Measuring Information in Timed Languages*

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Timed automata and timed languages [1] constitute a beautiful discovery that opened new perspectives to automata and language theory, as well as new applications to computer-aided verification. However the theory of timed regular languages is far from being achieved. Seven years ago, in [2], I argued that developing such a theory constituted an important research challenge, and I sketched a research program in this direction. Unfortunately, when listing research tasks on timed languages I have overlooked one interesting topic: measuring size of and information content in such languages. Catching up this omission became the focus of my research and the theme of this talk.

In this talk, I survey results obtained in [3–7] by my co-workers and myself on volume and entropy of timed regular languages.

To define size measures of timed languages we proceed as follows. A "slice" L_n of a timed language L, limited to words with n events, can be seen as subset of $\mathbb{R}^n \times \Sigma^n$, and its volume V_n can be defined in a natural way. We call the exponential growth rate of V_n volumic entropy H of the timed language L.

For a timed regular L, volumes V_n are rational, and relatively easy to compute for a fixed n. What is more interesting, computation of entropy H is based on functional analysis. Namely, to a timed automaton can be associated a positive linear operator Ψ (given by a matrix of integrals) on some Banach space. Then we prove that $H = \log \rho(\Psi)$, where ρ denotes the spectral radius. We discuss exact and approximate methods of computing the entropy.

Entropy H can be seen as a size of the language L or as information contents in its typical timed words. We formalize this last observation in terms of Kolmogorov complexity and in terms of symbolic dynamics.

Entropy brings new insights into classical questions on timed automata. We show that having a not-too-small entropy suffices to rule out Zeno pathologies in timed automata. Thus "non-vanishing" timed regular languages satisfy a weak pumping lemma and behave well under discretization.

For "degenerate" timed automata, it is often the case that the slice L_n is a union of polyhedra of different dimensions $\leq n$. We argue for a reasonable way of measuring volumes and entropies in this degenerate case.

We conclude this introduction to quantitative theory of timed languages by some open problems and speculate on future theoretical and practical applications.

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