Decidability of Weak Simulation on One-Counter Nets

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September 21, 2013

Weak Steps

For $a \neq \tau \in \mathrm{Act}$ and define

$$\overset{\tau}{\Longrightarrow} := \xrightarrow{\tau}^{*} \qquad \overset{a}{\Longrightarrow} := \xrightarrow{\tau}^{*} \xrightarrow{a} \xrightarrow{\tau}^{*}$$

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$$
\begin{tikzcd}
\alpha \arrow[d, "a"'] & \textit{vs.} & \beta \arrow[d, Rightarrow, "a"'] \\
\alpha' & \textit{vs.} & \beta'
\end{tikzcd}
$$

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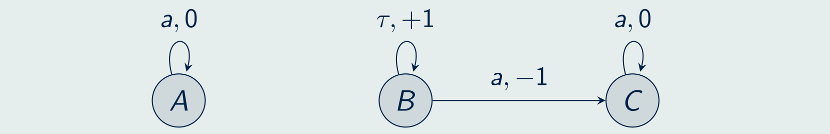
Def: Weak Simulation $\preceq$

$\alpha \preceq \beta$ iff Duplicator has a strategy to win from α vs. β.

Example



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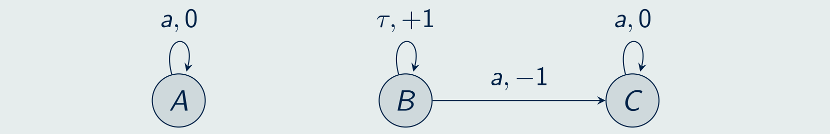

- $A0 \not\preceq B0$

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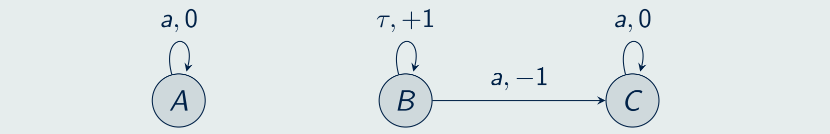

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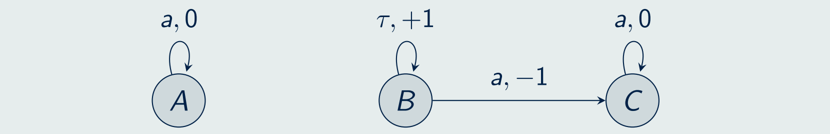

- $A0 \not\preceq B0$
- $A0 \preceq\!\!\!\equiv B0$
- $B0 \stackrel{a}{\Longrightarrow} Cn$ for every $n \in \mathbb{N}$

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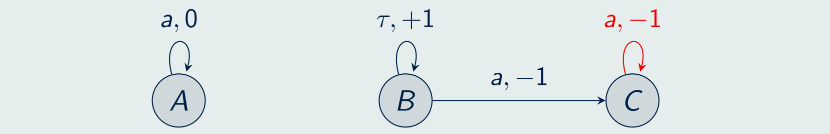

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Our Contribution

We show decidability of the

OCN Weak Simulation Problem

Input:	A net $\mathcal{N} = (Q, \mathrm{Act}, \delta)$ and configurations pm, qn.
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Theorem

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Why should you care?

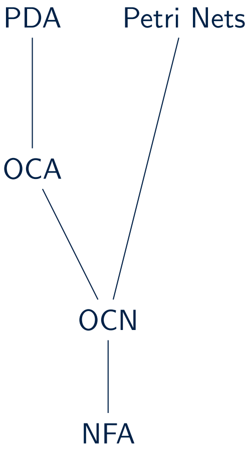
In practice, modelling might use both ∞-states and branching:

- network protocols/queues keeping track of their workload
- random guesses

Theoretically, surprising:

- rare positive result for behavioral preorder that is not finitely approximable $\preceq \neq \preceq_\omega$.
- goes against the usual ‘finer is easier’ trend

Some Context – Strong Case



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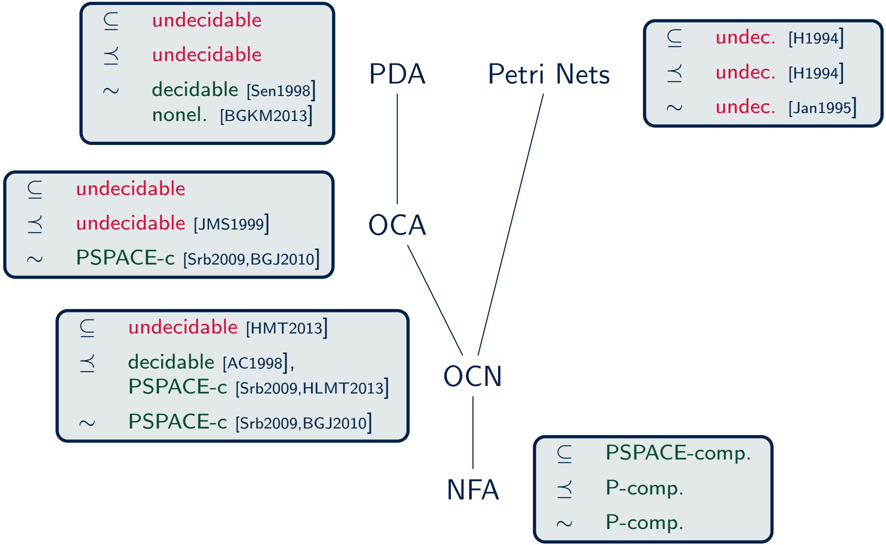
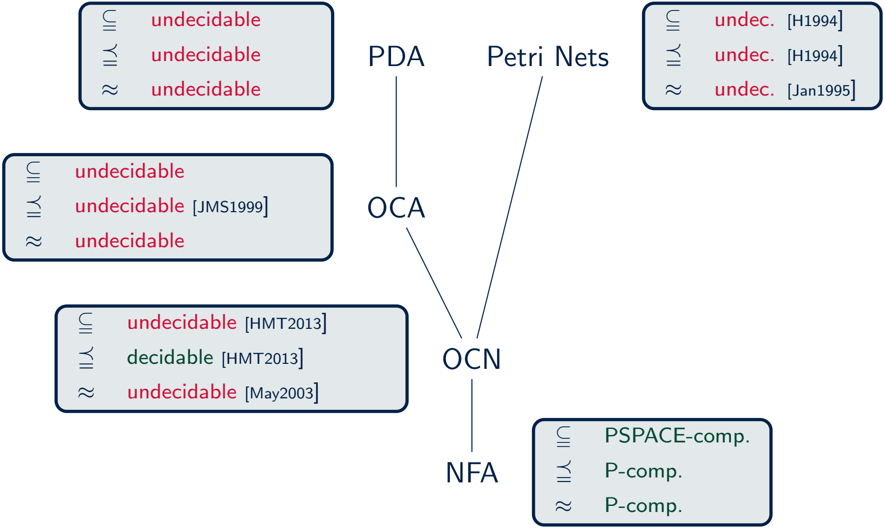


Some Context – Weak Case



Proof Overview

Symbolic infinite branching

1

Reduce (OCN $\preceq$ OCN) $\rightsquigarrow$ (OCN $\preceq$ ω-Net)

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Approximants for the new game 2

$\exists$ finite sequence $\preceq^0 \supseteq \preceq^1 \supseteq \preceq^2 \supseteq \cdots \supseteq \preceq^k = \preceq$

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Approximants for strong simulation (OCN vs. ω-Net)

$$\preceq^{k}$$

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...holds if Duplicator can guarantee to either

- enforce an infinite game or
- explicitly make use of ∞-branching k times.

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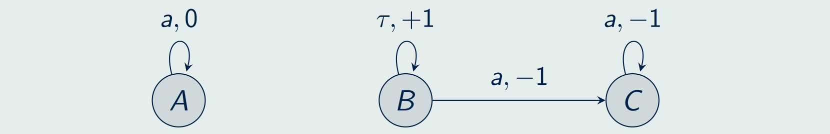

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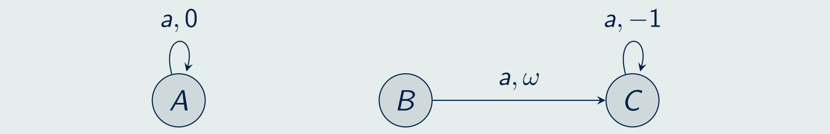

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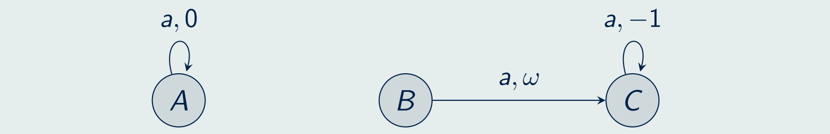

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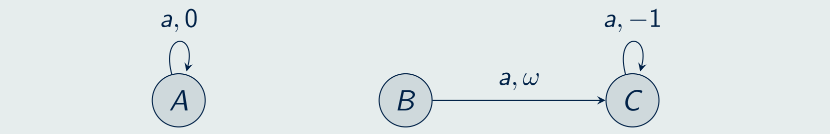

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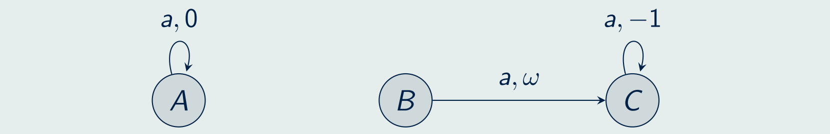

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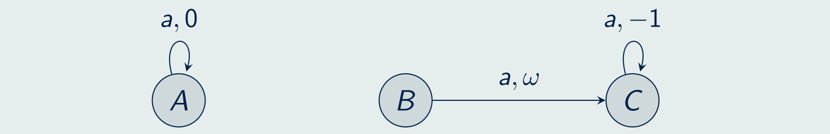

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