

# Types for 0, 1 or Many Uses

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**Abstract.** This paper will present an analysis for detecting single uses of values in functional programs during call-by-need reduction. There are several reasons why such information can be useful. The Clean language uses a uniqueness type system for detecting single-threaded uses which allow destructive updating. Single-use information has also been proposed for compile-time garbage collection. Turner, Wadler and Mossin have presented a single-use analysis which is intended to detect cases where call-by-need can safely be replaced by call-by-name.

This paper will focus on this last application of single-use analysis and present a type-based analysis which overcomes some limitations present in the abovementioned analysis.

*Keywords:* Program analysis, storage management, functional programming, types, constraints.

## 1 Introduction

This paper will present an analysis for detecting single uses of values in functional programs during call-by-need reduction. There are several reasons why such information can be useful. The Clean language [6] uses a uniqueness type system [1] for detecting single-threaded uses which allow destructive updating. In [3], single-use information is proposed for compile-time garbage collection using the observation that the any use of a single-use value will be the last use. In [10], Turner, Wadler and Mossin present an analysis based on linear types which is intended to detect cases where call-by-need can safely be replaced by call-by-name. The present paper will focus on this last application of single-use analysis and present a type-bases analysis which overcomes some limitations present in the analysis shown in [10].

The analysis is (like Turner *et al.*'s) based on a type system. We use the type rules to generate a set of constraints that are solved off-line using a linear-time constraint solver.

We do not attempt to prove our analysis correct, but it is expected that an approach similar to that used in [10] can be used.

## 1.1 Limitations in Turner *et al.*'s Analysis

Turner *et al.*'s analysis can handle simple cases of zero usage by use of structural rules similar to those found in linear logic. Since this method involves adding or deleting entire variables in the environment, it cannot handle zero usage of parts of variables, only whole variables.

Additionally, there is a restriction on usage-annotated types that means that whenever the spine of a list can be used repeatedly, it is assumed that the same is true for the elements. As an example, the function

$$\text{mean } l = \text{sum } l / \text{length } l$$

traverses the spine of  $l$  twice, but accesses the lements of  $l$  only once. The analysis cannot detect this, as it assumes each spine-traversal implies access to the elements.

In this paper, we introduce a type-based analysis that will be able to detect single use in such cases by removing the restriction on structured types and by introducing the ability to detect zero uses of parts of values by means of an explicit 0 usage annotation on types.

## 2 The Language

We use a simple functional language with integers, pairs and recursive datatypes and function definitions. This language is slightly larger than the language presented in [10], but this is mainly to show how general recursive data structures (*i.e.* more than lists) are handled.

### 2.1 Types

We assume that the programs are simply typed using the following types

$$t ::= \text{Int} \mid t_1 \times t_2 \mid \mu\alpha.c_1 t_1 + \dots + c_n t_n \mid \alpha \mid t_1 \rightarrow t_2$$

where  $\alpha$  is a type variable used for recursive types. We will later discuss possible extension to polymorphic types.

### 2.2 Expressions

The abstract syntax of the language is given below.  $n$  is any natural number, the  $x_i$  are variables and the  $c_i$  are constructors.

$$\begin{aligned} e ::= & x \mid n \mid e_1 + e_2 \mid (e_1, e_2) \mid \text{split } e_1 \text{ to } (x_1, x_2) \text{ in } e_2 \\ & \mid c_i e \mid \text{case } e_0 \text{ of } c_1 x_1 \rightarrow e_1; \dots; c_n x_n \rightarrow e_n \\ & \mid e_1 e_2 \mid \lambda x : t. e \mid \text{let } x : t = e_1 \text{ in } e_2 \\ & \mid \text{letrec } x_1 : t_1 = e_1, \dots, x_n : t_n = e_n \text{ in } e_0 \end{aligned}$$

Note the *split* expression which breaks down a pair into its components. We use this instead of *fst* and *snd* because these would require two uses of a pair to get at the components, where *split* uses only one. Hence, uses of *split* instead of *fst* and *snd* will give more cases of single use.

Pattern matching is naturally translated into uses of *case* and *split* (more so than with *fst* and *snd*) and it is easy to define *fst* and *snd* in terms of *split*. Hence, it is no serious limitation to use *split*.

The *case* expressions are assumed to be exhaustive (though this doesn't really matter for the analysis).

### 3 Adding Usage Annotations

We will now annotate types and expressions with usage counts. The count indicates an upper bound on the usage of values of the annotated expression or type. Usage counts can be any of 0, 1 or  $\infty$ . A count of 0 indicates that the value is certain never to be used. While this isn't likely to be the case for a value at the place of its construction, zero-counts are used to express that a variable (or part thereof) is unused in a subexpression. 1 means *at most* one use.  $\infty$  indicates that no upper bound on the use has been detected.

#### 3.1 Annotated Types

A count  $k$  on a type constructor in a well-typed program means that any value of that type is used at most  $k$  times during execution of the program.

When we use a type variable with a superscript count or count variable, e.g.  $t^1$  or  $t^k$  we mean that  $t$  ranges over all types with that count on the topmost type constructor. Count annotated types are written using the following notation

$$t ::= Int^k \mid t_1 \times^k t_2 \mid \mu\alpha.^k c_1 t_1 + \dots + c_n t_n \mid \alpha \mid t_1 \rightarrow^k t_2$$

where  $k$  is a count (0, 1 or  $\infty$ ).

#### 3.2 Annotated Expressions

The aim of the analysis is to annotate value introducing expressions in the program by an usage count, that gives an upper limit to the number of uses of the value created by the expression. The syntax of annotated expressions is

$$\begin{aligned} e ::= & x \mid n^k \mid e_1 +^k e_2 \mid (e_1,^k e_2) \mid \text{split } e_1 \text{ to } (x_1, x_2) \text{ in } e_2 \\ & \mid c_i^k e \mid \text{case } e_0 \text{ of } c_1 x_1 \rightarrow e_1; \dots; c_n x_n \rightarrow e_n \\ & \mid e_1 e_2 \mid \lambda^k x. t.e \mid \text{let } x : t = e_1 \text{ in } e_2 \\ & \mid \text{letrec } x_1 : t_1 = e_1, \dots, x_n : t_n = e_n \text{ in } e_0 \end{aligned}$$

where  $k$  is a count (0, 1 or  $\infty$ ). Note that the types used in abstractions and let(rec) expressions are now annotated types.

The annotations on value-producing expressions (*i.e.* constants, additions, pairs, injected values and lambda-abstractions) indicate upper bounds of the usage of the values produced by these expressions. Annotated types on lambda-bound and let(rec)-bound variables indicate how many times values bound to these variables are used *inside their scope*. Since the same value may be bound to several variables, a single-use annotation on one variable does not guarantee that the value bound to it is single-threaded. Only the annotation at the creation point of a value will reflect the total usage of that value throughout its lifetime.

For some kinds of optimization it is sufficient to know the number of uses of a value at the creation point of that value, but for other optimizations it is required at a usage point to know if this is the single use. The latter information is not present in annotations as shown, but it is a simple matter to propagate this information to the uses of the value. In this respect (as well as in our use of a 0 count and *split*-expressions), our analysis is similar to the analysis by Jensen and Mogensen from ESOP'90 [3].

### 3.3 Counts

Apart from the obvious ordering ( $0 \leq 1 \leq \infty$ ) we also need some operations on counts:

$k_1 \sqcup k_2$				$k_1 + k_2$				$k_1 \cdot k_2$				$k_1 \triangleright k_2$			
$k_1 \backslash k_2$	0	1	$\infty$	$k_1 \backslash k_2$	0	1	$\infty$	$k_1 \backslash k_2$	0	1	$\infty$	$k_1 \backslash k_2$	0	1	$\infty$
0	0	1	$\infty$	0	0	1	$\infty$	0	0	0	0	0	0	0	0
1	1	1	$\infty$	1	1	$\infty$	$\infty$	1	0	1	$\infty$	1	0	1	$\infty$
$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	0	$\infty$	$\infty$	$\infty$	0	1	$\infty$

$\sqcup$  is least upper bound.  $+$  and  $\cdot$  are addition and multiplication over counts, respectively.  $\triangleright$  is used as a guard to indicate that uses in an expression should not count if the result of the expression is never used. Hence, we use this operator to simulate laziness of evaluation: An expression is not evaluated if its result is never used.

We extend the ordering and the operations to work on annotated types and type environments as well as counts. The ordering extends pointwise: For a type  $t_1$  to be less than another  $t_2$ , the must have the same underlying type and the counts in  $t_1$  must be less than the counts on the corresponding positions in  $t_2$ . Note that this makes the ordering covariant on function types. Ordering on type environments  $E_1$  and  $E_2$  implies the same underlying structure (*i.e.* the same variable is bound to types in  $E_1$  and  $E_2$  that are identical except for annotations) and  $E_1 \leq E_2$  indicates that for all  $x$ , if  $E_1$  binds  $x$  to  $t_1$  and  $E_2$  binds  $x$  to  $t_2$  then  $t_1 \leq t_2$ .

$\sqcup$  (least upper bound) operator likewise works pointwisely on counts in equivalent positions in types and environments.

We will always use  $\cdot$  with a single count on the left-hand side, but may use either of count, type and environment on the right-hand side. If a type or environment occurs on the right-hand side, all count annotation in the type or

environment are multiplied by the left-hand count, e.g.  $\infty \cdot Int^1 \rightarrow^1 Int^0 \times^\infty Int^1 = Int^\infty \rightarrow^\infty Int^0 \times^\infty Int^\infty$ .

The same rules apply to  $\triangleright$ .

Addition is a bit more complex, as addition on function types uses least upper bound on the argument and result types. This models that, even though a function is used repeatedly, the argument and result of the function may still only be used once at each application. Since the argument and result may be constructed anew at each application, these may still be single-threaded even if the function itself is not. Hence, we define

$$\begin{array}{lll}
 int^{k_1} & + int^{k_2} & = int^{(k_1+k_2)} \\
 t_1 \times^{k_1} t_2 & + t_3 \times^{k_2} t_4 & = (t_1 + t_3) \times^{(k_1+k_2)} (t_2 + t_4) \\
 \mu\alpha.^{k_1} c_1 s_1 + \dots + c_n s_n + \mu\alpha.^{k_2} c_1 t_1 + \dots + c_n t_n = & & \\
 & \mu\alpha.^{(k_1+k_2)} c_1 (s_1 + t_1) + \dots + c_n (s_n + t_n) & \\
 t_1 \rightarrow^{k_1} t_2 & + t_3 \rightarrow^{k_2} t_4 & = (t_1 \sqcup t_3) \rightarrow^{(k_1+k_2)} (t_2 \sqcup t_4)
 \end{array}$$

Like the other operators, addition extends to type environment, and here it indicates point-wise addition of the annotated types.

We use the following precedence order on the operators, from tightest to weakest binding:  $\triangleright, \cdot, +, \sqcup, \leq$ .

## 4 Type Rules

We now present well-typedness rules for our language. The intention is that in any well-annotated program, the values created at an expression with annotation  $k$  will be used at most  $k$  times during evaluation under call-by-need. Unlike in [10], we allow the same variable to have different annotations on its type in different parts of the program. The annotations describe the usage in the given subexpression. It is hence the usage count in the type given at the creation time of a value that determines the total number of (potential) uses of the value.

The judgements are of the form

$$E \vdash e : t$$

where  $E$  is an environment mapping variables to annotated types,  $e$  is an annotated expression and  $t$  is an annotated type. The intuitive meaning is that if  $e$  is to be evaluated in a context given by  $t$ , then  $E$  describes the uses of the free variables of  $e$ .

If we assume that the result of the entire computation is printed, the natural context for the goal expression has usage 1 on all components of the result type.

In the rules below, we assume that the result of the entire expression is required. For non-strict uses of values, we use the  $\triangleright$  guard on the type environments of subexpressions that may not always be evaluated. In this way, laziness is modeled. In the explanation about how the information is passed around, we state an intuitive order of information flow. The actual analysis will not use this order of evaluation but generate a set of constraints that are solved off-line.

Hence, the flow-description is only intended to give an intuitive understanding of the type rules.

We will in general use inequality constraints between environments even in cases where equality would seem natural. The reason is that equality constraints are not in general efficiently solvable and a unique best solution may not exist. This point is explained in section 5.

The variable rule below simply indicates that if a variable is used in context  $t$ , then the environment reflects that use.

$$E[x : t] \vdash x : t$$

A constant expression indicates the number of uses of the constant.

$$E \vdash n^k : Int^k$$

If we add two numbers, these are used once each regardless of how many times we use the result.

$$\frac{E_1 \vdash e_1 : Int^1 \quad E_2 \vdash e_2 : Int^1 \quad E_1 + E_2 \leq E}{E \vdash e_1 +^k e_2 : Int^k}$$

The annotation on a pair expression must match the number of uses indicated by the type. The subexpressions are evaluated only if the corresponding parts of the pair are later accessed (indicated by a non-zero count on the types of the components). Hence, we guard the environments of the subexpressions by their types before adding them.

$$\frac{E_1 \vdash e_1 : t_1^{k_1} \quad E_2 \vdash e_2 : t_2^{k_2} \quad k_1 \triangleright E_1 + k_2 \triangleright E_2 \leq E}{E \vdash (e_1,^k e_2) : t_1^{k_1} \times^k t_2^{k_2}}$$

When analysing a *split* expression, we find the contexts of the components by analysing the body in the context of the entire expression. We then combine these to a product type with usage 1 (the usage implied by the *split*) and analyse the splitted expression. Since *split* is strict in both subexpressions, we use no guards.

$$\frac{E_1[x_1 : t_1, x_2 : t_2] \vdash e_2 : t \quad E_2 \vdash e_1 : t_1 \times^1 t_2 \quad E_1 + E_2 \leq E}{E \vdash \text{split } e_1 \text{ to } (x_1, x_2) \text{ in } e_2 : t}$$

When injecting a value into a sum-type, we annotate the constructor by the usage count indicated by the type. Since injection is lazy, we guard the environment for the injected expression by its usage.

$$\frac{E_1 \vdash e : t_i \quad t_i \triangleright E_1 \leq E}{E \vdash c_i^k e : \mu\alpha.^k c_1 t_1 + \dots + c_i t_i + \dots + c_n t_n}$$

When analysing a *case*-expression, we analyse each of the branches using the context of the entire expression. The yielded contexts for each of the pattern variables are used to construct a recursive type for the sum-type, constraining

each of the injected types to be conservative approximations of the pattern-variable contexts and adding the constraint that a value of that type may be used at least once (namely, by this *case*-expression). This type is used as the context for the inspected expression. The environment for the entire expression is found as the combined worst-case of the environments of the branches plus the environment of the inspected expression. The  $i$  subscript ranges from 1 to  $n$ .

$$\frac{E_i[x_i : s_i] \vdash e_i : t \quad s_i \leq t_i[\alpha \setminus (\mu\alpha.^k c_1 t_1 + \dots + c_n t_n)] \quad 1 \leq k}{E_0 \vdash e_0 : \mu\alpha.^k c_1 t_1 + \dots + c_n t_n \quad E_0 + E_i \leq E} \\ E \vdash \text{case } e_0 \text{ of } c_1 x_1 \rightarrow e_1; \dots; c_n x_n \rightarrow e_n : t$$

When analysing a function application, we find the type of the function to get a context for the argument. We also constrain the result type of the function to be at least as “bad” as the context of the application. Since function application is lazy, we guard the environment of the argument expression.

$$\frac{E_1 \vdash e_1 : t_1^k \rightarrow^1 t_2 \quad E_2 \vdash e_2 : t_1^k \quad t_3 \leq t_2 \quad E_1 + k \triangleright E_2 \leq E}{E \vdash e_1 e_2 : t_3}$$

An abstraction is annotated both with the number of uses of the function and by the type of the argument. The body is analysed to find the uses of variables in a single evaluation of the body. Since the body is evaluated at each application of the function, we multiply the environment for the body by the number of uses of the function.

$$\frac{E_1[x : t_1] \vdash e : t_2 \quad k \cdot E_1 \leq E}{E \vdash \lambda^k x : t_1. e : t_1 \rightarrow^k t_2}$$

We analyse the body of a *let*-expression to find the context for the bound variable. This is used as the context for the bound expression and to annotate the variable in the expression. Since *let*-expressions are lazy, we guard the environment of the bound expression by its type.

$$\frac{E_2[x : t_1^k] \vdash e_2 : t_2 \quad E_1 \vdash e : t_1^k \quad k \triangleright E_1 + E_2 \leq E}{E \vdash \text{let } x : t_1^k = e_1 \text{ in } e_2 : t_2}$$

We assume that strongly connected components of a *letrec* are found in an earlier stage of compilation and used to split complex *letrecs* into nested *lets* and *letrecs*, as described in [4]. This allows us to use a fairly simple conservative approximation and assume that the variables bound in a *letrec*-expression can be used arbitrarily often. Hence, we constrain the types of the bound expressions to have infinite counts if they are used at all. The subscript  $i$  ranges from 0 to  $n$  and  $j$  ranges from 1 to  $n$ .

$$\frac{E_i[x_1 : t_1, \dots, x_n : t_n] \vdash e_i : s_i \quad \infty \cdot t_j \leq s_j \quad E_0 + \dots + E_n \leq E}{E \vdash \text{letrec } x_1 : t_1 = e_1, \dots, x_n : t_n = e_n \text{ in } e_0 : s_0}$$

## 5 Usage Analysis

The type rules can be used to check whether a given annotation is valid, but it is more interesting to use the rules to find a minimal valid annotation of a program.

The approach we have chosen is to write a type derivation of the program where all counts are replaced by different variables. This is possible, as type derivations for different annotations of the same program only differ in the values of the counts used in types, environments and annotations. The type rules can then be seen as generating a set of constraints between counts (constraints between types and environments are expanded to collections of constraints over counts). Each constraint is in one of the forms  $k_1 = k_2$ ,  $1 \leq k_1$ ,  $k_1 \leq k_2$ ,  $k_1 + k_2 \leq k_3$ ,  $\infty \cdot k_1 \leq k_2$ ,  $k_1 \cdot k_2 \leq k_3$ ,  $k_1 \sqcup k_2 \leq k_3$  or  $k_1 \triangleright k_2 \leq k_3$ , where the  $k_i$  are variables that range over counts. Each solution to a set of constraints generated by the type rules corresponds to a type derivation for a well-annotated program.

We want minimal annotations, so we must find a minimal solution to the constraint set. We have carefully chosen the constraints to be meet-closed: Whenever we have two solutions, then the meet (greatest lower bound) of these is also a solution. This is the motivation for using  $k_1 + k_2 \leq k_3$  where  $k_1 + k_2 = k_3$  might seem more natural *etc.*, as the latter is not meet-closed. Hence, a unique minimal solution can be found by taking the meet of all solutions. That a solution must exist is not terribly difficult to see either: All occurrences of constants in the constraints will occur on the left of an inequality, so setting all count variables to  $\infty$  will yield a solution.

Each type rule generates a constant number of constraints between types and type environments. As we have chosen a language where variables are typed explicitly in the program text, the size (measured as number of nodes in the syntax tree) of a type is bounded by the size of the program. Furthermore, the sum of the sizes of types in a type environment is also bounded by the size of the program, as the type environment lists a subset of the variables from the program with their types. Hence, each constraint between types or environments expands to a number of primitive constraints that is bounded linearly by the size of the program. Since the number of inferences in a type derivation is proportional to the size of the program, the total number of primitive constraints is bounded by the square of the size of the typed program. As we will see that the constraints can be solved in time linear in the size of the constraint set, the analysis can be done in quadratic time.

### 5.1 Solving the Constraints

Rehof and Mogensen [7] show two different ways of solving meet-closed inequality constraints over finite lattices. One method solves the constraints directly, using a variant of Kildall's algorithm [5]. The other translates the constraints into Horn-clauses, which can be solved using the HORNSAT algorithm of Dowling and Gallier [2]. Both methods find the least solution in time linear in the size of



the constraint set. We will show the latter method here and refer readers to [7] for the first.

We represent each element in the count lattice by two boolean values:

count	representation
0	00
1	01
$\infty$	11

Hence, we replace each constraint variable  $k_i$  by two variables  $l_i$  and  $r_i$  over the binary domain, with the constraint  $l_i \leq r_i$  to reflect that the pair 10 is unused.

We translate each constraint over counts to a set of constraints over the boolean lattice. The translation below uses the the general scheme shown in [7] with local reduction in the cases where constants occur in the constraints:

constraint	translation
$k_1 = k_2$	$l_1 \leq l_2, r_1 \leq r_2, l_2 \leq l_1, r_2 \leq r_1$
$1 \leq k$	$1 \leq r$
$k_1 \leq k_2$	$l_1 \leq l_2, r_1 \leq r_2$
$k_1 + k_2 \leq k_3$	$l_1 \leq l_3, r_1 \leq r_3, l_2 \leq l_3, r_2 \leq r_3, r_1 \wedge r_2 \leq l_3$
$k_1 \sqcup k_2 \leq k_3$	$l_1 \leq l_3, r_1 \leq r_3, l_2 \leq l_3, r_2 \leq r_3$
$\infty \cdot k_1 \leq k_2$	$r_1 \leq l_2$
$k_1 \cdot k_2 \leq k_3$	$r_1 \wedge r_2 \leq r_3, l_1 \wedge r_2 \leq l_3, l_2 \wedge r_1 \leq l_3$
$k_1 \triangleright k_2 \leq k_3$	$r_1 \wedge l_2 \leq l_3, r_1 \wedge r_2 \leq r_3$

Note that we have reduced the number of different constraints to just ordering ( $\leq$  is the usual ordering on the binary domain) and  $x \wedge y \leq z$ , where  $x \wedge y$  is 0 if any of  $x$  or  $y$  is 0, and 1 otherwise.

It is shown in [7] that, given the extra constraints  $l_i \leq r_i$  for all variables  $k_i$ , the translation has a solution if and only if the original constraint has. Furthermore, any solution to the translation maps into a solution of the original constraint set using the inverse of the representation function.

## 6 Conclusion

We can solve these constraints in linear time using a variety of methods. The number of constraints is linear in the size of the type derivation for the program we analyse, i.e. in the worst case quadratic in the size of the typed program. This is somewhat worse than Turner *et al.*'s analysis [10] which is linear in the size of the typed program. Our analysis is, however, more precise: Whenever Turner *et al.*'s analysis detects single use, so will our, and there are cases (e.g. the *mean* example) where our analysis detects single use where Turner *et al.*'s analysis do not.

The presented version of the analysis finds usage counts at value creation times. This is fine if the update information is put into the values themselves in the form of self-updating closures, as is the case for most tagless implementations

of lazy functional languages. If usage counts are required at value destruction time, the presented analysis must be supplemented by a simple forwards analysis. This approach was also used in [3]. Note that this will actually reduce the precision of the analysis, as values with different usages may float to the same destruction point. If annotations are given at creation time, the destruction can treat these differently, but any annotation at the destruction point must treat the values the same way.

The analysis can detect that a partial application is single-use. In addition to optimizing call-by-need to call-by-name, that information can be used to avoid full-laziness transformation of the function, which can save both space and time.

Initial experiments with an implementation of the constraint solver from [7] indicate that solving the constraints directly takes approximately the same time as translating the constraints to Horn-clauses and then using the first of the linear-time HORNSAT algorithms from [2]. However, the latter method uses somewhat more space. These experiments use randomly generated constraints, not constraints generated by an actual usage count analysis, so it is too early to tell which will be best for an actual implementation.

Even though the worst case analysis time is quadratic, we believe this is rarely encountered in real programs, as types and environments typically grow less than linearly in program size. There is, however, little doubt that the analysis of [10] is faster, and it is not clear to what extent the added precision of the proposed analysis is worth the extra cost.

In addition to using the analysis to avoid updates during graph-reduction, it can be used for a limited kind of compile-time garbage collection. This was, indeed, the stated intention of the analysis presented in [3]. In a system with a good garbage collection, this kind of compile-time GC is probably not worth the effort. However, if the typing rules are modified to handle a call-by-value language (essentially eliminating the uses of  $\triangleright$ ), the analysis can be used in conjunction with region inference [9]. Experience with the ML-kit with regions [8] shows that a common space leak is when a function takes apart a constructor and builds a new constructor of the same type in the same region, causing this region to grow. If the analysis can detect that the original constructor is not used again, the same space can be reused for the new constructor, avoiding the growth.

Another related type system is the uniqueness typing of Clean [1]. This too has the restriction that if the spine of a list is used repeatedly, then it is assumed that this is also the case for the elements. However, the uniqueness type analysis takes evaluation order into account and can hence in some cases detect the last use of value even if there are multiple uses within the life time of the value, which our analysis can not. Furthermore, the uniqueness type system can handle polymorphism, which the analysis presented here does not.

A possible way of handling polymorphism is by abstracting over annotated types and include in each type schema a set of constraints over the type variables (both bound and free). These constraints will be added to the current set of constraints when a type schema is instantiated. When type variables are instantiated to actual types, constraints over the type variables are expanded

out into more primitive constraints. In the end (assuming the goal function of the program has a monomorphic type), all constraints are expanded fully into constraints over atomic counts which can be solved as described above. This approach is likely to have a high worst-case complexity, but may be O.K. in practice. It may be worthwhile to reduce the constraints in type schemas at the time they are constructed rather than wait until they are fully instantiated. This will, however, require a different solution mechanism, as the current constraint solver is inherently off-line.

## References

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