effect that $\hat{x}\phi$ is an element if in addition to being stratified ϕ is also *normal*; a formula is normal if all of its bound variables are restricted to be element variables. The condition of normality effectively obstructs Rosser's derivation of the Burali-Forti contradiction.

In the next part Wang points out that the classes of Quine's earlier *New foundations* system (II 86) correspond exactly to the elements of P. This leads to a relative consistency proof for P. For *New foundations* possesses a model in the domain of integers (by the Gödel extension of the Löwenheim-Skolem theorem) and hence, as Wang shows in detail, P possesses a model in the domain of classes of integers, that is in the real number system. Since *New foundations* is weaker than P and, moreover, has been subject to close scrutiny by Rosser (see V 32) the Wang system should be safe enough by usual standards.

THEODORE HAILPERIN

B. A. TRAHTÉNBROT. *Névozmožnost' algorifma dlá problémy razréšimosti na konéčnyh klassah* (Impossibility of an algorithm for the decision problem in finite classes). ***Doklady Akademii Nauk SSSR,*** vol. 70 (1950), pp. 569–572.

According to a well-known theorem of Church, there is no decision procedure which would enable one to test automatically whether an arbitrarily proposed formula A of the first-order functional calculus (with identity) is or is not identically valid in every non-void set I. In the paper under review Trahténbrot discusses an analogous problem arising from the problem just formulated by the substitution of the words "non-void finite set I" for the words "non-void set I." The author shows that the answer to this "decision problem in finite classes," as he calls it, is negative, just as in the case of the original decision problem investigated by Church.

The method of proof used by the author is not without interest in itself. Let A be a formula containing no free individual variables and containing the predicate variables $\mathrm{M}_1^1, \mathrm{M}_2^1, \ldots, \mathrm{N}_1^2, \mathrm{N}_2^2, \ldots$ (the upper indices indicate here the number of arguments). A finite model **M** for A is defined as a finite set P_1 together with the singulary, binary, . . . relations $M_1^1, M_2^1, \ldots, N_1^2, N_2^2, \ldots$ which satisfy A in P. Let $m_i(\mathbf{M})$ be the number of elements of M_i^1. The set of integers $m_i(\mathbf{M})$ where **M** ranges over all the finite models of A is called the *spectrum* of M_i^1. A number-theoretic function f is said to have a *spectral representation* if there is a formula A containing at least the two predicate variables M_1^1 and M_2^1 such that the spectrum of M_1^1 contains all non-negative integers and $m_2(\mathbf{M}) = f(m_1(\mathbf{M}))$ for every finite model **M**. The main theorem (the proof of which is briefly indicated) says that f has a spectral representation if and only if it is general recursive. The negative solution of the decision problem in finite classes follows easily from the main theorem.

In the final section Trahténbrot applies his theorem to the question of definitions of finiteness in systems of set theory formalized in the usual way. Each non-tautological formula A which is identically valid in every finite set gives rise to a formalized definition of the finiteness of a set q. To obtain this definition we express, by the means which we have at our disposal in the system under consideration, the proposition: A *is valid in* q. Let $A^+(q)$ be an abbreviation of the definition thus obtained. Trahténbrot shows that from the negative solution of the decision problem in finite classes it follows that to each definition of finiteness $A^+(q)$ there are two other definitions of finiteness $B^+(q)$ and $C^+(q)$ such that neither $B^+(q) \supset A^+(q)$ nor $A^+(q) \supset C^+(q)$ are provable in the formalized set theory. In the usual systems of set theory there is thus neither a strongest nor a weakest definition of finiteness.

In a note added during the proof the author remarks that a closely related result was announced by the reviewer in IV 30(1). Although Trahténbrot's paper does not contain proofs of the theorems pertaining to definitions of finiteness, it is clear from the indications given in the paper that his method is much simpler than the method used in the (unpublished) proof of the reviewer—which depended on a discussion of such forms of Gödel's incompleteness theorem as can be proved independently of the form of the axiom of infinity that is admitted in the system.

Errata: In the displayed formula on page 570 there should be "x" instead of "a". On page 572, line 2 from the top, read $\mathfrak{B}^*$ for $\mathfrak{B}$.

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