We tried and we tried, and we applied and implied, and still probabilistic automata we could not decide!

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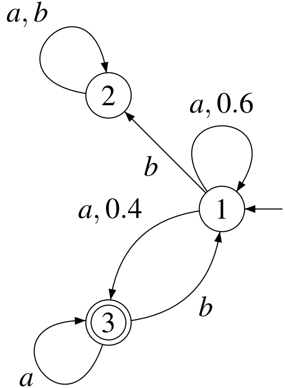
Highlights, September 21st, 2013

Probabilistic automata



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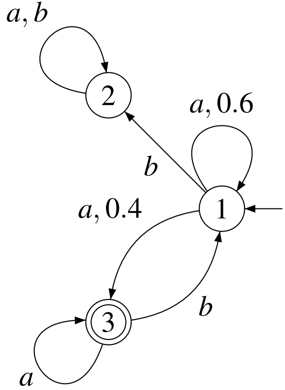

Early results

- (Paz, 71) The emptiness problem is undecidable;
- (Rabin, 69) If λ is isolated, L_λ is regular;
- (Bertoni, 74) The isolation problem is undecidable;
- (Condon-Lipton, 89) The approximation problem is undecidable.

Recent results

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- (Gimbert and Oualhadj, 2009) The value 1 problem is undecidable;
- Decidable classes: $\sharp$-acyclic $\subsetneq$ structurally simple $\subsetneq$ leaktight.

Undecidability results: towards fuzziness

- (Paz, 71) The emptiness problem:

$$\exists w,\ \mathbb{P}_{\mathcal{A}}(w) \geq \frac{1}{2}$$

- (Bertoni, 74) The isolation problem:

$$\forall \varepsilon, \exists w,\ \frac{1}{2} - \varepsilon \leq \mathbb{P}_{\mathcal{A}}(w) \leq \frac{1}{2} + \varepsilon$$

- (Condon-Lipton, 89) The approximation problem:

$$\exists w, \mathbb{P}_{\mathcal{A}}(w) \geq \frac{2}{3} \qquad \vee \qquad \forall w, \mathbb{P}_{\mathcal{A}}(w) \leq \frac{1}{3}$$

- (Gimbert-Oualhadj, 2009) The value 1 problem:

$$\forall \varepsilon, \exists w,\ \mathbb{P}_{\mathcal{A}}(w) \geq 1 - \varepsilon$$

Research questions

- What does the saturation algorithm compute?
- What is decidable for probabilistic automata?
  How much fuzziness is required to get decidability?

An example: numberless probabilistic automata.
Is the value 1 problem decidable?

Yet still PA we could not decide!

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The emptiness problem, the isolation problem and the approximation problems are all undecidable for randomized machines.

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Corollary

The value 1 *problem for numberless probabilistic automata are undecidable.*

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The value 1 *problem for numberless probabilistic automata are undecidable.*

Conclusion: the saturation algorithm is useless.