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MUSTARD WATCHES :

AN INTEGRATED APPROACH TO TIME AND FOOD.

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MUSTARD WATCHES :

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extended abstract

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The paper introduces the concept of mustard watch, a common generalisation of the concepts of watch and of mustard pot. The main property of mustard watches is that they can deliver mustard in any desired quantity (theorem 1) and still display time with a precision of 30 seconds (theorem 2). But the real superiority of mustard watches over classical ones is expressed by our theorem 3 : a mustard watch with no mustard in it is at least as precise as an ordinary one.

Classical watches display time, but can hardly do anything else. This limitation is artificial : for instance several people confessed to be often in want of mustard... and what is the point of knowing time if you cannot get mustard ?

The concept of mustard watches comes from this basic observation. They combine the advantages of watches and mustard pots : a quite accurate display of time together with the critical amount of mustard sufficient to cope with emergencies.

A (seemingly) similar remark was made by Pr. Costa di Meno from the university of La Sentenza [1]. Unfortunately his proposal of *ketchup watches* somehow misses the point : whereas Dijon mustard is so strong that we can content ourselves with very small quantities, a "ketchup watch" should be big enough to harbour at least one teaspoonful of ketchup. It would be more realistic to call those "watches" *clocks* for want of portability.

yann-joachim ringard

1. Mustard watches : definitions and main results

Unless otherwise stated we use the term "watch" as an abbreviation for "analog pocket watch".

DEFINITION 1

Let W be a classical watch ; a *mustard watch* derived from W is any W' obtained from W by adding a certain amount of mustard in the mechanism. (see fig. 1)

It is immediate from the definition that a given classical watch may have several extensions into a mustard watch, whereas given a mustard watch W' there is only one underlying classical watch W from which W' is derived.

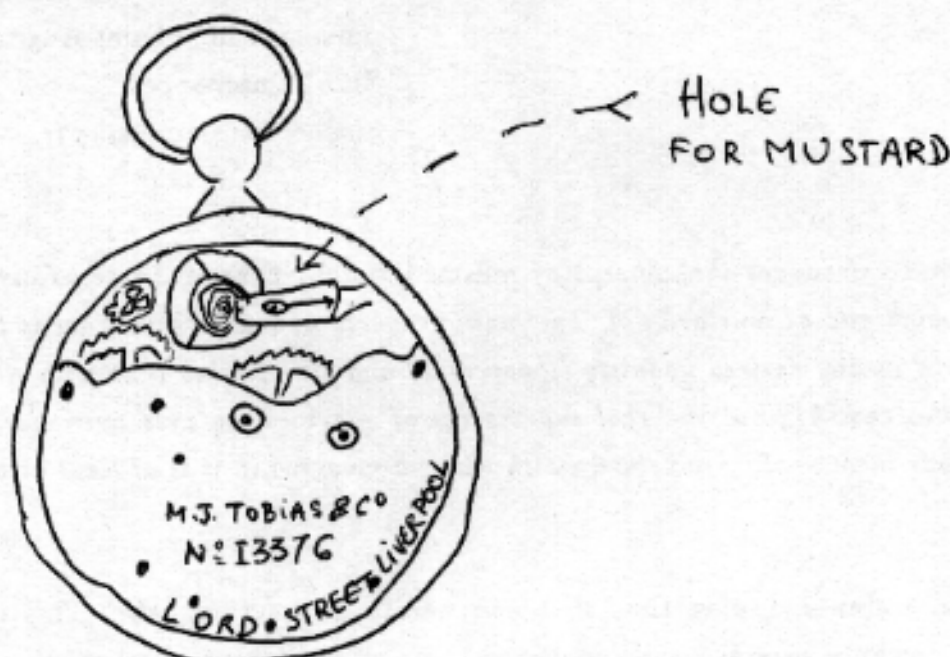


figure 1 : mustard watch just before refill

DEFINITION 2 :

A mustard watch is said to be *proper* when the amount of mustard in it is non zero ; it is said to be *degenerated* otherwise.

THEOREM 1 :

It is possible to get "as much mustard as wanted" from a mustard watch. More precisely, given any amount m of mustard, there is a mustard watch $W(m)$ containing mustard in quantity m .

PROOF : first observe that there are classical watches in any size ; in particular let W_0 be one with enough room in the mechanism to harbour m grams of mustard. Consider the completion $W(m)$ of W achieved by adding m grams of mustard to W . QED

THEOREM 2 :

A proper mustard watch can display time with a precision of 30 seconds.

PROOF : Let W be a proper mustard watch and let $T = (x, y, z)$ be the current value of time at the moment of display. Let $T_0 = (x_0, y_0, z_0)$ be the time displayed by W . Reset W , i.e. change (x_0, y_0, z_0) into $T_1 = (x_1, y_1, z_0)$ in such a way that $|T_1 - T_0|$ is less than 30 seconds. (This operation is made possible by the observation that z_0 is constant during the resetting of a proper mustard watch.). **QED**

THEOREM 3 :

There are mustard watches at least as precise as classical ones.

PROOF (sketch) : we must show that given any classical mustard watch W there is a mustard watch W_c with at least the same precision as W . The proof uses temporal logic Ω 7.2 : let W be a classical watch and let T be the theory of W in Ω 7.2. . Consider now the mustard watch W_c obtained from W by adding no mustard at all and let T_c be the theory of W_c in the same Ω 7.2. (but with an additional unary predicate M for mustard). It is possible to show, using cut-elimination, that T_c is conservative over T . **QED**

REMARK : the superiority of mustard watches over classical ones is clearly established by our proof, since *degenerated* mustard watches (like W_c) are already at least as good as classical watches.

2. Mustard watches : foundational issues

As pointed out by Pr. Costa di Meno (private communication), our theorem 2 is slightly problematic from the viewpoint of foundations : how do you know the time of display (x, y, z) ? Another watch, called a *metawatch* is needed. Now usual metawatches are degenerated, and the sensible thing to do is to make the metawatch benefit from the adjunction of mustard, in which case a *metametawatch* is required... In fact this process can be iterated so as to get a *predicative* hierarchy of watches indexed by ordinals up to Γ_0 . This opposes to the impredicative approach whose limitations are expressed by the following theorem :

THEOREM 4 :

A mustard watch which is its own metawatch is degenerated.

PROOF (sketch) : the basic idea is to imitate the proof of Gödel's second incompleteness theorem. In fact using a fixed point construction it is possible to show that in such a watch the value z_0 (seconds) cannot be kept constant during resetting, contradicting the conclusion of theorem 2. The use of a new temporal logic Ω 7.2.4. (recently introduced by the author, see [2]) is requested to take care of internalisation steps. **QED**

REMARK : from theorem 4 one could expect the notion of mustard watch to be undecidable ; very surprisingly this is not the case, and there are very efficient algorithms to decide whether or not a watch is proper. One for instance is based on the fact that a proper mustard watch is silent.

REFERENCES

- [1] Costa di Meno, Pandolfo : *On a new way of measuring time*, *Protocolli di Logica*, vol. 27 (1989).
- [2] Ringard, Yann-Joachim : *Five new temporal logics*, *Bordereaux de Logique Formelle*, vol. 48 (1990).