Doomsday Equilibria for Omega-Regular Games

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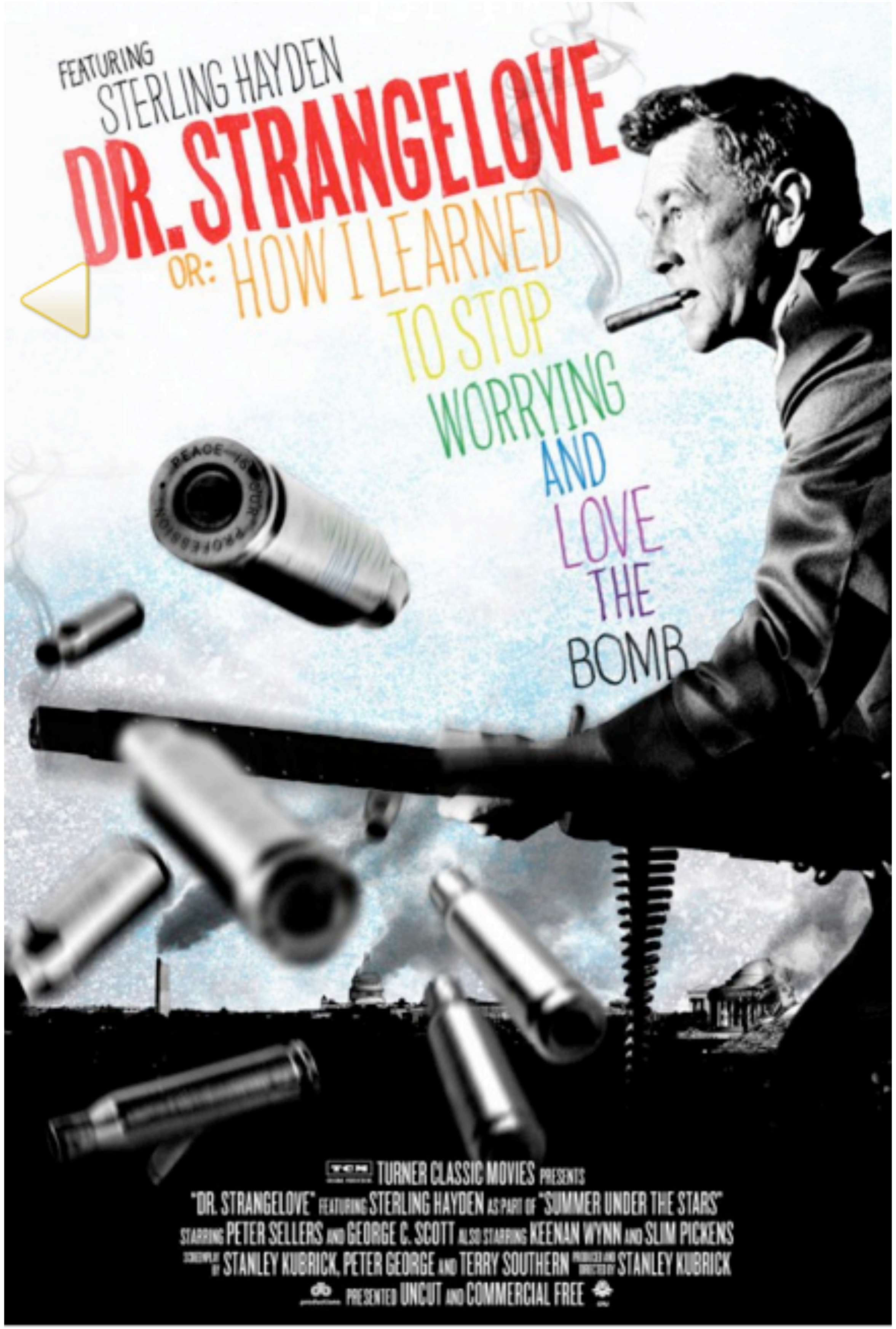
based on a joint work together with
K. Chatterjee (IST), L. Doyen (ENS), and E. Filiot (U Paris 12)



Freely inspired by

The **Doomsday machine**
of Dr. Strangelove
(Stanley Kubrick)

Motivations

- 2-player zero-sum games are useful for:
  - **controller synthesis** (system vs environment)
  - ...
  - relation with tree automata (emptiness algorithm)
  - notion of simulation relation
- to reason about **multi**-component systems, we need **multi-player non-zero sum games** (with **imperfect info**)
  - note that imperfect info and multi-player quickly lead to undecidability

Solution concepts

To predict/analyze how players behave in multi-player games, several notions have been proposed:

- **Admissibility** [Aum76,Ber07]:
  - A strategy of a player **dominates** another one if the outcome of the first strategy is better than the outcome of the second no matter how the other players play.
  - **Rationality**: a player does **not** play a strategy that is strictly dominated by another one
  - **Iterate**: once we know that some strategies are not played by other players, new strategies may become strictly dominated ➟ iterate up to a fixpoint.
    - see talk by Mathieu Sassolas

Solution concepts

To predict/analyze how players behave in multi-player games, several notions have been proposed:

- **Nash equilibria** [Nash51]:
  a strategy profile$(St_1,St_2,\dots,St_n)$ is a **NE**
  if no player has an incentive to **unitarily** deviate:
  $Out_1(\mathbf{St_1'},St_2,\dots,St_n) \leq Out_1(St_1,St_2,\dots,St_n)$
  with imperfect info: constrained existence is undecidable

- **Secure equilibria** (2 players) [CHJ06]:
  NE+deviation does **not** harm the other player:
  $Out_1(St'_1,St_2) \geq Out_1(St_1,St_2) \Longrightarrow Out_2(St'_1,St_2) \geq Out_2(St1,St2)$

Doomsday threatening equilibria: refinement/extension of secure equilibria to n players

Secure/Doomsday equilibria
An example

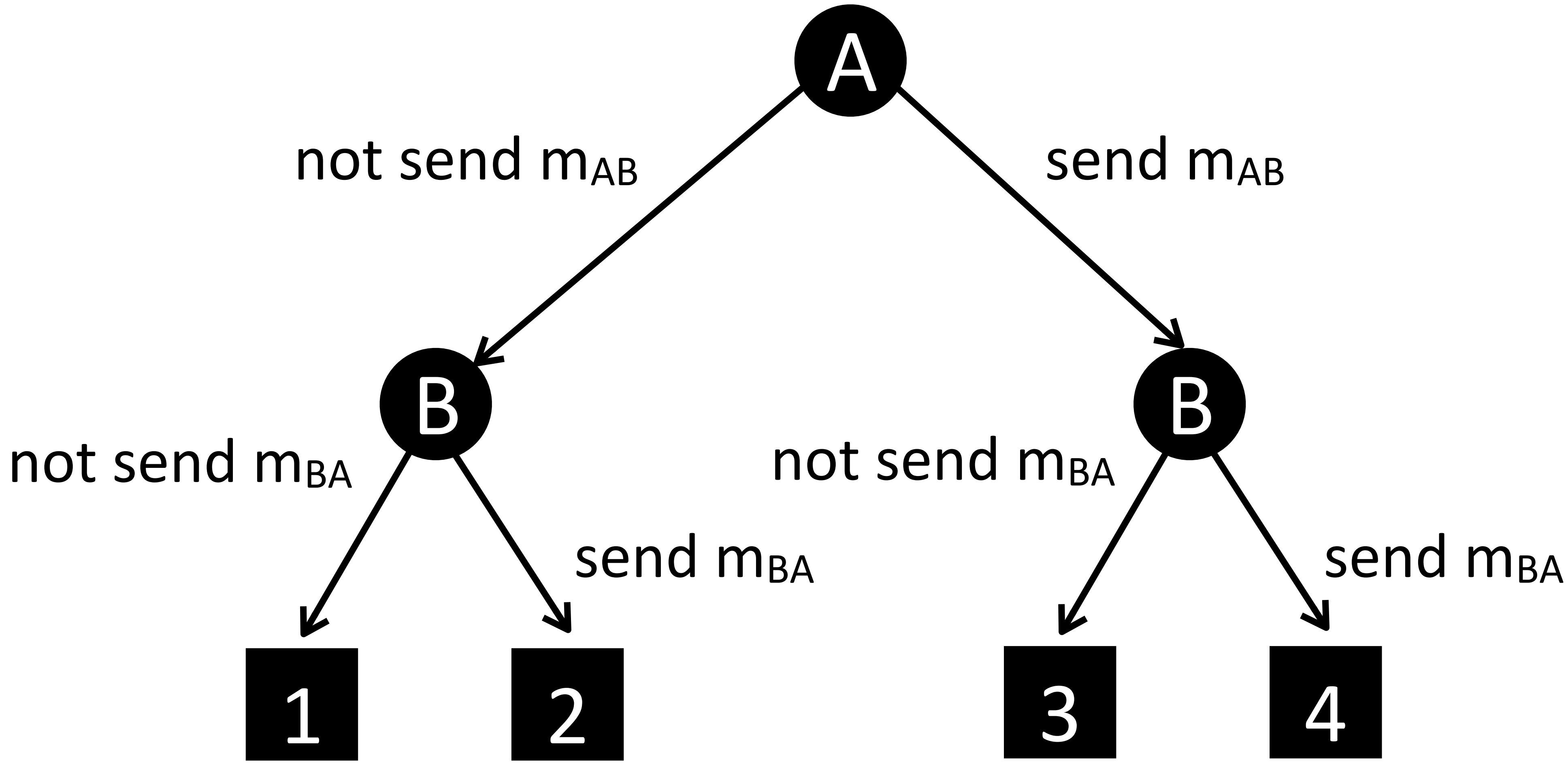
One simple example

- Alice and Bob want to exchange messages m_{AB} and m_{BA}
- Either
  - **both** have received their message (preferred)
  - or none (sub-optimal)
- If one receives and the other not, this is **not** acceptable (for the one that does not receive)

≈ spec. of "Fair Exchange Protocols"

  - no easy solution (e.g. need for a TTP)

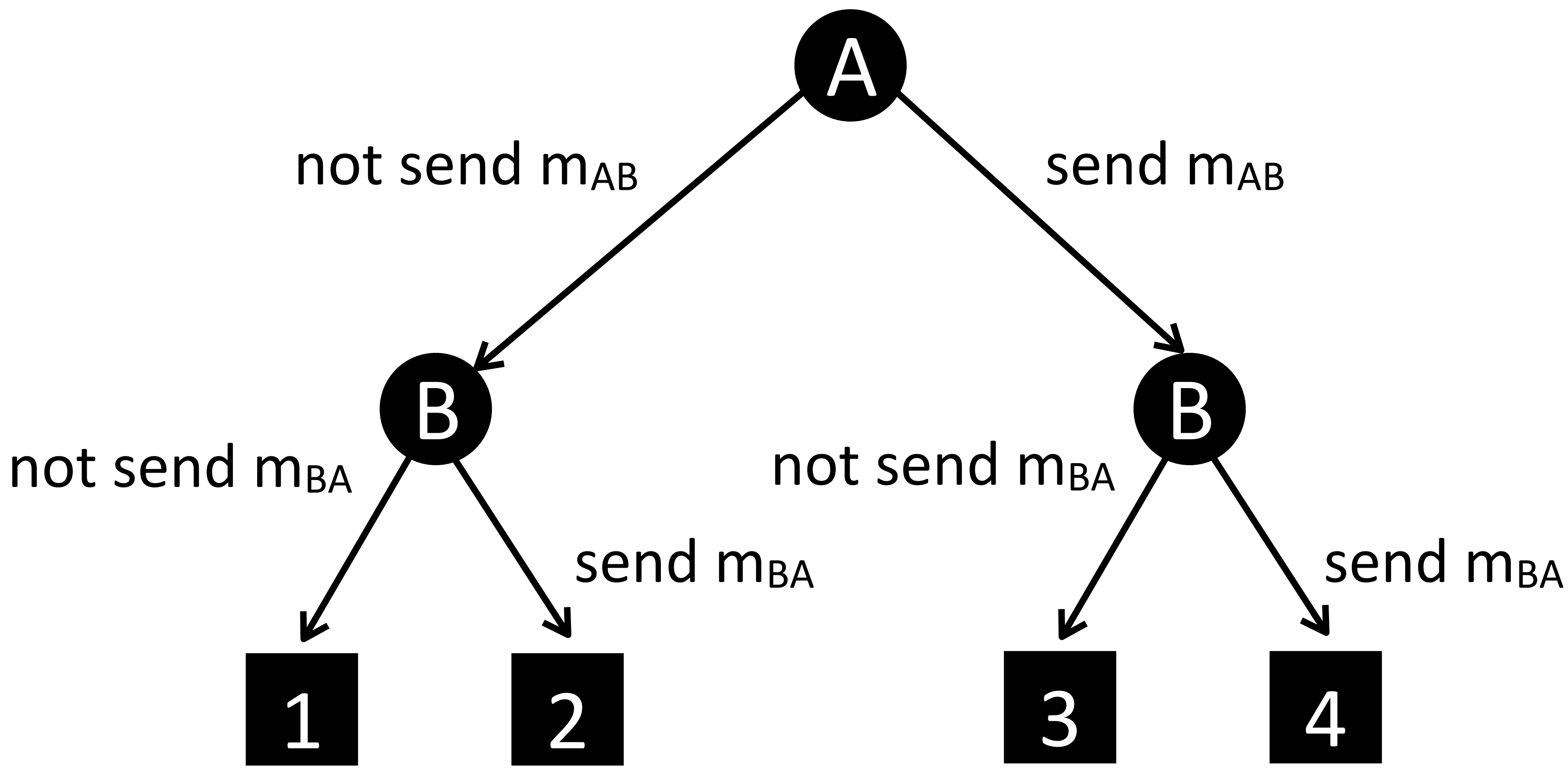
One simple example



A wants to reach {2,4}

B wants to reach {3,4}

One simple example



A preference: 2>4>1>3

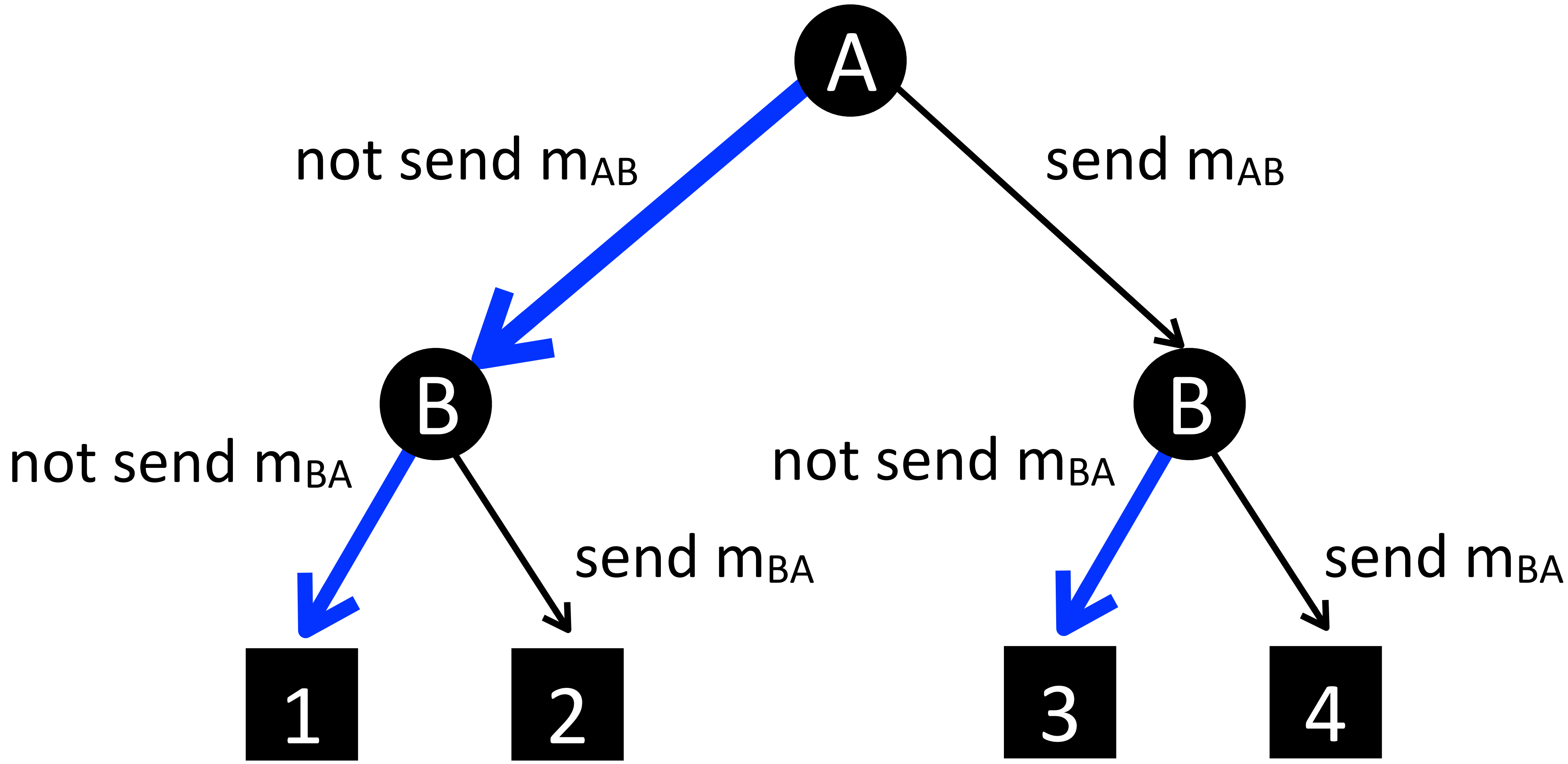
B preference: 3>4>1>2

Unique secure equilibrium:

not send m_{AB}, not send m_{BA}

Not satisfactory !

One simple example



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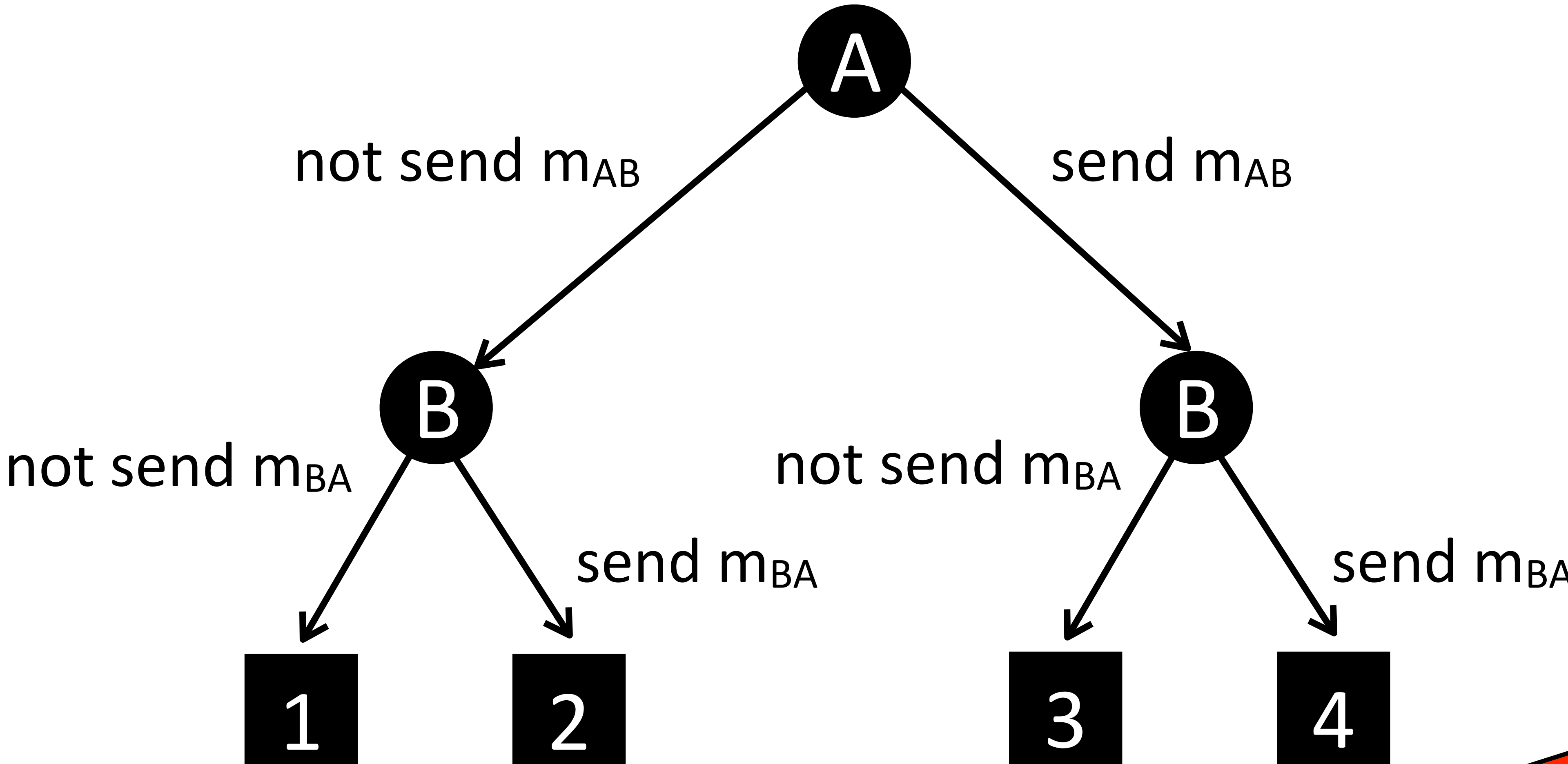


A preference: 2>4>1>3

B preference: 3>4>1>2

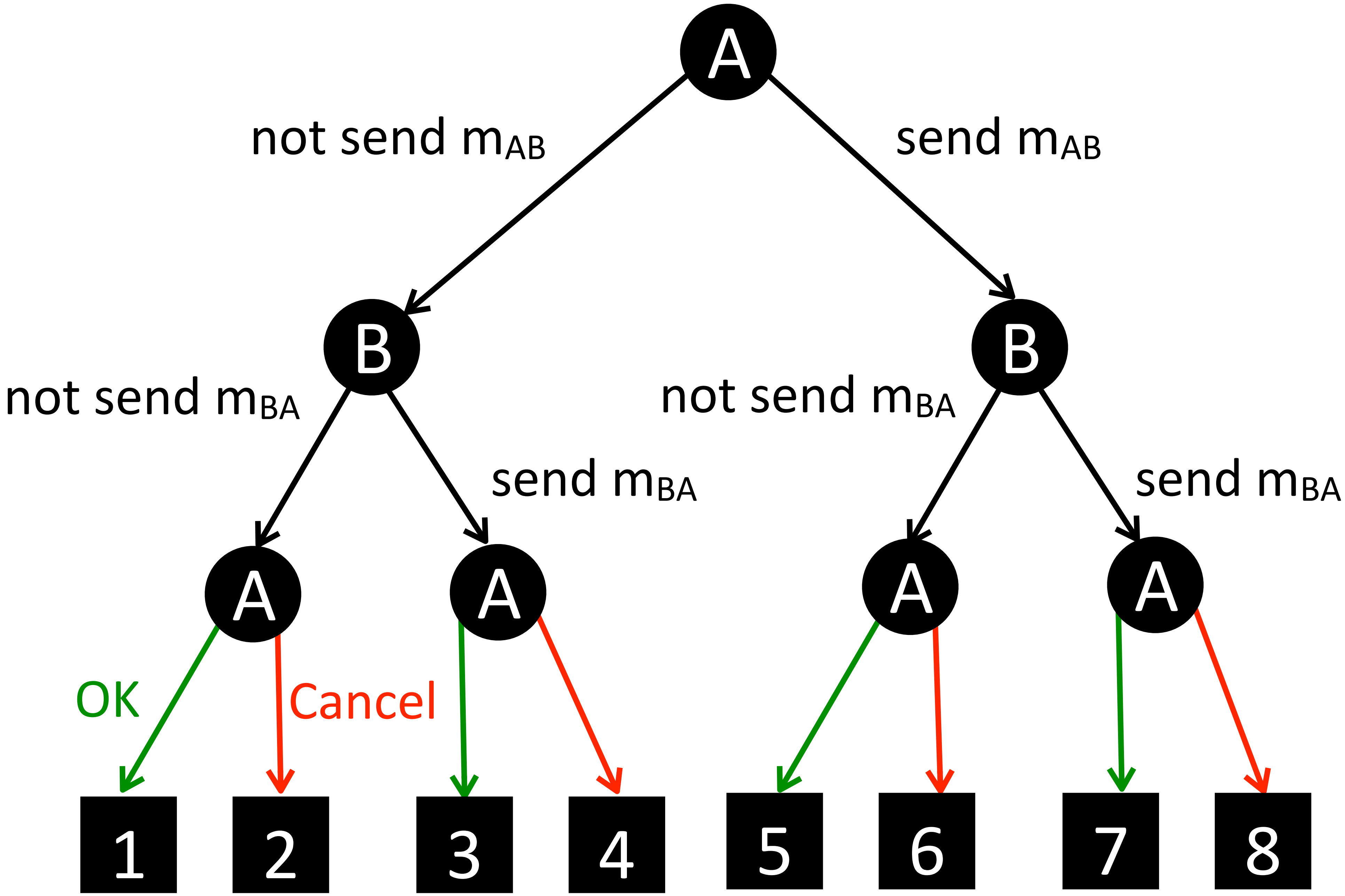
Un… …ilibrium:

… m_{AB}, **not send m_{BA}**

Idea: add **cancel** (≈TTP)

Not satisfactory !

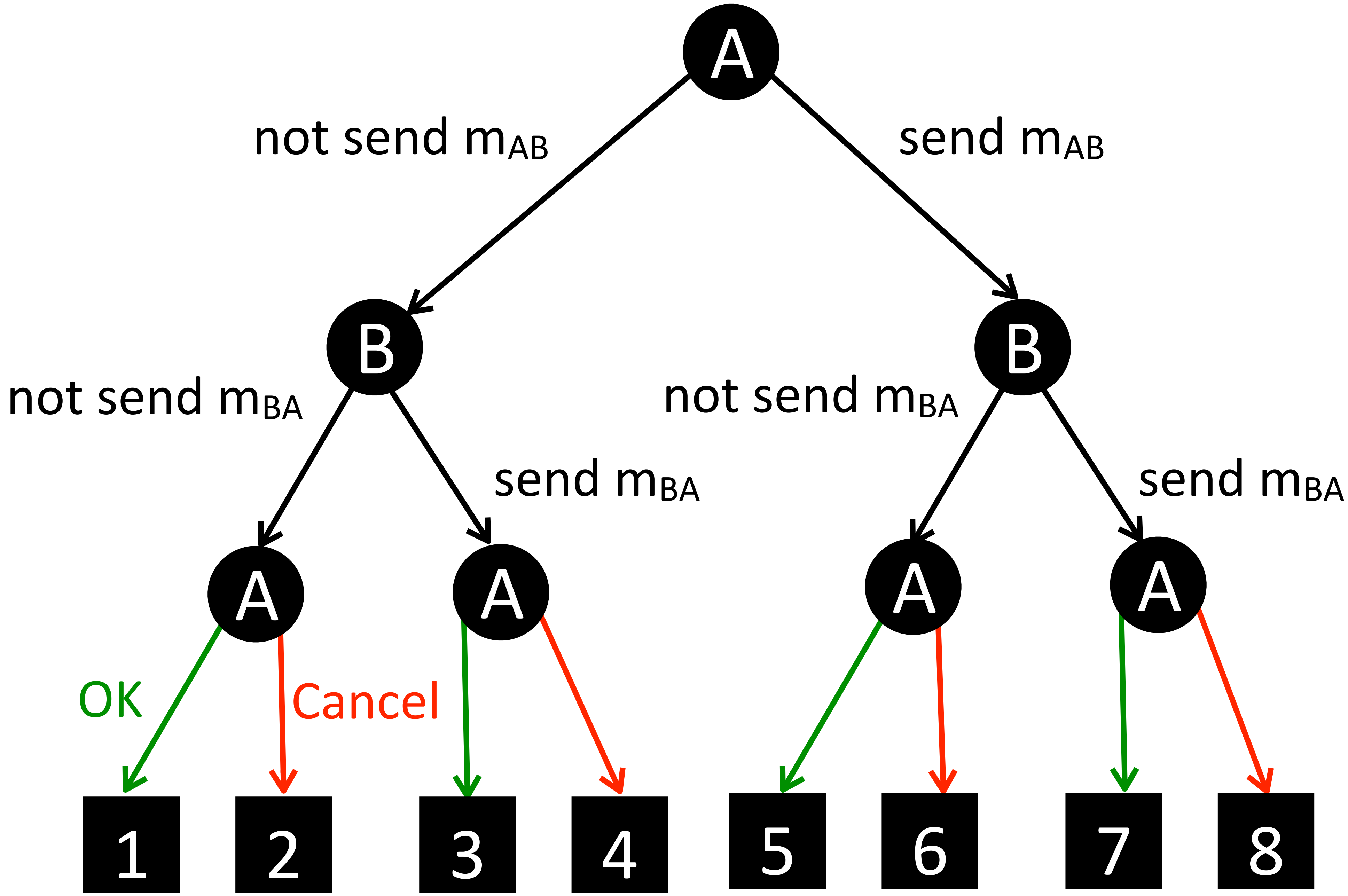
One simple example



A wants to reach {3,7}

B wants to reach {5,7}

One simple example



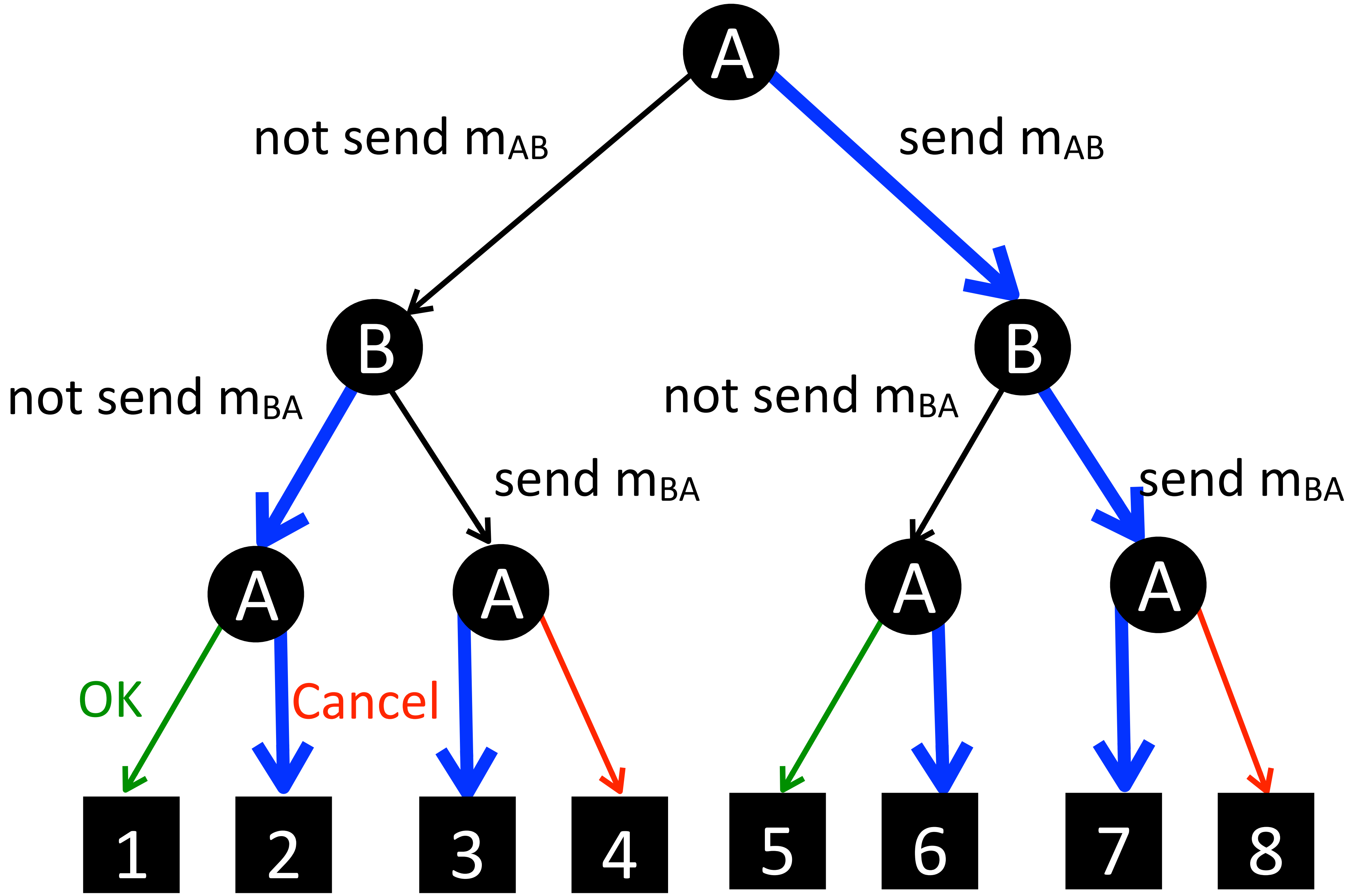
A preference: 3>7>1=2=4=6=8>5

B preference: 5>7>1=2=4=6=8>3

Unique secure equilibrium:

send m_{AB}, send m_{BA}, OK

One simple example



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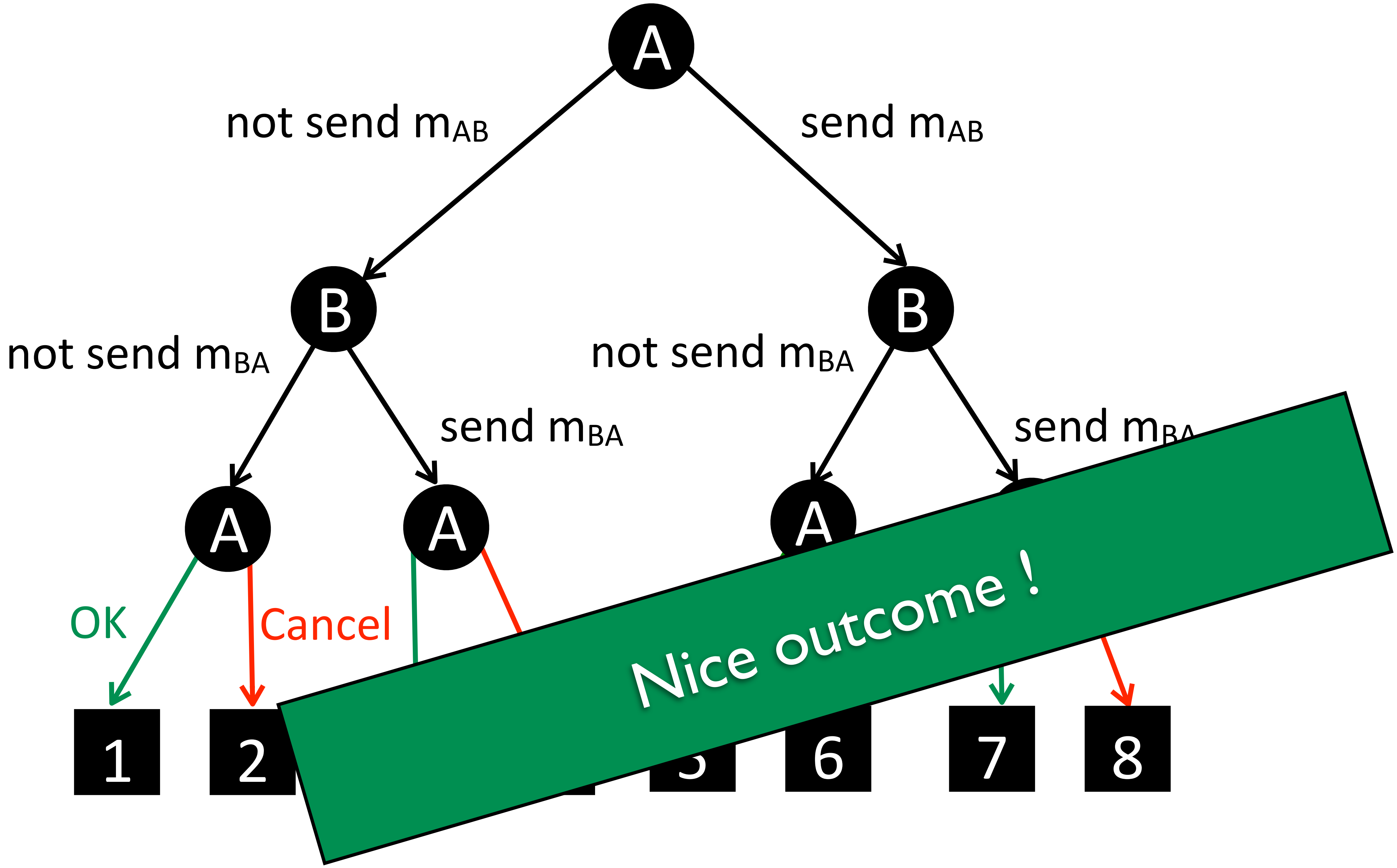

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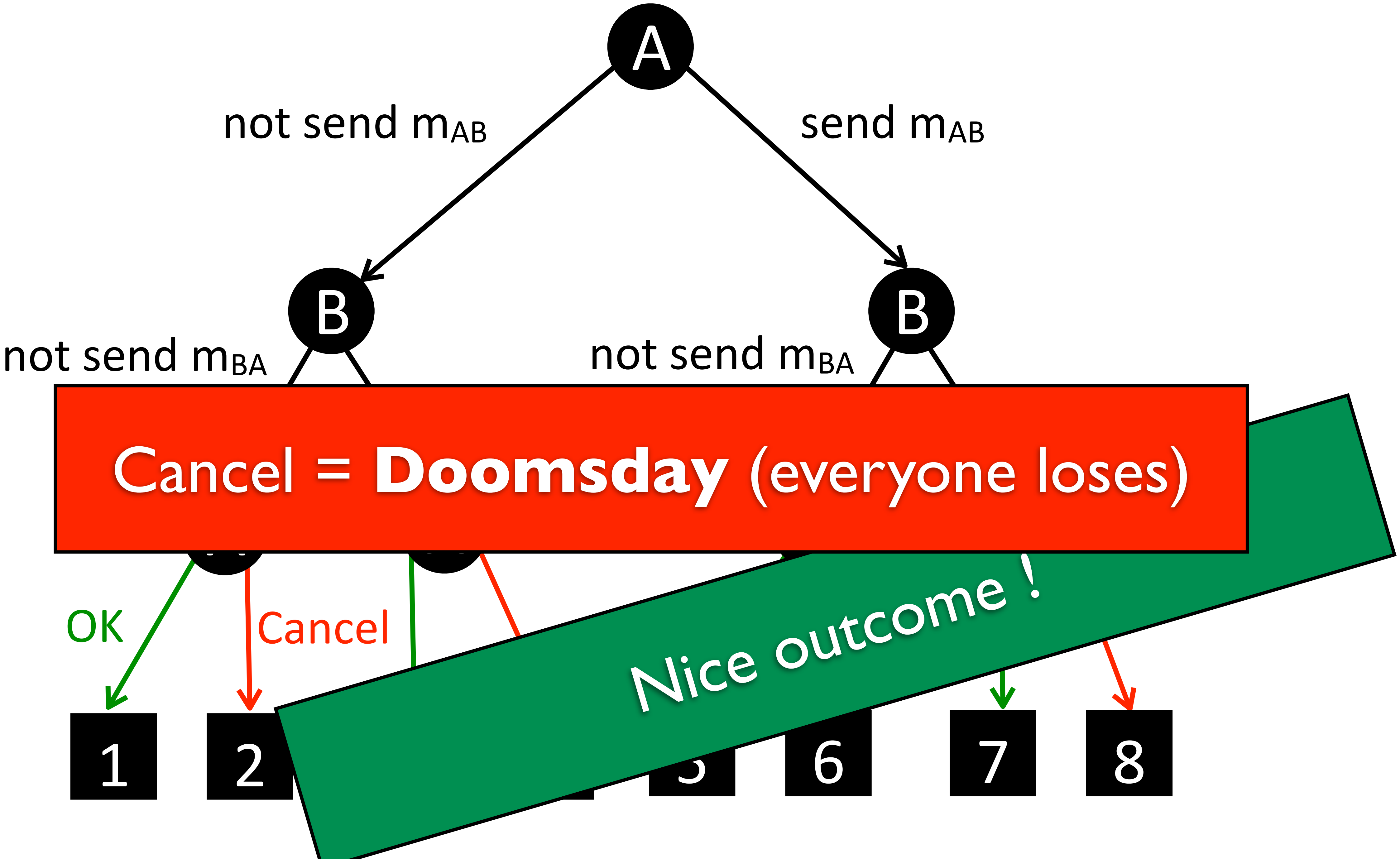

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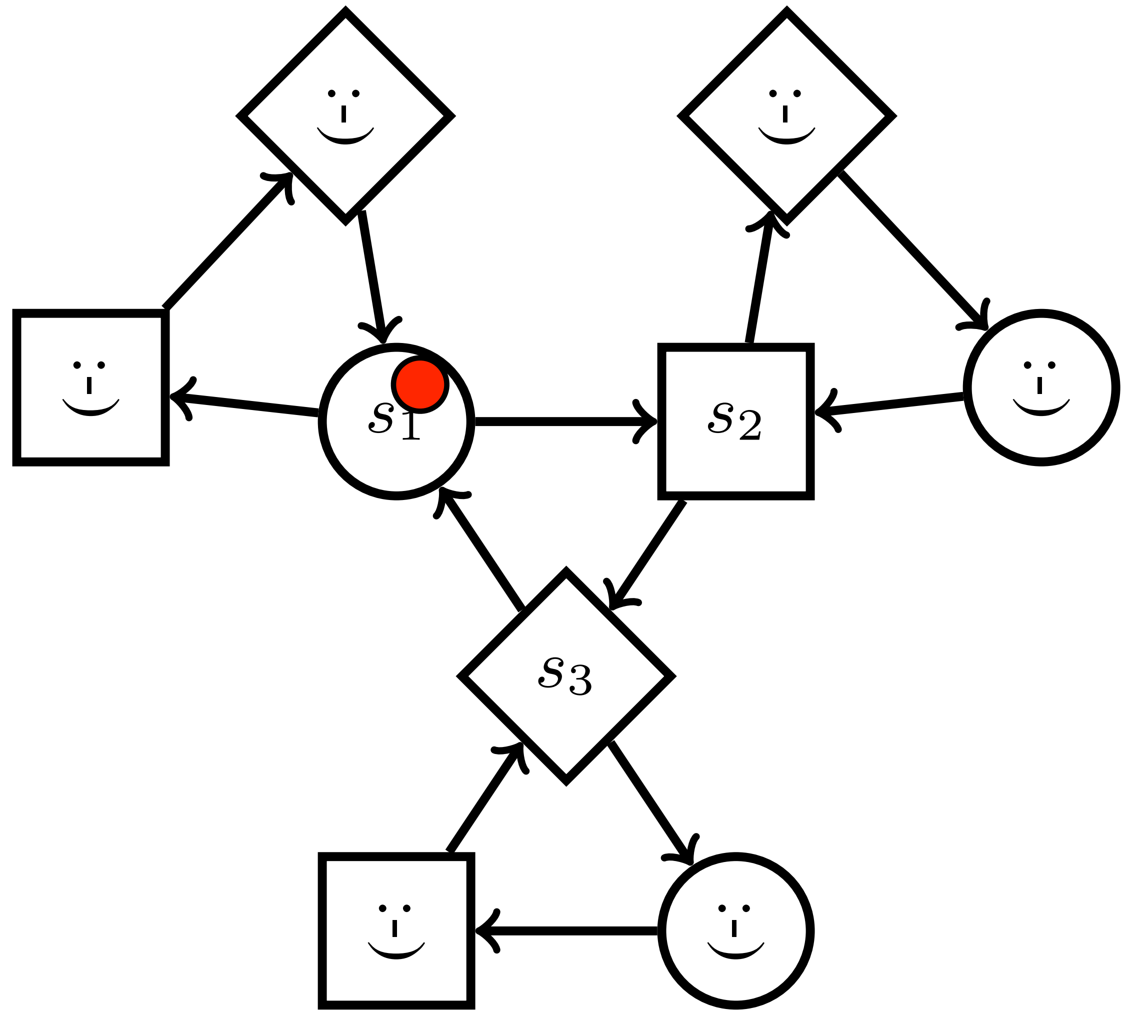
Doomsday threatening equilibria

A strategy profile $(St_1,St_2,\ldots,St_n)$ is a **doomsday threatening equilibrium (DE)** if:

1. $Outcome(St_1,St_2,\ldots,St_n)$ is "**winning**" for **all** players
2. For all player i, $Outcome(St_i)$ is such that: **either** player i wins **or** all players lose (doomsday) i.e. St_i is **good for retaliation**

Setting: n-player games



With **omega-regular objectives**: $Win_i \subseteq S^\omega$

$Win_i \in$ {safety, reachability, Büchi, coBüchi, parity, LTL}

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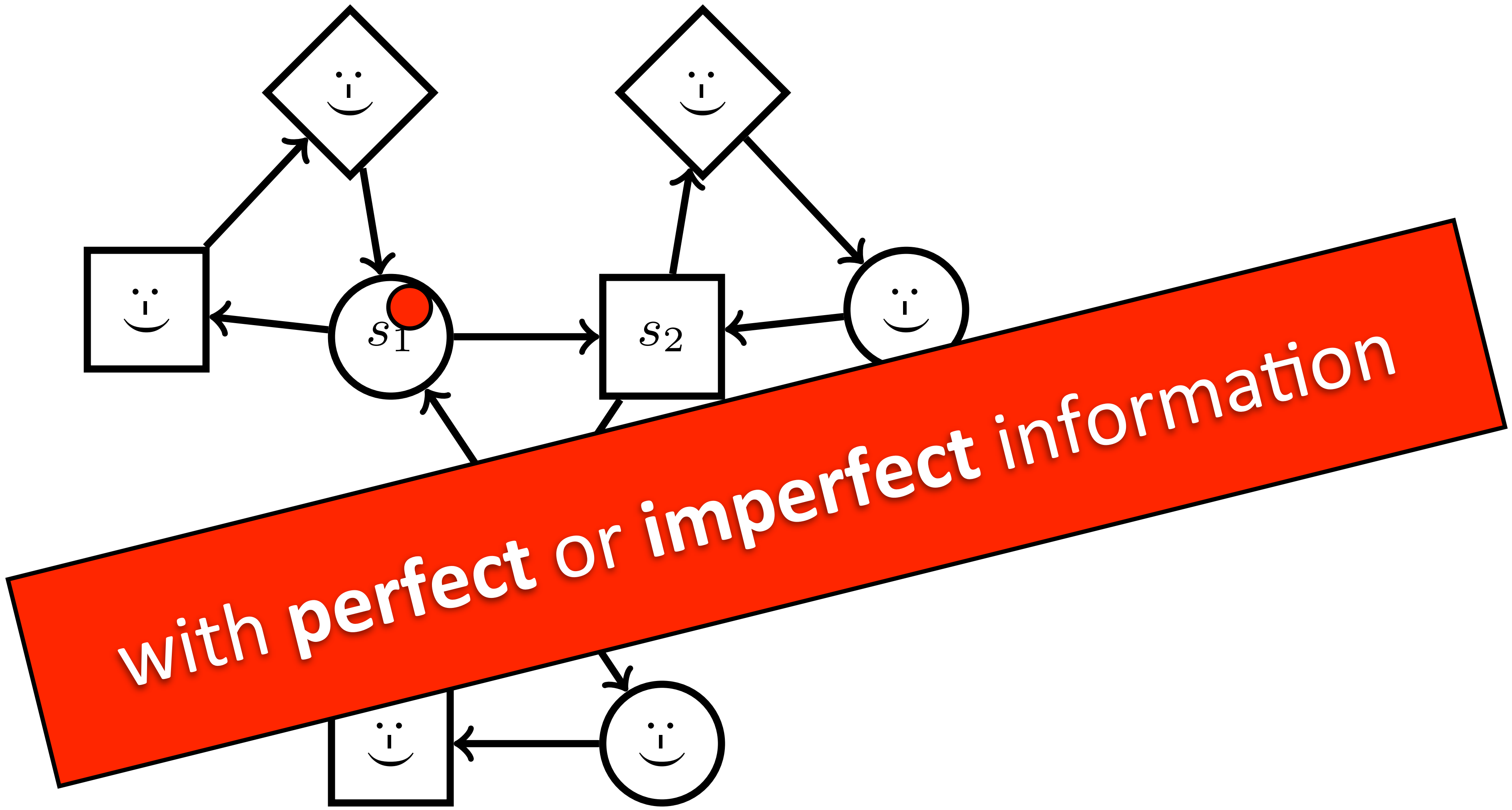

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Main results

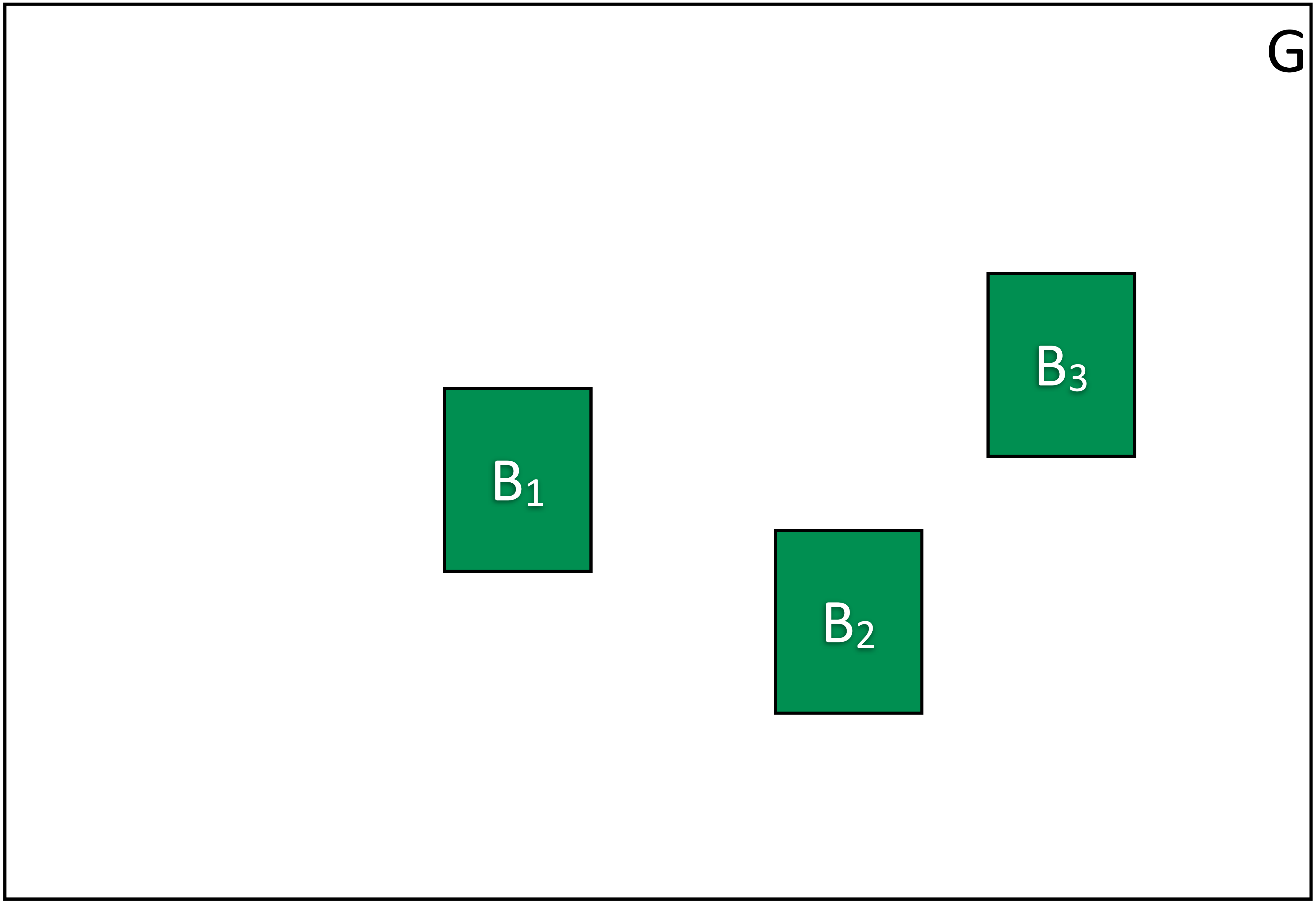
	Safety	Reach	Büchi	coBüchi	Parity	LTL
Perfect info	PSpace-C	PTime-C	PTime-C	PTime-C	in PSpace NP-Hard coNP-Hard	2ExpTimeC
Imperfect info	ExpTime-C	ExpTime-C	ExpTime-C	ExpTime-C	ExpTime-C	2ExpTimeC

Main results

	Safety	Reach	Büchi	coBüchi	Parity	LTL
Perfect info	2 PSpace-C	PTime-C	1 PTime-C	PTime-C	in PSpace NP-Hard coNP-Hard	2ExpTimeC
Imperfect info	ExpTime-C	3 ExpTime-C	ExpTime-C	ExpTime-C	ExpTime-C	2ExpTimeC

Doomsday Equilibria in Büchi Games

Büchi games

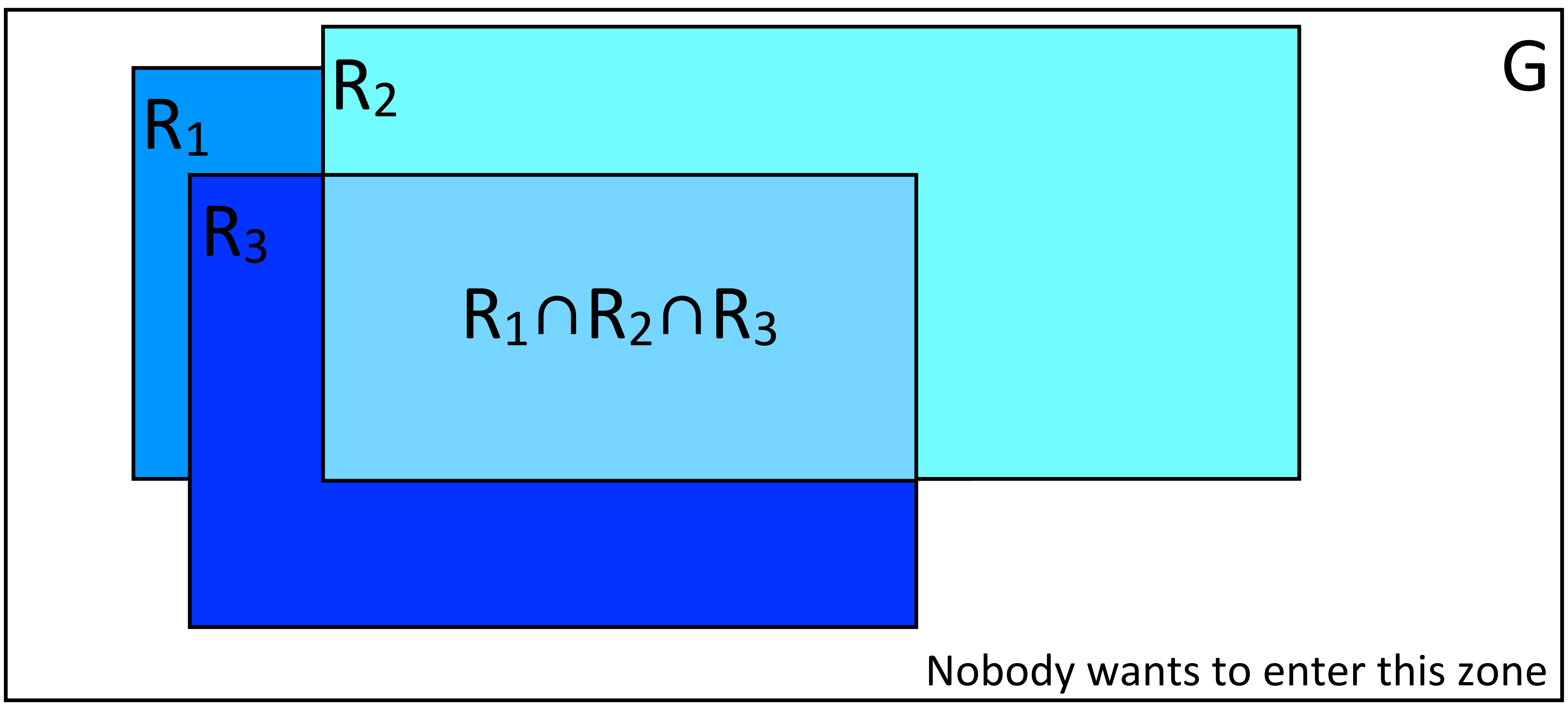


PTime algorithm for Büchi

Generic algorithm for **tail objectives**:

① compute $R_i = \langle\langle i \rangle\rangle \, Win_i \cup \bigcap_{j=1,..,n} \neg \, Win_j$

② check for a path in $\bigcap_{j=1,..,n} (\Box R_j \cap Win_j)$



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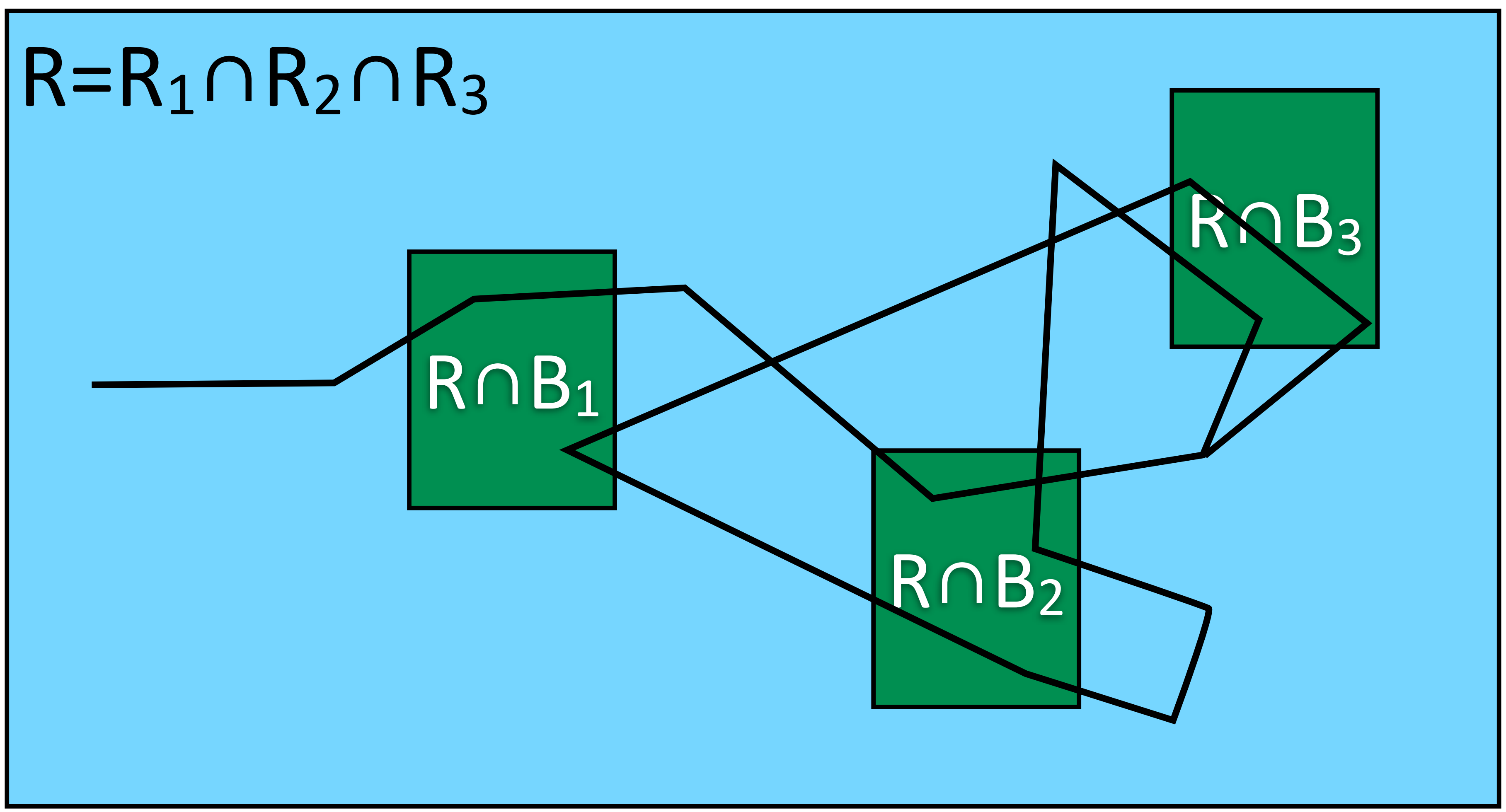


PTime algorithm for Büchi

Correctness:

(completness): Let $(St_1, St_2, \ldots, St_n)$ be a **DE then** $Out(St_1, St_2, \ldots, St_n)$ is winning for all players and never leaves $R_1 \cap \ldots \cap R_n$

(soundness): A *universally* winning path in $R_1 \cap \ldots \cap R_n$ witnesses a **DE**

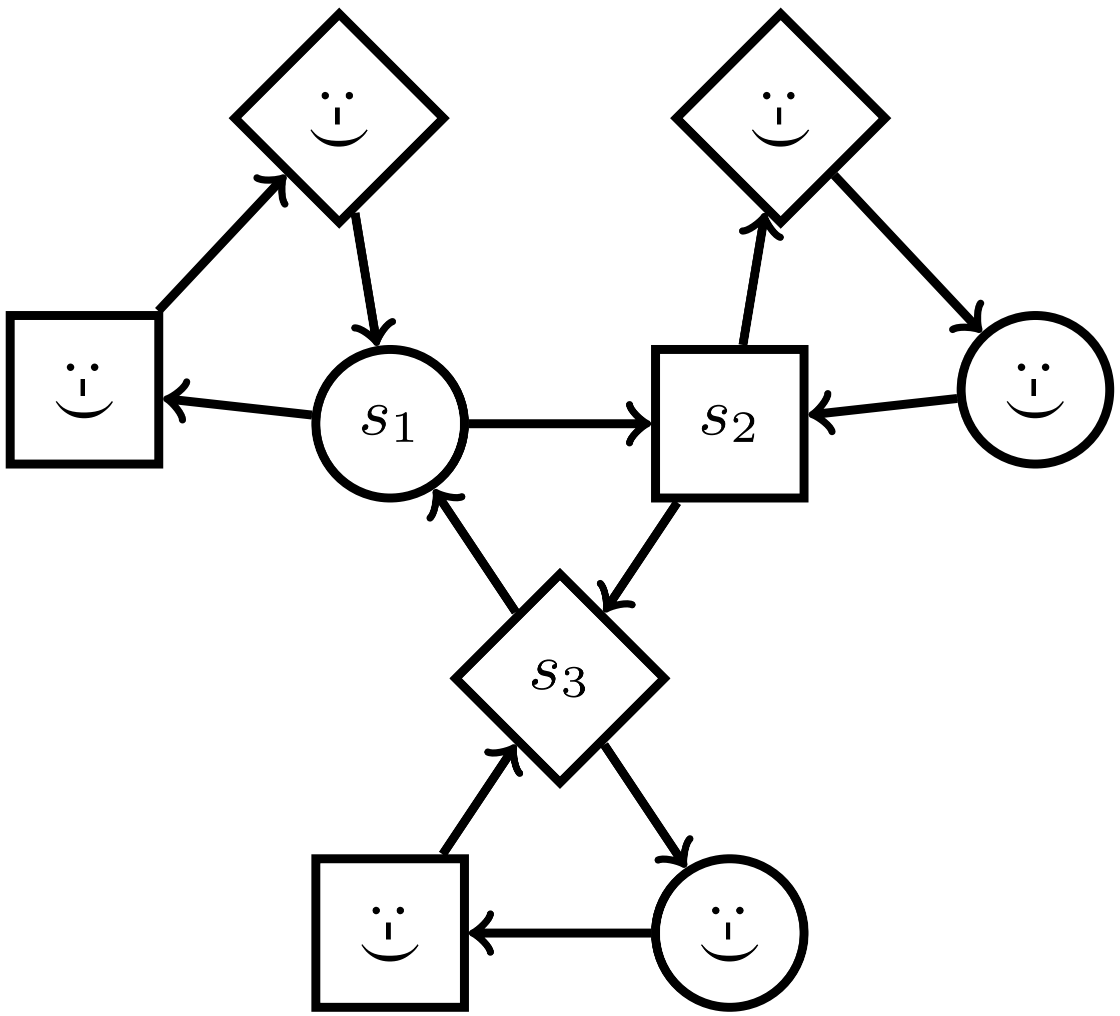
Complexity:

(easyness): Computing $R_i = \langle\langle i \rangle\rangle \, Win_i \cup \cap_{j=1,..,n} \neg \, Win_j$
in **PTIME** (Street game with one pair):

$$\langle\langle i \rangle\rangle \, \Box \Diamond B_i \vee \Diamond \Box \wedge_{j=1,..,n} \neg \, B_j$$

(hardness): **PTIME-hard** (reduc. from 2-pla. zero-sum Büchi games)

Büchi: an example



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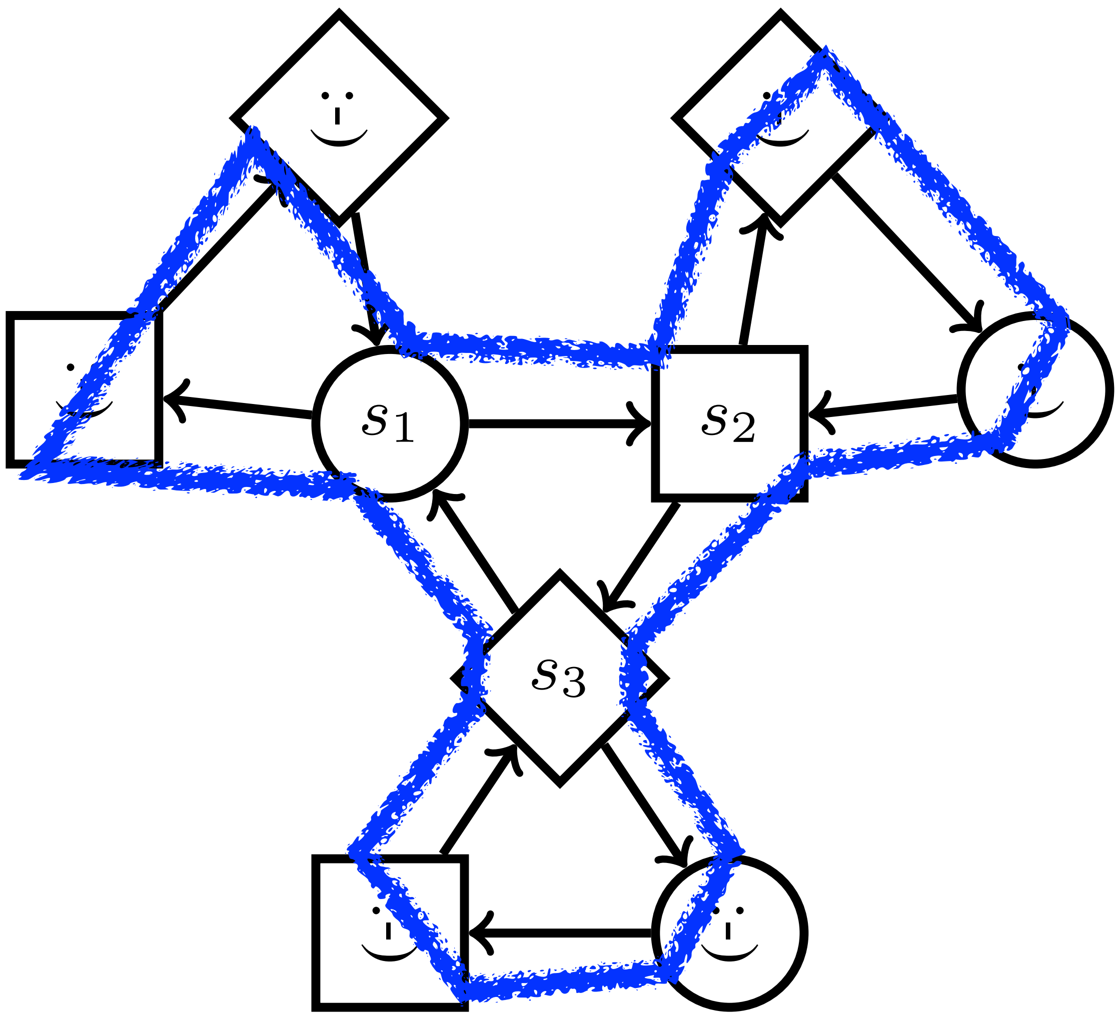
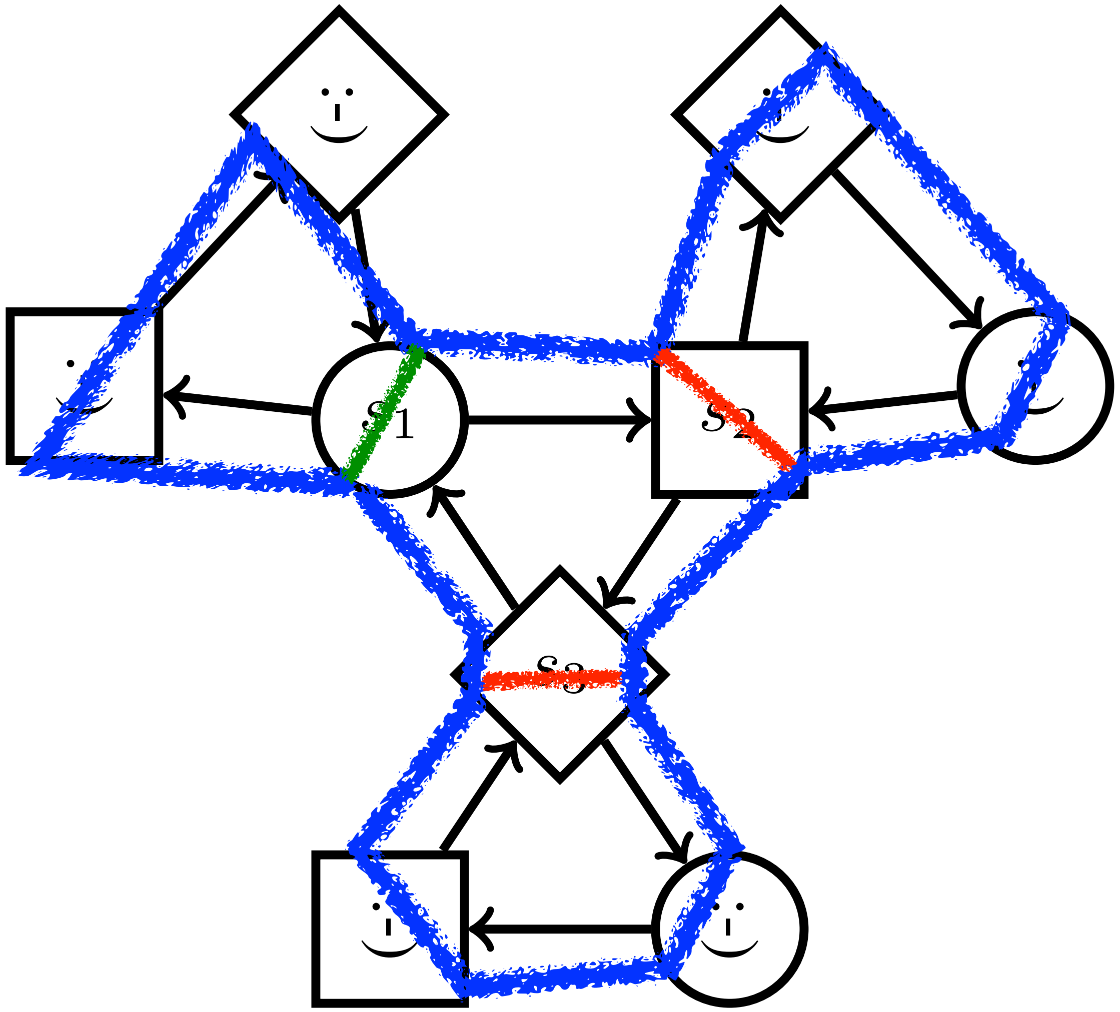


Witness a doomsday equilibrium if in addition, every player retaliates by skipping his loop when other players deviates.

Büchi: an example

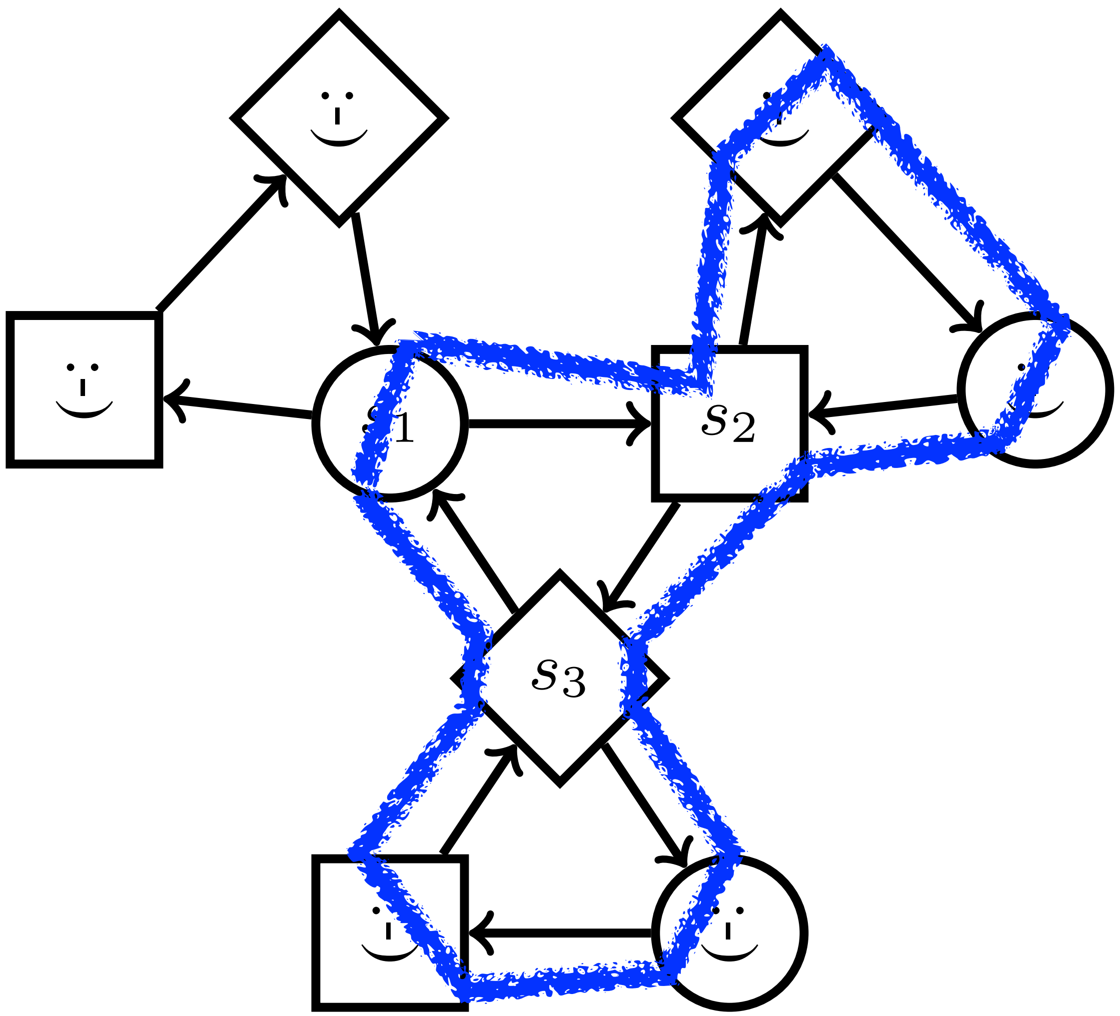


Witnesses a doomsday equilibrium if in addition, every player retaliates by skipping his loop when other players deviates.

Now consider strategies of Player 2 and 3 s.t. Player 1 loses.

Clearly, this appends only if Players 2 and 3 eventually never take their loop, but if it is the case, then Player 1 retaliates by avoiding his loop and **all** the players lose.

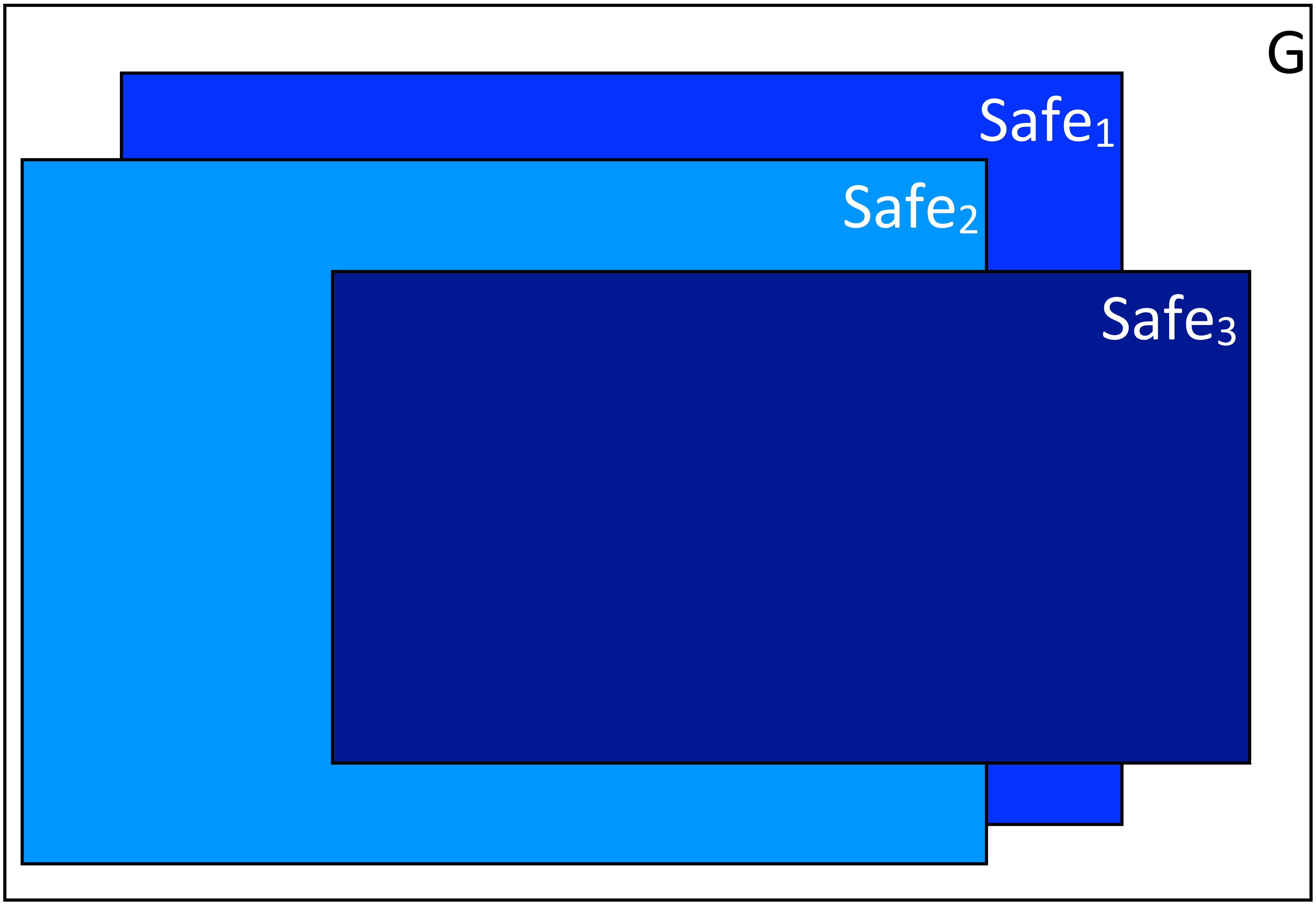
Büchi: an example



does also witness a DE !

Doomsday Equilibria in Safety Games

Safety games



Algorithm for safety

① compute $R_i = \langle\langle i \rangle\rangle \Box \mathit{Safe}_i \vee \bigwedge_{j=1..n} \Diamond \neg \mathit{Safe}_j$

This can be computed in PSpace [Alur et al. 04]

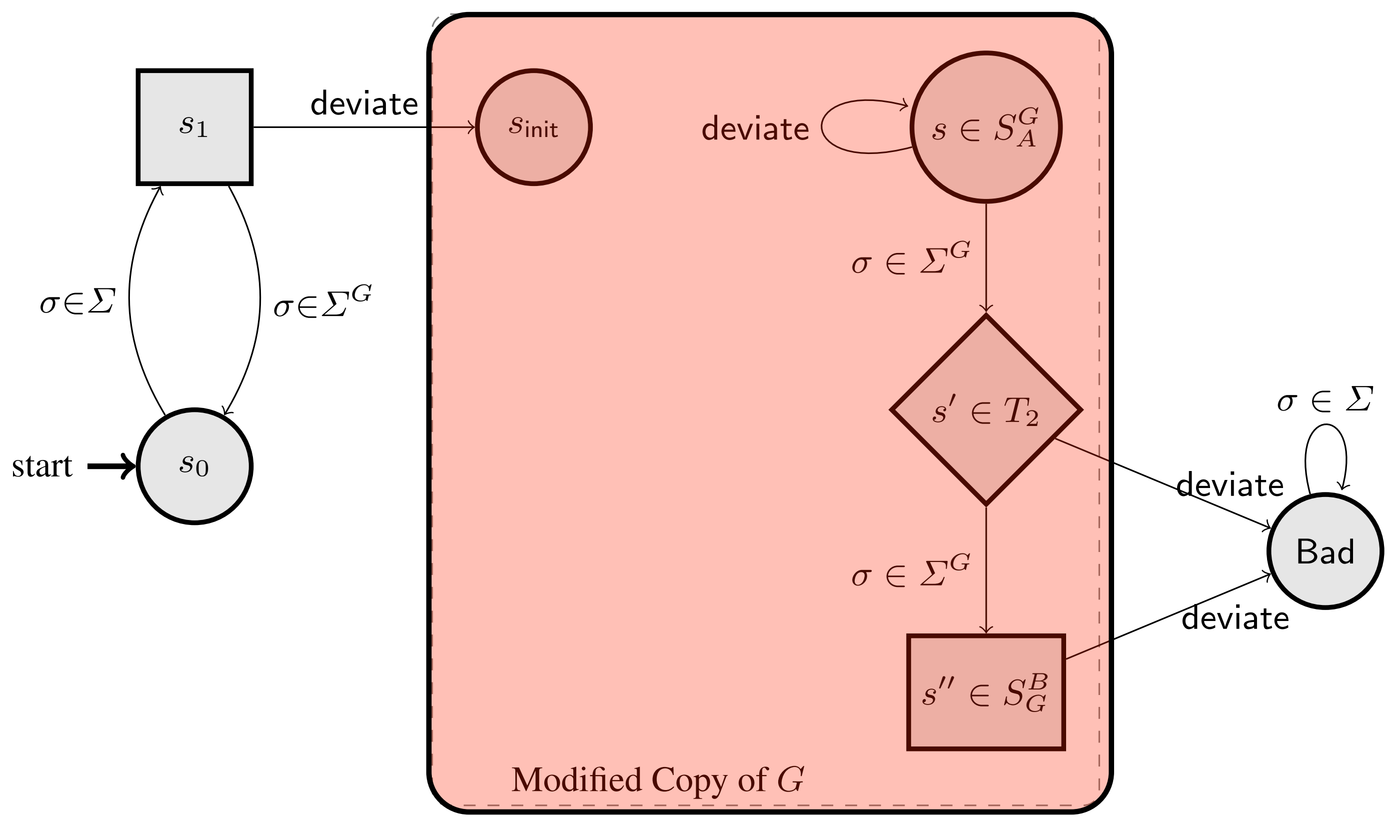
② check for a path in $\bigcap_{i=1,..,n} (\mathit{Safe}_i \cap R_i)$

This can be compute in PTime.

PSpace-Hardness for safety

Reduction from **generalized reachability games**

$\langle\langle 1 \rangle\rangle \Diamond T_1 \wedge \Diamond T_2 \wedge \ldots \wedge \Diamond T_n$ (PSpace-C) [AL04,FH11].

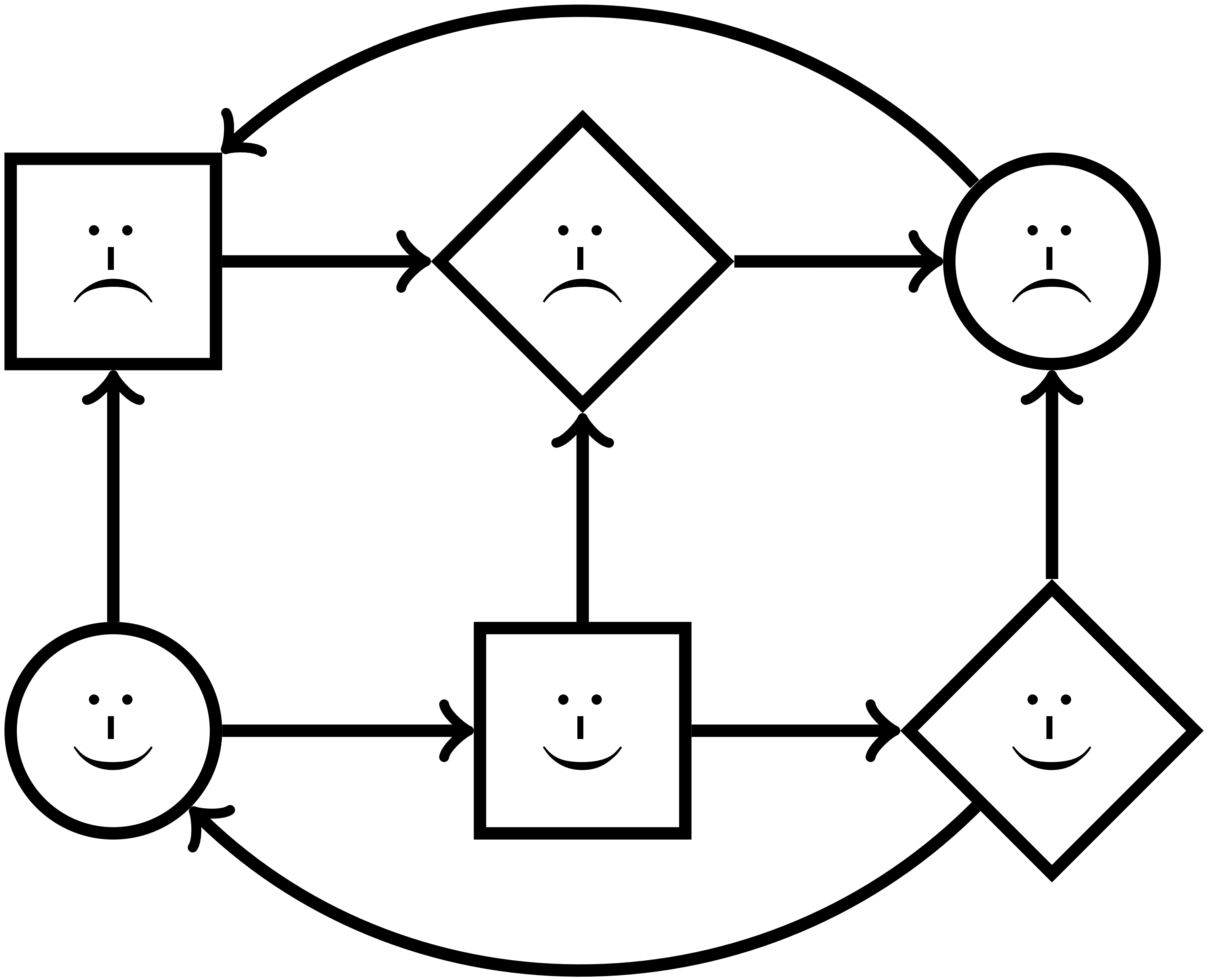


Unsafe for Player 1

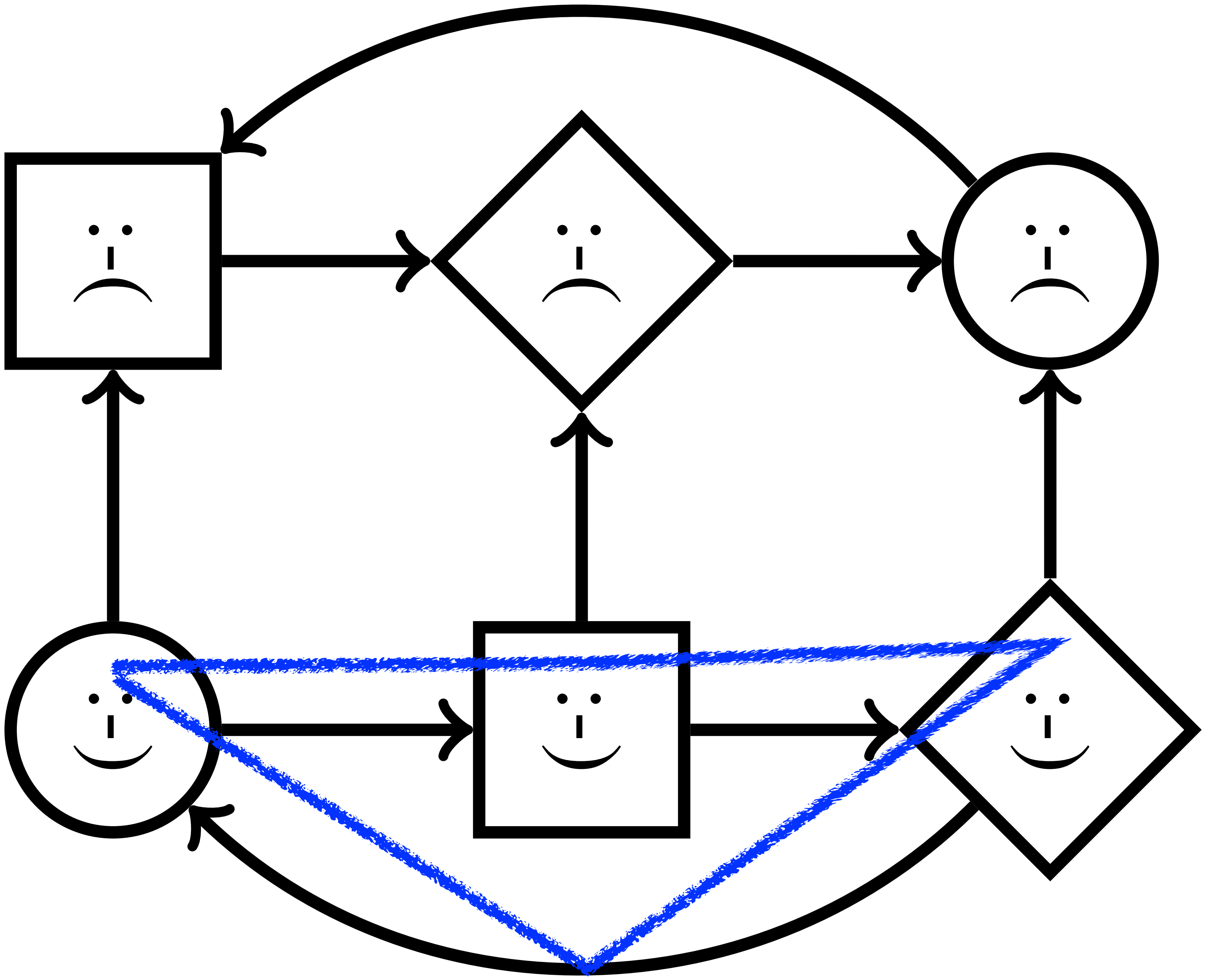
Player 1 can retaliate **iff** in modified copy of G, he can force a visit to $Unsafe_2 = T_1$, ..., $Unsafe_{n+1} = T_n$.

This is equivalent to ask if Player 1 wins the **generalized reachability game.**

Safety : an example



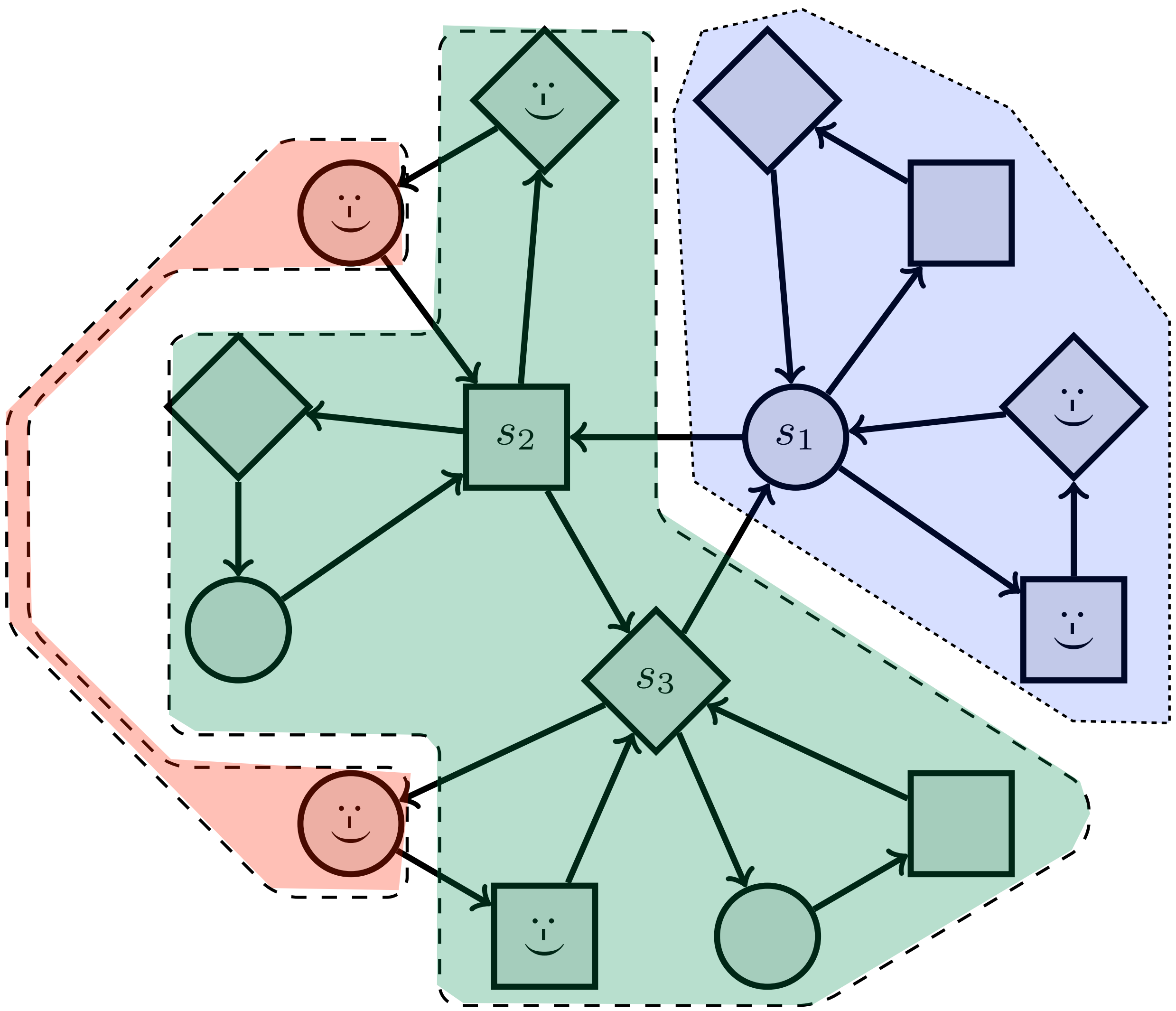
Safety : an example



Any deviation leads to **doomsday** !

Doomsday Equilibria in Games with Imperfect Info

Imperfect information



Ex: Observations of Player 1

For each player, there is a **partition** of the state space into **observation classes**

Strategies must be **observation-based**, i.e.

$\cancel{\lambda_i : S^* S_i \rightarrow \Sigma}$

$\lambda_i : Obs_i^* Obs_i \rightarrow \Sigma$

Imperfect information

An important difficulty:

A player can**not** always detect deviation, he can only detect **deviation from the expected observation sequence** !

Solution:

A seq. of obs. $\eta \in (\mathrm{Obs})^\omega$ is **doomsday compatible** (for Player i) if all plays that are compatible with η are either:

-winning for Player i,

-or losing for all the players (doomsday).

Imperfect information

When Player i **observes** deviation, he should be able to retaliate:

a prefix $\kappa \in (Obs_i)^* \cdot Obs_i$ of a seq. of obs. is
good for retaliation (for Player i)

if

there exists an **observation-based strategy** $\lambda_{i,R}$ of Player i s.t.
for all prefixes π compatible with κ:
outcome(π, $\lambda_{i,R}$) implies Player i wins or all players lose.

Solving games with imperfect info

Let G be an n-player game arena with imperfect information and winning objectives ϕ_i, $1 \leq i \leq n$.

There exists a **doomsday equilibrium** in G

iff

there exists a play ρ in G such that:

- $\rho \in \cap_{i=1..n} \phi_i$, i.e. ρ is **winning** for **all** the players,
- for all Player i, for all pref. κ of $Obs_i(\rho)$, κ is **i-good for retaliation**,
- for all Player i, $Obs_i(\rho)$ is **i-doomsday compatible**.

☛ leads to an **EXPTIME** algorithm!

Conclusion

- Introduction of **doomsday threatening equilibrium**
- **DE** refines and extends **secure equilibrium**
- Useful e.g. to reason on/synthesize **security protocols** (like fair exchange protocols)
- We have settled the **exact complexity** in most cases and Safraless approach for LTL
- DE leads to a **decidable** notion of equilibria in **imperfect information** games: DE avoids the usual undecidability results of n-player games with imperfect information