On Time with Minimal Expected Cost !

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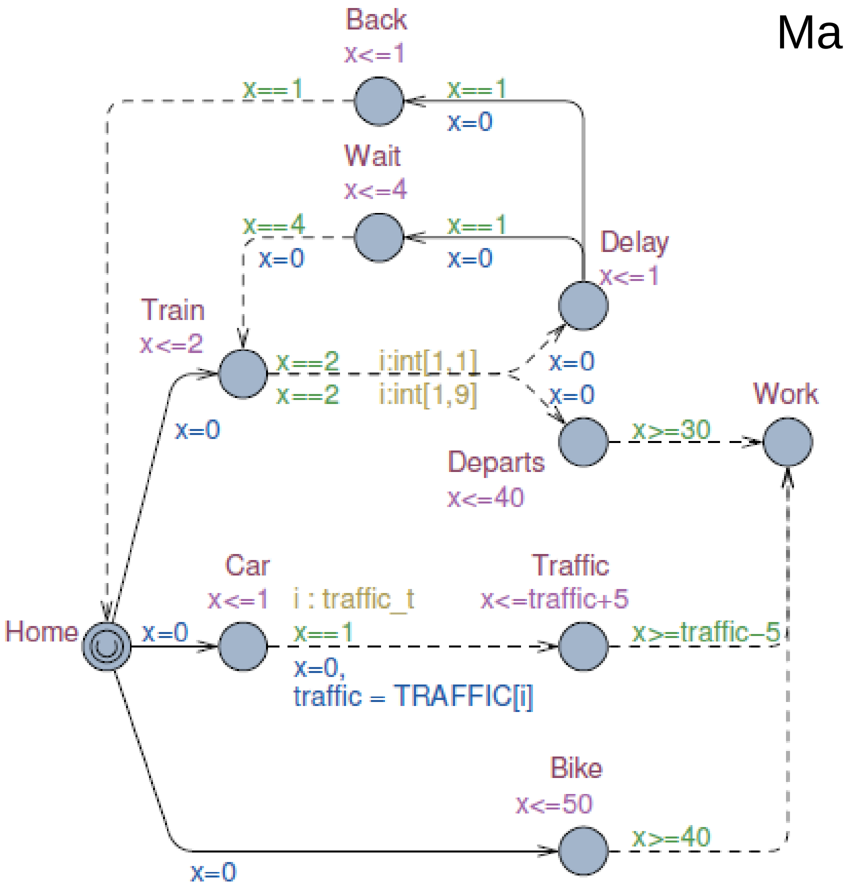
DENMARK

Motivation

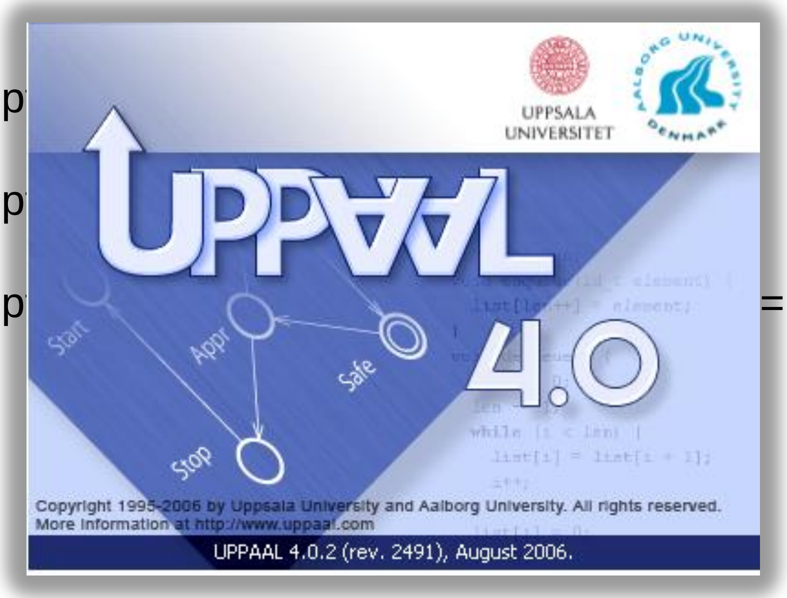
2-Player Game (Antagonistic opponent)
Markov Decision Process (probabilistic opponent)



Op

Op

Op =60:



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More information at http://www.uppaal.com

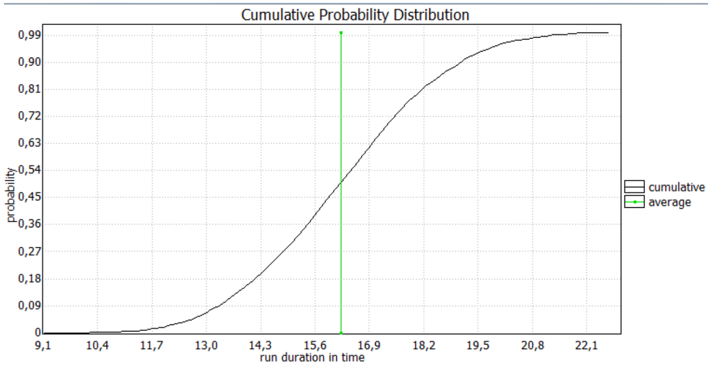
UPPAAL 4.0.2 (rev. 2491), August 2006.

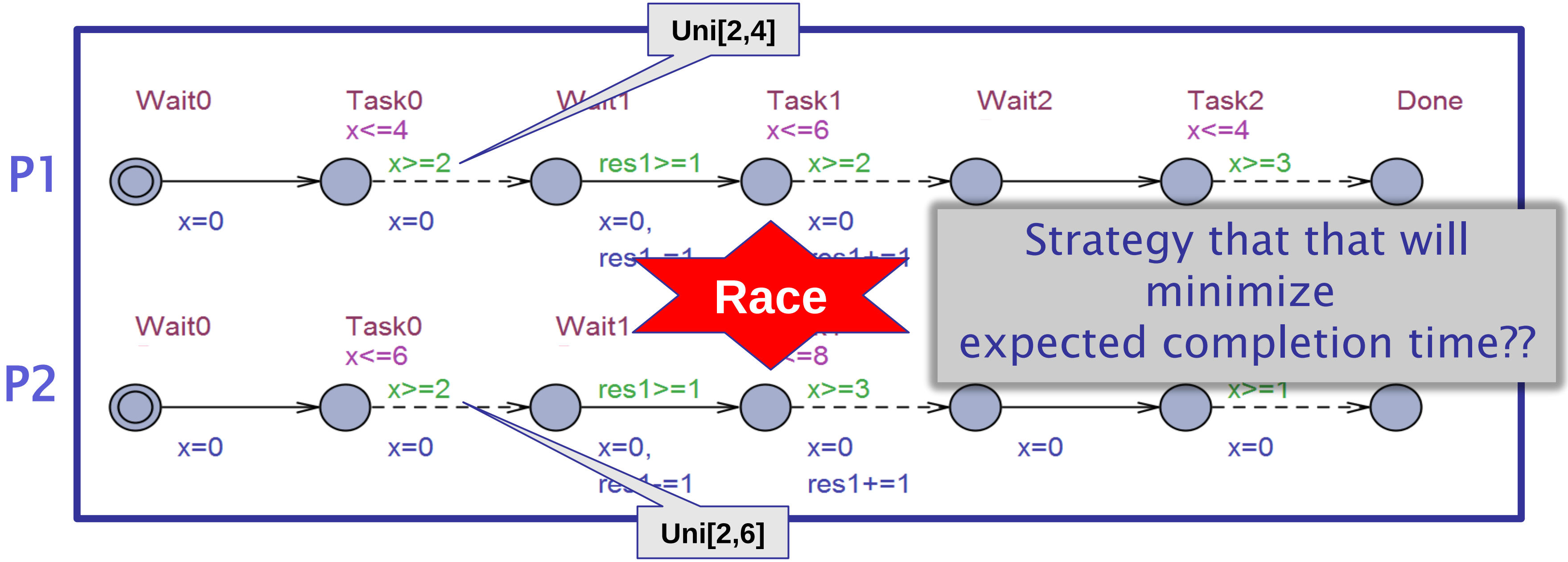
Bruyere, V., Filiot, E., Randour, M., Raskin, J.F.: Meet your expectations with guarantees: Beyond worst-case synthesis in quantitative games. STACS14

Duration Probabilistic Automata

```
/* Resources */  res1:1;
/* Processes */  P1:  [2,4].<res1:1>[2,6].[3,4];
                 P2:  [2,6].<res1:1>[3,8].[1,5];
```

Pr[<=1000](<> P1.Done && P2.Done)





Uni[2,4]

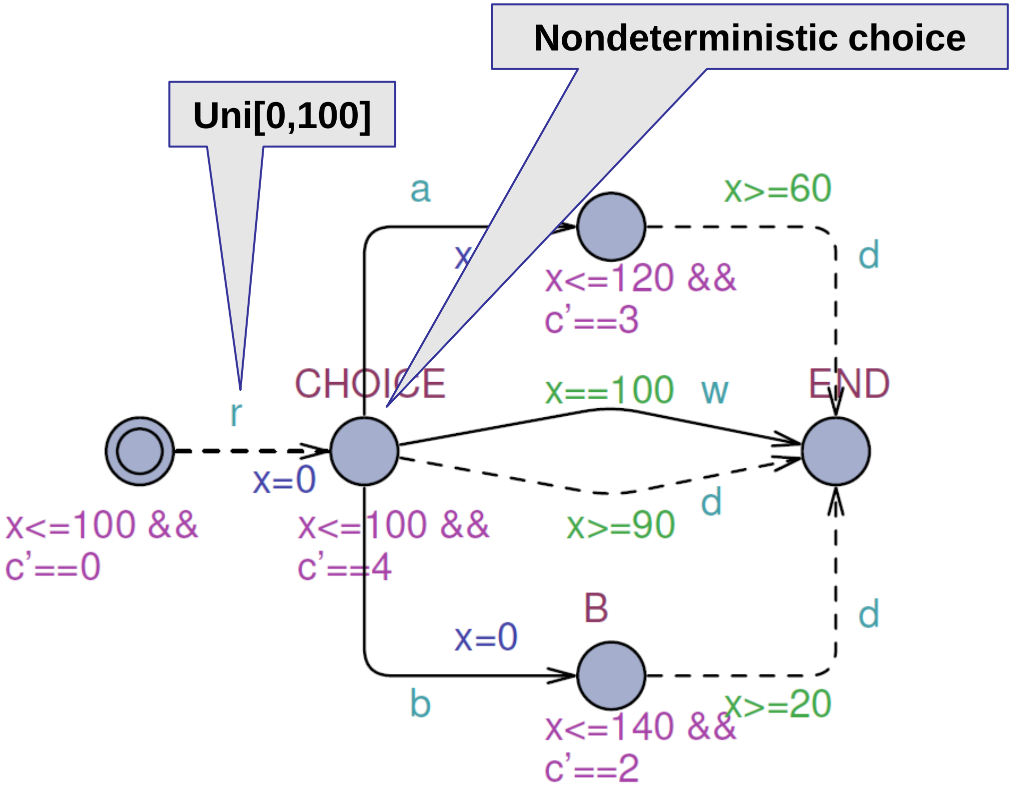
Uni[2,6]

Race

Strategy that that will minimize expected completion time??

Kempf, J.F., Bozga, M., Maler, O.: As soon as probable: Optimal scheduling under stochastic uncertainty. In: TACAS. pp. 385{400 (2013)

Motivation



× Priced Timed Game

✓ Timed Game TIGA

✓ Timed Automata UPPAAL

✓ Priced Timed Automata CORA

✓ Stochastic (P)TA SMC

× **Priced Timed MDP** TIGA/SMC

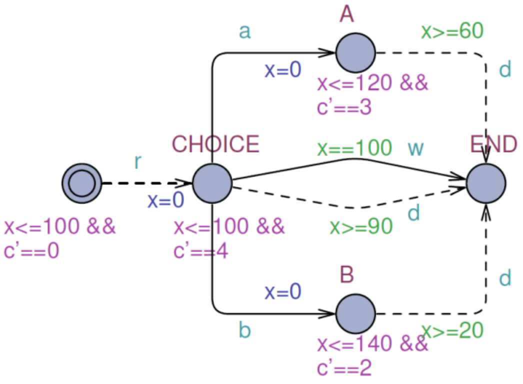
~ **Decision Stochastic Priced Timed Automata**

Minimize **expected cost**
subject to **guaranteed time-bound**

Overview

- Priced Timed Games
  - Time bounded reachability strategies
- Priced Timed Markov Decision Processes
  - Minimal expected cost reachability strategy
- Optimal Strategy Synthesis Using Reinforcement Learning
- Representation of Stochastic Strategies
- Experimental Results

PTA and PTG



$$\mathcal{A} = (L, \ell_0, X, \Sigma, E, P, Inv)$$

is a tuple where

- L is a finite set of locations,
- $\ell_0 \in L$ is the initial location,
- X is a finite set of non-negative real-valued clocks,
- Σ is a finite set of actions,
- $E \subseteq L \times \mathcal{B}(X) \times \Sigma \times 2^X \times L$ is a finite set of edges,
- $P : L \to \mathbb{N}$ assigns a price-rate to each location, and
- $Inv : L \to \mathcal{B}(X)$ sets an invariant for each location.

Priced Timed Game $\Sigma = \Sigma_c \uplus \Sigma_u$

PTA Semantics

$$S_{\mathcal{A}} = (Q, q_0, \Sigma, \rightarrow)$$

- $(\ell, v) \in Q$ for $\ell \in L$ and $v \in \mathbb{R}^{X}_{\geq 0}$ st $v \models Inv(\ell)\}$,
- $q_0 = (\ell_0, 0)$ is the in

and[1]

- $(\ell, v) \xrightarrow{a}_0 (\ell', v')$ if $(\ell \xrightarrow{g,a,r} \ell') \in E$ st.
- $(\ell, v) \xrightarrow{d}_p (\ell, v + d$ $p = P(\ell) \cdot d$, $v \models Inv(\ell)$ and $v + d \models Inv(\ell)$.

- Set of runs of: $Exec_{\mathcal{A}}$.
- Set of finite (maximal) runs: $Exec^f_{\mathcal{A}}$ ($Exec^m_{\mathcal{A}}$).
- $\pi[i]$ the state q_i,
- $\pi|_i$ ($\pi|^i$) the prefix (suffix) of π ending (starting) at q_i.
- $C(\pi)$ ($T(\pi)$) denotes total accumulated cost (time).

Run π:

$$q_0 \xrightarrow{d_0}_{p_0} q'_0 \xrightarrow{a_0}_0 q_1 \xrightarrow{d_1}_{p_1} q'_1 \xrightarrow{a_1}_0 \cdots \xrightarrow{d_{n-1}}_{p_{n-1}} q'_{n-1} \xrightarrow{a_{n-1}}_0 q_n \cdots$$

$a_i \in \Sigma$, $d_i, p_i \in \mathbb{R}_{\geq 0}$, and q_i is a state (ℓ_{q_i}, v_{q_i}).

Strategies & Outcome

Priced Timed Game $\Sigma = \Sigma_c \uplus \Sigma_u$

$$\sigma : Exec^f_{\mathcal{G}} \rightharpoonup \mathcal{P}\left(\Sigma_c \cup \{\lambda\}\right) \setminus \{\emptyset\}$$

such that for any finite run π, if $q = last(\pi)$ and $a \in \sigma(\pi) \cap \Sigma_c$, then $q \xrightarrow{a} q'$ fs q'.

$Out(\sigma) \subseteq Exec_{\mathcal{G}}$

- $q_0 \in Out(\sigma)$
- If $\pi \in Out(\sigma)$ then $\pi' = \pi \xrightarrow{e} q' \in Out(\sigma)$ if $\pi' = Exec_{\mathcal{G}}$ and either one of the following three conditions hold:
  1. $e \in \Sigma_u$, or
  2. $e \in \Sigma_c$ and $e \in \sigma(\pi)$, or
  3. $e \in \mathbb{R}_{\geq 0}$ and for all $e' < e$, $last(\pi) \xrightarrow{e'} q'$ for some q' st $\sigma(\pi \xrightarrow{e'} q') \ni \lambda$.

Cost Bounded Reachability Strategies

For $G \subseteq L$, $B \in \mathbb{R}_{\geq 0}$: (G, B) is a cost-bounded reachability objective.

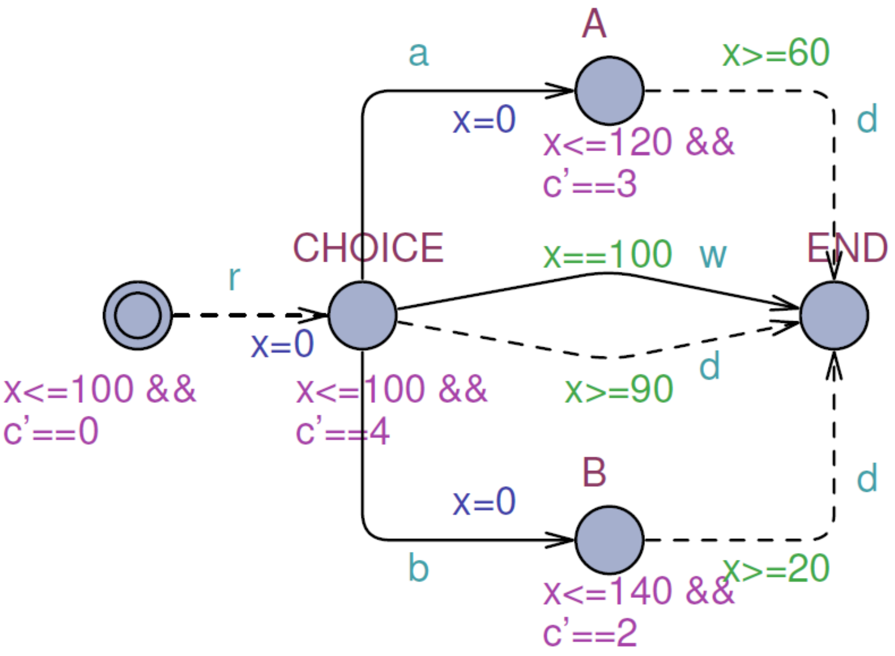
π is winning w.r.t. (G, B), if $\mathit{last}(\pi) \in G \times \mathbb{R}^{X}_{\geq 0}$ and $C(\pi) \leq B$.
A strategy σ over $\mathcal{G}$ is a winning strategy if all runs in $\mathit{Out}(\sigma)$ are winning.

Theorem (Memoryless, Most Permissive Strategies)

Let $\mathcal{G}$ be a non-Zeno, clocked TG. If a time-bounded reachability objective (G, T) has a winning strategy, then it has

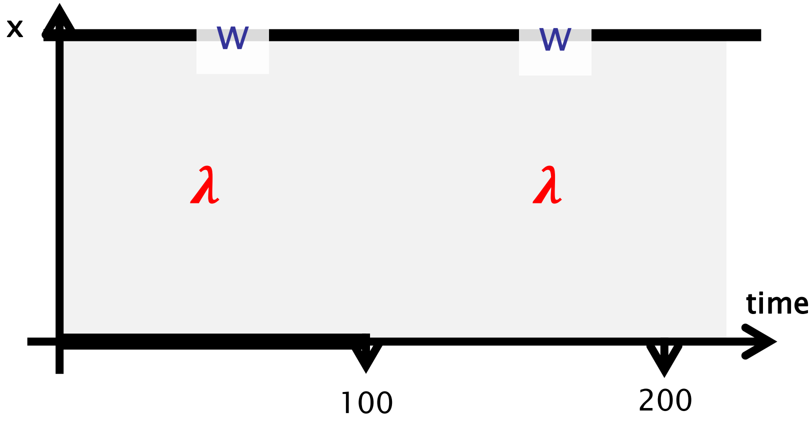
1. *a deterministic, memoryless winning strategies, and*
2. *a (unique) most permissive, memoryless winning strategy $\sigma^{p}_{\mathcal{G}}(G, T)$.*

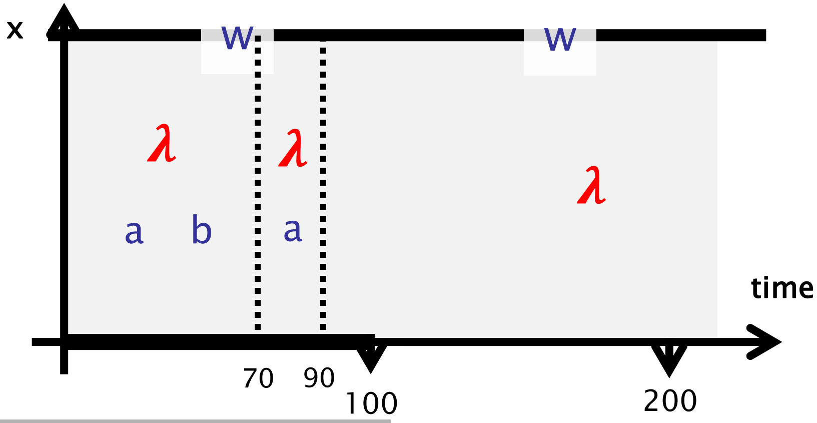
Motivation



Objective: $A\langle\rangle$ **(END $\wedge$ time ≤ 210)**

Deterministic, memoryless strategy:





Most permissive, memoryless strategy

Priced Timed MDPs

$$\mathcal{M} = \langle \mathcal{G}, \mu^u \rangle$$

where

- $\mathcal{G} = (L, \ell_0, X, \Sigma_c, \Sigma_u, E, P, Inv)$ is a PTG, and
- μ^u is a family of density-functions, $\{\mu^u_q : \exists \ell \exists v . q = (\ell, v)\}$, with $\mu^u_q(d, u) \in \mathbb{R}_{\geq 0}$ assigning the *density* of the environment aiming at taking the uncontrollable action $u \in \Sigma_u$ after a delay of d from state q.

Assumptions:

1. $\mu^u_q(d, u) > 0$ only if $q \xrightarrow{d,u}$ in $\mathcal{G}$.
2. $\sum_u (\int_{t \geq 0} \mu^u_q(t, u) dt) = 1$

Stochastic Strategies

μ^c for a PTMDP $\mathcal{M} = \langle \mathcal{G}, \mu^u \rangle$

is a family of density-functions, $\mu^c = \{\mu^c_q : \exists \ell \exists v . q = (\ell, v)\}$, where $\mu^c_q(d, c) \in \mathbb{R}_{\geq 0}$ assigns the *density* of the controller aiming at taking the controllable action $c \in \Sigma_c$ after a delay of d from state q.

- Repeated races between μ^u and μ^c,
- Induced probability measure $\mathbb{P}_{\langle \mathcal{G}, \mu^u \rangle, \mu^c}$ on (certain) sets of runs.

Induced Probability Measure

Cylinder set $\mathcal{C}(q, I_0\ell_0 I_1 \cdots I_n\ell_n)$ with $\ell_i \in L$ and $I_i = [l_i, u_i]$ with $l_i, u_i \in \mathbb{Q}$, $i = 0..n$, consists of all maximal runs having a prefix of the form:

$$q \xrightarrow{d_0}\xrightarrow{a_0} (\ell_0, v_0) \xrightarrow{d_1}\xrightarrow{a_1} \cdots \xrightarrow{d_n}\xrightarrow{a_n} (\ell_n, v_n)$$

where $d_i \in I_i$ for all $i < n$.

Probability Measure

$$\mathbb{P}_{\langle\mathcal{G},\mu^u\rangle,\mu^c}(\mathcal{C}(q, I_0\ell_0 I_1\ell_1 \cdots I_{n-1}\ell_n)) =$$

$$\sum_{p\in\{u,c\}} \sum_{\substack{a\in\Sigma_p \\ \ell_q \xrightarrow{a} \ell_1}} \int_{t\in I_0} \mu_q^p(t,a) \cdot \Big(\int_{\tau>t} \mu_q^{\overline{p}}(\tau) d\tau \Big) \cdot$$

$$\mathbb{P}_{\langle\mathcal{G},\mu^u\rangle,\mu^c}(\mathcal{C}((q^t)^a, \mathcal{C}(I_1 \cdots I_{n-1}\ell_n)) \, dt$$

where $\mu_q^p(\tau) = \sum_{a\in\Sigma_p} \mu_q^p(\tau, a)$.

Minimum Expected Cost

Let $\pi \in Exec^m$ and let G be as set of goal locations.

$$C_G(\pi) = min\{C(\pi|_i) \;:\; \pi[i] \in G\}$$

denotes the accumulated cost before π reaches G.

Expected Value of C_G given μ^c:

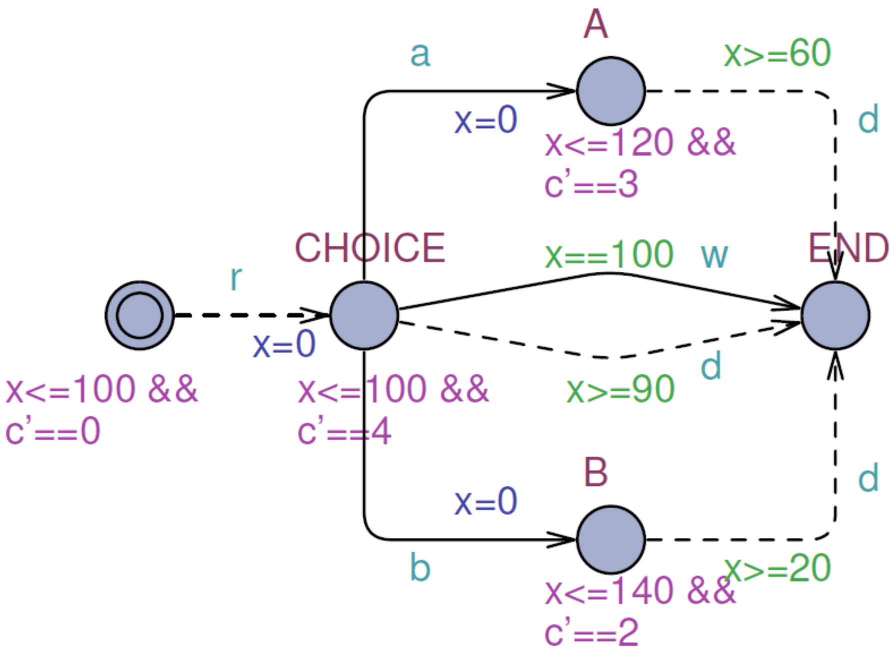
$$\mathbb{E}_{\mu^c}^{\langle \mathcal{G},\mu^u\rangle}(C_G) = \int_{\pi \in Exec^m} C_G(\pi)\mathbb{P}_{\langle \mathcal{G},\mu^u\rangle,\mu^c}(d\pi)$$

Optimal strategy μ^o

$$\mathbb{E}_{\mu^o}^{\langle \mathcal{G},\mu^u\rangle}(C_G) = \inf\left\{\, \mathbb{E}_{\mu^c}^{\langle \mathcal{G},\mu^u\rangle}(C_G) \mid \mu^c \prec \sigma^p(G,T) \,\right\}$$

where $\sigma^p(G,T)$ is the most permissive T time-bounded reachability strategy.

Motivation



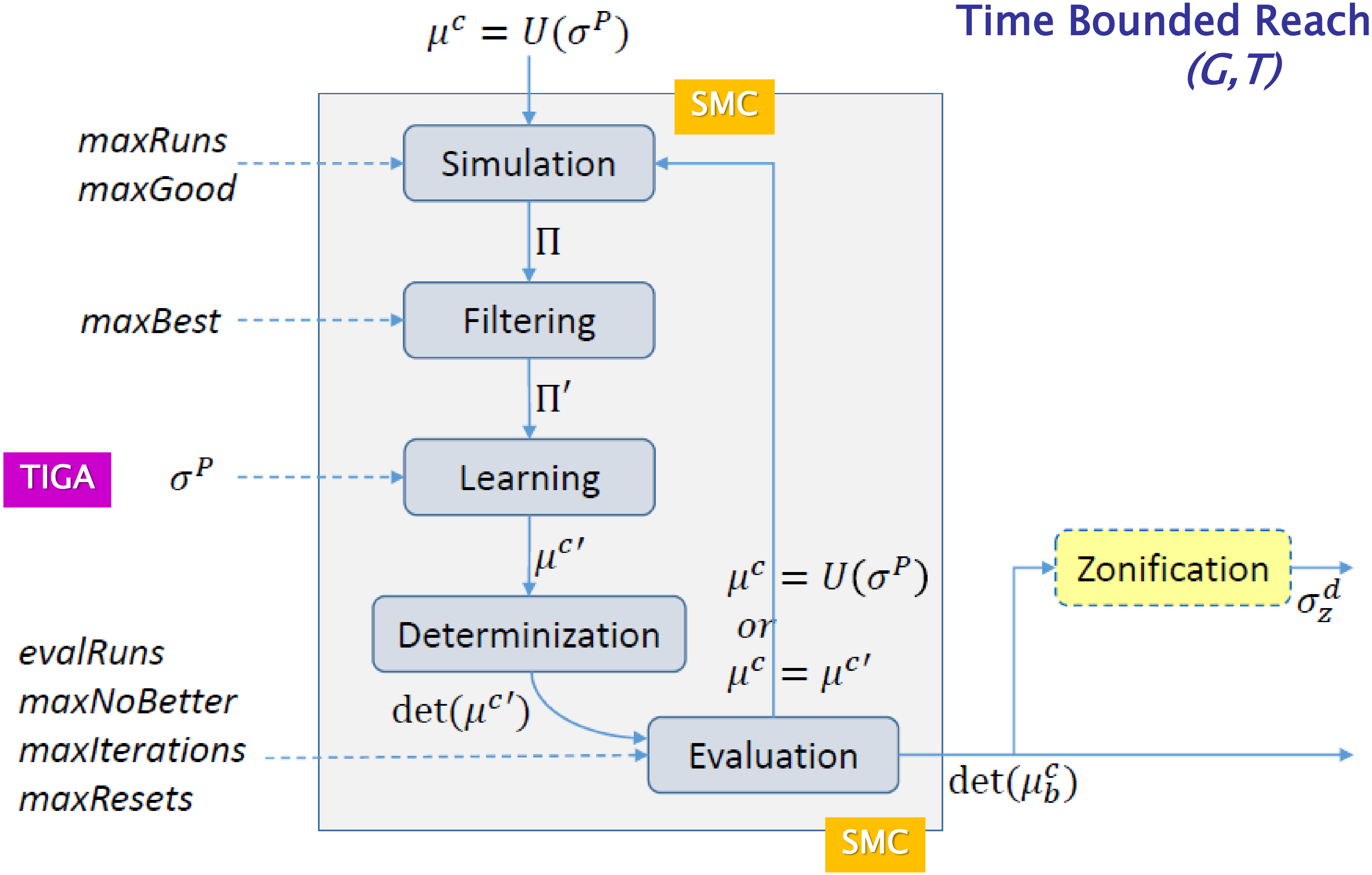
Minimal Expected Cost
Strategy (0,b) **2*80=160**

Expected Cost for TIGA
Strategy (100,w) **4*95=380**

Minimal Expected Cost while guaranteeing END is reached within time 210:

Strat.: t>90→ (100,w)
t>70→ (0,b)
ow → (0,a)
=
204

Reinforcement Learning



Time Bounded Reachability
(G,T)

Strategies

Nondeterministic Strategies (UPPAAL TIGA)

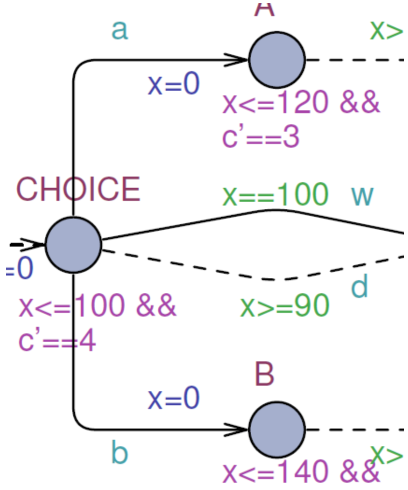
$R_\ell = \{(Z_1, a_1), \ldots, (Z_k, a_k)\}$, where $a_i \in \Sigma_c \cup \{\lambda\}$. Now R represents the strategy σ_R where $\sigma_R((\ell, v)) \ni a$ iff $(Z, a) \in R_\ell$ for some Z with $v \in Z$.

Stochastic Strategies (non-lazy *)

- Urgent: $\mu^c_{(\ell,v)}(d, a) = 0$ if $d > 0$, or
- Wait: $\mu^c_{(\ell,v)}(d, a) = 0$ whenever $\sigma^p(\ell, v + d) \ni \lambda$.

$$\mu^c_{(\ell,v)} : (\Sigma_c \cup \{\mathtt{w}\}) \rightarrow [0, 1].$$

Classes allowing for efficient representation and learning

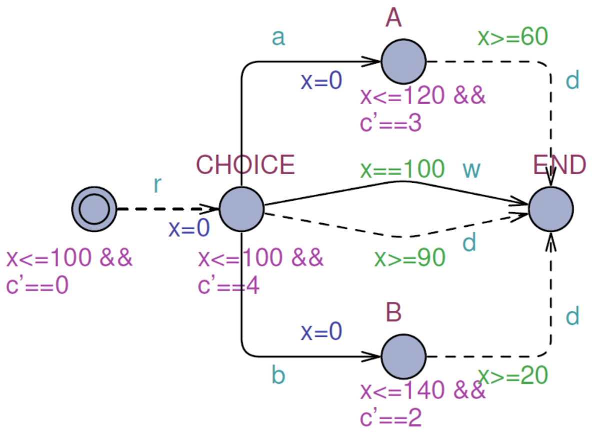


* Non-lazy strategies suffices for DPAs

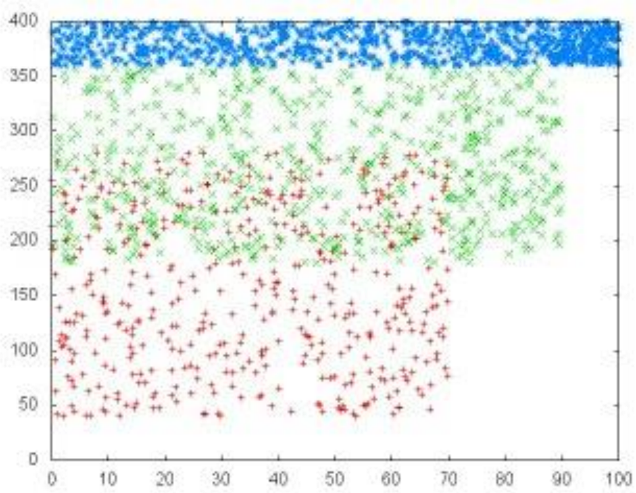
Learning

Given a set of runs Π the relevant information for the sub-strategy μ_ℓ^c is given as In_ℓ:

$$In_\ell = \{(s_n, v) \in (\Sigma_c \cup \mathbb{R}) \times \mathbb{R}^X_{\geq 0} \mid (q_0 \xrightarrow{s_0}_{p_0} \dots \xrightarrow{s_{n-1}}_{p_{n-1}} (\ell, v) \xrightarrow{s_n}_{p_n} \dots) \in \Pi\}$$



C(π)



wait

a

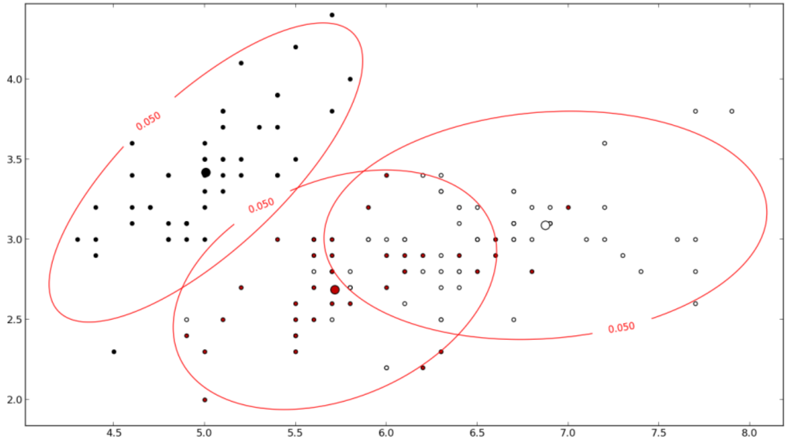
b

time(π)@Choice

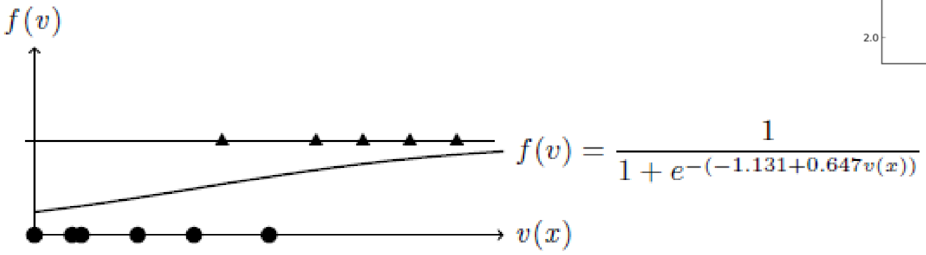
Simulation of $\boldsymbol{Uni}(\boldsymbol{\sigma^p})$ for $A\langle\rangle$ **(END** ∧ **time** ≤ 210)

Strategies – Representation & Manipulation

- Covariance Matrices

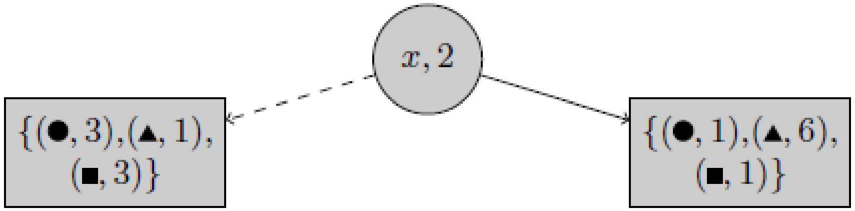


- Logistic Regression



$$f(v) = \frac{1}{1 + e^{-(-1.131+0.647v(x))}}$$

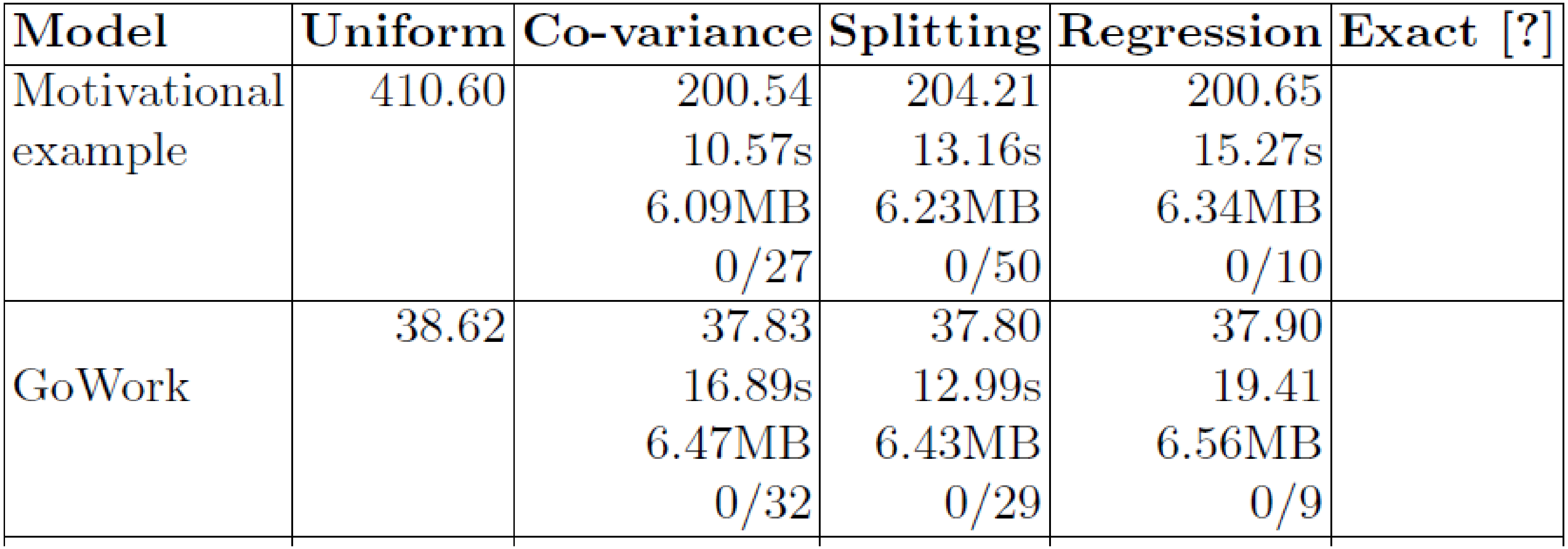
- Splitting



Using
Learning
Determinization

$$\mu^{c}_{(\ell,v)} : (\Sigma_c \cup \{\mathtt{w}\}) \to [0,1].$$

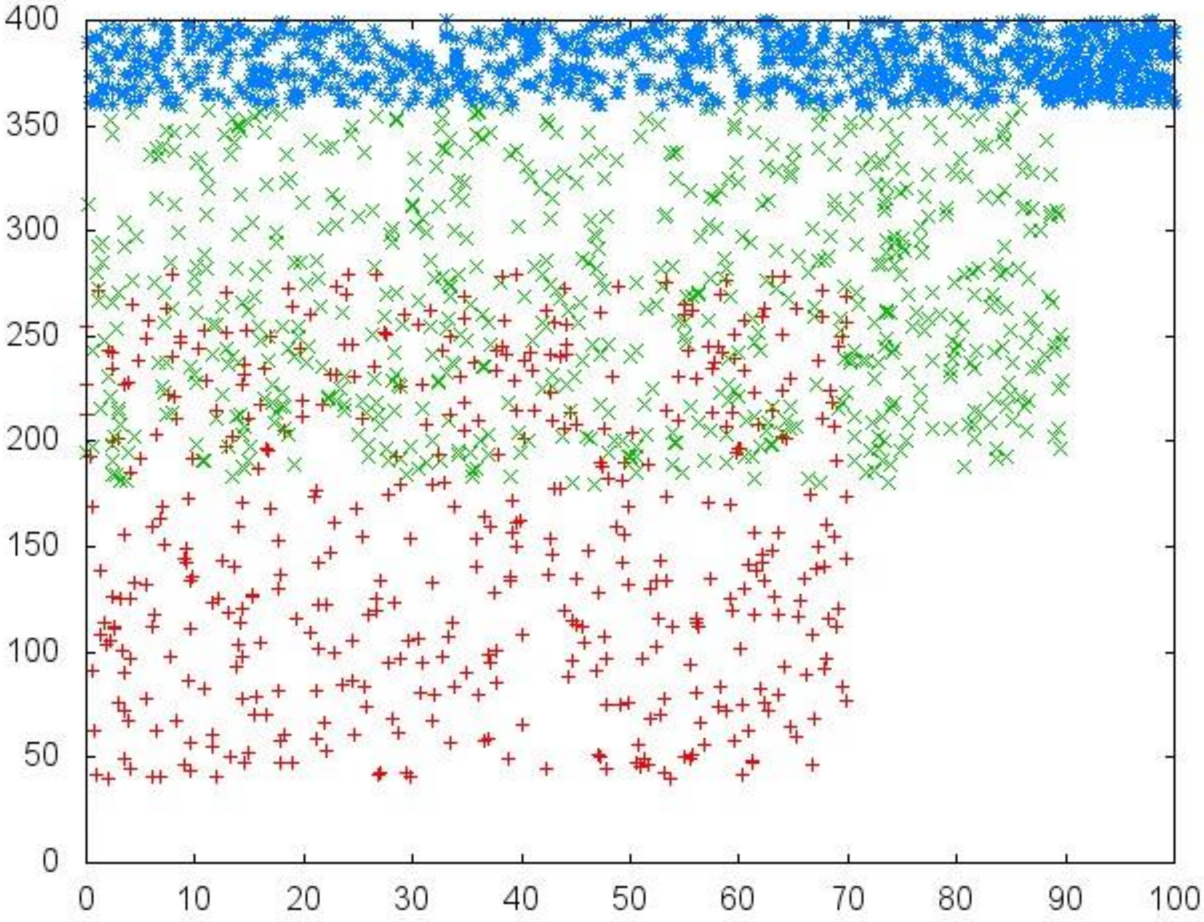
Experiments



Model	Uniform	Co-variance	Splitting	Regression	Exact [?]
Motivational example	410.60	200.54 10.57s 6.09MB 0/27	204.21 13.16s 6.23MB 0/50	200.65 15.27s 6.34MB 0/10	
GoWork	38.62	37.83 16.89s 6.47MB 0/32	37.80 12.99s 6.43MB 0/29	37.90 19.41 6.56MB 0/9	

Learned Strategies

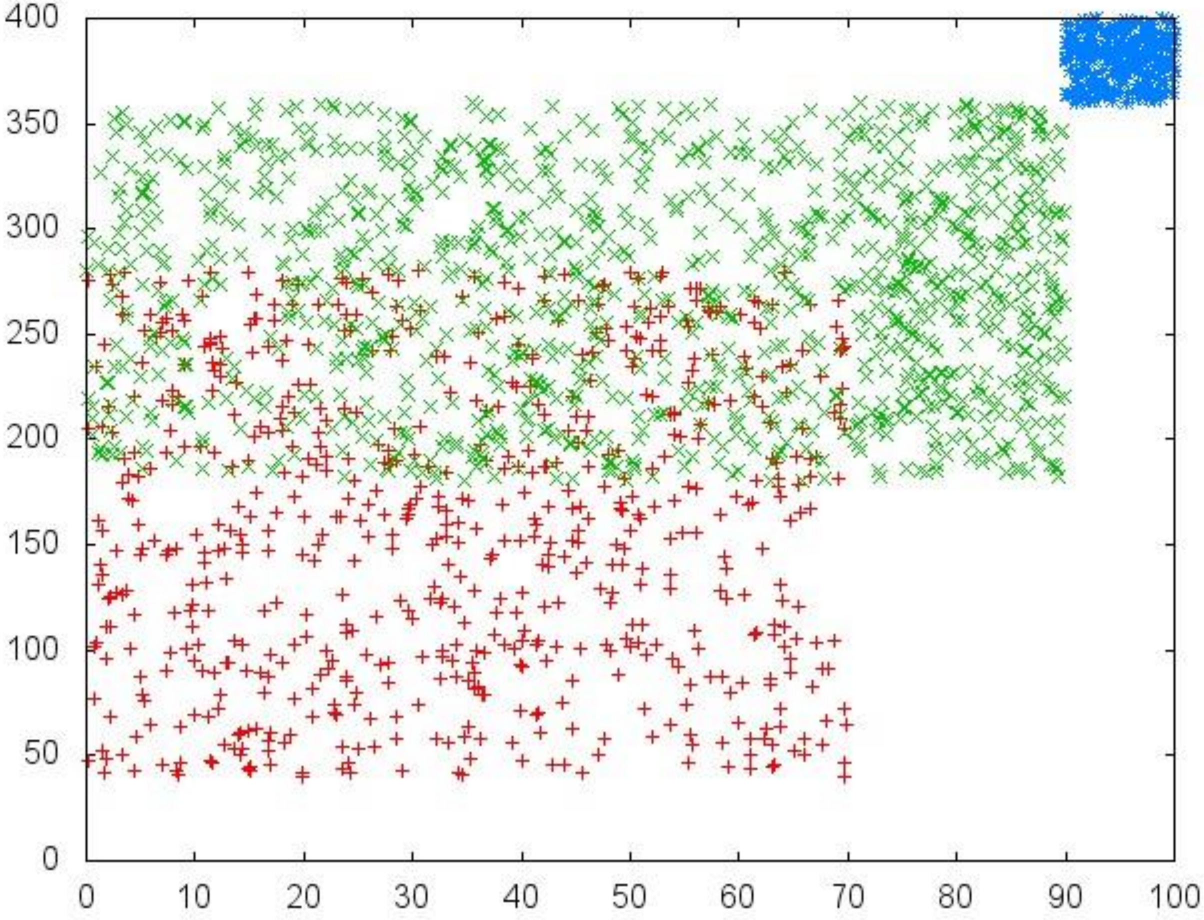
Covariance



e.

Learned Strategies

Covariance



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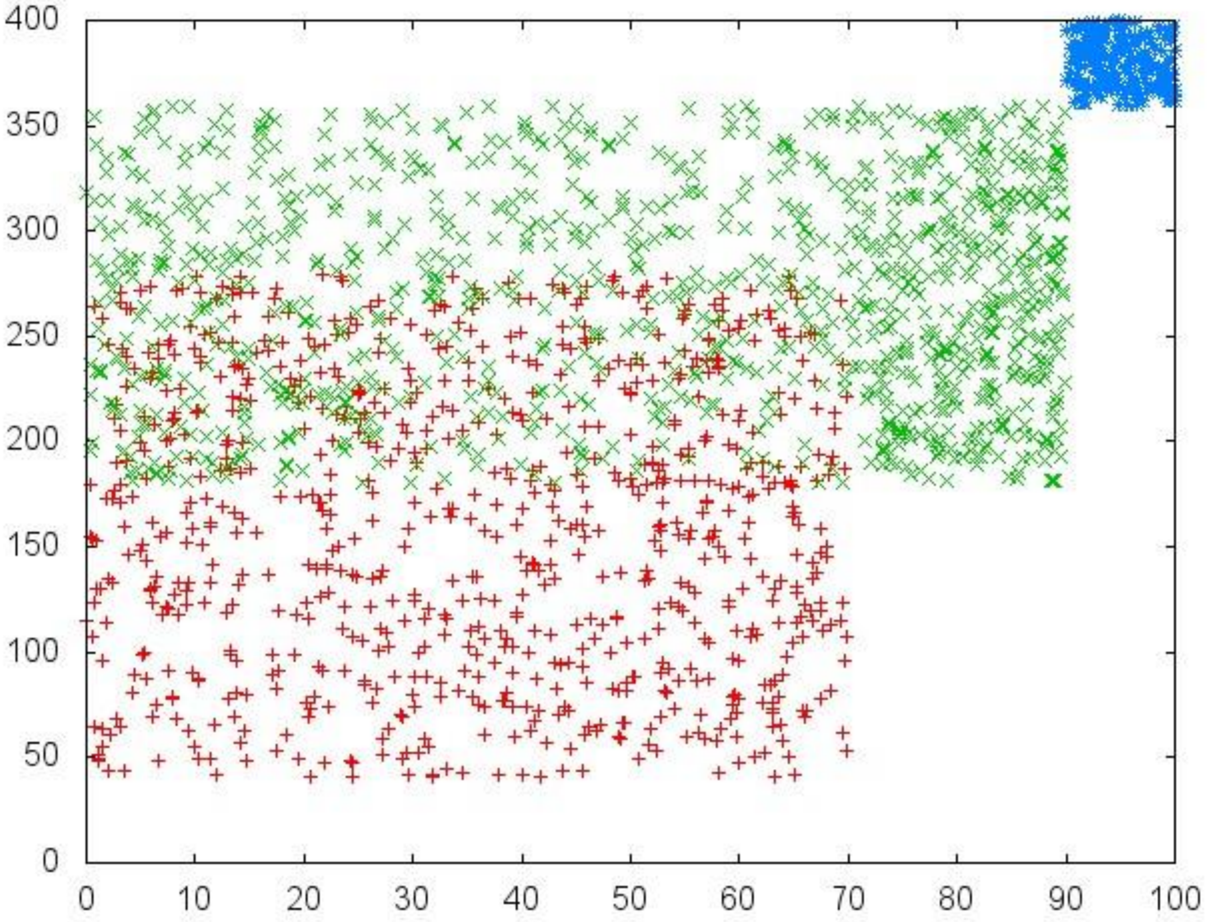
Learned Strategies



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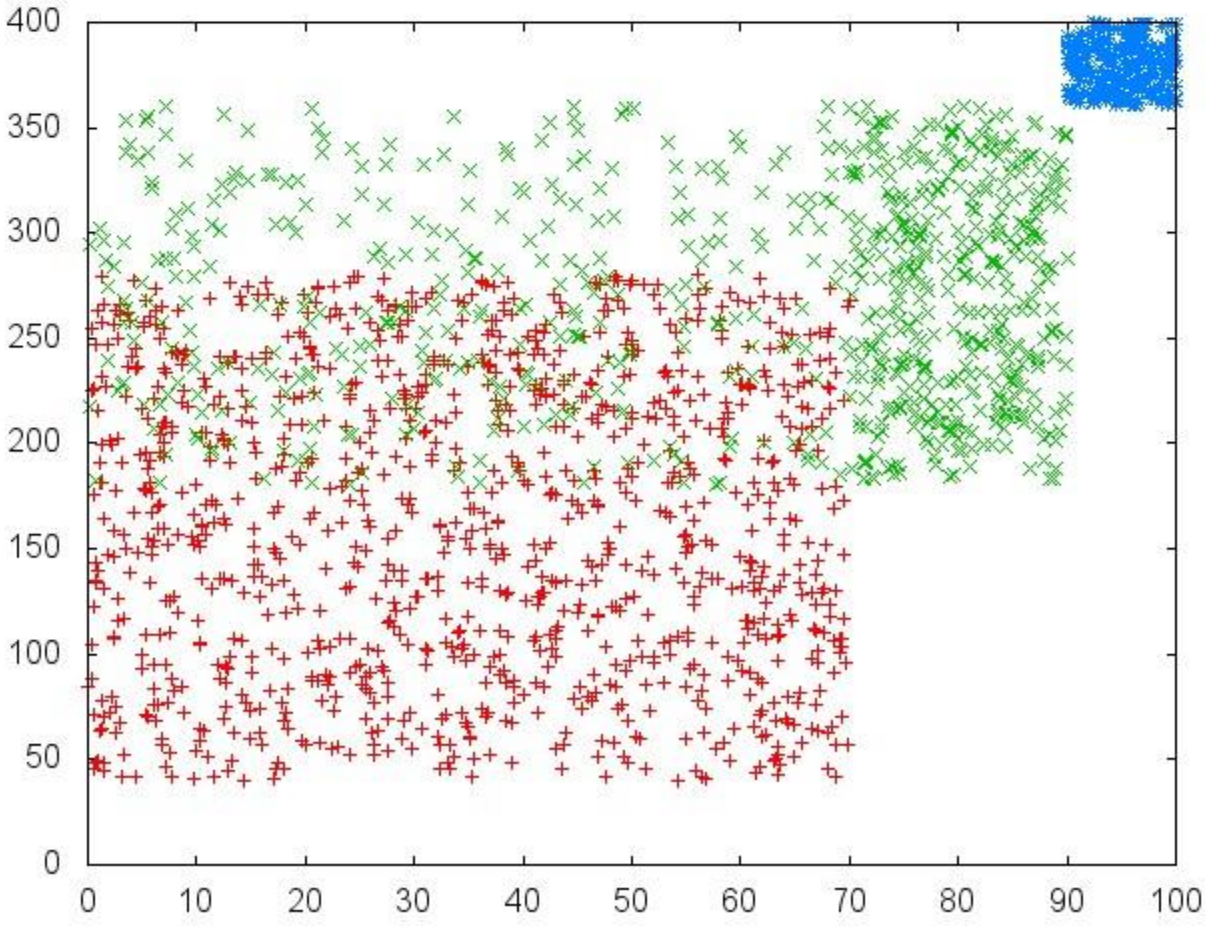
Learned Strategies

Covariance

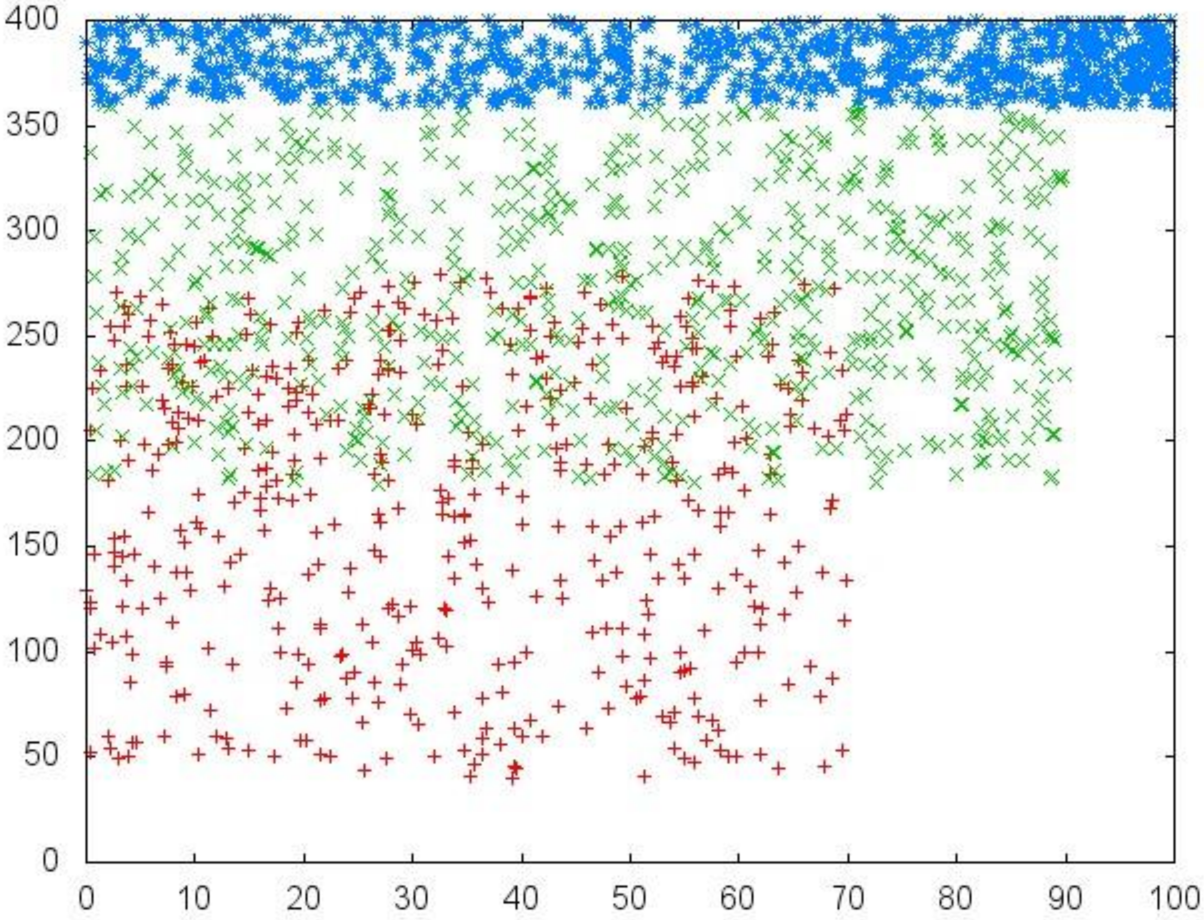


e.

Learned Strategies

Learned Strategies

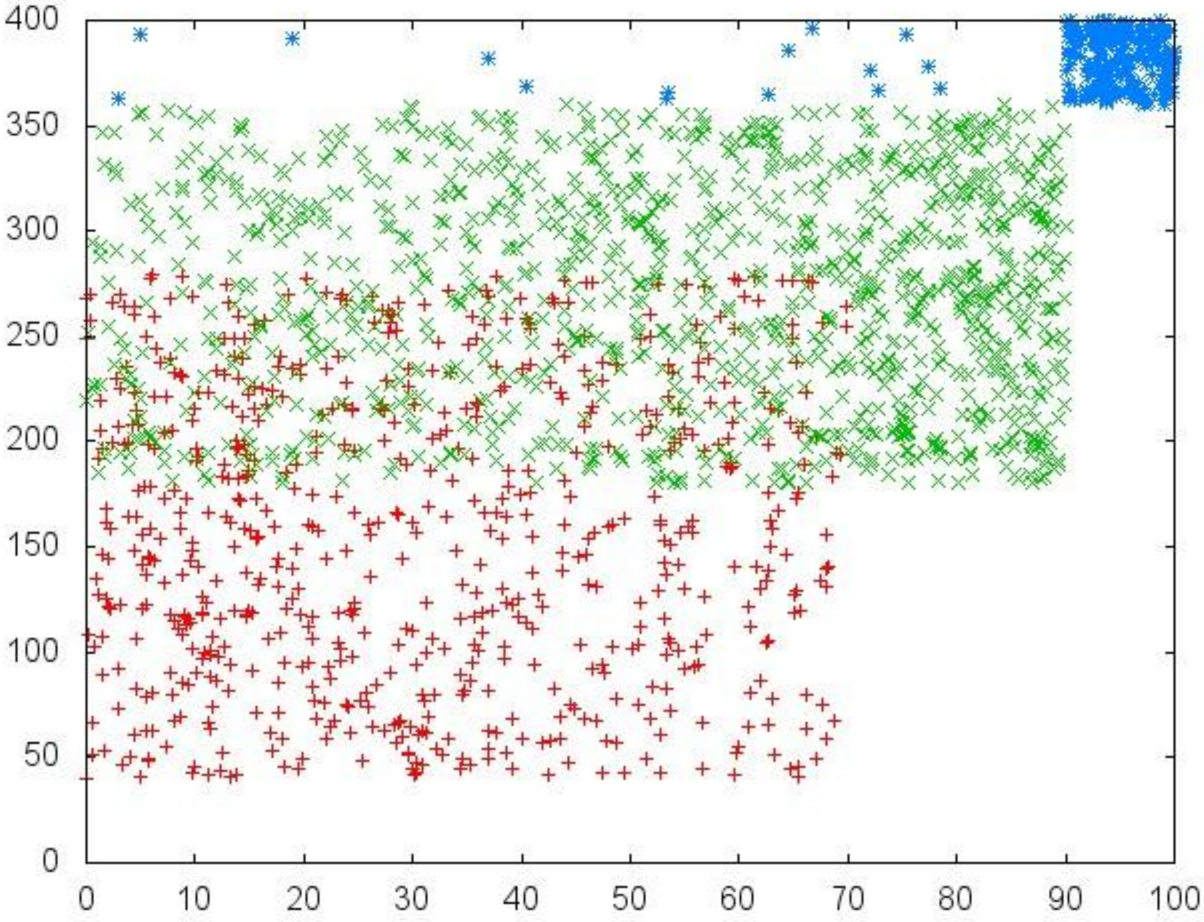
Regression



e.

Learned Strategies

Regression



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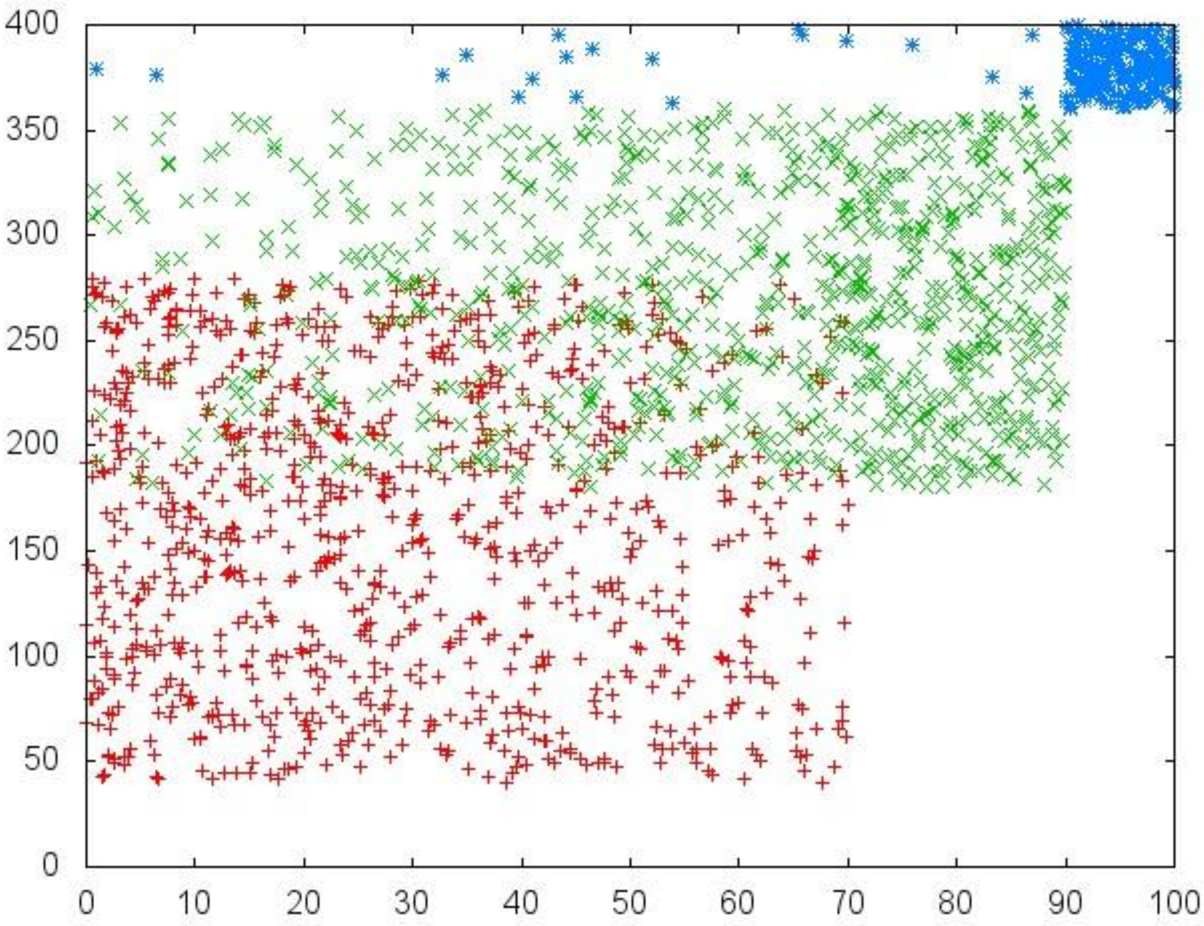
Learned Strategies



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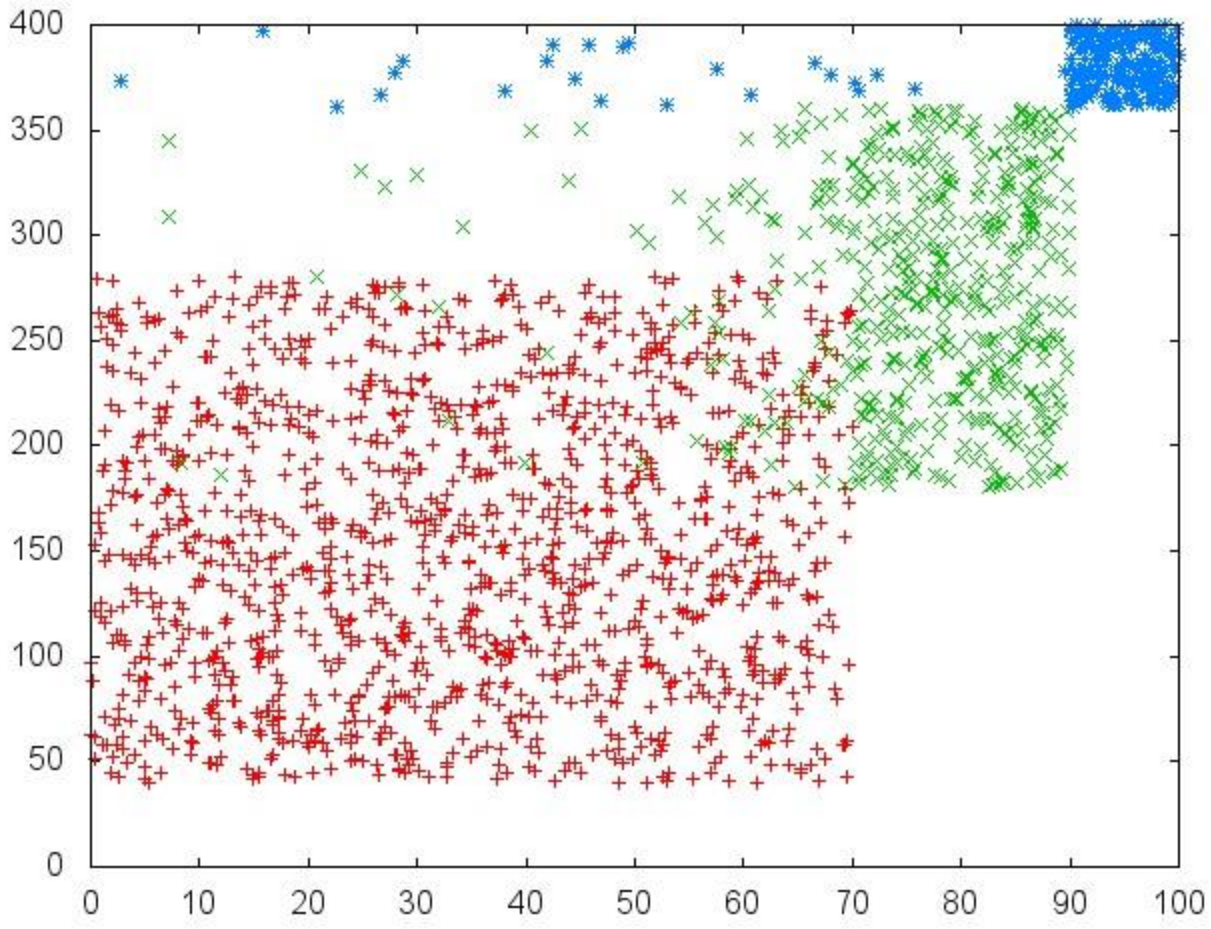
Learned Strategies

Regression



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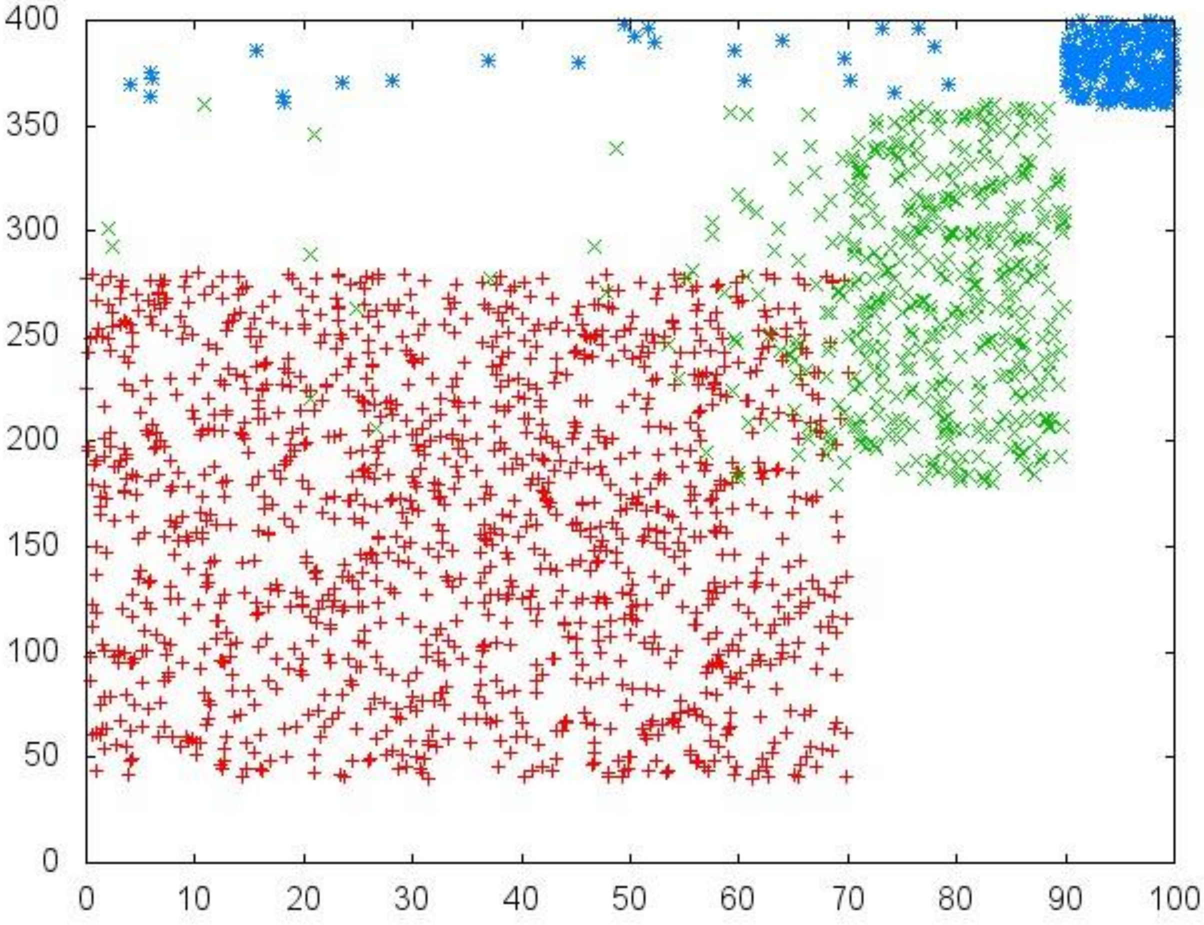
Learned Strategies



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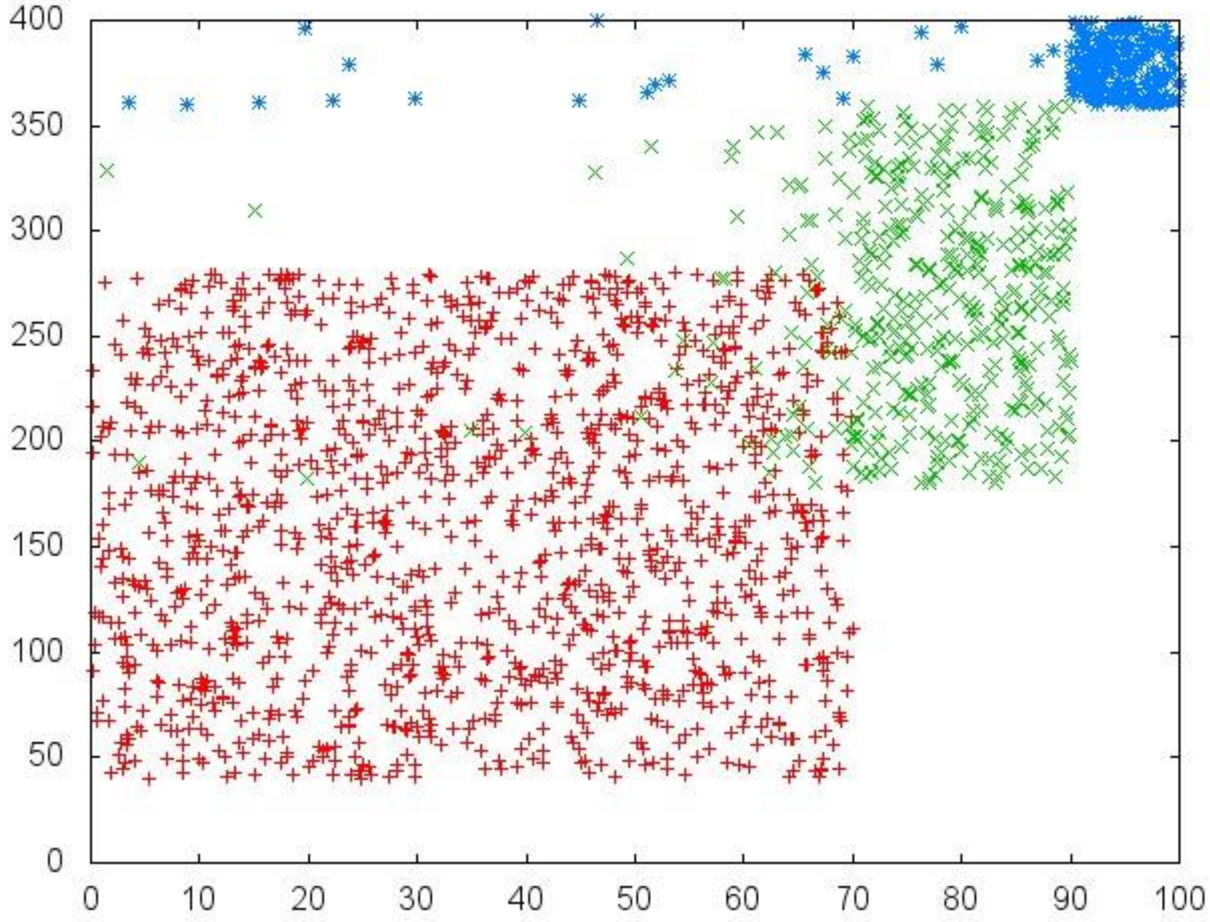
Learned Strategies

Regression



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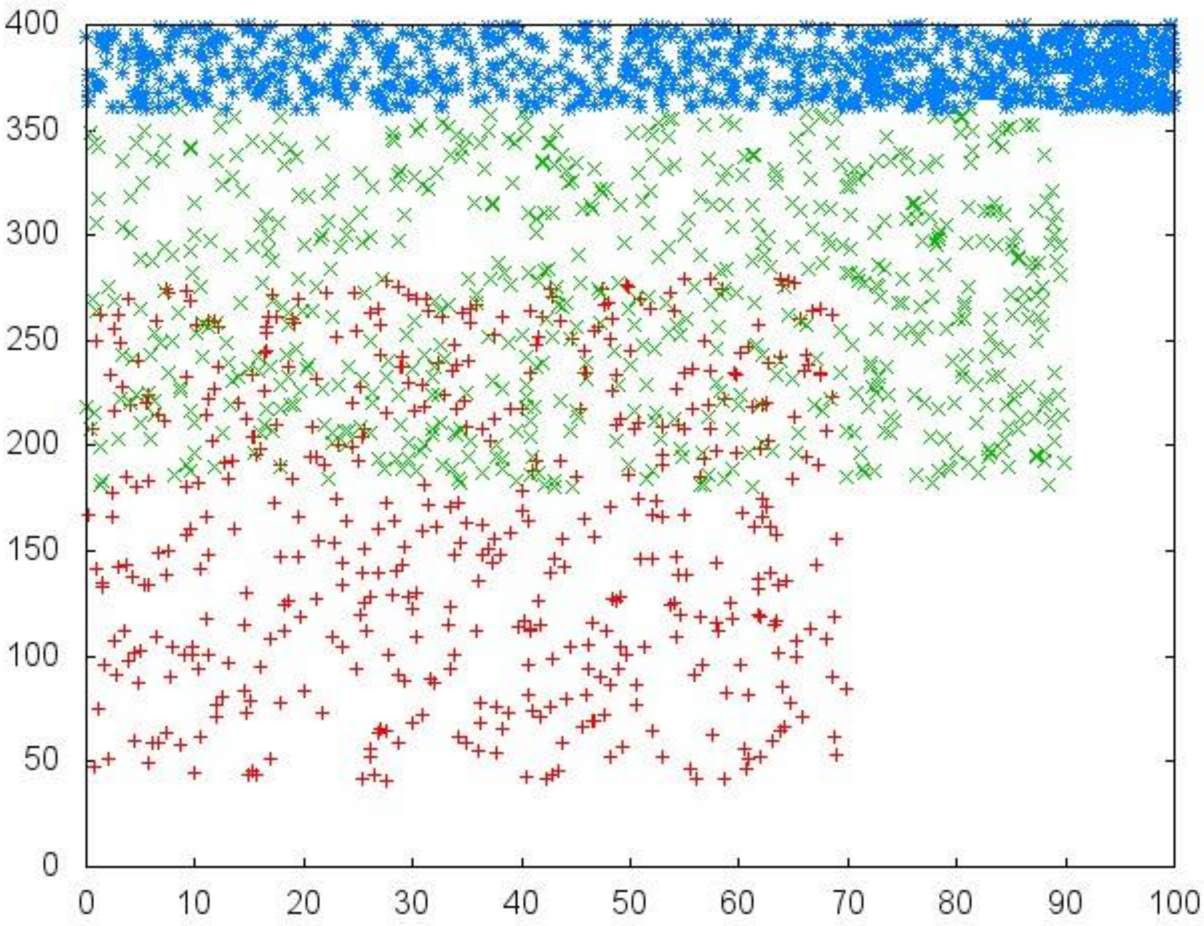
Learned Strategies

Learned Strategies

Splitting



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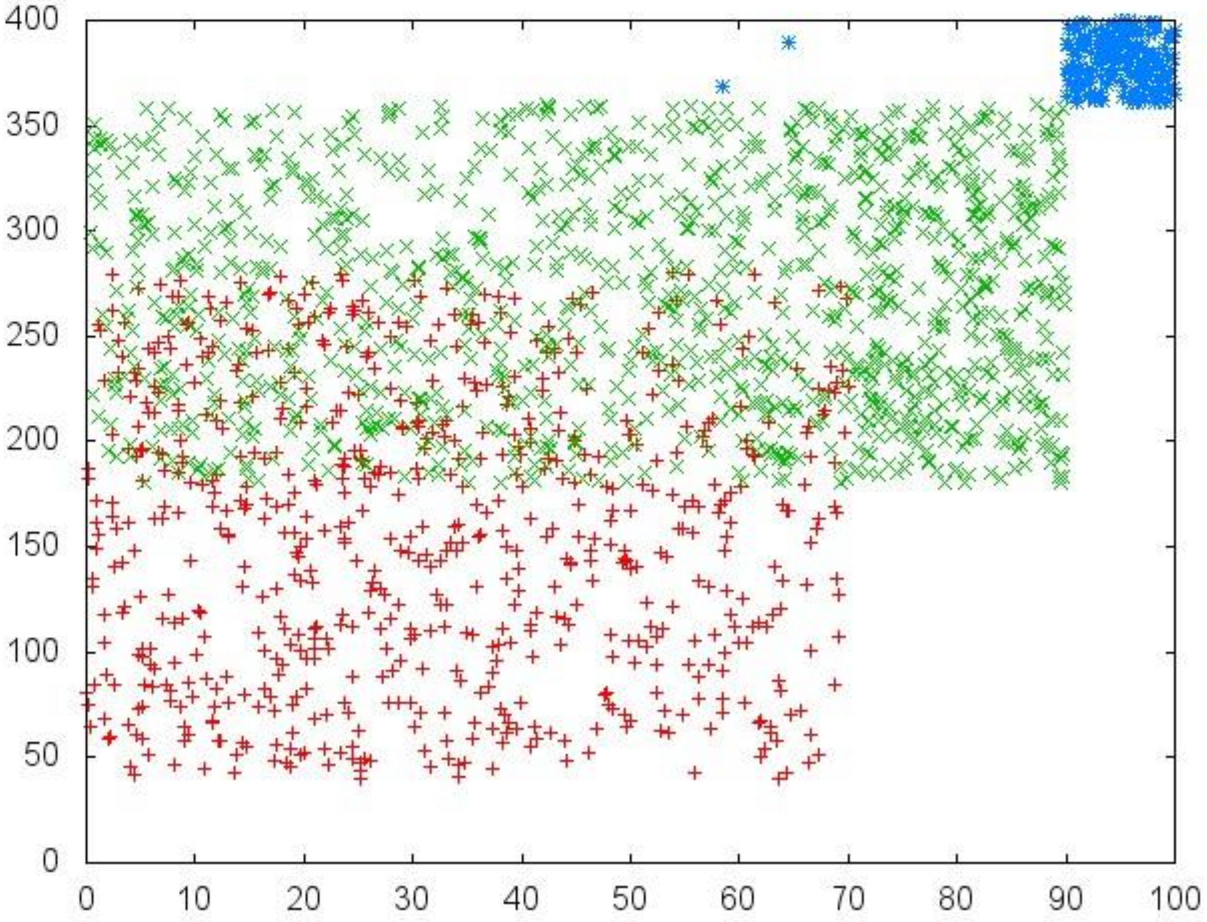
Learned Strategies

Splitting



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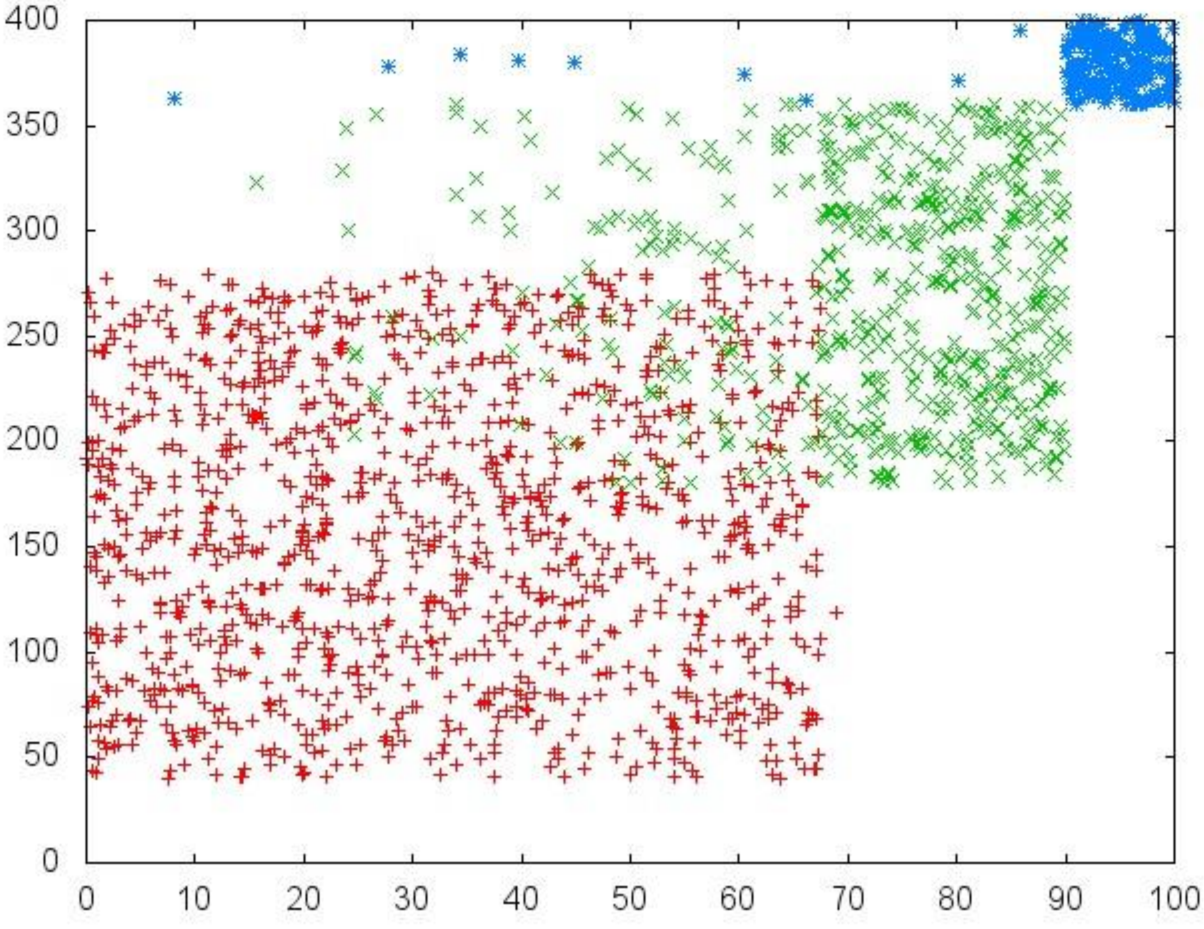
Learned Strategies

Splitting



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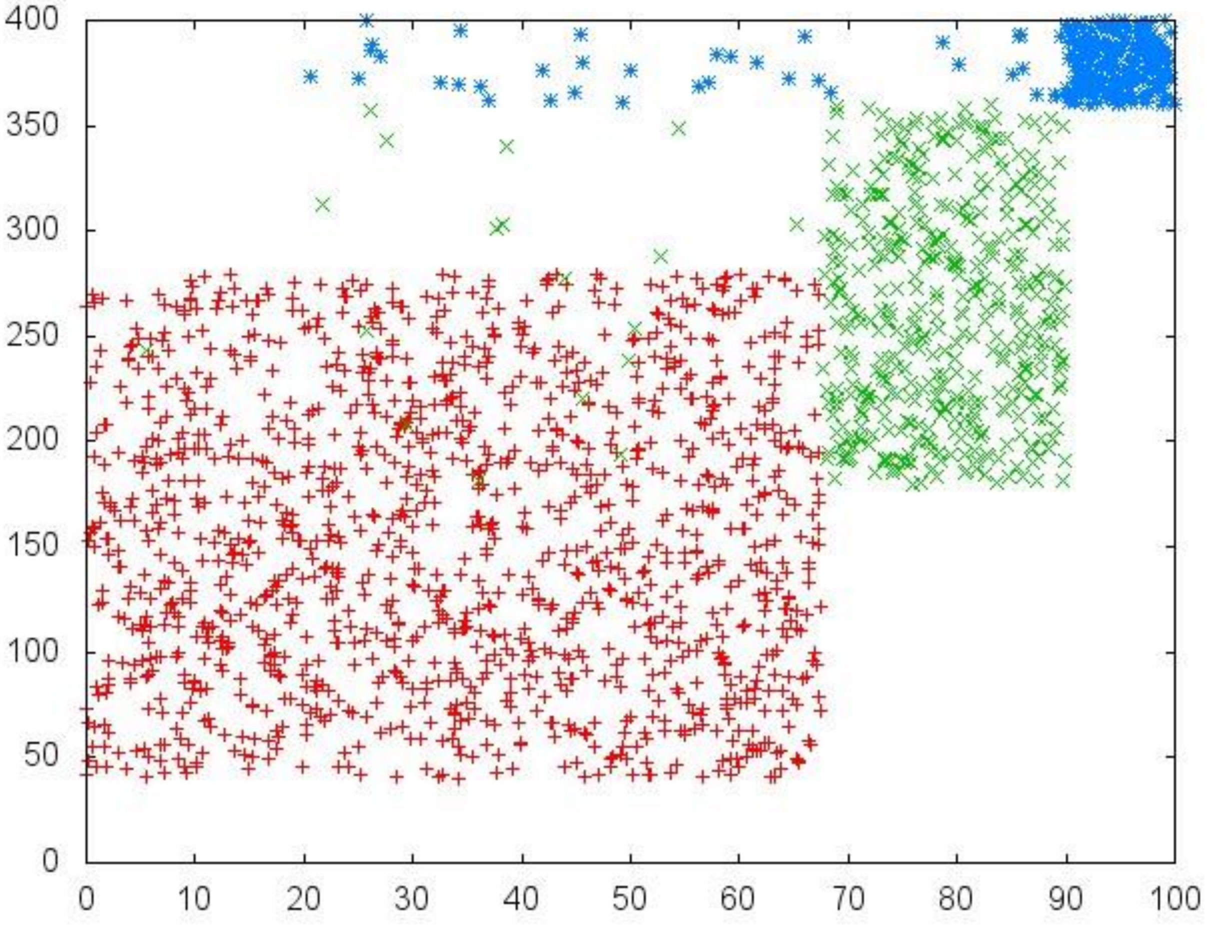
Learned Strategies

Splitting



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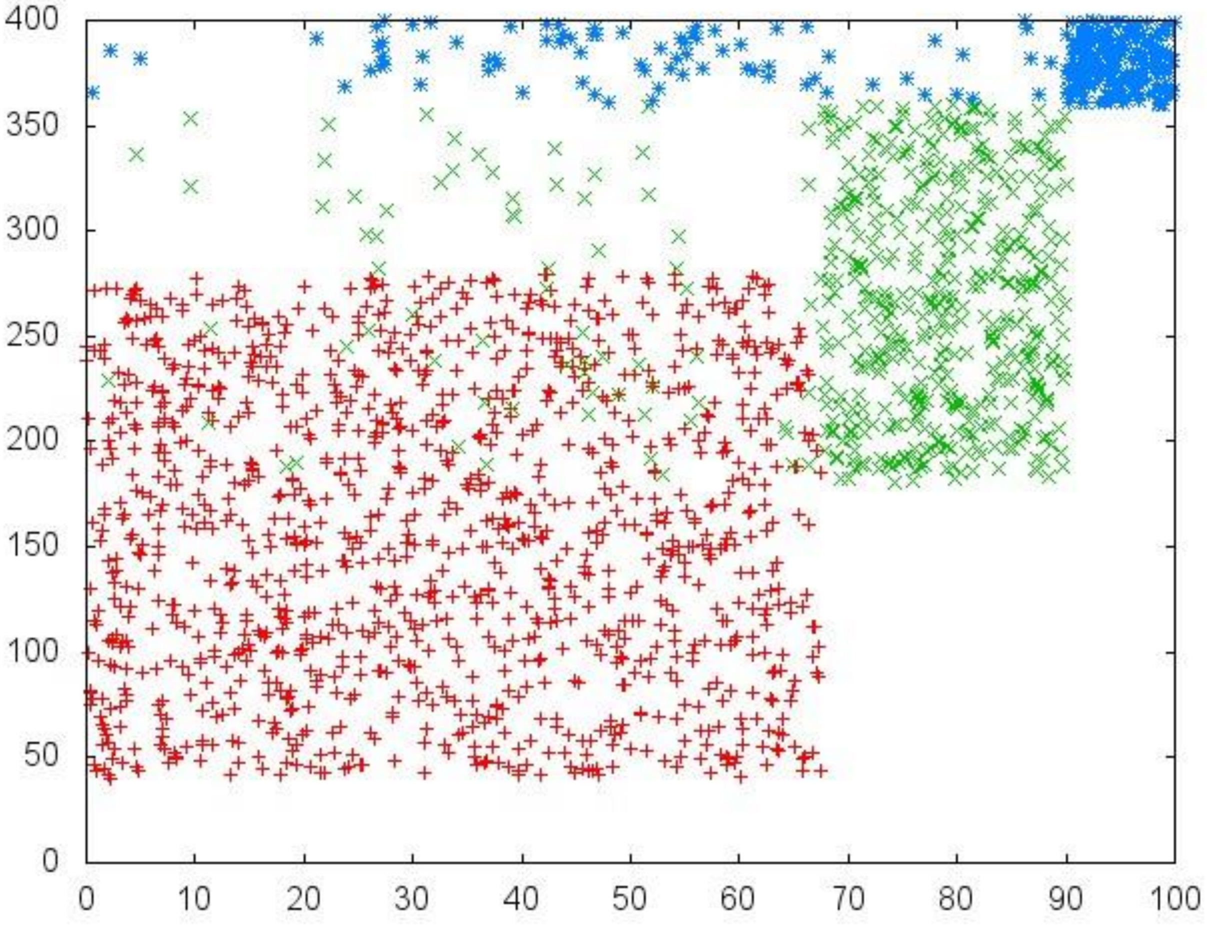
Learned Strategies

Splitting



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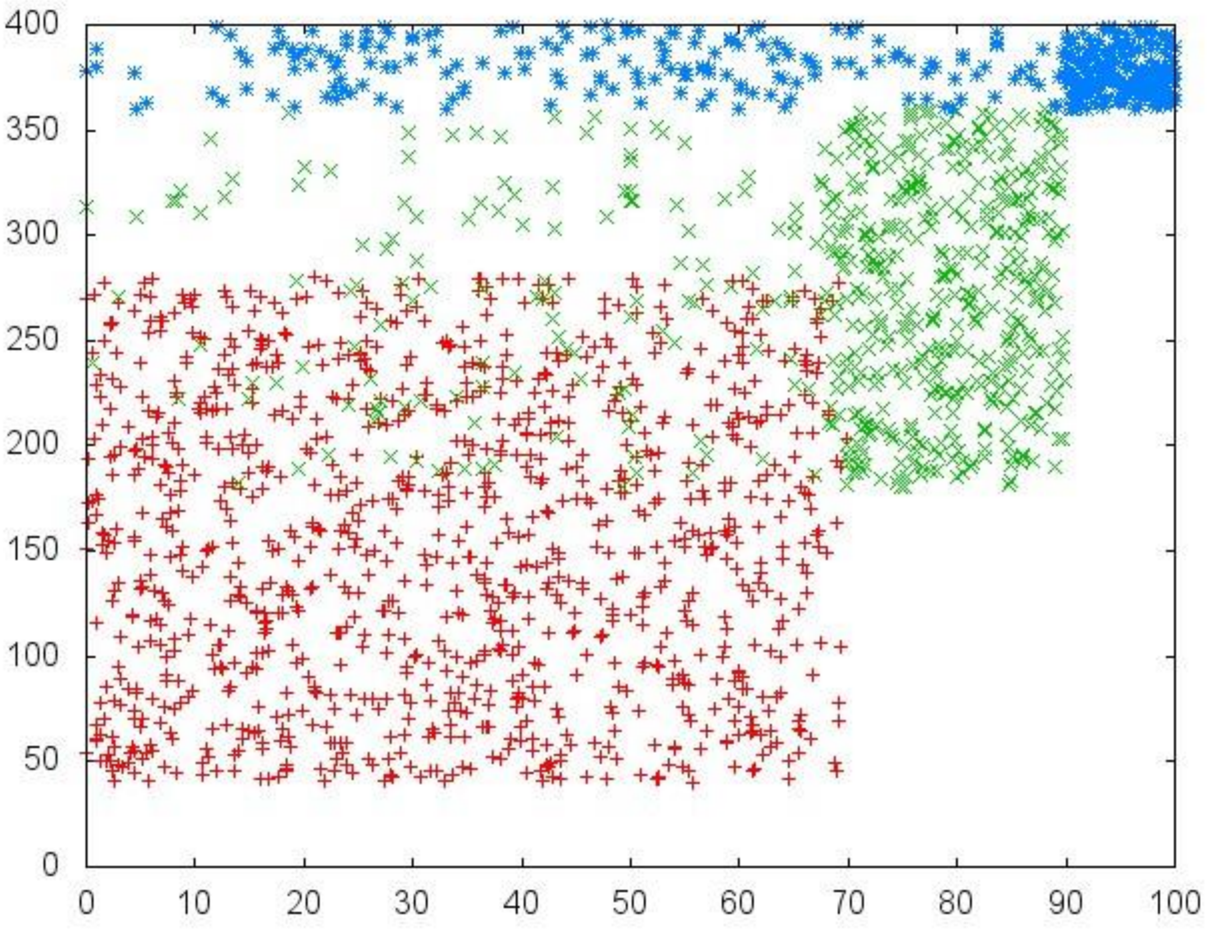
Learned Strategies

Splitting



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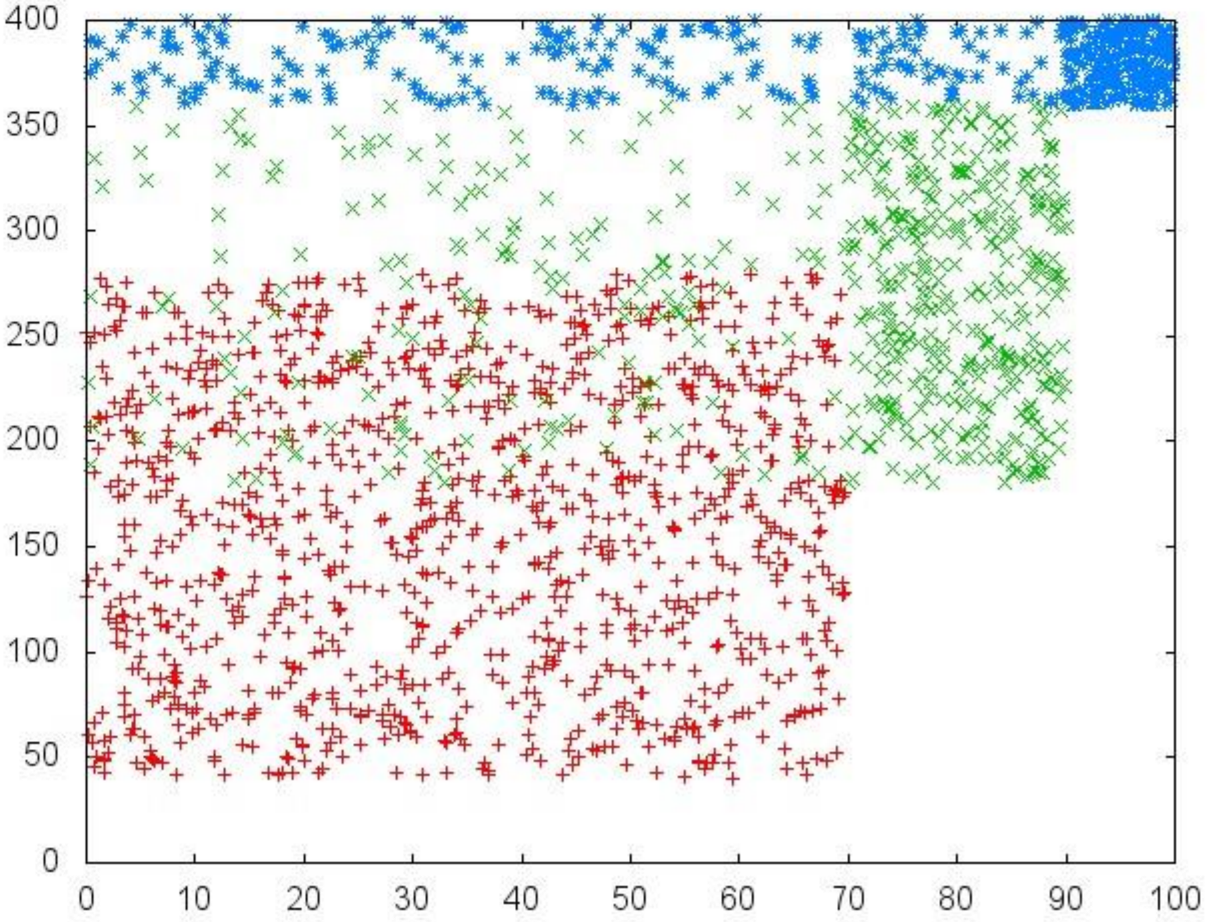
Learned Strategies

Splitting



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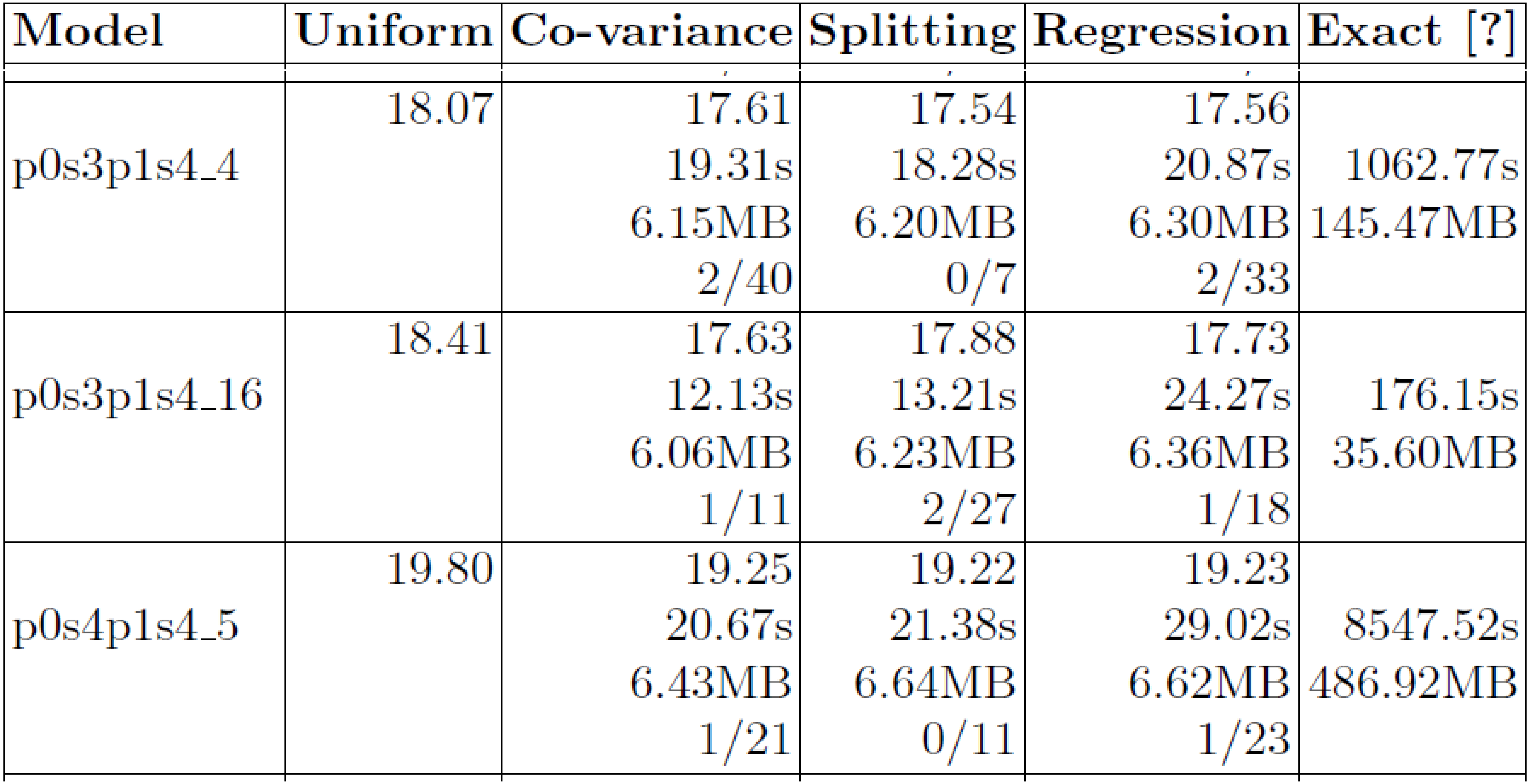
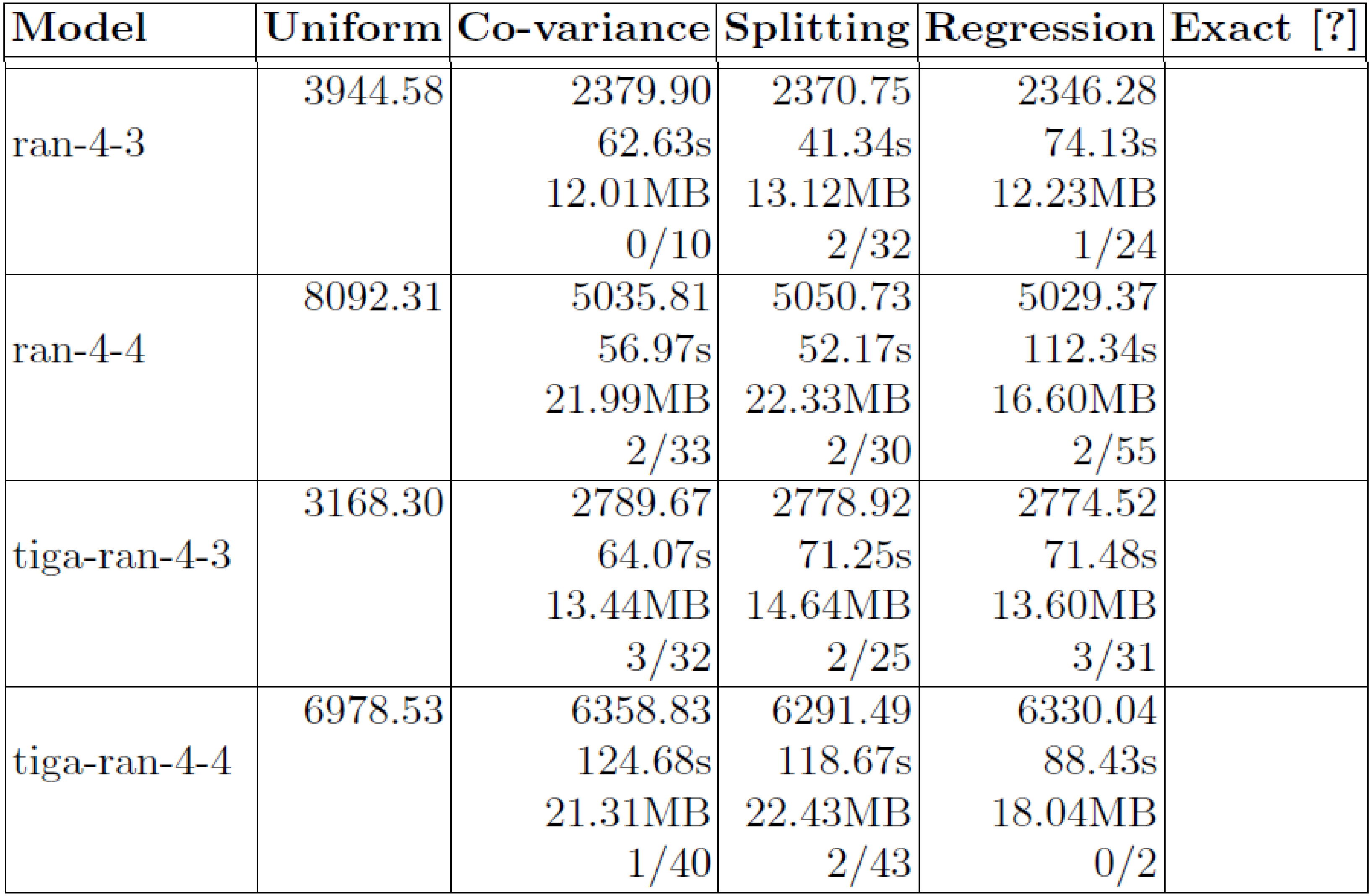
Experiments /DPA



Model	Uniform	Co-variance	Splitting	Regression	Exact [?]
p0s3p1s4_4	18.07	17.61 19.31s 6.15MB 2/40	17.54 18.28s 6.20MB 0/7	17.56 20.87s 6.30MB 2/33	1062.77s 145.47MB
p0s3p1s4_16	18.41	17.63 12.13s 6.06MB 1/11	17.88 13.21s 6.23MB 2/27	17.73 24.27s 6.36MB 1/18	176.15s 35.60MB
p0s4p1s4_5	19.80	19.25 20.67s 6.43MB 1/21	19.22 21.38s 6.64MB 0/11	19.23 29.02s 6.62MB 1/23	8547.52s 486.92MB

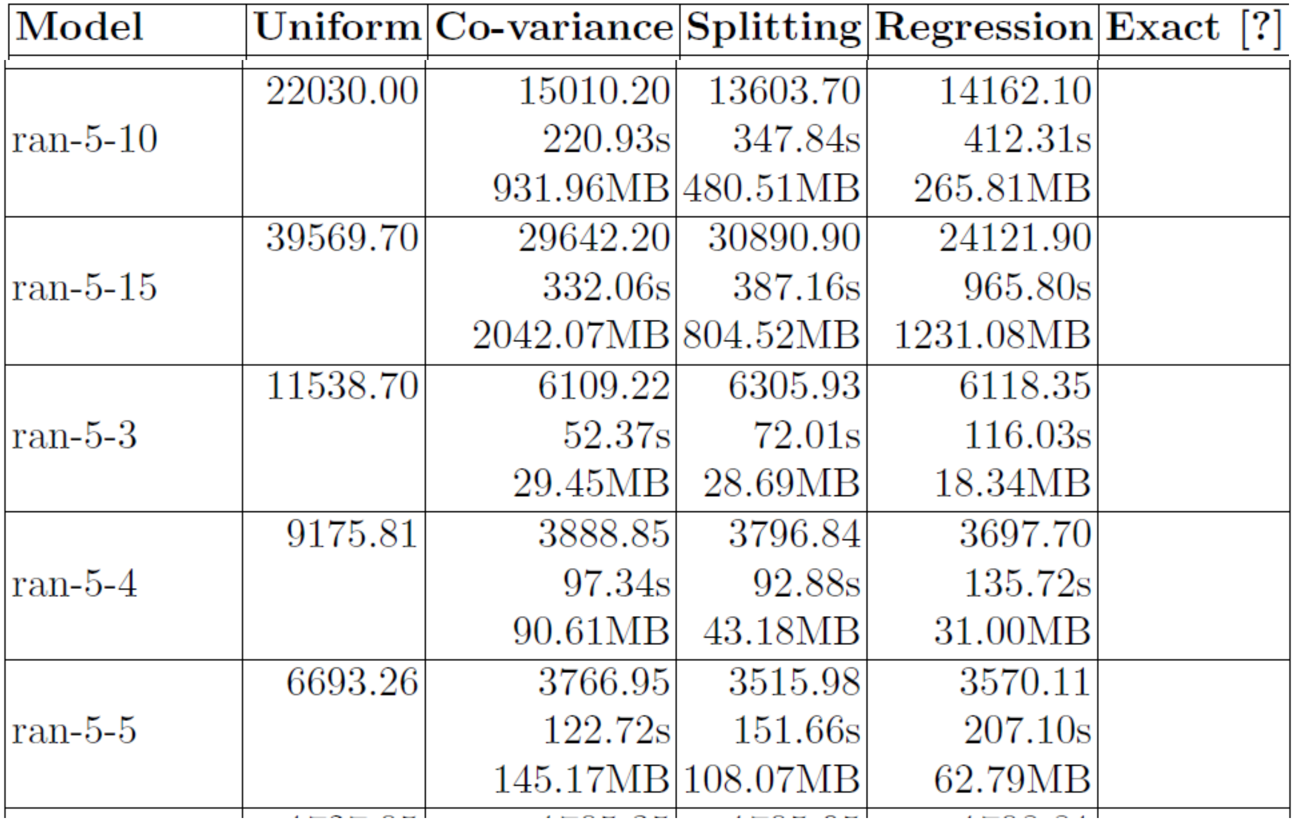
Kempf, J.F., Bozga, M., Maler, O.: As soon as probable: Optimal scheduling under stochastic uncertainty. In: TACAS. pp. 385{400 (2013)
http://www-verimag.imag.fr/PROJECTS/TEMPO/DATA/201304_dpa/

Experiments /DPA Random



Model	Uniform	Co-variance	Splitting	Regression	Exact [?]
ran-4-3	3944.58	2379.90 62.63s 12.01MB 0/10	2370.75 41.34s 13.12MB 2/32	2346.28 74.13s 12.23MB 1/24	
ran-4-4	8092.31	5035.81 56.97s 21.99MB 2/33	5050.73 52.17s 22.33MB 2/30	5029.37 112.34s 16.60MB 2/55	
tiga-ran-4-3	3168.30	2789.67 64.07s 13.44MB 3/32	2778.92 71.25s 14.64MB 2/25	2774.52 71.48s 13.60MB 3/31	
tiga-ran-4-4	6978.53	6358.83 124.68s 21.31MB 1/40	6291.49 118.67s 22.43MB 2/43	6330.04 88.43s 18.04MB 0/2	

Experiments /DPA Random



Model	Uniform	Co-variance	Splitting	Regression	Exact [?]
ran-5-10	22030.00	15010.20 220.93s 931.96MB	13603.70 347.84s 480.51MB	14162.10 412.31s 265.81MB	
ran-5-15	39569.70	29642.20 332.06s 2042.07MB	30890.90 387.16s 804.52MB	24121.90 965.80s 1231.08MB	
ran-5-3	11538.70	6109.22 52.37s 29.45MB	6305.93 72.01s 28.69MB	6118.35 116.03s 18.34MB	
ran-5-4	9175.81	3888.85 97.34s 90.61MB	3796.84 92.88s 43.18MB	3697.70 135.72s 31.00MB	
ran-5-5	6693.26	3766.95 122.72s 145.17MB	3515.98 151.66s 108.07MB	3570.11 207.10s 62.79MB	

Conclusion & Future Work

- Efficient synthesis of strategies for PTMDP ensuring time-bounds and minimizing expected cost.
- If not time-bound needed we can omit the UPPAAL TIGA synthesis
- Extension to Hybrid MDPs utilizing UPPAAL SMCs support for SHAs.

- Make TIGA/SMC available to you!
- Datastructures supporting general stochastic strategies – not just non-lazy ones.
- More clever filtrations of runs.