Compositional Verification and Optimization of Interactive Markov Chains

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Highlights, , September 21, 2013

Compositional Verification and Optimization of Interactive Markov Chains

Verification of ***Open*** Interactive Markov Chains

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Highlights, Paris
September 19, 2013

ger Herma
an Krčál
Křetínsk

Highlights

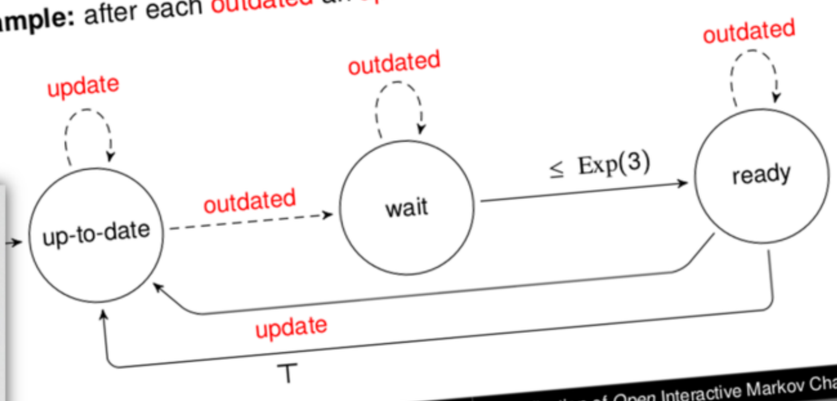
21. 2013

Specification formalism

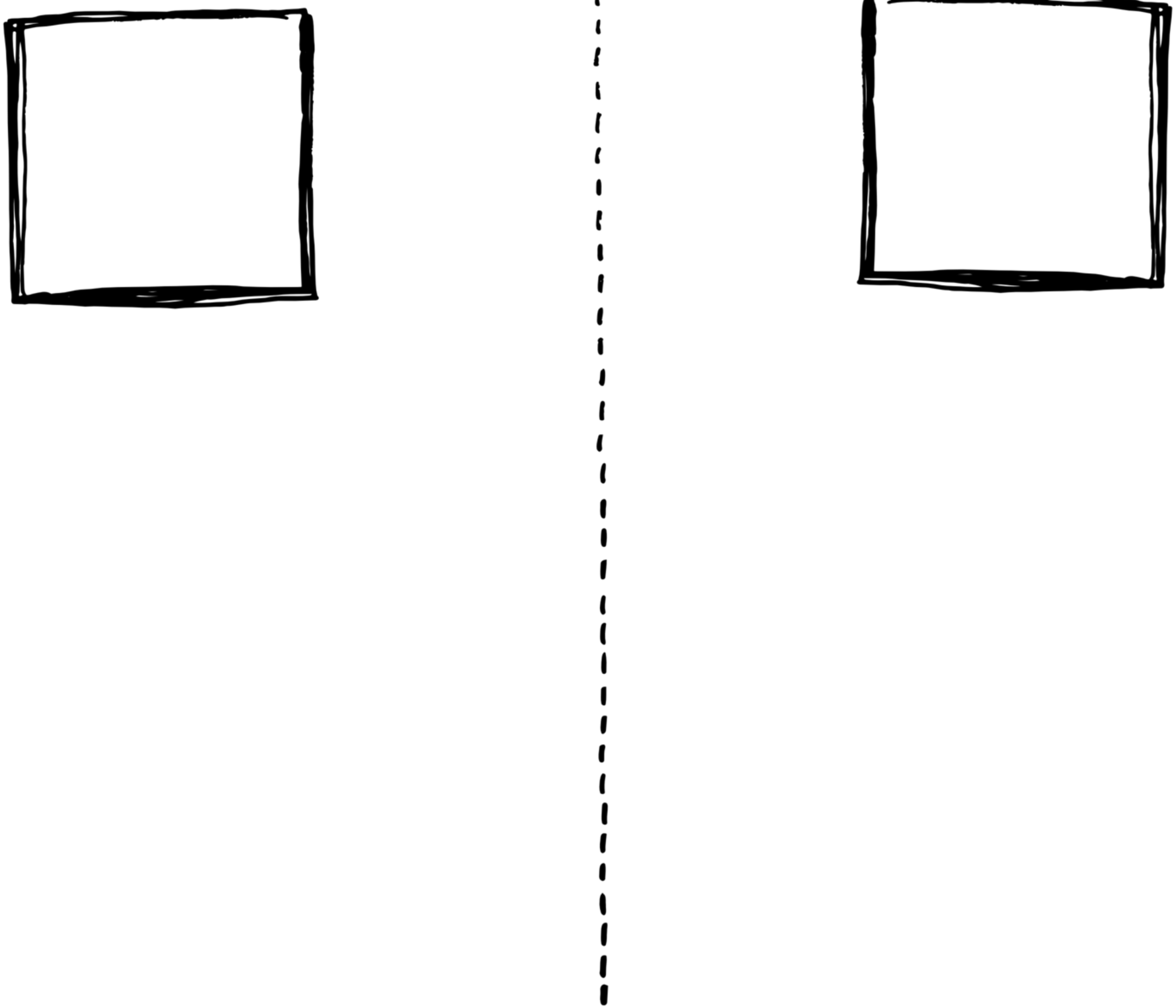
We introduce modal continuous-time automata (MCA)

- may/must transitions as in modal transition systems [Larsen&Thomsen'88]
- continuous time constraints extending timed automata [Alur,Courcoubetis&Dill'91]

Example: after each outdated an update is ready within time ~Exp(3)



Symmetry? (in continuous-time stochastic games)



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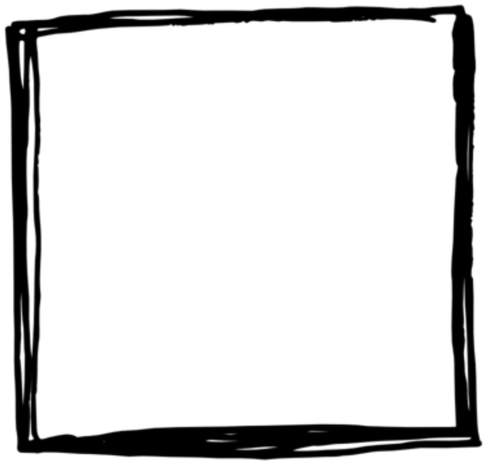
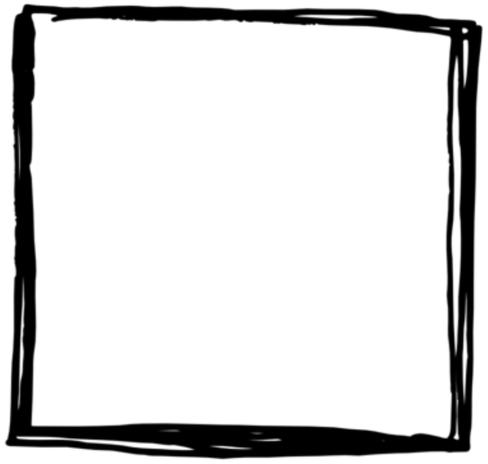
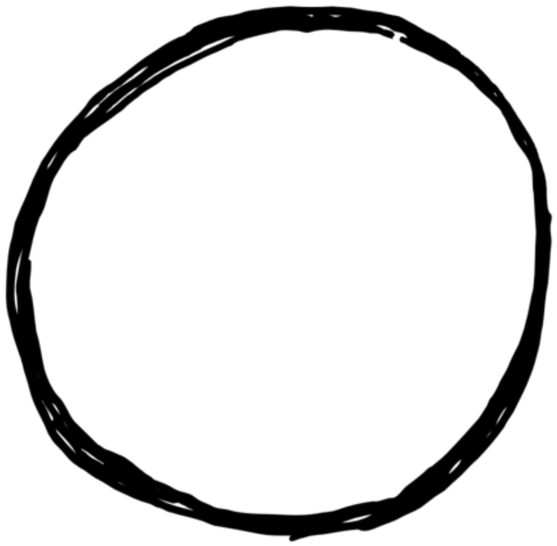
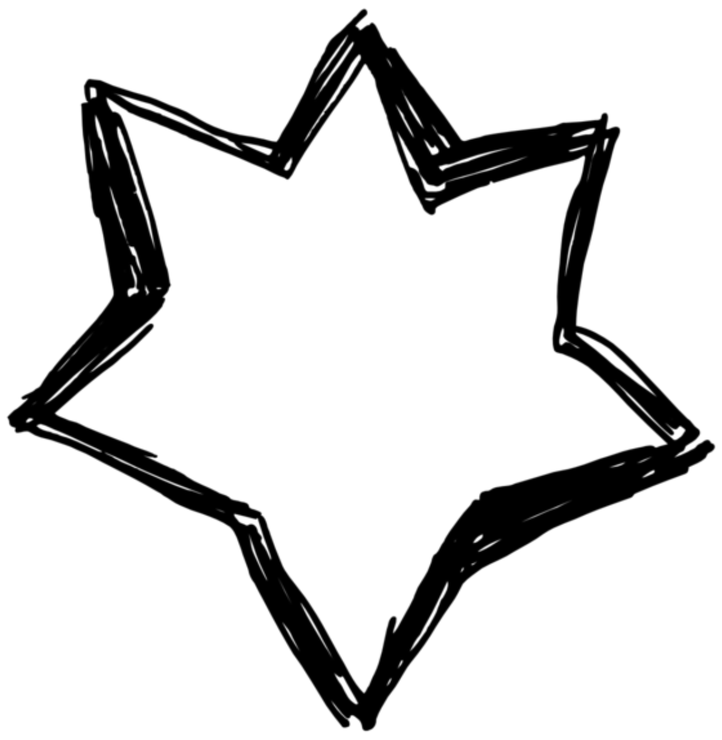
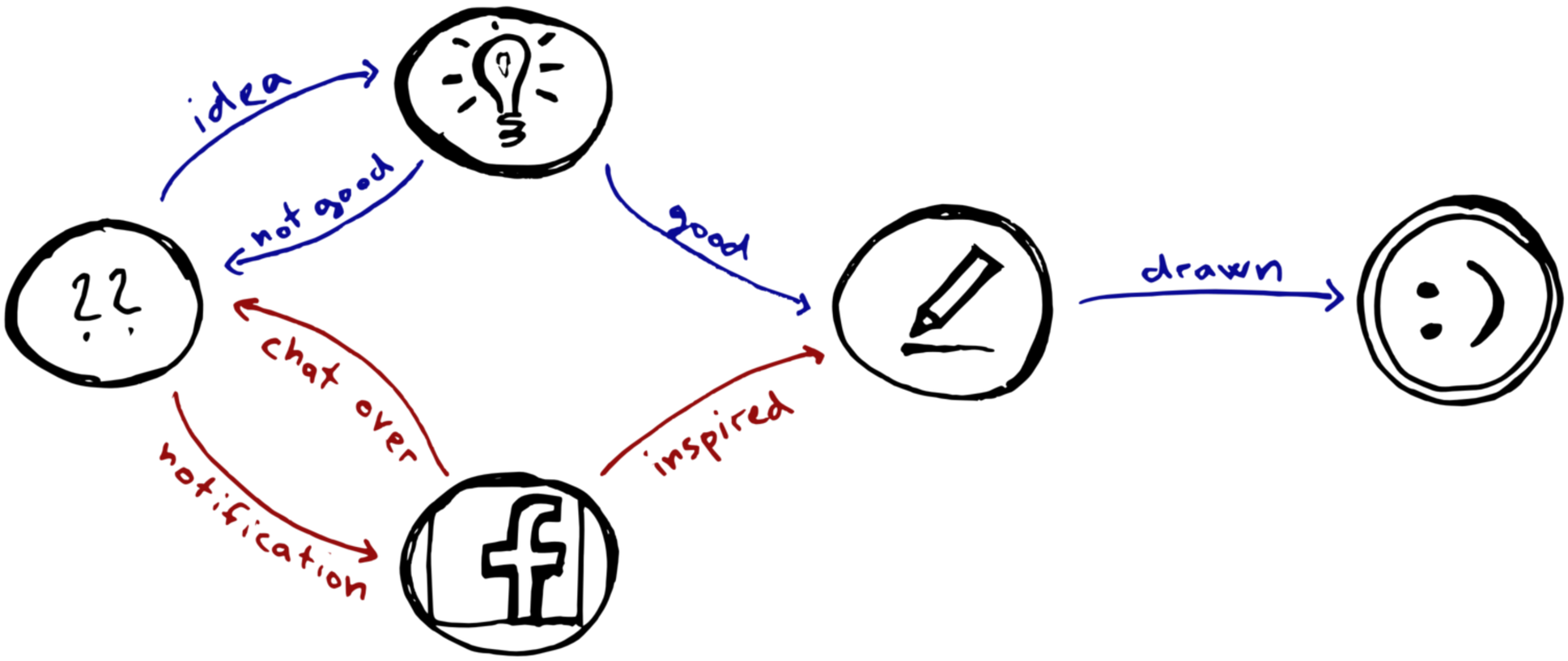








Game



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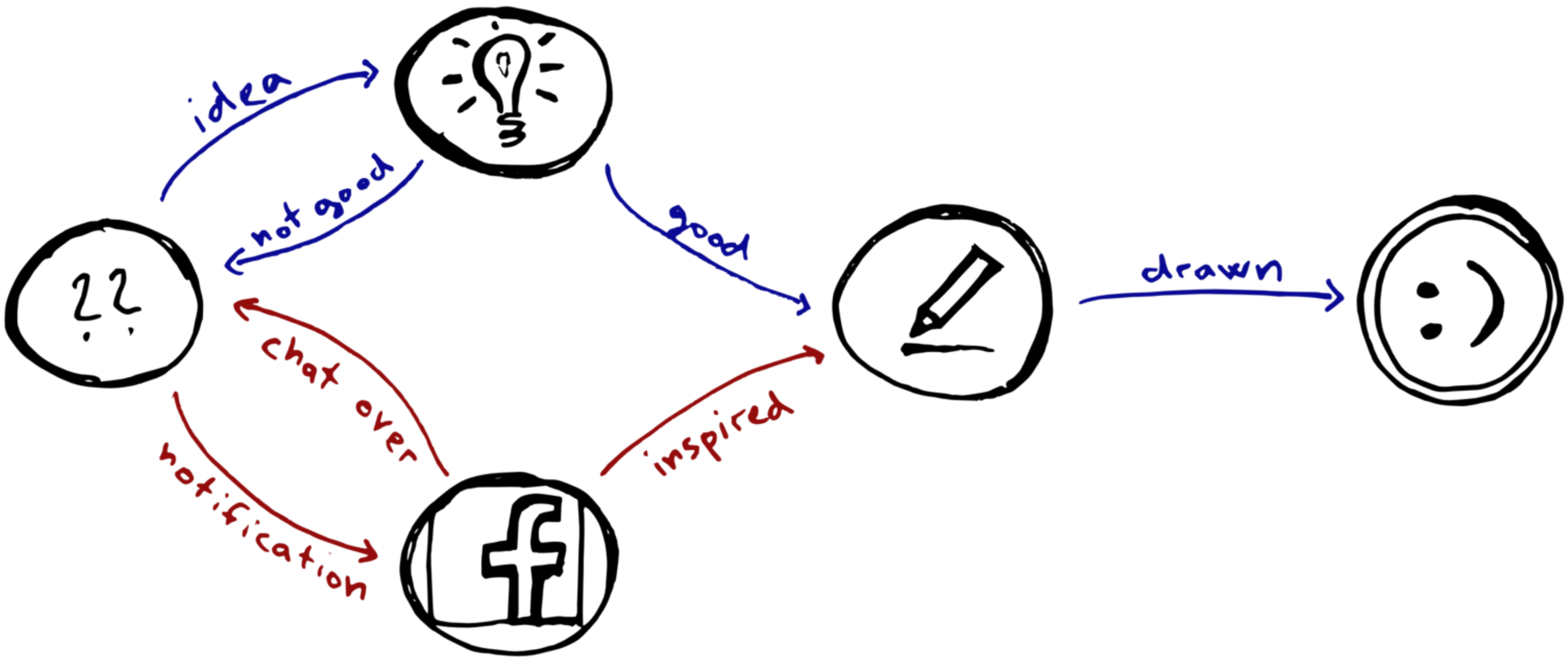

The Element of Surprise in Timed Games*

Luca de Alfaro[1], Marco Faella[1,2], Thomas A. Henzinger[3], Rupak Majumdar[3], and Mariëlle Stoelinga[1]

[1] Department of Computer Engineering, UC Santa Cruz, USA
[2] Dipartimento di Informatica ed Applicazioni, Università di Salerno, Italy
[3] Department of Electrical Engineering and Computer Sciences, UC Berkeley, USA

Abstract. We consider concurrent two-person games played in real time, in which the players decide both which action to play, and when to play it. Such timed games differ from untimed games in two essential ways. First, players can take each other by surprise, because actions are played with delays that cannot be anticipated by the opponent. Second, a player should not be able to win the game by preventing time from diverging. We present a model of timed games that preserves the element of surprise and accounts for time divergence in a way that treats both players symmetrically and applies to all ω-regular winning conditions. We prove that the ability to take each other by surprise adds extra power to the players. For the case that the games
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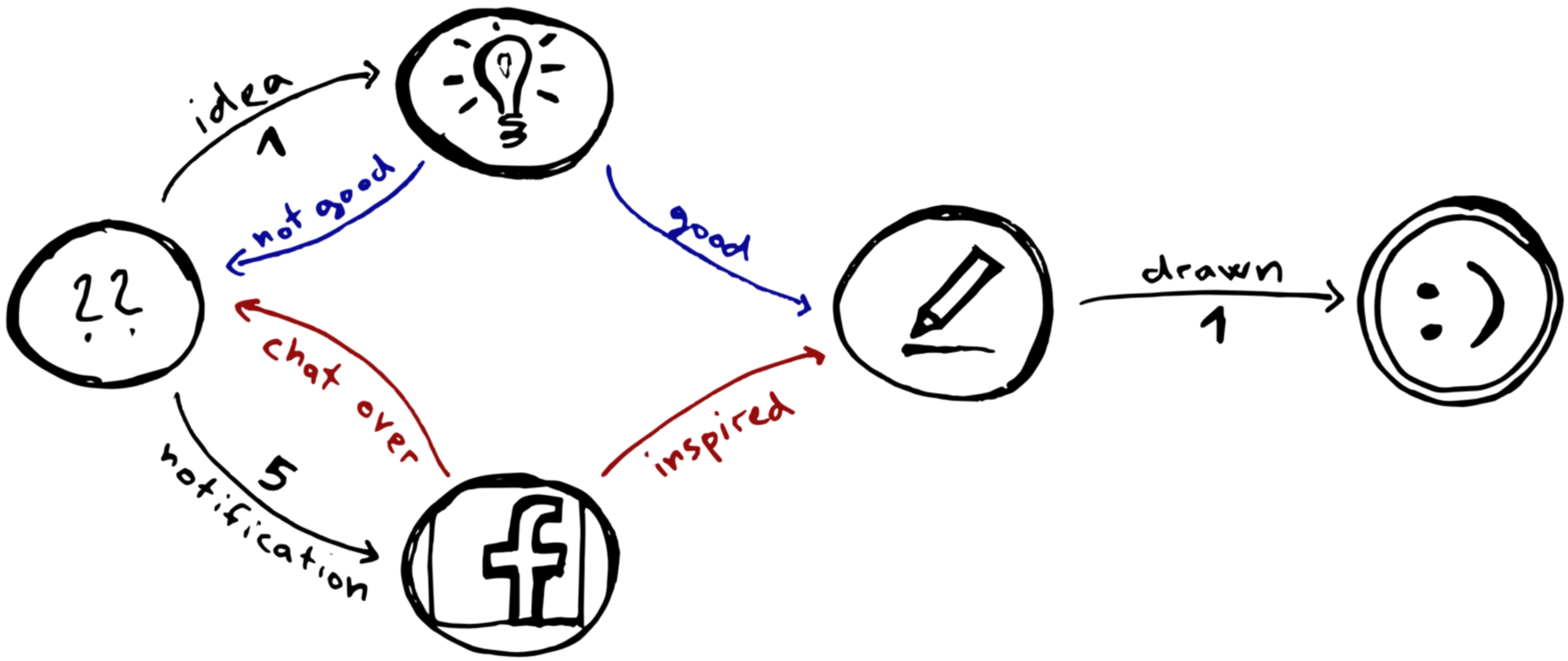

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Continuous-Time Stochastic Game with Time-Bounded Reachabilit

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ABSTRACT. We study continuous-time stochastic games with time-bounded reachability obje
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1 Introduction

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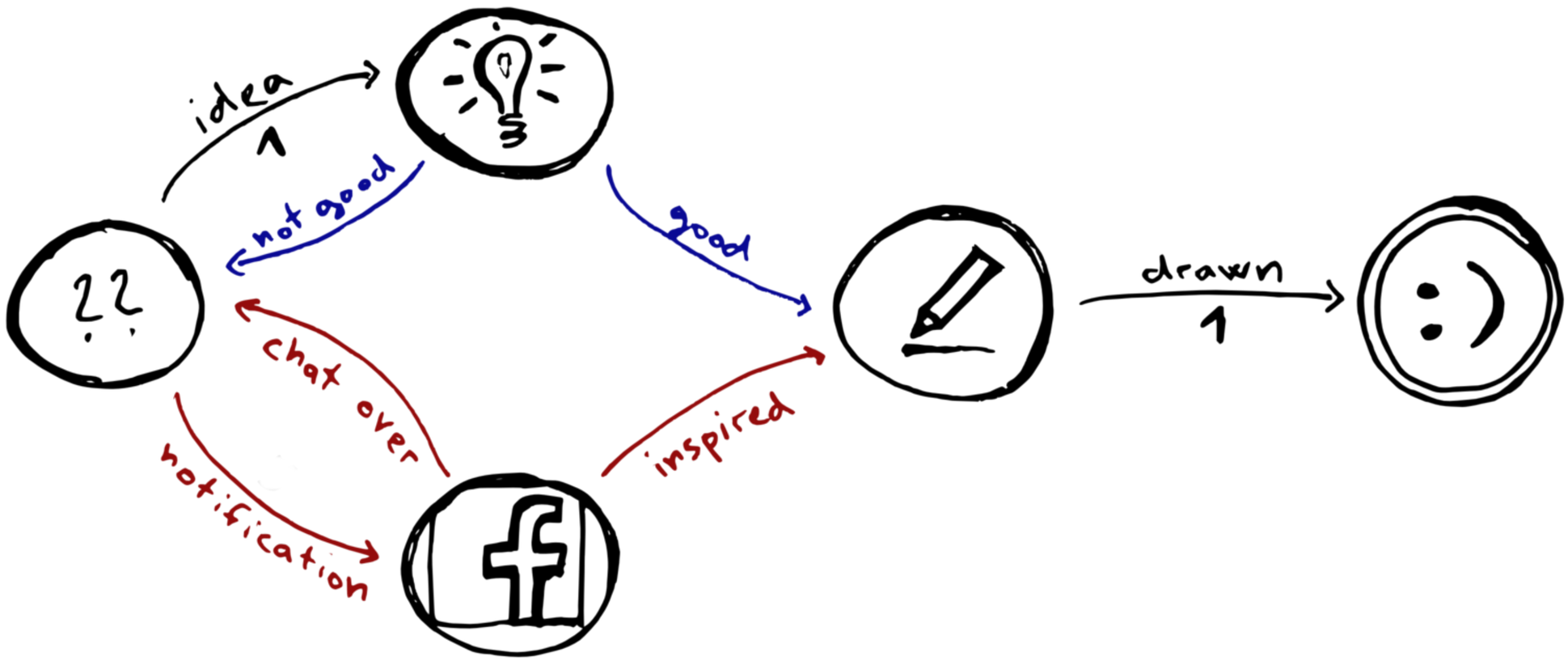

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Abstract. Interactive Markov chains (IMC) are compositional behavioural models extending labelled transition systems and continuous-time Markov chains. We provide a framework and algorithms for compositional verification and optimization of IMC with respect to time-bounded properties. Firstly, we give a specification formalism for IMC. Secondly, given a time-bounded property, an IMC component and the assumption that its unknown environment satisfies a given specification, we synthesize a

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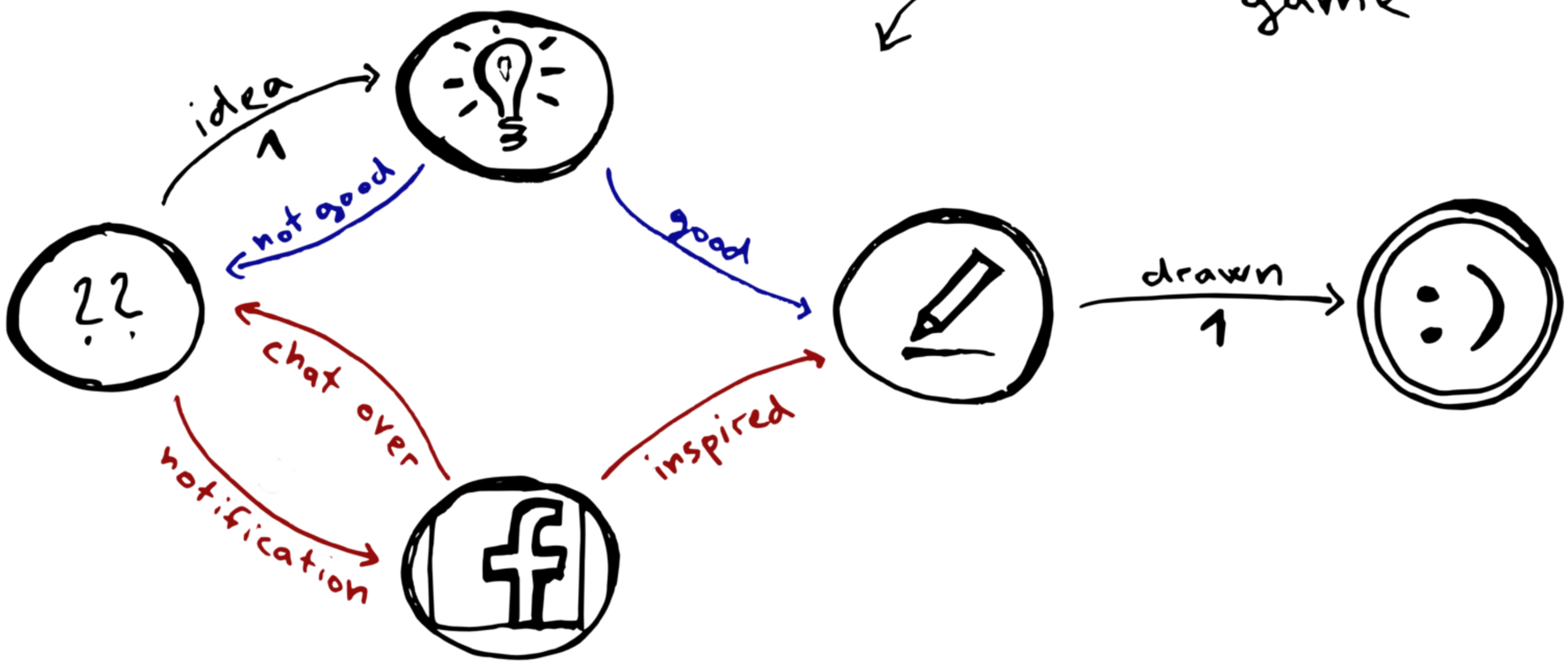

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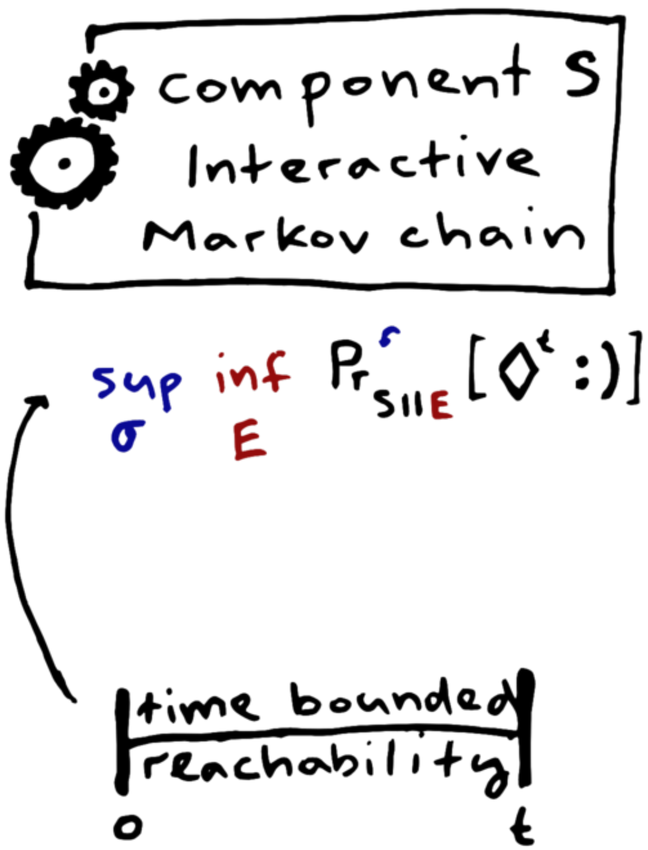

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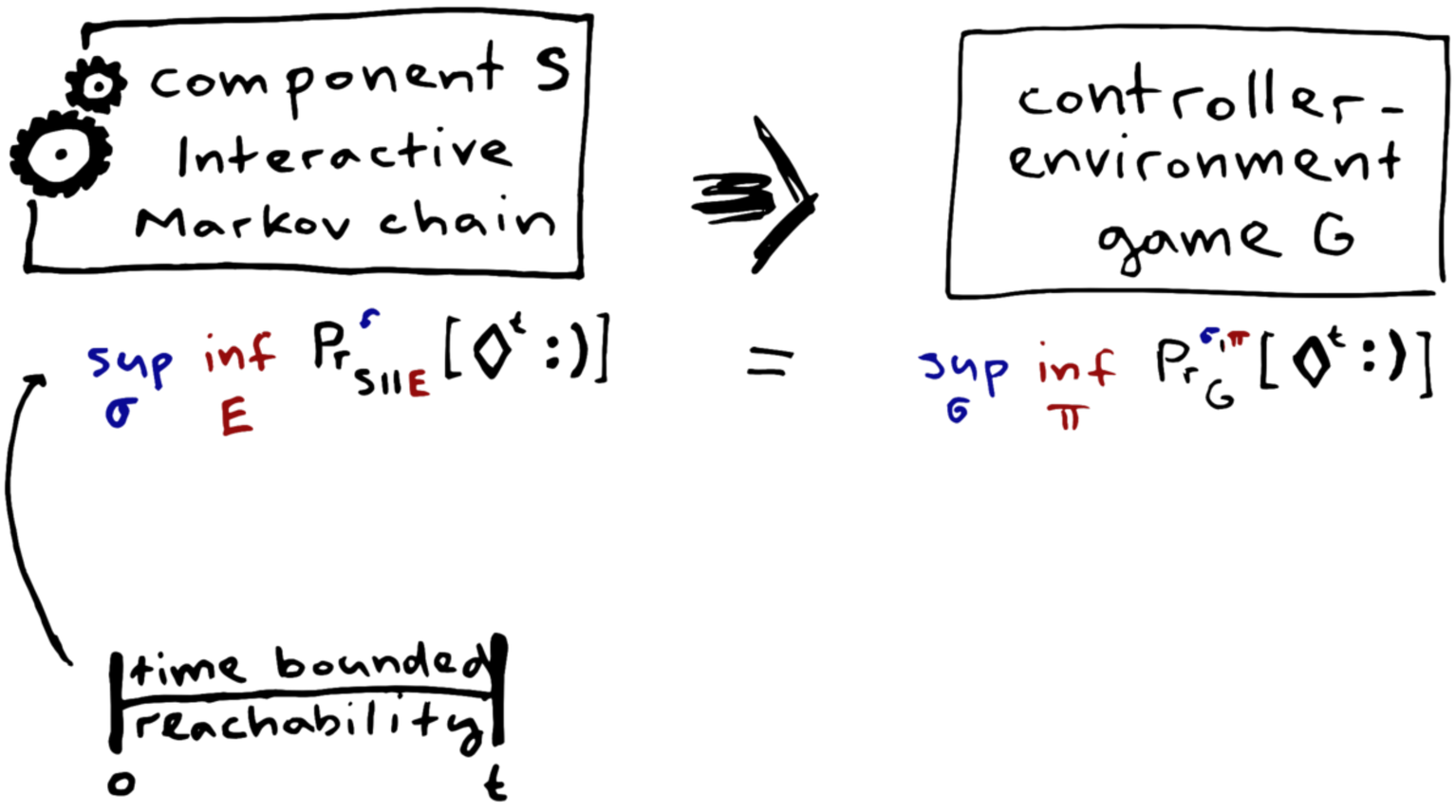

Application

component S
Interactive
Markov chain

$\sup_{\sigma} \inf_{E} Pr^{\sigma}_{S||E}[\Diamond^{\leq t}:)]$

time bounded reachability

0 t

$=$

controller-
environment
game G

$\sup_{\sigma} \inf_{\pi} Pr^{\sigma,\pi}_{G}[\Diamond^{\leq t}:)]$

$\approx$

discrete
game Δ

$\sup_{\sigma} \inf_{\pi} Pr^{\sigma,\pi}_{\Delta}[\Diamond^{\leq n}:)]$

with error bound

Application

specification φ
Modal continuous automaton

with (asymmetric) partial observation

component S
Interactive Markov chain

controller-environment game G

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$\sup_{\sigma} \inf_{E \models \varphi} Pr^{\sigma}_{S \| E}[\Diamond^{\leq t} :)]$ $=$ $\sup_{\sigma} \inf_{\pi} Pr^{\sigma,\pi}_{G}[\Diamond^{\leq t} :)]$ $\approx$ $\sup_{\sigma} \inf_{\pi} Pr^{\sigma,\pi}_{\Delta}[\Diamond^{\leq n} :)]$

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with error bound

time bounded reachability

0 — t

Theorem: For IMC S and MCA φ the guarantee $\sup_{\sigma} \inf_{E \vDash \varphi} Pr^{\sigma}_{S \| E}[\Diamond^{\leq t} :)]$ can be ε-approximated in exponential time

Summary

- first assume-guarantee reasoning on stochastic continuous-time systems
- crucial solution step: reduction to CE-game
  - a continuous-time stochastic game with asymmetric roles of players

Future work

- lowering the theoretical / practical complexity
- other properties

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Thank you for your attention!