Strategy Composition in Compositional Games

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RWTH AACHEN UNIVERSITY

Synthesis and Game Theory

Synthesis

Algorithmically derive a controller from a specification.

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Algorithmically derive a controller from a specification.

Input?

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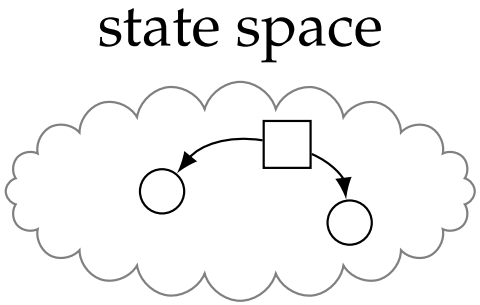

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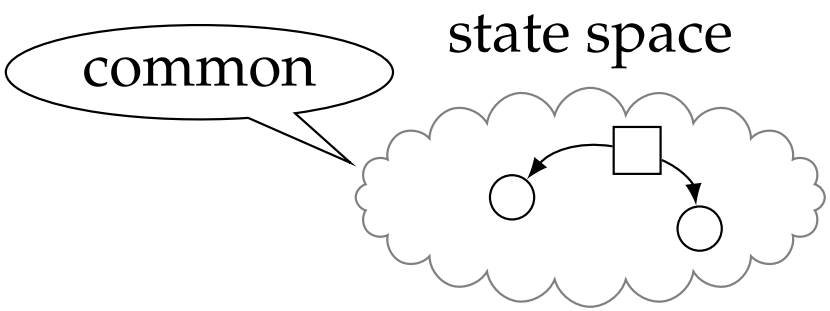

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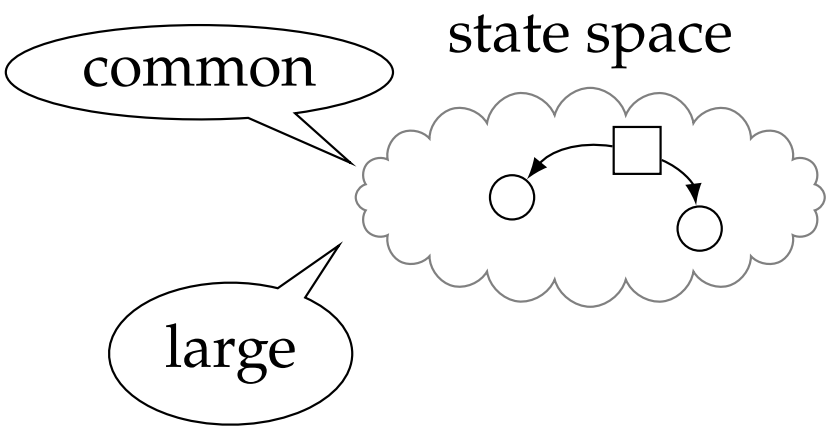

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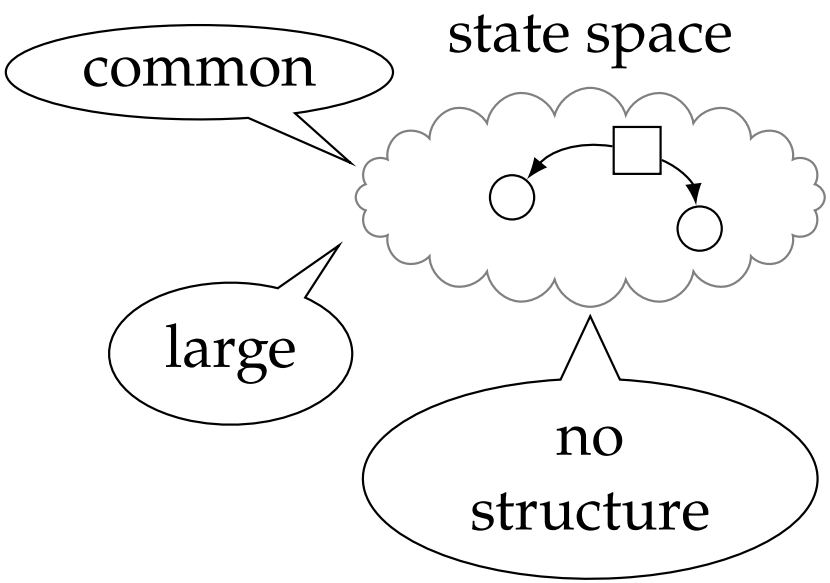

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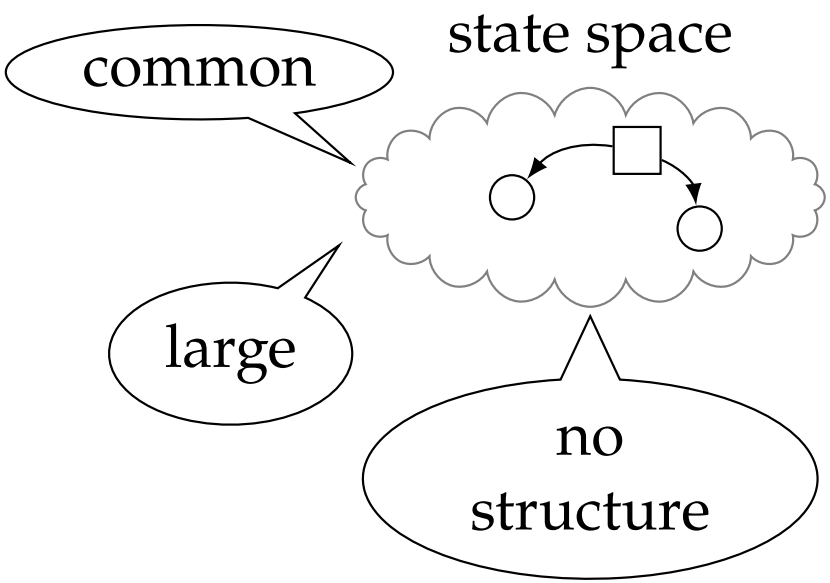

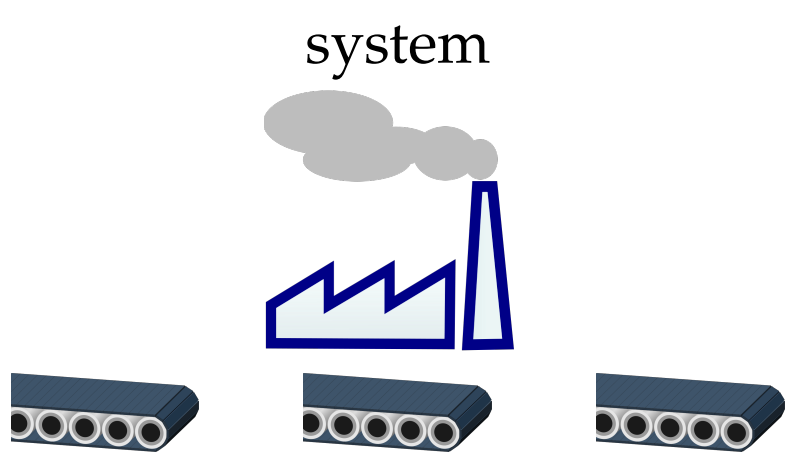

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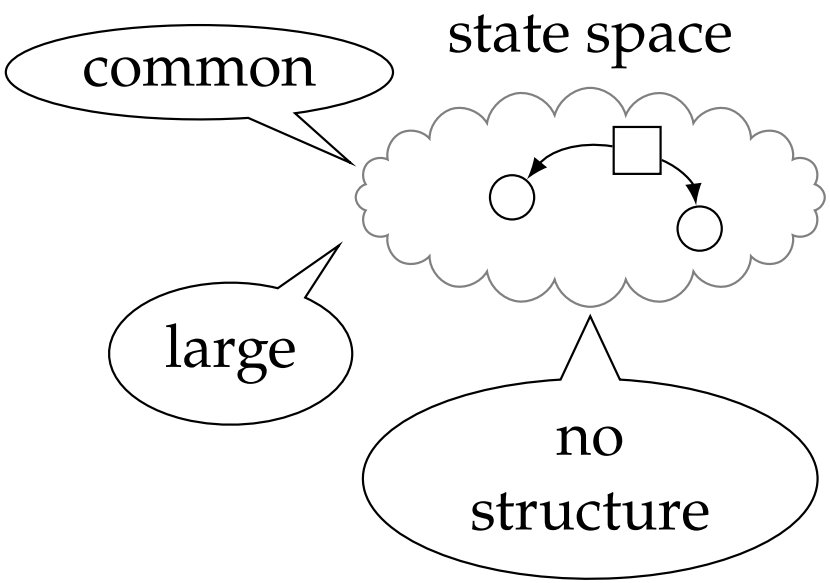
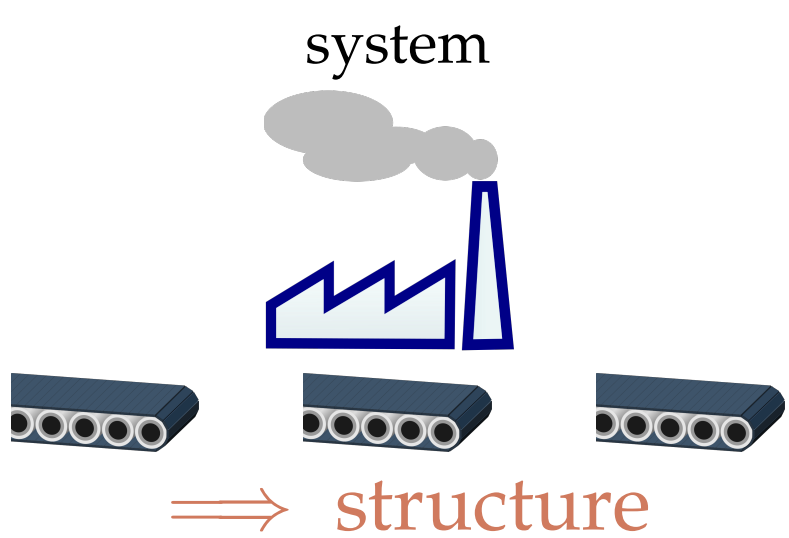
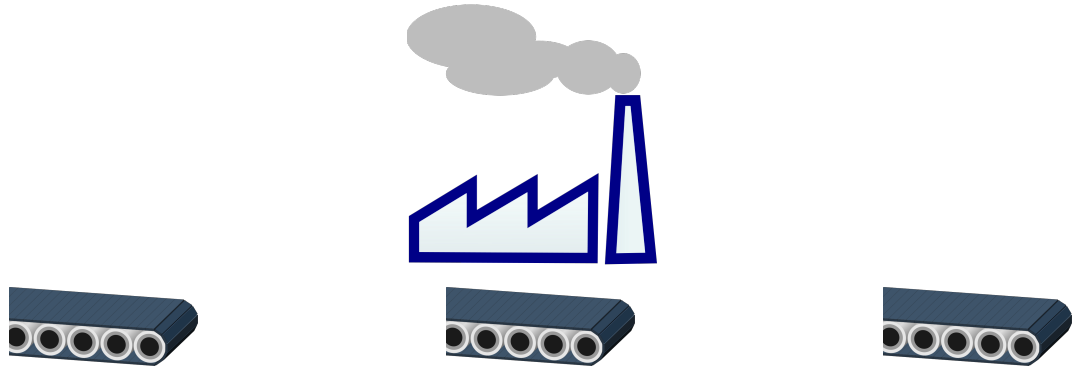




Controller Synthesis



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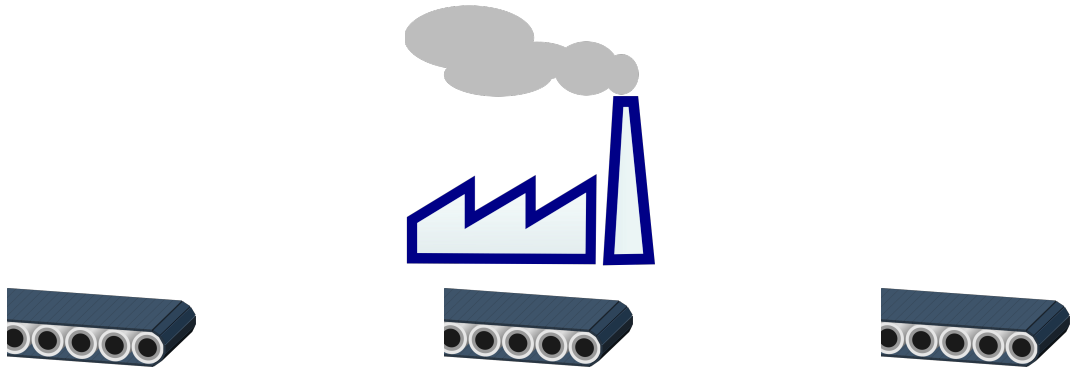

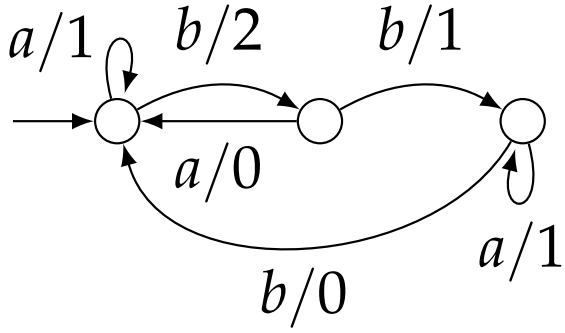

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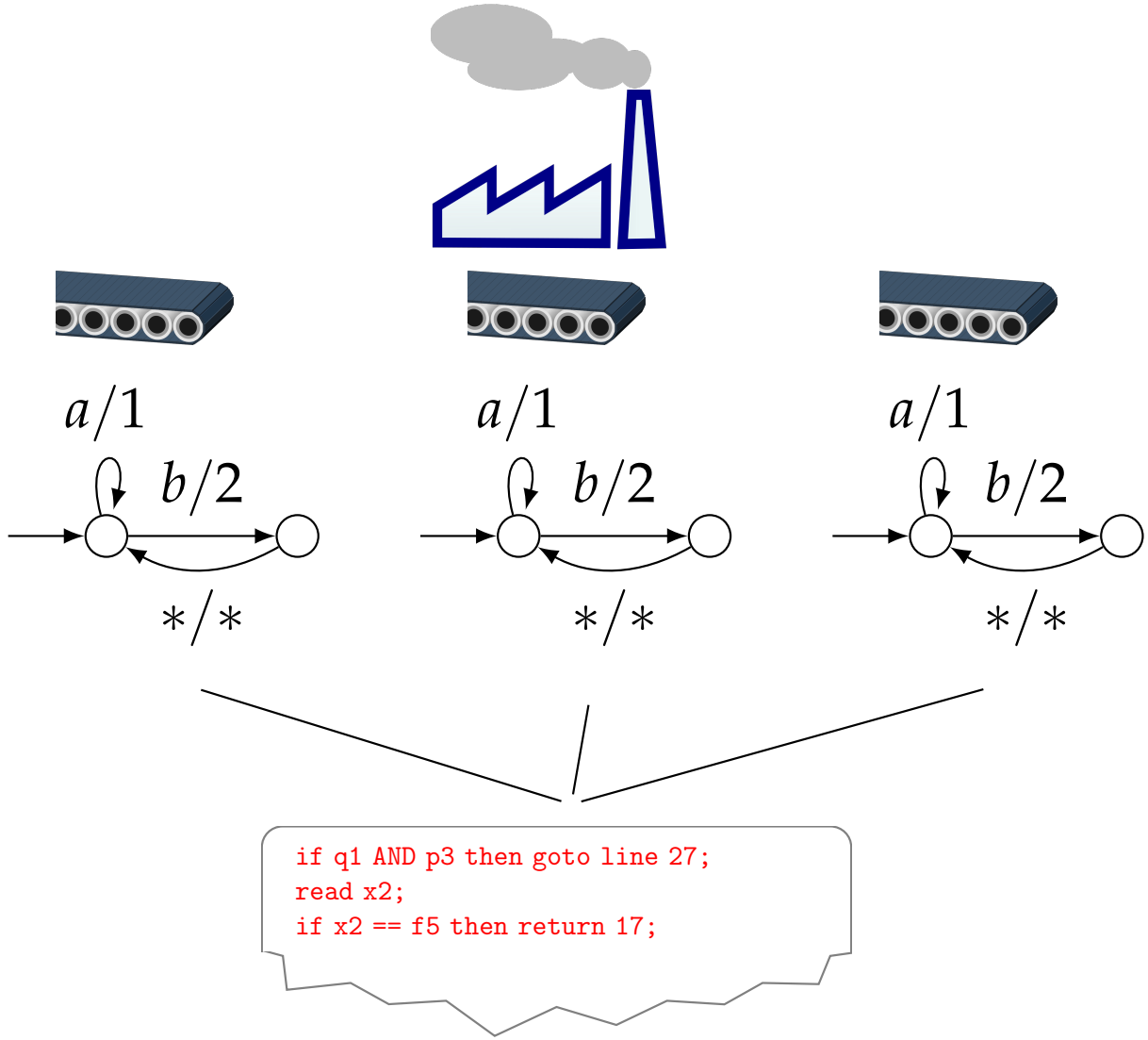
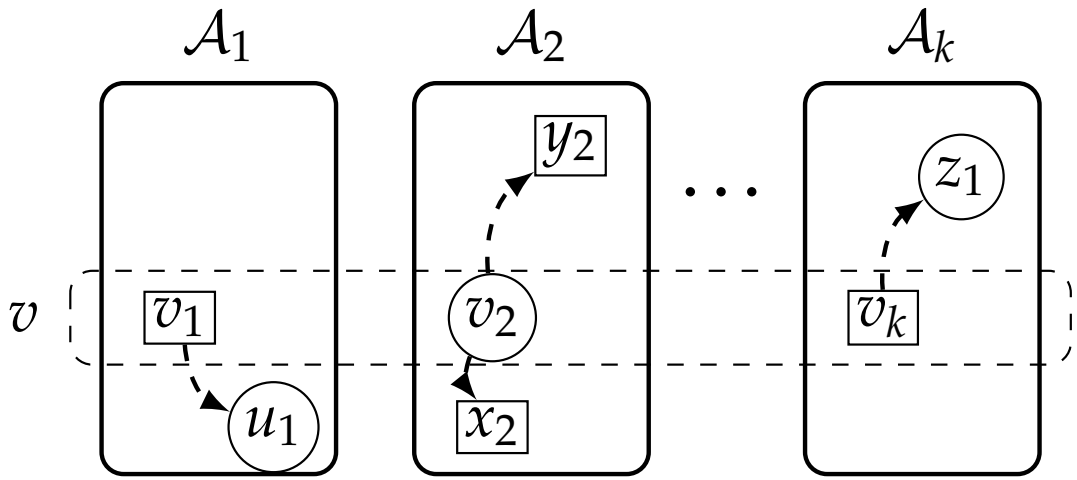
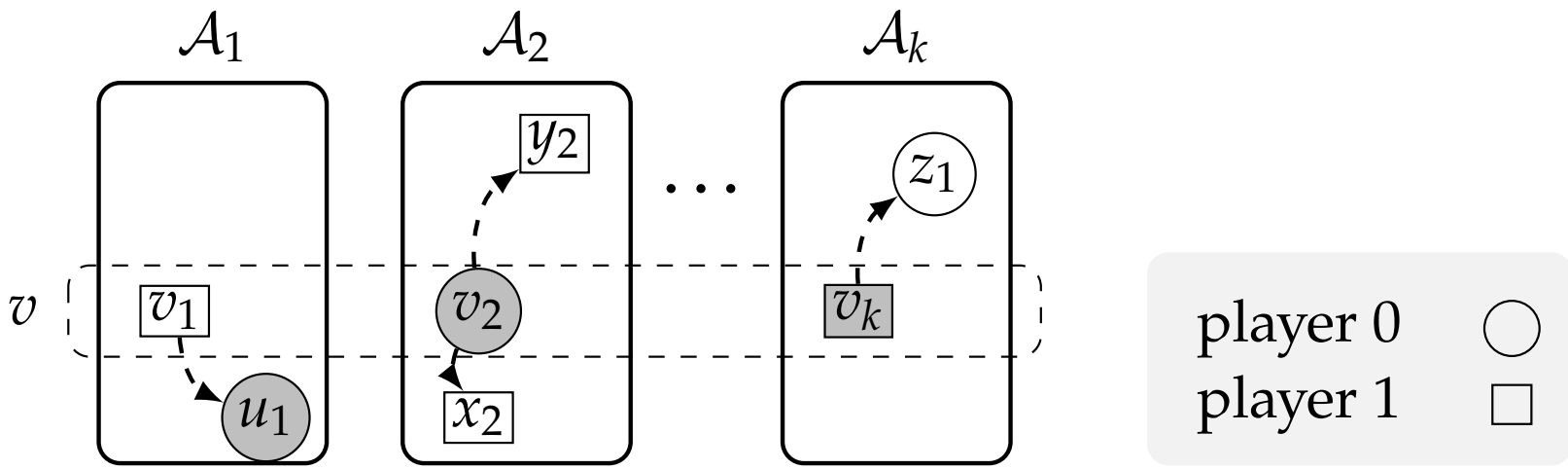


Parallel Product



player 0 ○
player 1 □

Parallel Product



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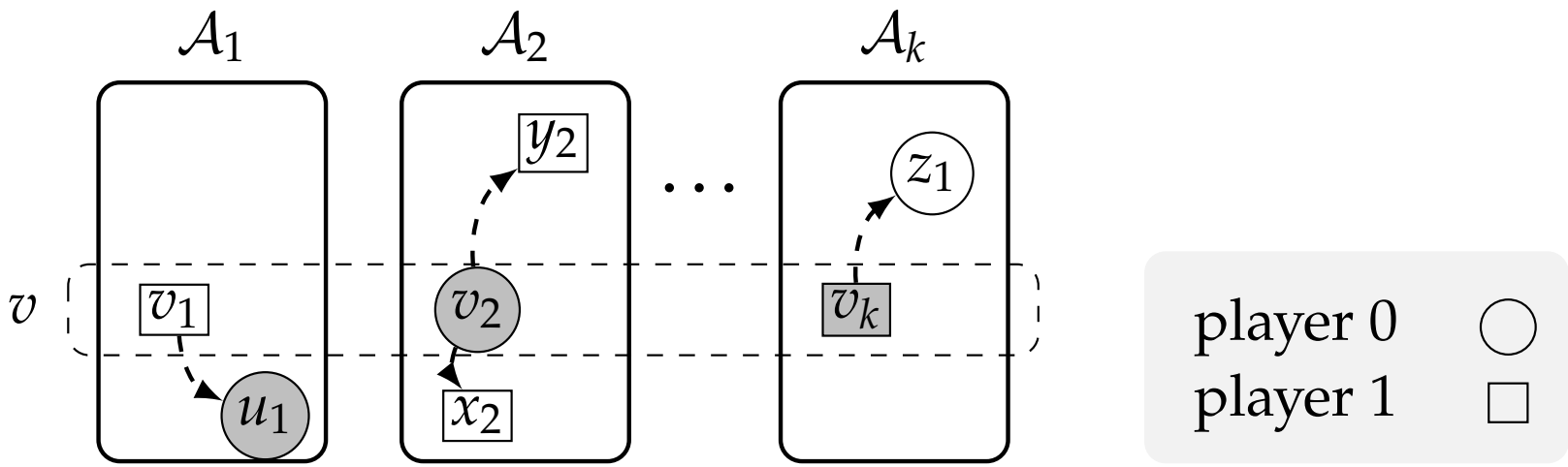
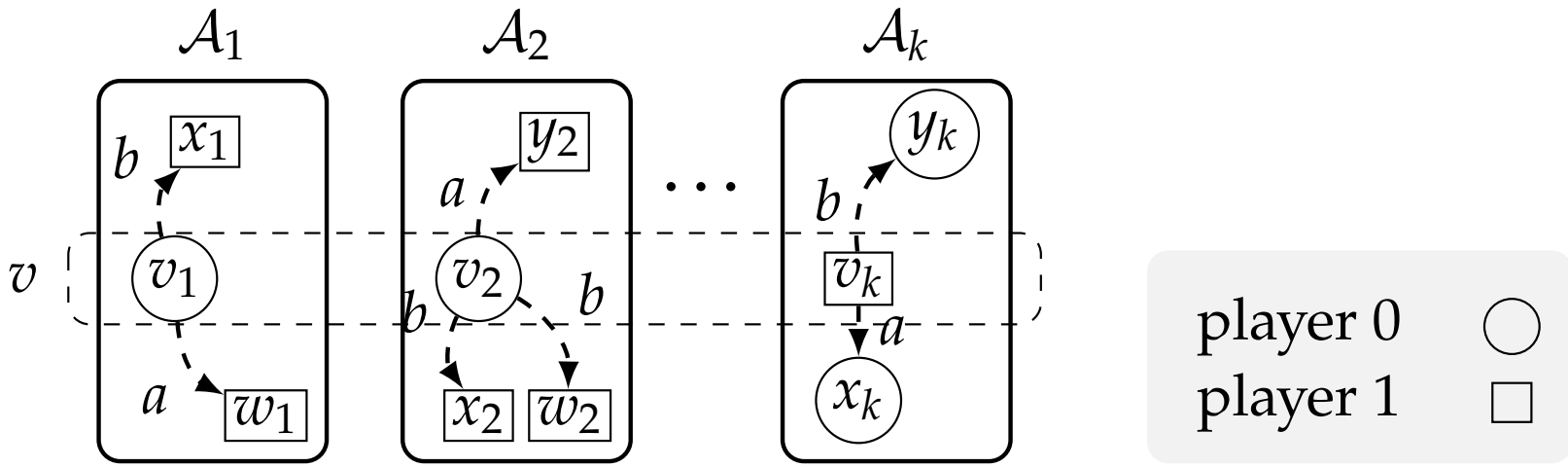


- player p picks one component i in a p-vertex
- player p picks a neighbour of v_i in $\mathcal{A}_i$

Synchronized Product



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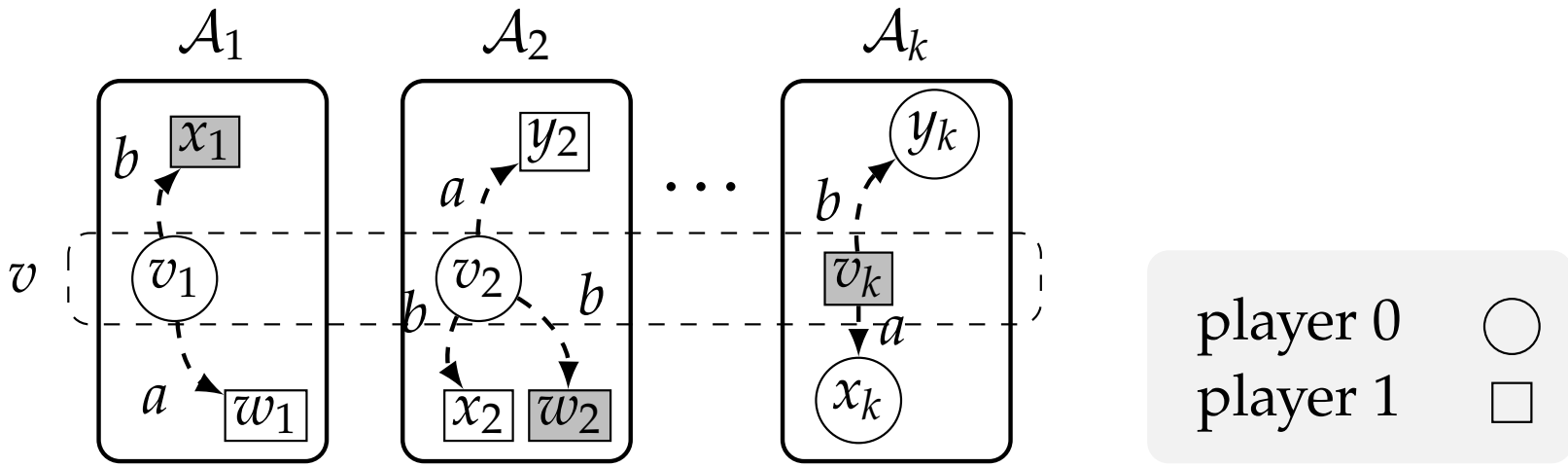

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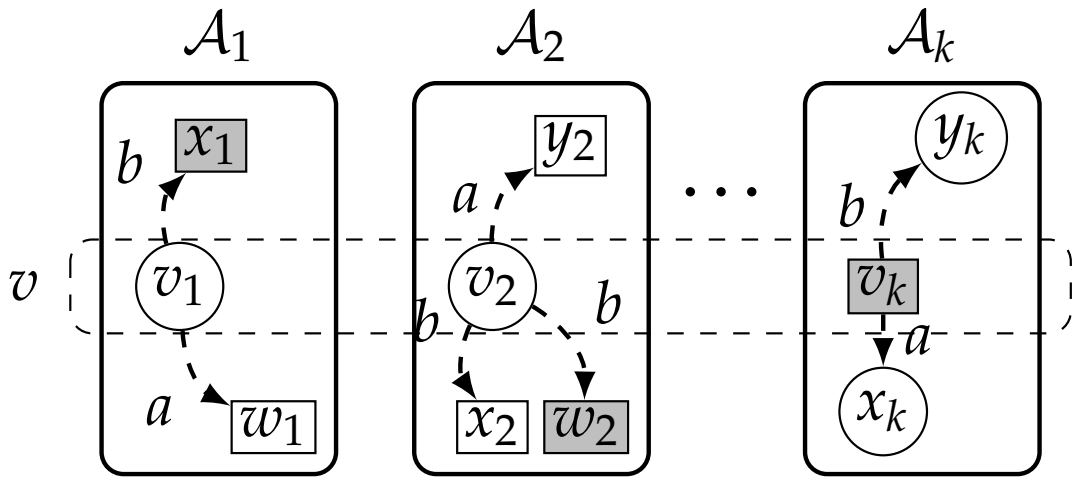

player 0 ◯
player 1 □

- labelled arenas $\mathcal{A}_i = (V_i, \Delta_i, \Sigma_i)$

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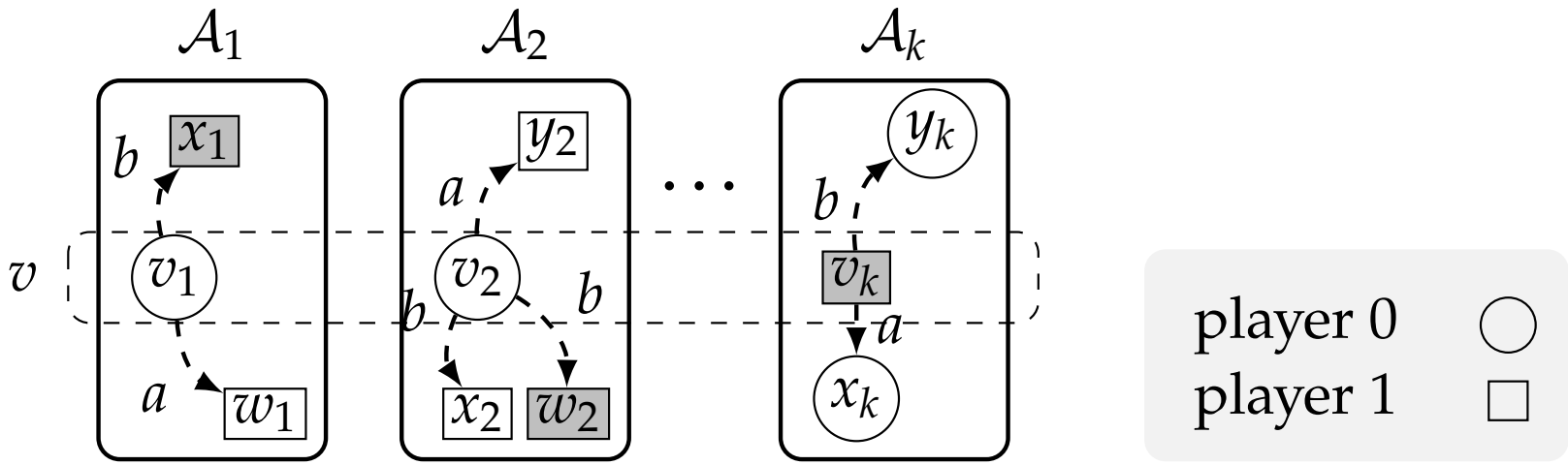

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- player p picks letter b

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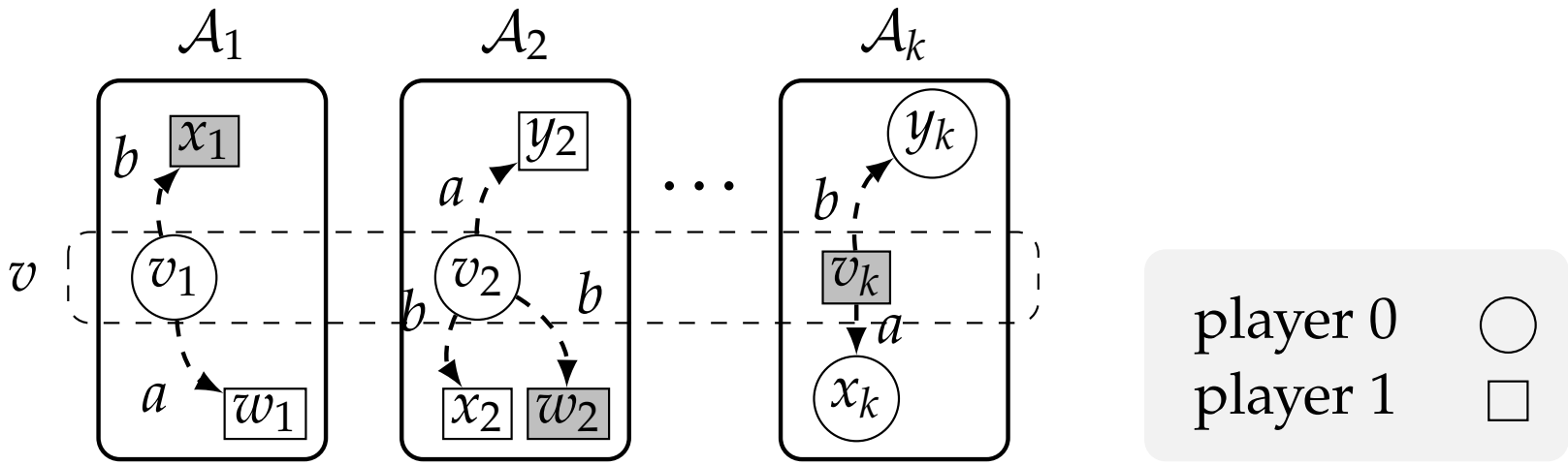
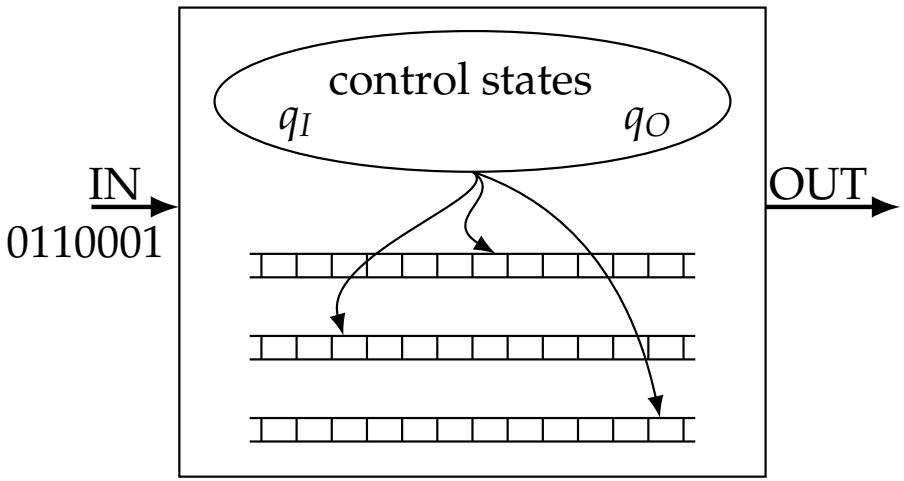


- labelled arenas $\mathcal{A}_i = (V_i, \Delta_i, \Sigma_i)$
- player p picks letter b
- player p picks b-transition in all components i where both
  - $v_i \in V_i^{(p)}$
  - and $b \in \Sigma_i$

Strategy Machines

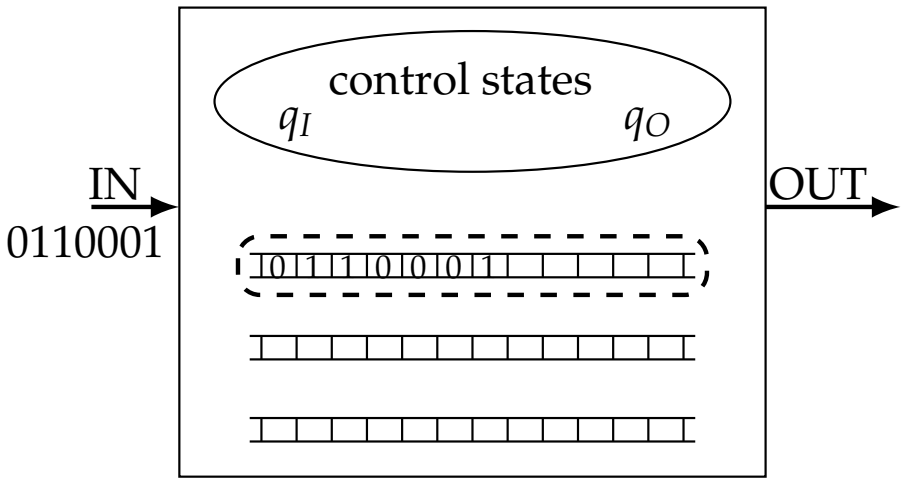


Definition (Strategy Machine)

3-tape Turing machine with

- input state q_I
- output state q_O

Strategy Machines



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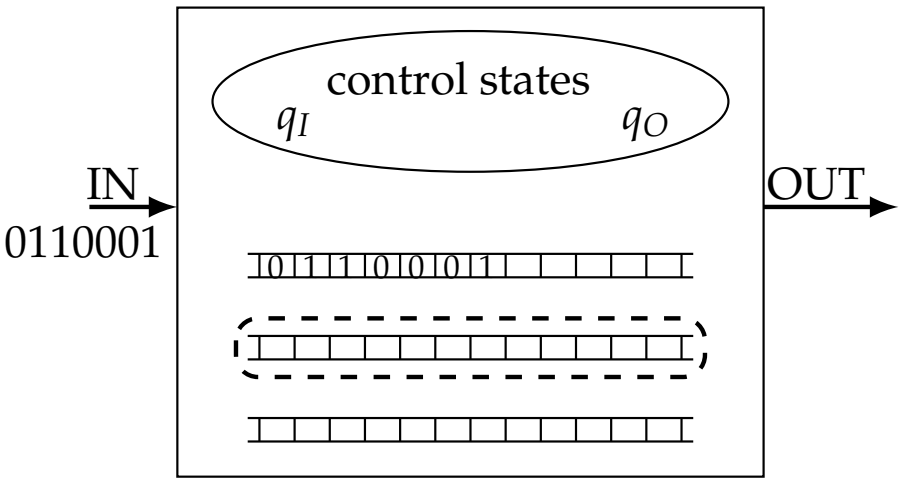

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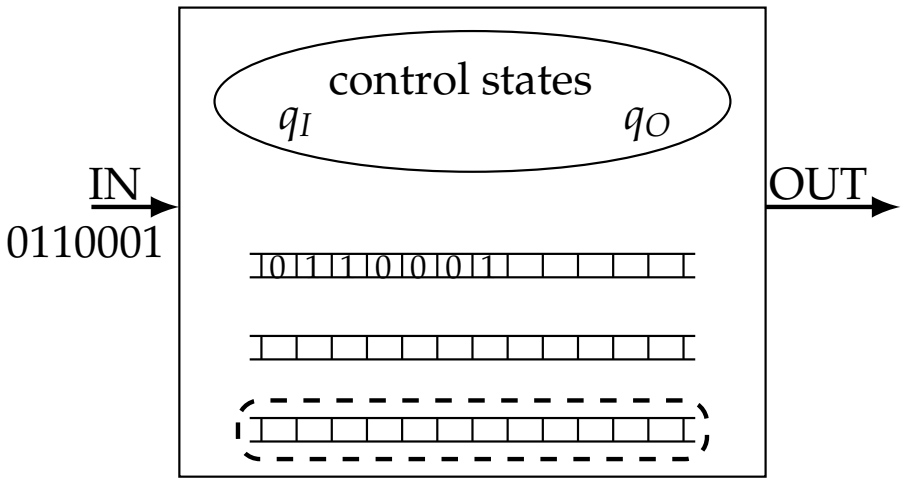

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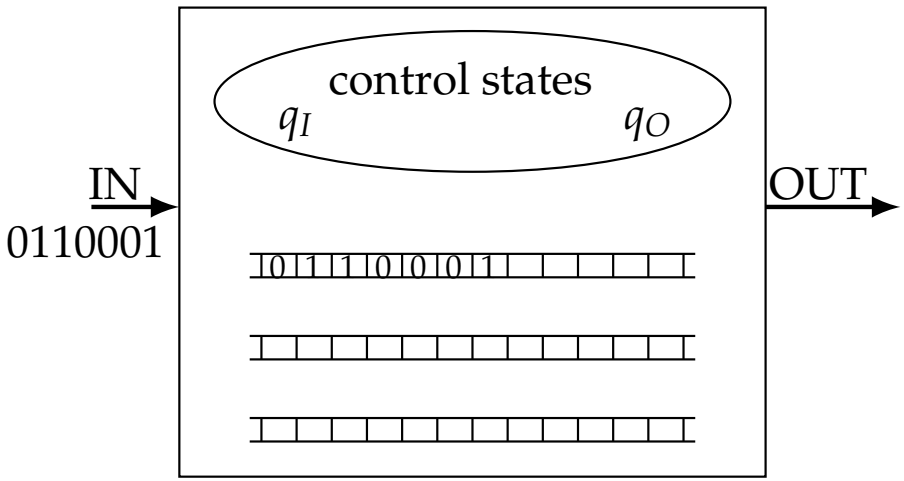

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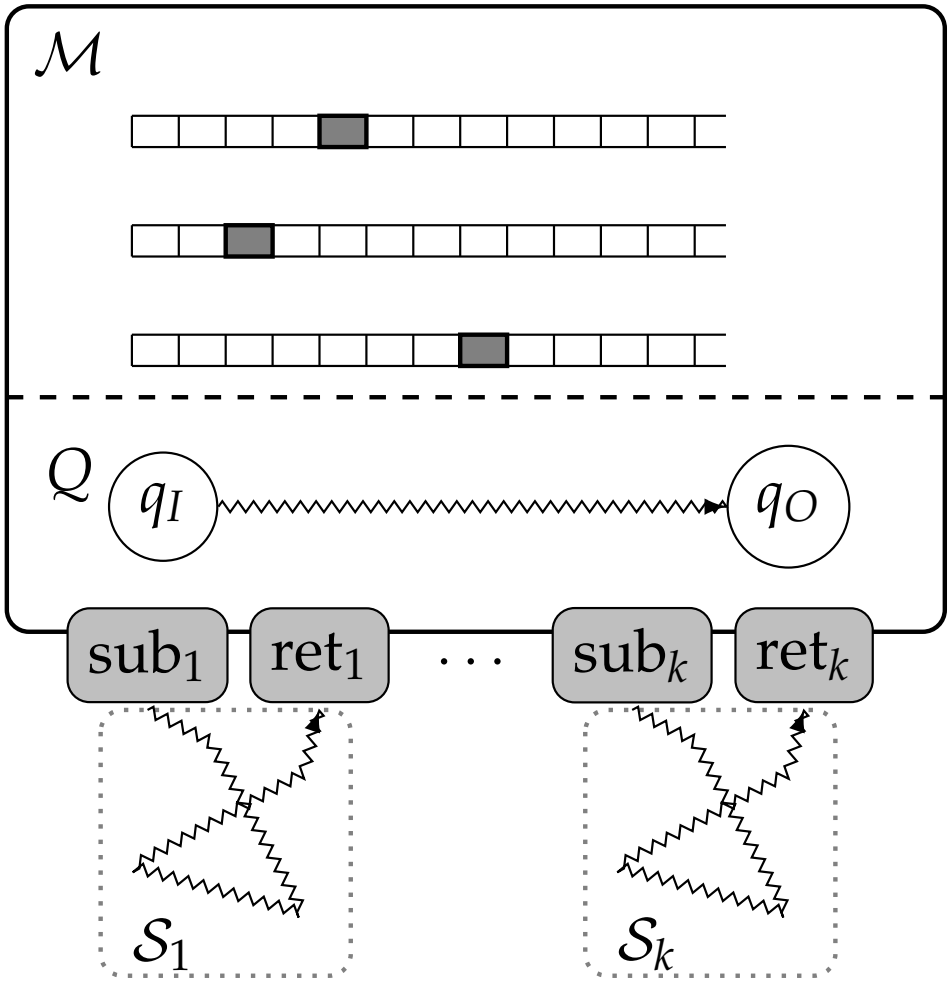
Definition

- size = $|Q|$ number of control states
- latency = length of longest iteration $q_I \rightsquigarrow q_O$
- space requirement = $\max_{\text{iterations}} \sum_{t \in \text{tapes}} \#$ cells used on t

Compositions



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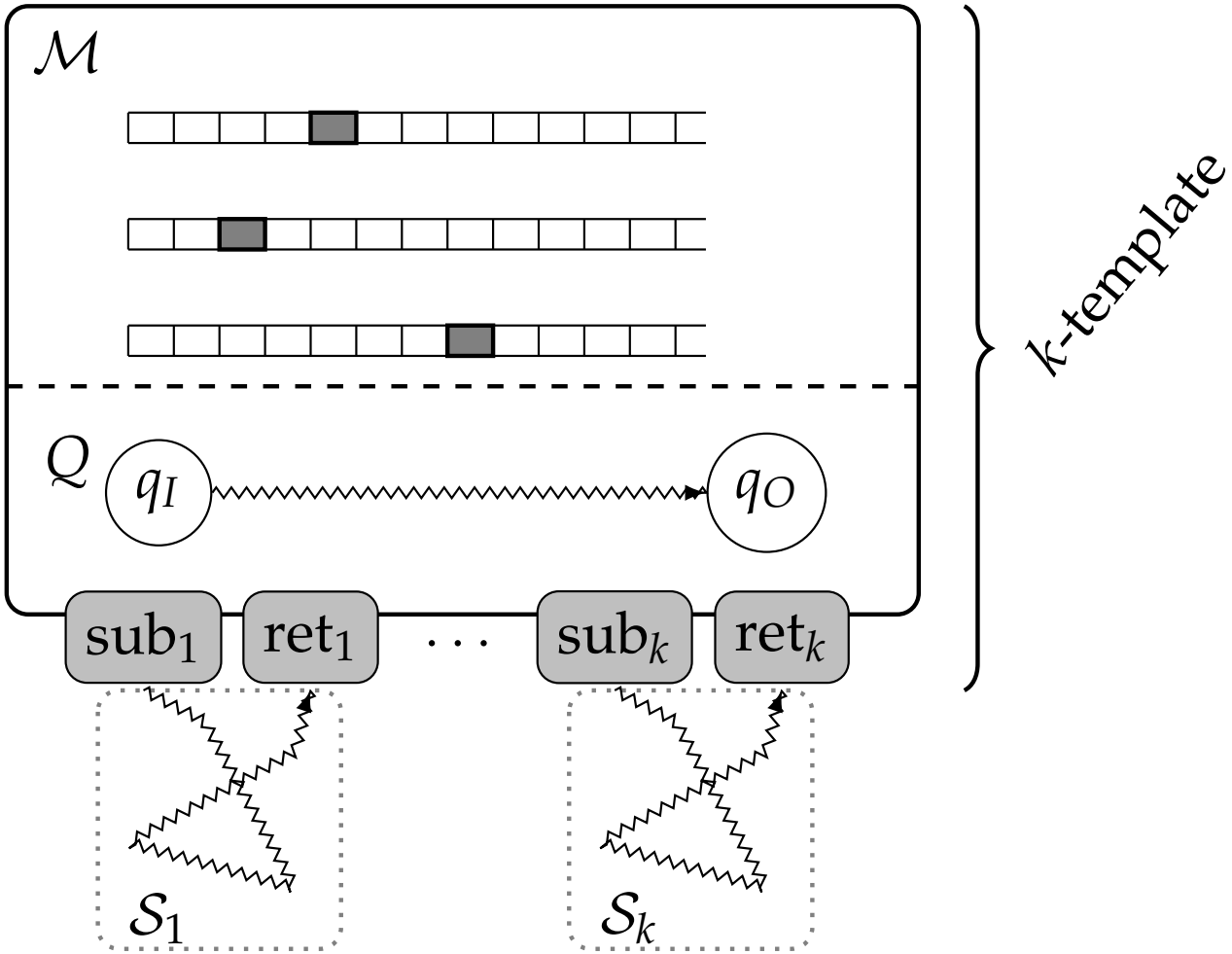

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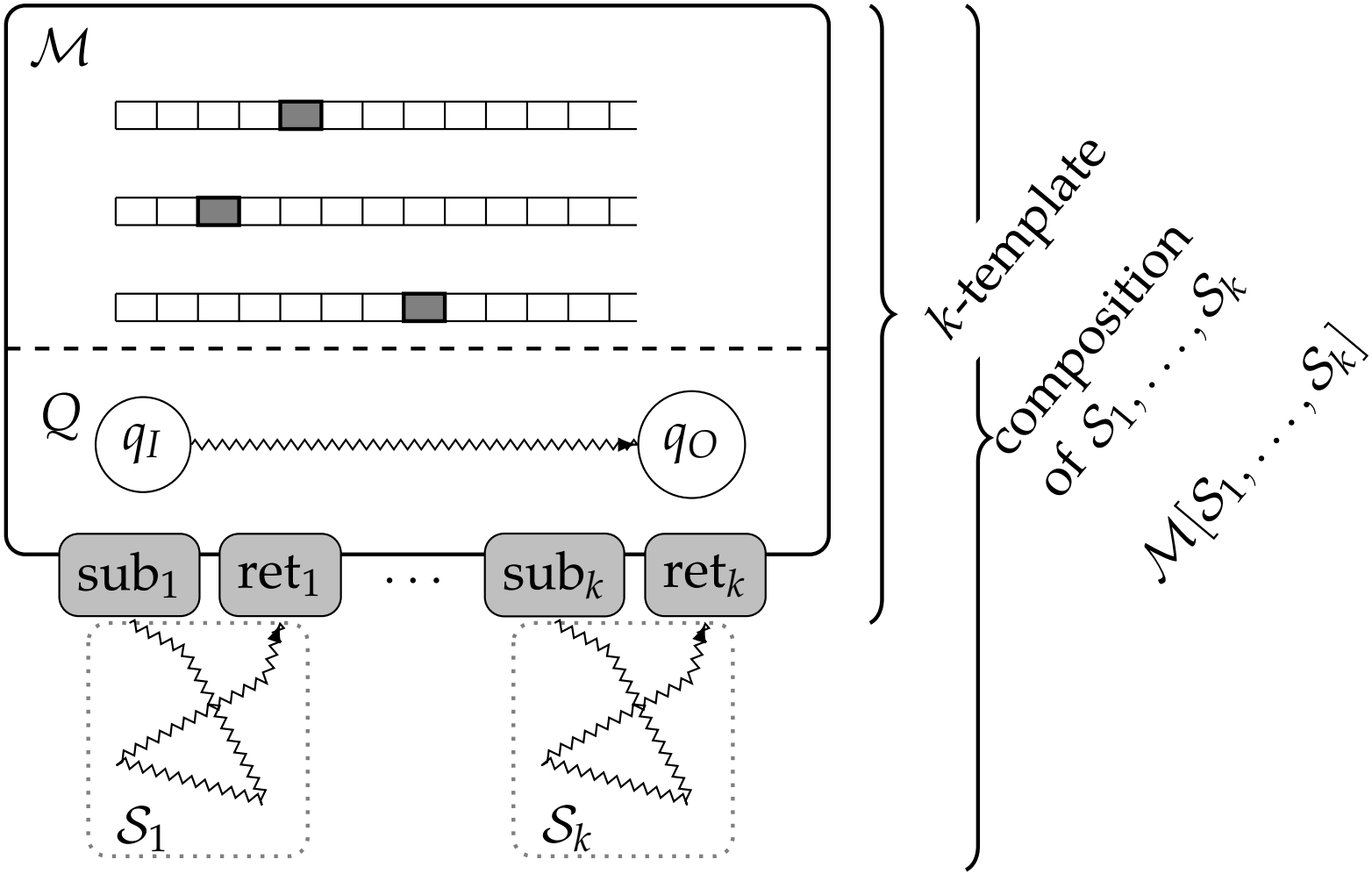

Reachability Games

Theorem

Local and synchronized reachability games on parallel products admit polynomial time computable polynomial compositions.
The component games of the composition are positionally determined.

Theorem

Local or synchronized reachability games on synchronized products admit poly-space compositions if and only if Pspace = Exptime.