Polynomial Guarded Transformation for the Modal Mu-Calculus Is Still Open

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Guarded Normal Form

Setting: modal μ-calculus and extensions

GNF: Every fixpoint variable under scope of a modal operator

Example: $\mu X.\Diamond(P \vee X)$ guarded, $\mu X.\nu Y.X \vee \Diamond Y$ not guarded

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Why GNF?
Synchronizes unfolding of fixpoints in tableaux, helps in constructions, translations to automata, etc.

can effectively transform any formula into guarded equivalent (BB89, Wal00,KVW00,Mat02)

Hence GNF commonly assumed when working with μ-calculus

Failure of Previous Results

Theorem: guarded transformation possible with no blowup (KVW00)/quadratic blowup (Mat02)

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Counterexample is

$$\Phi_n = \mu X_n \cdots \mu X_1.(X_n \vee \cdots \vee X_1 \vee \Box(X_n \vee \cdots \vee X_1))$$

Known GT procedures produce formulae of exponential modal depth

Reason: occurrence of variable at modal depth d will produce formula at modal depth $2d$ after unfolding

Vectorial Form and Hierarchical Equation Systems

Vectorial Form: allow formulae of form

$$\sigma \begin{Bmatrix} X_1.\varphi_1 \\ \vdots \qquad \vdots \\ X_n.\varphi_n \end{Bmatrix} \text{ where each } \varphi_i \text{ may refer to all other } X_i.$$

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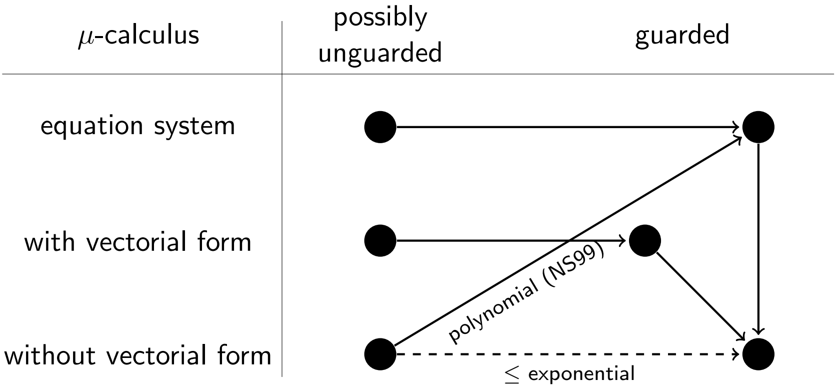
HES: Allow every fixpoint subformula to refer to every variable

Don't gain expressive power, but succinctness (best known algorithm to unfold is at least exponential)

Notion of guardedness can be generalized

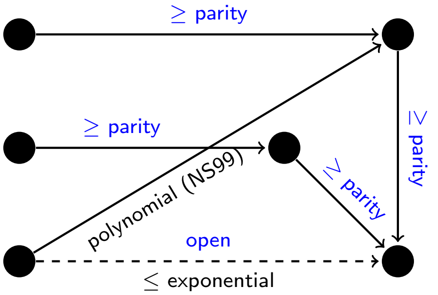
State of the Art



State of the Art

Our results in blue

μ-calculus	possibly unguarded	guarded
equation system		
with vectorial form		
without vectorial form		



Poly GT for Vect. Formulae $\rightarrow$ Poly PG Solving

1. Given μ-calc. formula φ and TS $\mathcal{T}, s$, can obtain vectorial φ' and $\mathcal{T}'$ s.t. φ' $\Diamond$,$\Box$-free, $\mathcal{T}'$ has only one state, both polynomial size, and

$$\mathcal{T}, s \models \varphi \leftrightarrow \mathcal{T}' \models \varphi'$$

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4. Consider PG, relevant Walukiewicz-formula: apply steps 1-3

Result: Polynomial GT for vectorial μ-calculus gives rise to polynomial solution procedure for parity games
Also holds for HES

Consequences and Outlook

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- Results in KVW00 still hold
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Open Questions:

- polynomial GT for μ-calculus $\rightarrow$ polynomial parity game solving?
- polynomial parity game solving $\rightarrow$ polynomial GT?
- relation between ϵ-transitions in alternating automata and GNF

The End

Thanks!

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Literature:

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