking, the Rosenfeld-Groebner algorithm returns a list (understand "intersection") of regular differential ideals $\mathcal{J}_1, \dots, \mathcal{J}_j$ presented by characteristic presentations $C_1, \dots, C_j$ so that

$$\sqrt{\mathcal{I}} = \mathcal{J}_1 \cap \mathcal{J}_2 \cap \dots \cap \mathcal{J}_j. \qquad (2)$$

In order to simplify the statement, we describe the algorithm when $j = 1$.

**Data** : $f$, $g$ and $h$ (the functions occuring in $\Sigma^\theta$).

**Step 1** computes the input-output characteristic presentation $C^{io}$.

**Step 2** computes the values of $\theta$ such that the assumptions of proposition 3.1 are not valid.

**Step 3** saves the coefficients $p_k^i(\theta)$ (see 3.3) in a list called exhaustive summary.

**Step 4** simplifies the exhaustive summary in order to extract its smallest generator system in terms of degree, number of monomials, ...

**Step 5** validates the method by checking the SARC or the independence of the monomials in $y$, $u$ and their derivatives occuring in the polynomials of $C^{io}$.

### 4.2 *examples*

The following *academic example* has been treated without control in (Denis-Vidal and Joly-Blanchard, 2000) :

$$\begin{cases} \dot{x}_1 &= \theta_1 x_1^2 + \theta_2 x_1 x_2 + u \\ \dot{x}_2 &= \theta_3 x_1^2 + \theta_4 x_1 x_2 \\ y &= x_1 \end{cases}$$

The algorithm distinguishes three cases :

- The general case gives
$\mathcal{U}_{ad} = \{(\theta_1, \theta_2, \theta_3, \theta_4) \in \mathbb{R}^4,\ \theta_2 \neq 0\}$
and is presented by the input-output equation:

$$y\ddot{y} = y\dot{u} + y^2\theta_1\dot{y} + y^4\theta_2\theta_3 - u\dot{y} \\ -uy^2\theta_4 + \dot{y}y^2 - y^4\theta_1\theta_4.$$

  The software verifies that the hypotheses of the proposition (3.1) are valid for all $\theta \in \mathcal{U}_{ad}$. If $(\theta_3, \theta_4) \neq (0,0)$, it checks the SARC to be satisfied for almost all values of initial conditions. Under this condition, the identifiability of $\theta_1$, $\theta_4$ and $\theta_2\theta_3$ is obtained. So these combinations only are structurally globally identifiable on $\mathcal{U}_{ad}$ and the system is not identifiable on $\mathcal{U}_{ad}$.
- $\theta_2 = 0$ is a particular case corresponding to the input-output relation :

$$-u + \dot{y} - y^2\theta_1 = 0$$

  With $\mathcal{U}_{ad} = \{(\theta_1, \theta_3, \theta_4) \in \mathbb{R}^3\}$ , the identifiability of $\theta_1$ is only obtained.
- $y = 0$ is a non-generic solution which doesn't give any identifiable parameter.

Now we consider *two real pharmacokinetic models* (Demignot and Domurado, 1987).

Here is the first one :

$$\begin{cases} \dot{x}_1 &= \alpha_1(x_2 - x_1) - \dfrac{V_m x_1}{1 + x_1} \\ \dot{x}_2 &= \alpha_2(x_1 - x_2) \\ y &= x_1 \end{cases}$$

The software rewrites the system as the following polynomial differential system :

$$\begin{cases} \dot{x}_1(1 + x_1) &= \alpha_1(x_2 - x_1)(1 + x_1) - V_m x_1 \\ \dot{x}_2 &= \alpha_2(x_1 - x_2) \\ y &= x_1 \\ 1 + x_1 &\neq 0 \end{cases}$$

From this system, the algorithm distinguishes two cases :

- The general case is presented by the input-output equation if $\alpha_1 \neq 0$:

$$2\ddot{y}y + \ddot{y} + (\alpha_1 + \alpha_2)\dot{y}y^2 + 2(\alpha_1 + \alpha_2)\dot{y}y + (\alpha_1 + \alpha_2 + V_m)\ddot{y} + \alpha_2 V_m y^2 + \alpha_2 V_m y = -\ddot{y}y^2$$

If $\mathcal{U}_{ad} = \{(\alpha_1, \alpha_2, V_m) \in \mathbb{R}^3, V_m\alpha_1 \neq 0\}$, the software returns the global identifiability of all the parameters after verifying $\det(\dot{y}y^2, \dot{y}y, \dot{y}, y^2, y) \neq 0$, for all $y$ solution of $\mathcal{I}^{io}$.
- $\alpha_1 = 0$ is a particular unidentifiable case with $\mathcal{U}_{ad} = \{(\alpha_2, V_m) \in \mathbb{R}^2, V_m\alpha_2 \neq 0\}$.

The second pharmacokinetic model is described by

$$\begin{cases} \dot{x}_1 = \alpha_1(x_2 - x_1) - \dfrac{k_a V_m x_1}{k_c k_a + k_c x_3 + k_a x_1} + u \\ \dot{x}_2 = \alpha_2(x_1 - x_2) \\ \dot{x}_3 = \beta_1(x_4 - x_3) - \dfrac{k_c V_m x_3}{k_c k_a + k_c x_3 + k_a x_1} + u \\ \dot{x}_4 = \beta_2(x_3 - x_4) \\ y_1 = x_1 \\ y_2 = x_3 \end{cases}$$

Firstly, the software rewrites the model as a polynomial differential system if $k_a k_c V_m \neq 0$. The algorithm returns a characteristic presentation. There are two input-output equations corresponding to the developed form of the following equations

$$\ddot{y}_1 = -\alpha_2(\dot{y}_1 + F_1 - u) - \alpha_1(\dot{y}_1) - \dot{F}_1 + \dot{u}$$
$$\ddot{y}_2 = -\beta_2(\dot{y}_2 + F_2 - u) - \beta_1(\dot{y}_2) - \dot{F}_2 + \dot{u}$$

where

$$F_1 = \frac{k_a V_m y_1}{k_c k_a + k_c y_2 + k_a y_1}$$
$$F_2 = \frac{k_a V_m y_2}{k_c k_a + k_c y_2 + k_a y_1}$$

The assumptions of the proposition 3.1 are satisfied for all $\theta \in \mathcal{U}_{ad}$ where $\mathcal{U}_{ad}$ is defined by $\{(\alpha_1, \alpha_2, \beta_1, \beta_2, k_a, k_c, V_m) \in \mathbb{R}^7, k_a k_c V_m \neq 0\}$. The computation of the SARC allows to conclude that the system has generic solutions and that the proposition (3.2) leads to an identifiability result.

## 5. CONCLUSION

The input-ouput approach allows us the identifiability of a wide class of models to be stated. Indeed, it can be applied to controlled or uncontrolled systems given in implicit form.

This approach is generally difficult to perform by pen and paper. But it has been used here to write an algorithm which checks easily the identifiability. This algorithm is based on notions of differential algebra and is implemented in MAPLE V within the package DIFFALG. Indeed, the idea of the algorithm is to analyse an ideal linked to the model equations by giving its characteristic presentation. So, from the data of the functions $f, g$ and $h$ occuring in the equations of the model, it computes the characteristic presentation. Then, after checking some specific conditions, it gives the identifiability result.

Even if the algorithm leads sometimes to cumbersome computation, the algorithm investigates efficiently many significant examples.

## 6. REFERENCES


Boulier, F. and F. Lemaire (2000). Computing canonical representatives of regular differential ideals. In: *proc. ISSAC 2000, International Symposium on Symbolic and Algebraic Computation.* St Andrews, Scotland.

Boulier, F., D. Lazard, F. Ollivier and M. Petitot (1995). Representation for the radical of a finitely generated differential ideal. In: *proc. ISSAC'95, International Symposium on Symbolic and Algebraic Computation.* Montréal, Canada. pp. 158–166.

Demignot, S. and D. Domurado (1987). Effect of prosthetic sugar groups on the pharmacokinetics of glucose-oxidase. *Drug Design and Delivery* **1(3)**, 333–348.

Denis-Vidal, L. and G. Joly-Blanchard (2000). An easy to check (un)identifiability criterion for uncontrolled systems and its applications. *IEEE Trans. on Automatic Control.*

Glad, S. T. (1998). Identifiability with constraints. In: *Proc. NOLCOS.* pp. 455–458.

Ljung, L. and S. T. Glad (1994). On global identifiability for arbitrary model parametrizations. *Automatica* **30**(2), 265–276.

Noiret, C. (2000). Utilisation du calcul formel pour l'identifiabilité de modèles paramétriques et nouveaux algorithmes en estimation de paramètres. PhD thesis. Université de technologie de Compiègne.

Ollivier, F. (1997). Identifiabilité des systèmes. Technical Report 97–04. GAGE, Ecole polytechnique.

van der Schaft, A. J. and H. Nijmeijer (1996). *Nonlinear Dynamical Control Systems.* Springer-Verlag.