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REVIEWS

Numerical cross references are to previous reviews in this JOURNAL or to *A bibliography of symbolic logic* (this JOURNAL, vol. 1, pp. 121–218), or to *Additions and corrections* to the latter (this JOURNAL, vol. 3, pp. 178–212).

References beginning with a Roman numeral are by volume and page to the place at which a publication has previously been reviewed or listed. When necessary in connection with such references, a third number will be added in parentheses, to indicate position on the page. Such a reference is ordinarily to the publication itself, but when so indicated the reference may be to the review or to both the publication and its review. Thus "XXVII 363" will refer to the review beginning on page 363 of volume 27 of this JOURNAL, or to the publication which is there reviewed; "XXVII 368" will refer to one of the reviews or one of the publications reviewed or listed on page 368 of volume 27, with reliance on the context to show which one is meant; "XXXI 283(1)" will refer to the first item listed on page 283 of volume 31, i.e., to Rivetti Barbò's ***L'antinomia del mentitore***; and "XXXI 284(1)" will refer to the first item listed on page 284 of volume 31, i.e., to Rivetti Barbò's *Prefazione*.

References such as 247***18***, 382***5*** are to the entries so numbered in the *Bibliography* Similar references preceded by the letter A or containing the fraction $\frac{1}{2}$ or a decimal point (as A548***1***, 21$\frac{1}{2}$***1***, 321***0.2***) are to the *Additions and corrections*. A reference followed by the letter A is a double reference to an entry of the same number in the *Bibliography* and in the *Additions and corrections*.

A. EHRENFEUCHT and A. MOSTOWSKI. *Models of axiomatic theories admitting automorphisms.* ***Fundamenta mathematicae,*** vol. 43 (1956), pp. 50–68.

This paper made a fundamental contribution to model theory by creating a method for applying in that theory the well-known theorem of Ramsey (295***12***): *If Y is an infinite set and* $Y^{(n)} = C_1 \cup \ldots \cup C_k$ (where $Y^{(n)}$ denotes the set of all n-element subsets of Y), *then there is an infinite subset Z of Y such that, for some $j \leqq k$,* $Z^{(n)} \subseteq C_j$. Other applications of Ramsey's theorem, by methods related to that developed here, were made almost at once in Ehrenfeucht's paper reviewed below, and later by various authors.

The main theorem of the paper concerns models of an arbitrary set Σ of first-order sentences: *If Σ has an infinite model and $(X, \prec)$ is an arbitrary ordering, then there is a model M of Σ such that X is a subset of the universe $|M|$ of M and every automorphism of $(X, \prec)$ can be extended to an automorphism of M.* In the first step of the proof, by "introducing Skolem functions," the theorem is reduced to the case in which Σ consists of universal sentences. Next, adding individual constants $\bar{x}$ for $x \in X$, one considers the set Σ' obtained by adding to Σ all sentences $\bar{x} \not\approx \bar{y}$ $(x, y \in X, x \neq y)$ and all sentences of the form $\varphi(\bar{x}_1, \ldots, \bar{x}_n) \leftrightarrow \varphi(\overline{a(x_1)}, \ldots, \overline{a(x_n)})$ (where a is an automorphism of $(X, \prec)$ and $x_i \in X$). Ramsey's theorem is used to show that every finite subset of Σ' has a model. An application of the compactness theorem then gives a model M' (in which $\bar{x}$ denotes x) whose subalgebra generated by X is the desired M.

At the end of the paper, a rather special strengthening of the main theorem (needed in Ehrenfeucht's paper reviewed below) is established by essentially the same proof. A more general statement, still having essentially the same proof, is the following: (*) In the main theorem, let the symbols of Σ include a unary relation symbol U and a binary relation symbol $<$, and assume Σ has a model M_0 in which $(\mathscr{U}^{M_0}, <^{M_0})$ is an infinite ordering; then in the conclusion we can specify that $(X, \prec)$ is a substructure of $(\mathscr{U}^M, <^M)$. (This observation is well known and not due to the reviewer.)

The style of the paper is precise and readable. However, in the discussion preparing the ground for the proof of the main theorem, notions or facts of general algebra are sometimes made less clear by formulating them in a way that unnecessarily involves metamathematical notions. On the other hand (and less trivially) Morley has noted (*Categoricity in power*, §3, see review below) that the main theorem can alternatively be given a more metamathematical formulation. Morley shows that the following theorem can be proved by methods which are quite close to those of the paper under review but less complicated, and very easily implies the main theorem: If Σ has an infinite model and $(X, \prec)$ is any ordering, then Σ has a model M such that $X \subseteq |M|$ and, for any $x_0 \prec x_1 \prec \ldots \prec x_{n-1}$, any $x'_0 \prec x'_1 \prec \ldots \prec x'_{n-1}$, and any φ, $M \models \varphi[x_0, \ldots, x_{n-1}]$ iff $M \models \varphi[x'_0, \ldots, x'_{n-1}]$. This statement can also be inferred from the main theorem, and indeed a closely related result was first obtained by Ehrenfeucht in such a way—cf. Ehrenfeucht, *Theories having at least continuum many non-isomorphic models in each power*, ***Notices of the American Mathematical Society***, vol. 5 (1958), pp. 680–681. The improvement (*) above, applies to Morley's formulation of the main theorem equally well. R. L. VAUGHT

A. EHRENFEUCHT. *On theories categorical in power.* Ibid., vol. 44 (1957), pp. 241–248.

In this article and, subsequently, in the abstract mentioned in the preceding review, Ehrenfeucht, making use of the results of the paper just reviewed, discovered some important facts regarding theories categorical in power. These discoveries eventually played a role in the (positive) solution by Morley (in the paper cited in the preceding review) of Łoś's problem: Must a theory categorical in one non-denumerable power be categorical in all?

The paper contains three theorems, in each of which T *is assumed to be a theory categorical in a certain infinite power* $2^{\mathfrak{n}}$. The main result of the paper is Theorem 1: *For any model M of T, any formula $\varphi(v_1, \ldots, v_m)$, and any infinite set $A \subseteq |M|$, the m-ary relation φ^M cannot order A and, more generally, cannot be antisymmetric and connected over A.* (An m-ary relation R is antisymmetric and connected over A, if, for any distinct $a_1, \ldots, a_m \in A$, $Ra_{p(1)} \cdots a_{p(m)}$ and $Ra_{q(1)} \cdots a_{q(m)}$ must hold for some permutations p, q of $\{1, \ldots, m\}$.) This result is proved here by an ingenious and very complicated argument. Later work, in Ehrenfeucht's cited abstract, and by the reviewer, Scott, and Morley, culminated in a simpler proof by Morley (loc. cit.) of Theorem 1 with $2^{\mathfrak{n}}$ replaced by any non-denumerable power.

The proof of Theorem 1 makes an unrecorded use of the generalized continuum hypothesis (GCH). However, as Jack Silver has pointed out to the reviewer, this can be avoided by the device of passing from $\mathfrak{n}$ to the smallest $\mathfrak{n}'$ such that $2^{\mathfrak{n}'} = 2^{\mathfrak{n}}$.

Theorem 2 states that, *if M is a model of T of power $2^{\mathfrak{n}}$, then there is a set $A \subseteq |M|$ of power $2^{\mathfrak{n}}$ such that every permutation of A extends to an automorphism of M.* The proof of Theorem 2 contains a serious error: no attention is paid to the fact that if T^* is obtained from T by adding Skolem functions, T^* need not be $2^{\mathfrak{n}}$-categorical. Indeed, the validity of Theorem 2 was not even established in Morley's cited opus, but was finally demonstrated (with the aid of some of Morley's results) by Silver (*Sets of absolutely indiscernible elements in models of theories categorical in power*, abstract, this JOURNAL, vol. 27 (1962), pp. 477–478).

Theorem 3 says that, *for any infinite $\mathfrak{p} < 2^{\mathfrak{n}}$, T has a model of power $\mathfrak{p}$ which is universal, i.e., has a submodel isomorphic to any model of T of power $\mathfrak{p}$.* The proof given here of Theorem 3 uses Theorem 2. However, Ehrenfeucht soon discovered a correct proof for the case $\mathfrak{p} = \aleph_0$ (and for some other $\mathfrak{p}$, assuming the GCH). Subsequently, Morley (loc. cit.) has shown (without assuming the GCH) that T has a universal model in *every* power $\mathfrak{p}$.

As might be inferred, the style is sometimes rather too brief. R. L. VAUGHT