Expressiveness modulo bisimilarity: a coalgebraic perspective

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- Janin & Walukiewicz: $\mu\mathsf{ML} \equiv \mathsf{MSO}/\underline{\leftrightarrow}$

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Proof sketch: J & W use automata:

1. $\mathsf{MSO} \equiv \mathsf{Aut}(\mathsf{FOE})$ on tree models
2. $\mu\mathsf{ML} \equiv \mathsf{Aut}(\mathsf{FO})$.

Hence it suffices to prove that

3. $\mathsf{Aut}(\mathsf{FO}) \equiv_s \mathsf{Aut}(\mathsf{FOE})/{\underline{\leftrightarrow}}$

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Point of talk: Item 3 follows from observations on relation FOE/FO

Preliminaries: Models

Fix a set X of proposition letters.

A model is a structure $\mathbb{S} = \langle S, R, V \rangle$ with $R \subseteq S \times S$ and $V : X \to \wp S$.

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  $R[s] := \{t \in S \mid Rst\}$ is the successor set of s
- $V : X \to \wp S$ induces $m_V : S \to \wp \mathrm{X}$, with
  $m_V(s) := \{p \in \mathrm{X} \mid s \in V(p)\}$ is the color of s.

Preliminaries: Automata

An automaton is a triple $\mathbb{A} = \langle A, \delta, \Omega \rangle$ with

- A a finite set of states/ propositional variables/monadic predicates
- $\Omega : A \to \mathbb{N}$ a priority map
- $\delta : A \times \wp X \to L(A)$, with $L(A)$ some set of one-step formulas in A.

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Acceptance game of an automaton $\mathbb{A}$ on a model $\mathbb{S}$:

Position	Player	Admissible moves	Priority
$(a, s) \in A \times S$	$\exists$	$\{I : A \to \wp S \mid$ "I makes $\delta(a, m_V(s))$ true"$\}$	$\Omega(a)$
$I : A \to \wp S$	$\forall$	$\{(b, t) \in A \times S \mid t \in I(b)\}$	0

One-step logic

- fix a finite set A
- think of A as a signature of monadic predicates
- structure for A: pair $\langle D, I \rangle$ with $I : A \to \wp D$ an interpretation
- corresponding monadic first-order language:

$$\varphi \ ::= \ a(x) \ | \ x = y \ | \ \neg\varphi \ | \ \varphi \vee \varphi \ | \ \exists x.\varphi$$

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Use in acceptance game:

- at position (a, s), $\exists$ needs to find an interpretation $I : A \to \wp R[s]$
  s.t. $\langle R[s], I \rangle \models \delta(a, m_V(s))$.

Acceptance Game

The acceptance game $\mathcal{A}(\mathbb{A}, \mathbb{S})$ of an automaton $\mathbb{A}$ on a model $\mathbb{S}$:

Position	Player	Admissible moves	Priority
$(a, s) \in A \times S$	$\exists$	$\{I : A \to \wp S \mid \langle R[s], I \rangle \models \delta(a, m_V(s))\}$	$\Omega(a)$
$I : A \to \wp S$	$\forall$	$\{(b, t) \in A \times S \mid t \in I(b)\}$	0

$(\mathbb{A}, a_I)$ accepts $(\mathbb{S}, s)$ if $(a_I, s) \in Win_{\exists}(\mathcal{A}(\mathbb{A}, \mathbb{S}))$.

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Focus on relation FOE/FO.

One-step bisimulation invariance

Definition

- Two A-structures (D, I) and (D', I') are P-similar if
  - $\forall d \in D \quad \exists d' \in D' \; m_I(d) = m_{I'}(d')$
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Theorem $\mathrm{FO} \equiv_s \mathrm{FOE}/\mathrm{P}$:
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If $M \equiv_s L/\mathrm{P}$ then $\mathrm{Aut}(M) \equiv_s \mathrm{Aut}(L)/\underline{\leftrightarrow}$.

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Point of talk

$$\mu\text{ML} \equiv_s \text{MSO}/\underline{\leftrightarrow} \text{ because } \text{FO} \equiv_s \text{FOE}/\text{P}.$$

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Proof

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   (b) μML $\equiv$ Aut(FO).
2. FO $\equiv_s$ FOE/P
3. If $M = L/\text{P}$ then $\text{Aut}(M) = \text{Aut}(L)/\underline{\leftrightarrow}$

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  What about an FMT version of the Janin-Walukiewicz Theorem?
- Approach works for restricted classes of automata:

  $$\mathsf{WMSO}/\underline{\leftrightarrow} = ?$$

  (Carreiro, Facchini, Venema & Zanasi, forthcoming)

Reference

Y. Venema,
Expressiveness modulo bisimilarity: a coalgebraic perspective,
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Downloadable from http://staff.science.uva.nl/~yde