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Erratum

Erratum to "What's so special about Kruskal's theorem and the ordinal Γ_0? A survey of some results in proof theory" [Ann. Pure Appl. Logic 53 (1991) 199–260]

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Gopalan Nadathur has pointed out that there is a gap in the proof of Theorem 4.5, pp. 207–208. Specifically, there is a gap in the proof of the claim that $\preceq$ is a *wqo* on $\mathscr{D}$ (line 11 of p. 208). The problem is that even though $t_k \preceq t_h$, the proof does not ensure that $k < h$ (line 16 of p. 208). However, the proof of the claim can be repaired as follows:

Correction: Let

$$\mathscr{D} = \{s_{\varphi(i)}/j \mid i \geqslant 1,\ 1 \leqslant j \leqslant rank(s_{\varphi(i)})\}.$$

We claim that $\preceq$ is a *wqo* on $\mathscr{D}$. Otherwise, let $r = \langle r_1, r_2, \ldots, r_j, \ldots \rangle$ be a bad sequence in $\mathscr{D}$. Because r is bad, it contains a bad subsequence $r' = \langle r'_1, r'_2, \ldots, r'_j, \ldots \rangle$ with the following property: if $i < j$, then r'_i is a subtree of a tree t_p and r'_j is a subtree of a tree t_q such that $p < q$. Indeed, every t_i only has finitely many subtrees, and r being bad must contain an infinite number of distinct trees. Thus, we consider a bad sequence r with the additional property that if $i < j$, then r_i is a subtree of a tree t_p and r_j is a subtree of a tree t_q such that $p < q$. Let n be the index of the first tree in the sequence t such that $t_n/j = r_1$ for some j. If $n = 1$, since $|r_1| < |t_1|$ and the sequence r is bad, this contradicts the fact that t is a minimal bad sequence. If $n > 1$, then the sequence

$$\langle t_1, t_2, \ldots, t_{n-1}, r_1, r_2, \ldots, r_j, \ldots \rangle$$

is bad, since by clause (ii) of the definition of $\preceq$, for any k s.t. $1 \leqslant k \leqslant n-1$, $t_k \preceq r_j$ would imply that $t_k \preceq t_h$ for some t_h and l such that $r_j = t_h/l$ and $k < h$, since each r_i is a subtree of some t_p such that $n - 1 < p$. But since $|r_1| < |t_n|$, this contradicts the fact that t is a minimal bad sequence. Hence, $\mathscr{D}$ is a *wqo*.

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