A Compositional Proof Rule for Coordination Logic

Highlights 2013, Paris

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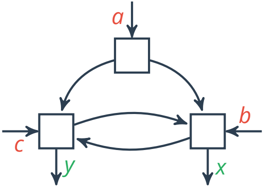
joint work with Bernd Finkbeiner

What is Coordination Logic?

Logic of the *Distributed Synthesis Problem*



$\exists$ *implementation* s.t. φ holds?

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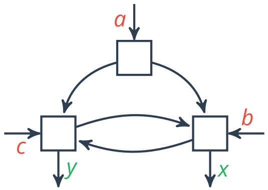

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Set of strategies for output variables

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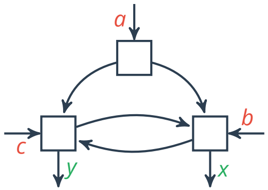

$\exists$ *implementation* s.t. φ holds?

Set of strategies for output variables

LTL formula over input/output variables

Syntax

LTL $\quad x \mid \neg x \mid \varphi \vee \varphi \mid \varphi \wedge \varphi \mid \bigcirc \varphi \mid \varphi \,\mathcal{U}\, \varphi \mid \varphi \,\overline{\mathcal{U}}\, \varphi$

$x \in \mathcal{C} \cup \mathcal{S}$

+

Strategic Quantification $\quad \exists C \triangleright s.\, \varphi \mid \forall C \triangleright s.\, \varphi$

$C \subseteq \mathcal{C}, s \in \mathcal{S}$

Coordination variables

represent information given by the environment

$\mathcal{C}$

Strategy variables

represent strategic choices made based on visible information

$\mathcal{S}$

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Strategic Quantification

$$\exists C \triangleright s.\, \varphi \mid \forall C \triangleright s.\, \varphi$$

$$C \subseteq \mathcal{C}, s \in \mathcal{S}$$

synthesize strategy

no control about strategy

Coordination variables

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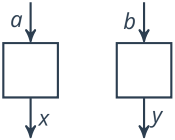
Decidability



- Distributed Synthesis is undecidable
- ➤ Coordination Logic is **undecidable**

Decidability

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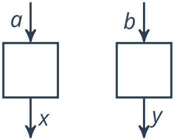
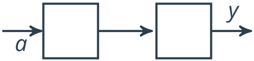
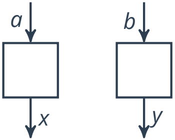


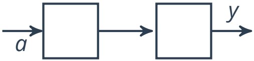
- Special cases are decidable
- ➤ Syntactic restricted fragment of CL



Decidability



- Distributed Synthesis is undecidable
- ➤ Coordination Logic is **undecidable**



- Special cases are decidable
- ➤ Syntactic restricted fragment of CL

- Many practical synthesis problems are not in the fragment
- ➤ **Goal:** Complete Proof Framework for CL

A Compositional Proof Rule

- CL formula $\Phi = \mathcal{H}(\mathcal{S}).\varphi = QC_1 \triangleright s_1 \dots QC_n \triangleright s_n.\varphi$ in PNF
- Suitable cut-set $S_{cut} = \{s_1, \dots, s_k\} \subseteq \mathcal{S}$

$$
\frac{
\begin{array}{lll}
(R_1) & \models & \mathcal{H}(S_{cut})\,.\,\psi \\
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simplified

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\hline
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\end{array}
$$

Completeness

The proof rule is complete for formulas

- in the *universal-hierarchical fragment* of Coordination Logic, and
- in Prenex Normal Form (PNF)

Example

$$\exists\{b,c\} \triangleright x_1 .\, \forall\{a\} \triangleright y_1 .\, \exists\{a,c\} \triangleright x_2 .\, \exists\{a,d\} \triangleright x_3 .\, \forall\{a,c\} \triangleright y_2 .\, \varphi$$

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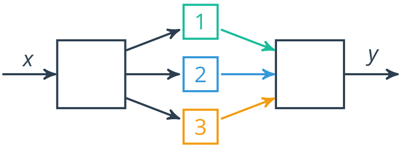
$$\{a\} \subseteq \{a,c\}, \{a\} \subseteq \{a,d\}$$

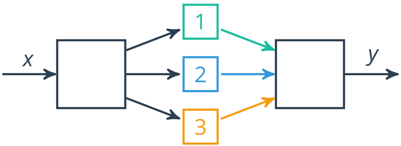
Example



$$\varphi := \big(y = f(x)\big)$$

$$\begin{aligned} & \left(\text{operational}_{2,3} \rightarrow \square\, \varphi\right) \\ \wedge & \left(\text{operational}_{1,3} \rightarrow \square\, \varphi\right) \\ \wedge & \left(\text{operational}_{1,2} \rightarrow \square\, \varphi\right) \end{aligned}$$

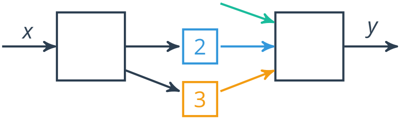
Example



$$\square\,(y = \text{majority vote})$$
$$\wedge\ \square\,(p_1 = f(x))$$
$$\wedge\ \square\,(p_2 = f(x))$$
$$\wedge\ \square\,(p_3 = f(x))$$

Example

$\square\,(p_1 = f(x))$

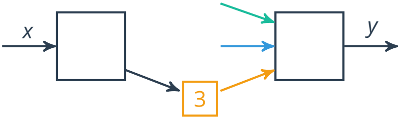


$$\begin{aligned} &\square\,(y = \text{majority vote}) \\ \wedge\ &\square\,(p_2 = f(x)) \\ \wedge\ &\square\,(p_3 = f(x)) \end{aligned}$$

Example

$\Box\,(p_2 = f(x))$

$\Box\,(p_1 = f(x))$



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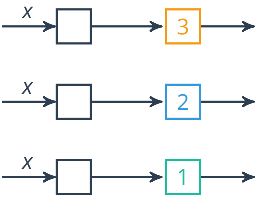

$\Box\, (p_3 = f(x))$

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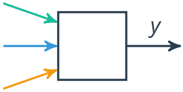
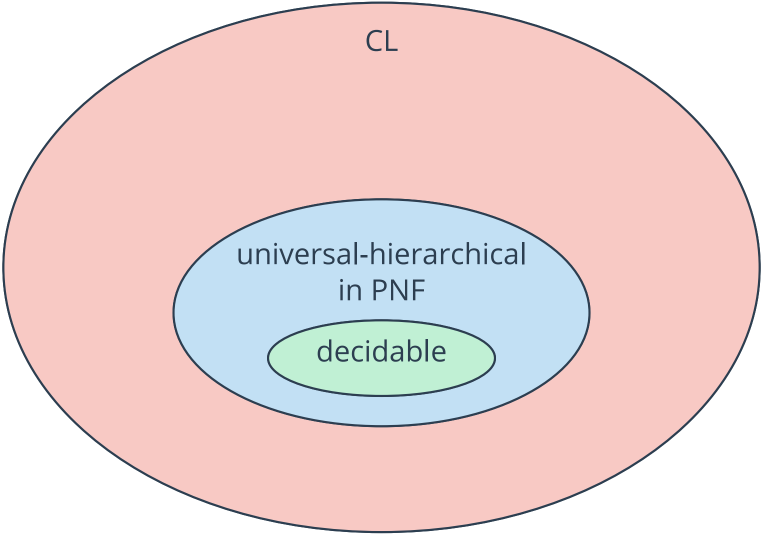


$\Box\, (y = \text{majority vote})$

Improvements



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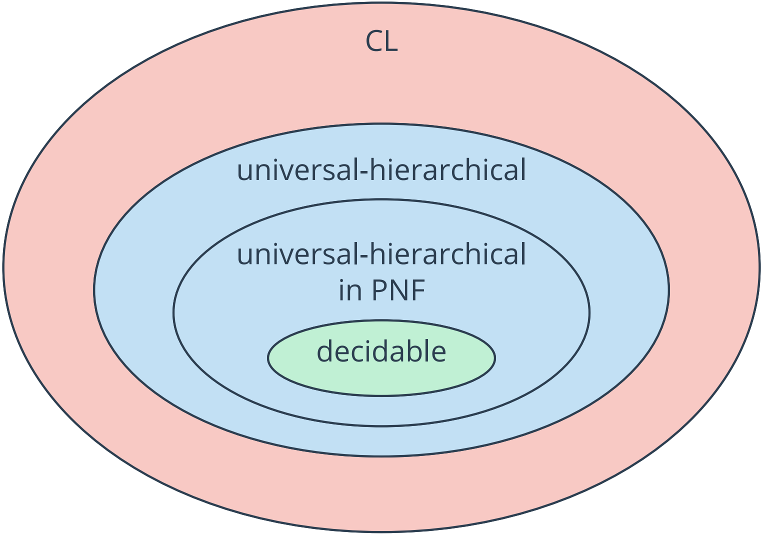

Prenex Normal Form

Theorem

Every CL formula can be transformed into an equivalent CL formula with only prenex quantification.

- Unlike FOL and other logics, prenex normal form transformation is not trivial

Example $\forall\{a,b\} \triangleright x. \bigcirc \exists\{a\} \triangleright y. \varphi$

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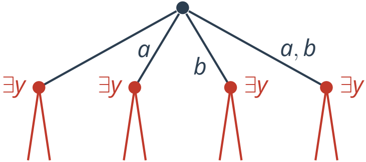

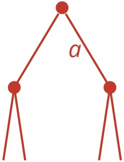

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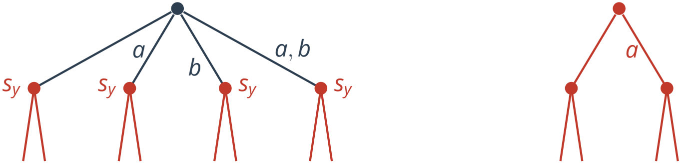

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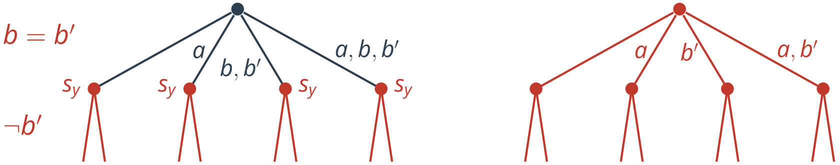
Example $\forall\{a,b\} \triangleright x.\, \exists\{a,b,b'\} \triangleright s_y.\, \exists\{a,b'\} \triangleright y.\, \varphi'$



Conclusion
and Future Work

- A complete proof system for CL formulas with hierarchical universal quantification
- This includes all distributed synthesis problems with Pnueli/Rosner architectures
- Open Problem: complete proof system for non-hierarchical universal quantification?