A Survey of Two-Dimensional Automata Theory

Katsushi Inoue and Itsuo Takanami
Department of Electronics, Faculty of Engineering
Yamaguchi Univesity,Ube, 755 Japan

Abstract. The main purpose of this paper is to survey several properties of alternating, nondeterministic, and deterministic two-dimensional Turing machines (including two-dimensional finite automata and marker automata), and to briefly survey cellular types of two-dimensional automata.

1. Introduction

During the past thirty years, many investigations about automata on a one-dimensional tape (i.e., string) have been made (for example, see [25]). On the other hand, since Blum and Hewitt [3] studied two-dimensional finite automata and marker automata, several researchers have been investigating a lot of properties about automata on a two-dimensional tape .

The main purpose of this paper is to survey main results of two-dimensional sequential automata obtained since [3], and to give several open problems. Chapter 2 concerns alternating, nondeterministic, and deterministic two-dimensional Turing machines (including finite automata and marker automata). Section 2.1 gives preliminaries necessary for the subsequent discussions. Section 2.2 gives a difference among alternating, nondeterministic, and deterministic machines. Section 2.3 gives a difference between three-way and four-way machines. Section 2.4 states space complexity results of two-dimensional Turing machines. Sections 2.5 and 2.6 states closure properties and decision problems, respectively. Section 2.7 concerns recognition of connected pictures. Section 2.8 states other topics. Chapter 3 briefly surveys cellular types of two-dimensional automata.

2. Alternating, Nondeterministic, and Deterministic Turing Machines

This chapter concerns alternating, nondeterministic, and deterministic two-dimensional Turing machines, including two-dimensional finite automata and marker automata.

2.1. Preliminaries

Let Σ be a finite set of symbols. A two-dimensional tape over Σ is a two-dimensional rectangular array of elements of Σ. The set of all two-dimensional tapes over Σ is denoted by $\Sigma^{(2)}$.

For a tape $x \in \Sigma^{(2)}$, we let $\ell_1(x)$ be the number of rows of x and $\ell_2(x)$ be the number of columns of x. If $1 \leq i \leq \ell_1(x)$ and $1 \leq j \leq \ell_2(x)$, we let $x(i,j)$ denote the symbol in x with coordinates (i,j). Furthermore, we define

$$x[(i,j),(i',j')],$$

when $1 \leq i \leq i' \leq \ell_1(x)$ and $1 \leq j \leq j' \leq \ell_2(x)$, as the two-dimensional tape z satisfying the following: (i) $\ell_1(z)=i'-i+1$ and $\ell_2(z)=j'-j+1$, (ii) for each k,r $[1 \leq k \leq \ell_1(z), 1 \leq r \leq \ell_2(z)]$, $z(k,r)=x(k+i-1,r+j-1)$.

We now give some definitions of two-dimensional alternating Turing machines.

Definition 2.1. A two-dimensional alternating Turing machine (ATM) is a seven-tuple $M=(Q,q_0,U,F,\Sigma,\Gamma,\delta)$, where (1) Q is a finite set of states, (2) $q_0 \in Q$ is the initial state, (3) $U \subseteq Q$ is the set of universal states, (4) $F \subseteq Q$ is the set of accepting states, (5) Σ is a finite input alphabet ($\# \notin \Sigma$ is the boundary symbol), (6) Γ is a finite storage tape alphabet ($B \in \Gamma$ is the blank symbol), and (7) $\delta \subseteq (Q \times (\Sigma \cup \{\#\}) \times \Gamma) \times (Q \times (\Gamma-\{B\}) \times \{\text{left,right,up,down,no move}\} \times \{\text{left,right,no move}\})$ is the next move relation.

A state q in Q-U is said to be existential. As shown in Fig.1, the machine M has

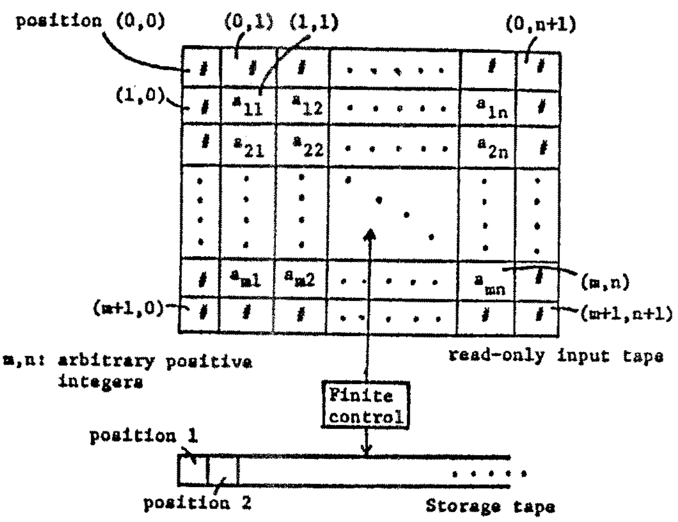

Fig.1. Two-dimensional alternating Turing machine

a read-only rectangular input tape with boundary symbols "#" and one semi-infinite storage tape, initially blank. Of course, M has a finite control, an input head, and a storage tape head. A position is assigned to each cell of the storage tape, as shown in Fig.1. A step of M consists of reading one symbol from each tape, writing a symbol on the storage tape, moving the input and storage heads in specified directions (left,right,up,down,or no move for input head, and left,right, or no move for storage head),and entering a new state, in accordance with the next move relation δ.

A configuration of an ATM $M=(Q,q_0,U,F,\Sigma,\Gamma,\delta)$ is an element of $\Sigma^{(2)}\times(N\cup\{0\})^2\times S_M$, where $S_M=Q\times(\Gamma-\{B\})^*\times N$, and N denotes the set of all positive integers. The first component x of a configuration $c=(x,(i,j),(q,\alpha,k))$ represents the input to M. The second component (i,j) of c represents the input head position. The third component (q,α,k) of c represents the state of the finite control, nonblank contents of the storage tape, and the storage-head position. If q is the state associated with configuration c, then c is said to be universal (existential, accepting) configuration if q is a universal (existential, accepting) state. The initial configuration of M on input x is $I_M(x)=(x,(1,1),(q_0,\lambda,1))$, where λ denotes the empty string. We write $c \vdash_M c'$ and say c' is a successor of c if configuration c' follows from configuration c in one step of M, according to the transition rules δ. A computation tree of M is a finite, nonempty labeled tree with the properties,

(1) each node π of the tree is labeled with a configuration $\ell(\pi)$,

(2) if π is an internal node (a nonleaf) of the tree, $\ell(\pi)$ is universal, and $\{c \mid \ell(\pi) \vdash_M c\}=\{c_1,\dots,c_k\}$, then π has exactly k children $\rho_1,\dots,\rho_k$ such that $\ell(\rho_i)=c_i$,

(3) if π is an internal node of the tree and $\ell(\pi)$ is existential, then π has exactly one child ρ such that $\ell(\pi)\vdash_M \ell(\rho)$.

An accepting computation tree of M on x is a computation tree whose root is labeled with $I_M(x)$ and whose leaves are all labeled with accepting configurations. We say that M accepts x if there is an accepting computation tree of M on input x. Define $T(M)=\{x\in\Sigma^{(2)} \mid M \text{ accepts } x\}$.

A three-way two-dimensional alternating Turing machine (TATM) is an ATM whose input head can move left, right, or down, but not up.

A two-dimensional nondeterministic Turing machine (NTM) (a three-way two-dimensional nondeterministic Turing machine (TNTM)) is an ATM (TATM) which has no universal state. A two-dimensional deterministic Turing machine (DTM) (a three-way two-dimensional deterministic Turing machine (TDTM)) is an ATM (TATM) whose configurations each have at most one successor.

Let $L(m,n):N^2\rightarrow R$ be a function with two variables m and n, where R denotes all non-negative real numbers. With each ATM (TATM,NTM,TNTM,DTM,TDTM) M we associate a space complexity function SPACE which takes configuration $c=(x,(i,j),(q,\alpha,k))$

to natural numbers. Let SPACE(c)=the length of α. We say that M is <u>L(m,n) space-bounded</u> if for all $m,n\geq 1$ and for all x with $\ell_1(x)=m$ and $\ell_2(x)=n$, if x is accepted by M, then there is an accepting computation tree of M on input x such that, for each node π of the tree, SPACE($\ell(\pi)$)$\leq \lceil L(m,n) \rceil$. By "ATM(L(m,n))" ("TATM(L(m,n))", "NTM(L(m,n))", "TNTM(L(m,n))", "DTM(L(m,n))", "TDTM(L(m,n))") we denote an L(m,n) space bounded ATM (TATM, NTM, TNTM, DTM, TDTM).

We are also interested in two-dimensional Turing machines M whose input tapes are restricted to square ones. Let $L(m):N\rightarrow R$ be a function with one variable m. We say that M is <u>L(m) space-bounded</u> if for all $m\geq 1$ and for all x with $\ell_1(x)=\ell_2(x)=m$, if x is accepted by M, then there is an accepting computation tree of M on x such that, for each node π of the tree, SPACE($\ell(\pi)$)$\leq L(m)$. By "$ATM^s(L(m))$" ("$TATM^s(L(m))$", "$NTM^s(L(m))$", "$TNTM^s(L(m))$", "$DTM^s(L(m))$", "$TDTM^s(L(m))$") we denote an L(m) space-bounded ATM (TATM, NTM, TNTM, DTM, TDTM) whose input tapes are restricted to square ones.

For any constant $k\geq 0$, a k space-bounded ATM (NTM, DTM) is called a <u>two-dimensional alternating (nondeterministic, deterministic) finite automaton</u>, denoted by "AFA" ("NFA", "DFA"). A three-way AFA (NFA, DFA) is denoted by "TAFA" ("TNFA", "TDFA"). For any positive integer k, a <u>two-dimensional alternating (nondeterministic, deterministic) k-marker automaton</u>, denoted by "AMA(k)" ("NMA(k)", "DMA(k)"), is an AFA (NFA, DFA) which can use k markers on the input tape. By "AFA^s" we denote an AFA whose input tapes are restricted to square ones. NFA^s, DFA^s, etc., have the same meaning. Define

$\mathcal{L}$[ATM(L(m,n))]={T | T=T(M) for some ATM(L(m,n)) M}, and

$\mathcal{L}$[$ATM^s(L(m))$]={T | T=T(M) for some $ATM^s(L(m))$ M}.

$\mathcal{L}$[NTM(L(m,n))], $\mathcal{L}$[$NTM^s(L(m))$], $\mathcal{L}$[AFA], $\mathcal{L}$[AFA^s], etc., have the same meaning.

The following concepts are used in the subsequent discussions.

<u>Definition 2.2</u>. A function $L(m):N\rightarrow R$ ($L(m,n):N^2\rightarrow R$) is called <u>two-dimensionally space constructible</u> if there is a DTM^s (DTM) M such that (i) for each $m\geq 1$ ($m,n\geq 1$) and for each input tape x with $\ell_1(x)=\ell_2(x)=m$ ($\ell_1(x)=m$ and $\ell_2(x)=n$), M uses at most $\lceil L(m) \rceil$ ($\lceil L(m,n) \rceil$) cells of the storage tape, (ii) for each $m\geq 1$ ($m,n\geq 1$), there exists some input tape x with $\ell_1(x)=\ell_2(x)=m$ ($\ell_1(x)=m$ and $\ell_2(x)=n$) on which M halts after its storage head has marked off exactly $\lceil L(m) \rceil$ ($\lceil L(m,n) \rceil$) cells of the storage tape, and (iii) for each $m\geq 1$ ($m,n\geq 1$), when given any input tape x with $\ell_1(x)=\ell_2(x)=m$ ($\ell_1(x)=m$ and $\ell_2(x)=n$), M never halts without marking off exactly $\lceil L(m) \rceil$ ($\lceil L(m,n) \rceil$) cells of the storage tape.

<u>Definition 2.3</u>. A function $L(m):N\rightarrow R$ ($L(m,n):N^2\rightarrow R$) is called <u>two-dimensionally fully space constructible</u> if there exists a DTM^s (DTM) M which, for each $m\geq 1$ ($m,n\geq 1$) and for each input tape x with $\ell_1(x)=\ell_2(x)=m$ ($\ell_1(x)=m$ and $\ell_2(x)=n$), makes use of exactly $\lceil L(m) \rceil$ ($\lceil L(m,n) \rceil$) cells of the storage tape and halts.

<u>Notation 2.1</u>. Let f(n) and g(n) be any functions with one variable n. We write $f(n) \ll g(n)$ when $\lim_{n\to\infty} f(n)/g(n)=0$.

2.2. A Difference among Alternating, Nondeterministic, and Deterministic Machines

This section states a difference among the accepting powers of alternating, nondeterministic, and deterministic machines. For the one-dimensional case, it is well known [11,24,69] that the following theorem holds.

Theorem 2.1. For any function $L(n) \ll \log\log n$, $L(n)$ space-bounded two-way alternating, nondeterministic, and deterministic Turing machines are all equivalent to one-way deterministic finite automata in accepting power.

We first show that a different situation occurs for the two-dimensional case. Let $T_1=\{x \in \{0,1\}^{(2)} \mid \exists m \geq 1 [\ \ell_1(x)=\ell_2(x)=m\ \&\ \exists i (1 \leq i \leq m-1)[x[(i,1),(i,m)]=x[(m,1),(m,m)]]]\}$ and $T_2=\{x \in \{0,1\}^{(2)} \mid \exists m \geq 0 [\ \ell_1(x)=\ell_2(x)=2m+1\ \&\ x(m+1,m+1)=1$ (i.e., the center symbol of x is 1)$]\}$. It is shown in [58,59] that $T_1 \in \mathcal{L}[\mathrm{TAFA^s}]-\mathcal{L}[\mathrm{NTM^s}(L(m))]$ and $T_2 \in \mathcal{L}[\mathrm{TNFA^s}]-\mathcal{L}[\mathrm{DTM^s}(L(m))]$ for any function $L(m) \ll \log m$. Thus we have

Theorem 2.2. For any function $L(m) \ll \log m$, (1) $\mathcal{L}[\mathrm{DTM^s}(L(m))] \subsetneq \mathcal{L}[\mathrm{NTM^s}(L(m))] \subsetneq \mathcal{L}[\mathrm{ATM^s}(L(m))]$, and (2) $\mathcal{L}[\mathrm{TDTM^s}(L(m))] \subsetneq \mathcal{L}[\mathrm{TNTM^s}(L(m))] \subsetneq \mathcal{L}[\mathrm{TATM^s}(L(m))]$.

Corollary 2.1 [3,58,59,89]. $\mathcal{L}[\mathrm{DFA^s}] \subsetneq \mathcal{L}[\mathrm{NFA^s}] \subsetneq \mathcal{L}[\mathrm{AFA^s}]$, and $\mathcal{L}[\mathrm{TDFA^s}] \subsetneq \mathcal{L}[\mathrm{TNFA^s}] \subsetneq \mathcal{L}[\mathrm{TAFA^s}]$.

For the three-way case, we can show that the following stronger results hold.

Theorem 2.3. (1) $\mathcal{L}[\mathrm{TDTM^s}(L(m))] \subsetneq \mathcal{L}[\mathrm{TNTM^s}(L(m))] \subsetneq \mathcal{L}[\mathrm{TATM^s}(L(m))]$ for any function $L(m) \ll m^2$, (2) $\mathcal{L}[\mathrm{TDTM}(L(m,n))] \subsetneq \mathcal{L}[\mathrm{TNTM}(L(m,n))] \subsetneq \mathcal{L}[\mathrm{TATM}(L(m,n))]$ for each $L(m,n) \in \{f(m) \times g(n),\ f(m)+g(n)\}$, where $f(m):N \rightarrow R$ is a function such that $f(m) \ll m$, and $g(n):N \rightarrow R$ is a monotone nondecreasing function which is fully space constructible [25], and (3) $\mathcal{L}[\mathrm{TDTM}(L(m,n))] \subsetneq \mathcal{L}[\mathrm{TNTM}(L(m,n))] \subsetneq \mathcal{L}[\mathrm{TATM}(L(m,n))]$ for each $L(m,n) \in \{f(m) \times g(n),\ f(m)+g(n)\}$, where $f(m):N \rightarrow R$ is a function, and $g(n):N \rightarrow R$ is a function such that $g(n) \ll n$.

Proof. (1): See [44,58].

(2): In [44], it is shown that $\mathcal{L}[\mathrm{TDTM}(L(m,n))] \subsetneq \mathcal{L}[\mathrm{TNTM}(L(m,n))]$. Below, we show that $\mathcal{L}[\mathrm{TNTM}(L(m,n))] \subsetneq \mathcal{L}[\mathrm{TATM}(L(m,n))]$. Let $T[g]=\{x \in \{0,1\}^{(2)} \mid \exists n \geq 1 [\ \ell_1(x)=2 \times 2^{\lceil g(n) \rceil}\ \&\ \ell_2(x)=n\ \&$ (the top and bottom halves of x are the same)$]\}$. It is easy to show that $T[g] \in \mathcal{L}[\mathrm{TATM}(g(n))]$. The claim follows from this and from the fact [44] that $T[g] \notin \mathcal{L}[\mathrm{TNTM}(L(m,n))]$ for each $L(m,n) \in \{f(m) \times g(n),\ f(m)+g(n)\}$.

(3): In [44], it is shown that $\mathcal{L}[\mathrm{TDTM}(L(m,n))] \subsetneq \mathcal{L}[\mathrm{TNTM}(L(m,n))]$. Below, we show that $\mathcal{L}[\mathrm{TNTM}(L(m,n))] \subsetneq \mathcal{L}[\mathrm{TATM}(L(m,n))]$. Let $T_3=\{x \in \{0,1\}^{(2)} \mid \ell_1(x)=2\ \&$ (the first and second rows of x are the same)$\}$. It is easy to show that $T_3 \in \mathcal{L}[\mathrm{TAFA}]$. The claim follows from this and from the fact [44] that $T_3 \notin \mathcal{L}[\mathrm{TNTM}(L(m,n))]$ for each $L(m,n) \in \{f(m) \times g(n),\ f(m)+g(n)\}$.

For four-way Turing machines on nonsquare tapes, we have

Theorem 2.4. (1) $\mathcal{L}[\mathrm{NTM}(L(m,n))] \subsetneq \mathcal{L}[\mathrm{ATM}(L(m,n))]$ for each $L(m,n) \in \{f(m) \times g(n),\ f(m)+g(n)\}$, where $f(m):N \rightarrow R$ is a function such that $f(m) \ll \log m$, and $g(n):N \rightarrow R$ is a monotone nondecreasing function which is fully space constructible. (2) $\mathcal{L}$

$[NTM(L(m,n))] \subsetneq \mathcal{L}[ATM(L(m,n))]$ for each $L(m,n) \in \{f(m) \times g(n),\ f(m)+g(n)\}$, where $f(m): N \to R$ is a monotone nondecreasing function which is fully space constructible, and $g(n): N \to R$ is a function such that $g(n) \ll \log n$.

Proof. We only prove (1), because the proof of (2) is similar. Let $x \in \{0,1\}^{(2)}$ and $\ell_2(x)=n$ $(n \geq 1)$. When $\ell_1(x)$ is divided by $2^{\lceil g(n) \rceil}$, we call $x[((j-1)2^{\lceil g(n) \rceil}+1,1),(j2^{\lceil g(n) \rceil},n)]$ the j-th g(n)-block of x for each j $(1 \leq j \leq \ell_1(x)/2^{\lceil g(n) \rceil})$. We say that x has exactly k g(n)-blocks if $\ell_2(x)=n$ and $\ell_1(x)=k2^{\lceil g(n) \rceil}$ for some positive integer $k \geq 1$. Let $T(g)=\{x \in \{0,1\}^{(2)} \mid (\exists n \geq 1)(\exists k \geq 2)[(x$ has exactly k g(n)-blocks$)\ \&\ \exists j(2 \leq j \leq k)$[the first and j-th g(n)-blocks of x are identical]]$\}$. It is easy to show that $T(g) \in \mathcal{L}[ATM(g(n))]$. On the other hand, we can show, by using the same technique as in the proof of Lemma 3.3 in [45], that $T(g) \notin \mathcal{L}[NTM(L(m,n))]$ for each $L(m,n) \in \{f(m) \times g(n),\ f(m)+g(n)\}$. Thus (1) follows.

It is well known [3] that one-dimensional 1-marker automata are equivalent to one-dimensional finite automata. For the two-dimensional case, a different situation occurs. Let T_1 be the set described above. We can show that $T_1 \in \mathcal{L}[DMA(1)]-\mathcal{L}[NFA]$. Let $T_4=\{x \in \{0,1\}^{(2)} \mid \exists m \geq 1[\ell_1(x)=2m\ \&\ \ell_2(x)=m\ \&$ (the top and bottom halves of x are the same)$]\}$. It is shown in [29,113] that $T_4 \in \mathcal{L}[NMA(1)]-\mathcal{L}[DMA(1)]$. Thus we have

Theorem 2.5. (1) There exists a set in $\mathcal{L}[DMA(1)]$, but not in $\mathcal{L}[NFA]$, and (2) $\mathcal{L}[DMA(1)] \subsetneq \mathcal{L}[NMA(1)]$.

Savitch [91] showed that for any fully space constructible function $L(n) \geq \log n$, L(n) space-bounded one-dimensional nondeterministic Turing machines can be simulated by $L^2(n)$ space-bounded one-dimensional deterministic Turing machines. By using the same technique as in [91], we can show that a similar result also holds for the two-dimensional case.

Theorem 2.6. For any two-dimensionally fully space constructible function $L(m) \geq \log m$ $(L(m,n) \geq \log m + \log n)$, $\mathcal{L}[NTM^s(L(m))] \subseteq \mathcal{L}[DTM^s(L^2(m))]$ $(\mathcal{L}[NTM(L(m,n))] \subseteq \mathcal{L}[DTM(L^2(m,n))])$.

Open problems: (1) For any two-dimensionally fully space constructible function $L(m) \geq \log m$ $(L(m,n) \geq \log m + \log n)$, $\mathcal{L}[DTM^s(L(m))] \subsetneq \mathcal{L}[NTM^s(L(m))] \subsetneq \mathcal{L}[ATM^s(L(m))]$ $(\mathcal{L}[DTM(L(m,n))] \subsetneq \mathcal{L}[NTM(L(m,n))] \subsetneq \mathcal{L}[ATM(L(m,n))])$? (2) Let f(m) and g(n) be the functions described in Theorem 2.4(1) or Theorem 2.4(2). Then $\mathcal{L}[DTM(L(m,n))] \subsetneq \mathcal{L}[NTM(L(m,n))]$ for each $L(m,n) \in \{f(m) \times g(n),\ f(m)+g(n)\}$? (3) Is there a set in $\mathcal{L}[NFA]$, but not in $\mathcal{L}[DMA(1)]$? (4) For any $k \geq 1$, $\mathcal{L}[DMA(k)] \subsetneq \mathcal{L}[NMA(k)] \subsetneq \mathcal{L}[AMA(k)]$?

2.3. Three-way versus Four-way

This section states a relationship between the accepting powers of three-way machines and four-way machines.

As shown in Theorem 2.1, for the one-dimensional case, $L(n)$ space-bounded one-way and two-way Turing machines are equivalent for any $L(n) \ll \log\log n$. We shall below show that a different situation occurs for the two-dimensional case. Let $T_5=\{x \in \{0,1\}^{(2)} \mid \exists m \geq 1 \ [\ \ell_1(x)=\ell_2(x)=2m\ \&\ (x[(1,1),(1,m)]$ is the reversal of $x[(1,m+1),(1,2m)])]\}$. It is shown in [64] that $T_5 \in \mathcal{L}[\mathrm{DFA}^s]-\mathcal{L}[\mathrm{TATM}^s(L(m))]$ for any function $L(m) \ll \log m$. On the other hand, as stated in Section 2.2, $T_1 \in \mathcal{L}[\mathrm{TAFA}^s]-\mathcal{L}[\mathrm{NTM}^s(L(m))]$ for any $L(m) \ll \log m$. From these facts, for example, we have

Theorem 2.7. For any function $L(m) \ll \log m$, (1) $\mathcal{L}[\mathrm{TXTM}^s(L(m))] \subsetneq \mathcal{L}[\mathrm{XTM}^s(L(m))]$ for each $X \in \{D,N,A\}$, (2) $\mathcal{L}[\mathrm{DTM}^s(L(m))]$ is incomparable with $\mathcal{L}[\mathrm{TNTM}^s(L(m))]$ and $\mathcal{L}[\mathrm{TATM}^s(L(m))]$, and (3) $\mathcal{L}[\mathrm{NTM}^s(L(m))]$ is incomparable with $\mathcal{L}[\mathrm{TATM}^s(L(m))]$.

Remark 2.1. It is shown in [44] that Theorem 2.7(1) can be strengthened as follows: " $\mathcal{L}[\mathrm{TXTM}^s(L(m))] \subsetneq \mathcal{L}[\mathrm{XTM}^s(L(m))]$ for each $X \in \{D,N\}$ and each function $L(m) \ll m^2$." It is obvious that $\mathcal{L}[\mathrm{TXTM}^s(L(m))]=\mathcal{L}[\mathrm{XTM}^s(L(m))]$ for each $L(m) \geq m^2$.

Remark 2.2. By using the same technique as in the proof of the fact [74] that $L(n)$ space-bounded one-way and two-way alternating Turing machines are equivalent for any $L(n) \geq \log n$, we can show that $\mathcal{L}[\mathrm{TATM}^s(L(m))]=\mathcal{L}[\mathrm{ATM}^s(L(m))]$ for any function $L(m) \geq \log m$.

For nonsquare tapes, we have

Theorem 2.8. (1) $\mathcal{L}[\mathrm{TXTM}(L(m,n))] \subsetneq \mathcal{L}[\mathrm{XTM}(L(m,n))]$ for each $X \in \{D,N\}$ and each $L(m,n) \in \{f(m) \times g(n),\ f(m)+g(n)\}$, where $f(m)$ and $g(n)$ are the functions described in Theorem 2.3(2) or Theorem 2.3(3), (2) $\mathcal{L}[\mathrm{TATM}(L(m,n))] \subsetneq \mathcal{L}[\mathrm{ATM}(L(m,n))]$ for each $L(m,n) \in \{f(m) \times g(n),\ f(m)+g(n)\}$, where $f(m): N \rightarrow R$ is a function such that $f(m) \ll \log m$, and $g(n): N \rightarrow R$ is a monotone nondecreasing function which is fully space constructible, and (3) $\mathcal{L}[\mathrm{TATM}(L(m,n))]=\mathcal{L}[\mathrm{ATM}(L(m,n))]$ for any function $L(m,n) \geq \log m$.

Proof. See [44] for (1). We leave the proof of (3) to the reader. We below show that (2) holds. Let $T(g)$ be the set described in the proof of Theorem 2.4 (1). As stated in the proof of Theorem 2.4(1), $T(g) \in \mathcal{L}[\mathrm{ATM}(g(n))]$. On the other hand, we can show, by using the same technique as in the proof of Lemma 4.2 in [64], that $T(g) \notin \mathcal{L}[\mathrm{TATM}(L(m,n))]$ for each $L(m,n) \in \{f(m) \times g(n),\ f(m)+g(n)\}$. Thus it follows that (2) holds.

It is natural to ask how much space is required for three-way machines to simulate four-way machines. The following two theorems answer this question.

Theorem 2.9. (1) $n \log n$ (n^2) space is necessary and sufficient for TDTM's to simulate DFA's (NFA's) (see [48,83]). (2) n space is necessary and sufficient for TNTM's to simulate DFA's and NFA's (see [57]). (3) $2^{\theta(n \log n)}$ ($2^{\theta(n^2)}$) space is necessary and sufficient for TDTM's to simulate DMA(1)'s (NMA(1)'s) (see [67]). (4) $n \log n$ (n^2) space is necessary and sufficient for TNTM's to simulate DMA(1)'s (NMA(1)'s) (see [67]). (In this theorem, note that n denotes the number of columns of tapes.)

Open problems: (1) $\mathcal{L}$[AFA]$\subseteq$ $\mathcal{L}$[TNTM(n)] ? (2) $\mathcal{L}$[AMA(1)]$\subseteq$ $\mathcal{L}$[TNTM($2^{O(n)}$)] ?

2.4. Two-Dimensionally Space Constructible Functions and Space Complexity Results

This section concerns two-dimensionally space constructible functions and space complexity hierarchy. We state these subjects only for square tapes. (See [78,80,82] for the case of nonsquare tapes.) It is well known [24] that in the one-dimensional case, there exists no space constructible function which grows more slowly than the order of loglog n, thus no space hierarchy of language acceptability exists below space complexity loglog n. Below, we state that a different situation occurs for the two-dimensional case.

We consider the following three functions:

(i) $\log^{(1)} m = \begin{cases} 0 & (m=0) \\ \lceil \log_2 m \rceil & (m \geq 1) \end{cases}$

$\log^{(k+1)} m = \log^{(1)}(\log^{(k)} m)$

(ii) $\exp^* 0 = 1$, $\exp^*(m+1) = 2^{\exp^* m}$

(iii) $\log^* m = \min\{x \mid \exp^* x \geq m\}$.

The following theorem demonstrates that there exist two-dimensionally space constructible functions which grow more slowly than the order of loglog m.

Theorem 2.10 [78,82]. The functions $\log^{(k)} m$ (k: any natural number) and $\log^* m$ are two-dimensionally space constructible.

More generally, we have

Theorem 2.11 [78,82]. Let $f(m): N \to N$ be any monotone nondecreasing total recursive function such that $\lim_{m\to\infty} f(m) = \infty$. Then, there exists a two-dimensionally space constructible and monotone nondecreasing function L(m) such that (i) $L(m) < f(m)$ and (ii) $\lim_{m\to\infty} L(m) = \infty$.

It is shown in [105] that there exists no fully space constructible function which grows more slowly than the order of log m. It is unknown whether or not there exists a two-dimensionally fully space constructible function which grows more slowly than the order of log m.

For the one-dimensional case, the following three important theorems concerning space complexity hierarchy of Turing machines are known. (By $\mathcal{L}$[1NTM(L(n))] ($\mathcal{L}$[1DTM(L(n))]) we denote the class of languages accepted by L(n) space-bounded one-dimensional nondeterministic (deterministic) Turing machines [25].)

Theorem 2.12 [102].Let L(n) be a space function. For any constant c>0 and each X $\in$ {D,N}, $\mathcal{L}$[1XTM(L(n))]= $\mathcal{L}$[1XTM($c\cdot$L(n))].

Theorem 2.13 [102]. Let $L_1(n)$ and $L_2(n)$ be any space constructible functions such that $\lim_{i\to\infty} L_1(n_i)/L_2(n_i)=0$ and $L_2(n_i)/\log n_i > k$ $(i=1,2,\cdots)$ for some increasing sequence of natural numbers $\{n_i\}$ and for some constant k>0. Then there exists a language in $\mathcal{L}$[1DTM(L_2(n))], but not in $\mathcal{L}$[1DTM(L_1(n))].

Theorem 2.14 [24]. Let $L_1(n)$ and $L_2(n)$ be space constructible functions such that $\lim_{i\to\infty} L_1(n_i)/L_2(n_i)=0$ and $L_2(n_i)/\log n_i<1/2$ for some increasing sequence of natural numbers $\{n_i\}$. Then there exists a language in $\mathcal{L}[\mathrm{1DTM}(L_2(n))]$, but not in $\mathcal{L}[\mathrm{1DTM}(L_1(n))]$.

By using the ideas similar to those of the proofs of Theorems 2.12 and Theorem 2.13, we can prove the following two-dimensional analogues to these theorems.

Theorem 2.15. Let $L(m)$ be a space function. For any constant $c>0$ and each $X\in\{D,N,A\}$,

$$\mathcal{L}[\mathrm{XTM}^s(L(m))]=\mathcal{L}[\mathrm{XTM}^s(cL(m))].$$

Theorem 2.16 [78,80]. Let $L_2(m)$ be a two-dimensionally space constructible function. Suppose that $\lim_{i\to\infty} L_1(m_i)/L_2(m_i)=0$ and $L_2(m_i)>k\cdot\log m_i$ $(i=1,2,\cdots)$ for some increasing sequence of natural numbers $\{m_i\}$ and for some constant $k>0$. Then there exists a set in $\mathcal{L}[\mathrm{DTM}^s(L_2(m))]$ but not in $\mathcal{L}[\mathrm{DTM}^s(L_1(m))]$.

Recently, It is shown in [28,103] that for each space constructible function $L(n)\geq\log n$, $\mathcal{L}[\mathrm{1NTM}(L(n))]$ is closed under complementation. This result can be extended to the two-dimensional case. By using these facts, we can extend Theorem 2.13 and Theorem 2.16 to the nondeterministic case [21].

The following theorem, which is a two-dimensional analogue to Theorem 2.14, cannot be proved by the same idea as in the proof of Theorem 2.14.

Theorem 2.17 [78,80]. Let $L_2(m)$ be a two-dimensionally space constructible function. Suppose that $\lim_{i\to\infty} L_1(m_i)/L_2(m_i)=0$, $\lim_{i\to\infty} L_2(m_i)=\infty$, and $L_2(m_i)<k\cdot\log m_i$ $(i=1,2,\cdots)$ for some increasing sequence of natural numbers $\{m_i\}$ and for some constant $k>0$. Then there exists a set in $\mathcal{L}[\mathrm{DTM}^s(L_2(m))]$, but not in $\mathcal{L}[\mathrm{DTM}^s(L_1(m))]$.

The following theorem, which is a nondeterministic version of Theorem 2.17, is proved in [60].

Theorem 2.18 [60]. Let $L_2(m)$ be a two-dimensionally space constructible function such that $L_2(m)\leq\log m$. Suppose that $\lim_{m\to\infty} L_1(m)/L_2(m)=0$. Then there exists a set in $\mathcal{L}[\mathrm{NTM}^s(L_2(m))]$ (in fact, in $\mathcal{L}[\mathrm{DTM}^s(L_2(m))]$) but not in $\mathcal{L}[\mathrm{NTM}^s(L_1(m))]$.

From Theorem 2.10 and Theorem 2.18, we have the following corollary, which implies that in the two-dimensional case, there is an infinite hierarchy of acceptabilities even for space complexity classes below loglog m.

Corollary 2.2. For any constant $c>0$, each $k\in N$, and each $X\in\{D,N\}$,

$$\mathcal{L}[\mathrm{XFA}^s]=\mathcal{L}[\mathrm{XTM}^s(c)]\subsetneq\cdots\subsetneq\mathcal{L}[\mathrm{XTM}^s(\log^{(k+1)}m)]\subsetneq\mathcal{L}[\mathrm{XTM}^s(\log^{(k)}m)]\cdots.$$

Open problem: Do results analogous to Theorems 2.16 and 2.17 hold for ATM^s ?

2.5 Closure properties

This section presents only closure properties of the classes of sets accepted by several types of two-dimensional finite automata. (See [41,44,45,48,106] for closure properties of the classes of sets accepted by space-bounded two-dimensional Turing machines.) It is well known [25] that the class of sets accepted by one-dimensional finite automata is closed under many operations , including Boolean operations. We below demonstrate that a different situation occurs for two-dimensional finite automata. We first define several operations over two-dimensional tapes.

<u>Definition 2.4</u>. Let

$$x=\begin{matrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{matrix}, \text{ and } y=\begin{matrix} b_{11} & \cdots & b_{1n'} \\ \vdots & \ddots & \vdots \\ b_{m'1} & \cdots & b_{m'n'} \end{matrix}.$$

Then the <u>rotation</u> x^R of x and the <u>row reflection</u> x^{RR} of x are given by Fig.2 and Fig.3, respectively. A <u>row cyclic shift</u> of x is any two-dimensional tape of the form of Fig.4 for some $1\leq k\leq m$ (not that for k=m this is x itself), and a <u>column cyclic shift</u> of x is any two-dimensional tape of the form of Fig.5 for some $1\leq k\leq n$ (not that for k=n this is x itself). The <u>row catenation</u> $x\ominus y$ is defined only when n=n' and is given by Fig.6, and the <u>column catenation</u> $x\oplus y$ is defined only when m=m' and is given by Fig.7.

<u>Definition 2.5</u>. Let S and S' be two sets of two-dimensional tapes. Then

$S^R=\{x^R \mid x\in S\}$ (<u>rotation</u> of S),

$S^{RR}=\{x^{RR} \mid x\in S\}$ (<u>row reflection</u> of S),

$S^{RC}=\{y \mid$ y is a row cyclic shift of some $x\in S\}$ (<u>row cyclic closure</u> of S),

$S^{CC}=\{y \mid$ y is a column cyclic shift of some $x\in S\}$ (<u>column cyclic closure</u> of S).

$S\ominus S'=\{x\ominus y \mid x \text{ in } S, y \text{ in } S'\}$ (<u>row catenation</u>),

$S\ \ S'=\{x\oplus y \mid x \text{ in } S, y \text{ in } S'\}$ (<u>column catenation</u>),

$S_+=\bigcup_{i\geq 1} S_i$ (<u>row closure</u>),

$S^+=\bigcup_{i\geq 1} S^i$ (<u>column closure</u>),

$$\begin{matrix} a_{m1} & \cdots & a_{11} \\ \vdots & \ddots & \vdots \\ a_{mn} & \cdots & a_{1n} \end{matrix}$$

Fig.2

$$\begin{matrix} a_{m1} & \cdots & a_{mn} \\ \vdots & \ddots & \vdots \\ a_{11} & \cdots & a_{1n} \end{matrix}$$

Fig.3

$$\begin{matrix} a_{1,k+1} & \cdots & a_{1n} & a_{11} & \cdots & a_{1k} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ a_{m,k+1} & \cdots & a_{mn} & a_{m1} & \cdots & a_{mk} \end{matrix}$$

Fig.5

$$\begin{matrix} a_{k+1,1} & \cdots & a_{k+1,n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \\ a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{k1} & \cdots & a_{kn} \end{matrix}$$

Fig.4

$$\begin{matrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \\ b_{11} & \cdots & b_{1n} \\ \vdots & \ddots & \vdots \\ b_{m'1} & \cdots & b_{m'n} \end{matrix}$$

Fig.6

$$\begin{matrix} a_{11} & \cdots & a_{1n} & b_{11} & \cdots & b_{1n'} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} & b_{m1} & \cdots & b_{mn'} \end{matrix}$$

Fig.7

where $S_1=S$, $S_2=S\ominus S,\ldots$, $S_{i+1}=S_i\ominus S$, and $S^1=S$, $S^2=S\oplus S,\ldots$, $S^{i+1}=S^i\oplus S$.

For three-way finite automata, we have

Theorem 2.19. (1) $\mathcal{L}$[TDFA] is not closed under union, intersection, rotation, row reflection, row and column cyclic closures, row and column catenations, or row and column closures [44,45,48,56,106]. (2) $\mathcal{L}$[TNFA] is closed under union, row catenation, and row closure, but not closed under intersection, complementation, rotation, row and column cyclic closures, column catenation, or column closure [44,45,56,106]. (3) $\mathcal{L}$[TAFA] is closed under union and intersection, but not closed under rotation, row reflection, row and column cyclic closures, row and column catenations, or row and column closures [64,68].

Open problems: (1) Are $\mathcal{L}$[TDFA] and $\mathcal{L}$[TAFA] closed under complementation ? (2) Is $\mathcal{L}$[TNFA] closed under row reflection ?

For four-way finite automata, we have

Theorem 2.20. (1) $\mathcal{L}$[DFA] is closed under Boolean operations, rotation and row reflection, but not closed under row and column cyclic closures, row and column catenations, or row and column closures [41,42,51]. (2) $\mathcal{L}$[NFA] is closed under union, intersection, rotation, and row reflection, but not closed under row and column cyclic closures, row and column catenations, or row and column closures [41,51,52]. (3) $\mathcal{L}$[AFA] is closed under union and intersection, rotation, and row reflection.

Remark 2.3. That $\mathcal{L}$[DFA] is closed under Boolean operations can be proved by using the technique in [96].

Open problems: (1) Is $\mathcal{L}$[NFA] closed under complementation ? (2) Is $\mathcal{L}$[AFA] closed under complementation, row and column cyclic closures, row and column catenations, and row and column closures ?

2.6. Decision Problems

This section concerns decision problems of two-dimensional finite automata. It is well known [25] that many decision problems of one-dimensional finite automata are decidable. As suggested by the following theorem, most of decision problems of four-way two-dimensional finite automata are undecidable.

Theorem 2.21 [3,111]. The emptiness and universe problems for DFA's are undecidable even for a one-letter alphabet.

We below state some decision problems of three-way finite automata. For each X$\in$ {D,N,A}, let TXFA(0) denote a TXFA which operates on two-dimensional tapes over a one-letter alphabet. The following two theorems are all that have been obtained for three-way finite automata by now.

Theorem 2.22 [49]. (1) The emptiness and universe problems for TDFA(0)'s are decidable. (2) The emptiness problem for TNFA(0)'s is decidable. (3) The universe, inclusion, and equivalence problems for TNFA's are undecidable.

Theorem 2.23 [70]. (1) The disjointness, inclusion, and equivalence problems for TDFA(0)'s are decidable. (2) The disjointness and inclusion problems for TDFA's are undecidable.

Open problems: (1) Are the emptiness, universe, and equivalence problems for TDFA's decidable ? (2) Are the universe, inclusion, and equivalence problems for TNFA(0)'s and TAFA(0)'s decidable ? (3) Is the emptiness problem for TAFA(0)'s decidable ?

2.7. Recognizability of Connected Pictures

Let T_c be the set of all two-dimensional connected pictures [53,89]. It is interesting to investigate how much space is required for two-dimensional Turing machines to accept T_c. For this problem, we have

Theorem 2.24. (1) n space is necessary and sufficient for TDTM's and TNTM's to accept T_c (see [116]). (2) $T_c \in \mathcal{L}$[AFA] (see [58]). (3) $T_c \in \mathcal{L}$[DMA(1)] (see [3,89]). (4) $T_c^s \notin \mathcal{L}[\mathrm{TATM}^s(L(m))]$ for any $L(m) \ll \log m$, where T_c^s denotes the set of all the square connected pictures (see [64]).

Open problem: $T_c \in \mathcal{L}$[DFA] or $T_c \in \mathcal{L}$[NFA] ?

2.8. Other Topics

In this section, we list up other topics and related references about sequential automata on a two-dimensional tape.

(1) Maze (or labyrinth) search problems: see [1,4,5,7,8,9,10,22,75,104].

(2) Characterizations of one-dimensional languages by two-dimensional automata: see [20,29,32,33].

(3) A relationship between two-dimensional automata and two-dimensional array grammars: see [19,30,73,76,79,84,89,99,115].

(4) Properties of special types of two-dimensional Turing machines (two-dimensional pushdown automata, stack automata, multi-counter automata, multihead automata, and marker automata): see [3,27,46,47,55,56,78,81,89,94,95,113].

(5) Parallel, time, space, and reversal complexities of two-dimensional alternating multihead Turing machines: see [26,50,58,59].

(6) Properties of two-dimensional finite automata over a one-letter alphabet: see [36,40,70].

(7) Properties of two-dimensional automata on a nonrectangular tape: see [77,88,89].

(8) A relationship between two-dimensional alternating finite automata and cellular types of two-dimensional automata: see [62,63,65,66].

The most interesting problem in the future is to investigate time complexity hierarchy of two-dimensional Turing machines.

Two-dimensional (or array) grammars are not discussed here. For this subject, see the excellent book of Rosenfeld [89] and the excellent surveys of Siromoney [97,98].

3. Cellular Types of Two-Dimensional Automata

Many authors investigated language acceptability of one-dimensional cellular automata (for example, see [6,12,14,101,114]). On the other hand, cellular automata on a two-dimensional tape are being investigated not only in the viewpoint of formal language theory but also in the viewpoint of pattern recognition. Cellular automata on a two-dimensional tape can be classified into three types.

The first type, called a <u>two-dimensional cellular automaton</u> (CA for short), is investigated in [2,13,17,29,31,34,35,37,39,53,61-63,65,71,72,87,89,100,112]. CA's make use of two-dimensional cellular arrays. It is shown, for example, that (1) the set T_c of all two-dimensional connected pictures can be accepted by deterministic CA's in linear time [2], (2) the majority problem can be solved by deterministic CA's in linear time, and thus the set of all the two-dimensional tapes over {0,1} with positive Euler number can be accepted by deterministic CA's in linear time [100], (3) the two-dimensional packing problem can be solved by deterministic CA's in linear time [71], (4) NFA's can be simulated by deterministic CA's in linear time [72], and (5) AFA's can be simulated by deterministic CA's in constant state change [62]. (The notion of state change complexity was first introduced in [114]). Many properties of <u>two-dimensional on-line tessellation acceptors</u> (OTA's for short) introduced in [29,35] are investigated in [29,31,34,35,37,39,53,65,112]. The OTA is a restricted type of CA in which cells do not make transitions at every time step; rather, a transition 'wave' passes once diagonally across the array. It is shown, for example, that (1) nondeterministic OTA's are more powerful than NFA's, and deterministic OTA's are incomparable (in accepting power) with NFA's and DFA's [29,35], (2) the set T_c described above cannot be accepted by deterministic OTA's [53], and (3) deterministic OTA's can be used as two-dimensional pattern matching machines [112]. In [17], a generalization of CA's in which each cell is a space-bounded Turing machine rather than a finite automaton, is introduced. Fast algorithms are given for performing various basic image processing tasks by such automata.

The second type of cellular automata on a two-dimensional tape is investigated in [15,30-33,37,38,57,66,78,89,90,92,93,99,107-110]. Two typical models of this type are parallel/sequential array automata (PSA's) [90] and one-dimensional bounded cellular acceptors (BCA's) [92,107-110]. The PSA makes use of one-dimensional (e.g., horizontal) cellular array which can move, as a unit, in the vertical direction, and accepts a tape if the leftmost cell (i.e., the cell which reads the first column of the tape) enters an accepting state in some time. The BCA is a restricted type of one-way PSA in which the cellular array moves downwards each time step, and the BCA accepts a tape if the state configuration of the cellular array just after it has completely scanned the tape is an element of the specified regular set (called the accepting configuration set). It is shown, for example, that (1) nondeterministic one-way PSA's are more powerful than deterministic ones, two-way PSA's are more powerful than one-way PSA's, and T_c is accepted by deterministic one-way PSA's [90], (2) deterministic one-way PSA's are incomparable with NFA's and DFA's [15,48], (3) one-way PSA's are more powerful than OTA's [35], (4) nondeterministic BCA's are equivalent to nondeterministic OTA's, and deterministic BCA's are incomparable with deterministic OTA's and DFA's [31,57]. See [30,99] for a relationship between PSA's and two-dimensional grammars, and see [37,38] for closure properties of PSA's. An extension of BCA's in which the accepting configuration set is a context-free language, context-sensitive language, or phrase structure language, is introduced in [107-110].

The third type, called a pyramid cellular acceptor (PCA), is investigated in [16,18,43,54,85,86,89]. The PCA is a pyramid stack of two-dimensional cellular arrays, where the bottom array has size 2^n by 2^n, the next lowest 2^{n-1} by 2^{n-1}, and so forth, the (n+1)st layer consisting of a single cell, called the root. Each cell has nine neighbors -- four son cells in a 2-by-2 block in the level below, four brother cells in the current level, and one father cell in the level above. The transition function of each cell maps 10-tuples of states into states -- or sets of states, in the nondeterministic case. An input tape is stored as initial states of the bottom array; the upper-level cells are initialized to a quiescent state. The root is the accepting cell. A bottom-up pyramid cellular acceptor (UPCA) is a PCA in which the next state of a cell depends only on the current states of that cell and its four sons. It is shown, for example, that (1) both nondeterministic PCA's and nondeterministic UPCA's are equivalent to nondeterministic CA'S [16,85,89], (2) nondeterministic UPCA's are more powerful than deterministic UPCA's [85,89], (3) nondeterministic UPCA's can simulate nondeterministic OTA's, thus NFA's in O(diameter) time [54,86], and (4) O(diameter $\times$ log diameter) time (O((diameter)^2) time) is necessary for deterministic UPCA's to simulate DFA's (NFA's) [54]. See the excellent book [89] of Rosenfeld for image processing task by PCA's and UPCA's.

Open problems:

(1) Can AFA's be simulated by deterministic CA's in linear time ?

(2) Can AFA's be simulated by nondeterministic OTA's ?

(3) Are deterministic CA's equivalent to nondeterministic CA's ?

(4) Is T_c accepted by nondeterministic OTA's or deterministic UPCA's ?

(5) Is T_c accepted by nondeterministic UPCA's in diameter time ?

(6) Can DFA's, NFA's, or AFA's be simulated by deterministic UPCA's ?

4. Conclusions

In this paper, we surveyed several aspects of two-dimensional automata theory. We believe that there are many problems about two-dimensional automata to solve in the future. We hope that this survey will activate the investigation of two-dimensional automata theory.

References

[1] H.Antelmann, L.Budach and H.A.Rollik, On universal traps, EIK 15, 3 (1979),123-131.

[2] T.Beyer, Recognition of topological invariants by iterative arrays, Ph.D.Thesis, MIT, 1970.

[3] M.Blum and C.Hewitt, Automata on a two-dimensional tape, IEEE Symposium on Switching and Automata Theory, 1967, 155-160.

[4] M.Blum and D.Kozen, On the power of the compass, Proc. of the 19th Annual Symp. on Foundations of Computer Science, 1978,

[5] M.Blum and W.J.Sakoda, On the capability of finite automata in 2 and 3 dimensional space, Proc. of the 18th Annual Symp. on Foundations of Computer Science, 1977, 147-161.

[6] W.Bucher and K.Culik II, On real time and linear time cellular automata, Research Report 115, Institute fur Informationsverarbeitung, Technical University of Graz, 1983.

[7] L.Budach, Environments, labyrinths and Automata, Lecture Notes in Computer Science 56, Springer Verlag, 1977.

[8] __, Automata and Labyrinths, Math. Nachr. 86 (1978), 195-282.

[9] L.Budach and C.Meinel, Environments and automata I, EIK 18, 1/2 (1982), 13-40.

[10] __, Environments and automata II, EIK 18, 3 (1982), 115-139.

[11] A.K.Chandra, D.C.Kozen and L.J.Stockmeyer, Alternation, J.Assoc.Comput.Mach. 28, 1 (1981), 114-133.

[12] C.Choffrut and K.Culik II, On real-time cellular automata and trellis automata, Research Report F114, Institute fur Informationsverarbeitung, Technical University of Graz, 1983.

[13] P.Dietz and S.R.Kosaraju, Recognition of topological equivalence of patterns by array automata, JCSS 2 (1980), 111-116.

[14] C.R.Dyer, One-way bounded cellular automata, Information and Control 44 (1980),261-281.

[15] __, Relation of one-way parallel/sequential automata to 2-d finite automata, Information Sciences 23 (1981), 25-30.

[16] C.R.Dyer and A.Rosenfeld, Cellular pyramids for image analysis, University of Maryland, Computer Science Center, TR-544, AFOSR-77-3271, 1977.

[17] __, Parallel image processing by memory-augmented cellular automata, IEEE Trans. on Pattern Analysis and Machine Intelligence, PAMI-3, 1 (1981), 29-41.

[18] __, Triangle cellular automata, Information and Control 48 (1981), 54-69.

[19] E.M.Ehlers and S.H.Von Solms, A hierarchy of random context grammars and automata, Information Sciences 42 (1987), 1-29.

[20] M.J.Fisher, Two characterizations of the context sensitive languages, IEEE Symp. on Switching and Automata Theory (1969), 149-156.

[21] J.Hartmanis, The structural complexity column, Bulletin of the EATCS, 33 (October 1987), 26-39.

[22] A.Hemmerling, Normed two-plane traps for finite systems of cooperating compass automata, EIK 23, 8/9 (1987),, 453-470.

[23] F.Hoffmann, One pebble does not suffice to search plane labyrinths, Lecture Notes in Computer Science 117 (Fundamentals of Computation Theory), 1981, 433-444.

[24] J.E.Hopcroft and J.D.Ullman, Some results on tape-bounded Turing machines, J.Assoc.Comput.Math. 19, 2 (1972), 283-295.

[25] __, Introduction to Automata Theory, Languages, and Computation, Addison-Wesley, Reading, Mass. 1979.

[26] J.Hromkovic, K.Inoue and I.Takanami, Lower bounds for language recognition on two-dimensional alternating multihead machines, To appear in JCSS.

[27] O.H.Ibarra and R.T.Melson, Some results concerning automata on two-dimensional tapes, Intern.J.Computer Math. 4-A (1974), 269-279.

[28] N.Immerman, Nondeterministic space is closed under complement, submitted for publication.

[29] K.Inoue, Investigations of two-dimensional on-line tessellation acceptors (in Japanese), Ph.D.Thesis, Nagoya University, 1977.

[30] K.Inoue and A.Nakamura, Some notes on parallel sequential array acceptors (in Japanese), IECE of Japan Trans.(D), March 1975, 167-169.

[31] __, On the relation between two-dimensional on-line tessellation acceptors and one-dimensional bounded cellular acceptors (in Japanese), IECE of Japan Trans.(D), September 1976, 613-620.

[32] __, Some properties of one-way parallel sequential array acceptors and two-dimensional one-marker automata (in Japanese), IECE of Japan Trans.(D), September 1976, 682-683.

[33] __, Some properties of parallel sequential array acceptors and two-dimensional two-marker automata (in Japanese), IECE of Japan Trans.(D), September 1976, 680-681.

[34] __, Some properties of two-dimensional on-line tessellation acceptors (in Japanese), IECE of Japan Trans.(D), October 1976, 695-702.

[35] __, Some properties of two-dimensional on-line tessellation acceptors, Information Sciences 13 (1977), 95-121.

[36] __, Some properties of two-dimensional automata with a one-letter alphabet -Recognizability of functions by two-dimensional automata- (in Japanese), IECE of Japan Trans.(D), September 1977, 679-686.

[37] __, Nonclosure properties of two-dimensional on-line tessellation acceptors and one-way parallel sequential array acceptors, IECE of Japan Trans.(E), September 1977, 475-476.

[38] __, Some properties on two-dimensional nondeterministic finite automata and parallel sequential array acceptors (in Japanese), IECE of Japan Trans.(D), November 1977, 990-997.

[39] __, Two-dimensional multipass on-line tessellation acceptors, Information and Control 41, 3 (June 1979), 305-323.

[40] __, Two-dimensional finite automata and unacceptable functions, Intern.J.Computer Math. Section A, 7 (1979), 207-213.

[41] K.Inoue and I.Takanami, A note on closure properties of the classes of sets accepted by tape-bounded two-dimensional Turing machines, Information Sciences 15, 1 (1978), 143-158.

[42] __, Cyclic closure properties of automata on a two-dimensional tape, Information Sciences 15, 1 (1978), 229-242.

[43] __, A note on bottom-up pyramid acceptors, Information Processing Letters 8, 1 (1979), 34-37.

[44] __, Three-way tape-bounded two-dimensional Turing machines, Information Sciences 17, 3 (1979), 195-220.

[45] __, Closure properties of three-way and four-way tape-bounded two-dimensional Turing machines, Information Sciences 18, 3 (1979), 247-265.

[46] __, Three-way two-dimensional multicounter automata, Information Sciences 19, 1 (1979), 1-20.

[47] __, Some properties of three-way two-dimensional multicounter automata over square tapes (in Japanese), IECE of Japan Trans.(D), October 1979, 673-680.

[48] __, A note on deterministic three-way tape-bounded two-dimensional Turing machines, Information Sciences 20, 1 (1980), 41-55.

[49] __, A note on decision problems for three-way two-dimensional finite automata, Information Processing Letters 10, 5 (July 1980), 245-248.

[50] K.Inoue, I.Takanami and J.Hromkovic, A leaf-size hierarchy of two-dimensional alternating Turing machines, Discrete Algorithms and Complexity, Academic Press (1987), 389-404.

[51] K.Inoue, I.Takanami and A.Nakamura, A note on two-dimensional finite automata, Information Processing Letters 7, 1 (1978), 49-52.

[52] __, Nonclosure property of nondeterministic two-dimensional finite automata under cyclic closure, Information Sciences 22, 1 (1980), 45-50.

[53] __, Connected pictures are not recognizable by deterministic two-dimensional on-line tessellation acceptors, Computer Vision, Graphics, and Image Processing 26 (1984), 126-129.

[54] __, A note on time-bounded bottom-up pyramid cellular acceptors, To appear in Information Sciences.

[55] K.Inoue, I.Takanami and H.Taniguchi, Three-way two-dimensional simple multihead finite automata -Hierarchical properties- (in Japanese), IECE of Japan Trans.(D), February 1979, 65-72.

[56] __, Three-way two-dimensional simple multihead finite automata -Closure properties- (in Japanese), IECE of Japan Trans.(D), April 1979, 273-280.

[57] __, The accepting powers of two-dimensional automata over square tapes (in Japanese), IECE of Japan Trans.(D), February 1980, 113-120.

[58] __, Two-dimensional alternating Turing machines, Theoretical Computer Science 27 (1983), 61-83.

[59] A.Ito, k.Inoue, I.Takanami and H.Taniguchi, Two-dimensional alternating Turing machines with only universal states, Information and Control 55, 1-3 (1982), 193-221.

[60] __, A note on space complexity of nondeterministic two-dimensional Turing machines, IECE of Japan Trans.(E), August 1983, 508-509.

[61] __, Hierarchy of the accepting power of cellular space based on the number of state changes (in Japanese), IECE of Japan Trans.(D), September 1985, 1553-1561.

[62] __, Relationships of the accepting powers between cellular space with bounded number of state-changes and other automata (in Japanese), IECE of Japan Trans.(D), September 1985, 1562-1570.

[63] __, State-change bounded rectangular array cellular space acceptors with three-neighbor (in Japanese), IEICE of Japan Trans.(D), December 1987, 2339-2347.

[64] A.Ito, K.Inoue and I.Takanami, A note on three-way two-dimensional alternating Turing machines, Information Sciences 45, 1 (1988), 1-22.

[65] __, Deterministic on-line tessellation acceptors are equivalent to two-way two-dimensional alternating finite automata through 180° rotations, to appear in Theoretical Computer Science.

[66] __, A relationship between one-dimensional bounded cellular acceptors and two-dimensional alternating finite automata, manuscript (1987).

[67] __, The simulation of two-dimensional one-marker automata by three-way two-dimensional Turing machines, to appear in the fifth International Meeting of Young Computer Scientists, November 14-18, 1988, Czechoslovakia.

[68] __, Some closure properties of the class of sets accepted by three-way two-dimensional alternating finite automata, submitted for publication.

[69] K.Iwama, ASPACE(o(loglog n)) is regular, Research Report, KSU/ICS, Institute of Computer Sciences, Kyoto Sangyo University, March 1986.

[70] E.B.Kinber, Three-way automata on rectangular tapes over a one-letter alphabet, Information Sciences 35 (1985), 61-77.

[71] S.R.Kosaraju, On some open problems in the theory of cellular automata, IEEE Trans. on Computers C-23, 6 (1974), 561-565.

[72] __, Fast parallel processing array algorithms for some graph problems, Proc. of the 11th Annual ACM Symp. on Theory of Computing (1979), 231-236.

[73] K.Krithivasan and R.Siromoney, Array automata and operations on array languages, Intern.J.Computer Math. 4-A (1974), 3-30.

[74] R.E.Ladner, R.J.Lipton and L.J.Stockmeyer, Alternating pushdown automata, Proc. of the 19th IEEE Symp. on Foundations of Computer Science (1978), 92-106.

[75] C.Meinel, The importance of plane labyrinths, EIK 18, 7/8 (1982), 419-422.

[76] D.L.Milgram and A.Rosenfeld, Array automata and array grammars, IFIP Congress 71 (1971), Booklet Ta2, 166-173.

[77] D.L.Milgram, A region crossing problem for array-bounded automata, Information and Control 31 (1976), 147-152.

[78] K.Morita, Computational complexity in one- and two-dimensional tape automata, Ph.D. Thesis, Osaka University, 1978.
[79] K.Morita and K.Sugata, Three-way horizontally context-sensitive array grammars (in Japanese), Technical Report No.AL80-66, IECE of Japan (1981).
[80] K.Morita, H.Umeo and K.Sugata, Computational complexity of L(m,n) tape-bounded two-dimensional tape Turing machines (in Japanese), IECE of Japan Trans.(D), November 1977, 982-989.
[81] __, Language recognition abilities of several two-dimensional tape automata and their relation to tape complexities (in Japanese), IECE of Japan Trans.(D), December 1977, 1077-1084.
[82] K.Morita, H.Umeo, H.Ebi and K.Sugata, Lower bounds on tape complexity of two-dimensional tape Turing machines (in Japanese), IECE of Japan Trans.(D), 1978, 381-386.
[83] K.Morita, H.Umeo and K.Sugata, Accepting capability of offside-free two-dimensional marker automata -the simulation of four-way automata by three-way tape-bounded Turing machines-(in Japanese), Technical Report No.AL79-2, IECE of Japan, 1979.
[84] A.Nakamura and K.Aizawa, Acceptors for isometric parallel context-free array languages, Information Processing Letters 13, Nos4-5 (1981), 182-186.
[85] A.Nakamura and C.R.Dyer, Bottom-up cellular pyramids for image analysis, Proc. of the 4th Int.Joint Conf.Pattern Recognition, 1978.
[86] K.Nakazono, K.Morita and K.Sugata, Accepting ability of linear time nondeterministic bottom-up pyramid cellular automata (in japanese), IEICE of Trans.(D), February 1988, 458-461.
[87] J.Pecht, T-recognition of T-languages, a new approach to describe and program the parallel pattern recognition capabilities of d-dimensional tessellation structures, Pattern Recognition 19, 4 (1986), 325-338.
[88] A.Rosenfeld, Some notes on finite-state picture languages, Information and Control 31 (1976), 177-184.
[89] __, Picture Languages (Formal Models for Picture Recognition), Academic Press, New York, 1977.
[90] A.Rosenfeld and D.L.Milgram, Parallel/sequential array automata, Information Processing Letters 2 (1973), 43-46.
[91] W.J.Savitch, Relationships between nondeterministic and deterministic tape complexities, JCSS 4 (1970), 177-192.
[92] S.Seki, Real-time recognition of two-dimensional tapes by cellular automata, Information Sciences 19 (1979), 179-198.
[93] S.M.Selkow, One-pass complexity of digital picture properties, J.Assoc.Comput.Math. 19(2) (1972), 283-295.
[94] A.N.Shah, Pebble automata on arrays, Computer Graphics and Image Processing 3 (1974), 236-246.
[95] __, Pushdown automata on arrays, Information Sciences 25 (1981), 175-193.
[96] M.Sipser, Halting space-bounded computatiopns (Note), Theoretical Computer Science 10 (1980), 335-338.
[97] R.Siromoney, Array languages and Lindenmayer systems -a survey, in 'The Book of L' (eds. G.Rozenberg and A.Salomaa), Springer-Verlag, Berlin, 1985.

[98] __, Advances in array languages, Lecture Notes in Computer Science 291 (Graph-Grammars and Their Application to Computer Science), Ehrig et al. (Eds.), 1987, 549-563.

[99] R.Siromoney and G.Siromoney, Extended controlled table L-arrays, Information and Control 35 (1977), 119-138.

[100] A.R.Smith III, Two-dimensional formal languages and pattern recognition by cellular automata, Proc. of the 12th Switching and Automata Theory (1971), 144-152.

[101] __, Real-time language recognition by one-dimensional cellular automata, JCSS 6 (1972), 233-253.

[102] R.E.Stearns, J.Hartmanis and P.M.Lewis II,, Hierarchies of memory limited computations, IEEE Conf.Rec.on Switching Circuit Theory and Logical Design (1965),, 179-190.

[103] R.Szelepcsenyi, The method of forcing for nondeterministic automata, submitted for publication.

[104] A.Szepietowski, A finite 5-pebble automaton can search every maze, Information Processing Letters 15, 5 (December 1982), 199-204.

[105] __, There are no fully space constructible functions between loglog n and log n, Information Processing Letters 24, 6 (April 1987), 361-362.

[106] __, On three-way two-dimensional Turing machines, manuscript (1987).

[107] H.Taniguchi, K.Inoue, I.Takanami and S.Seki, (k,l)-neighborhood template -type bounded cellular acceptors (in Japanese), IECE of Japan Trans.(D), March 1981, 244-251.

[108] __ , (k,l)-neighborhood template -type bounded cellular acceptors -refinements of hierarchical properties-(in Japanese), IECE of Japan Trans.(D), September 1983, 1062-1069.

[109] __ , Relationship between the accepting powers of (k,l)-neighborhood template -type 1-dimensional bounded cellular acceptors and other types of 2-dimensional automata (in Japanese), IECE of Japan Trans.(D), October 1985, 1711-1718.

[110] __ , Closure properties of (k,l)-neighborhood template -type 1-dimensional bounded cellular acceptors (in Japanese), IECE of Japan Trans.(D), March 1986, 279-290.

[111] K.Taniguchi and T.Kasami, Some decision problems for two-dimensional nonwriting automata (in Japanese), IECE of Japan Trans.(C), 1971, 578-585.

[112] M.Toda, K.Inoue and I.Takanami, Two-dimensional pattern matching by two-dimensional on-line tessellation acceptors, Theoretical Computer Science 24 (1983), 179-194.

[113] H.Umeo, K.Morita and K.Sugata, Pattern recognition by automata on a two-dimensional tape, IECE of Japan Trans.(D), November 1976, 817-824.

[114] R.Vollmar, On cellular automata with a finite number of state changes, Comput.Suppl. 3 (1981),181-191.

[115] P.S.P.Wang, Finite-turn repetitive checking automata and sequential/parallel matrix languages, IEEE Trans. on Computers C-30, 5 (May 1981), 366-370.

[116] Y.Yamamoto, K.Morita and K.Sugata, Space complexity for recognizing connectedness in three-dimensional patterns, IECE of Japan Trans.(E), 1981, 778-785.