

# Strong simulation for one counter nets is PSPACE-complete

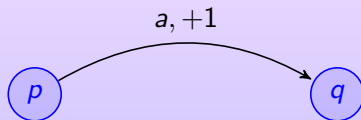
Piotr Hofman, Sławomir Lasota, Richard Mayr and Patrick Totzke  
Institute of Informatics - University of Warsaw  
School of Informatics - University of Edinburgh

# Plan

- 1 Model
- 2 Simulation
- 3 History
- 4 Results
- 5 Future work

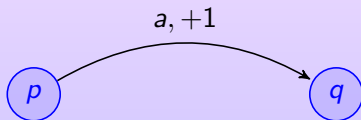
# One-Counter Nets

$(Q, Act, \delta) \quad \delta \subseteq (Q \times Act \times \{-1, 0, +1\} \times Q)$

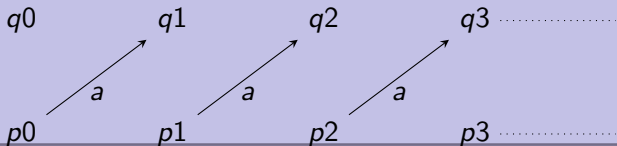


# One-Counter Nets

$(Q, Act, \delta) \quad \delta \subseteq (Q \times Act \times \{-1, 0, +1\} \times Q)$

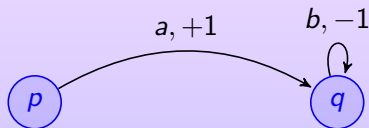


## Induced LTS over $Q \times \mathbb{N}$

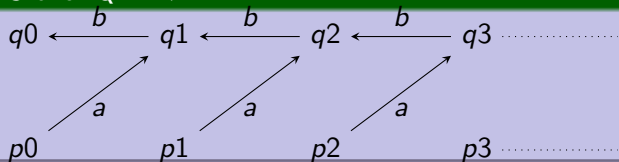


# One-Counter Nets

$(Q, Act, \delta) \quad \delta \subseteq (Q \times Act \times \{-1, 0, +1\} \times Q)$

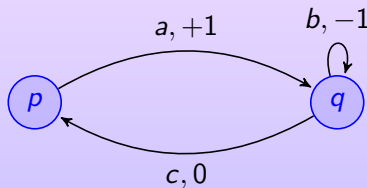


## Induced LTS over $Q \times \mathbb{N}$

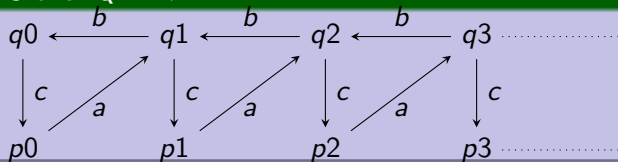


# One-Counter Nets

$(Q, Act, \delta) \quad \delta \subseteq (Q \times Act \times \{-1, 0, +1\} \times Q)$

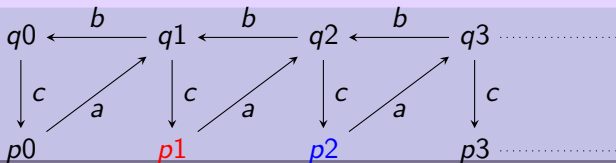


## Induced LTS over $Q \times \mathbb{N}$



# Simulation Games

... are played in rounds between **Spoiler** and **Duplicator** on the pair of configurations:



Round description on the next slide.

## Winning conditions:

- 1 If a player cannot move the other wins.
- 2 Infinite plays are won by Duplicator.

# Simulation Games

...are played in rounds between Spoiler and Duplicator.

In each round

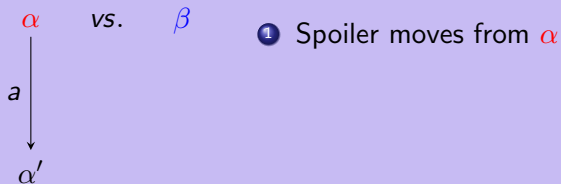
$\alpha$  vs.  $\beta$



# Simulation Games

... are played in rounds between **Spoiler** and **Duplicator**.

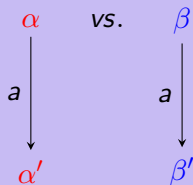
In each round



# Simulation Games

... are played in rounds between **Spoiler** and **Duplicator**.

In each round

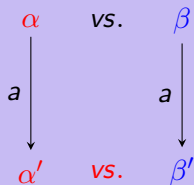


- 1 Spoiler moves from  $\alpha$
- 2 Duplicator responds from  $\beta$

# Simulation Games

... are played in rounds between **Spoiler** and **Duplicator**.

In each round



- 1 Spoiler moves from  $\alpha$
- 2 Duplicator responds from  $\beta$
- 3 game continues from  $\alpha'$  vs.  $\beta'$

# History

Previous results

# History

## Previous results

- Simulation is Decidable for One-counter Nets  
by *Parosh Aziz Abdulla* , *Karlis Cerans*

# History

## Previous results

- Simulation is Decidable for One-counter Nets  
by *Parosh Aziz Abdulla , Karlis Cerans*
- Simulation and Bisimulation over One-Counter Processes  
by *Petr Jančar , Antonín Kučera , and Faron Moller*

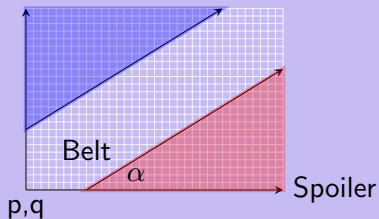
# History

## Previous results

- Simulation is Decidable for One-counter Nets  
by *Parosh Aziz Abdulla , Karlis Cerans*
- Simulation and Bisimulation over One-Counter Processes  
by *Petr Jančar , Antonín Kučera , and Faron Moller*

## Description of the relation

Duplicator



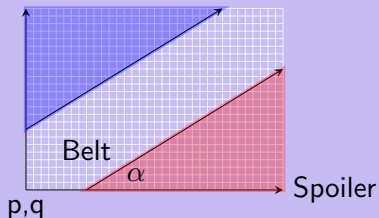
# History

## Previous results

- Simulation is Decidable for One-counter Nets  
by *Parosh Aziz Abdulla , Karlis Cerans*
- Simulation and Bisimulation over One-Counter Processes  
by *Petr Jančar , Antonín Kučera , and Faron Moller*

## Description of the relation

Duplicator



- Slope  $\alpha$  is  $\text{poly}(|\text{Net}|)$ .



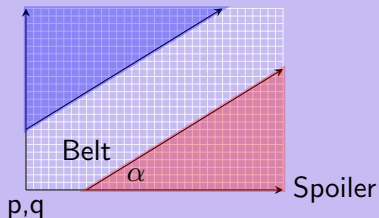
# History

## Previous results

- Simulation is Decidable for One-counter Nets  
by *Parosh Aziz Abdulla , Karlis Cerans*
- Simulation and Bisimulation over One-Counter Processes  
by *Petr Jančar , Antonín Kučera , and Faron Moller*

## Description of the relation

Duplicator

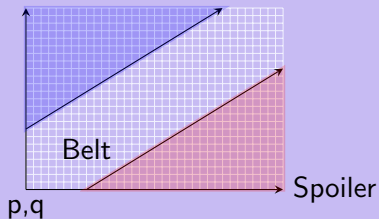


- Slope  $\alpha$  is  $\text{poly}(|\text{Net}|)$ .
- Pattern inside the belt is regular.

# Our contribution

## Description of the relation

Duplicator

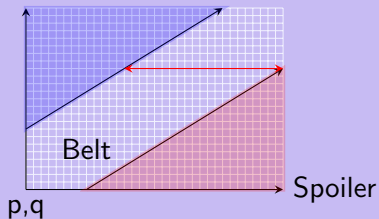


- New proof of the belt theorem.

# Our contribution

## Description of the relation

Duplicator

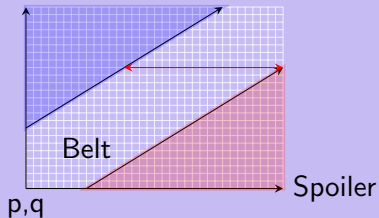


- New proof of the belt theorem.
- **Width** of the belt is  $\text{poly}(|Net|)$ .

# Our contribution

## Description of the relation

Duplicator



- New proof of the belt theorem.
- **Width** of the belt is  $\text{poly}(|Net|)$ .

## One Counter Automata

OCA vs. OCN is decidable.

Thank You.

# Questions?