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Analytical redundancy relations for fault detection and isolation in algebraic dynamic systems☆

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Abstract

This paper extends the analytical redundancy techniques, well developed for linear systems, to FDI in non-linear dynamic systems modeled by polynomial differential algebraic equations. The residual evaluation form is developed for sensor, actuator and process component faults. Fault detectability and isolability conditions are given, and the design of robust structured residuals is addressed. © 2001 Elsevier Science Ltd. All rights reserved.

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1. Introduction

Most work on fault detection and isolation in non-linear systems is based on non-linear observers (Seliger & Frank, 1991; Hammouri, Kinnaert, & Yaagoubi, 1999; Yu & Shields, 1995). Parity space methods have only been investigated for linear (Chow & Willsky, 1984), bi-linear (Yu, Williams, Shields, & Gomm, 1995) or state affine (Comtet-Varga, Cassar, & Staroswiecki, 1997) systems. The aim of this paper is to extend the parity space approach to non-linear systems, focusing on the issues of robustness, fault detectability and isolability.

Analytical redundancy relations (ARR) have to be sensitive to faults and insensitive to perturbations. The effects of different faults have to be different. This can be achieved by designing robust structured residuals. Considerable attention has been paid to this problem for linear systems, including optimization approaches when only approximate properties can be obtained (Patton, 1994). In this paper, ARR design is discussed for the class of algebraic dynamic systems. Continuous-time models are considered, but the approach obviously extends to discrete-time models.

The paper is organized as follows. Section 2 recalls some results on analytical redundancy relations for polynomial dynamic systems. Section 3 concerns fault detection and isolation. The residual evaluation form is developed for sensor, actuator or component faults, and the issues of fault detectability and fault isolability are discussed. Section 4 addresses the design of robust structured residuals. An induction motor example is given in Section 5.

2. Analytical redundancy relations in algebraic dynamic systems

We consider continuous-time, time-invariant systems described by

$$\dot{x} = f(x, z, \theta), \tag{1}$$

$$y = h(x, z, \theta), \tag{2}$$

where $x \in R^n$ is the state vector, $z \in R^m$ is the input vector, $y \in R^p$ is the output vector, $\theta \in R^l$ is a constant parameter vector; $z = (u^t, v^t, \zeta^t, \varepsilon^t)^t$ where u is the control vector, v is some unknown input vector, ζ is some fault vector and ε is some stochastic vector ($v = 0$ means no perturbation, $\zeta = 0$ means no fault and $\varepsilon = 0$ means no noise). The components of f and h are polynomial functions over R of their arguments. The following notation will be used:

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ϑ being some vector, $\bar{\vartheta}^{(k)}$ is the vector whose components are ϑ and its time derivatives up to order k; $\bar{\vartheta}$ stands for ϑ and its time derivatives up to some (unspecified) order.

2.1. Existence of analytical redundancy relations

Analytical redundancy relations (ARR) involve only $\bar{y}$ and $\bar{z}$. For linear systems, the unknown state is eliminated from (1), (2) using the parity space approach. Projection techniques can also be used for state-affine systems which are polynomial in the input (Comtet-Varga, Cassar, & Staroswiecki, 1997). Such a class obviously includes bi-linear systems. In the case of algebraic dynamic systems, elimination theory provides the most straightforward and understandable tool for state elimination. Note that ARR are input–output system descriptions (although in some cases output–output relations exist, e.g. when redundancy is present among the system sensors). Many authors have addressed their existence and computation (Fliess, 1989; Glad, 1989; Diop, 1991), their use for the analysis of structural properties (Diop & Wang, 1993; Ljung & Glad, 1994) and some recent papers have considered fault detection and isolation issues (Guernez, Cassar, & Staroswiecki, 1997; Zhang, Basseville, & Benveniste, 1998; Shumsky, 2000). This section recalls some results about the existence of ARR for polynomial dynamic systems.

2.1.1. Auto-redundancy

For the jth output, $j \in J = \{1, \ldots, p\}$, Eq. (2) is

$$y_j = h_j(x, z, \theta). \tag{3}$$

From (1) and (3) one has $\dot{y}_j = (\partial h_j/\partial x)f + (\partial h_j/\partial z)\dot{z}$ and it can be proven that for all $(j, s) \in J \times N$ there exists g_s^j a polynomial function over R such that $y_j^{(s)} = g_s^j(x, \bar{z}^{(s)}, \theta)$. Let $g_0^j = h_j$ and $G_s^j = ((g_0^j)^t (g_1^j)^t \ldots (g_{s_j}^j)^t)^t$. Let r_s^j be the generic rank of the Jacobian matrix $J_x(G_s^j)$.

Theorem 1. *For any fixed $s \in N$ there are exactly $s + 1 - r_s^j$ ARR*:

$$w_k(\bar{y}_j^{(s)}, \bar{z}^{(s)}, \theta) = 0, \quad k = 1, \ldots, s + 1 - r_s^j \tag{4}$$

which are independent in the following sense: *the Jacobian matrix*

$$J_{\bar{y}_j}(W) = \begin{bmatrix} \left(\dfrac{\partial w_1}{\partial \bar{y}_j^{(s)}}\right)(\bar{y}_j^{(s)}, \bar{z}^{(s)}, \theta) \\ \vdots \\ \left(\dfrac{\partial w_{s+1-r_s^j}}{\partial \bar{y}_j^{(s)}}\right)(\bar{y}_j^{(s)}, \bar{z}^{(s)}, \theta) \end{bmatrix}$$

is of rank $s + 1 - r_s^j$ for almost any vector $(\bar{y}_j^{(s)}, \bar{z}^{(s)}, \theta)$ which satisfies (4).

Corollary 1. $\exists r^j \in N$, $r^j \le n$ *such that*

$$r_s^j = \begin{cases} s + 1, & s < r^j, \\ r^j, & s \ge r^j. \end{cases}$$

Therefore, ARR exist only if $s \ge r^j$ (obviously, $s + 1 - r_s^j = 0$ when $s < r^j$).

Remark 1. In Fliess (1989) r^j is the algebraic dimension of the single output system (1), (3) i.e. the dimension of the minimal state according to realization theory.

Remark 2. Corollary 1 is the analogue of the Cayley–Hamilton theorem for non-linear algebraic systems.

Remark 3. The ARR of a polynomial system form a differential ideal.

2.1.2. Inter-redundancy

ARR which involve several outputs are called *inter-redundancy* relations when they cannot be inferred from auto-redundancy relations.

Theorem 2. *Let $(s_1, s_2, \ldots, s_p) \in N^p$ with $s_j < r^j$ $\forall j \in J$ and let $r_{(s_1, s_2, \ldots, s_p)}$ be the generic rank of $[J_x^t(G_{s_1}^1) \ \ldots \ J_x^t(G_{s_p}^p)]^t$. Then there are exactly $\eta = \sum_{j=1}^p s_j + p - r_{(s_1, s_2, \ldots, s_p)}$ independent ARR involving $\bar{y}_1^{(s_1)}, \ldots, \bar{y}_p^{(s_p)}$, θ and $\bar{z}$ in the form*

$$w_k(\bar{y}_1^{(s_1)}, \ldots, \bar{y}_p^{(s_p)}, \bar{z}, \theta) = 0, \qquad k = 1, \ldots, \eta \tag{5}$$

with the following rank property:

$$\mathit{rank} \begin{bmatrix} \left(\dfrac{\partial w_1}{\partial \bar{y}}\right)(\bar{y}, \bar{z}, \theta) \\ \vdots \\ \left(\dfrac{\partial w_\eta}{\partial \bar{y}}\right)(\bar{y}, \bar{z}, \theta) \end{bmatrix} = \eta$$

for almost any $\bar{y} = (\bar{y}_1^{(s_1)}, \ldots, \bar{y}_p^{(s_p)})$, $\bar{z}$ and θ which satisfy (5).

Corollary 2. $\forall (s_1, \ldots, s_p) \in N^p$ *with $s_j < r^j$ $\forall j \in J$,*

$$r_{(s_1, \ldots, s_p)} \le \mathrm{Min}\left(\sum_{j=1}^{p} s_j + p, n\right).$$

When equality holds, the choice $\sum_{j=1}^p s_j + p = n + 1$ minimizes the highest order of derivatives for each output. Only one ARR exists.

2.2. Computational issues

2.2.1. (Off-line) state elimination

Computing the ARR amounts to eliminating the state in the system:

$$\bar{y}^{(s_j)} = G_{s_j}^j(x, \bar{z}^{(s_j)}, \theta), \qquad j = 1, \ldots, p. \tag{6}$$

There are, basically, three techniques. All three require the components of the state to be eliminated according to some selected order.

Elimination theory rests on Euclidean division and derivation. Since (6) contains no derivative of the variables to be eliminated, only Euclidean division is used for ARR computation. Details on elimination theory may be found in Diop (1991) and Seidenberg (1956).

Gröbner bases rest on Buchberger's algorithm (Buchberger, 1985) which uses Euclidean division and the computation of the so-called S-polynomials. Details and definitions can be found in Cox, Little, and O'Shea (1992). For an application of Gröbner bases to the calculation of ARR for discrete-time polynomial systems, see Guernez, Cassar, & Staroswiecki (1997).

Characteristic sets (also called Ritt's algorithm) rest on Euclidean division and derivation. The state is directly eliminated from system (1), (2), and ARR with minimum derivative order can be obtained. The reader is referred to Glad (1989) for an application to the calculation of input–output relations and to Zhang, Basseville, and Benveniste (1998) for an application to FDI.

2.2.2. (*On-line*) *estimation of derivatives*

ARR link the system inputs and outputs and their derivatives. In the next section, they will be used to build non-linear parity-space residuals. Because of the presence of noise, it is difficult in practice to compute the signal derivatives, and one could argue that observer-based approaches would not present this drawback. However, when discrete-time models are available, the previous developments still hold and lead to similar results, with derivatives replaced by time-shifted variables. Note that a discrete-time model is indeed used in observer-based approaches, since the practical software used for integration actually rests on time discretization.

Evaluating derivatives from noisy signals can be done using either observers or specific algorithms. Non-linear observers can obviously be used to feed parity space residuals with derivative estimates. Specific algorithms are extensively reported in the literature (Fox & Parker, 1968; David, 1975; Dierckx, 1993). In the induction motor example of Section 5, the non-linear parity space approach has been implemented using a polynomial approximation-based scheme, which has proved to work satisfactorily.

Even when signal derivatives are estimated by non-linear observers, parity space residuals still deserve some interest. On the one hand, although the structuring of observer-based residuals is considered in Hammouri, Kinnaert, and El Yaagoubi (1999) for additive faults, using parity residuals provides degrees of freedom (especially for the structuring w.r.t. parameters) which are easy to obtain and understand using elimination theory (see Section 4). On the other hand, ARR are a good tool for the analysis of fault detectability conditions, which are the concern of the next part of this paper. In the linear case, parity space and observer-based approaches have been shown to be equivalent (Magni & Mouyon, 1994) and this result has been extended to non-linear state-affine systems (Kaboré, Staroswiecki, & Wang, 1999) and to a class of non-linear systems (Cocquempot & Christophe, 2000). For general non-linear systems such a result is not available, but it can be conjectured that faults which are found to be non-detectable through the ARR analysis will also be non-detectable using observer-based instead of ARR-based residuals.

3. Fault detection and isolation

3.1. ARR decomposition

Consider a set of ARR $w(\bar{y}, \bar{z}, \theta) = 0$. Remember that $z = (u^t, v^t, \zeta^t, \varepsilon^t)^t$ and $w(\bar{y}, \bar{z}, \theta)$ is a polynomial in the components of $\bar{z}$. The following decomposition holds:

$$w(\bar{y}, \bar{u}, \bar{v}, \bar{\zeta}, \bar{\varepsilon}, \theta) = w_d(\bar{y}, \bar{u}, \bar{v}, \bar{\zeta}, \theta) - w_s(\bar{y}, \bar{u}, \bar{v}, \bar{\zeta}, \bar{\varepsilon}, \theta), \tag{7}$$

where $w_d(\bar{y}, \bar{u}, \bar{v}, \bar{\zeta}, \theta) = w_0(\bar{y}, \bar{u}, \theta) - w_1(\bar{y}, \bar{u}, \bar{\zeta}, \theta) - w_2(\bar{y}, \bar{u}, \bar{v}, \bar{\zeta}, \theta)$ is the deterministic part of $w(\bar{y}, \bar{u}, \bar{v}, \bar{\zeta}, \bar{\varepsilon}, \theta)$ (a polynomial of degree zero in the components of $\bar{\varepsilon}$), w_s is the stochastic part (a polynomial of degree at least one in some component of $\bar{\varepsilon}$). $w_0(\bar{y}, \bar{u}, \theta) - w_1(\bar{y}, \bar{u}, \bar{\zeta}, \theta)$ is the perturbation-free part of $w(\bar{y}, \bar{u}, \bar{v}, \bar{\zeta}, \bar{\varepsilon}, \theta)$ and $w_0(\bar{y}, \bar{u}, \theta)$ is the perturbation and fault-free part. Note that

$$\begin{aligned} &w_1(\bar{y}, \bar{u}, 0, \theta) = 0, \quad w_2(\bar{y}, \bar{u}, 0, \bar{\zeta}, \theta) = 0, \\ &w_s(\bar{y}, \bar{u}, \bar{v}, \bar{\zeta}, 0, \theta) = 0. \end{aligned} \tag{8}$$

ARR insensitivity w.r.t. unknown inputs results from $w_2(\bar{y}, \bar{u}, \bar{v}, \bar{\zeta}, \theta)$ being the null polynomial and $(\partial w_s/\partial \bar{v})(\bar{y}, \bar{u}, \bar{v}, \bar{\zeta}, \bar{\varepsilon}, \theta) = 0$. ARR might enjoy this property either because there is no unknown input or because they have been decoupled w.r.t. the unknown inputs (this is a highly desirable property, and Section 4 addresses this issue). From now on, only ARR insensitive w.r.t. unknown inputs are considered. They are of the form

$$w(\bar{y}, \bar{u}, \bar{\zeta}, \bar{\varepsilon}, \theta) = w_0(\bar{y}, \bar{u}, \theta) - w_1(\bar{y}, \bar{u}, \bar{\zeta}, \theta) - w_s(\bar{y}, \bar{u}, \bar{\zeta}, \bar{\varepsilon}, \theta) = 0$$

and the following is always true:

$$w_0(\bar{y}, \bar{u}, \theta) = w_1(\bar{y}, \bar{u}, \bar{\zeta}, \theta) + w_s(\bar{y}, \bar{u}, \bar{\zeta}, \bar{\varepsilon}, \theta). \tag{9}$$

3.1.1. Residual computation form

Let $\rho = w_0(\bar{y}, \bar{u}, \theta)$. ρ is called the *residual*, and $w_0(\bar{y}, \bar{u}, \theta)$ is the *residual computation form*. It involves only known variables and thus it can be computed on-line. From (8), in the absence of noise and faults, ρ is identically zero for any triple $(\bar{y}, \bar{u}, \theta)$ which satisfies the state and measurement equations (1), (2).

3.1.2. Residual evaluation form

In real life, the behavior of ρ is unknown (because of ζ) and stochastic (because of ε). The right-hand side of (9) explains why ρ might be different from zero. It is called the *residual evaluation form*, and $w_1(\bar{y},\bar{u},\bar{\zeta},\theta)$ is its deterministic part.

3.2. D-detectability

Fault detection procedures have to decide between the two hypotheses H_0: $\zeta = 0$ (healthy system) and H_1: $\zeta \neq 0$ (faulty system). For linear systems and additive faults, the residual vector is $\rho = w_1(\bar{\zeta},\theta) + w_s(\bar{\varepsilon},\theta)$ and $w_s(\bar{\varepsilon},\theta)$ is linear w.r.t. $\bar{\varepsilon}$, so that, assuming $E(\bar{\varepsilon}) = 0$, hypotheses H_0 and H_1 are characterized by

H_0: $\rho = w_s(\bar{\varepsilon},\theta)$ and $E(\rho) = 0$,
H_1: $\rho = w_1(\bar{\zeta},\theta) + w_s(\bar{\varepsilon},\theta)$ and $E(\rho) = w_1(\bar{\zeta},\theta)$.

Non-detectable faults are such that the residual vector exhibits the same stochastic behavior under both hypotheses, thus: $w_1(\bar{\zeta},\theta) = 0$ in spite of $\zeta \neq 0$. In the non-linear situation $\rho = w_s(\bar{y},\bar{u},0,\bar{\varepsilon},\theta)$ under H_0, since $w_1(\bar{y},\bar{u},0,\theta) = 0$ from (8). However, no simple characterization of the residual's stochastic behavior can be provided since $w_s(\bar{y},\bar{u},0,\bar{\varepsilon},\theta)$ is non-linear w.r.t. $\bar{\varepsilon}$. Note that in general $E(\rho) = 0$ is not true. The faulty situation H_1 is characterized by $\rho = w_1(\bar{y},\bar{u},\bar{\zeta},\theta) + w_s(\bar{y},\bar{u},\bar{\zeta},\bar{\varepsilon},\theta)$ which shows that the fault ζ influences both the deterministic and the stochastic parts of the residual vector. Defining non-detectable faults as above seems thus to be most impractical, and a stronger definition, namely D-detectability, is proposed by analogy with the linear case.

Definition. A D-detectable fault is such that the deterministic part of its residual evaluation form differs from zero.

D-detectability is indeed a weaker concept, since non-D-detectable faults might, nevertheless, influence the stochastic part of the residual in such a way that they could be detected.

3.3. The residual evaluation form

In this section, different fault hypotheses are considered. Faults ζ are modeled by additive signals on the inputs (f_u), the outputs (f_y) or the system parameters (f_θ). Since D-detectability is investigated, only deterministic systems are considered.

3.3.1. Sensor faults

Let y be the output associated with healthy sensors and let y_a be the actual output. $f_y = y_a - y$ is the fault model. Based on the actual outputs, the residual computation form produces $\rho = w_0(\bar{y}_a,\bar{u},\theta)$. Since Taylor expansions of polynomials are exact, valid and finite (Lelong-Ferrand & Armandiès, 1978), and since $w_0(\bar{y},\bar{u},\theta) = 0$, the residual evaluation form is

$$\rho = \sum_{k=1}^{\gamma} \left(\frac{1}{k!}\right)\left[\left(\frac{\partial w(\bar{y},\bar{u},\theta)}{\partial \bar{y}}\right)\bar{f}_y\right]^{(k)} = w_1(\bar{y},\bar{u},\bar{f}_y,\theta). \tag{10}$$

3.3.2. Actuator faults

A simple model of a faulty actuator is that of the non-faulty one driven by some unknown signal. Let $u_a = u + f_u$, where u is the (known) theoretical actuator input and f_u is some unknown input deviation. More realistic approaches would introduce parameterized actuator models in the state equations and consider system component faults described by parameter deviations (see next section).

Actuator faults result in both state and output deviations. Let y_a be the actual output. Using the notation $\bar{y}_a = \bar{y} + \delta\bar{y}$, the evaluation form is

$$\rho = w_0(\bar{y}_a,\bar{u},\theta) = w_0(\bar{y} + \delta\bar{y},\bar{u},\theta) = w_1(\bar{y},\bar{u},\delta\bar{y},\theta). \tag{11}$$

Note that in (10) f_y represents the sensor faults, which are independent exogenous variables, while in (11) the independent exogenous variables are f_u, and $\delta\bar{y}$ represents the effect of actuator faults on the system outputs.

Because of the chosen fault model, the following redundancy relations hold:

$$w_0(\bar{y}_a,\bar{u}_a,\theta) = 0 \tag{12}$$

and using Taylor expansion, one obtains

$$w_0(\bar{y}_a,\bar{u}_a,\theta) - w_0(\bar{y},\bar{u},\theta) = P(\delta\bar{y},\bar{f}_u) = 0, \tag{13}$$

where P is a polynomial vector in the components of $\delta\bar{y}$ and $\bar{f}_u$, whose coefficients are polynomials in the variables $(\bar{y},\bar{u},\theta)$, with the fundamental property $\bar{f}_u = 0 \Rightarrow \delta\bar{y} = 0$.

3.3.3. System component faults

System component faults are modeled as parameter deviations $\theta_a = \theta + f_\theta$ where θ_a is the actual parameter vector, and θ is its nominal value. This model only considers component faults which leave the system structure unchanged. Parametric faults produce deviations in the state: $x_a = x + \delta x$ and in the outputs: $y_a = y + \delta y$, and the residual computation and evaluation forms are $\rho = w_0(\bar{y}_a,\bar{u},\theta) = w_1(\bar{y},\bar{u},\delta\bar{y},\theta)$. Again, δy is the output deviation which results from the parameter deviations, and since

$$w_0(\bar{y}_a,\bar{u},\theta_a) = 0, \tag{14}$$

then

$$w_0(\bar{y}_a,\bar{u},\theta_a) - w_0(\bar{y},\bar{u},\theta) = Q(\delta\bar{y},f_\theta) = 0, \tag{15}$$

where Q is a polynomial in the components of $\delta\bar{y}$ and f_θ whose coefficients are polynomials in $(\bar{y},\bar{u},\theta)$ with the fundamental property $f_\theta = 0 \Rightarrow \delta\bar{y} = 0$.

3.4. Fault sensitivities

For each fault, the deterministic part of the residual evaluation form is a polynomial in the deviation vector. A unified representation is

$$\begin{aligned}&\rho = w_0(\bar{y}_a, \bar{u}, \theta) \quad \text{computation form,}\\ &\rho = w_1(\bar{y}, \bar{u}, \theta, \delta\bar{y}) \quad \text{D-evaluation form}\end{aligned} \tag{16}$$

with

$$\delta y - f_y = 0 \quad \text{for sensor faults,} \tag{17a}$$

$$P(\delta\bar{y}, \bar{f}_u) = 0 \quad \text{for actuator faults,} \tag{17b}$$

$$Q(\delta\bar{y}, f_\theta) = 0 \quad \text{for component faults} \tag{17c}$$

and properties:

$$w_1(\bar{y}, \bar{u}, \theta, 0) = 0, \tag{18a}$$

$$f_y \equiv 0 \;\Rightarrow\; \delta\bar{y} = 0, \tag{18b}$$

$$f_u \equiv 0 \;\Rightarrow\; \delta\bar{y} = 0, \tag{18c}$$

$$f_\theta \equiv 0 \;\Rightarrow\; \delta\bar{y} = 0. \tag{18d}$$

The presence of a fault is expected to deviate the D-evaluation form from zero. The analysis of D-detectability is concerned with two cases in which no such deviation exists, namely output insensitivity and residual insensitivity. Output insensitive faults are characterized by $\delta\bar{y} = 0$, so that, by (18a) the D-evaluation form remains equal to zero in spite of the fault. On the contrary, residual insensitivity is characterized by $w_1(\bar{y}, \bar{u}, \theta, \delta\bar{y}) = 0$ in spite of $\delta\bar{y}$ being non-zero.

3.4.1. Output sensitivity

Consider (17), and define the following three sets:

$$Z_y(\bar{y}, \bar{u}, \theta) = \{f_y \neq 0 \text{ s.t. } \delta\bar{y} = 0\},$$

$$Z_u(\bar{y}, \bar{u}, \theta) = \{f_u \neq 0 \text{ s.t. } P(\delta\bar{y}, \bar{f}_u) = 0 \Rightarrow \delta\bar{y} = 0\},$$

$$Z_\theta(\bar{y}, \bar{u}, \theta) = \{f_\theta \neq 0 \text{ s.t. } Q(\delta\bar{y}, f_\theta) = 0 \Rightarrow \delta\bar{y} = 0\},$$

$Z_y(\bar{y}, \bar{u}, \theta)$ is obviously empty (sensor faults are always output sensitive), while $Z_u(\bar{y}, \bar{u}, \theta)$ (resp. $Z_y(\bar{y}, \bar{u}, \theta)$) are the sets of output insensitive actuator faults (resp. the set of output insensitive component faults). A necessary condition for a fault f_u (resp. for a fault f_θ) to belong to $Z_u(\bar{y}, \bar{u}, \theta)$ (resp. to $Z_\theta(\bar{y}, \bar{u}, \theta)$) is that it satisfies the differential equations $P(0, \bar{f}_u) = 0$ (resp. $Q(0, f_\theta) = 0$).

More generally, divide the outputs into two subsets $y = ((y')^t\ (y'')^t)^t$ and define output sensitivity with respect to y':

$$Z_y(\bar{y}, \bar{u}, \theta)(y') = \{f_y \neq 0 \text{ s.t. } \delta\bar{y}' = 0\},$$

$$Z_u(\bar{y}, \bar{u}, \theta)(y') = \{f_u \neq 0 \text{ s.t. } P(\delta\bar{y}, \bar{f}_u) = 0 \Rightarrow \delta\bar{y}' = 0\},$$

$$Z_\theta(\bar{y}, \bar{u}, \theta)(y') = \{f_\theta \neq 0 \text{ s.t. } Q(\delta\bar{y}, f_\theta) = 0 \Rightarrow \delta\bar{y}' = 0\}.$$

Then $Z_y(\bar{y}, \bar{u}, \theta)(y') = \{f_y \neq 0 \text{ s.t. } f_{y'} = 0\}$ and necessary conditions for f_u (resp. f_θ) to belong to $Z_u(\bar{y}, \bar{u}, \theta)(y')$ (resp. $Z_\theta(\bar{y}, \bar{u}, \theta)(y')$) are

$$P(0, \delta\bar{y}'', \bar{f}_u) = 0 \text{ (resp. } Q(0, \delta\bar{y}'', \bar{f}_\theta) = 0) \quad \forall \delta\bar{y}''.$$

3.4.2. Residual sensitivity

Residual sensitivity is concerned with the transfer between $\delta\bar{y}$ and the D-evaluation form. Divide the residual vector ρ into two sub-vectors ρ' and ρ''. The set of output deviations to which ρ' is insensitive is defined by

$$\Omega(\bar{y}, \bar{u}, \theta)(\rho') = \{\delta\bar{y} \neq 0 \text{ s.t. } w_1'(\bar{y}, \bar{u}, \theta, \delta\bar{y}) = 0\},$$

where $w_1'(\bar{y}, \bar{u}, \theta, \delta\bar{y})$ is the D-evaluation form of the sub-residual ρ'.

3.4.3. Structured residuals

Consider the residual ρ_i and let $S_y(i)$ (resp. $S_u(i), S_\theta(i)$) be the subset of the sensors (resp. the actuators, the system components) that appear in its computation form. Let y_i (resp. u_i, θ_i) be the corresponding input (resp. output, parameter) sub-vectors. $S(i) = S_y(i) \cup S_u(i) \cup S_\theta(i)$ is residual ρ_i's structure (Gertler & Luo, 1989). One has

$$\frac{\partial \rho_i}{\partial \bar{y}_k}(\bar{y}, \bar{u}, \theta) \equiv 0 \quad \forall y_k \notin S_y(i),$$

$$\frac{\partial \rho_i}{\partial \bar{u}_k}(\bar{y}, \bar{u}, \theta) \equiv 0 \quad \forall u_k \notin S_u(i),$$

$$\frac{\partial \rho_i}{\partial \theta_k}(\bar{y}, \bar{u}, \theta) \equiv 0 \quad \forall \theta_k \notin S_\theta(i).$$

It is postulated that structured residuals are insensitive to faults which do not belong to their structure. This is quite obvious for sensor faults in the linear case, and has been widely used in the DOS and GOS observer schemes (Frank, 1993). For general non-linear systems, this property has been used in the structural analysis approach (Cocquempot, Cassar, & Staroswiecki, 1991) but not demonstrated. Though the proof is very simple, the property cannot be directly derived from (16) and (17) except for sensor faults since for any subset of faulty sensors, none of which belongs to $S_y(i)$, (17a) gives $\delta y_i = f_{y_i} \equiv 0$.

Consider now a subset of faulty actuators none of which belongs to $S_u(i)$. In this case, $f_{u_i} \equiv 0$ but (17b) does not imply $\delta y_i \equiv 0$. In fact,

$$P_i(\delta\bar{y}_i, 0) = 0,$$

$$P_j(\delta\bar{y}_j, \bar{f}_{u_j}) = 0, \quad j \neq i$$

shows that such faulty actuators might, nevertheless, lead to $\delta\bar{y}_i \neq 0$ (indeed, one might have $\exists j \neq i$ s.t. $S_y(j) \cap S_y(i) \neq \emptyset$ and $\delta\bar{y}_j \neq 0$). However, Eq. (12), which is of the form $w_{0i}(y_i^- + \delta\bar{y}_i, \bar{u}_i, \theta_i) = 0$, shows that the D-evaluation form of ρ_i would still be zero. When component faults are considered, similar conclusions are derived from (14).

3.5. D-detectability condition

For simplicity, only homogeneous subsets of faults (sensor, actuator or component) are considered. Let v stand for y, u or θ and consider a subset of faults such that $f_v = (f^t_{v'}\ f^t_{v''})^t, f_{v'} \neq 0, f_{v''} \equiv 0$. A necessary and sufficient condition for the subset of faults $f_{v'}$ to be D-detectable is

(1) $\exists y'$ s.t. $(f^t_{v'}\ 0^t)^t \notin Z_v(\bar{y}, \bar{u}, \theta)(y')$,
(2) $\delta\bar{y}' \notin \Omega(\bar{y}, \bar{u}, \theta)(\rho)$,

where y' is a subset of sensors and $\delta y'$ is the output deviation which results from the fault vector $f_{v'}$. The first condition expresses that there exists at least one output which is sensitive to the faults $f_{v'}$, while the second condition expresses that at least one residual is sensitive to the resulting output deviation.

D-detectability conditions define the set of faults which cannot be detected since they have no effect on the outputs or on the residuals. The residual evaluation form is indeed a non-linear function of the system trajectories and faults, therefore ARR-based residuals might fail to detect some faults. D-detectability conditions can be used in different ways, e.g. to characterize the set of faults which cannot be detected, for given nominal trajectories; to characterize the set of faults which can (or cannot) be detected for *any* system trajectory; to check the FDI sensitivity along the nominal system trajectories (or on-line along the current system trajectories), for given faults to be detected.

3.6. Fault isolability

In the structured residual approach, a given fault signature is defined by the subset of those residuals whose structure the faulty element belongs to (Gertler & Luo, 1989). Consider the set of all the possible fault signatures, and define a dissimilarity index between signatures (for example the Hamming distance). The set of fault signatures can be decomposed into equivalence classes, each of them gathering signatures with dissimilarity index zero. These classes are the subsets of faults which are not isolable from each other using the given residual. In other words, a necessary and sufficient condition for two faults to be isolable by a given residual is that they do not belong to the same equivalence class.

4. The design of robust structured residuals

The fundamental problem of residual generation (FPRG) consists of the design of residuals which do not depend on any unknown input and whose structure is defined a priori. Robust structured residuals avoid false alarms due to unknown inputs and uncertain parameters, and solve the fault isolation problem in a simple way.

4.1. Robustness with respect to unknown inputs

The deterministic state and measurement equations (1), (2) are

$$\dot{x} = f(x, u, v, \zeta, \theta), \tag{19}$$

$$y = h(x, u, v, \zeta, \theta). \tag{20}$$

The goal is to design ARR that do not depend on v, i.e. for some fixed multi-index $(s_1, s_2, \ldots, s_p) \in N^p$, both x and $\bar{v}^{(s)}$, where $s = \max_i s_i$, are to be eliminated from system (6).

Theorem 3. *Let* $r^\star_{(s_1, s_2, \ldots, s_p)} = rank[J^t_{x,\bar{v}}(G^1_{s_1}) \ \ldots \ J^t_{x,\bar{v}}(G^p_{s_p})]^t$, *then there are exactly* $\eta = \sum_{i=1}^p s_i + p - r^\star_{(s_1, s_2, \ldots, s_p)}$ *independent ARR involving* $\bar{y}_1^{(s_1)}, \ldots, \bar{y}_p^{(s_p)}, \bar{u}, \bar{\zeta}$ *and* θ *in the form*

$$w_k(\bar{y}_1^{(s_1)}, \ldots, \bar{y}_p^{(s_p)}, \bar{u}, \bar{\zeta}, \theta) = 0, \quad k = 1, \ldots, \eta \tag{21}$$

with the following rank property:

$$rank\begin{bmatrix} \left(\dfrac{\partial w_1}{\partial \bar{y}}\right)(\bar{y}_1^{(s_1)}, \ldots, \bar{y}_p^{(s_p)}, \bar{u}, \bar{\zeta}, \theta) \\ \vdots \\ \left(\dfrac{\partial w_\eta}{\partial \bar{y}}\right)(\bar{y}_1^{(s_1)}, \ldots, \bar{y}_p^{(s_p)}, \bar{u}, \bar{\zeta}, \theta) \end{bmatrix} = \eta$$

for almost any vector $(\bar{y}_1^{(s_1)}, \ldots, \bar{y}_p^{(s_p)}, \bar{u}, \bar{\zeta}, \theta)$ *which satisfies* (21).

Remark 4. The existence of robust ARR is not ensured, even for high orders of derivation, since there is no equivalent of Corollary 2 to give an upper bound of $r^\star_{(s_1, s_2, \ldots, s_p)}$

4.2. Structured residuals

Let us consider a subset of sensors $J' \subset J$ and, for simplicity, renumber them so that $J' = \{1, \ldots, p'\}$. The output vector is split into y' and y''. Consider also a subset of actuators (resp. of parameters) such that the input vector u (resp. the parameter vector θ) is split into two sub-vectors u' and u'' (resp. θ' and θ''). A first (weak) setting of the FPRG is to design residuals whose structure does not contain y'', u'' and θ''. A second (strong) setting is to ensure that the residual structure contains all components of y', u' and θ'.

4.2.1. Weak residual generation problem

In order to address the WRGP, the extended state $(x^t, \theta^t)^t$ is considered:

$$\dot{x} = f(x, u', u'', v, \zeta, \theta', \theta''),$$

$$\dot{\theta} = 0,$$

$$y' = h'(x, u', u'', v, \zeta, \theta', \theta'').$$

Gathering x and θ'' in a single vector ψ (v and u'' in a single vector ϑ), one has

$$\dot{\psi} = \varphi(\psi, u', \vartheta, \zeta, \theta'),$$

$$\dot{\theta}' = 0,$$

$$y' = h'(\psi, u', \vartheta, \zeta, \theta') \tag{22}$$

and the WRGP simply consists in eliminating the state ψ and the unknown inputs ϑ in (22).

Theorem 4. *A residual with weak parameter structure* (y', u', θ') *exists at the orders of derivation* $(s_j, j \in J')$ *of the outputs* y' *if and only if*

$$\eta(J', u', \theta') = \sum_{j \in J'} s_j + p' - r^{\#}_{(s_j,\ j \in J')} > 0,$$

where

$$r^{\#}_{(s_j,\ j \in J')} = rank \begin{bmatrix} J_x(G^1_{s_1}) & J_{\theta''}(G^1_{s_1}) & J_{\bar{v},\ \bar{u}''}(G^1_{s_1}) \\ \vdots & \vdots & \vdots \\ J_x(G^{p'}_{s_{p'}}) & J_{\theta''}(G^{p'}_{s_{p'}}) & J_{\bar{v},\ \bar{u}''}(G^{p'}_{s_{p'}}) \end{bmatrix}.$$

Note that, from Theorem 2 and Corollary 2, the state x and the parameters θ'' can be eliminated, since the rank of the Jacobian sub-matrix $J_{x,\ \theta''}$ is bounded, while nothing can be said about the actual number of ARR which are robust w.r.t. v and u'' (see Remark 4).

4.2.2. Strong residual generation problem

The SRGP is concerned with the actual presence of each component of (y', u', θ') in the residuals. Since most of the development is similar, only the design of strongly structured residuals w.r.t. (y', θ') will be considered, starting with the part of the system robust w.r.t. the unknown inputs.

For each component θ_k of θ', define the following ranks:

$$r^{\theta''}_{(s_j,\ j \in J')} = rank \begin{bmatrix} J_x(G^1_{s_1}) & J_{\theta''}(G^1_{s_1}) \\ \vdots & \vdots \\ J_x(G^{p'}_{s_{p'}}) & J_{\theta''}(G^{p'}_{s_{p'}}) \end{bmatrix},$$

$$r^{\theta''}_{(s_j,\ j \in J')}(\theta_k) = rank \begin{bmatrix} J_x(G^1_{s_1}) & J_{\theta''}(G^1_{s_1}) & J_{\theta_k}(G^1_{s_1}) \\ \vdots & \vdots & \vdots \\ J_x(G^{p'}_{s_{p'}}) & J_{\theta''}(G^{p'}_{s_{p'}}) & J_{\theta_k}(G^{p'}_{s_{p'}}) \end{bmatrix}.$$

Theorem 5. *A residual with strong parameter structure* (y', θ') *exists at the orders of derivation* $(s_j, j \in J')$ *of the outputs* y' *if and only if*

(a) $\sum_{j \in J'} s_j + 1 > r^{\theta''}_{(s_j,\ j \in J')}$,

(b) $r^{\theta''}_{(s_j,\ j \in J')}(\theta_k) = r^{\theta''}_{(s_j,\ j \in J')} + 1 \quad \forall \theta_k \in \theta'$,

$$\text{(c)} \quad \begin{cases} \textit{There exists no subset } \tilde{y}' \subset y' \\ \textit{and multi-index } (\sigma_j,\ j \in \tilde{J}') \\ \textit{such that properties (a) and (b)} \\ \textit{hold for } \sigma_j \le s_j \quad \forall j \in J' \cap \tilde{J}' \end{cases} \tag{23}$$

Indeed, let l'' be the dimension of θ''. There exist two integers $r'' \le n + l''$ and $r''_k \le n + l'' - 1$ such that

$$r^{\theta''}_{(s_i,\ i \in I')} = \begin{cases} \sum_{i \in I'} s_i + 1 & \textit{if } \sum_{i \in I'} s_i < r'', \\ r'' & \textit{if } \sum_{i \in I'} s_i \ge r'', \end{cases} \tag{24}$$

$$r^{\theta''}_{(s_i,\ i \in I')}(\theta_k) = \begin{cases} \sum_{i \in I'} s_i + 1 & \textit{if } \sum_{i \in I'} s_i < r''_k, \\ r'_k & \textit{if } \sum_{i \in I'} s_i \ge r''_k. \end{cases} \tag{25}$$

From (24), it appears that (23a) can hold only if $\sum_{j \in J'} s_j \ge r''$, which means that there exist exactly $\eta''_{J'} = \sum_{j \in J'} s_j + 1 - r''$ independent ARR possibly involving $(\bar{y}^{(s_j)}_j, j \in J')$, $\bar{u}, \bar{\zeta}, \theta'$ and no others. Property (23b) means that for each parameter θ_k of θ', there exist $\eta''_{J'} - 1$ independent ARR possibly involving $(\bar{y}^{(s_j)}_j, j \in J')$, $\bar{u}, \bar{\zeta}, \theta' \setminus \{\theta_k\}$ and no others.

For $\sum_{j \in J'} s_j = r''$ (23a) and (23b) give a necessary and sufficient condition for the existence of one residual designed from the subset $(y_j, j \in J')$ at orders of derivation $(s_j, j \in J')$ whose structure contains all the parameters of θ' and none of the parameters of θ''. Condition (23c) means that this result cannot be obtained from any subset of sensors $\tilde{y}' \subset y'$, except possibly using higher orders of derivation. Thus, all the sensors of y' really belong to the structure of the ARR.

Remark 5. The existence of a residual with parameter structure θ' is a generalization of the identifiability problem, in which θ' contains only one component (for a reference study on identifiability, see Ljung and Glad (1994))

Remark 6. From (24) and (25), if there exists a parameter θ_k such that $r''_k = r''$, then θ_k cannot belong to any structure which would contain no parameter of θ'', at any order of derivation of the considered outputs. In the special case $\theta' = \{\theta_k\}$ (i.e. $\theta'' = \theta \setminus \{\theta k\}$), this corresponds to θ_k being non-identifiable through the observation of the outputs y', at any order of derivation of these outputs.

Remark 7. Suppose there exists a subset of parameters θ'' such that $r'' = r$. Then no parameter of θ'' could belong to a structure which would not contain any component of x. The conclusion is that it is not possible to design any parity-space residual sensitive to a parameter fault in θ'', using the subset of outputs y', at any order of derivation of these outputs.

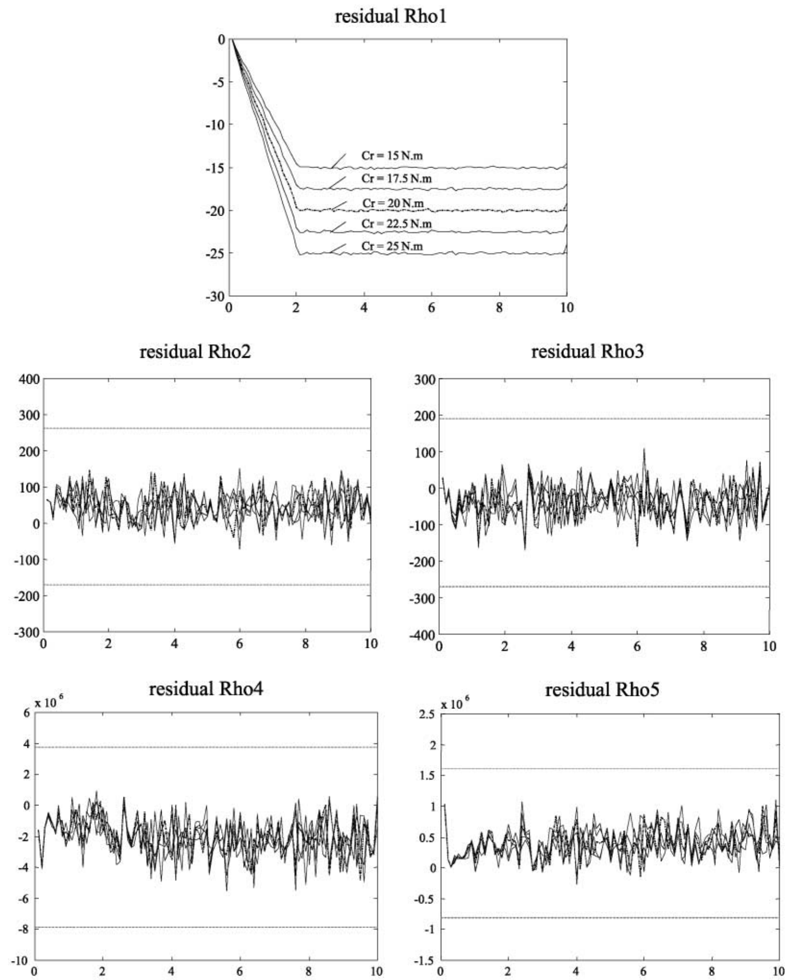

Fig. 1. Residual evolutions: non-faulty case; C_{r} from 15 to 25 N m.

5. Example

5.1. System model

Consider the (d, q) model of the induction motor (for an extensive development of this example, see Christophe, Cocquempot, and Staroswiecki (1999) and Comtet-Varga, Christophe, Cocquempot, and Staroswiecki (1999)). The state variables are the rotor angular speed Ω and the (d, q) projections of the stator current and rotor flux: i_{sd}, i_{sq}, ϕ_{rd} and ϕ_{rq}. The control inputs are the (d, q) projections of the stator voltage: v_{sd} and v_{sq}. The load torque C_{r} is unknown. The outputs are the only state variables that can be measured, i.e. Ω, i_{sd} and i_{sq}. Under usual hypotheses the model equations are

$$\begin{aligned}
\dot{x}_1 &= \frac{pM}{JL_{\mathrm{r}}}(x_5 x_2 - x_4 x_3) - \frac{K}{J}x_1 - \frac{C_{\mathrm{r}}}{J}, \\
\dot{x}_2 &= \frac{MR_{\mathrm{r}}}{L_{\mathrm{r}}}x_4 - \frac{R_{\mathrm{r}}}{L_{\mathrm{r}}}x_2 - px_3 x_1, \\
\dot{x}_3 &= \frac{MR_{\mathrm{r}}}{L_{\mathrm{r}}}x_5 - \frac{R_{\mathrm{r}}}{L_{\mathrm{r}}}x_3 + px_2 x_1, \\
\dot{x}_4 &= \gamma x_4 + \frac{M}{L_{\mathrm{r}}L_{\mathrm{s}}\sigma}\left(\frac{R_{\mathrm{r}}}{L_{\mathrm{r}}}x_2 + px_1 x_3\right) + \frac{1}{L_{\mathrm{s}}\sigma}v_{\mathrm{s}d}, \\
\dot{x}_5 &= \gamma x_5 + \frac{M}{L_{\mathrm{r}}L_{\mathrm{s}}\sigma}\left(\frac{R_{\mathrm{r}}}{L_{\mathrm{r}}}x_3 - px_2 x_3\right) + \frac{1}{L_{\mathrm{s}}\sigma}v_{\mathrm{s}q}
\end{aligned} \tag{26}$$

with state vector

$$x = (x_1, x_2, x_3, x_4, x_5)^{\mathrm{t}} = (\Omega, \phi_{\mathrm{r}d}, \phi_{\mathrm{r}q}, i_{\mathrm{s}d}, i_{\mathrm{s}q})^{\mathrm{t}},$$

and measurement vector

$$y = (y_1, y_2, y_3)^{\mathrm{t}} = (x_1, x_4, x_5)^{\mathrm{t}}.$$

$R_{\mathrm{s}}, L_{\mathrm{s}}$ are the stator resistance and inductance, $R_{\mathrm{r}}, L_{\mathrm{r}}$ are the rotor resistance and inductance, M is the mutual inductance between stator and rotor, p is the number of pole pairs, K is the damping coefficient, J is the moment of inertia, $\sigma = 1 - M^2/L_{\mathrm{r}}L_{\mathrm{s}}$ and $\gamma = -(1/\sigma)(R_{\mathrm{s}}/L_{\mathrm{s}} + (1-\sigma)R_{\mathrm{r}}/L_{\mathrm{r}})$.

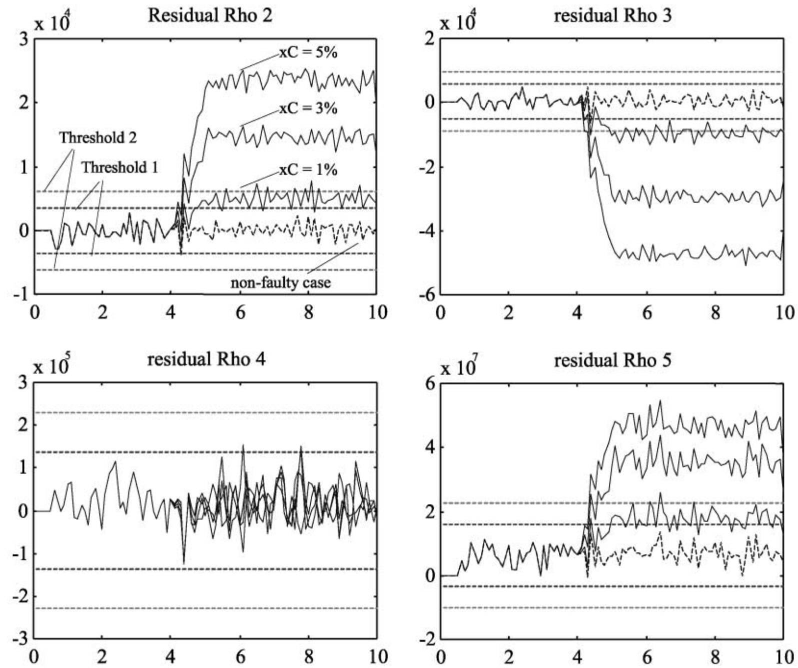

Fig. 2. Residual mean values: non-faulty case and speed sensor fault (1, 3 and 5%).

5.2. Residual design

5.2.1. State elimination

Deriving speed and currents at order 1, the Jacobian matrix

$$\begin{bmatrix} J_x(G_1^1) \\ J_x(G_1^2) \\ J_x(G_1^3) \end{bmatrix}$$

has rank 5. Thus, according to Theorem 2, there exists exactly one residual involving $(\bar{y}_1^{(1)}, \bar{y}_2^{(1)}, \bar{y}_3^{(1)})$:

$$\rho_1 = p^2 L_s \sigma A_1 (py_1 y_2 - \beta y_3) + p^2 L_s \sigma A_2 (py_1 y_3 + \beta y_2) + [\beta^2 + (py_1)^2][Jp\dot{y}_1 + Kpy_1 + pC_r]$$

with $A_1 = \dot{y}_2 - \gamma y_2 - (1/L_s\sigma)v_{sd}$, $A_2 = \dot{y}_3 - \gamma y_3 - (1/L_s\sigma)v_{sq}$ and $\beta = R_r/L_r$. Note that ρ_1 (unit: A^2 rad s^{-2}) is not robust w.r.t. the load torque.

5.2.2. Robustness w.r.t. the unknown input

Using higher-order derivatives (speed at order 1 and currents up to order 2), the Jacobian matrix

$$\begin{bmatrix} J_x(G_1^1) & J_{C_r}(G_1^1) \\ J_x(G_2^2) & J_{C_r}(G_2^2) \\ J_x(G_2^3) & J_{C_r}(G_2^2) \end{bmatrix}$$

has rank 6: there exist 2 ARR robust w.r.t. the load torque C_r:

$$\begin{aligned} \rho_2 = {} & \dot{A}_1[\beta^2 + (py_1)^2] \\ & - \frac{1-\sigma}{\sigma}[\beta^2 y_2 + \beta y_1 y_3][\beta^2 + (py_1)^2] \\ & - [\beta A_2 + A_1 py_1][-2\beta py_1 + p\dot{y}_1] \\ & - [\beta A_1 - A_2 py_1][(py_1)^2 - \beta^2], \end{aligned}$$

$$\begin{aligned} \rho_3 = {} & \dot{A}_2[\beta^2 + (py_1)^2] \\ & - \frac{1-\sigma}{\sigma}[\beta^2 y_3 + \beta y_1 y_2][\beta^2 + (py_1)^2] \\ & - [\beta A_1 + A_2 py_1][-2\beta py_1 + p\dot{y}_1] \\ & - [\beta A_2 - A_1 py_1][(py_1)^2 - \beta^2]. \end{aligned}$$

The units of ρ_2 and ρ_3 are A rad s^{-3}.

5.2.3. Structured residuals

Let us design residuals robust w.r.t. the load torque, whose structure contains either parameter R_r or parameter R_s. In Christophe, Cocquempot, and Staroswiecki (1999), it is shown that the residual with structure R_r (unit: A^2 rad^2 s^{-2}), is given by the determinant of the *Sylvester* matrix:

$$\rho_4 = \begin{vmatrix} S_{1,1}^{\alpha} & S_{1,2}^{\alpha} \\ S_{2,1}^{\alpha} & S_{2,2}^{\alpha} \end{vmatrix},$$

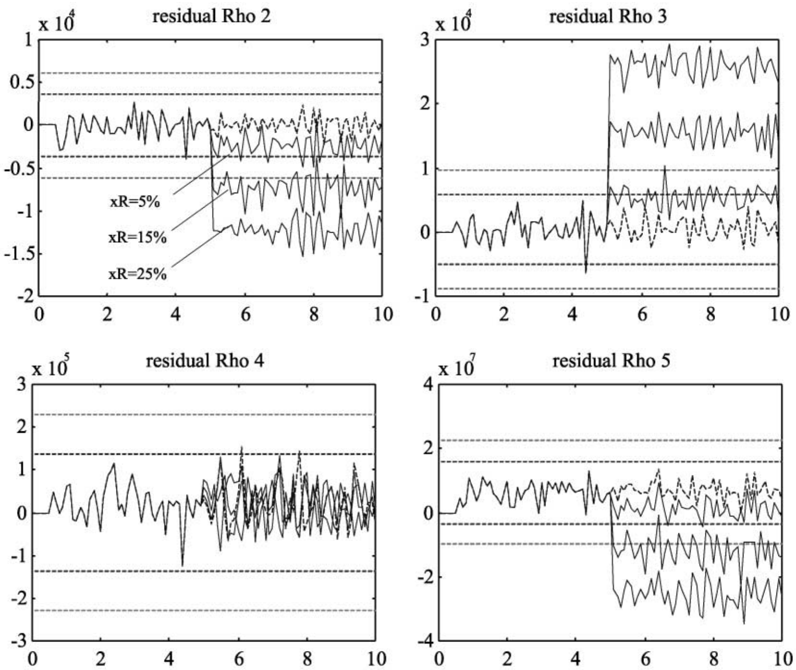

Fig. 3. Residual mean values: non-faulty case and rotor resist. fault (5, 15 and 25%).

where

$$S^{\alpha}_{1,1} = (\beta^2 + (py_1)^2)(-\dot{y}_3 - \beta y_3 + py_1 y_2) + p\dot{y}_1(py_1 y_3 - \beta y_2),$$

$$S^{\alpha}_{1,2} = (\beta^2 + (py_1)^2)(-\beta y_2 - \dot{y}_2 - py_1 y_3) + p\dot{y}_1(py_1 y_2 + \beta y_3),$$

$$S^{\alpha}_{2,1} = (\beta^2 + (py_1)^2)(-\beta\dot{y}_3 - \sigma\ddot{y}_3 + p\sigma y_1 \dot{y}_2) + (\beta^2 + (py_1)^2)\left(\frac{\dot{v}_{sq}}{L_s} - py_1\frac{v_{sd}}{L_s} + \beta\frac{v_{sq}}{L_s}\right) + p\dot{y}_1\left(\beta(1-\sigma)(py_1 y_3 - \beta y_2) + \left(\beta\frac{v_{sd}}{L_s} - py_1\frac{v_{sq}}{L_s}\right)\right),$$

$$S^{\alpha}_{2,2} = (\beta^2 + (py_1)^2)(-\beta\dot{y}_2 - \sigma py_1\dot{y}_3 - \sigma\ddot{y}_2) + (\beta^2 + (py_1)^2)\left(\frac{\dot{v}_{sd}}{L_s} + \beta\frac{v_{sd}}{L_s} + py_1\frac{v_{sq}}{L_s}\right) + p\dot{y}_1(\beta(1-\sigma)py_1 y_2 + \beta y_3) - \left(\beta\frac{v_{sq}}{L_s} + py_1\frac{v_{sd}}{L_s}\right)\Bigg)$$

and the residual with structure R_s (unit: $A^3 s^{-3}$) is given by the determinant

$$\rho_5 = \begin{vmatrix} S^{\beta}_{1,1} & 0 & 0 & S^{\beta}_{1,4} & 0 & 0 \\ S^{\beta}_{2,1} & S^{\beta}_{1,1} & 0 & S^{\beta}_{2,4} & S^{\beta}_{1,4} & 0 \\ S^{\beta}_{3.1} & S^{\beta}_{2,1} & S^{\beta}_{1,1} & S^{\beta}_{3,4} & S^{\beta}_{2.4} & S^{\beta}_{1,4} \\ S^{\beta}_{4,1} & S^{\beta}_{3.1} & S^{\beta}_{2,1} & S^{\beta}_{4,4} & S^{\beta}_{3,4} & S^{\beta}_{2.4} \\ 0 & S^{\beta}_{4,1} & S^{\beta}_{3,1} & 0 & S^{\beta}_{4,4} & S^{\beta}_{3.4} \\ 0 & 0 & S^{\beta}_{4,1} & 0 & 0 & S^{\beta}_{4,4} \end{vmatrix},$$

where

$$S^{\beta}_{1,1} = -\alpha y_3 - \dot{y}_3 + \frac{v_{sq}}{L_s},$$
$$S^{\beta}_{2,1} = py_1 B_1 - \dot{B}_2 - p\dot{y}_1 y_2(1-\sigma),$$
$$S^{\beta}_{3,1} = -p\dot{y}_1 B_1 + (py_1)^2(S^{\beta}_{1,1}) + p^2\dot{y}_1 y_3(1-\sigma),$$
$$S^{\beta}_{4,1} = (py_1)^3 B_1 + p^2\dot{y}_1 B_2 - (py_1)^2\dot{B}_2,$$
$$S^{\beta}_{1,4} = -\alpha y_2 - \dot{y}_2 + v_{sd}/L_s,$$
$$S^{\beta}_{2,4} = -py_1 B_2 - \dot{B}_1 + p\dot{y}_1 y_3(1-\sigma),$$
$$S^{\beta}_{3,4} = p\dot{y}_1 B_2 + (py_1)^2(S^{\beta}_{4,1}) + p^2\dot{y}_1 y_2(1-\sigma),$$
$$S^{\beta}_{4,4} = -(py_1)^3 B_2 + p^2\dot{y}_1 B_1 - (py_1)^2\dot{B}_1,$$

and $B_1 = \alpha y_2 + \sigma\dot{y}_2 - v_{sd}/L_s$ $B_2 = \alpha y_3 + \sigma\dot{y}_3 - v_{sq}/L_s$.

5.3. Simulation results

5.3.1. Simulation parameters

The control strategy is a vector control law which cancels the rotor flux ϕ_{rd} in order to control the angular

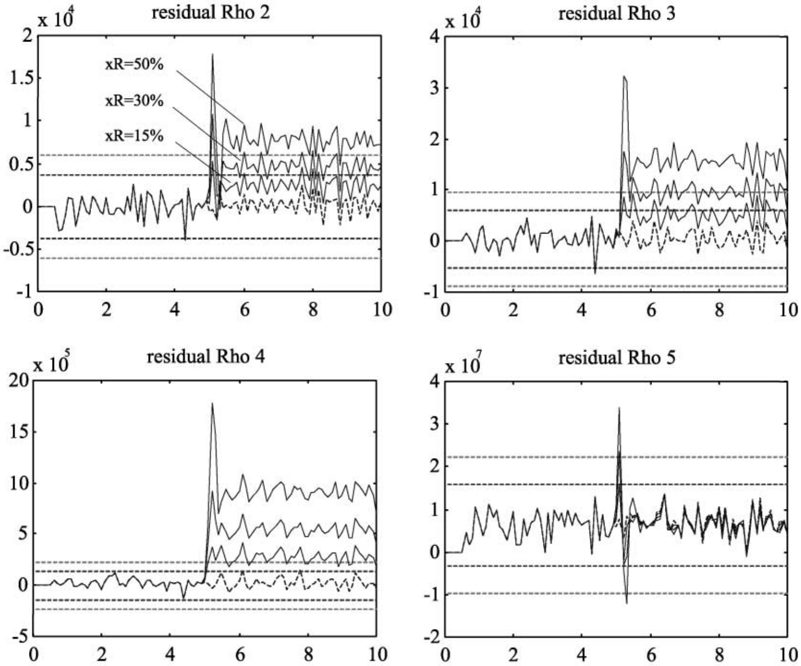

Fig. 4. Residual mean values: non-faulty case and stator resist. fault (15, 30 and 50%).

speed by acting on the stator current. The parameters are

$$R_s = 10\ \Omega,\quad L_s = 0.38\ \text{H},\quad R_r = 3.5\ \Omega,\quad L_r = 0.3\ \text{H},$$

$$M = 0.3\ \text{H},$$

$$J = 0.02\ \text{kg m}^2,$$

$$K = 0.04\ \text{N m s rad}^{-1}\ \text{and}\ p = 2.$$

The sampling interval is 0.1 ms and the simulation is performed for 10 s. Gaussian white noise with zero mean is added to the sensor outputs. The noise standard deviation equals 1 for the current and speed sensors and 10 for the voltage sensors, leading to the signal-to-noise ratios:

$$\left(\frac{s}{n}\right)_{\text{current}} = 18\ \text{dB},\quad \left(\frac{s}{n}\right)_{\text{speed}} = 25.5\ \text{dB},$$

$$\left(\frac{s}{n}\right)_{\text{voltage}} = 38.4\ \text{dB}.$$

The speed measurement is filtered with a lowpass nineth-order Butterworth filter. The cut-off frequency is 200 Hz. The (unknown) load torque is normally less than 20 N m.

5.3.2. Estimation of the derivatives

The signal derivatives are calculated using polynomial approximation, i.e. the variables are approximated in the least-squares sense, on a given time window, by a polynomial function of time. In this example, the time window is 1.51e-2 s long and the approximating polynomial is of degree 5. The signal derivative is estimated by the polynomial derivative in the middle of the time window.

5.3.3. FDI performance

Five different situations have been considered:

1. normal load torque ($C_r \leq 20$ N m)
   15, 17.5 and 20 N m,
2. external fault ($C_r > 20$ N m)
   22.5 and 25 N m,
3. speed sensor fault (at $C_r = 20$ N m)
   1, 3 and 5% error,
4. rotor resistance fault (at $C_r = 20$ N m)
   5, 15 and 25% variation,
5. stator resistance fault (at $C_r = 20$ N m)
   15, 30 and 50% variation.

In all runs, the load torque increases from zero to its steady-state value during the first 2 s. Rotor and stator resistance faults are abrupt changes at $t = 5$ s. The speed sensor fault increases from zero to the fault value from $t = 4$ to 5 s and remains constant after $t = 5$ s.

Fig. 1 displays the time evolution of residuals ρ_1–ρ_5 in the first two cases. Residual ρ_1 can be used to estimate C_r and to detect external faults, while residuals ρ_2–ρ_5, which are robust w.r.t. C_r, are not affected by its variations (all trajectories overlap).

Figs. 2, 3 and 4 show the time evolution of the residual mean values (computed over a successive 1000 sample points) in fault cases 3, 4 and 5, for different fault sizes. Only the robust residuals ρ_2–ρ_5 are used. It can be checked that ρ_4 is insensitive to the rotor resistance deviation and that ρ_5 is insensitive to the stator resistance deviation. It also appears that the fault on the speed sensor does not affect the mean value of residual ρ_4. It can indeed be proved that the speed sensor fault is not D-detectable using this residual.Two decision thresholds (resp. 3σ and 5σ, with σ the standard deviations in the non-faulty case) are displayed. The differences in the mean values between the non-faulty and the faulty cases are significant enough, which gives in practice very good false-alarm and missed-detection rates, in spite of the heuristic decision procedure.

6. Conclusion

Robust structured analytical redundancy relations can be designed for algebraic dynamic systems, thus extending the parity-space approach. Further extensions to rational systems and algebraic discrete-time systems can be done. It should also be noticed that non-linear systems that are not algebraic can be transformed into algebraic systems, especially in cases where the non-linearities appear through sines, cosines, logarithms, or exponentials.

Detectability issues have been discussed, and the definition of D-detectability has been introduced. Conditions for the design of structured residuals and their decoupling w.r.t. unknown inputs have been developed. The approach has been illustrated on the induction motor example.

FDI theory for non-linear systems still requires more research. For practical applications, the calculation of the residuals arises two kinds of problems. On the one hand, the time requirements of formal computational software heavily depend on the a priori chosen order of elimination of the system variables (Faugère et al., 1993). In some cases, the problem is unmanageable with the available software. On the other hand, analytical redundancy residuals involve successive derivations of the system inputs and outputs. For practical applications, the real-time estimation of these derivatives still requires tools to be developed.

The design of structured residuals and the problem of unknown inputs are solved through strong decoupling approaches. However, in some cases, approximate decoupling has to be considered. The theory of approximate decoupling has been well developed for linear systems, because of the simple definition of sensitivity and robustness indices, which are constant. The non-linear situation is much more complex, since sensitivities are non-linear functions of the system trajectories and faults.

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