How to be both rich and happy: Combining quantitative and qualitative strategic reasoning in multi-player games

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Builds on several existing types of models and logics.

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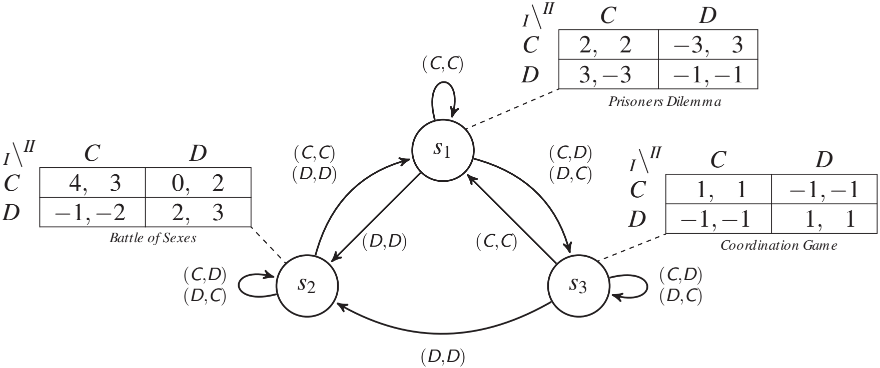
CGMPGs: games with qualitative and quantitative objectives.

Guarded concurrent game model with payoffs: example



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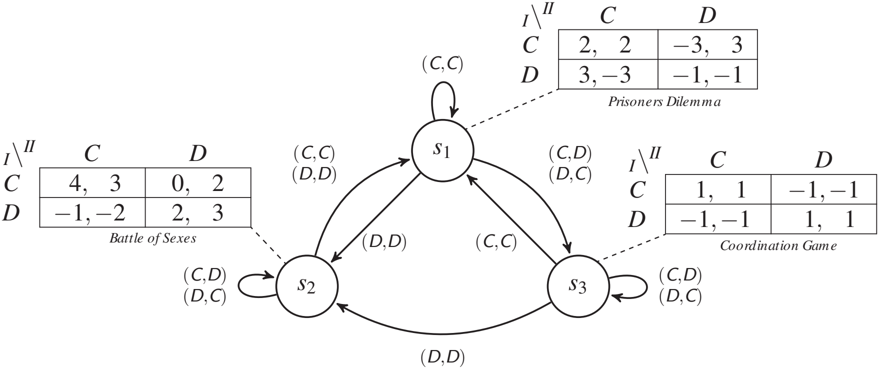

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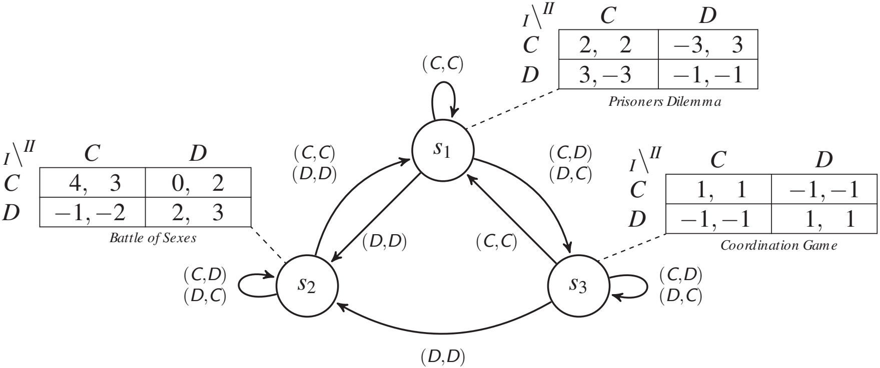

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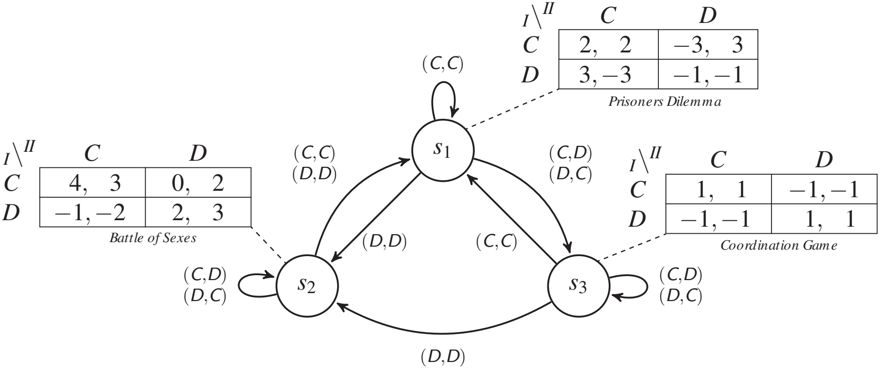

The guards for both players are defined at each state so that the player may:

- apply any action if she has a positive current accumulated utility,
- only apply action C if she has accumulated utility 0,
- must play an action maximizing her minimum payoff in the current game if she has a negative accumulated utility.

Configurations, plays and histories in a GCGMP

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Configuration in $\mathfrak{M} = (\mathcal{S}, \text{payoff}, \{g_{\mathbf{a}}\}_{\mathbf{a}\in\mathbb{A}}, \{d_{\mathbf{a}}\}_{\mathbf{a}\in\mathbb{A}})$:
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Effective strategies: bounded memory strategies determined by transducers with transitions and outputs defined by arithmetical constraints on the current configurations.

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An extension: with arithmetic formulae over entire plays. Requires adding discounting factors on payoffs. Will not be discussed here.

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Ultimately, we define $\mathfrak{M}, c \models \varphi$ iff $\mathfrak{M}, c, 0 \models \varphi$.

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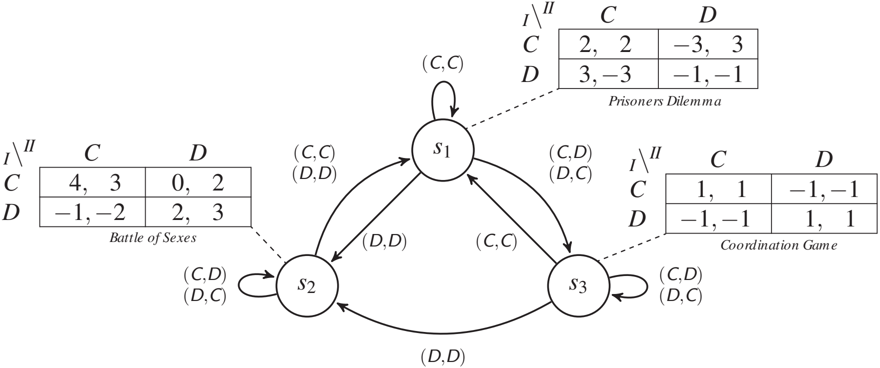
Expressing properties in QATL*: more examples



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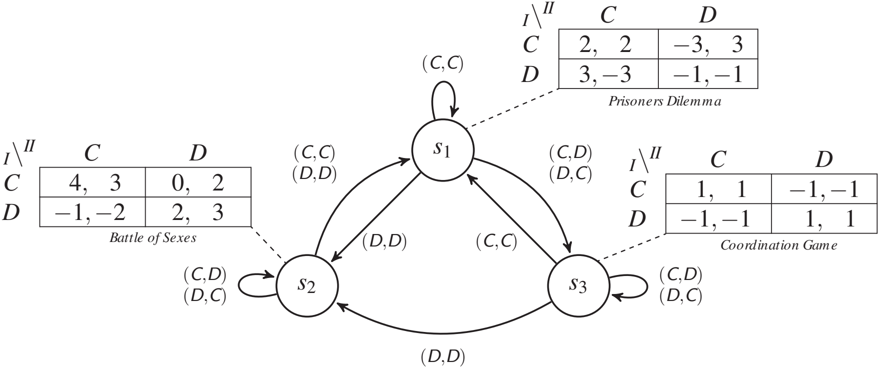

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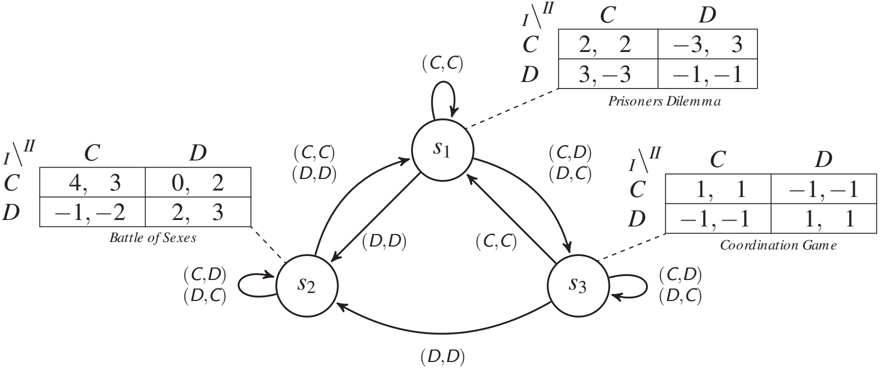

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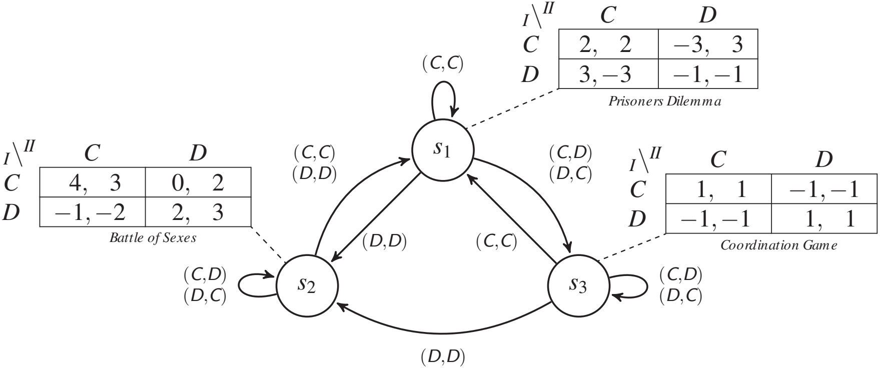

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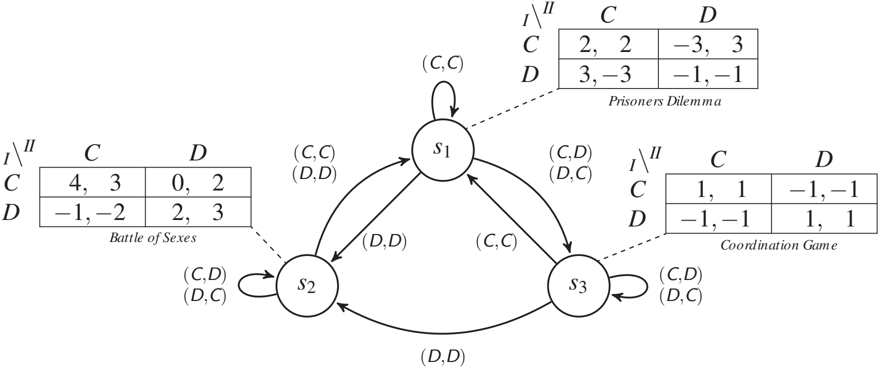

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For any Minsky machine (2-counter automaton) A a finite 2-player GCGMP $\mathfrak{M}^A$ using a proposition `halt` can be constructed so that:

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3. Three players, no guards, non-negative payoffs only.

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