WHO VERIFIES THE VERIFIERS? A COMPUTER-CHECKED IMPLEMENTATION OF THE DPLL ALGORITHM IN DAFNY *

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ABSTRACT. We build a SAT solver implementing the DPLL algorithm in the verification-enabled programming language Dafny. The resulting solver is fully verified (soundness, completeness and termination are computer checked). We benchmark our Dafny solver and we show that it is just as efficient as an equivalent DPLL solver implemented in C# and roughly two times less efficient than an equivalent solver written in C++. We conclude that auto-active verification is a promising approach to increasing trust in SAT solvers, as it combines a good trade-off between execution speed and degree of trustworthiness of the final product.

1. INTRODUCTION

Modern high-performance SAT solvers quickly solve large satisfiability instances that occur in practice. If the instance is satisfiable, then the SAT solver can provide a witness in the form of a satisfying truth assignment, which can be checked independently.

If the instance is unsatisfiable, the situation is less clear. Brummayer and others [BLB10] have shown using fuzz testing that in 2010 many state-of-the-art SAT solvers contained bugs, including soundness bugs. Since 2016, in order to mitigate this issue, the annual SAT competition requires solvers competing in the main track to output UNSAT certificates [BHJ17]; these certificates are independently checked in order to ensure soundness.

These certificates could be exponentially large and the SAT solver might not even be able to output them due to various resource constraints. The implementation of the SAT solver should then be trusted not to contain bugs. However, typical high-performance SAT solvers contain data structures and algorithms complex enough to allow for subtle programming errors.

To handle these potential issues, we propose a *verified SAT solver* using the Dafny system [Lei13]. Dafny is a high-level imperative language with support for object oriented features. It features methods with preconditions, postconditions and invariants, which are

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checked at compilation time by relying on the Z3 SMT solver [dMB08]. If a postcondition cannot be established (either due to a timeout or due to the fact that it does not hold), compilation fails. Therefore, we can place a high degree of trust in a program verified using the Dafny system.

A modern high-performance SAT solver implements a backtracking search for a satisfying assignment. The search space is pruned by using several algorithmic tricks: (1) unit propagation; (2) fast data structures (e.g., to identify unit clauses); (3) variable ordering heuristics; (4) back-jumping; (5) conflict analysis; (6) clause learning; (7) restart strategy. In addition, careful engineering of the implementation is required for high performance.

The first three items are usually referred to as the DPLL algorithm [DP60, DLL62], and all items together are the core of the state-of-the-art CDCL algorithm [MS99, BS97]. We have implemented and verified in Dafny the first three items, constituting the DPLL algorithm, and we leave the other items for future work. We implement the MOMS variable ordering heuristic [HV95]. We note that our Dafny solver is computer checked for soundness, completeness and termination. We assume that the input is already in CNF form. The parser, which reads a file in the well-known DIMACS format, is also written in Dafny and hence verified against, e.g., out of bounds errors. However, there is no specification for the parser.

Our work is part of the larger trend towards producing more trustworthy software artifacts, ranging among certified compilers [Ler09], system software [HP10, HHL+14, BFK16, ZBPB17], or logic [BFLW18]. The main conceptual difference to previous work on verified or certified SAT solvers is that we propose to check directly the imperative algorithm using deductive verification, instead of, e.g., verifying functional code and relying on a refinement mechanism to extract imperative code, which could hurt performance.

Structure. In Section 2, we briefly go over the DPLL algorithm, as presented in the literature. In Section 3, we present our verified implementation in Dafny of the algorithm. We start by presenting the main data structures and their invariants (Section 3.1). We continue with the operations supported by the data structures in Section 3.2. Finally, in Section 3.3, we present the implementation of the core DPLL algorithm, together with the verified guarantees that it provides. In Section 4, we benchmark the performance of our solver. In Section 5, we discuss related work. We conclude in Section 6. We also discuss the main challenge in verifying our implementation of DPLL, along with some methodological tricks that we have used to make the verification effort tractable.

Contributions. We present the first (to our knowledge) assertional proof of the DPLL algorithm. The implementation is competitive in running time with an equivalent C++ solver.

Comparison with the workshop version. This paper is a revised extended version of our previous work [AC19] published in EPTCS. We feature an improved presentation, additional explanations and a benchmark of the performance of our solver. In addition, the solver improvements over the workshop version are:

(1) The new implementation features machine integers, which improve performance approximately 10 times in our tests. Going to machine integers from unbounded integers requires proving upper bounds on indices throughout the code.

(2) The new implementation features mutable data structures for identifying unit clauses. Our previous approach used Dafny sequences (`seq`), which are immutable and cause a performance drawback because they are updated frequently. The new mutable data

structures make the solver significantly faster, but they are more difficult to reason about and verify.

(3) We implement and verify the MOMS variable ordering heuristic.
(4) We also improve the methodology of our verification approach and in particular we significantly reduce verification time. By carefully specifying invariants and separating concerns in the implementation, the verification time is now approximately 13 minutes for the entire project.

In contrast, in our previous implementation, one method (`setLiteral`) took approximately 10 minutes to verify on its own (the entire project used to take about 2 hours to verify in its entirety).
(5) We benchmark our Dafny implementation against similar DPLL implementations written in C# and C++ and we show it is competitive in terms of performance.

2. The Davis-Putnam-Logemann-Loveland Algorithm

The DPLL procedure is an optimization of backtracking search. The main improvement is called unit propagation. A unit clause has the property that its literals are all *false* in the current assignment, except one, which has no value yet. If this literal would be set to *false*, the clause would not be satisfied; therefore, the literal must necessarily be *true* for the entire formula to be true. This process of identifying unit clauses and setting the unknown literal to true is called unit propagation.

Example 2.1. We consider a formula with 7 variables and 5 clauses:
$$(x_1 \lor x_2 \lor x_3) \land (\neg x_1 \lor \neg x_2) \land (x_2 \lor \neg x_3) \land (x_2 \lor x_4 \lor x_5) \land (x_5 \lor x_6 \lor x_7)$$
The formula is satisfiable, as witnessed by the truth assignment *(true, false, false, true, true, false, true)*.

Algorithm 2.2 describes the DPLL procedure [GKSS08] that we implement and verify, presented slightly differently in order to match our implementation more closely:

Algorithm 2.2. Function `DPLL-recursive`(*F, tau*)

> **input** : A CNF formula F and an partial assignment *tau*
> **output** : SAT/UNSAT, depending on where there exists an assignment extending
> *tau* that satisfies F
> **while** $\exists$ *unit clause* $\in F$ **do**
> > $\ell \leftarrow$ the unset literal from the unit clause
> > $tau \leftarrow tau[\ell := true]$
>
> **if** F *contains the empty clause* **then return** *UNSAT*;
> **if** F *has no clauses left* **then**
> > Output *tau*
> > **return** *SAT*
>
> $\ell \leftarrow$ some unset literal
> **if** $DPLL\text{-}recursive(F, tau[\ell := true]) = SAT$ **then return** *SAT*;
> **return** $DPLL\text{-}recursive(F, tau[\ell := false])$

We describe how the algorithm works on this example: first, the algorithm chooses the literal x_1 and sets it to *true* (arbitrarily; if *true* would not work out, then the algorithm would backtrack here and try *false*). At the next step, it finds that the second clause is unit and sets $\neg x_2$ to *true*, which makes the third clause unit, so $\neg x_3$ is set to *true*. After

unit propagation, the next clause not yet satisfied is the fourth one, and the first unset literal is x_4. At the branching step, x_4 is assigned to *true*. Furthermore, only one clause is not satisfied yet, and the next decision is to choose x_5 and set it to *true*, which makes the formula satisfied, even if x_6 and x_7 are not set yet.

Next, we recall some well-known terminology in SAT solvers. Choosing and assigning an unset literal to *true* or *false* is called a branching step or a *decision*. Every time the algorithm makes a *decision*, the decision level is incremented by one and some more literals are assigned to *true* or *false* by unit propagation.

The *trace* of assignments is split into layers, one layer per decision. Multiple literals can be set at the same decision level (the decision literal, and the literals assigned by unit propagation). Every time the algorithm backtracks it must revert an entire layer of assignments. A possible assignments trace corresponding to Example 2.1 is shown in Figure 1.

Example formula:	Example assignments trace:
1) $x_1 \vee x_2 \vee x_3$	$(x_1, true), (x_2, false), (x_3, false)$
2) $\neg x_1 \vee \neg x_2$	$(x_4, true)$
3) $x_2 \vee \neg x_3$	$(x_5, true)$
4) $x_2 \vee x_4 \vee x_5$	
5) $x_5 \vee x_6 \vee x_7$	

Figure 1: Assignments trace representation for Example 2.1 divided into layers. The first layer corresponds to setting the decision variable x_1 to *true*, followed by unit propagation, which sets x_2 and x_3. This trace occurs just before the algorithm stops with an answer of SAT, after setting the decision variables x_4 and x_5 to *true*.

3. A Verified Implementation of the DPLL Algorithm

In this section, we present the main ingredients of our verified solver. The full source code, along with instruction on how to compile it and reproduce our benchmarks, can be found at

https://github.com/andricicezar/sat-solver-dafny-v2.

3.1. **Data Structures.** We first discuss the data structures for representing the formula, for quickly identifying unit clauses and for recalling the current truth assignment.

3.1.1. *Representing the CNF formula.* The main class in our Dafny development is `Formula`, which extends `DataStructures` (Figure 2). This class is instantiated with the number of propositional variables (`variablesCount`) and with the `clauses` of the formula to be checked for satisfiability. Propositional variables are represented by values between 0 and `variablesCount`−1, positive literals are represented by values between 1 and `variablesCount`, and negative integers between −1 and −`variablesCount` represent negative literals. Variables and literals are represented by values of type `Int32.t`, which we define to model machine integers and which is extracted to `int`.

```
trait DataStructures {
    var variablesCount : Int32.t;
    var clauses : seq< seq<Int32.t> >;
    var decisionLevel : Int32.t;
...
    var traceVariable : array<Int32.t>;
    var traceValue : array<bool>;
    var traceDLStart : array<Int32.t>;
    var traceDLEnd : array<Int32.t>;
    ghost var assignmentsTrace : set<(Int32.t, bool)>;
...
    var truthAssignment : array<Int32.t>;
    var trueLiteralsCount : array<Int32.t>;
    var falseLiteralsCount : array<Int32.t>;
    var positiveLiteralsToClauses : array< seq<Int32.t> >;
    var negativeLiteralsToClauses : array< seq<Int32.t> >;
}
```

Figure 2: The most important fields in our data structures (file solver/data_structures.dfy).

Clauses are sequences of literals and the entire formula is represented by a sequence of clauses (`var clauses : seq< seq<Int32.t> >`). Using sequences for clause (sequences are immutable in Dafny) has no significant performance impact, since they are set at the beginning once and never changed.

3.1.2. *Representing the current assignment and the assignments trace.* The member variable `decisionLevel` recalls the current decision level, which has an initial value of -1. The assignments trace is represented at computation time by using the arrays `traceVariable`, `traceValue`, `traceDLStart` and `traceDLEnd` and at verification time also by the ghost construct `assignmentsTrace` (see Figure 2).

The arrays `traceVariable` and `traceValue` have the same actual length. They recall, in order, all variables that have been set so far, together with their value. The arrays `traceDLStart` and `traceDLEnd` recall at what index in `traceVariable` and `traceValue` each decision layer starts and ends, respectively.

The ghost construct `assignmentsTrace` recalls the same information as a set of (variable, value) pairs. This set is used for the convenience of specifying some of the methods and it only lives at verification time; it is erased before running time and therefore it entails no performance penalty.

Note that `traceVariable`, `traceValue`, `traceDLStart` and `traceDLEnd` are arrays, and they are extracted to C# as such. Therefore, lookups and updates in these arrays take constant time. The link between the ghost construct `assignmentsTrace` and its imperative counterparts (`traceVariable`, `traceValue`, `traceDLStart` and `traceDLEnd`) is computer checked as the following class invariant:

```
(decisionLevel ≥ 0 ⟹ (
  (∀ i • 0 ≤ i < traceDLEnd[decisionLevel] ⟹
    (traceVariable[i], traceValue[i]) in assignmentsTrace)
  ∧
  (∀ x • x in assignmentsTrace ⟹ (
```

```
∃ i  •  0 ≤ i < traceDLEnd[decisionLevel] ∧
   (traceVariable[i], traceValue[i]) = x))))
```

The array `truthAssignment` is indexed from 0 to `variablesCount` − 1 and it recalls the current truth assignment. The value `truthAssignment`[v] is −1 if the propositional variable v is unset, 0 if v is false, and 1 if v is true. At the beginning, it is initialized to −1 at all indices. The following class invariant describing the expected link between the assignments trace and the current truth assignment is computer checked:

```
truthAssignment.Length = variablesCount ∧
(∀ i  •  0 ≤ i < variablesCount ⟹ -1 ≤ truthAssignment[i] ≤ 1) ∧
(∀ i  •  0 ≤ i < variablesCount ∧ truthAssignment[i] ≠ -1 ⟹
  (i, truthAssignment[i]) in assignmentsTrace) ∧
(∀ i  •  0 ≤ i < variablesCount ∧ truthAssignment[i] = -1 ⟹
  (i, false) ∉ assignmentsTrace ∧ (i, true) ∉ assignmentsTrace)
```

Note that the invariant makes use of the ghost construct `assignmentsTrace` for brevity.

3.1.3. *Quickly identifying unit clauses.* The array `trueLiteralsCount` (`falseLiteralsCount`) is used to recall how many literals in each clause are currently true (resp. false). They are indexed from 0 to |`clauses`| − 1. The value `trueLiteralsCount`[i] denotes the number of literals set to true in `clauses`[i] and `falseLiteralsCount`[i] the number of false literals in `clauses`[i]. These are used to quickly identify which clauses are satisfied, which clauses are unit or which clauses are false. For example, to check whether `clauses`[i] is satisfied, we simply evaluate `trueLiteralsCount`[i] > 0. The following class invariant involving these arrays is computer checked:

```
|trueLiteralsCount| = |clauses| ∧
∀ i  •  0 ≤ i < |clauses| ⟹
  0 ≤ trueLiteralsCount[i] = countTrueLiterals(truthAssignment, clauses[i])
```

and analougously for `falseLiteralsCount`. Note that `countTrueLiterals` is a function (not a method, hence it is used for specification only) that actually computes the number of true literals by walking through all literals in the respective clause.

In order to quickly update `trueLiteralsCount` and `falseLiteralsCount` when a new literal is (un)set, we use `positiveLiteralsToClauses` and `negativeLiteralsToClauses`. These are arrays indexed from 0 to `variablesCount` − 1. The first array contains the indices of the clauses in which a given variable occurs. The second array contains the indices of the clauses in which the negation of the given variable occurs. They provably satisfy the following invariant:

```
|positiveLiteralsToClauses| = variablesCount ∧ (
  ∀ variable  •  0 ≤ variable < |positiveLiteralsToClauses| ⟹
    ghost var s := positiveLiteralsToClauses[variable];
    ...
    (∀ clauseIndex  •  clauseIndex in s ⟹ variable+1 in clauses[clauseIndex]) ∧
    (∀ clauseIndex  •  0 ≤ clauseIndex < |clauses| ∧ clauseIndex ∉ s ⟹
      variable+1 ∉ clauses[clauseIndex]))
```

(analogously for `negativeLiteralsToClauses`).

To represent class invariants, Dafny encourages a methodology of defining a class predicate `valid`. In our development, `valid` consists of the conjunction of the above invariants, plus several other lower-level predicates that we omit for brevity. The predicate

`valid` is used as a precondition and postcondition for all class methods, and therefore plays the role of a class invariant. This way, it is guaranteed that the data structures are consistent.

3.2. **Verified Operations over the Data Structures.** From the initial (valid) state, we allow one of these four actions:

(1) increase the decision level,
(2) set a variable,
(3) set a literal and perform unit propagation, and
(4) revert the assignments done on the last decision level.

Each of the actions is implemented as a method and we show that these four methods preserve the data structure invariants above.

3.2.1. *The Method* `increaseDecisionLevel`. This method increments the decision level by one and creates a new layer. The method guarantees that the new state is valid, and nothing else changes. Its signature and its specification are:

```
method increaseDecisionLevel()
  requires validVariablesCount();
  requires validAssignmentTrace();
  requires decisionLevel < variablesCount - 1;
  requires decisionLevel ≥ 0 ⟹
    traceDLStart[decisionLevel] < traceDLEnd[decisionLevel];

  modifies `decisionLevel, traceDLStart, traceDLEnd;

  ensures decisionLevel = old(decisionLevel) + 1;
  ensures validAssignmentTrace();
  ensures traceDLStart[decisionLevel] = traceDLEnd[decisionLevel];
  ensures getDecisionLevel(decisionLevel) = {};
  ensures ∀ i • 0 ≤ i < decisionLevel ⟹
    old(getDecisionLevel(i)) = getDecisionLevel(i);
```

The predicates `validVariablesCount` and `validAssignmentTrace` are used as conjuncts in the class invariant. The function `getDecisionLevel` returns all assignments at a given decision level as a set.

3.2.2. *The Method* `setVariable`. This method takes a variable that is not yet set and it updates is value. Because the trace of assignments and `truthAssignment` are changed, `trueLiteralsCount` and `falseLiteralsCount` have to be updated. We use the arrays `positiveLiteralsToClauses` and `negativeLiteralsToClauses` to efficiently update them, and prove that the clauses that are not mentioned in these arrays are not impacted. The signature of `setVariable` and its specification are:

```
method setVariable(variable : Int32.t, value : bool)
  requires valid();
  requires validVariable(variable);
  requires truthAssignment[variable] = -1;
  requires 0 ≤ decisionLevel;

  modifies truthAssignment, traceVariable, traceValue,
           traceDLEnd, `assignmentsTrace, trueLiteralsCount,
           falseLiteralsCount;
```

```
ensures valid();
ensures traceDLStart[decisionLevel] < traceDLEnd[decisionLevel];
ensures traceVariable[traceDLEnd[decisionLevel]-1] = variable;
ensures traceValue[traceDLEnd[decisionLevel]-1] = value;

// post conditions that ensure that only a position of the arrays
// has been updated.
ensures value = false ⟹ old(truthAssignment[..])[variable := 0]
  = truthAssignment[..];
ensures value = true ⟹ old(truthAssignment[..])[variable := 1]
  = truthAssignment[..];
ensures ∀ i • 0 ≤ i < variablesCount ∧ i ≠ decisionLevel ⟹
  traceDLEnd[i] = old(traceDLEnd[i]);
ensures ∀ i • 0 ≤ i < variablesCount ∧ i ≠ old(traceDLEnd[decisionLevel]) ⟹
  traceVariable[i] = old(traceVariable[i]) ∧ traceValue[i] = old(traceValue[i]);
ensures ∀ x • 0 ≤ x < old(traceDLEnd[decisionLevel]) ⟹
  traceVariable[x] = old(traceVariable[x]);

ensures ∀ i • 0 ≤ i < decisionLevel ⟹
  old(getDecisionLevel(i)) = getDecisionLevel(i);

ensures assignmentsTrace = old(assignmentsTrace) + { (variable, value) };

ensures countUnsetVariables(truthAssignment[..]) + 1 =
  old(countUnsetVariables(truthAssignment[..]));
```

3.2.3. *The Method* setLiteral. This method uses setVariable as a primitive, so the preconditions and postconditions are similar. The main difference is that after it makes the first update, it also performs unit propagation, possibly recursively. This means that it calls setLiteral again with new values. So, at the end of a call, truthAssignment might change at several positions. To prove termination, we use as a variant the number of unset variables, which provably decreases at every recursive step. Its signature and its specification are:

```
method setLiteral(literal : Int32.t, value : bool)
  requires valid();
  requires validLiteral(literal);
  requires getLiteralValue(truthAssignment[..], literal) = -1;
  requires 0 ≤ decisionLevel;

  modifies truthAssignment, trueLiteralsCount,
           falseLiteralsCount, traceDLEnd, traceValue,
           traceVariable, 'assignmentsTrace;

  ensures valid();
  ensures traceDLStart[decisionLevel] < traceDLEnd[decisionLevel];
  ensures ∀ x • 0 ≤ x < old(traceDLEnd[decisionLevel]) ⟹
    traceVariable[x] = old(traceVariable[x]);
  ensures assignmentsTrace = old(assignmentsTrace) +
    getDecisionLevel(decisionLevel);
  ensures ∀ i • 0 ≤ i < decisionLevel ⟹
    old(getDecisionLevel(i)) = getDecisionLevel(i);
  ensures countUnsetVariables(truthAssignment[..]) <
    old(countUnsetVariables(truthAssignment[..]));
```

```
  ensures (
    ghost var (variable, val) := convertLVtoVI(literal, value);
    isSatisfiableExtend(old(truthAssignment[..])[variable as int := val]) ⟺
            isSatisfiableExtend(truthAssignment[..])
  );

  decreases countUnsetVariables(truthAssignment[..]), 0;
```

In the code above, the function getLiteralValue(tau, ℓ) returns the value of the literal ℓ in the truth assignment tau. Note that the variable truthAssignment is an array, while truthAssignment[..] converts the array to a sequence. The sequence (immutable) is used to represent truth assignments at specification level.

3.2.4. *The Method* revertLastDecisionLevel. This method reverts the assignments from in the last layer by changing the value of the respective literals to −1. The proof of this method requires several helper proofs that confirm that the data structures are updated correctly. To quickly update trueLiteralsCount and falseLiteralsCount, we again use the two arrays positiveLiteralsToClauses and negativeLiteralsToClauses. As part of postcondition, we prove that the literals not on the last decision level remain unchanged:

```
method revertLastDecisionLevel()
  requires valid();
  requires 0 ≤ decisionLevel;

  modifies `assignmentsTrace, `decisionLevel, truthAssignment, trueLiteralsCount,
           falseLiteralsCount, traceDLEnd;

  ensures decisionLevel = old(decisionLevel) - 1;
  ensures assignmentsTrace = old(assignmentsTrace) -
    old(getDecisionLevel(decisionLevel));
  ensures valid();
  ensures ∀ i • 0 ≤ i ≤ decisionLevel ⟹
    old(getDecisionLevel(i)) = getDecisionLevel(i);
  ensures decisionLevel > -1 ⟹
    traceDLStart[decisionLevel] < traceDLEnd[decisionLevel];
```

3.3. **Proof of Functional Correctness for the Main Algorithm.** The entry point called to solve the SAT instance is solve:

```
method solve() returns (result : SAT_UNSAT)
  requires formula.valid();
  requires formula.decisionLevel > -1 ⟹
    formula.traceDLStart[formula.decisionLevel] <
      formula.traceDLEnd[formula.decisionLevel];

  modifies formula.truthAssignment, formula.traceVariable, formula.traceValue,
           formula.traceDLStart, formula.traceDLEnd, formula`decisionLevel,
           formula`assignmentsTrace, formula.trueLiteralsCount,
           formula.falseLiteralsCount;

  ensures formula.valid();
  ensures old(formula.decisionLevel) = formula.decisionLevel;
  ensures old(formula.assignmentsTrace) = formula.assignmentsTrace;
  ensures ∀ i • 0 ≤ i ≤ formula.decisionLevel ⟹
```

```
      old(formula.getDecisionLevel(i)) = formula.getDecisionLevel(i);
  ensures formula.decisionLevel > -1 ⟹
    formula.traceDLStart[formula.decisionLevel] <
      formula.traceDLEnd[formula.decisionLevel];

  ensures result.SAT? ⟹ formula.validValuesTruthAssignment(result.tau);
  ensures formula.countUnsetVariables(formula.truthAssignment[..]) =
    formula.countUnsetVariables(old(formula.truthAssignment[..]));

  ensures result.SAT? ⟹
    formula.isSatisfiableExtend(formula.truthAssignment[..]);
  ensures result.UNSAT? ⟹
    ¬formula.isSatisfiableExtend(formula.truthAssignment[..]);

  decreases formula.countUnsetVariables(formula.truthAssignment[..]), 1;
```

It implements the *DPLL-procedure* given in Algorithm 2.2 using recursion. However, for efficiency, the data structures are kept in the instance of a class instead of being passed as arguments. The most important postconditions stating the functional correctness are: if it returns SAT then the current `truthAssignment` can be extended to satisfy the formula, and if returns UNSAT it means that no truth assignment extending the current `truthAssignment` satisfies it. We use the predicate `isSatisfiableExtend(tau, clauses)`, which tests whether there exists a complete assignment that extends the partial truth assignment `tau` and that satisfies the formula.

We also show as a postcondition that `solve` ends in the same state as where it starts. This means that we chose to undo the changes even if we find a solution. Otherwise, the preconditions and postconditions for solve would need to change accordingly and become more verbose and less elegant. For simplicity, we chose to revert to the initial state every time.

A flowchart that shows graphically the main flow of the `solve` method, together with the most important statements that hold after each line, is presented in Figure 3.

Once a literal is chosen, the updates to the data structures are delegated to the `step` method. This removes some duplication in the code, but it also makes the verification take less time. The preconditions and postconditions of `step` are the same as in `solve`, but taking into account that `step` additionally takes as arguments a literal and a desired value for this literal. The method `step` calls `setLiteral` to set the literal and perform unit propagation, and then calls `solve` recursively:

```
method step(literal : Int32.t, value : bool) returns (result : SAT_UNSAT)
  requires formula.valid();
  requires formula.decisionLevel < formula.variablesCount - 1;
  requires formula.decisionLevel > -1 ⟹
    formula.traceDLStart[formula.decisionLevel] <
      formula.traceDLEnd[formula.decisionLevel];
  requires ¬formula.hasEmptyClause();
  requires ¬formula.isEmpty();
  requires formula.validLiteral(literal);
  requires formula.getLiteralValue(formula.truthAssignment[..], literal) = -1;

  modifies formula.truthAssignment, formula.traceVariable, formula.traceValue,
           formula.traceDLStart, formula.traceDLEnd, formula`decisionLevel,
           formula`assignmentsTrace, formula.trueLiteralsCount,
           formula.falseLiteralsCount;
```
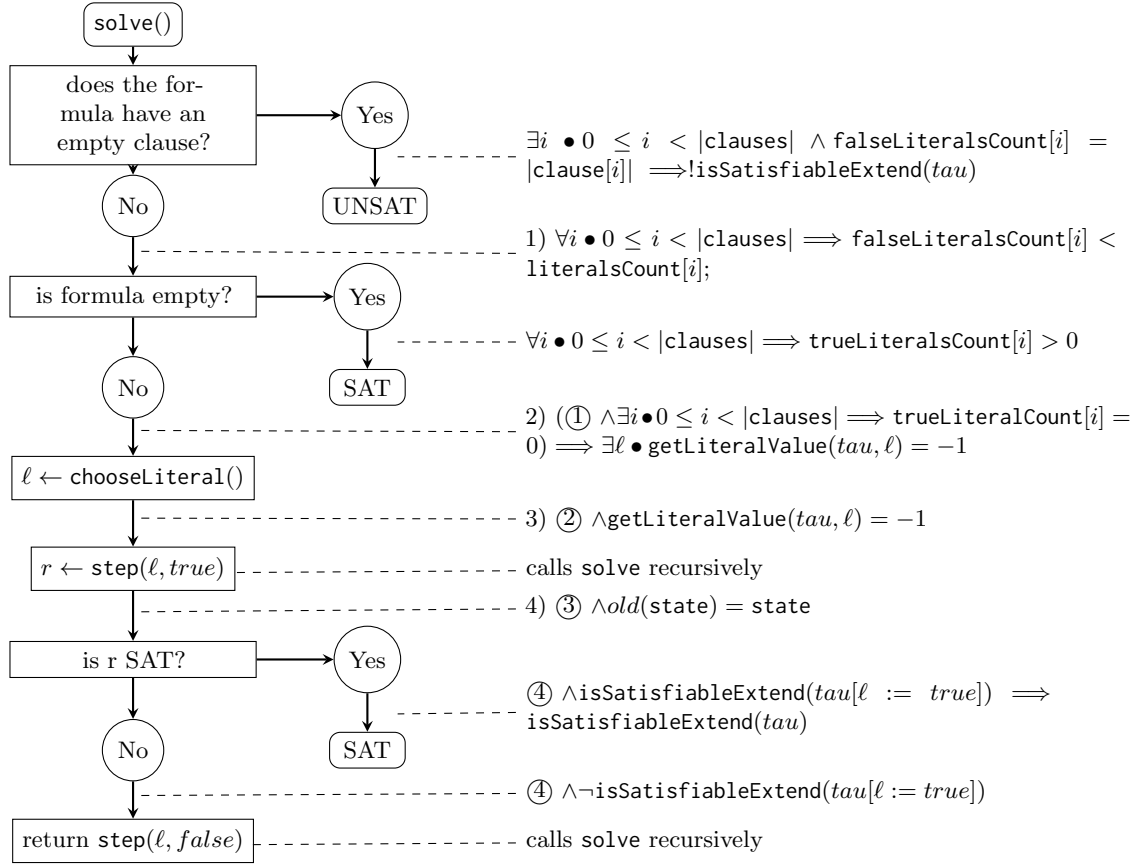
Figure 3: Flowchart of method solve. For simplicity, when the initial state is reached, we use the notation state = old(state).

```
ensures formula.valid();
ensures old(formula.decisionLevel) = formula.decisionLevel;
ensures old(formula.assignmentsTrace) = formula.assignmentsTrace;
ensures ∀ i • 0 ≤ i ≤ formula.decisionLevel ⟹
  old(formula.getDecisionLevel(i)) = formula.getDecisionLevel(i);
ensures formula.decisionLevel > -1 ⟹
  formula.traceDLStart[formula.decisionLevel] <
    formula.traceDLEnd[formula.decisionLevel];

ensures result.SAT? ⟹ formula.validValuesTruthAssignment(result.tau);
ensures result.SAT? ⟹ (
  var (variable, val) := formula.convertLVtoVI(literal, value);
  formula.isSatisfiableExtend(formula.truthAssignment[..][variable := val]));

ensures result.UNSAT? ⟹ (
  var (variable, val) := formula.convertLVtoVI(literal, value);
  ¬formula.isSatisfiableExtend(formula.truthAssignment[..][variable := val]));

ensures formula.countUnsetVariables(formula.truthAssignment[..]) =
  formula.countUnsetVariables(old(formula.truthAssignment[..]));
```

```
decreases formula.countUnsetVariables(formula.truthAssignment[..]), 0;
```

4. Benchmarks

Dafny code can be extracted to C#, and then compiled and executed as regular C# code. In this section, we present the results obtained by benchmarking the C# code extracted from our verified solver (we refer to this code as the Dafny solver) to see how it performs against other solvers.

Benchmark used. For benchmarking, we use some of the tests in SATLIB - Benchmark Problems[1]. We select the sets `uf100`, `uuf100` up to `uf200`, `uuf200`. These sets of SAT problems all contain instances in 3-CNF, with `uf` denoting satisfiable instances and `uuf` denoting unsatisfiable instances. The numbers in the names (e.g., `100`, `200`) denote the number of propositional variables. The number of clauses in each set is chosen such that the problems sit at the satisfiability threshold [CA96]. We choose these sets of SAT instances because they are small enough for DPLL to solve in reasonable time, but big enough so that the search dominates the execution time (and not, e.g., reading the input).

Benchmarking methodology. We run the tests using the benchmarking framework BenchExec [BLW19], a solution that reliably measures and limits resource usage of the benchmarked tool[2]. We used BenchExec to limit resource usage to set the following for each run: time limit to 5000s, memory limit to 1024 MB, CPU core limit to 1. We used a Intel Core i7-9700K CPU @ 3.60GHz machine (cores: 4, threads: 8, frequency: 4900 MHz, Turbo Boost: enabled; RAM: 8290 MB, Operating System Linux-5.3.0-40-generic-x86_64-with-Ubuntu-18.04-bionic, Dafny 2.3.0.10506, Mono JIT compiler 6.8.0.105, G++ 7.5.0).

Benchmark 1. We first check whether the extracted code has any added overhead compared to a implementation written directly in C#. For this purpose, we write in C# a solver implementing the same algorithm and data structures as the Dafny solver.

We find that there is a negligible overhead coming from the method we use to read files in Dafny, and not from the extraction process itself. In our results, the reading and parsing of the input file in Dafny takes at least twice as long as in C#. On small inputs, the C# solver therefore outperforms the Dafny solver. On larger inputs, the performance is the same.

Benchmark 2. The language C# is not popular in SAT solving, with C++ being the language of choice because of performance. Therefore, we implement the same DPLL algorithm directly in C++. We benchmark our verified Dafny solver against the C++ implementation. The results show that the (unverified) C++ solver is approximately twice as fast on large tests as our verified Dafny solver.

Benchmark 3. To put the performance of our verified Dafny solver into context, we also benchmark against the solver MiniSat [3] (with the default settings). As MiniSat implements the full CDCL algorithm, which can be exponentially faster than DPLL, it outperforms our solver significantly. However, the correctness guarantee offered by our verified solver is higher than the unverified C code of MiniSat.

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[1]https://www.cs.ubc.ca/~hoos/SATLIB/benchm.html

[2]https://github.com/sosy-lab/benchexec

[3]http://minisat.se/

In Table 1, we summarize the running times of all solvers on the respective sets of tests. We report the average running time, the standard deviation and the sum over all running times for SAT instances in each particular set of tests.

	Dafny SAT Solver			C# DPLL Solver			C++ DPLL Solver			MiniSat v2.2.0		
	avg	sd	sum	avg	sd	sum	avg	sd	sum	avg	sd	sum
uf100	0.07	0.02	72.72	0.05	0.02	55.75	0.02	0.01	21.73	0.00	0.00	2.70
uuf100	0.13	0.03	130.58	0.11	0.03	111.65	0.04	0.01	49.94	0.00	0.00	4.03
uf150	1.29	1.30	129.24	1.22	1.26	122.22	0.63	0.66	63.54	0.00	0.00	0.84
uuf150	3.48	1.56	348.44	3.34	1.51	334.52	1.76	0.79	176.18	0.01	0.00	1.87
uf175	6.90	6.90	690.60	6.88	6.87	688.59	3.50	3.50	350.75	0.02	0.02	2.58
uuf175	21.66	10.60	2166.94	21.80	10.67	2180.41	10.77	5.25	1077.03	0.05	0.02	5.13
uf200	43.22	39.99	4322.50	47.07	43.69	4707.89	21.92	20.35	2192.21	0.06	0.05	6.48
uuf200	110.64	48.33	10953.40	120.07	52.64	11887.17	55.98	22.55	5542.17	0.18	0.08	17.82

Table 1: The CPU time required to solve each set of instances by each SAT Solver in seconds. The first three solvers are implemented by us.

Figures 4 and 5 present the running times (log scale) of all four solvers mentioned above on all SAT instances in the `uf200` and `uuf200` sets, respectively. The running times are sorted by the time it takes for the Dafny solver to finish.
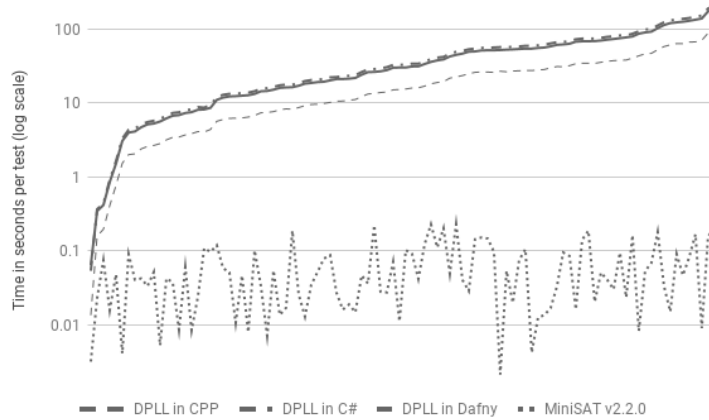

Figure 4: CPU time on each instance in the set `uf200` (all instances are satisfiable in this set). The Dafny solver and the C# solver are essentially indistinguishable. The C++ solver is approximately twice as fast. MiniSAT is much faster, since it implements the full CDCL algorithm.

We conclude that our verified Dafny solver is competitive with an equivalent implementation in C++ (it is only two times slower), but the correctness guarantee offered by our verified solver makes it significantly more trustworthy.

5. Related Work

The SAT solver `versat` [OSOC12] was implemented and verified in the Guru programming language using dependent types. As our solver, it also implements efficient data structures.
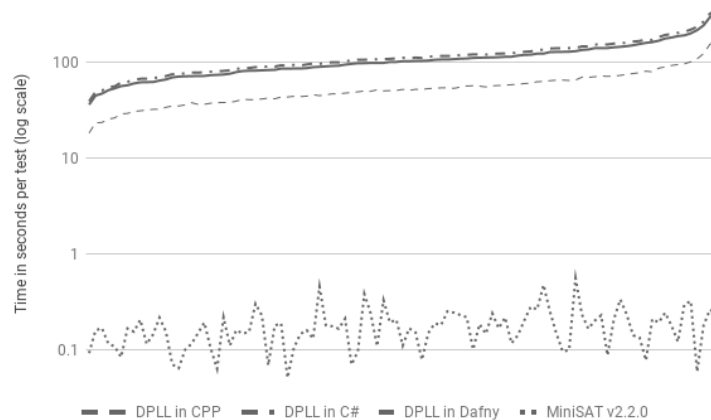
Figure 5: CPU time on each instance in the set `uuf200` (all instances are unsatisfiable in this set). The Dafny solver and the C# solver are essentially indistinguishable. The C++ solver is approximately twice as fast. MiniSAT is much faster, since it implements the full CDCL algorithm.

However, it relies on a translation to C where data structures are implemented imperatively by using reference counting and a statically enforced read/write discipline. Unlike our approach, the solver is only verified to be sound: if it produces an `UNSAT` answer, then the input formula truly is unsatisfiable. However, termination and completeness (if the solver produces SAT, then the formula truly is satisfiable) are not verified. Another small difference is the verification guarantee: `versat` is verified to output `UNSAT` only if a resolution proof of the empty clause exists, while in our approach we use a semantic criterion: our solver always terminates and produces UNSAT only if there is no satisfying model of the input formula. Of course, in the case of propositional logic these criteria are equivalent and therefore this difference is mostly a matter of implementation. Unlike our solver, some checks are not proved statically and must be checked dynamically, so they could be a source of incompleteness. An advantage of `versat` over our approach is that is implements more optimizations, like conflict analysis and clause learning, which enable it to be more competitive in terms of running time. Blanchette and others [BFLW18] present a certified SAT solving framework verified in the Isabelle/HOL proof assistant. The proof effort is part of the *Isabelle Formalization of Logic* project. The framework is based on refinement: at the highest level sit several calculi like CDCL and DPLL, which are formally proved. Depending on the strategy, the calculi are also shown to be terminating. The calculi are shown to be refined by a functional program. Finally, at the lowest level is an imperative implementation in Standard ML, which is shown to be a refinement of the functional implementation. Emphasis is also placed on meta-theoretical consideration. The final solver can still two orders of magnitude slower than a state-of-the-art C solver and therefore additional optimizations [Fle19] are desirable. In contrast, in our own work we do not investigate any meta-theoretical properties of the DPLL/CDCL frameworks; we simply concentrate on obtaining a verified SAT solver. We investigate to what extent directly proving the imperative algorithm is possible in an auto-active manner. A key challenge is that the verification of Dafny code may take a lot of time in certain cases and we have to optimize our code for verification time as

well. Another SAT solver verified in Isabelle/HOL, is by Marić [Mar09]. In contrast to previous formalization, the verification methodology is not based on refinement. Instead, the Hoare triples associated to the solver pseudo-code are verified in Isabelle/HOL. In subsequent work [MJ11], Marić and Janičić prove in Isabelle the functional correctness of a SAT solver represented as an abstract transition system. Another formalization of a SAT solver (extended with linear arithmetic) is by Lescuyer [Les11], who verifies a DPLL-based decision procedure for propositional logic in Coq and exposes it as a reflexive tactic. Finally, a decision procedure based on DPLL is also verified by Shankar and Vaucher [SV11] in the PVS system. For the proof, they rely on subtyping and dependent types. Berger et al. have used the Minlog proof assistant to extract a certified SAT solver [BLFS15]. For these last approaches, performance considerations seem to be secondary.

6. Conclusion and Further Work

We have developed a formally verified implementation of the DPLL algorithm in the Dafny programming language. Our implementation is competitive in terms of execution time, but it is also trustworthy: all specifications are computer checked by the Dafny system. Other approaches to SAT solvers that rely on type checkers [BFLW18] are arguably even more trustworthy, since they are verified by a software system satisfying the de Bruijn criterion. However, we believe that our approach can strike a good balance between efficiency and trustworthiness of the final product.

Our implementation incorporates data structures to quickly identify unit clauses and perform unit propagation. The formalization consists of around 3088 lines of Dafny code, including the parser. The code was written by the first author in approximately one year and a half of part time work. The author also learned Dafny during that time. The ratio between lines of proof and lines of code is approximately 4/1. Table 2 contains a summary of our verified solver in numbers.

Lines of code	3088 (without whitespace)	Preconditions	420
Classes	4 (and 1 trait)	Postconditions	181
Methods	33	Invariants	173
Verification time	approx. 13 minutes (entire project)	Variants	44
	most expensive: `SATSolver.solve`	Predicates	37
	(approx. 175s)	Functions	20
	more than 60s:	Ghost variables	24
	6 methods/lemmas	Lemmas	42
	between 10s and 60s:	Assertions	169
	2 methods/lemmas		
Ratio specification/code	2488 lines of specification/proofs to 600 lines of code	`reads` annotations	41
		`modifies` annotations	26

Table 2: Various statistics for our verified DPLL solver.

In addition to coming up with the right invariants, the main challenge in the development of the verified solver is the large amount of time required by the Dafny system to discharge the verification conditions. In order to minimize this verification time, we develop and use the following development/verification methodology:

(1) Avoid nested loops in methods. Nested loops usually require duplicating invariants, which decreases elegance and increases verification time.

   An example of applying this tip is the `revertLastDecisionLevel` method (in the file `solver/formula.dfy`), whose purpose is to backtrack to the previous decision level. The code of the method is currently very simple: it calls `removeLastVariable` repeatedly in a while loop. However, because it is so simple, it is tempting to inline `removeLastVariable` – this would lead to a significant increase in verification time.

(2) In the same spirit, avoid multiple quantifications in specifications.

   We have found it useful, whenever having a specification of the form `forall x :: forall y :: P(x, y)`, to try to extract the subformula `forall y :: P(x, y)` as a separate predicate of `x`. This helps in two distinct ways: it forces the programmer to name the subformula, thereby clearing their thought process and making their intention more clear, and it enables the Z3 pattern-based quantifier instantiation to perform better [Mos09].

(3) Use very small methods. We find that it is better to extract as a method even code that is only a few lines of code long. In a usual programming language, such methods would be inlined (by the programmer). In our development, it is not unusual for such methods (with very few lines of code) to require many more helper annotations (invariants, helper assertions, etc.) and take significant time to verify.

(4) Use minimal `modifies` clauses in methods and `reads` clauses in functions. In particular, we make extensive use of the less well-known backtick operator in Dafny.

(5) During development, use Dafny to verify only the lemma/method currently being worked on. Run Dafny on the entire project at the end. To force Dafny to check only one method, we use the `-proc` command line switch.

(6) Finally, we have found that using the rather nice Z3 axiom profiler [BMS19] to optimize verification time does not scale well to projects the size of our solver.

Our project shows that it is possible to obtain a fully verified SAT solver written in assertional style, solver that is competitive in terms of running time with similar solvers written in non-verifiable languages. However, our experience with the verified implementation of the solver is that it currently takes significant effort and expertise to achieve this. We consider that three directions of action for the development of Dafny (and other similar auto-active verification tools) would be beneficial in order to improve this situation:

(1) Improve verification time of individual methods/lemmas,
(2) Make failures of verification obligations to check more explainable, and
(3) Devise a method better than `assert`s to guide the verifier manually.

As future work, we would like to verify an implementation of the full CDCL algorithm, thereby obtaining a verified solver that is competitive against state-of-the-art SAT solvers. In order to upgrade to a competitive CDCL solver, we need to modify the algorithm to implement a back-jumping and clause learning strategy, but also implement the two watched literals data structure [GKSS08], which becomes more important for performance when the number of clauses grows.

Acknowledgments

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