Static Analysis of Functional Programs using Tree Automata

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Outline

1. Motivating example
2. Background on tree automata completion
3. What is missing for a decent static analysis of functional programs?

... Related work scattered in subsections

Motivating example

OCaml type checking

```
let rec append l1 l2 = match l1 with
  | [] -> l2
  | h::t -> h :: (append t l2);;
# val append: 'a list -> 'a list -> 'a list = <fun>

let rec rev l = match l with
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We would like to have... more than simple types

```
# val rev: 'a list -> empty list
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Motivating example (II)

OCaml type checking

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We would like to have...

```
val rev: list of As then Bs -> list of Bs then As
```

Background: Term Rewriting

Sets of symbols and variables

- Set of ranked symbols $\mathcal{F} = \{app : 2, cons : 2, nil : 0, a : 0\}$
- Set of variables $\mathcal{X} = \{x, y, z, \ldots\}$

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Term Rewriting Systems (TRS)

Set of rewrite rules $l \to r$ with $l, r \in \mathcal{T}(\mathcal{F}, \mathcal{X})$ and $Var(r) \subseteq Var(l)$ *e.g.*

$$\mathcal{R} = \left\{ \begin{array}{l} app(nil, x) \to x \\ app(cons(x, y), z) \to cons(x, app(y, z)) \end{array} \right\}$$

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- Rewriting term $app(cons(a, nil), cons(b, nil))$ using

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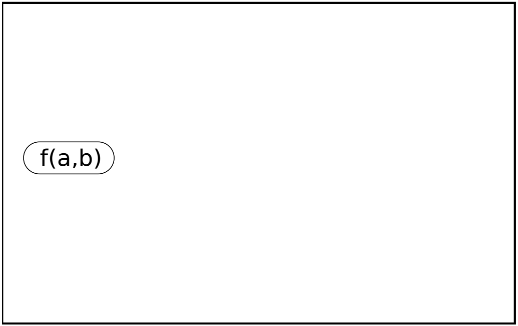
Equational abstraction [Meseguer & al. 03] [Takai 04]

$$\mathcal{R} = \begin{cases} (1)\ f(x,y) \rightarrow f(g(x),y) \\ (2)\ f(x,y) \rightarrow f(x,h(y)) \end{cases}$$

prove that $f(a,b) \not\rightarrow_{\mathcal{R}}{}^{*} f(a,h(g(b)))$?



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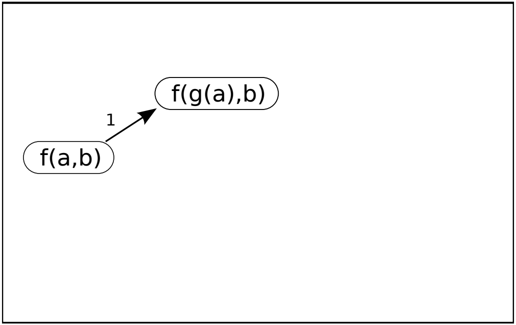

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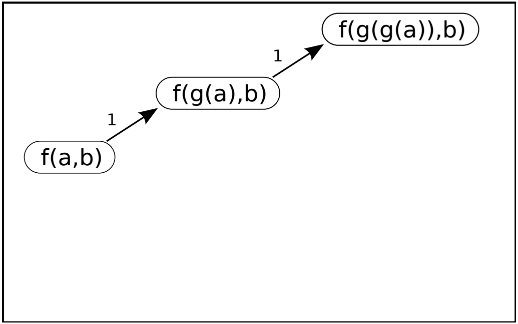

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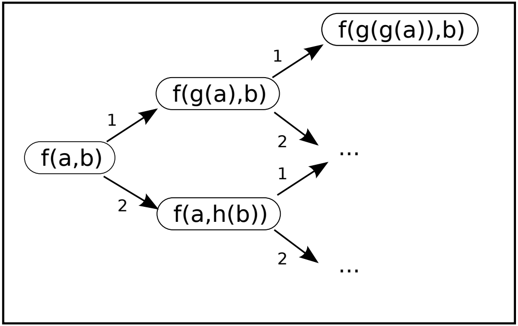

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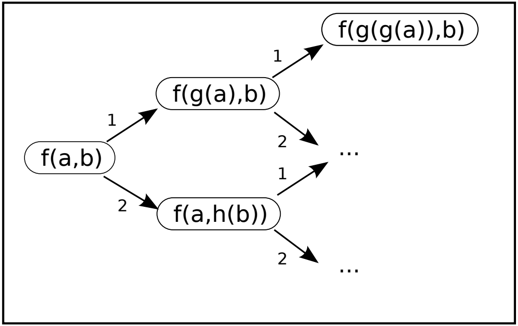

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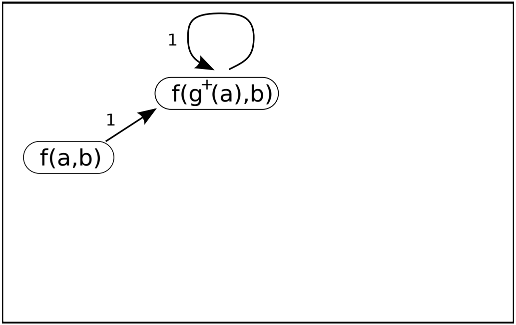

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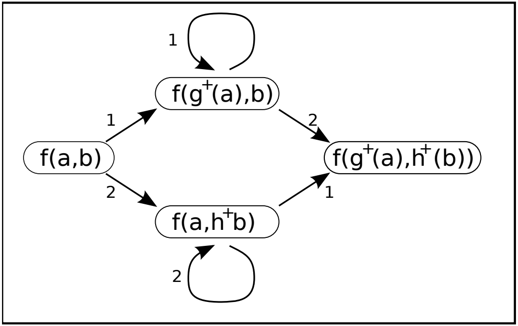

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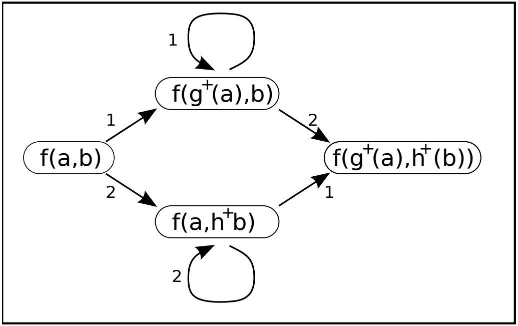

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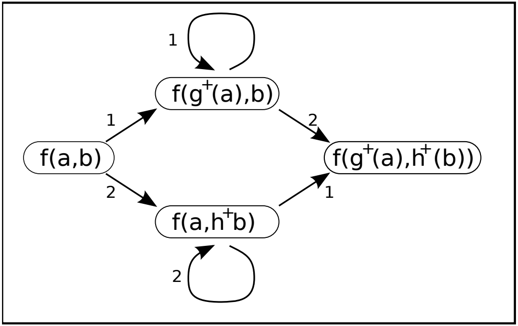

$$f(a,b) \not\to_{\mathcal{R}/E}^{*} f(a,h(g(b))) \quad \Rightarrow \quad f(a,b) \not\to_{\mathcal{R}}^{*} f(a,h(g(b)))$$

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Recognized language $\mathcal{L}(\mathcal{A}, q)$

$\mathcal{A} = \langle \mathcal{F}, \mathcal{Q}, \mathcal{Q}_f, \Delta \rangle$
with $\mathcal{Q} = \{q_a, q_b, q_{la}, q_{lb}, q_f\}$
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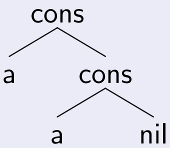

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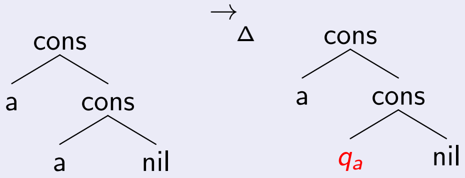

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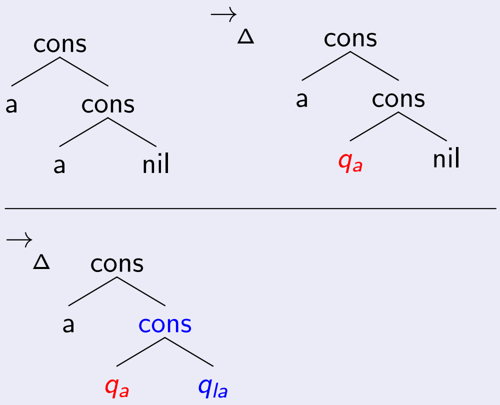

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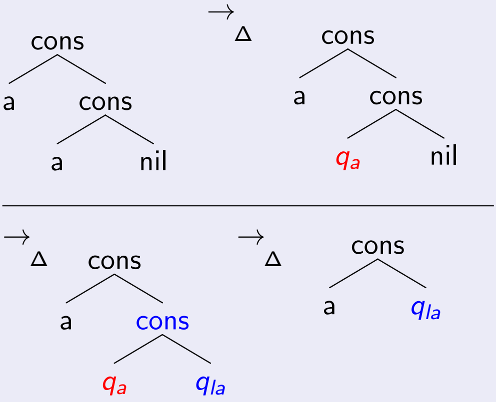

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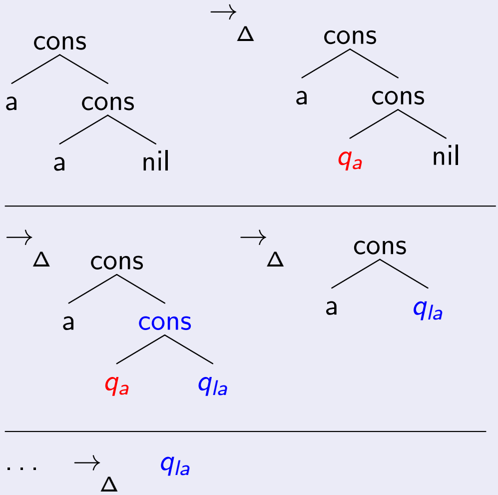

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$$\mathcal{L}(\mathcal{A}) = \{s \in \mathcal{T}(\mathcal{F}) \mid s \to_\Delta^* q \wedge q \in \mathcal{Q}_f\}$$

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Tree automata completion semi-algorithm (particular ARTMC)

- Input: a TRS $\mathcal{R}$, a tree automaton $\mathcal{A}$ and approximation equations E
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$$\mathcal{R}^*(\mathcal{L}(\mathcal{A})) \subseteq \mathcal{L}(\mathcal{A}^*_{\mathcal{R},E}) \subseteq \mathcal{R}^*_E(\mathcal{L}(\mathcal{A}))$$ [with V. Rusu, 2010]

Theorem 1 (Upper bound)

Given a left-linear TRS $\mathcal{R}$, a tree automaton $\mathcal{A}$ and a set of equations E, if completion terminates on $\mathcal{A}^*_{\mathcal{R},E}$ then $\mathcal{L}(\mathcal{A}^*_{\mathcal{R},E}) \supseteq \mathcal{R}^*(\mathcal{L}(\mathcal{A}))$.

Theorem 2 (Lower bound)

Given a left-linear TRS $\mathcal{R}$, a tree automaton $\mathcal{A}$ and a set of equations E, if $\mathcal{A}$ is R/E-coherent then $\mathcal{L}(\mathcal{A}^i_{\mathcal{R},E}) \subseteq \mathcal{R}^*_E(\mathcal{L}(\mathcal{A}))$.

Tree Automata Completion Demo : Timbuk

[with V. Viet Triem Tong, Y. Boichut, B. Boyer, V. Murat]
(Around 13000 lines of Ocaml)

Timbuk provides

- Tree automata completion
- Equational approximations
- Coq checker for completion results
- Beta: CEGAR, Abstract Domains (*e.g. integer intervals*)

Used for Cyptographic Protocol, Java and JavaScript verification

Demo:

- `demo_basic.txt`
- `demo_reverseBug.txt`

What is missing for static analysis of functional languages?

1. Define equations guaranteeing termination of completion
2. Deal with higher order functions
3. Take evaluation strategies into account
   - call by value (*e.g.* Ocaml) $\approx$ innermost rewrite strategy
   - call by need (*e.g.* Haskell) $\approx$ outermost rewrite strategy + sharing
   - order in pattern matching $\approx$ priority rewrite strategy
4. Deal with built-in types (`int`,`float`,`char`,`strings`, ...)
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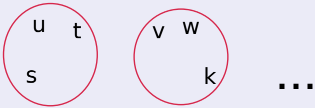
Equations guaranteeing termination of completion

Intuition: finite set of E-equivalence classes $\Rightarrow$ completion terminates

$\mathcal{T}(\mathcal{F})/_{=_E}$ is



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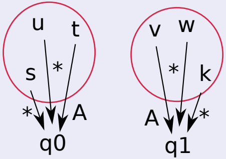

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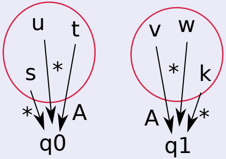

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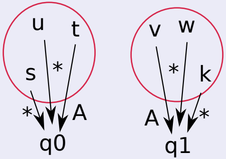

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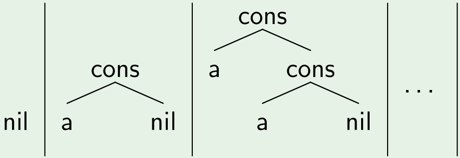
Example 3 (Equations for the append function)

Let $\mathcal{F} = \{app : 2, cons : 2, nil : 0, a : 0\}$.



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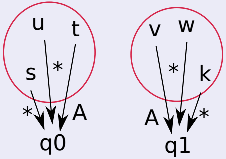

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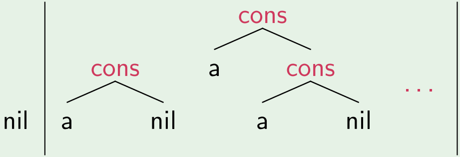

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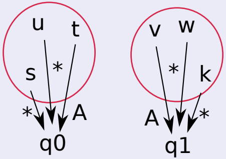

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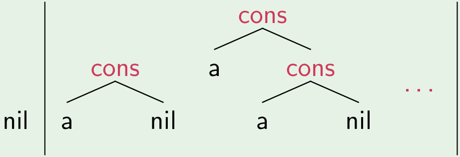


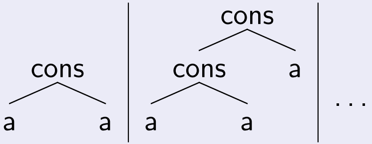
With $E = \{cons(x, cons(y, z)) = cons(x, z)\}$

But $\mathcal{T}(\mathcal{F})/_{=_E}$ is not finite!

Equations guaranteeing termination of completion (II)

With $E = \{cons(x, cons(y, z)) = cons(x, z)\}$, $\mathcal{T}(\mathcal{F})/_{=_E}$ is not finite!

Infinitely many classes of ill-typed terms



are all in different classes!

Ill-typed terms incompatible with `cons:`α `->` α `list ->` α `list`

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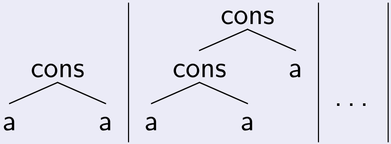

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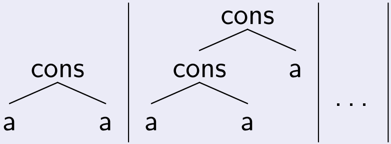

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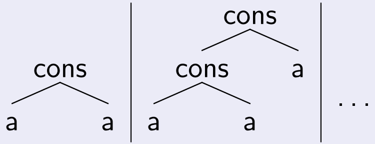

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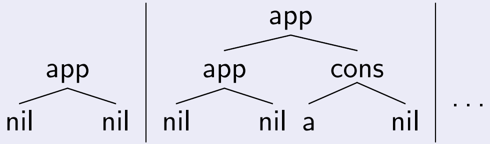
Infinitely many classes of partially evaluated terms



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Partially evaluated terms

Equations guaranteeing termination of completion (III)

Proposed solution: use $\mathcal{F} = \mathcal{C} \uplus \mathcal{D}$ and $E = E^c_{\mathcal{C}} \cup E_{\mathcal{R}}$

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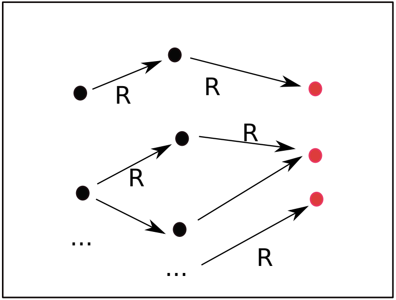

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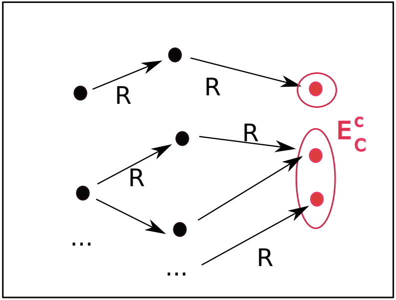

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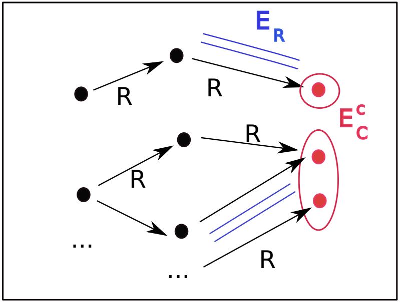

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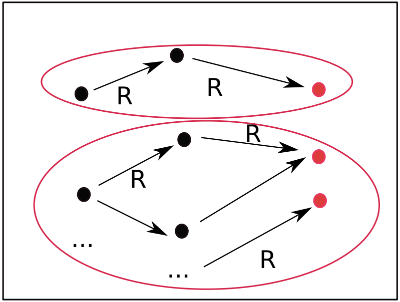

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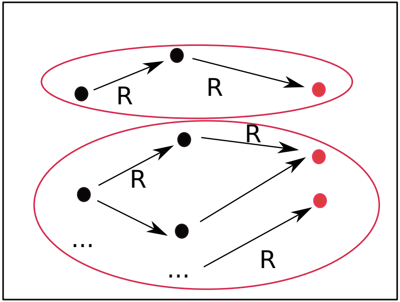

A word about Higher-Order functions

Static analysis of higher-order functional programs

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Example 4 (Encoding of H.O. functions into TRS)

Use an explicit function application operator '@'.

```
let rec map f l1 = match l1 with
  | [] -> []
  | h::t -> (f h) :: (map f t);;
```

becomes

$$@(@(map, f), nil) \to nil$$

$$@(@(map, f), cons(h, t)) \to cons(@(f, h), @(@(map, f), t))$$

A word about Higher-Order functions (II)

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Example 5 (filter nz on any nat list, results in a list without 0)

```
let if2 c t e = match c with
  | true -> t
  | false -> e;;
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let nz i= match i with
  | 0 -> false
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let rec filter p l= match l with
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Successful on some examples but needs to be investigated further!

A word about evaluation strategies

Example 6 (Terminating with call-by-need but not for call-by-value)

```
let rec sumList(x,y)= (x+y)::sumList(x+y,y+1);;
let rec nth i (x::l)= if i<=0 then x else nth (i-1) l;;
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Adapted tree automata completion for innermost strategy [with Y. Salmon]

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let sum x= nth x (sumList(0,0));;
```

$(sum\, 4) = 10$ with call by need and diverges with call-by-value

Completion covers all reachable terms (for all strategies)

$\mathcal{R}^*((sum\, 4)) \subseteq \mathcal{L}(\mathcal{A}^*_{\mathcal{R},E})$ contains 10 (and intermediate computations)

Call-by-value $\leftrightarrow$ innermost strategy for TRSs

Adapted tree automata completion for innermost strategy [with Y. Salmon]

$\mathcal{R}^*_{in}((sum\, x)) \subseteq \mathcal{L}(\mathcal{A}^*_{\mathcal{R}_{in},E})$ contains no normal form (no result)

A word about built-in types

Recall this example:

Example 7 (`filter nz` on any `nat list`, results in a list without 0)

```
let if2 c t e = match c with
  | true -> t
  | false -> e;;
```

```
let nz i= match i with
  | 0 -> false
  | S(x) -> true;;
```

Programs usually use machine integers instead of Peano numbers

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e.g. integer lists with no zero:

$cons(q_i, q_l) \rightarrow q_l$	$[-\infty; -1] \rightarrow q_i$
$nil \rightarrow q_l$	$[1; +\infty] \rightarrow q_i$

What about the presentation of the results/annotations?

A simple automaton for the *A* then *B* lists

```
Automaton A0
States qA, qB, qnil, qlB, qlAB
Final States qlAB
Transitions
  A -> qA
  B -> qB
  nil -> qnil
  cons(qB,qnil) -> qlB
  cons(qB,qlB) -> qlB
  cons(qA,qlB) -> qlAB
  cons(qA, qlAB) -> qlAB
```

Any suggestion for a short textual/graphical format is welcome!

What about the presentation of the results/annotations?

Contracts [D. Xu, 2009]

```
contract rev = {l | ab l} -> {l | ba l};;
```

where `ab` and `ba` are user defined functions discriminating the ≪A then B lists≫ etc. Contracts can be dynamically or statically checked.

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```
rev :: [a]<{\h v -> h <= v}> -> [a]<{\h v -> h >= v}
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Two remarks and one question

+ Those techniques prove stronger properties (*e.g.* `quicksort` sorts)

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Two remarks and one question

+ Those techniques prove stronger properties (*e.g.* `quicksort` sorts)
– (Co)-Domains annotations are given by the user (we infer them)
• Can we define user friendly "language annotations" close to types?

Conclusion

1. Define equations guaranteeing termination of completion ✓ ⚠
2. Deal with higher order functions ⚠
3. Take evaluation strategies into account
    - call by value (*e.g.* Ocaml) $\approx$ innermost rewrite strategy ✓
    - call by need (*e.g.* Haskell) $\approx$ outermost rewrite strategy $+$ sharing
    - order in pattern matching $\approx$ priority rewrite strategy
4. Deal with built-in types ✓
5. Modularity of the analysis
6. User friendly way to display/define language annotations ...

Further research

- Find a translation from OCaml to TRS s.t.
  - Typing is preserved
  - Higher-order functions can be encoded
  - OCaml pattern matching exhaustivity $\Rightarrow$ TRS sufficient completeness

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  e.g. Discard the "sufficient completeness" requirement

Example 8 (sumList is not sufficiently complete)

```
let rec sumList(x,y)= (x+y)::sumList(x+y,y+1);;
let rec nth i (x::l)= if i<=0 then x else nth (i-1) l;;
let sum x= nth x (sumList(0,0));;
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Completion algorithm

Tree automata completion principle

1. complete $\mathcal{A}$ with new transitions into $\mathcal{A}^1_{\mathcal{R}}, \mathcal{A}^2_{\mathcal{R}}, \ldots$

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   $\forall l \to r \in \mathcal{R}$, $\forall q \in \mathcal{Q}$, $\forall \sigma : \mathcal{X} \mapsto \mathcal{Q}$:

$$\begin{array}{ccc} l\sigma & \xrightarrow[\mathcal{R}]{} & r\sigma \\ {\scriptstyle \mathcal{A}_{\mathcal{R}}^i}\big\downarrow{\scriptstyle *} & & \\ q & & \end{array}$$

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$$\begin{tikzcd} l\sigma \arrow[r, "\mathcal{R}"'] \arrow[d, "\mathcal{A}_{\mathcal{R}}^i"', "*"] & r\sigma \arrow[d, "*"', "\mathcal{A}_{\mathcal{R}}^{i+1}"] \\ q & q' \arrow[l, "\mathcal{A}_{i+1}"] \end{tikzcd}$$

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q & q' \arrow[l, "\mathcal{A}_{i+1}"]
\end{tikzcd}
$$

2. use approximation equations of E to (possibly) converge on $\mathcal{A}^*_{\mathcal{R},E}$

Completion algorithm (II)

Definition 9 (Set $E^c_{\mathcal{C}}$ of contracting equations)

The set of well-sorted equations $E^c_{\mathcal{C}}$ is *contracting* if its equations are of the form $u = u|_p$ with u linear and $p \neq \Lambda$ and if the set of normal forms of $\mathcal{T}(\mathcal{C})^{\mathcal{S}}$ w.r.t. the TRS $\overrightarrow{E^c_{\mathcal{C}}} = \{u \to v \mid u = v \in E^c_{\mathcal{C}}\}$ is finite.

R/E-coherence

Languages recognized by states of $\mathcal{A}$ (ϵ-free) are E-equivalent terms.