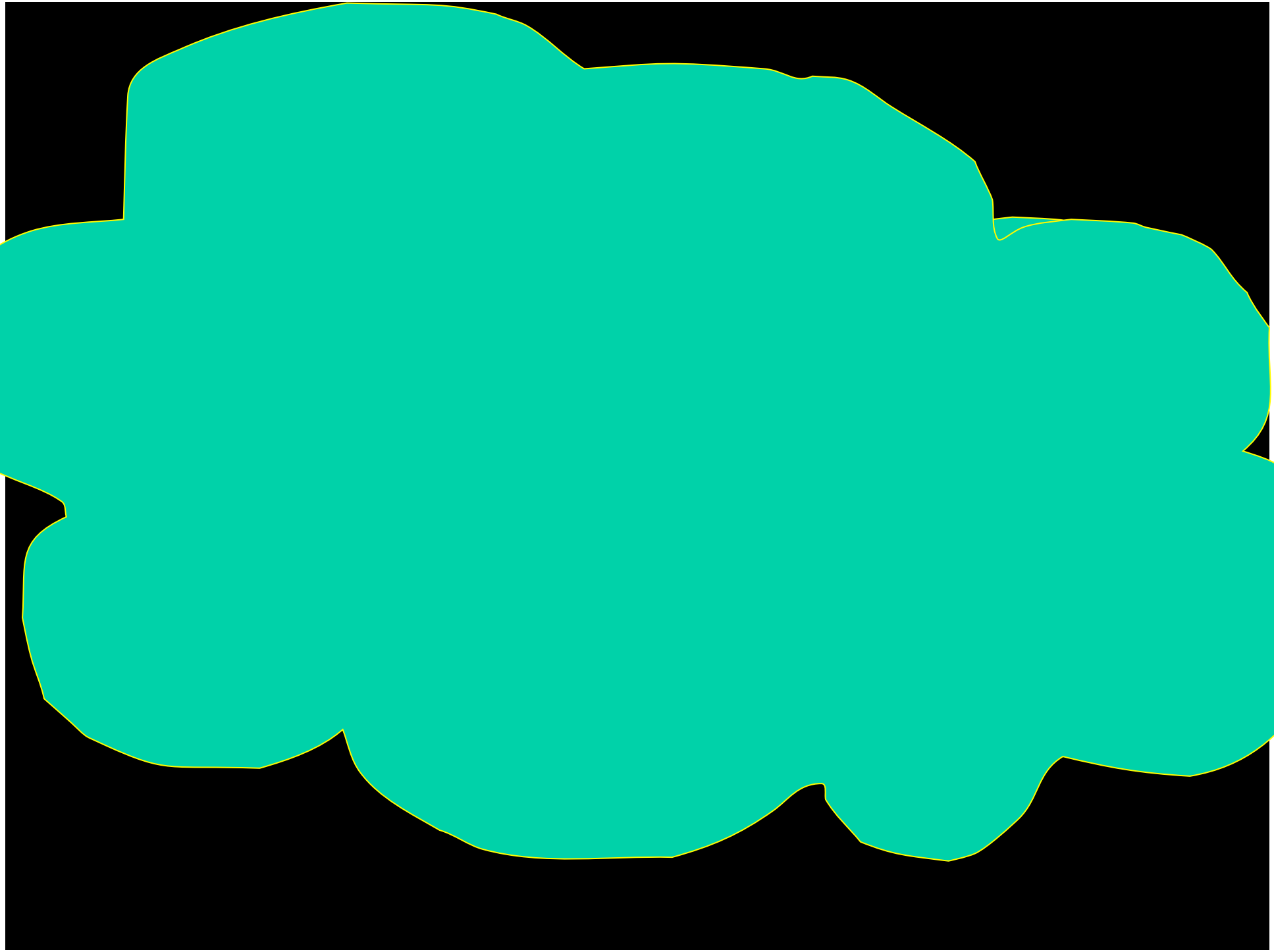
Algorithms, Games, and the Internet

Christos H. Papadimitriou
UC Berkeley



The Internet is Changing CS

Science in Crisis → Scientific Revolution

[Thomas Kuhn 1962]

In what ways?

- The computer is no longer the main focus of CS…

The Internet turned CS into a natural science

The first computational artifact that was ***never designed***, and hence must be approached by the *scientific method:*

- Observations
- Experiments
- Falsifiable theories
- Specialized applied mathematics

…and a social science

- Economics and Game Theory

  "The Internet is an equilibrium, we just have to identify the game"

  Scott Shenker

- Sociology

  The Internet cannot be studied in isolation from the complex social system it enables and serves. And it is an ideal test bed for sociological analysis and experimentation.

Algorithmic Game Theory: Four Vignettes

- Equlibria and Complexity
- Equilibria and Risk
- The Price of Anarchy
- Mechanism Design

Behavior predictions in Economics: Equilibria

- Nobody has an incentive to change, as long as everybody else stays put

The story of equilibria

[von Neumann 1928]: They always exist, as long as the game is two-player zero-sum

[Nash 1950]: Even in nonzero-sum, multiplayer games

[Arrow-Debreu 1954]: In markets too

Question: Can they be computed efficiently?

But why should we care about algorithms for equilibria?

- Equilibria are predictions of behavior
- *Computational tractability is an important modeling prerequisite*

"If your laptop can't find it, then neither can the market." Kamal Jain

…and indeed…

Theorem [Daskalakis, Goldberg & P, 2006]:

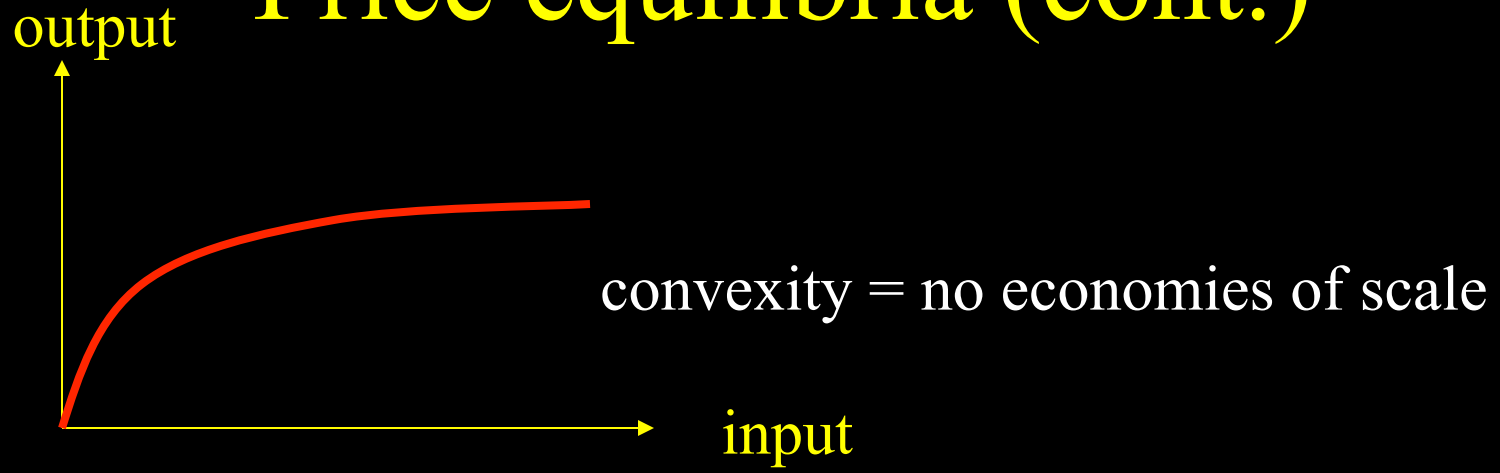
Finding a Nash equilibrium is an intractable problem

[Myerson 1999]: The universality of Nash equilibrium lies in the foundations of modern economic thought

Price equilibria

- The Arrow-Debreu theorem: producers, consumers, production functions, utilities
- Convexity $\Rightarrow$ prices
- Prices $\Rightarrow$ Pareto efficiency
- ***Convex production?!?***

Price equilibria (cont.)



- How do you get efficiency when you have economies of scale?
- Microeonomics has struggled with this conundrum for 50 years

Complexity!

- (Joint work with Chris Wilkens)
- Finding a Pareto optimum is harder than NP-complete
- ***Complexity equilibria:*** Economy can improve, but everybody is stuck at a trough and it is intractable to get unstuck

Risk: the plot **thickens**

- But what if risk is taken into account?
- (Joint work with Amos Fiat)
- General risk valuation V: $\Delta R \rightarrow R$
- Expectation is one example
- Defines a new concept, V-Nash equilibrium
- Does Nash's Theorem hold?

Some examples

- E
- E – var
- E + var
- Prob[X ≥ 100,000]
- x: prob[X ≥ x] = .95
- ½[max + min]

Does Nash's theorem hold under all these risk valuations?

- E **Yes**
- E – var **No!**
- E + var **Yes!!**
- Prob[X ≥ 100,000] **Yes!**
- x: prob[X ≥ x] = .95 **No!**
- ½[max + min] **No!**

Yes: exists and is as easy to find as Nash eq.

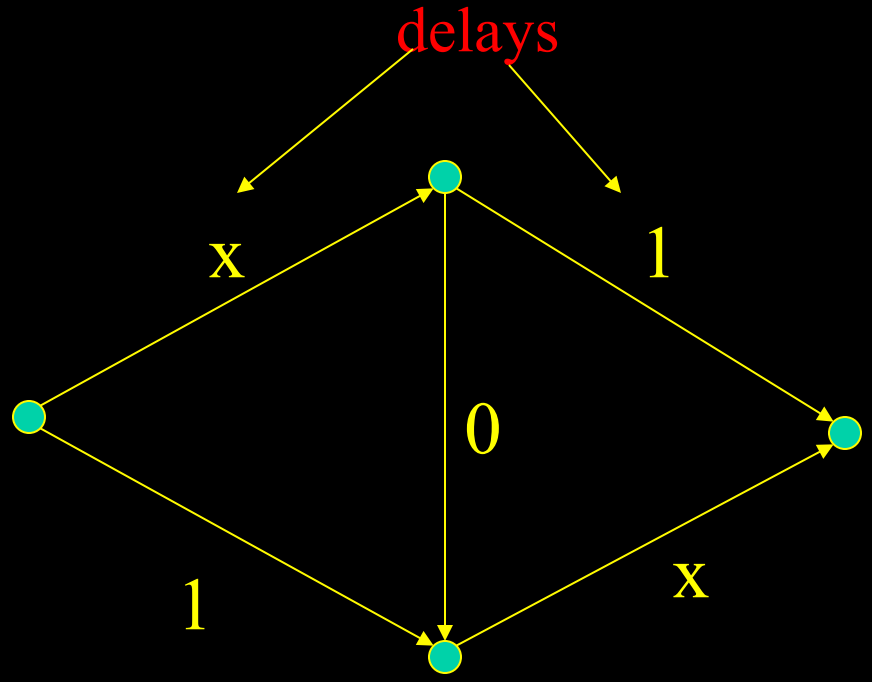
No: may not exist, and it's NP-hard to tell…

Equilibria can be Inefficient: "The price of anarchy"

$$\text{p. of } (A) = \frac{\text{cost of worst Nash equilibrium}}{\text{"socially optimum" cost}}$$

[Koutsoupias and P. 1999]

Selfishness can hurt you!



Social optimum: 1.5

Anarchical equilibrium: 2

How much worse does it get?

Price of anarchy = ¾ !!! [Roughgarden and Tardos, 2000; Roughgarden 2002]

But in the Internet flows don't choose routes…

- (Joint work with Greg Valiant, 2010)
- If routing decisions are made by selfish routers, price of anarchy can be unbounded
- However, if routers charge for routing then (under assumptions) price of anarchy becomes one!

Mechanism design (or *inverse* game theory)

- agents have utilities – but these utilities are known *only to them*
- game designer prefers certain outcomes *depending on players' utilities*
- designed game (mechanism) has designer's goals as equilibrium

e.g., Vickrey auction

- sealed-highest-bid auction encourages gaming and speculation
- Vickrey auction: Highest bidder wins, *pays second-highest bid*
- Participants are incentivized to tell the truth: *Incentive compatible (truthful) mechanism*
- *Think of it as the new max algorithm!*

The new computing paradigm?

- Today, the inputs to your algorithm are selfish agents
- Your algorithm must contain incentives for them
- *What can be computed efficiently in this world?*

The new complexity theory

- $P^{IC} = P$
- $NP\text{-complete}^{IC} = NP\text{-complete}$
- *But is $APX^{IC} = APX$?*
- [P., Schapira, Singer 2008]: No!

Turing:
An Internet Fantasy

...and then at last (I stirred with joy)
my sons and daughters wove a net
they wrote the code that weaves the nets
(...) how could I stay out of this feast?

it's good to be again, to play again, to stare at the future
a future so complex and bright you have to squint

Happy Birthday, Tom!