On the Complexity of Path Checking in Temporal Logics

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Given LTL formula $(\mathrm{X}p) \wedge ((\mathrm{G}q)\ \mathrm{U}\ r))$

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and finite trace

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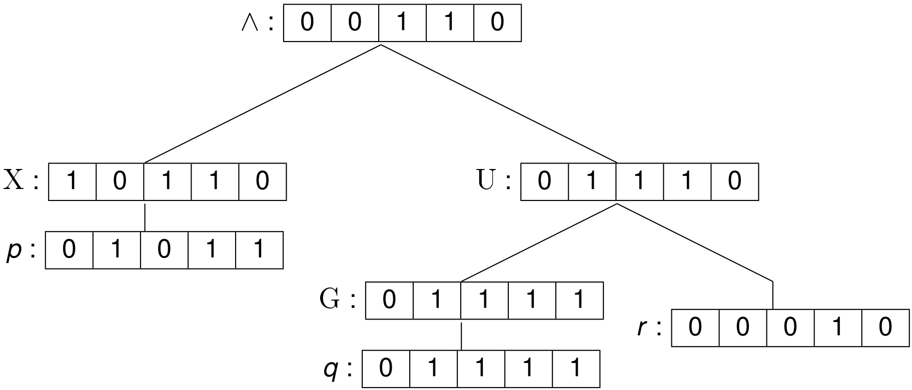
Does it satisfy the formula? What is the complexity of checking?

$$(\mathrm{X}p) \wedge ((\mathrm{G}q)\,\mathrm{U}\,r))$$



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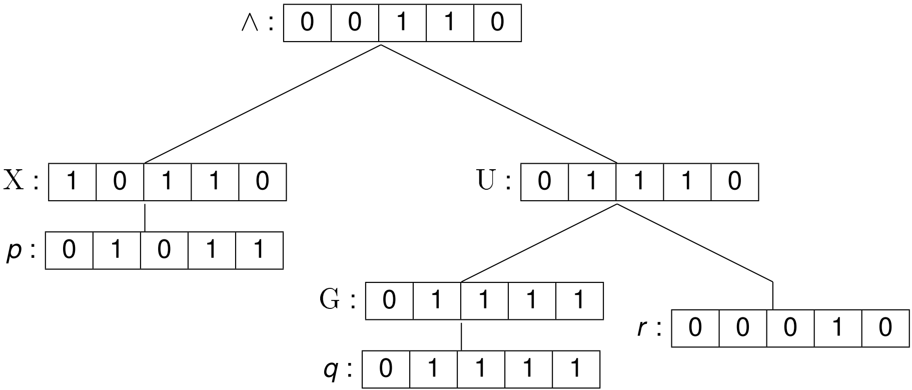

Upper bound: polynomial time: $O(|\mathrm{formula}||\mathrm{trace}|)$.

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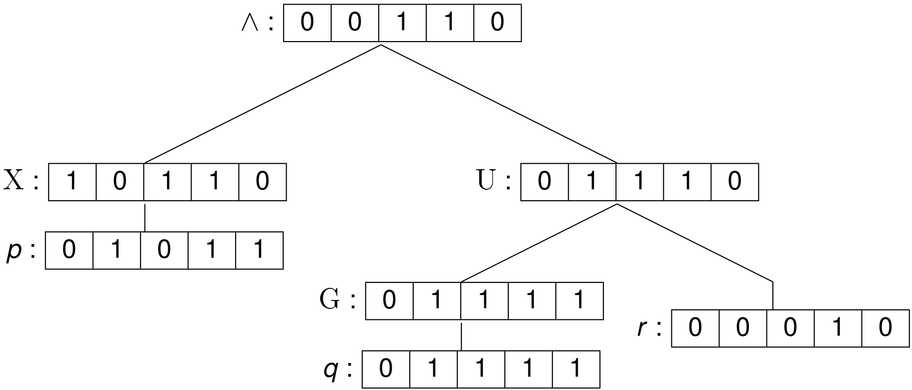

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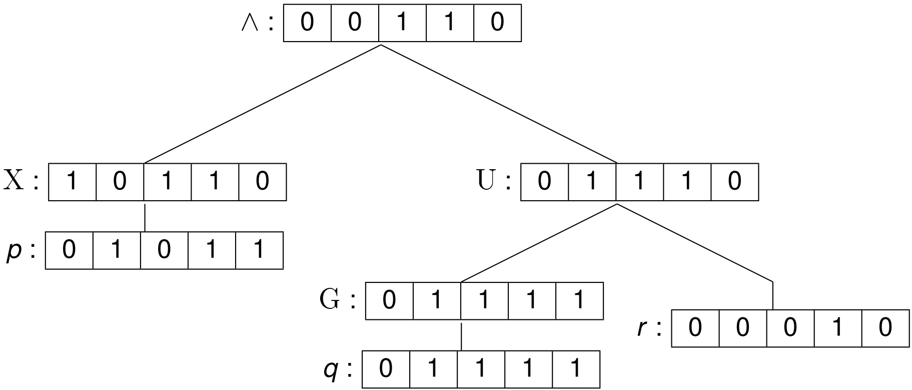

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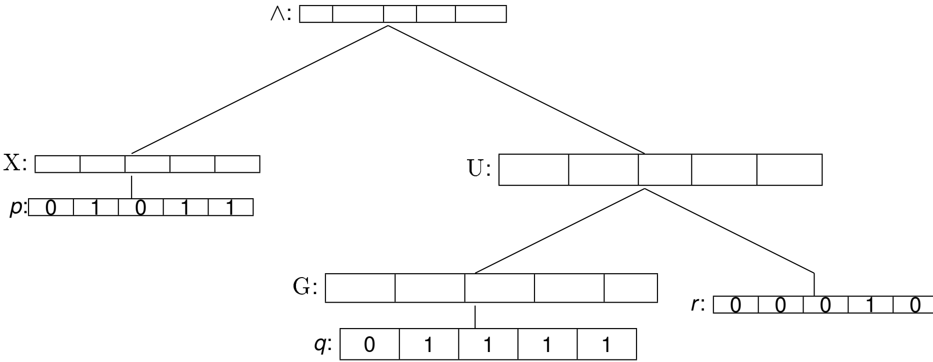
Can we do better than P?

Tree Contraction Algorithm, Kuhtz and Finkbeiner, ICALP'09



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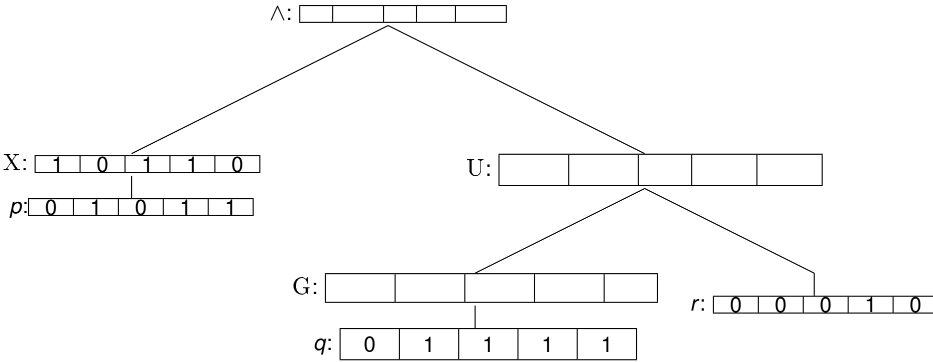

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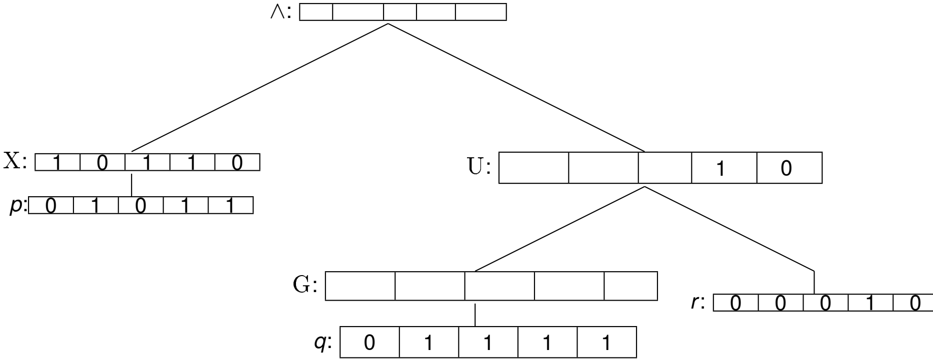

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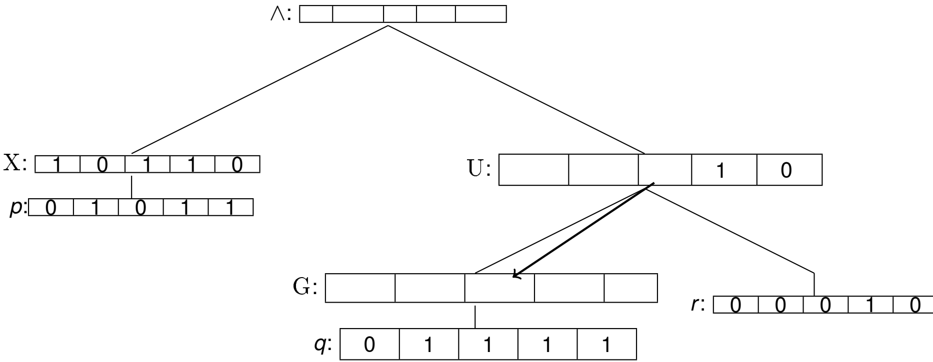

$$\varphi \, \mathrm{U} \, \psi = \psi \vee (\varphi \wedge \mathrm{X}(\varphi \, \mathrm{U} \, \psi))$$

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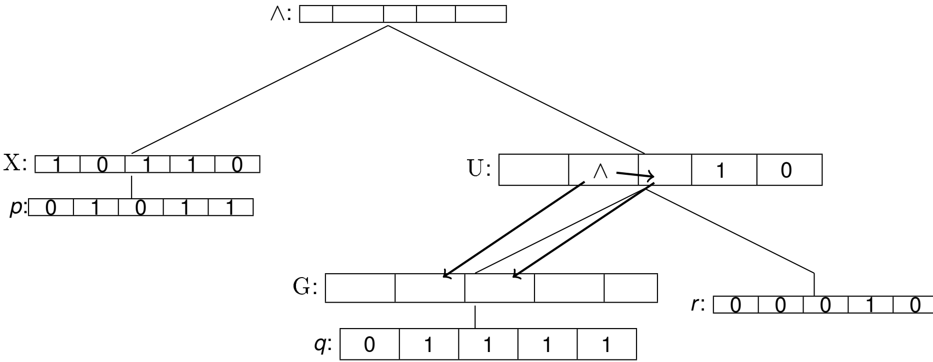

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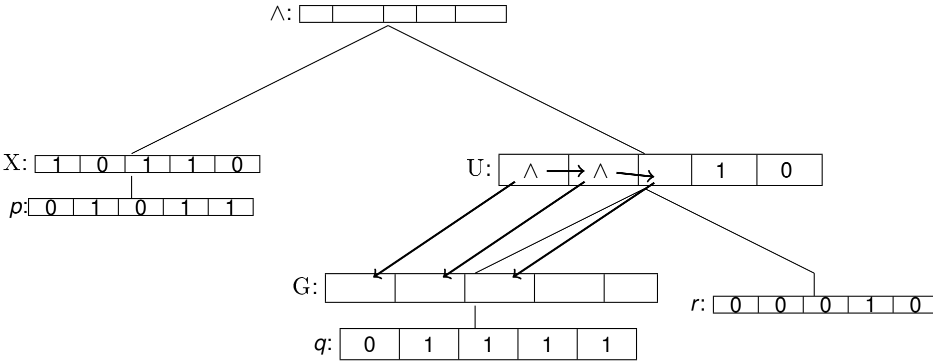

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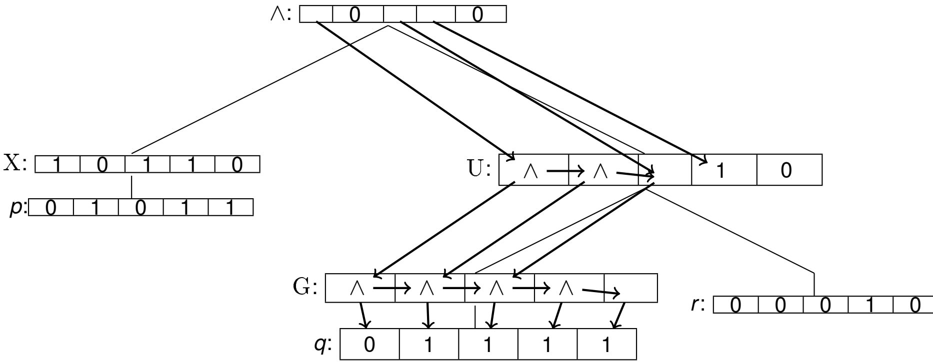

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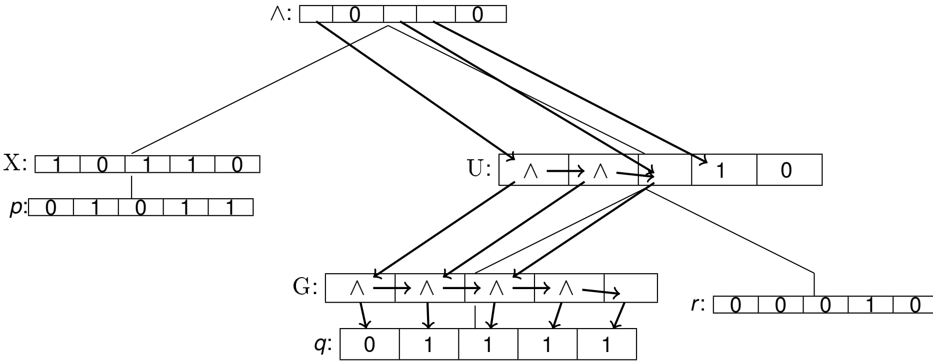

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Build circuits for independent leaves in parallel.

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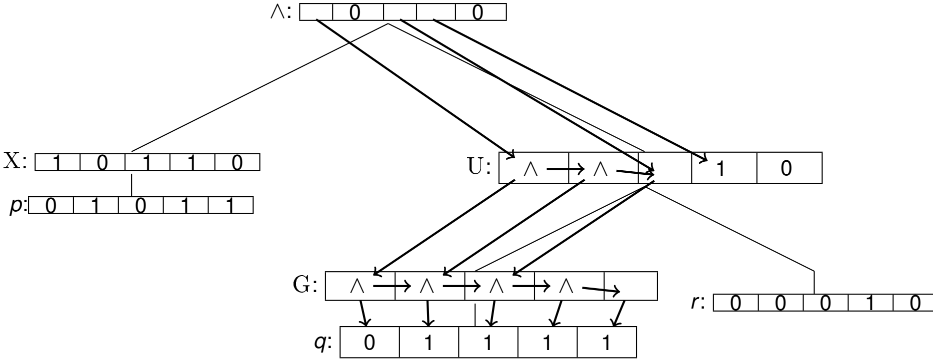

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Circuits:

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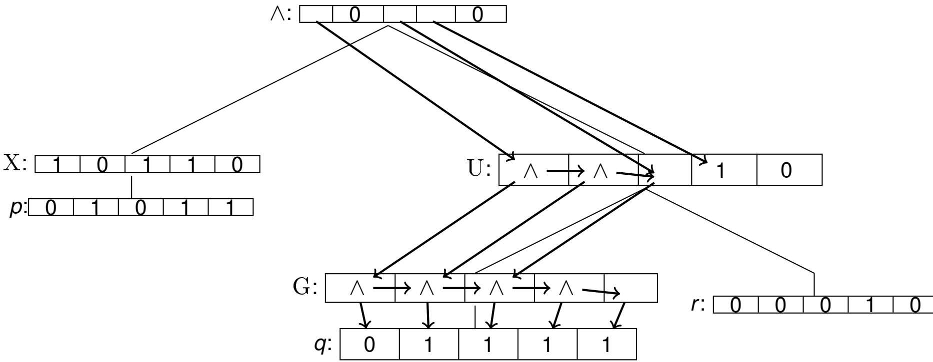

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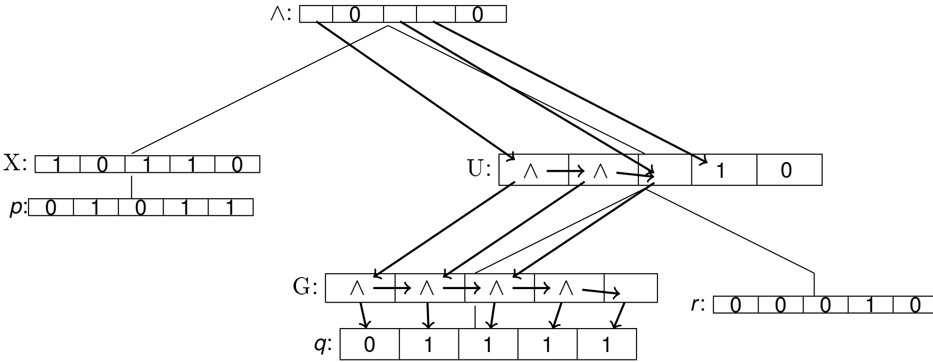

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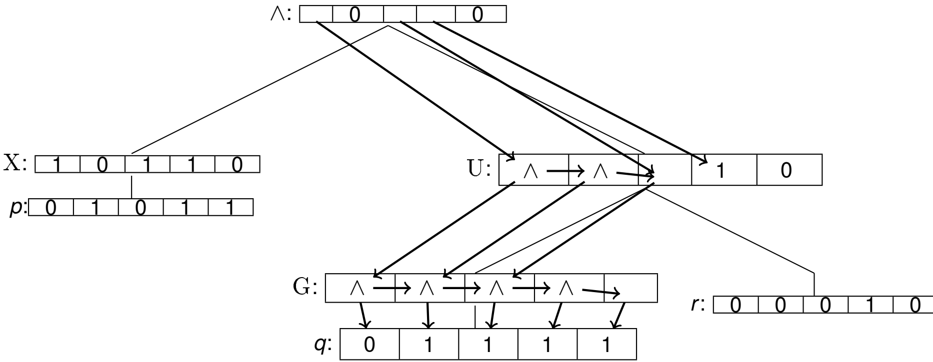

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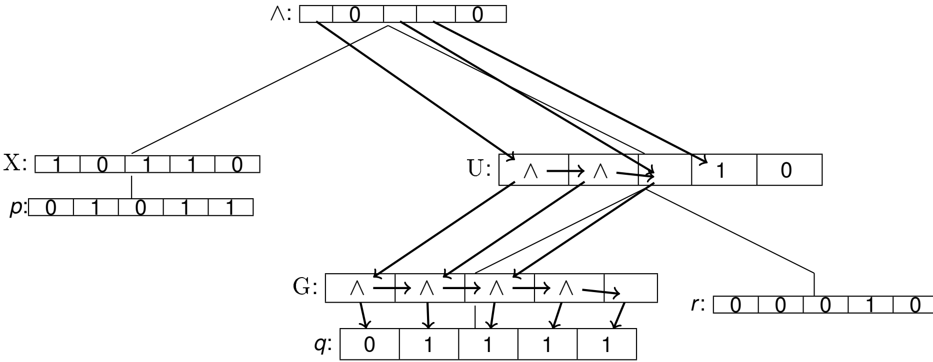

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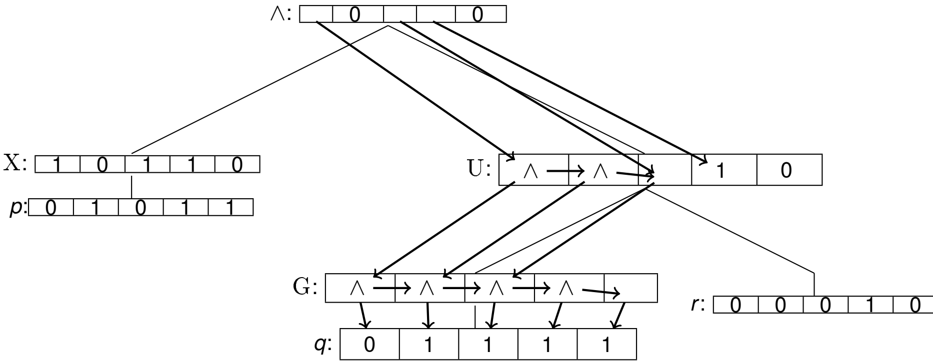

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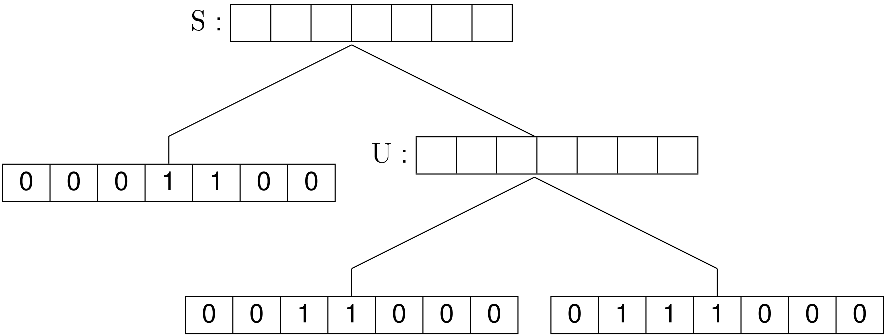
AC^1[logDCFL] $\subseteq$ AC^2 $\Longrightarrow$ efficient parallel algorithm for LTL path checking. LTL path checking unlikely P-hard

Circuit Evaluation



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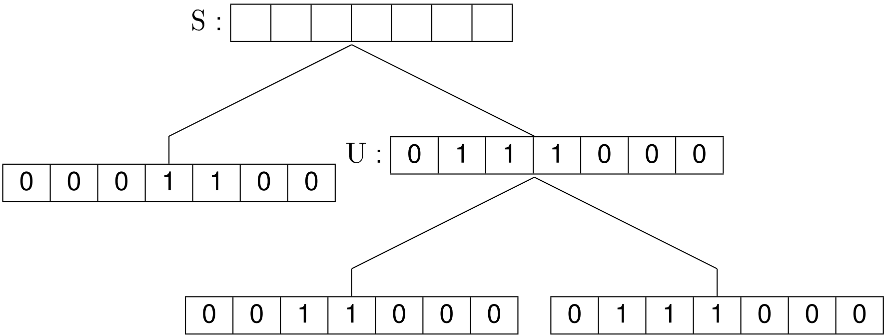

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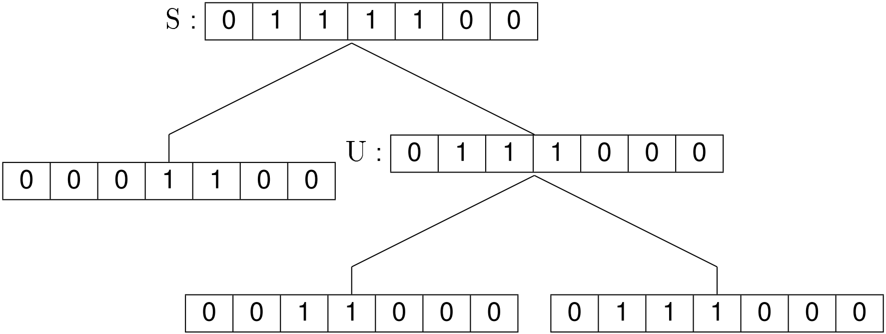

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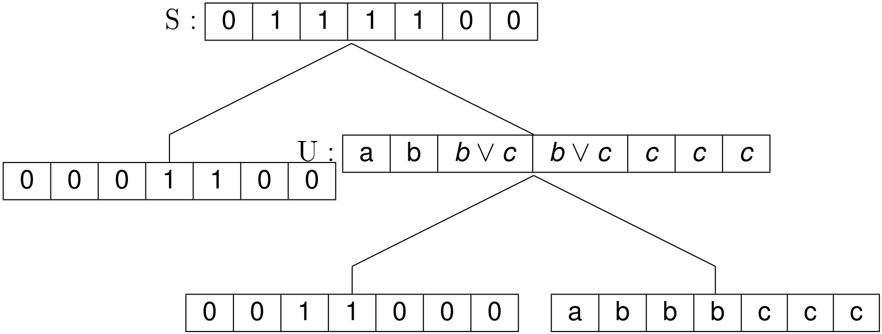

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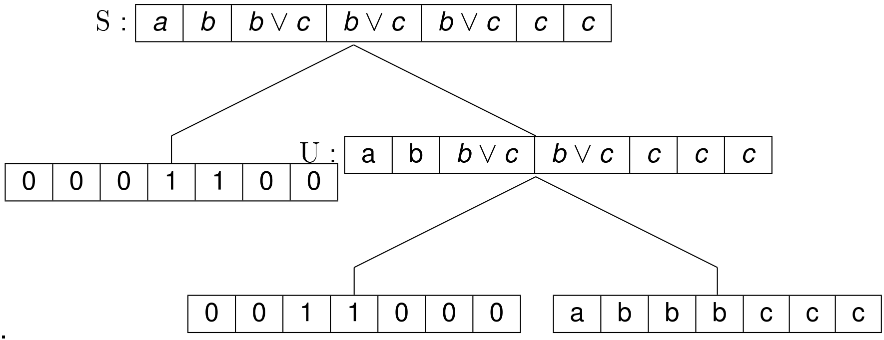
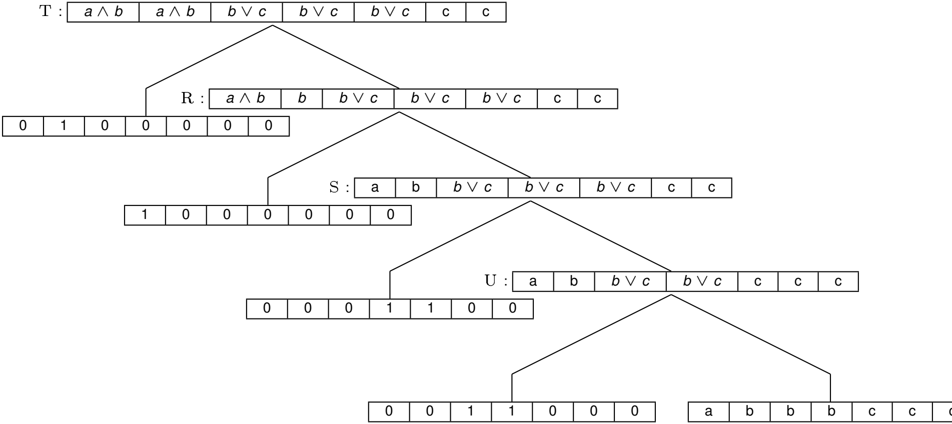


Circuit Evaluation Continued



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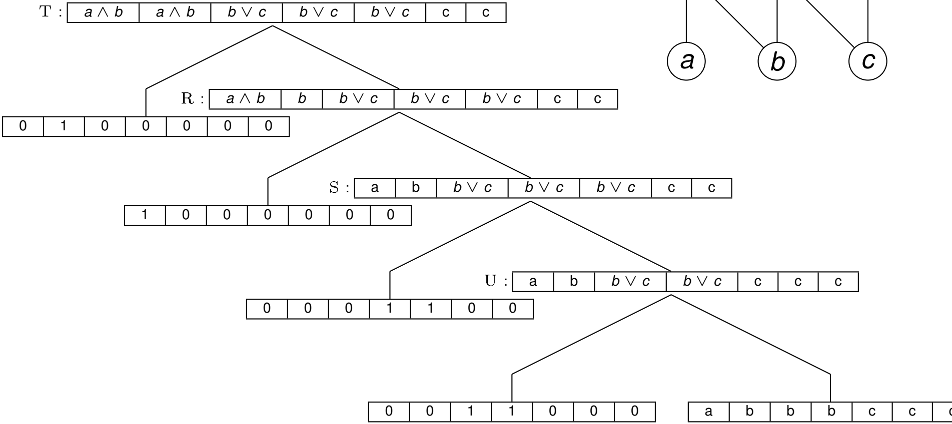
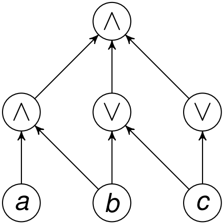
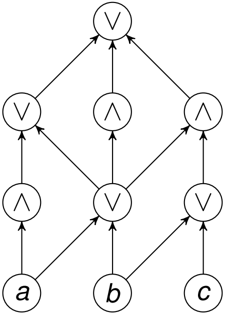




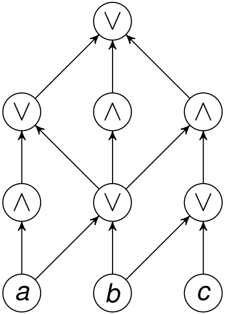
Evaluated a single layer of the circuit

Circuit Evaluation Continued



Inductively, circuit evaluation reduces to LTL path checking

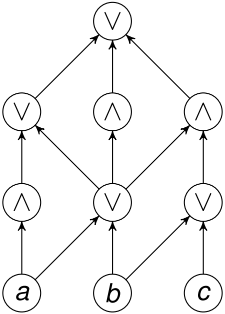
Circuit Evaluation Continued



Inductively, circuit evaluation reduces to LTL path checking

Matching upper bounds ($AC^1[\log DCFL]$)

Circuit Evaluation Continued



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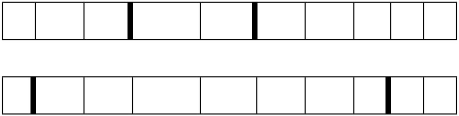
Nonmonotone circuits: PTIME-Complete



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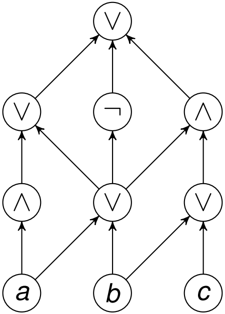

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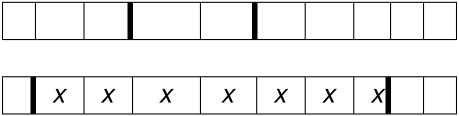

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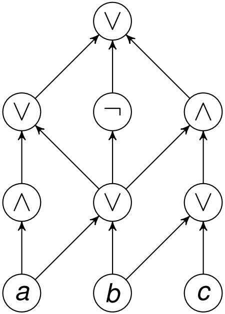

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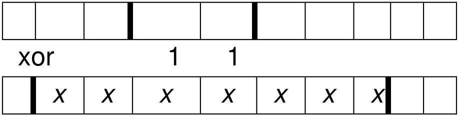

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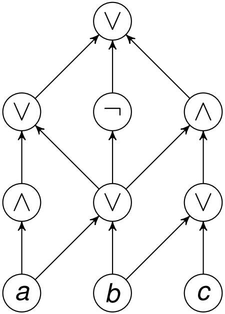

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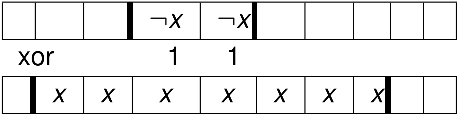

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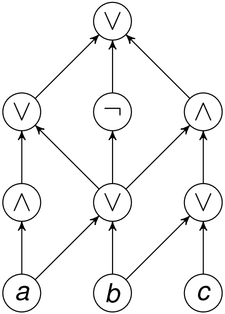

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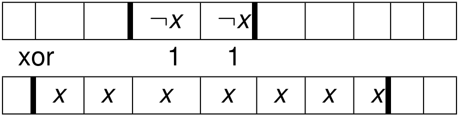

$\Longrightarrow$ LTL+XOR path checking is PTIME-complete

$$\mathrm{UTL} \subseteq \mathrm{LTL} \subseteq \mathrm{MTL}$$

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MTL

Formula $p \,\mathrm{U}_{[1,3]}\, q$:

U :	?	?	?	?	?	?	?	?

p :	*	*	*	*	*	*	*	*
q :	0	0	0	0	1	0	1	1
t :	1	2	3	3.5	3.8	4	4.5	5

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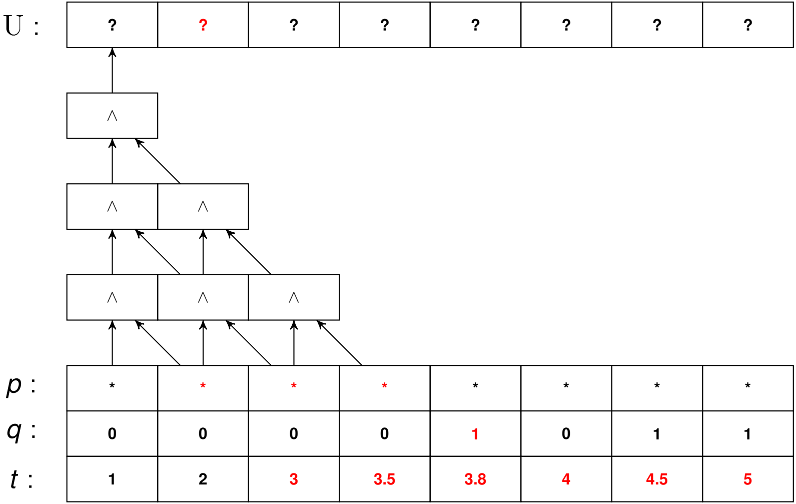

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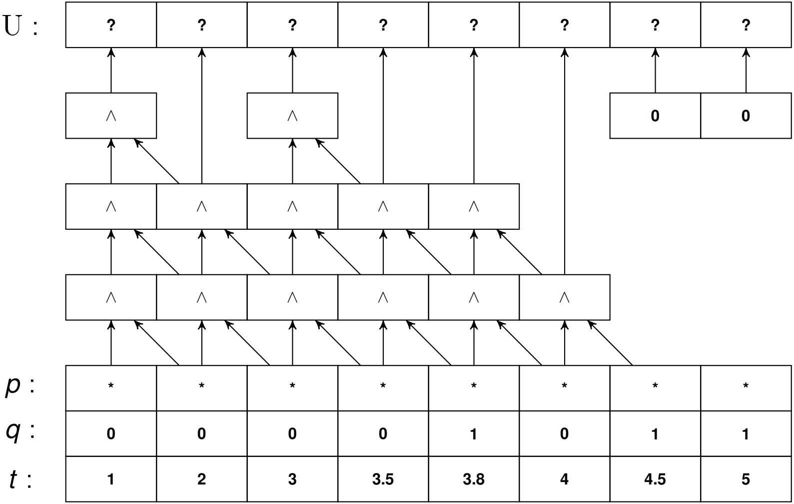

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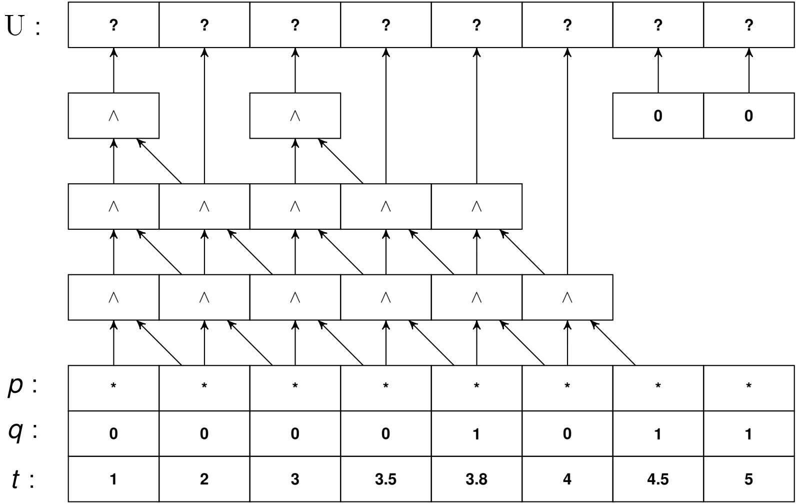

Similar circuits for other operators

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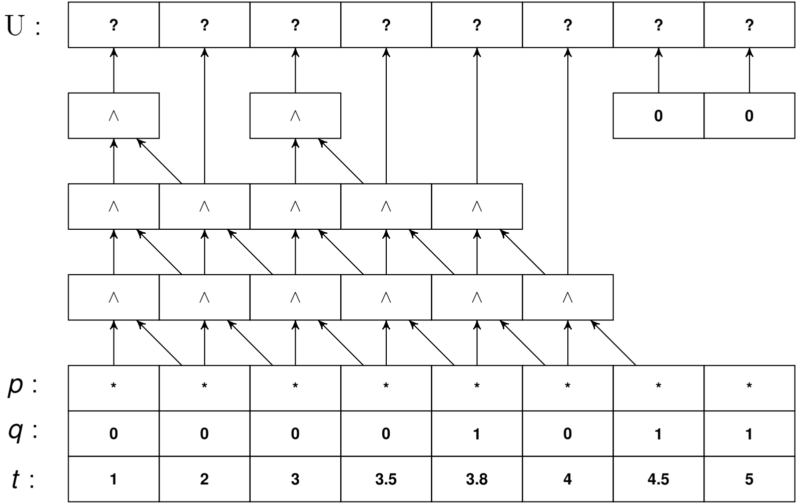

Similar circuits for other operators

$\Longrightarrow$ MTL path checking in AC^1[logDCFL].

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UTL - $\mathrm{G}, \mathrm{F}, \wedge, \vee, \mathrm{X}$

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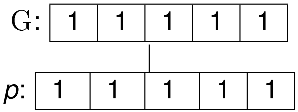

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Leads to AC^1 algorithm for UTL

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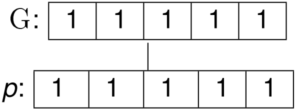
$$NC^1 \subseteq AC^1 \subseteq AC^1[\text{logDCFL}] \subseteq AC^2 \subseteq \cdots \subseteq \mathit{PTIME}$$

UTL LTL MTL LTL+Xor

Can we close the gaps?

UTL - $\mathrm{G}, \mathrm{F}, \wedge, \vee, \mathrm{X}$



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Questions?