On the Elementary Theory of Linear Order by H. Läuchli; J. Leonard
Review by: Julia Robinson
The Journal of Symbolic Logic, Vol. 33, No. 2 (Jun., 1968), p. 287
Published by: Association for Symbolic Logic
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The paper concludes with some open problems (and a conjecture which Morley has informed the reviewer to be false).

The style and balance of the paper are very good. E. G. K. LOPEZ-ESCOBAR

H. JEROME KEISLER. *Unions of relational systems.* ***Proceedings of the American Mathematical Society,*** vol. 15 (1964), pp. 540–545.

JAN MYCIELSKI. *On unions of denumerable models.* ***Algébra i logika, Séminar,*** vol. 4 no. 2 (1965), pp. 57–58.

Keisler proves his characterization theorem for $\forall_n\exists$-classes, i.e., classes of all models of a set of $\forall\exists$-sentences each of which has exactly n universal quantifiers. The proof, which employs a quite general technique, uses the "special structures" of Morley-Vaught (XXXII 535). Keisler's discussion here includes proofs of almost everything needed except for an existence theorem for special structures and some results from Tarski's study of universal classes (XXI 405).

Let $n < \omega$. Definition: A structure $\mathscr{A}$ is the n-union of a class of structures $\mathbf{K}$ if $\mathscr{A} = \bigcup \mathbf{K}$ and for any $x \in {}^n|\mathscr{A}|$ there is $\mathscr{B} \in \mathbf{K}$ with $x \in {}^n|\mathscr{B}|$. ($\mathscr{A} = \bigcup\mathbf{K}$ implies $\mathbf{K} \subseteq \mathbf{S}(\mathscr{A})$, the class of substructures of $\mathscr{A}$.)

THEOREM. Let $\mathbf{K} \in \mathbf{EC}_\Delta$. Then (i) if $\mathscr{A}$ is the n-union of a subset of $\mathbf{K}$ then $\mathscr{A} \in \mathbf{K}$ iff (ii) $\mathbf{K}$ is the class of all models of a set of $\forall_n\exists$-sentences.

The crux of the proof that (i) implies (ii) is Lemma 4. Let κ be the cardinal of the set of all sentences in a language for $\mathbf{K}$. Let $\mathscr{B}$ be either a finite structure or a special structure of power at least κ. Then if $\mathscr{B}$ is a model of the $\forall_n\exists$-theory of $\mathbf{K}$, then $\mathscr{B}$ is the n-union of $\mathbf{S}(\mathscr{B}) \cap \mathbf{K}$. The proof of this lemma is elegant.

By the existence theorem of Morley-Vaught stated as Lemma 2, any model $\mathscr{A}$ of the $\forall_n\exists$-theory of $\mathbf{K}$ is elementarily equivalent to a $\mathscr{B}$ as in Lemma 4. Hence the difficult half of the theorem.

Mycielski proves the following result. Let $\mathbf{K}$ be a class of denumerable systems such that $\bigcup\mathbf{K}$ exists. If $\mathbf{K}$ is closed under denumerable union, then any elementary statement holding throughout $\mathbf{K}$ holds in $\bigcup\mathbf{K}$; in fact, for each $\mathscr{A} \in \mathbf{K}$ there is an extension $\mathscr{B} \in \mathbf{K}$ which is an elementary substructure of $\bigcup\mathbf{K}$. It is observed that there is an analogue for higher powers.

MARTIN HELLING

H. LÄUCHLI and J. LEONARD. *On the elementary theory of linear order.* ***Fundamenta mathematicae,*** vol. 59 (1966), pp. 109–116.

Let L be the first-order language with identity and one binary predicate $<$. Let T be the theory over L with the axioms: $\sim u < u$, $u < v \wedge v < w \rightarrow u < w$, and $u = v \vee u < v \vee v < u$. The authors prove that T is decidable by showing that (1) every sentence of L consistent with T has a model with order type belonging to a particular set M of denumerable order types and (2) there is a method for deciding the truth-value of an arbitrary sentence Φ of L in $\langle A, <\rangle$ for any A whose order type is in M. M is the least class which contains 1 and is closed under the operations $\alpha + \beta$, $\alpha\cdot\omega$, $\alpha\cdot\omega^*$, and σF. Here F is a non-empty set of order types of M and σF is the order type of a set A such that there is a partition of A of type η into segments whose order types belong to F and such that between any two of the segments there is some segment of each type in F. Skolem 2474 showed that σF is uniquely determined for each F.

The proofs are based on Fraïssé's theorem that two theories A and B are elementarily equivalent if and only if $A \equiv_n B$ for every n (XXII 371). Ramsey's combinatorial theorem is also used. The decidability of T was first obtained by a different method by Ehrenfeucht (see ***Notices of the American Mathematical Society,*** vol. 6 (1959), pp. 268–269).

Corrections. Page 110, line 16, for "refutable," read "unprovable," and page 113, next-to-last line of text, for Σ_{α_j}, read Σ_{α_i}. JULIA ROBINSON

DANA SCOTT and PATRICK SUPPES. *Foundational aspects of theories of measurement.* ***The journal of symbolic logic,*** vol. 23 no. 2 (for 1958, pub. 1959), pp. 113–128. Reprinted in ***Readings in mathematical psychology,*** Volume I, edited by R. Duncan Luce, Robert R. Bush, and Eugene Galanter, John Wiley and Sons, Inc., New York and London 1963, pp. 212–227.

In §1 the authors define a *theory of measurement.* They are concerned with quantitative concepts of empirical sciences. The measurability of such a concept is supposed to depend on

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