Aperiodic two-way transducers

Luc Dartois [1] ,
joint work with Olivier Carton[1]

Highlights of Logic, Games and Automata,
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Equivalence between two formalisms

Theorem [Engelfriet and Hoogeboom (2001)]

A function is *MSO*-definable if and only if
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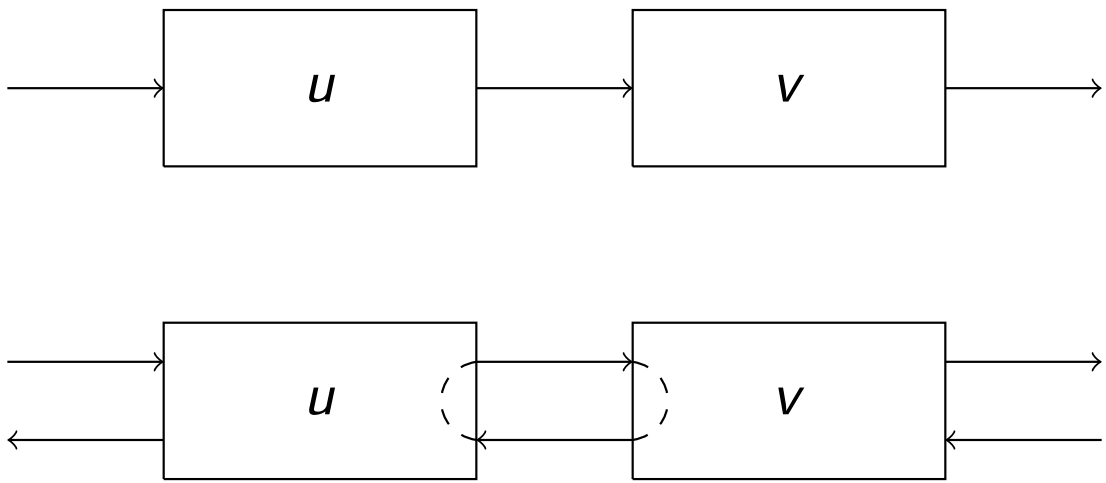
Our result

A function is *FO*-definable if and only if
it is realized by a deterministic aperiodic two-way transducer.

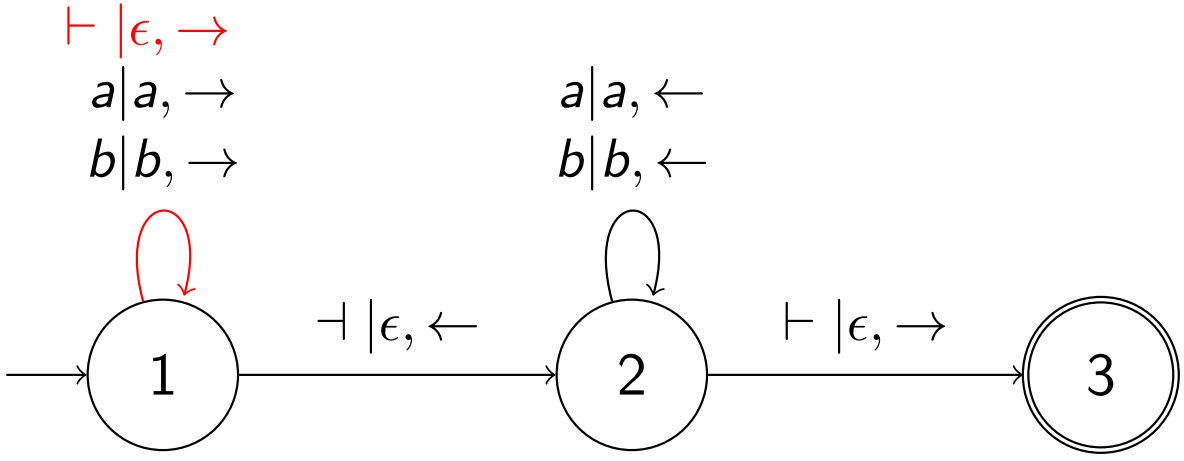
Aperiodicity

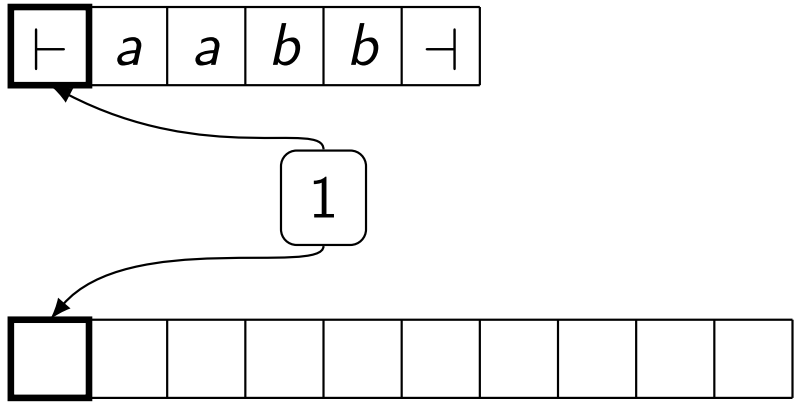
The notion of transition monoid can be extended to two-way automata, but composition is more difficult.



Deterministic Two-way transducers

A two-way transducer has a two-way input tape and a one-way output tape.This deterministic transducer realizes the mirror function $w \rightarrow w\tilde{w}$.





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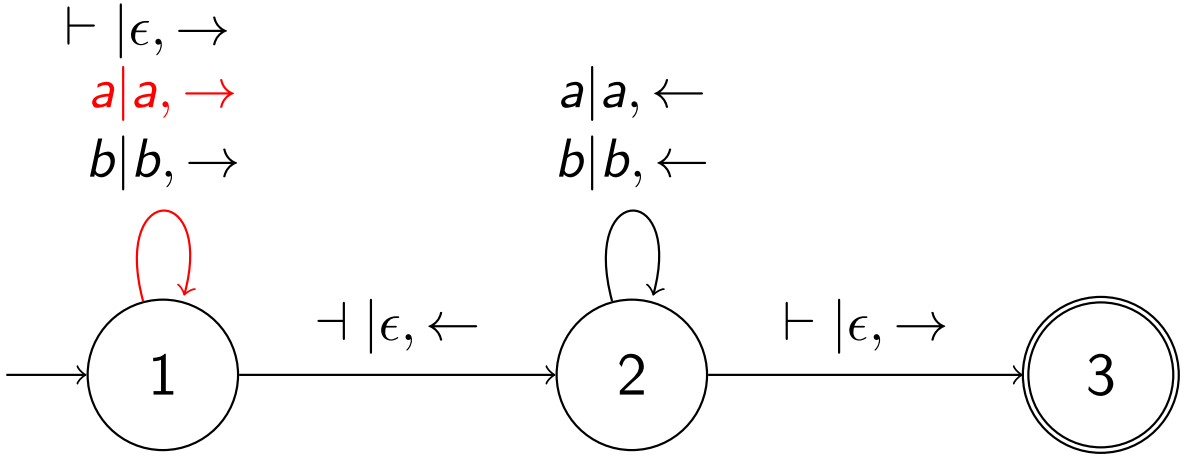

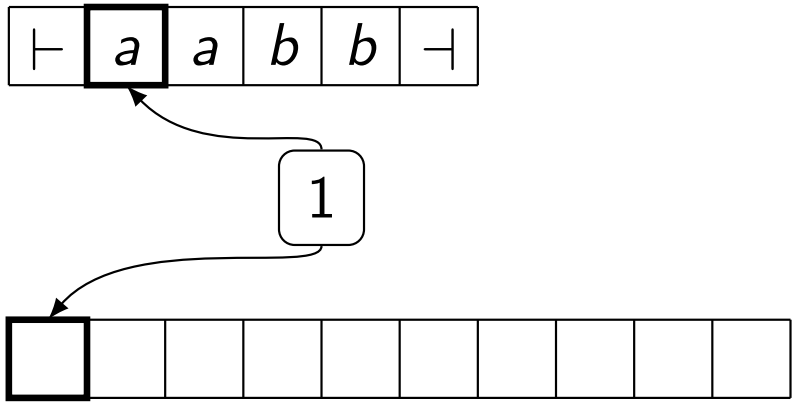

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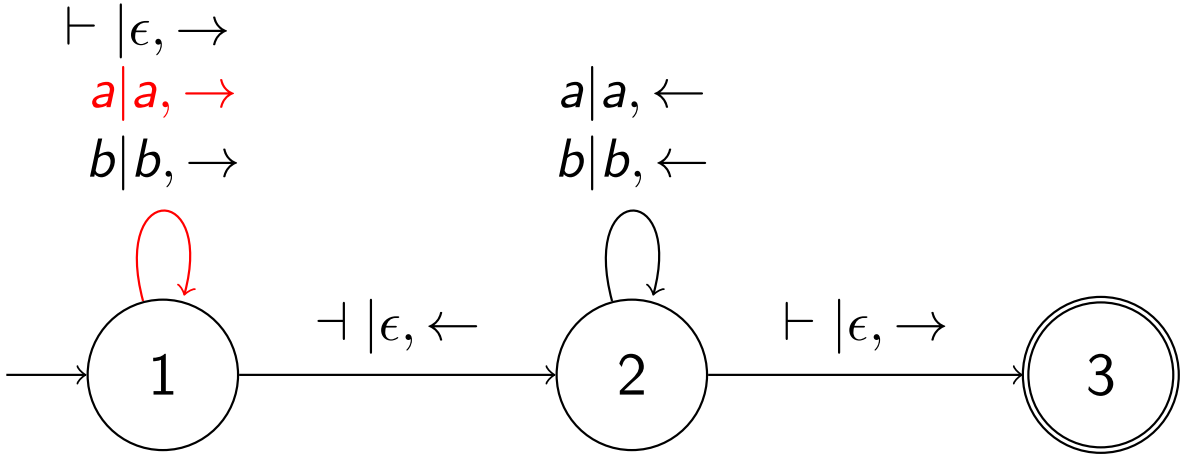

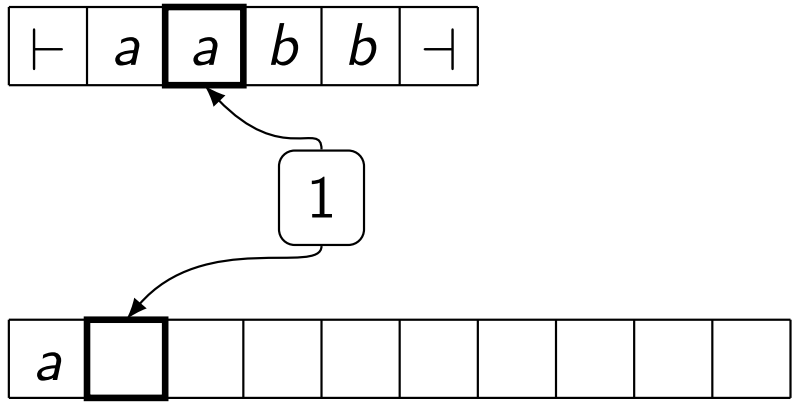

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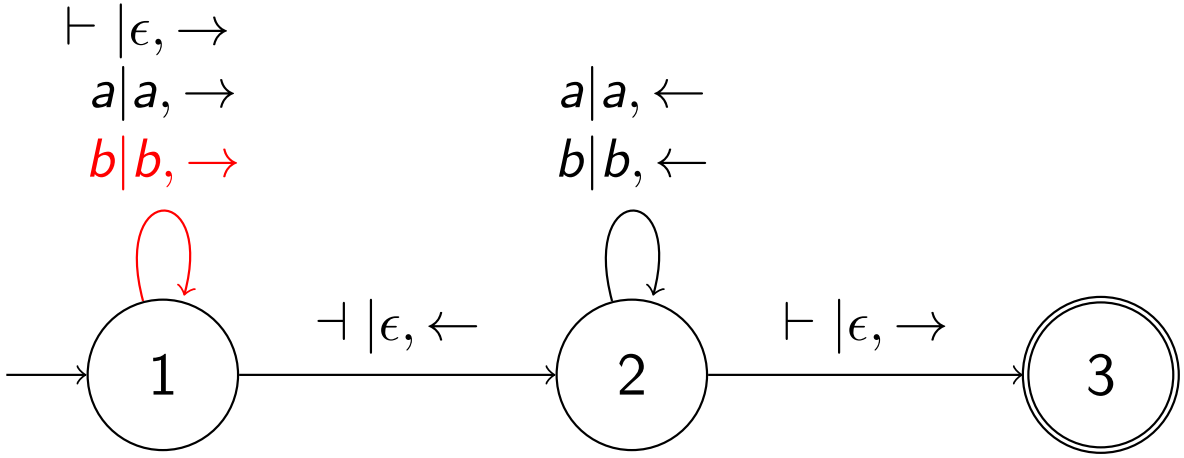

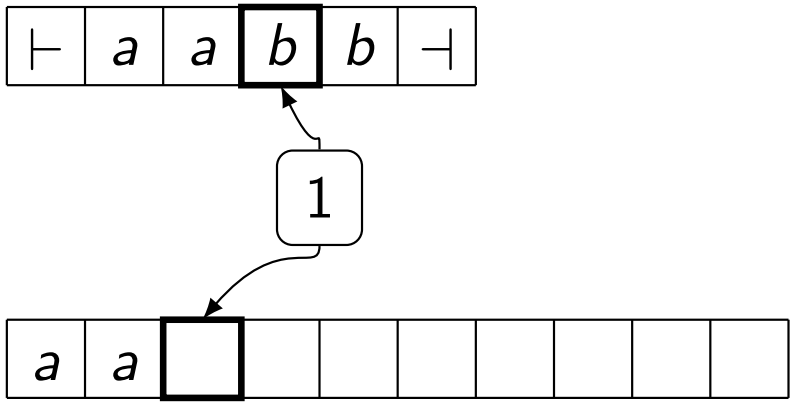

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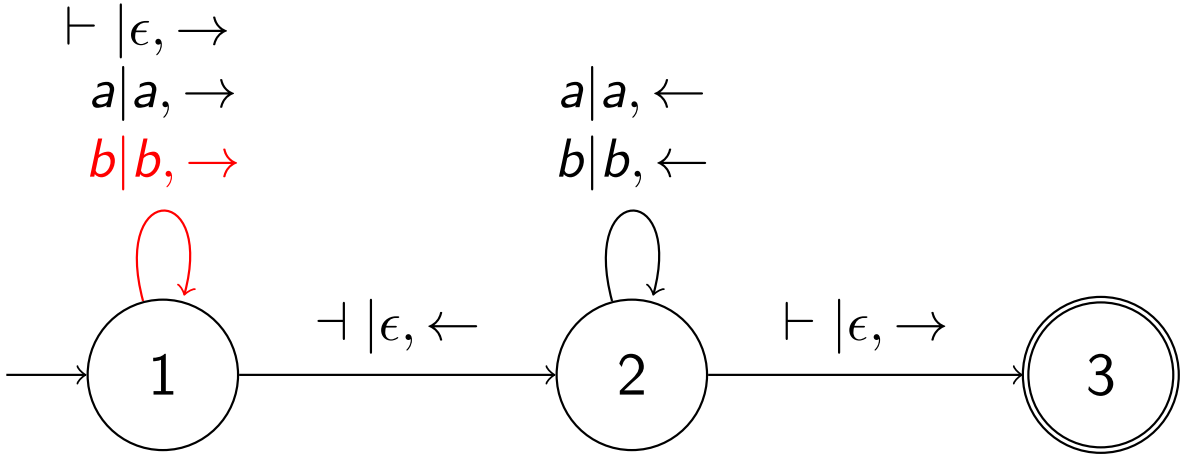

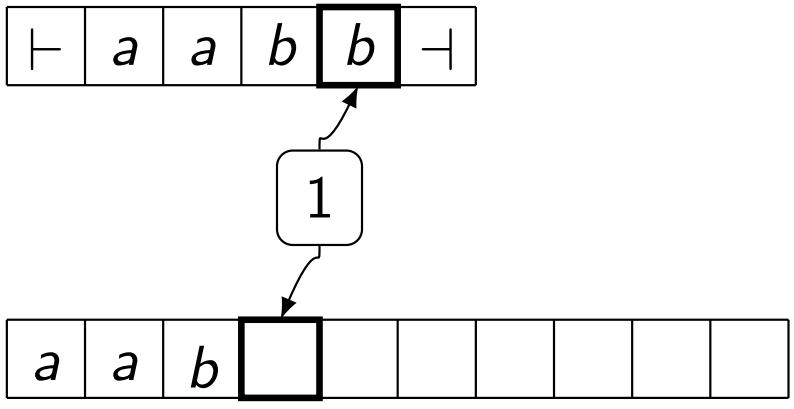

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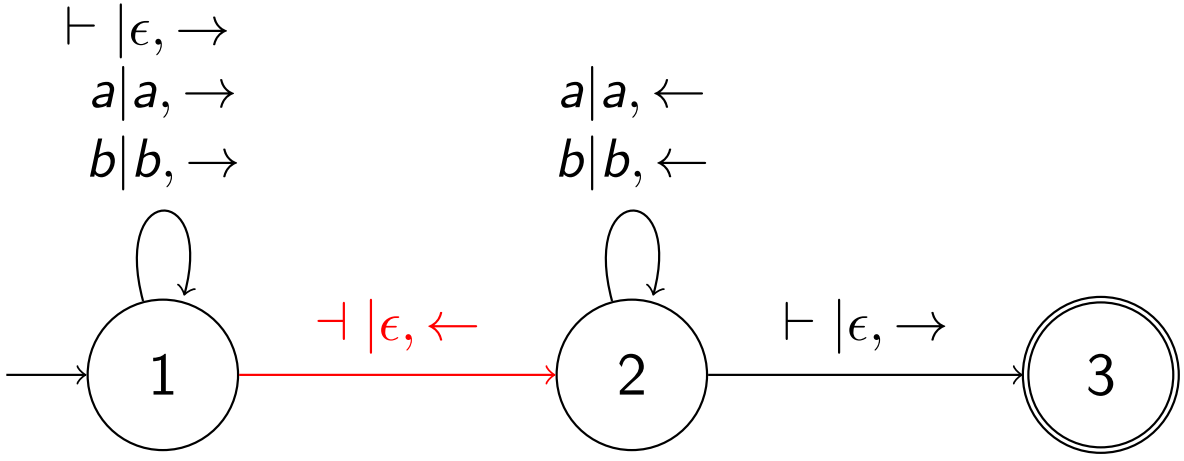

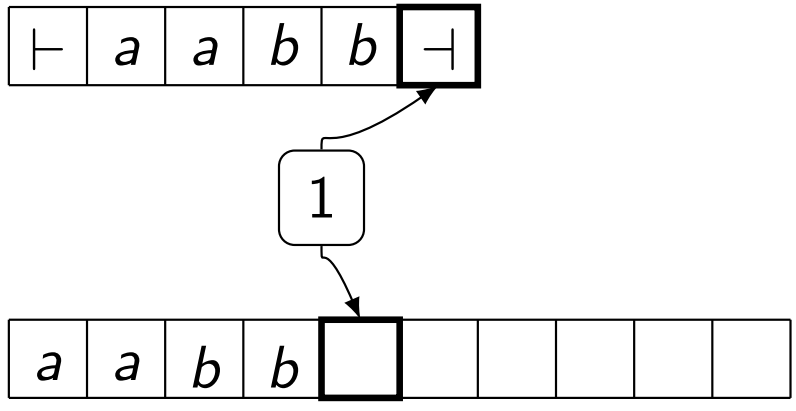

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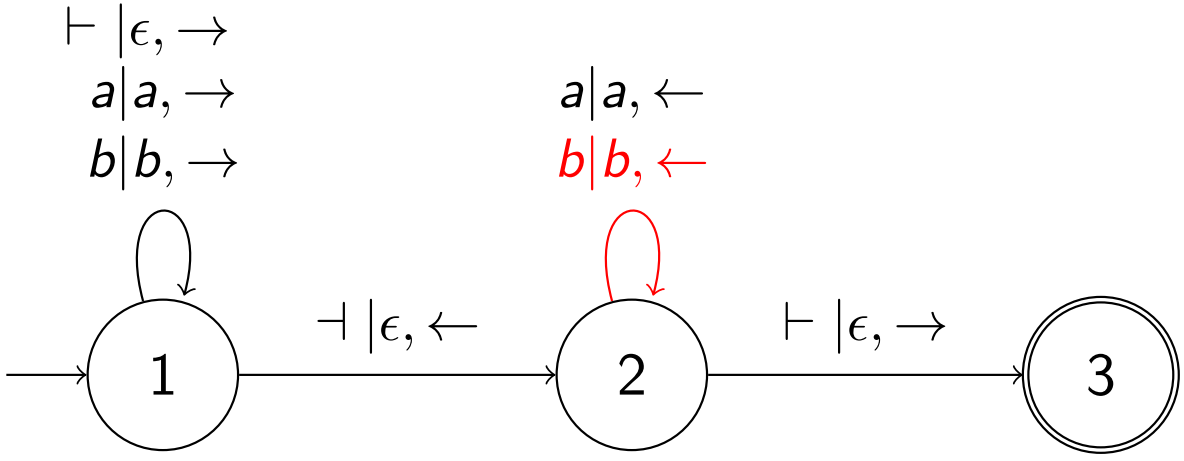

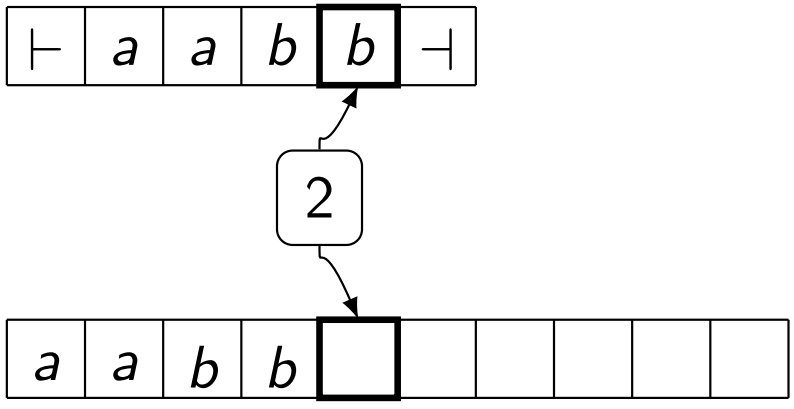

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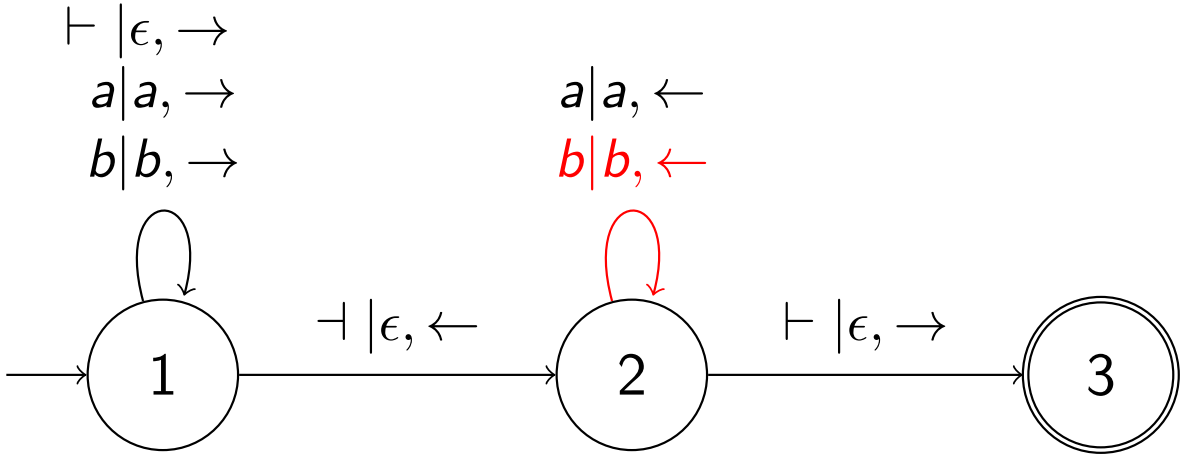

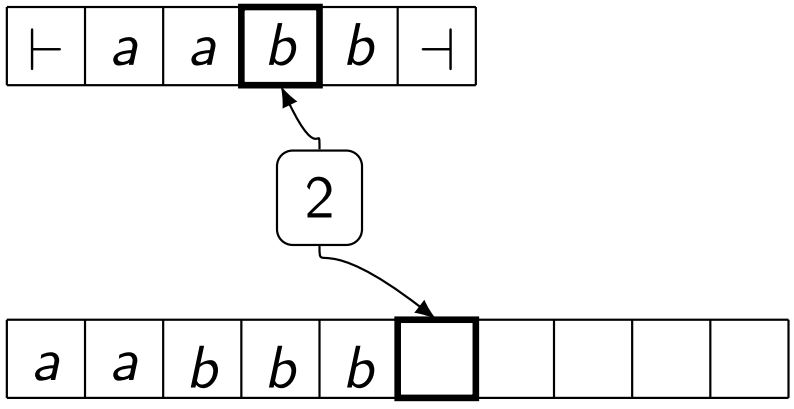

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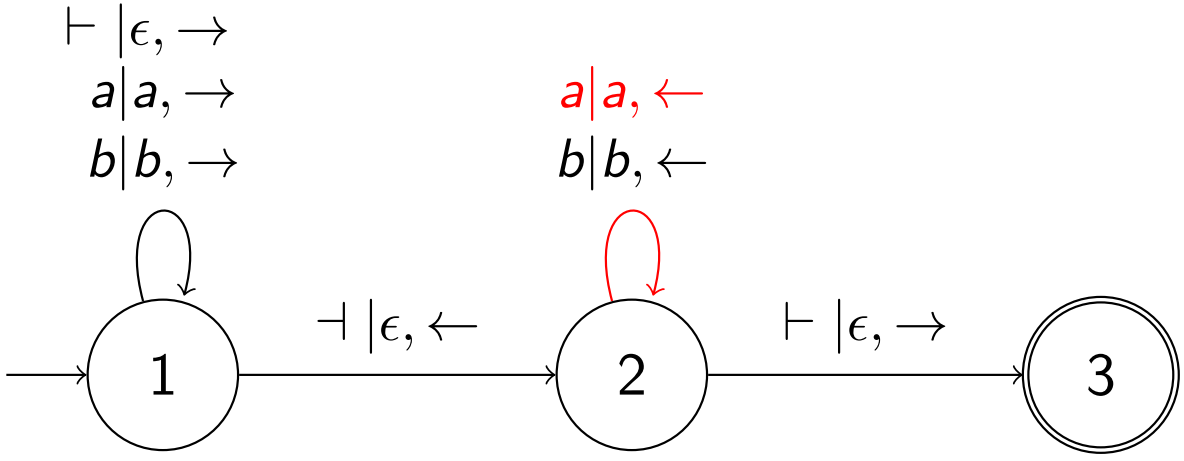

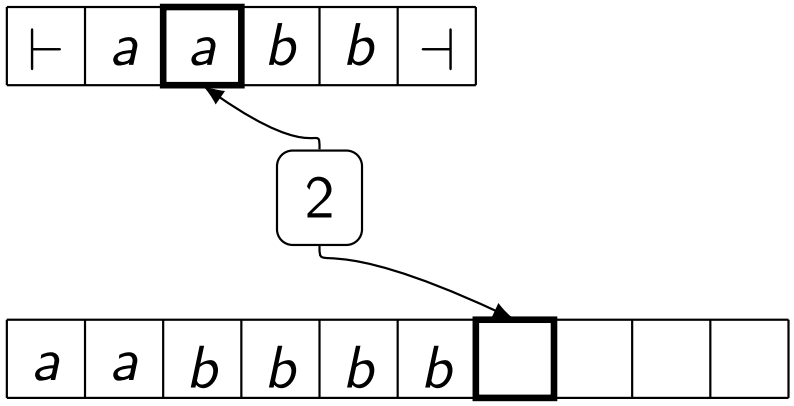

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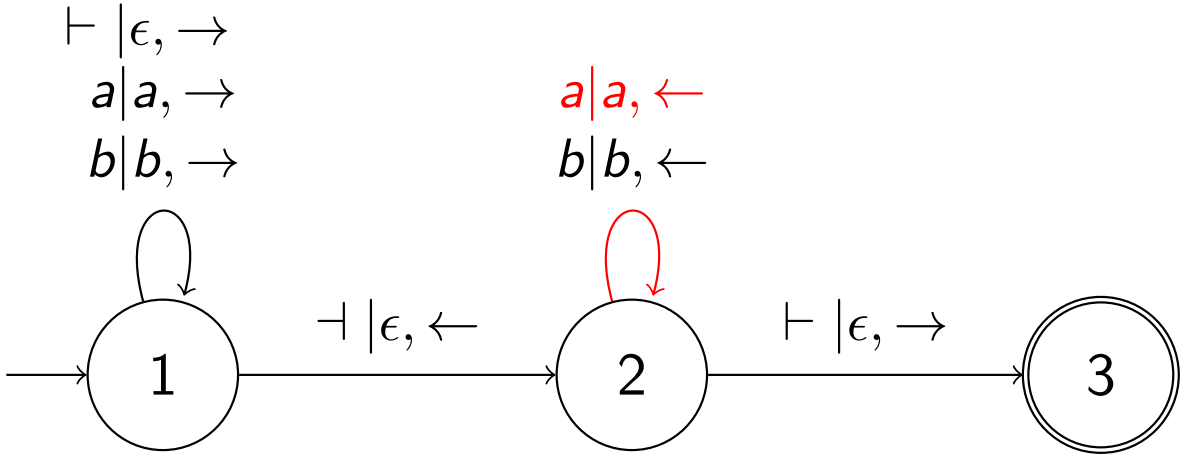

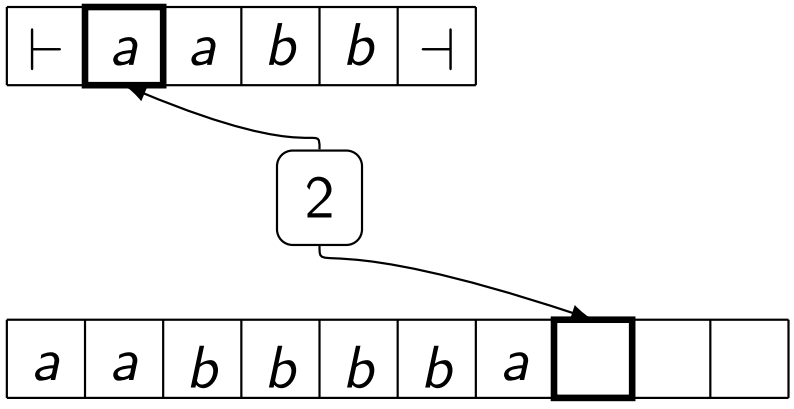

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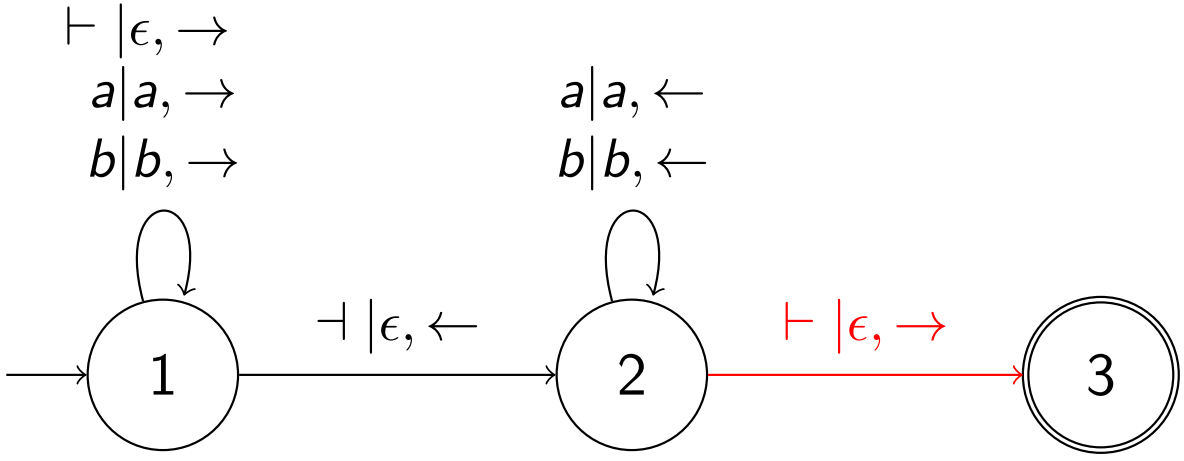

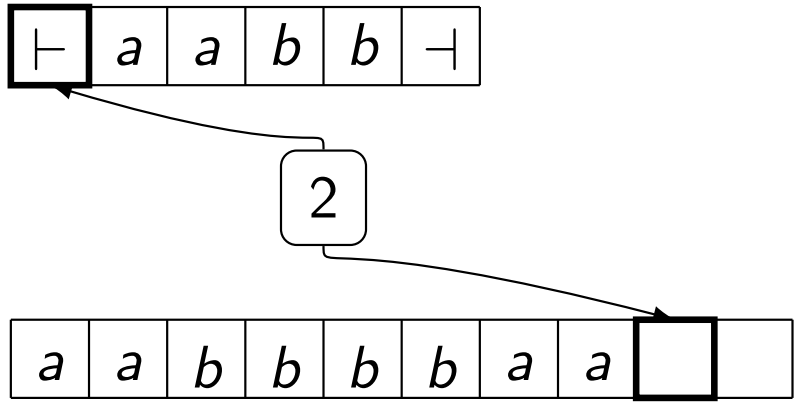

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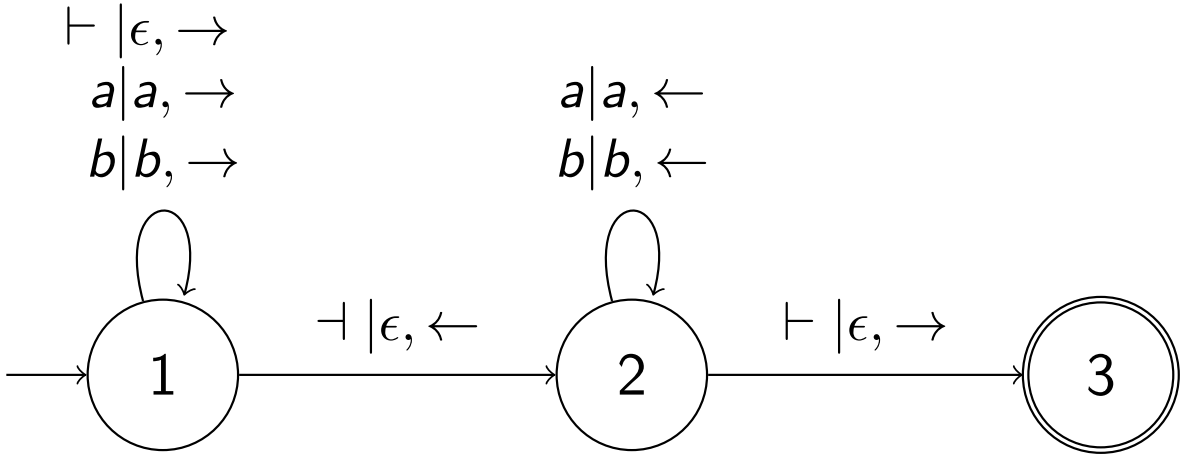
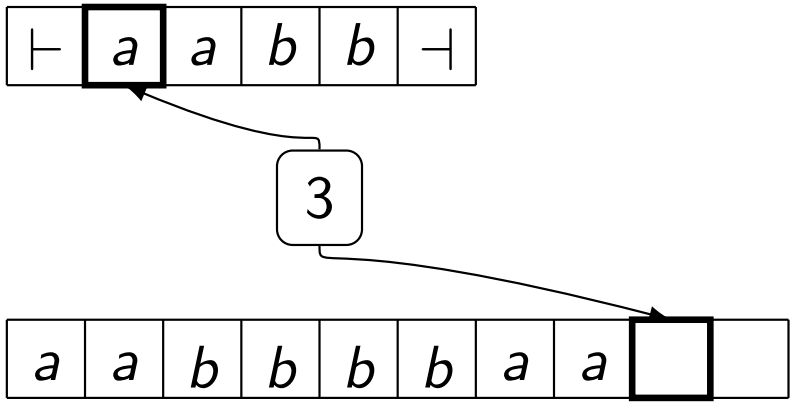
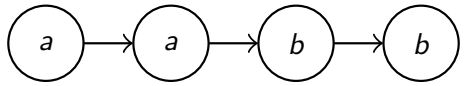




MSO-definable function

word w $\rightsquigarrow$ linear graph G_w

aabb $\rightsquigarrow$ 

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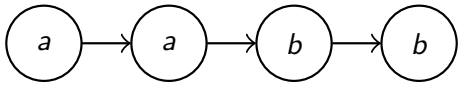
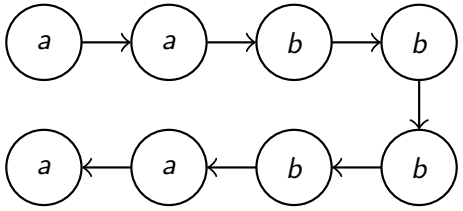
$aabb$ $\rightsquigarrow$ 

$f(aabb) = aabbbbaa$ $\rightsquigarrow$ 

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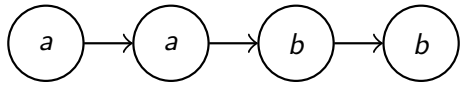
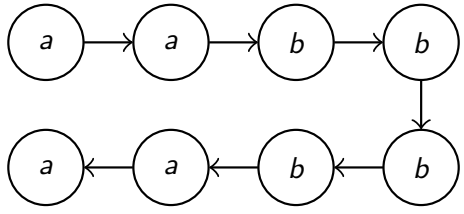
$aabb$ $\rightsquigarrow$ 

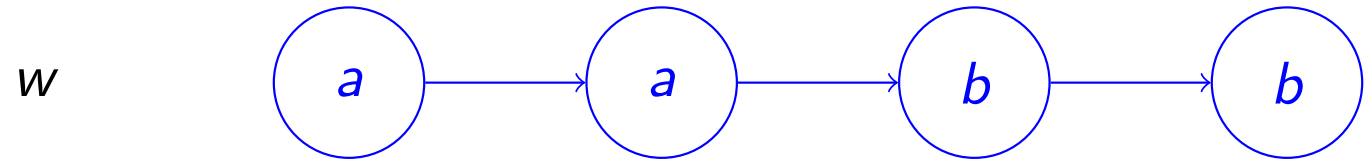
$f(aabb) = aabbbbaa$ $\rightsquigarrow$ 

Definition

A function f is *MSO*-definable iff there exist an integer k and a graph interpretation φ such that for any word w, $G_{f(w)}$ is the interpretation of k copies of G_w by φ .

Example of *MSO*-definable function

The same function $f : w \to w\tilde{w}$ is a *MSO* definable function. We construct the image of the word *aabb*.



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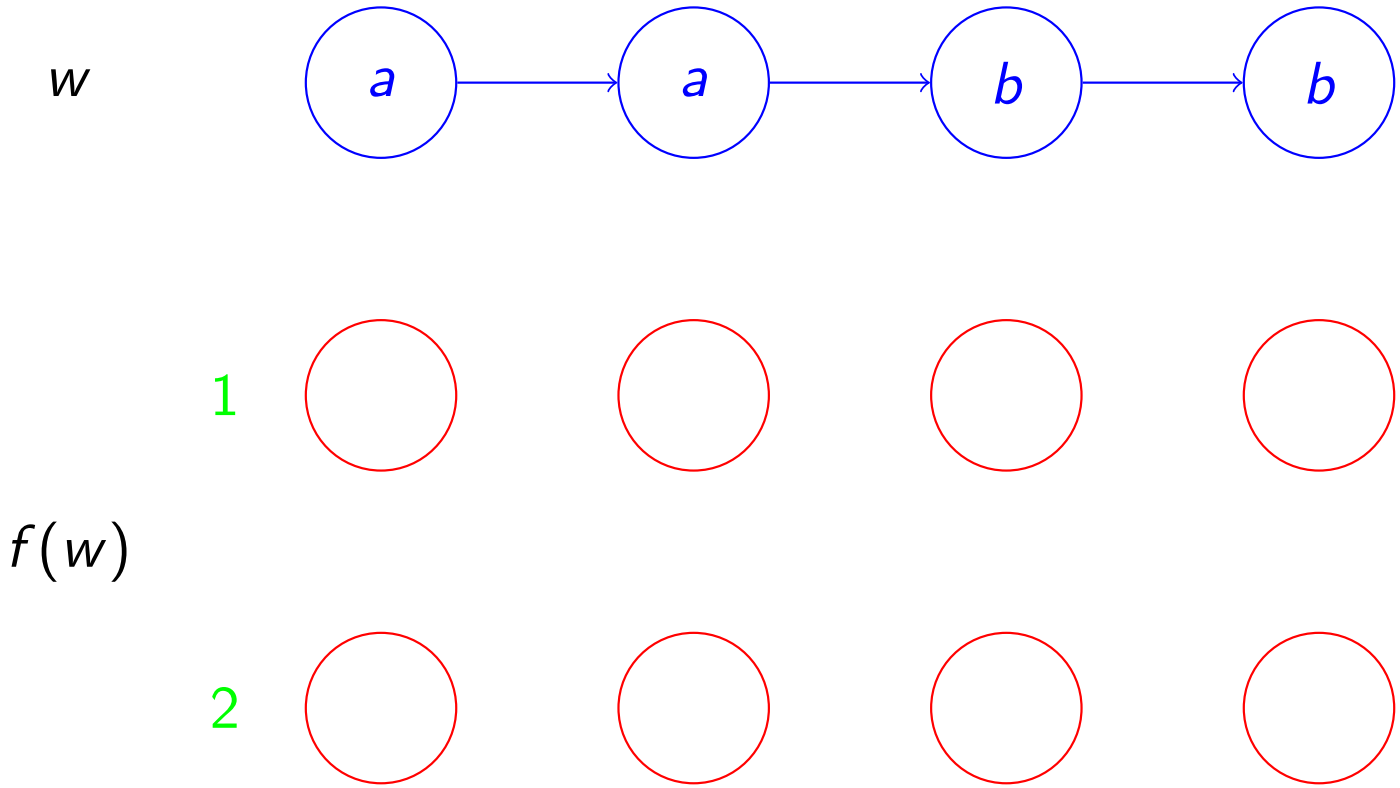

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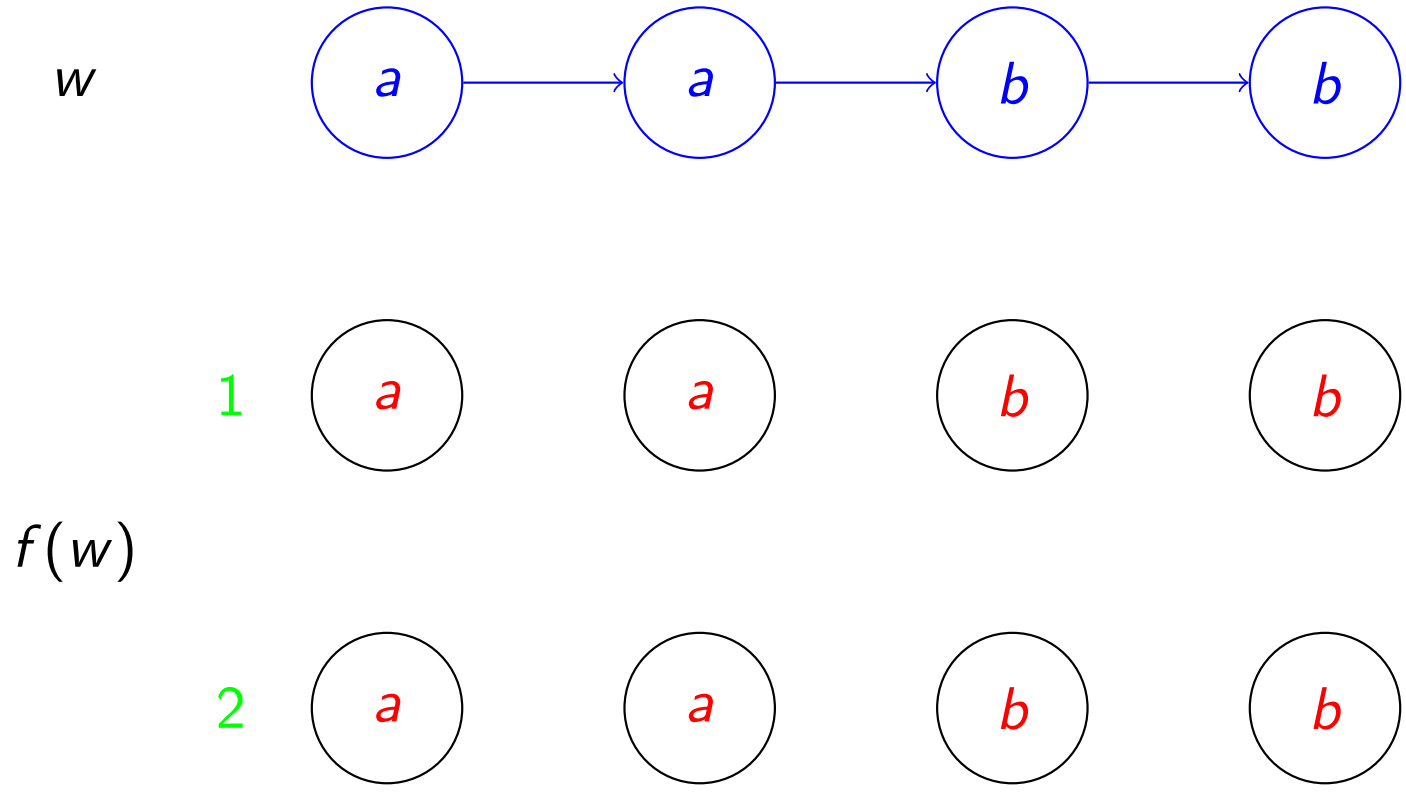

$lab^1(x) \equiv lab^2(x) \equiv lab(x)$

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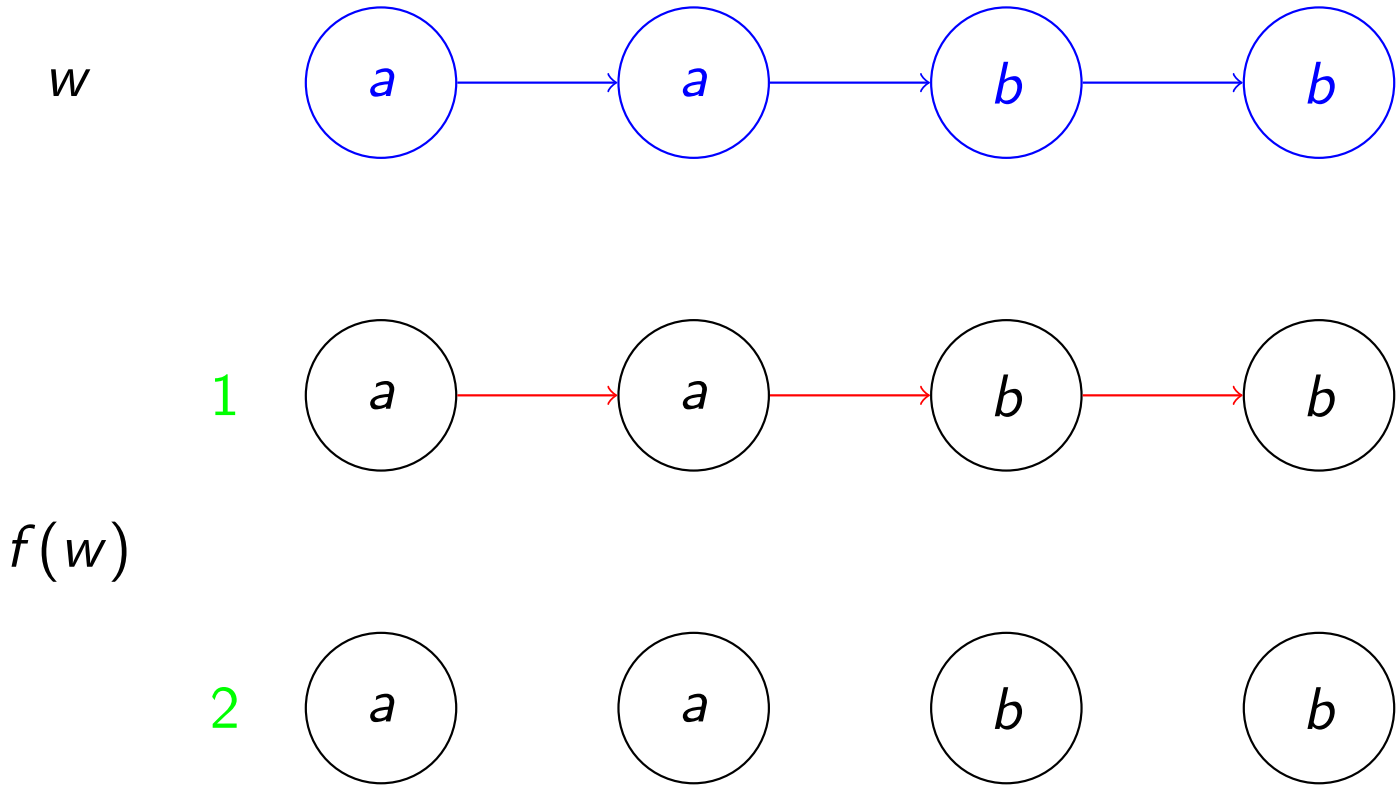

$$edge^{1,1}(x,y) \equiv edge(x,y)$$

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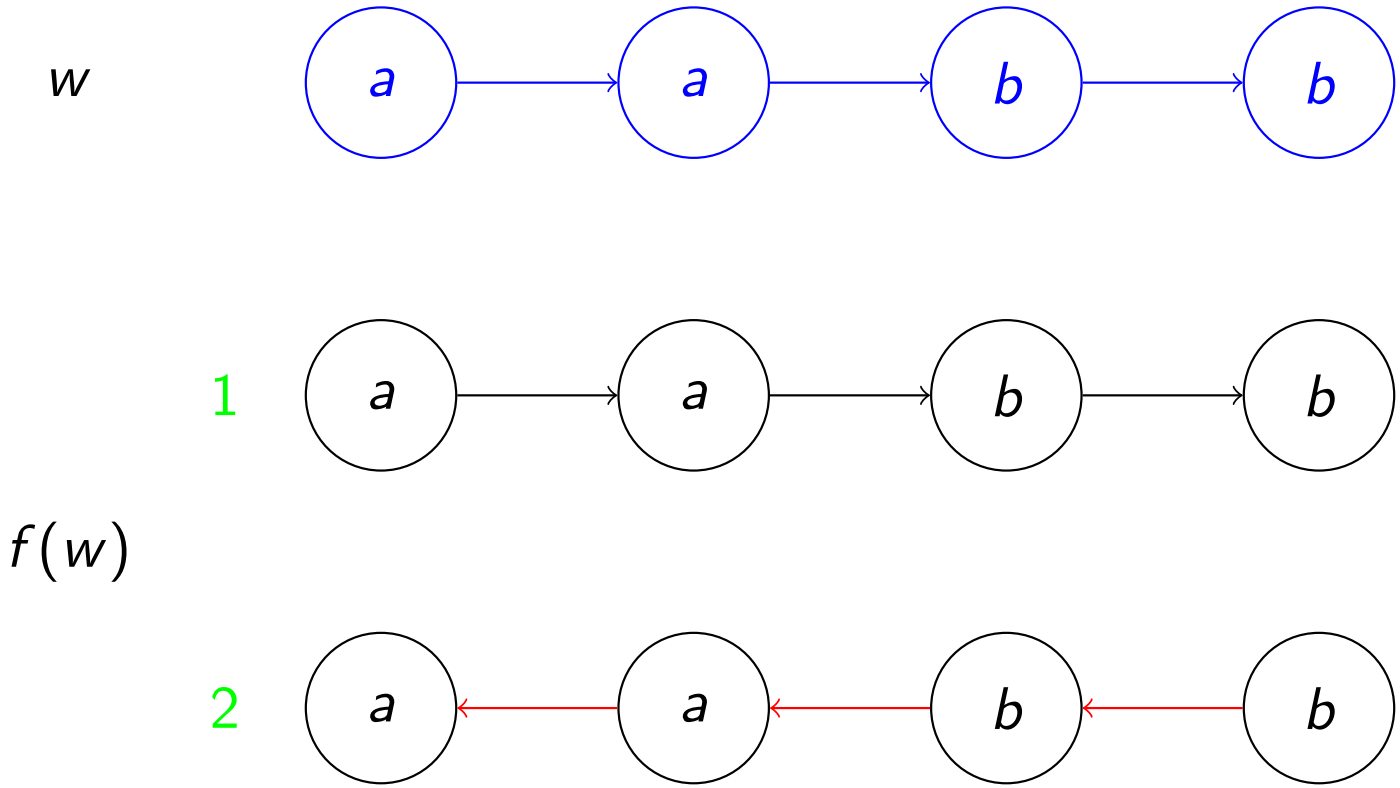

$edge^{2,2}(x, y) \equiv edge(y, x)$

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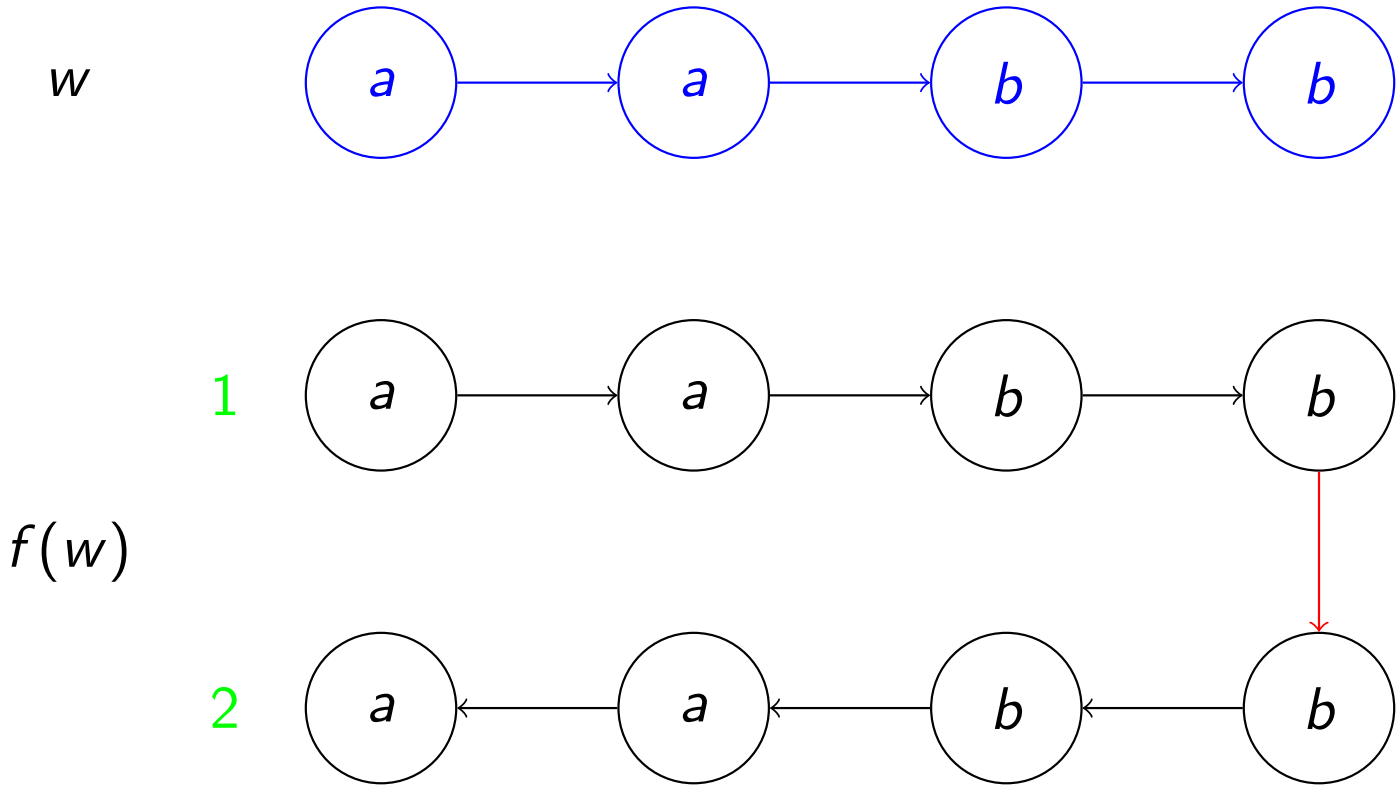

$$edge^{1,2}(x, y) \equiv (x = y) \wedge \neg(\exists z\ edge(x, z))$$

Conclusion

Our result

A function is *FO*-definable if and only if
it is realized by a deterministic aperiodic two-way transducer.

We constructed a robust subclass of two-way transducers that

- is stable by composition,
- translates naturally to a subclass of logic transductions.

Some questions

- As minimization is an issue here, is this subclass decidable ?
- Can we get a natural restriction of other equivalent formalisms like the streaming strings transducers of Alur & Černý ?