

Efficient Separability of Regular Languages by Subsequences and Suffixes

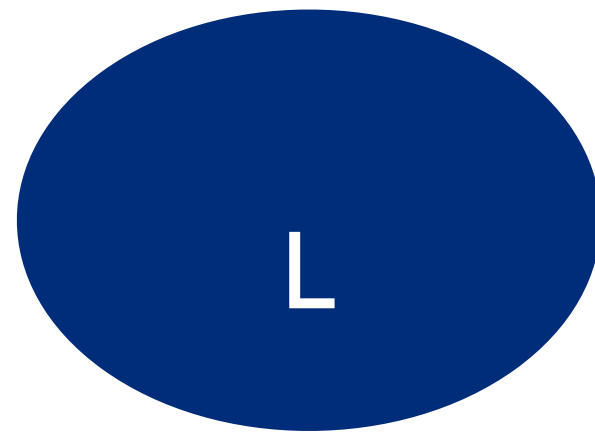
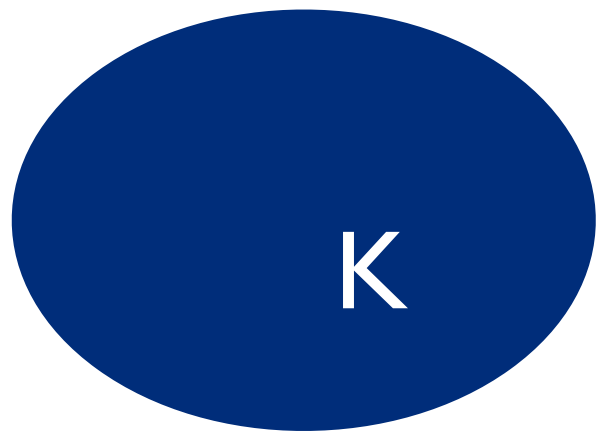
Wojciech Czerwiński

Tomáš Masopust

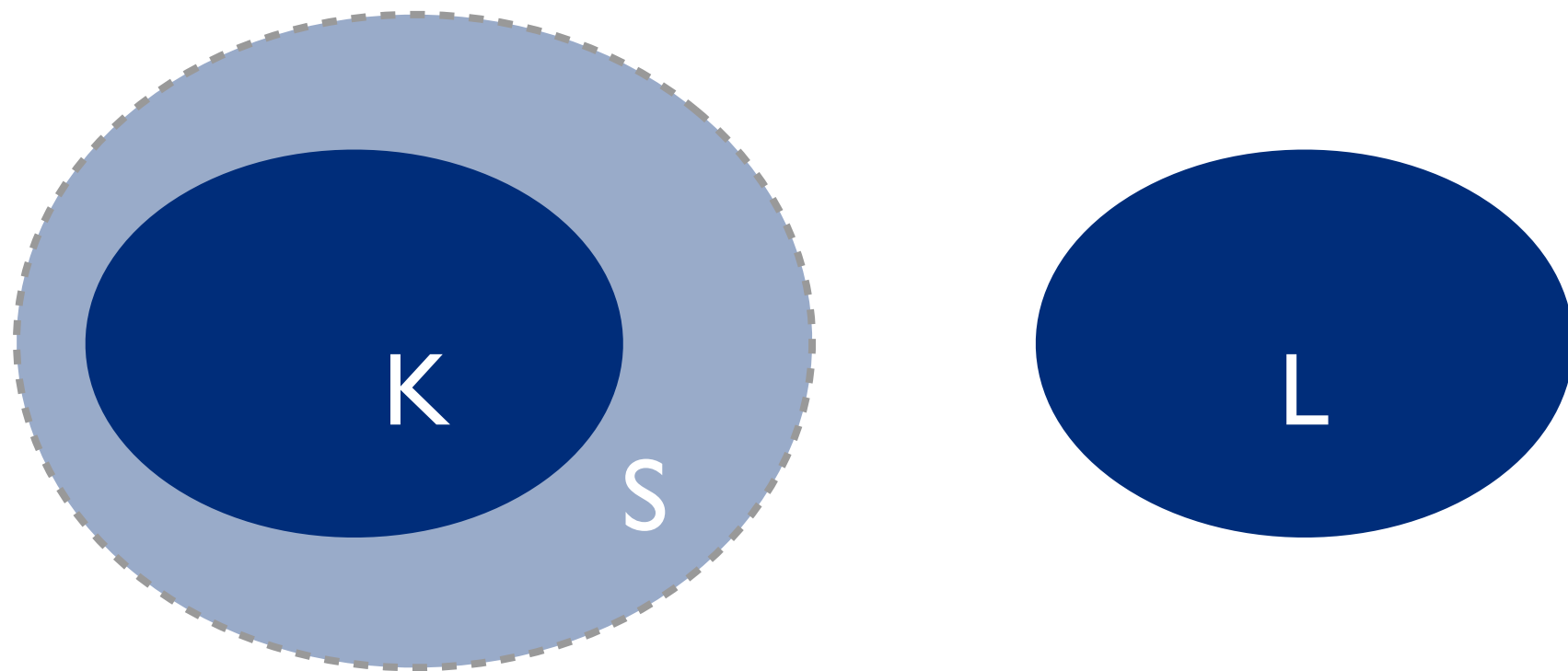
Wim Martens

Separability

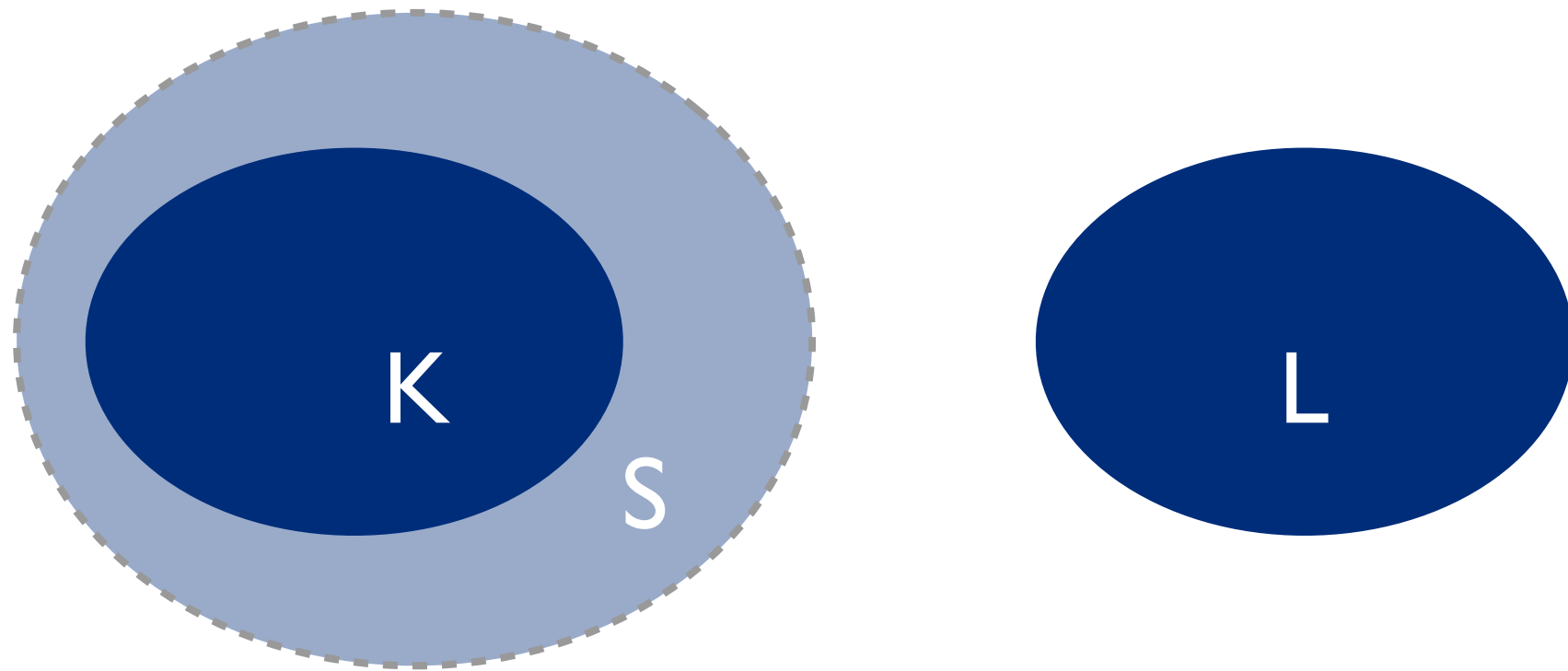
Separability



Separability

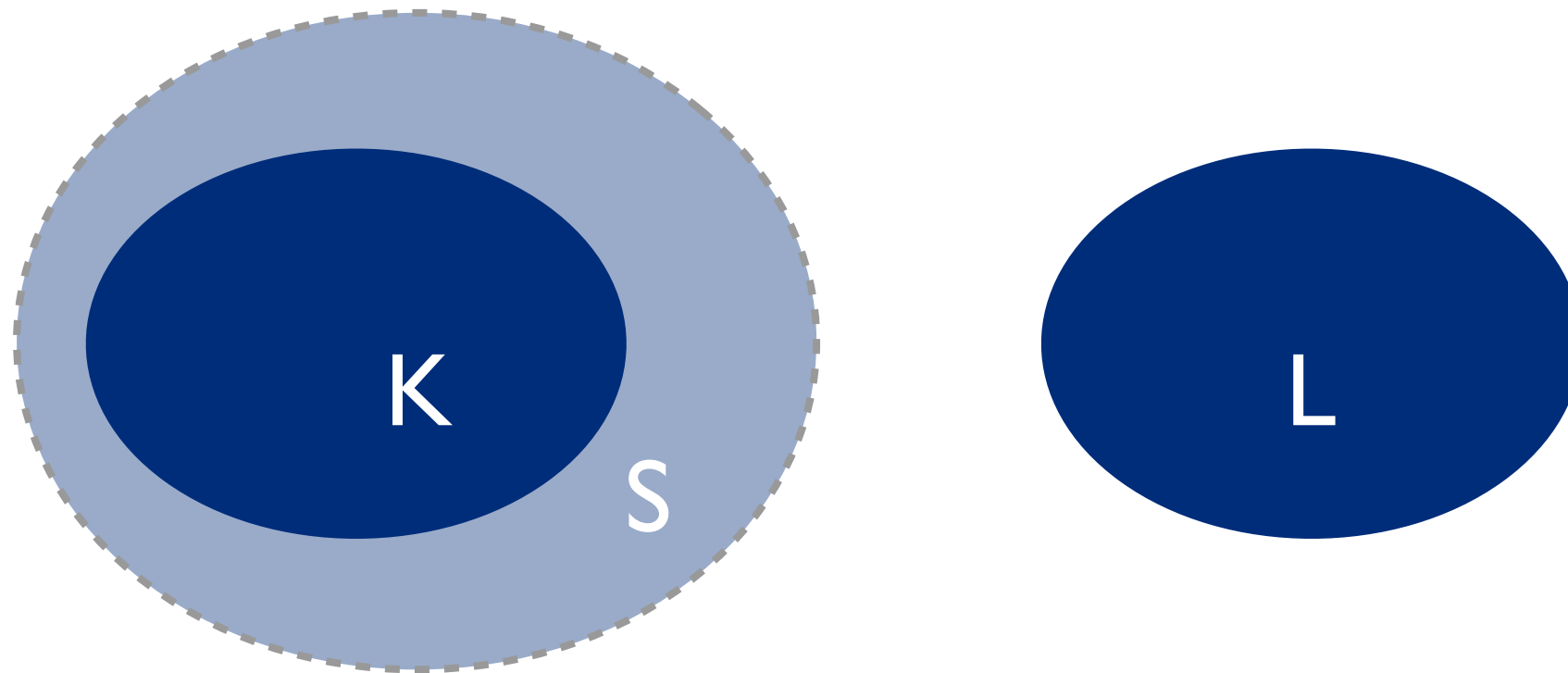


Separability



S separates K and L

Separability



S separates K and L

K and L are separable by family F
if some *S* from *F* separates them

Problem

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Given: nondeterministic automata for
languages K and L

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Question: are K and L separable by Piecewise Testable Languages (PTL)?

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piece language

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piece language

$$\Sigma^* a_1 \Sigma^* a_2 \Sigma^* \dots \Sigma^* a_n \Sigma^*$$

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Given: nondeterministic automata for
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piecewise testable language

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Question: are K and L separable by
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piece language

$$\Sigma^* a_1 \Sigma^* a_2 \Sigma^* \dots \Sigma^* a_n \Sigma^*$$

piecewise testable language

bool. comb. of pieces

What is known?

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- Simon 1975, piecewise testable = j-trivial

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- Stern 1985, deciding if a language is piecewise testable is in PTIME

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- Stern 1985, deciding if a language is piecewise testable is in PTIME
- Almeida, Zeitoun 1997, exponential algorithm for separability by PTL

First main result

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Theorem:

Separability of Regular Languages
by Piecewise Testable Languages
can be decided in PTIME

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obtained independently
by Place, van Rooijen, Zeitoun
MFCS '13

First main result

Theorem:

Separability of Regular Languages
by Piecewise Testable Languages
can be decided in PTIME

Second main result

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For **any** two word languages K and L the following conditions are equivalent:

Second main result

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I) K and L are separable by piecewise testable languages

Second main result

For **any** two word languages K and L the following conditions are equivalent:

- 1) K and L are separable by piecewise testable languages
- 2) there is no infinite zigzag between K and L

Infinite zigzag

Infinite zigzag

$abcd \preceq dbabacbdb$

Infinite zigzag

Infinite zigzag

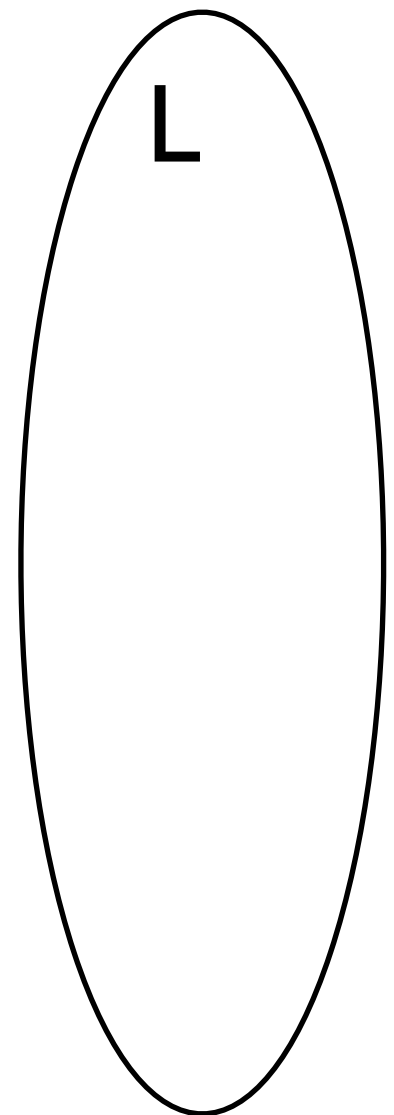
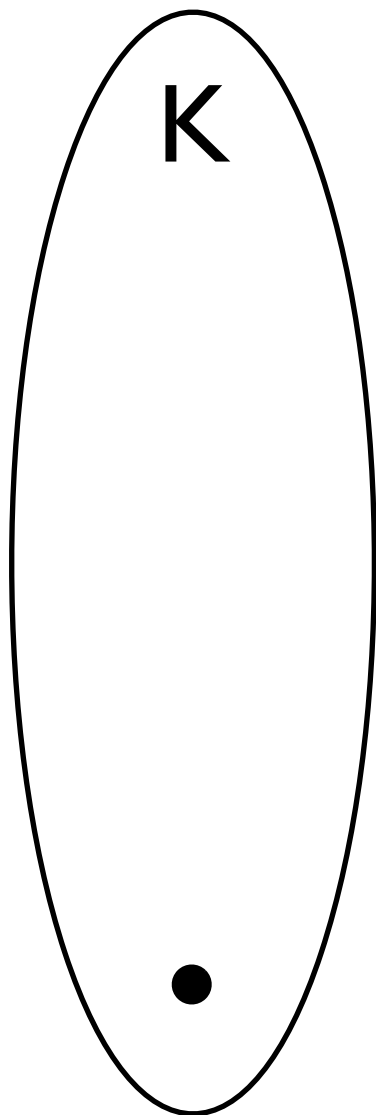


K

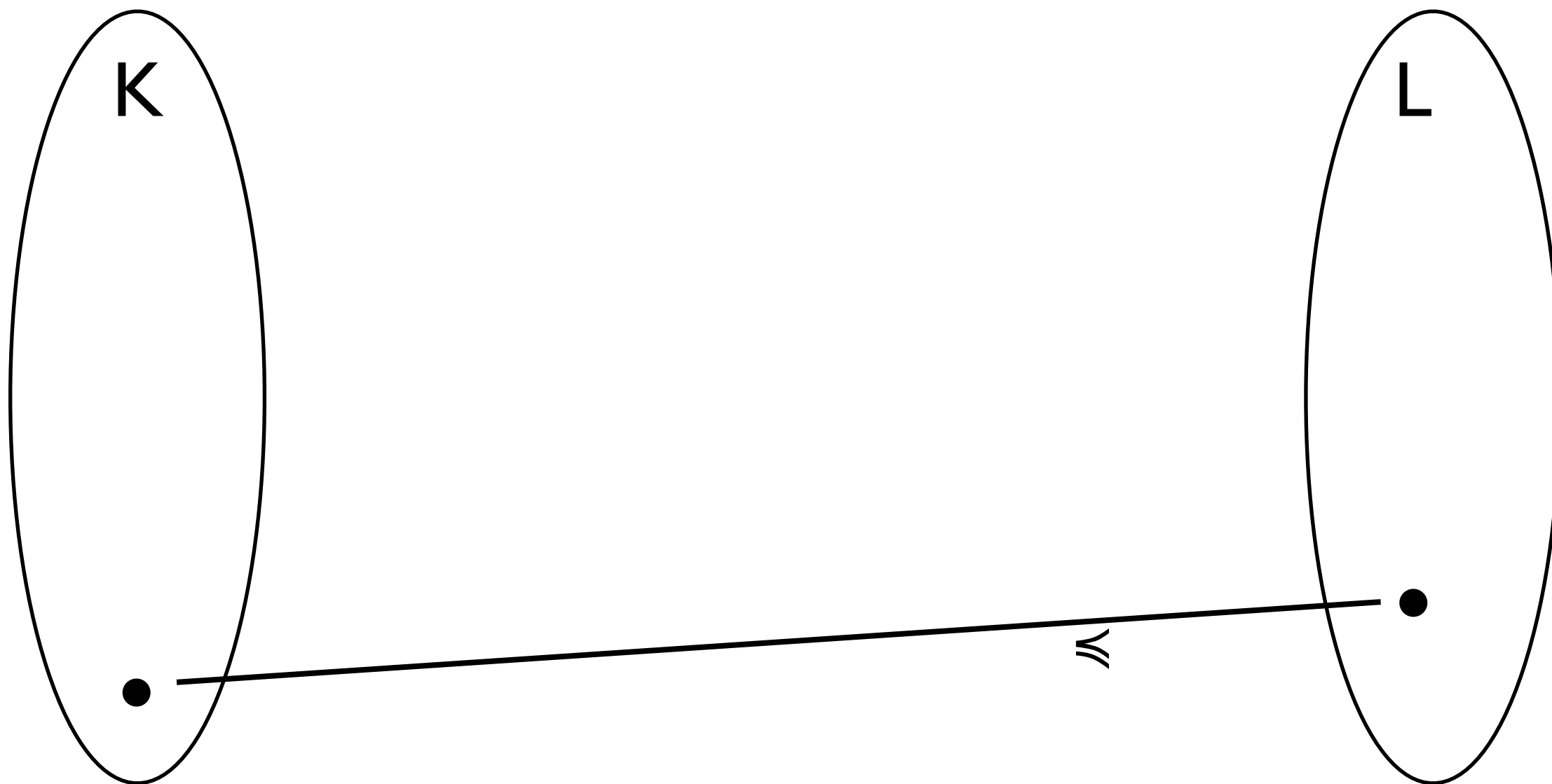


L

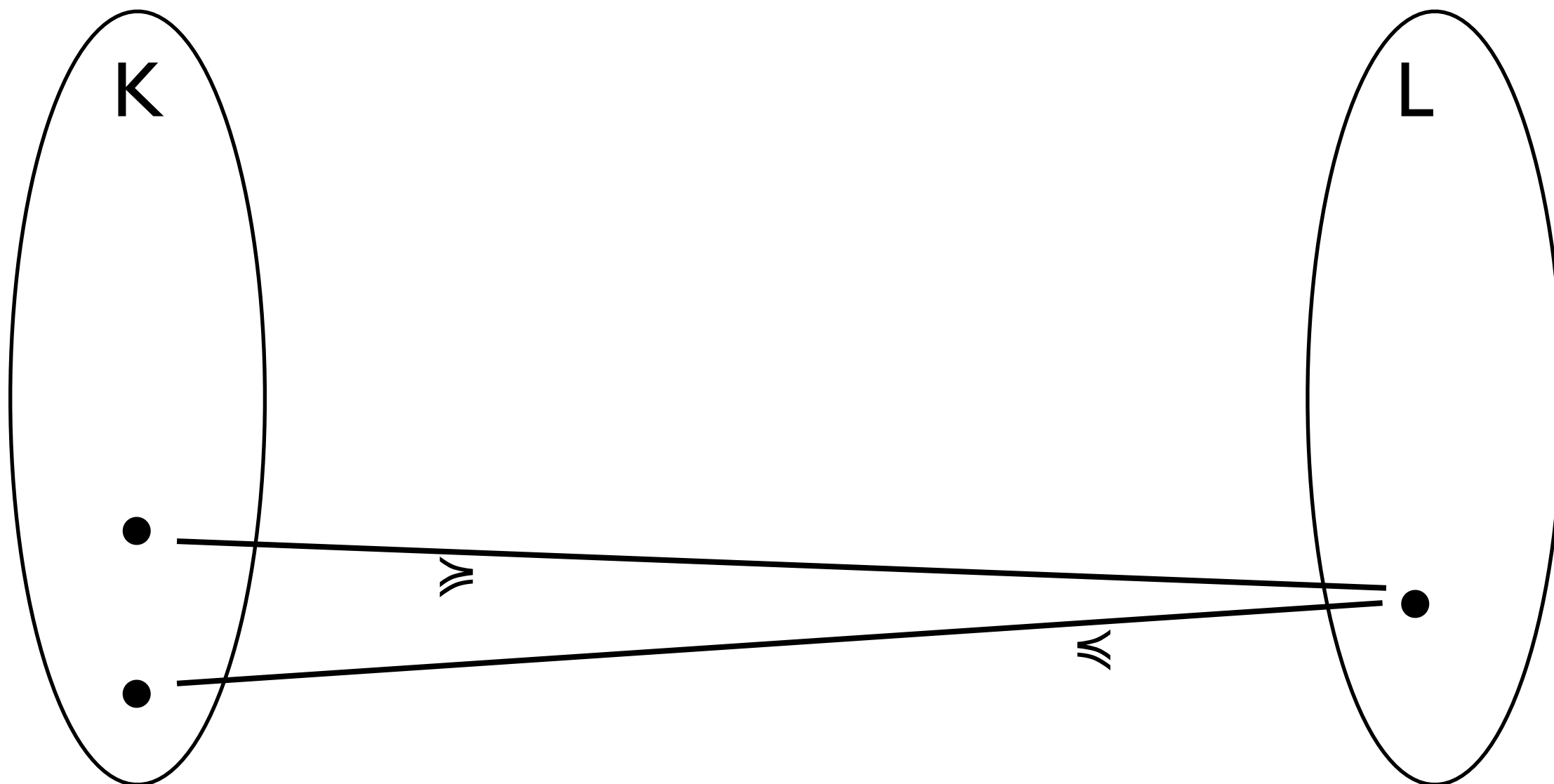
Infinite zigzag



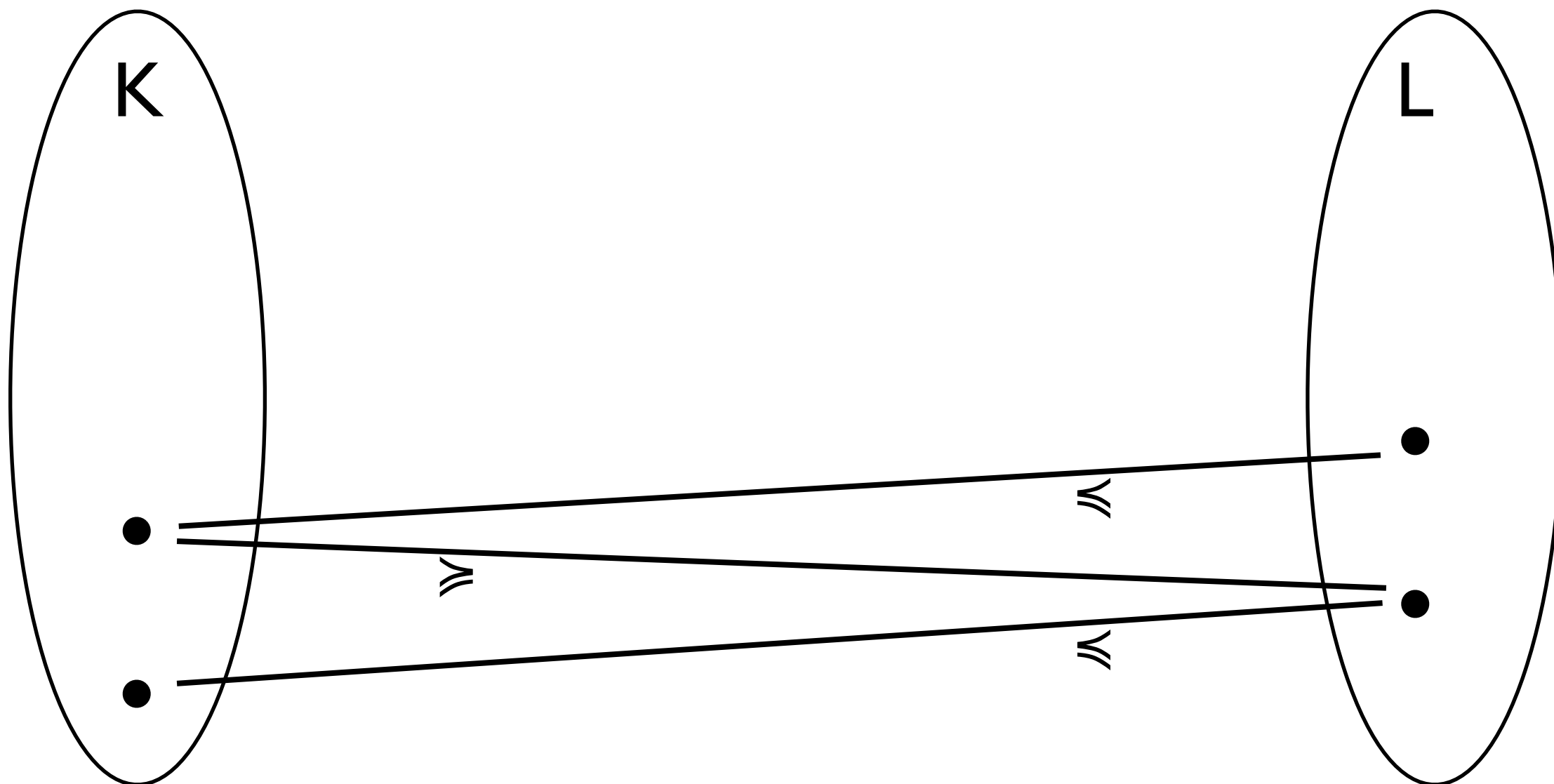
Infinite zigzag



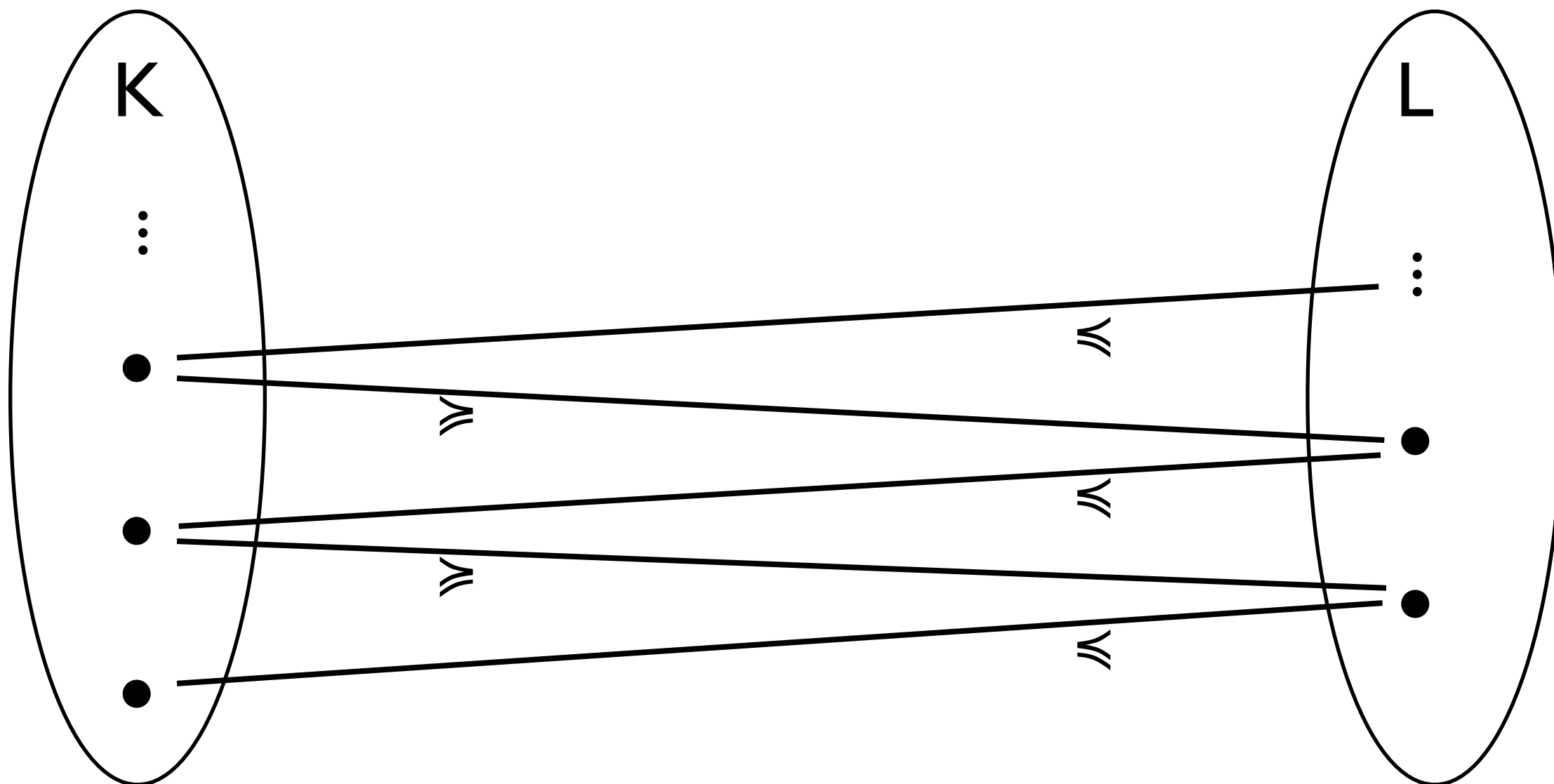
Infinite zigzag



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For **any** two word languages K and L the following conditions are equivalent:

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Second main result

For **any** two word languages K and L the following conditions are equivalent:

- 1) K and L are separable by piecewise testable languages
- 2) there is no infinite zigzag between K and L
- 3) K and L are layered separable by pieces

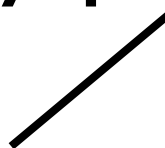
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$$\Sigma^* a_1 \Sigma^* a_2 \Sigma^* \dots \Sigma^* a_n \Sigma^*$$

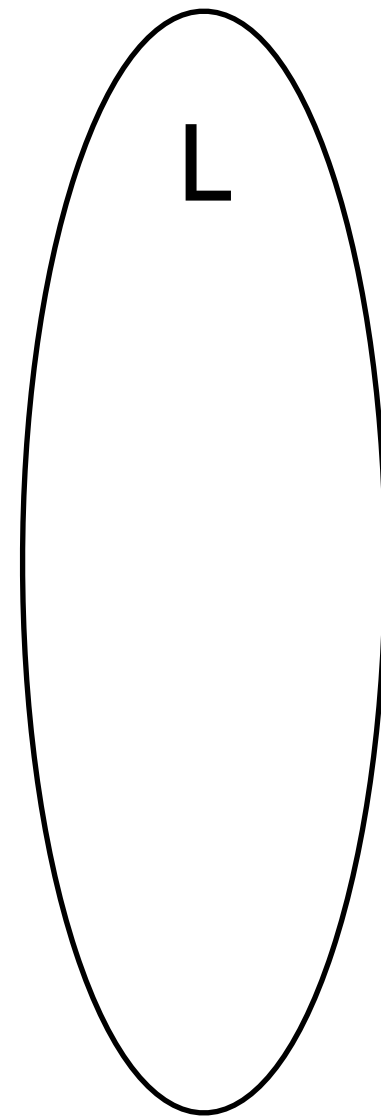
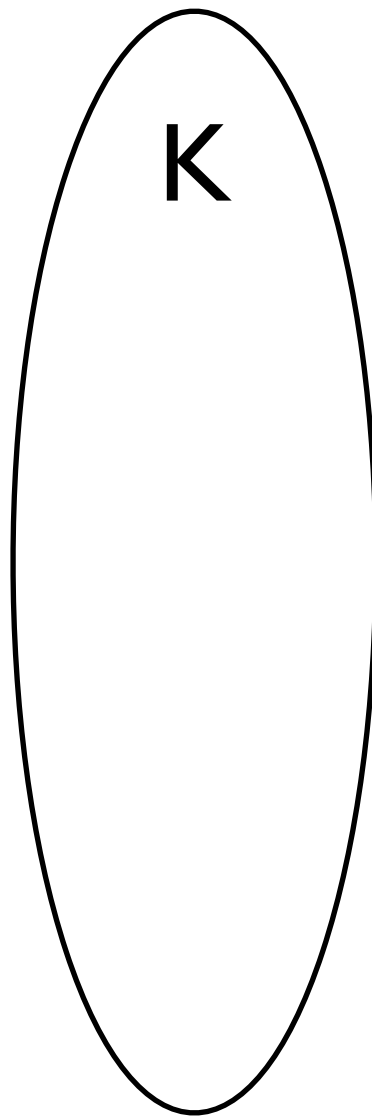
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Layered separability

Layered separability



S_I

Layered separability

K

L



The diagram illustrates the concept of layered separability. It features two large, overlapping, light-blue rounded rectangles. The top-left rectangle is labeled S_1 in white text, and the top-right rectangle is labeled S_2 in white text. The intersection of these two rectangles is shaded in a darker blue. Within the intersection, the text "Layered separability" is written in a large, bold, dark blue font. Below the intersection, there are two vertical, black-outlined ellipses. The left ellipse is labeled "K" in black text, and the right ellipse is labeled "L" in black text. Both ellipses are positioned such that they overlap with the intersection of S_1 and S_2 .

Layered separability

S_1

S_2

K

L

Layered separability



The diagram consists of three overlapping, horizontally-oriented ellipses labeled S_1 , S_2 , and S_3 . S_1 is the top-most ellipse, S_2 is the middle one, and S_3 is the bottom-most one. They overlap in a way that creates a central region where all three intersect. Below these three ellipses are two vertical ellipses, labeled K and L . Ellipse K is on the left and ellipse L is on the right. Both K and L overlap with the bottom portion of the S_1 , S_2 , and S_3 ellipses.

S_1

S_2

S_3

K

L

Layered separability



The diagram shows four overlapping ellipses labeled S_1 , S_2 , S_3 , and S_4 . S_1 is at the top, S_2 is to the right, S_3 is to the left, and S_4 is at the bottom. The central area where all four ellipses overlap is the darkest blue. Two vertical ellipses, labeled K and L , are drawn over the diagram. K is on the left, enclosing the intersection of S_1 and S_3 . L is on the right, enclosing the intersection of S_2 and S_4 .

S_1

S_2

S_3

S_4

K

L

Layered separability



The diagram consists of five overlapping ellipses labeled S_1 , S_2 , S_3 , S_4 , and S_5 . S_1 is at the top, S_2 is to the right, S_3 is to the left, S_4 is to the right, and S_5 is at the bottom. The central area where all ellipses overlap is the darkest blue. Two vertical ellipses, labeled K and L , are drawn over the diagram. K is on the left, enclosing the intersection of S_3 and S_5 . L is on the right, enclosing the intersection of S_2 and S_4 .

S_1

S_2

S_3

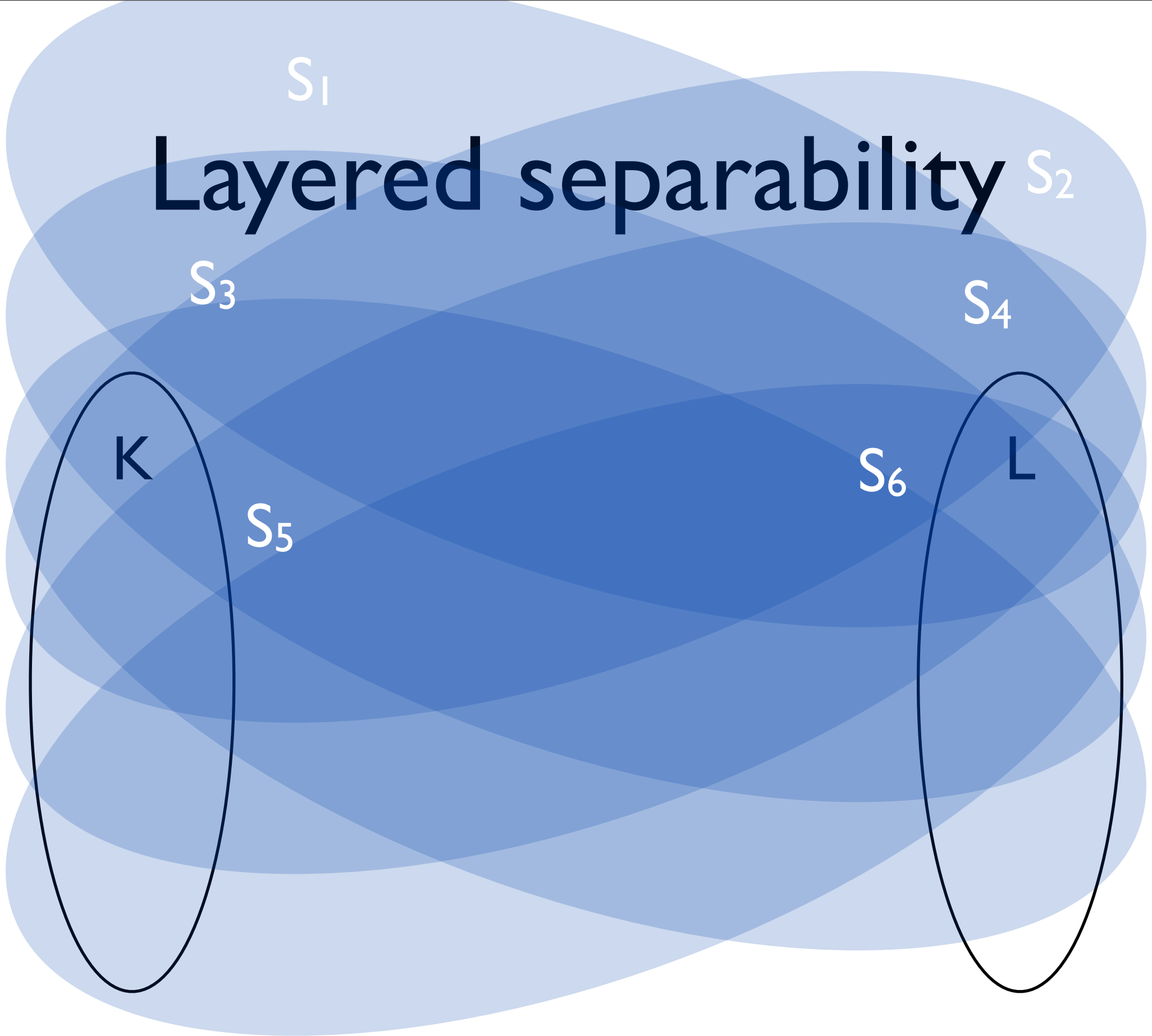
S_4

K

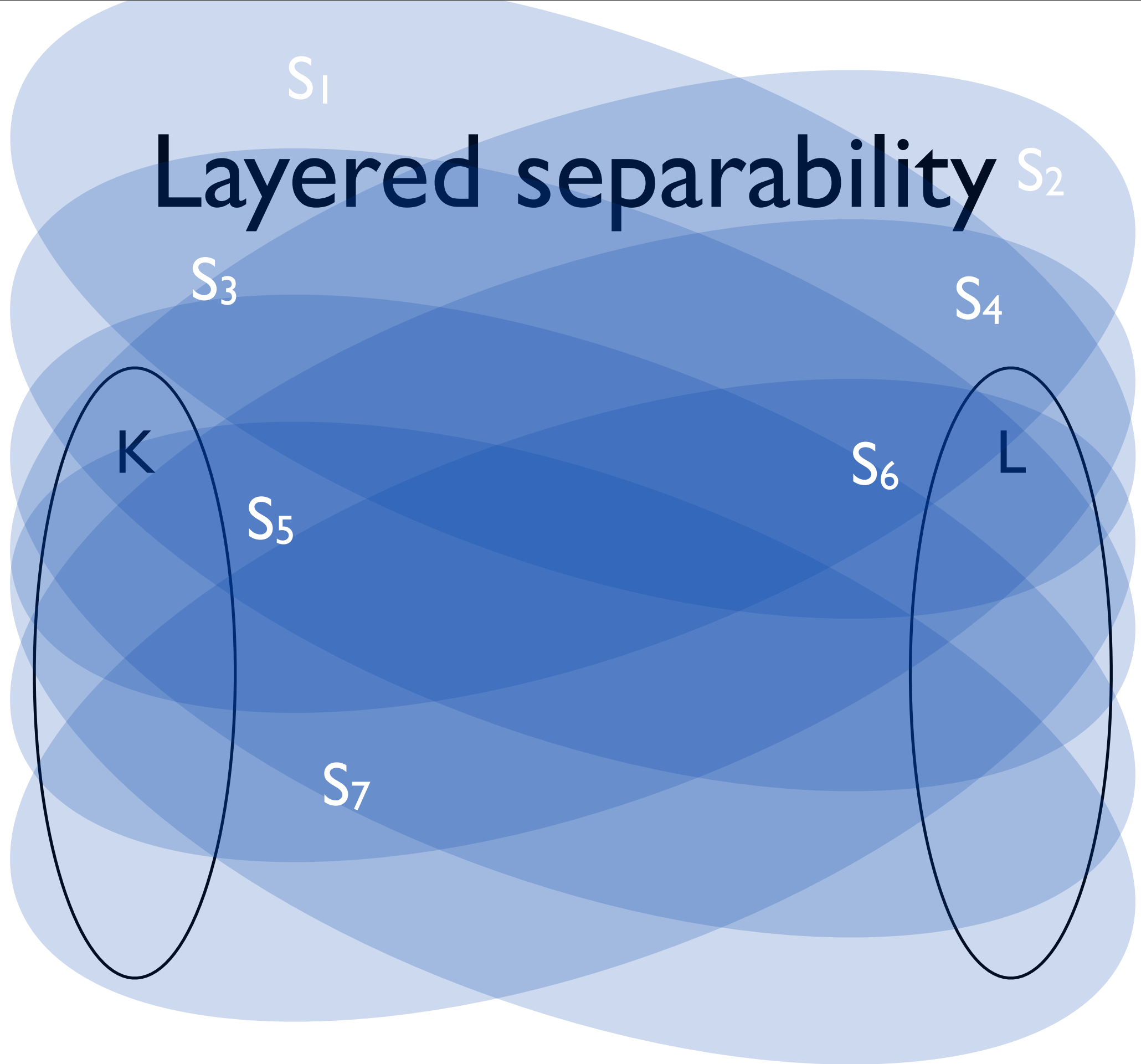
L

S_5

Layered separability



Layered separability



Second main result

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- 3) K and L are layered separable by pieces

Effective characterization

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Effective characterization

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Effective characterization

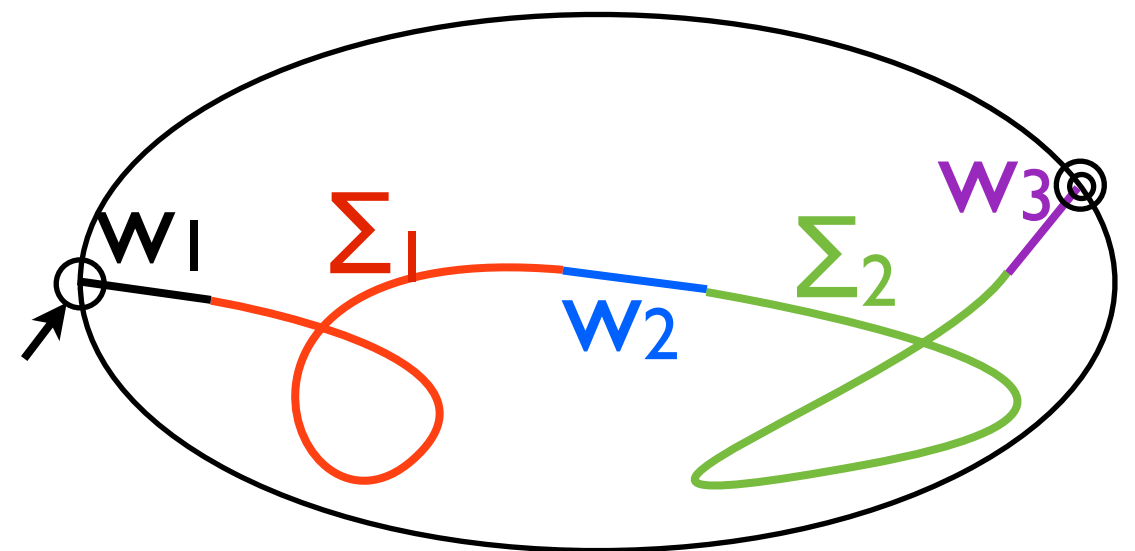
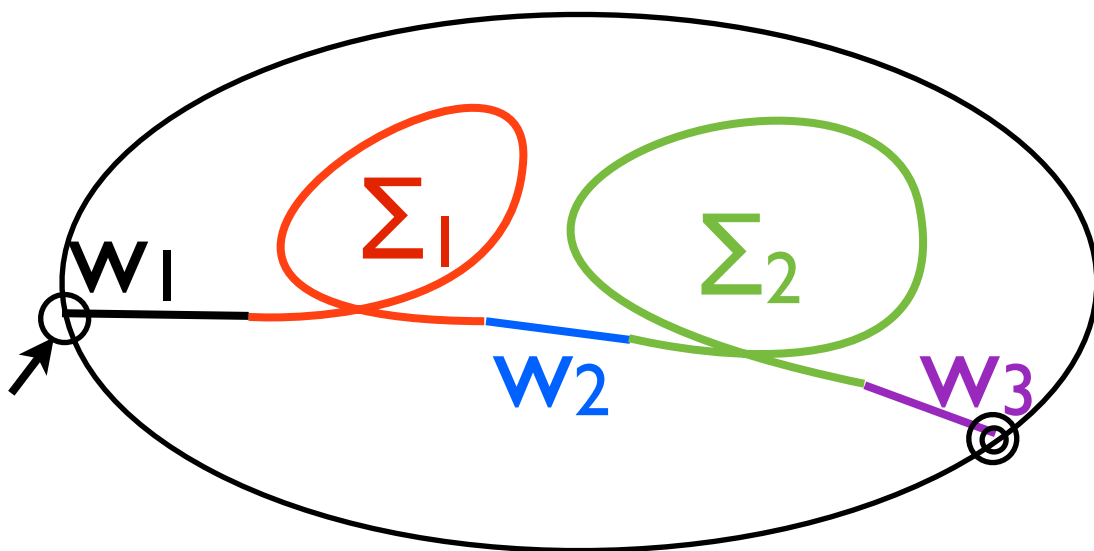
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- 2) in both automata of K and L the following pattern occurs:

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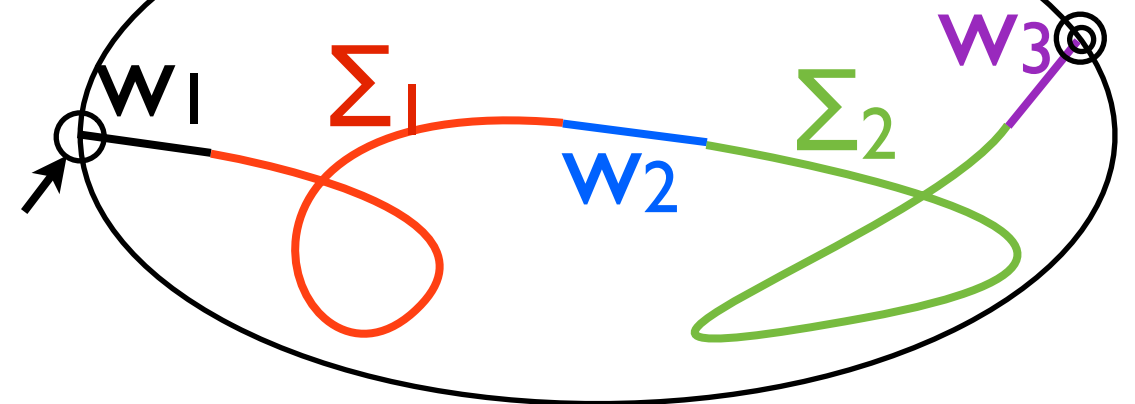
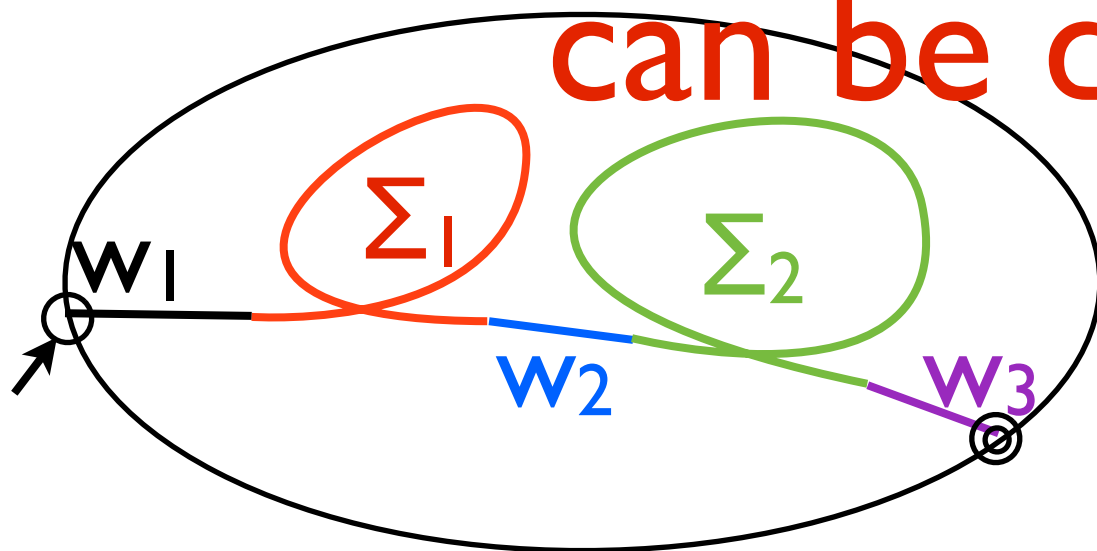


Effective characterization

For two regular languages K and L
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- 1) there is an infinite zigzag between K and L
- 2) in both automata of K and L the following pattern occurs:

can be decided in PTIME



Other variants

Other variants

Separability Problem

Other variants

Separability Problem

Other variants

Separability Problem

pieces			

Other variants

Separability Problem

	single		
pieces			

Other variants

Separability Problem

	single	unions	
pieces			

Other variants

Separability Problem

	single	unions	boolean combinations
pieces			

Other variants

Separability Problem

	single	unions	boolean combinations
pieces			PTIME

Other variants

Separability Problem

	single	unions	boolean combinations
pieces		PTIME	PTIME

Other variants

Separability Problem

	single	unions	boolean combinations
pieces	NP-comp.	PTIME	PTIME

Other variants

Separability Problem

	single	unions	boolean combinations
pieces	NP-comp.	PTIME	PTIME
suffixes			

Other variants

Separability Problem

	single	unions	boolean combinations
pieces	NP-comp.	PTIME	PTIME
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$\Sigma^* w$

Other variants

Separability Problem

	single	unions	boolean combinations
pieces	NP-comp.	PTIME	PTIME
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Other variants

Separability Problem

	single	unions	boolean combinations
pieces	NP-comp.	PTIME	PTIME
suffixes	PTIME	PTIME	PTIME

Thank you!