MATRIX-GEOMETRIC SOLUTIONS TO STOCHASTIC MODELS

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This is a survey of material on matrix-geometric solutions to stochastic models. The main result is that the class of irreducible, positive recurrent Markov chains which have a block-partitioned structure of the form

$$\begin{vmatrix} B_0 & A_0 & 0 & 0 & 0 & 0 & \cdots \\ B_1 & A_1 & A_0 & 0 & 0 & 0 & \cdots \\ B_2 & A_2 & A_1 & A_0 & 0 & 0 & \cdots \\ & \cdots\cdots & & & & & \end{vmatrix}$$

have an invariant probability vector that is matrix-geometric, that is of the form

$$x_0,\ x_0R,\ x_0R^2,\ x_0R^3,\ \ldots$$

where the matrix R is the minimal nonnegative solution to a nonlinear matrix equation. The matrix R has an interesting probabilistic interpretation and many other quantities related to the Markov chain may be expressed in terms of the matrix R and the vector x_0. The matrix R may be evaluated by numerical procedures. This result has direct applications in a rich variety of stochastic models, particularly in the theory of queues and dams.

The utility of the general results for the analysis and numerical solution of such models will be illustrated by the consideration of a message queue in which some messages leave behind a secondary job, stored in a finite buffer of capacity N. Whenever the buffer becomes full, the server must process a number K of the secondary jobs. By use of the matrix-geometric form of the steady-state probabilities in an interactive algorithm, it is possible to determine values of K and N which meet certain desirable design criteria.

Operations Research Proceedings 1983