Constructive interpolation for guarded logics

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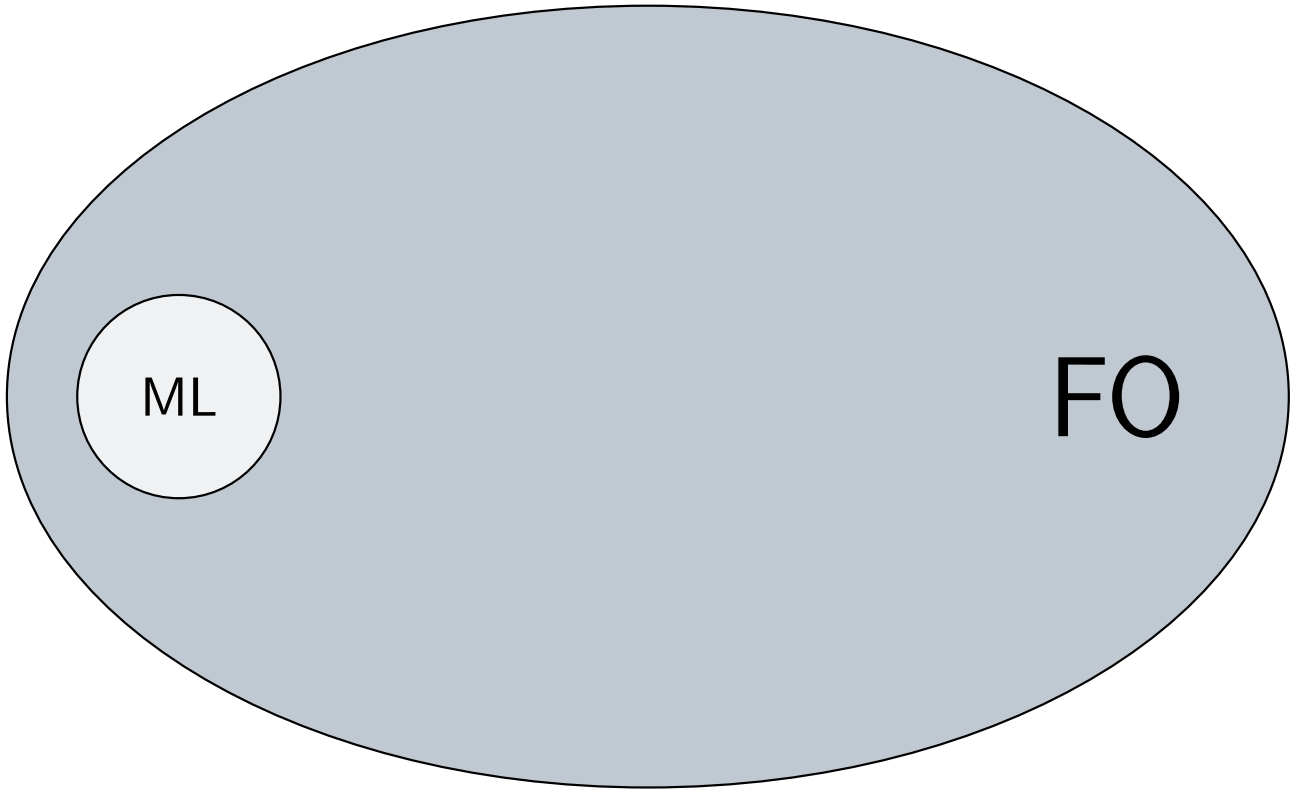
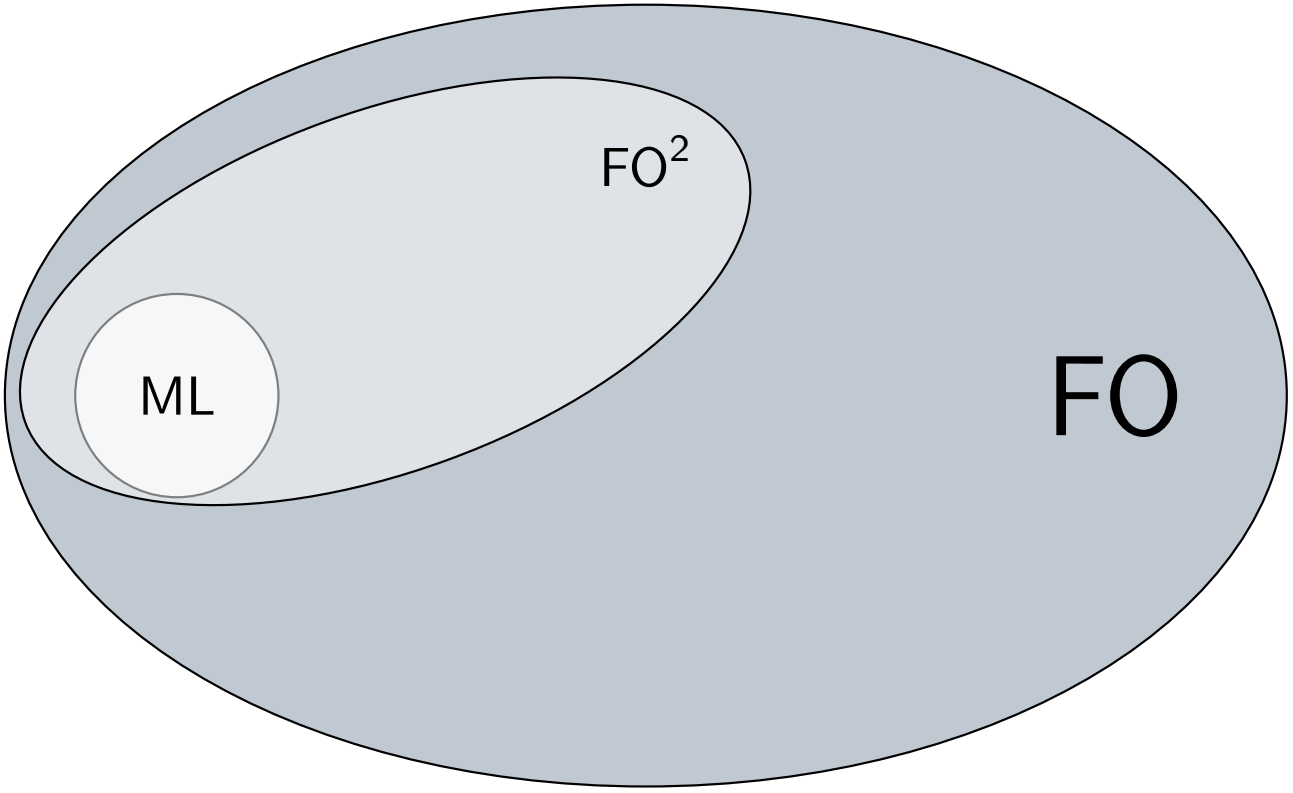
Some decidable fragments of first order logic



	ML
finite model property	✓
tree-like model property	✓
Craig interpolation	✓

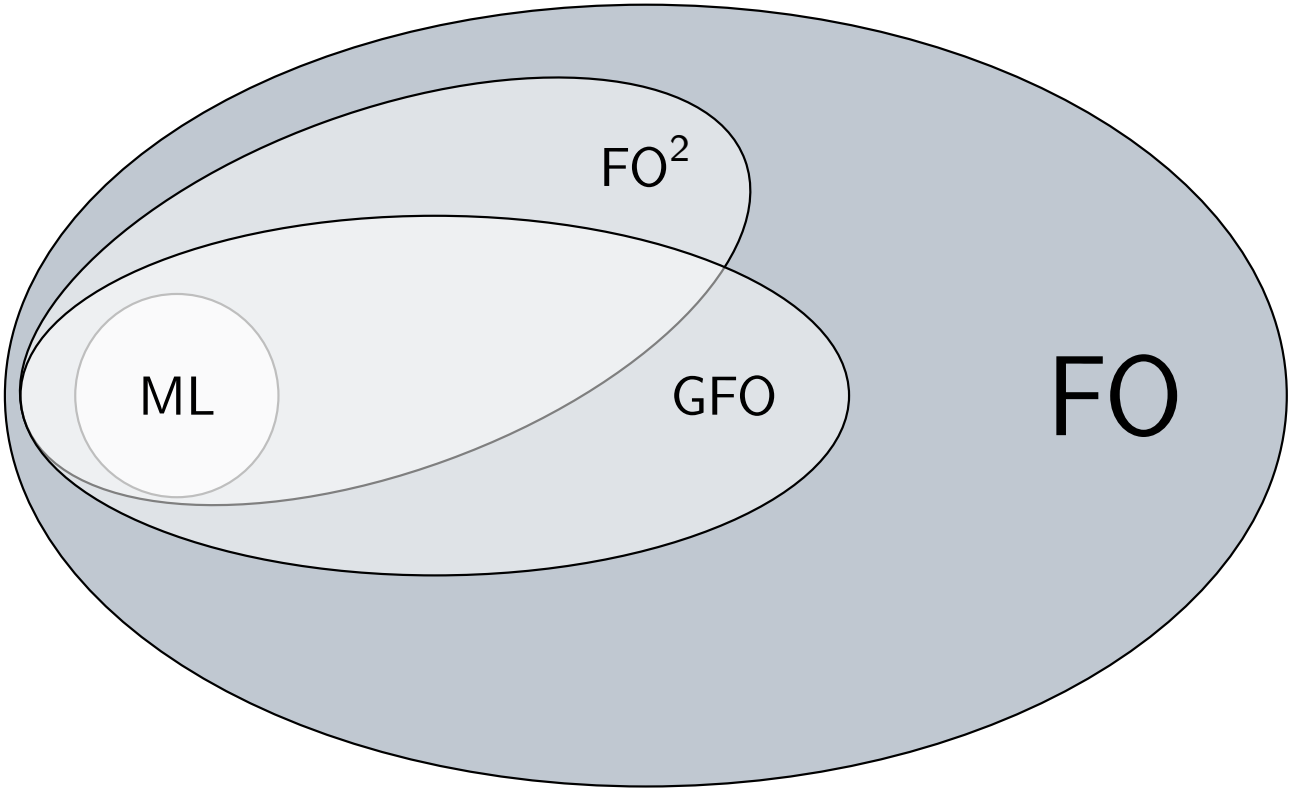
Some decidable fragments of first order logic



constrain
number of variables

	ML	FO^2
finite model property	✓	✓
tree-like model property	✓	✗
Craig interpolation	✓	✗

Some decidable fragments of first order logic



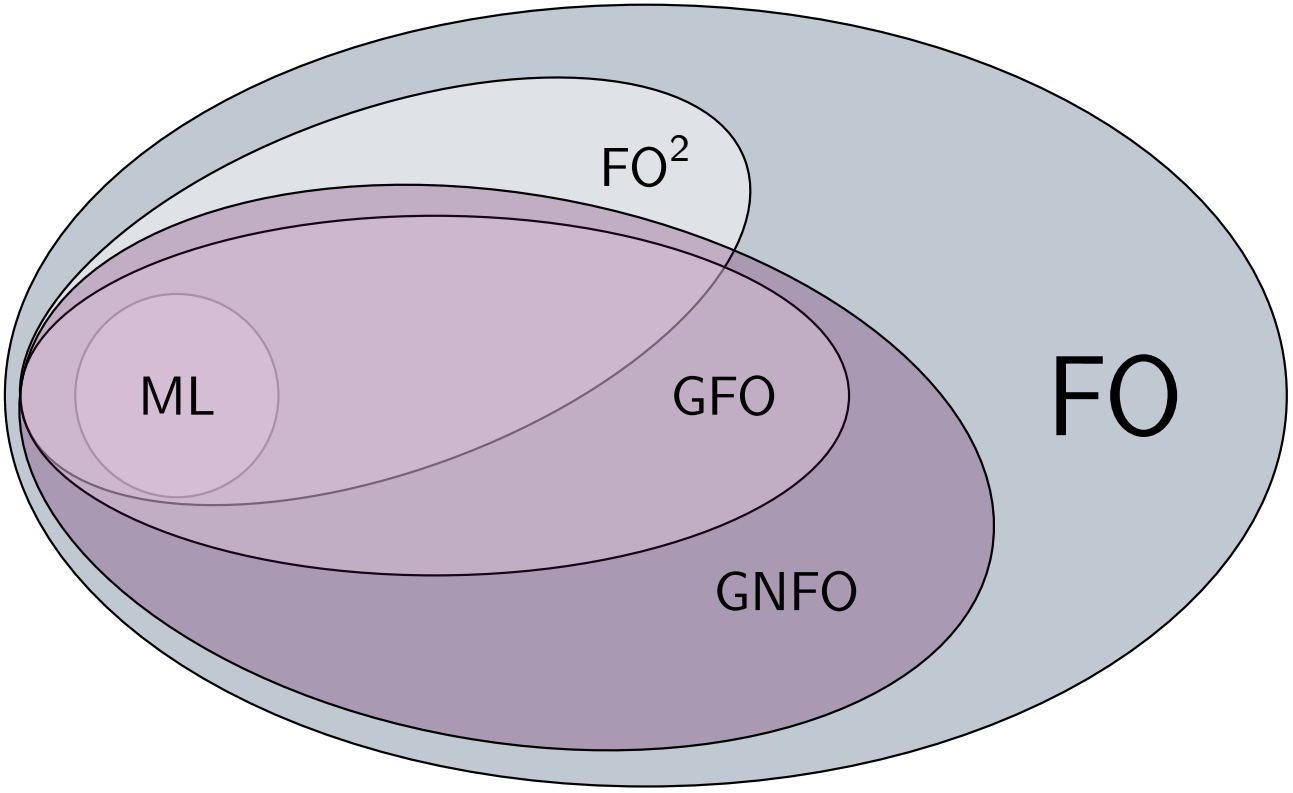
constrain
number of variables

constrain
quantification

$\exists \vec{x}.G(\vec{x}\vec{y}) \wedge \psi(\vec{x}\vec{y})$
$\forall \vec{x}.G(\vec{x}\vec{y}) \rightarrow \psi(\vec{x}\vec{y})$

	ML	FO^2	GFO
finite model property	✓	✓	✓
tree-like model property	✓	✗	✓
Craig interpolation	✓	✗	✗

Some decidable fragments of first order logic



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constrain
negation

$\exists \vec{x}.\psi(\vec{x}\vec{y})$
$G(\vec{x}\vec{y}) \wedge \neg\psi(\vec{x}\vec{y})$

	ML	FO^2	GFO	GNFO
finite model property	✓	✓	✓	✓
tree-like model property	✓	✗	✓	✓
Craig interpolation	✓	✗	✗	✓

Interpolation

$$\varphi \quad \models \quad \psi$$

Interpolation

$$\varphi \models \overset{\text{interpolant}}{\chi} \models \psi$$

only uses relations in both φ and ψ

Interpolation example

$$\exists xyz(Gxyz \wedge Rxy \wedge Ryz \wedge Rzx) \quad \models \quad \exists xy(Rxy \wedge ((Sx \wedge Sy) \vee (\neg Sx \wedge \neg Sy)))$$

"there is a G-guarded 3-cycle using R"

"there is an odd-length cycle using R"

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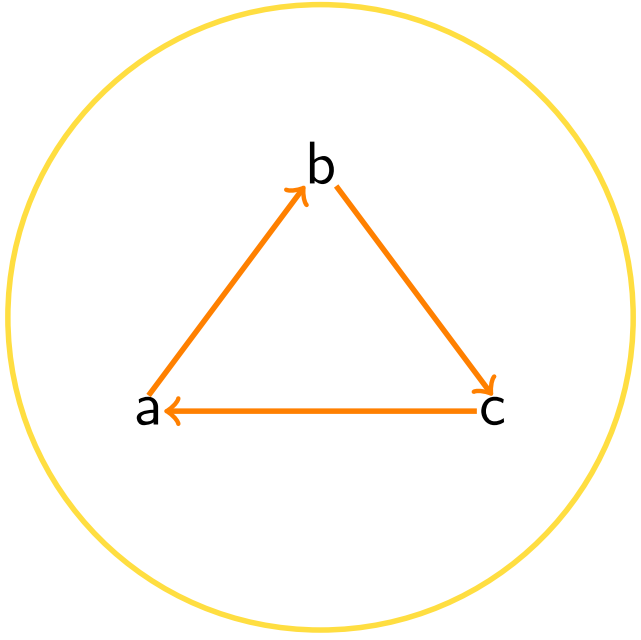

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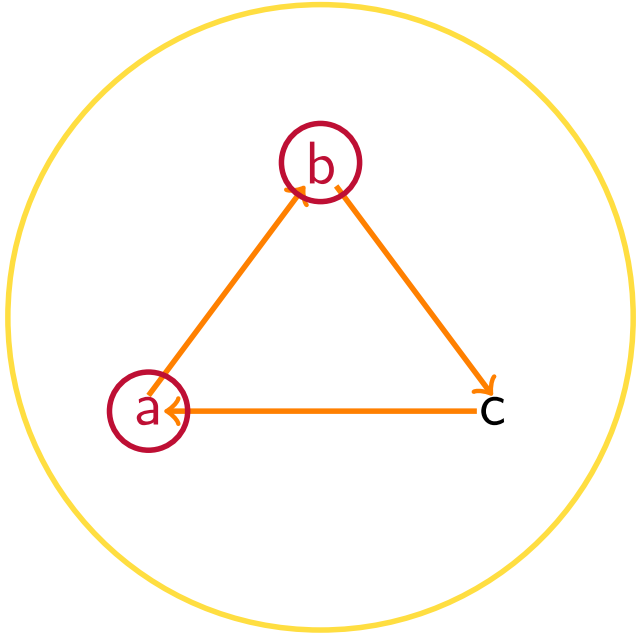

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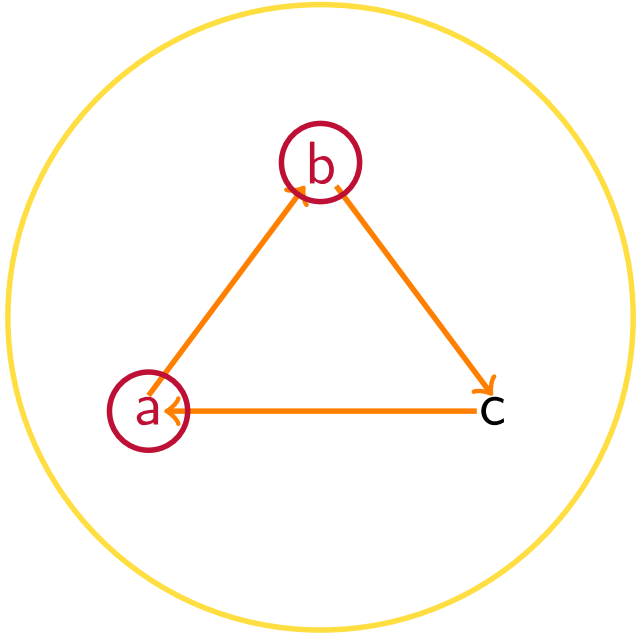

interpolant $\chi := \exists xyz(Rxy \wedge Ryz \wedge Rzx)$

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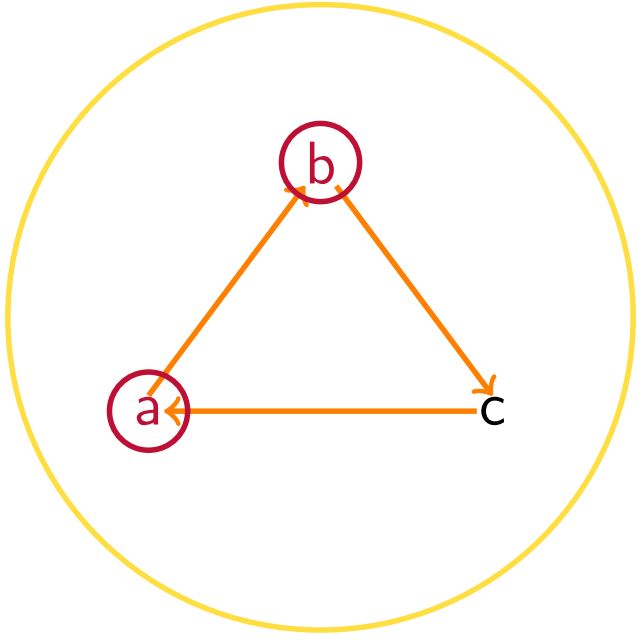
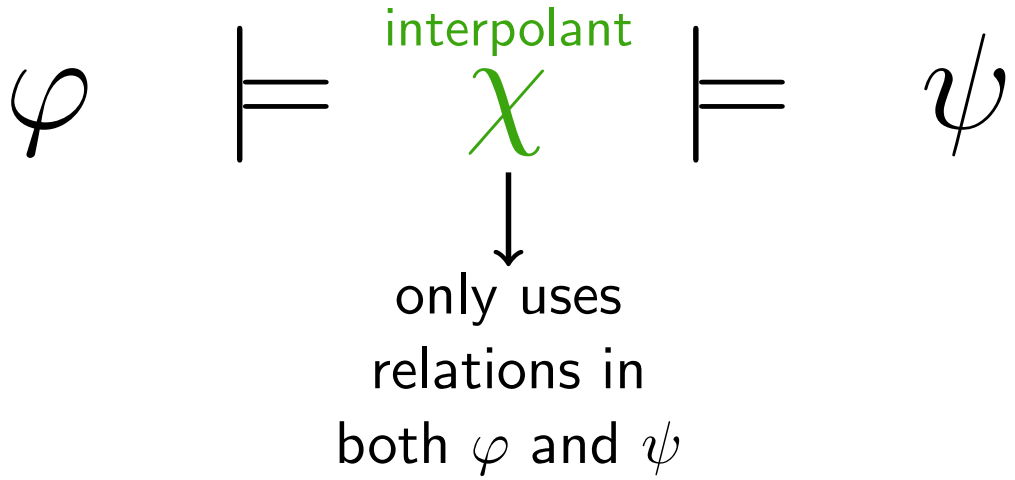


GNFO interpolant $\chi := \exists xyz(Rxy \wedge Ryz \wedge Rzx)$

"there is a 3-cycle using R"

Interpolation



Theorem [Barany+Benedikt+ten Cate '13]

Given GNFO φ and ψ such that $\varphi \models \psi$, there is a GNFO interpolant χ (but model theoretic proof implies no bound on size of χ).

Interpolation

$$\varphi \models \overset{\text{interpolant}}{\chi} \models \psi$$

χ only uses relations in both φ and ψ

Theorem [Barany+Benedikt+ten Cate '13]

Given GNFO φ and ψ such that $\varphi \models \psi$, there is a GNFO interpolant χ (but model theoretic proof implies no bound on size of χ).

Theorem [VB, unpublished]

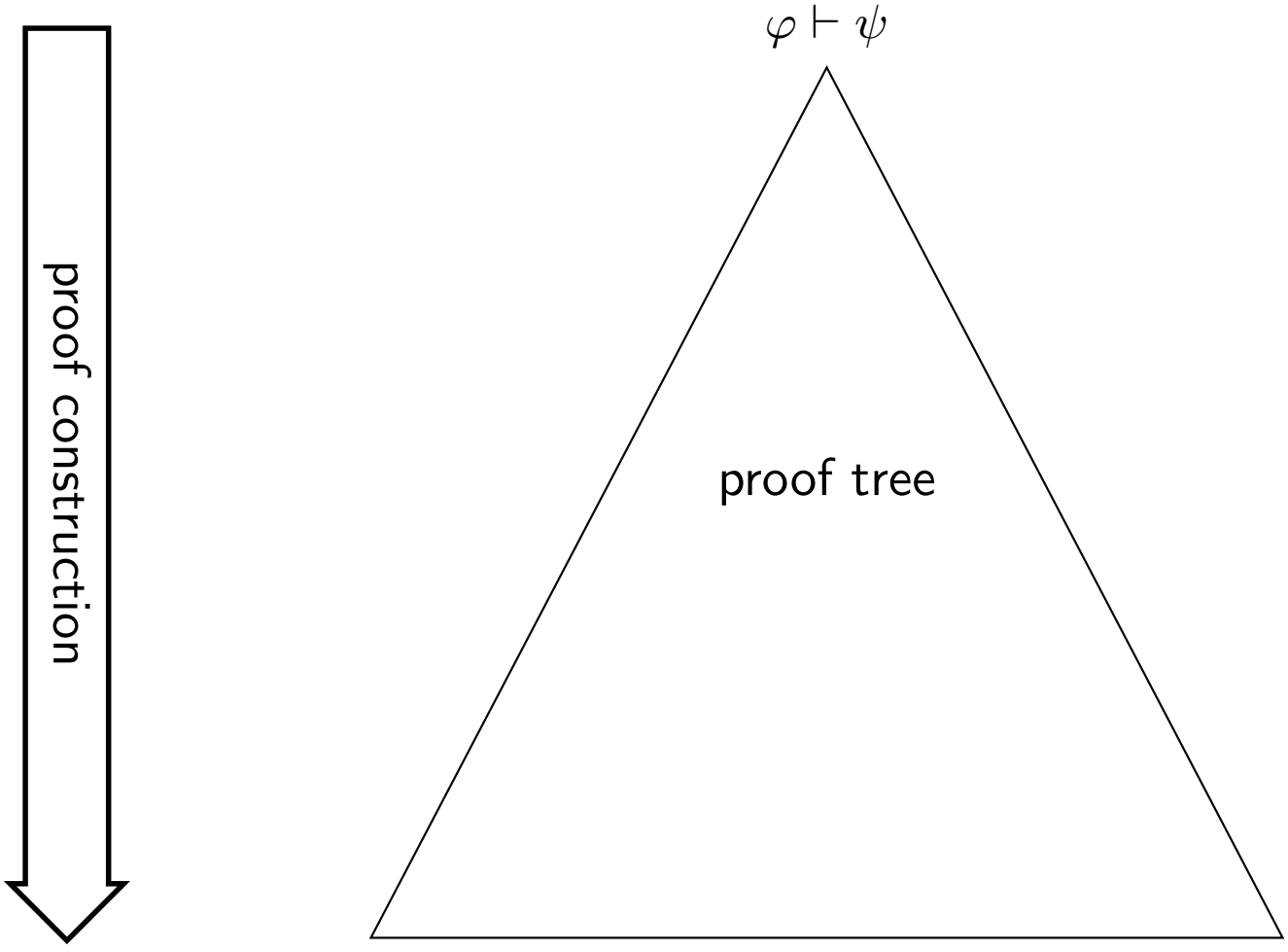
Given GNFO φ and ψ such that $\varphi \models \psi$, we can construct a GNFO interpolant χ of size $f(|\varphi| + |\psi|)$

(where $f(n)$ is a tower of exponentials of height $2^{2^{2^{2^n}}}$).

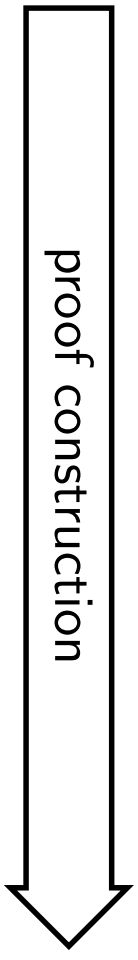
Constructive interpolation

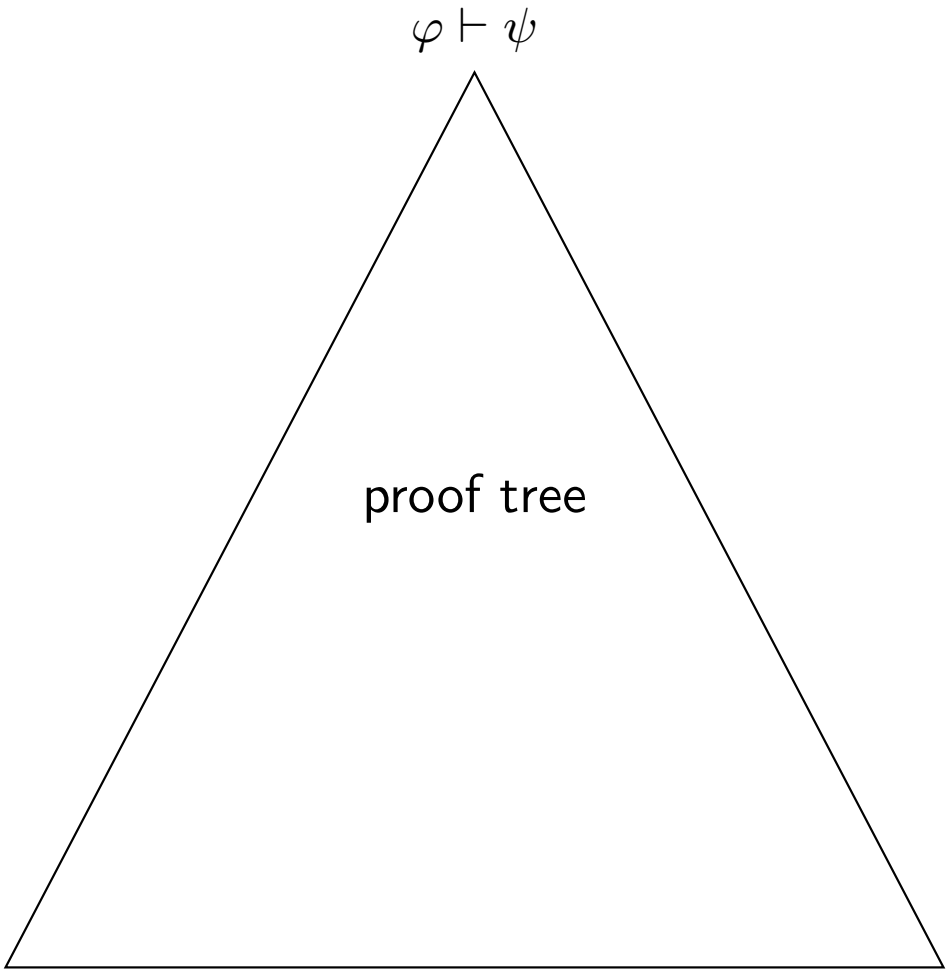
$$\varphi \vdash \psi$$

Constructive interpolation



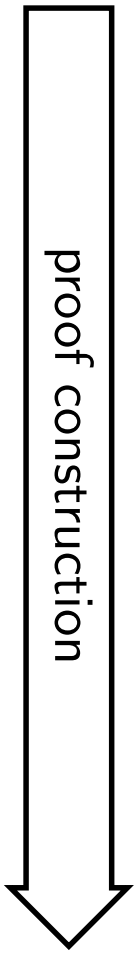
Constructive interpolation





bound proof size in terms of size of φ and ψ

Constructive interpolation



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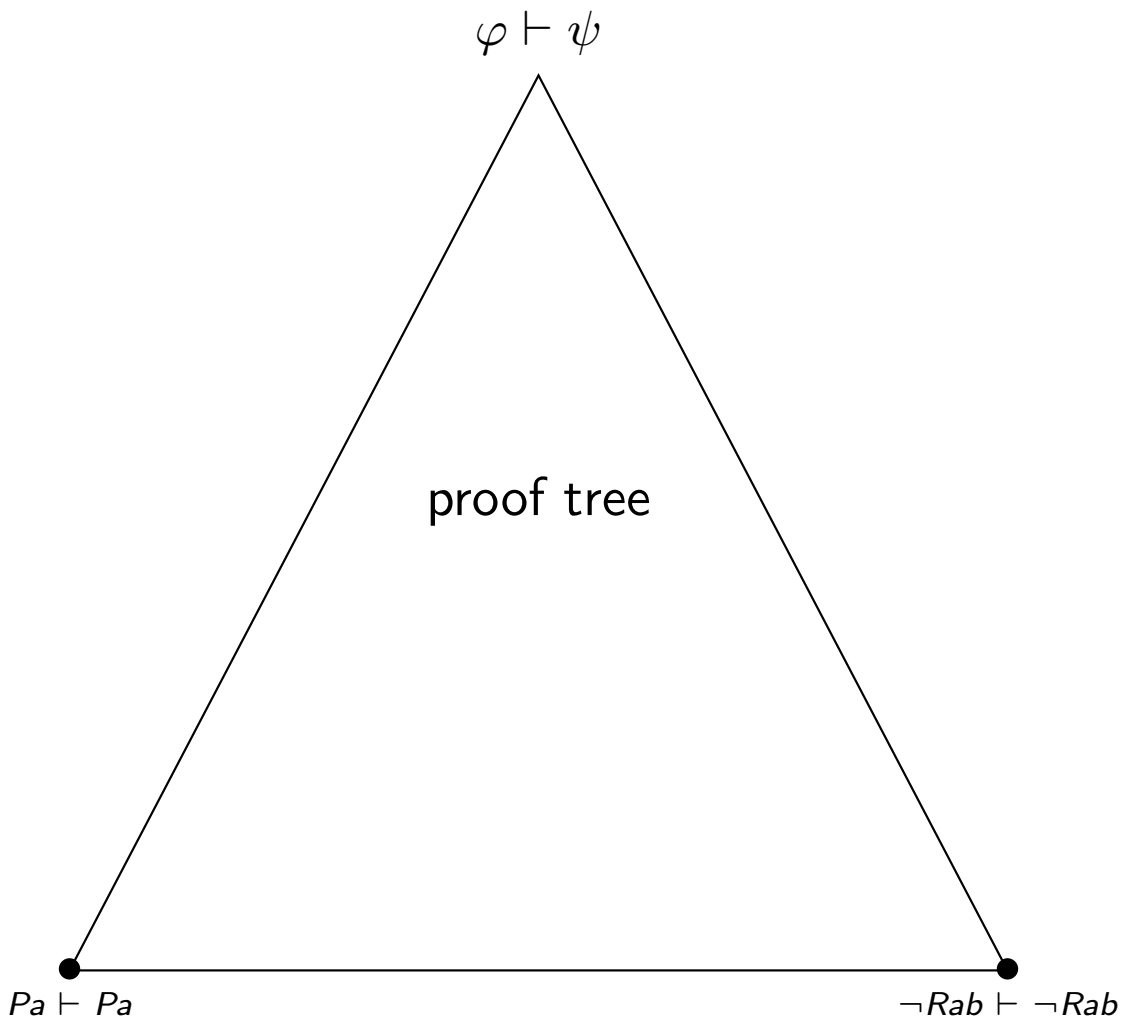

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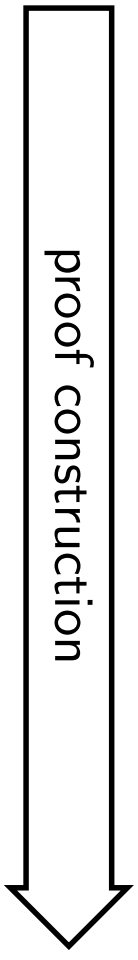

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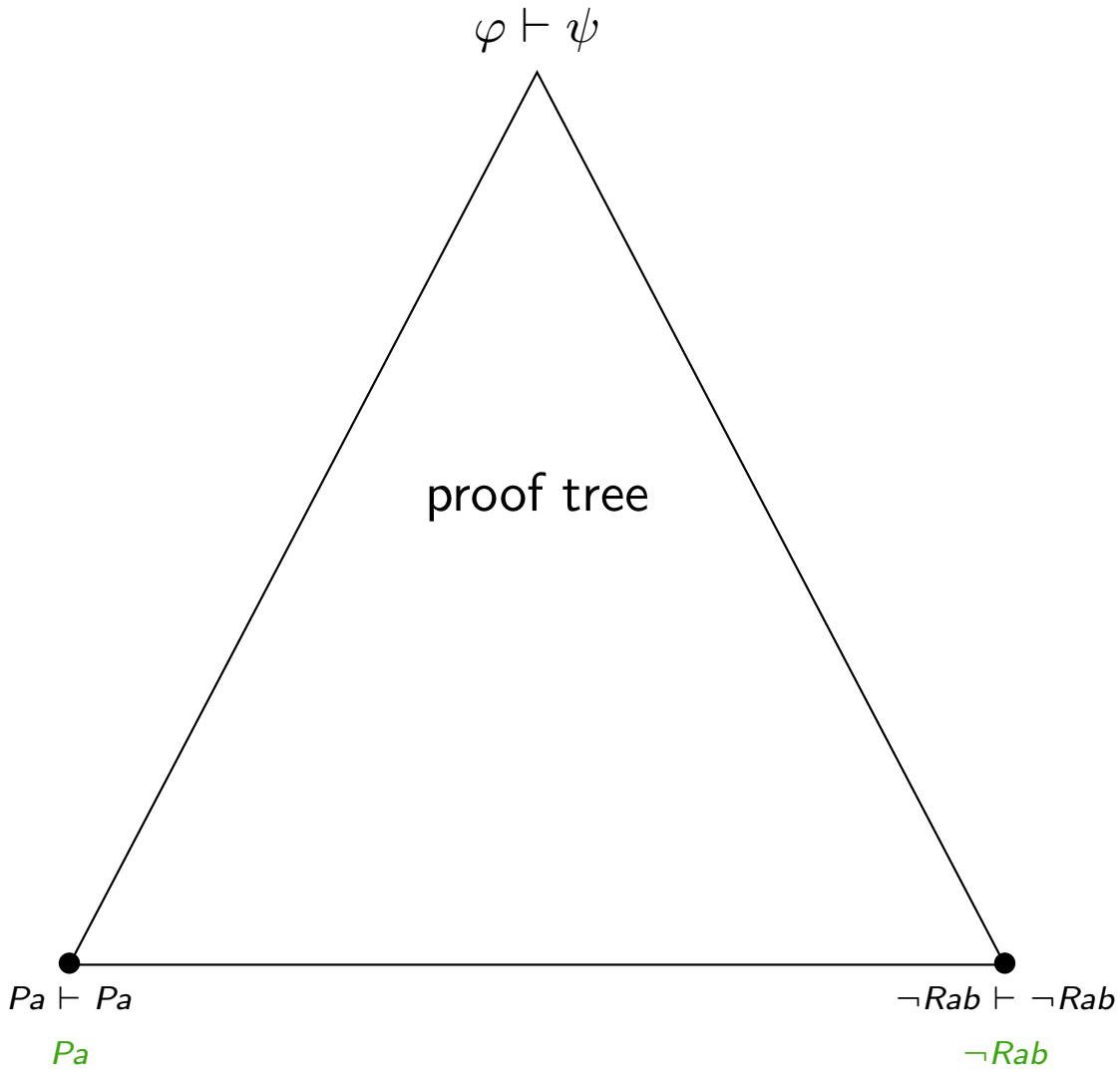

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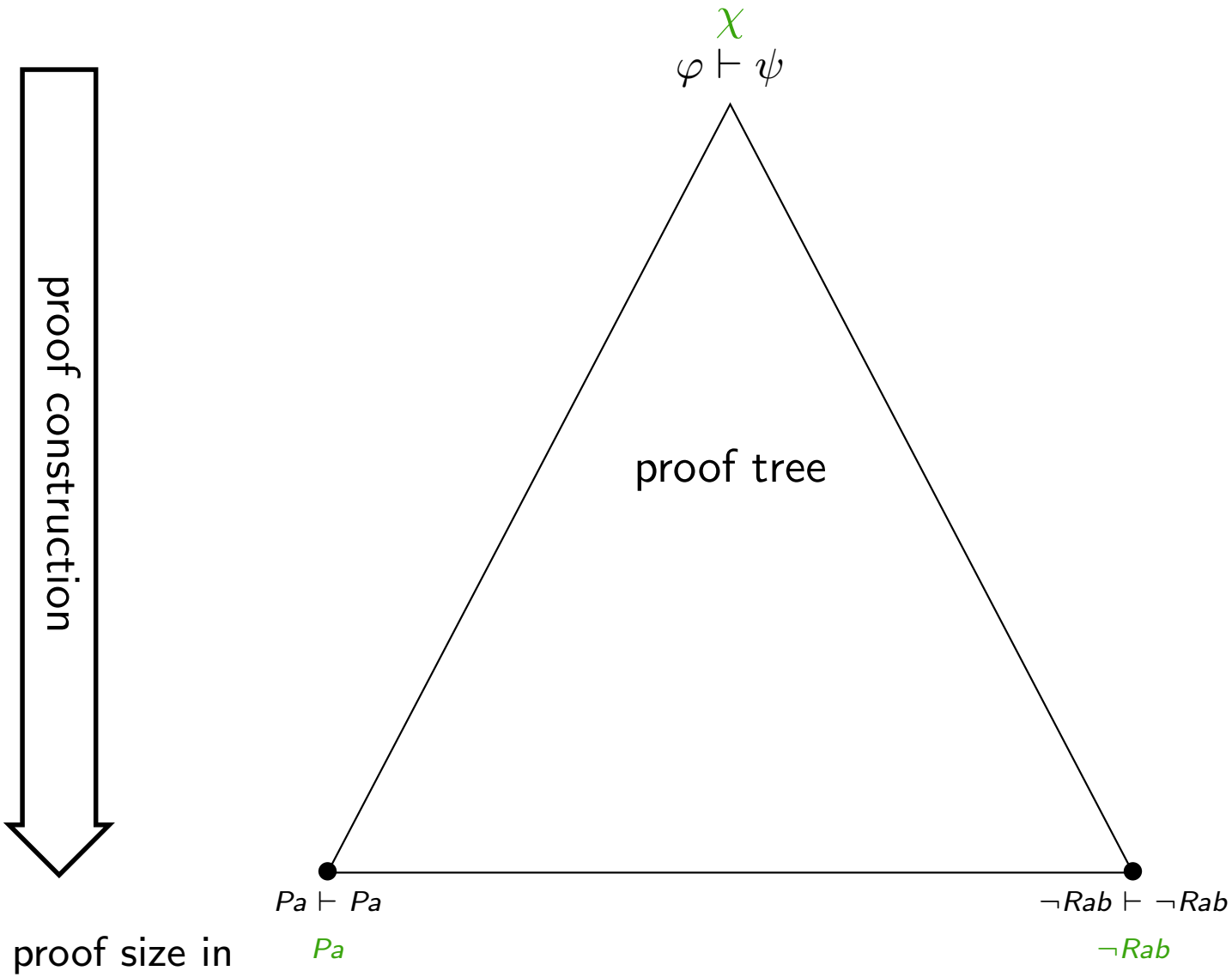
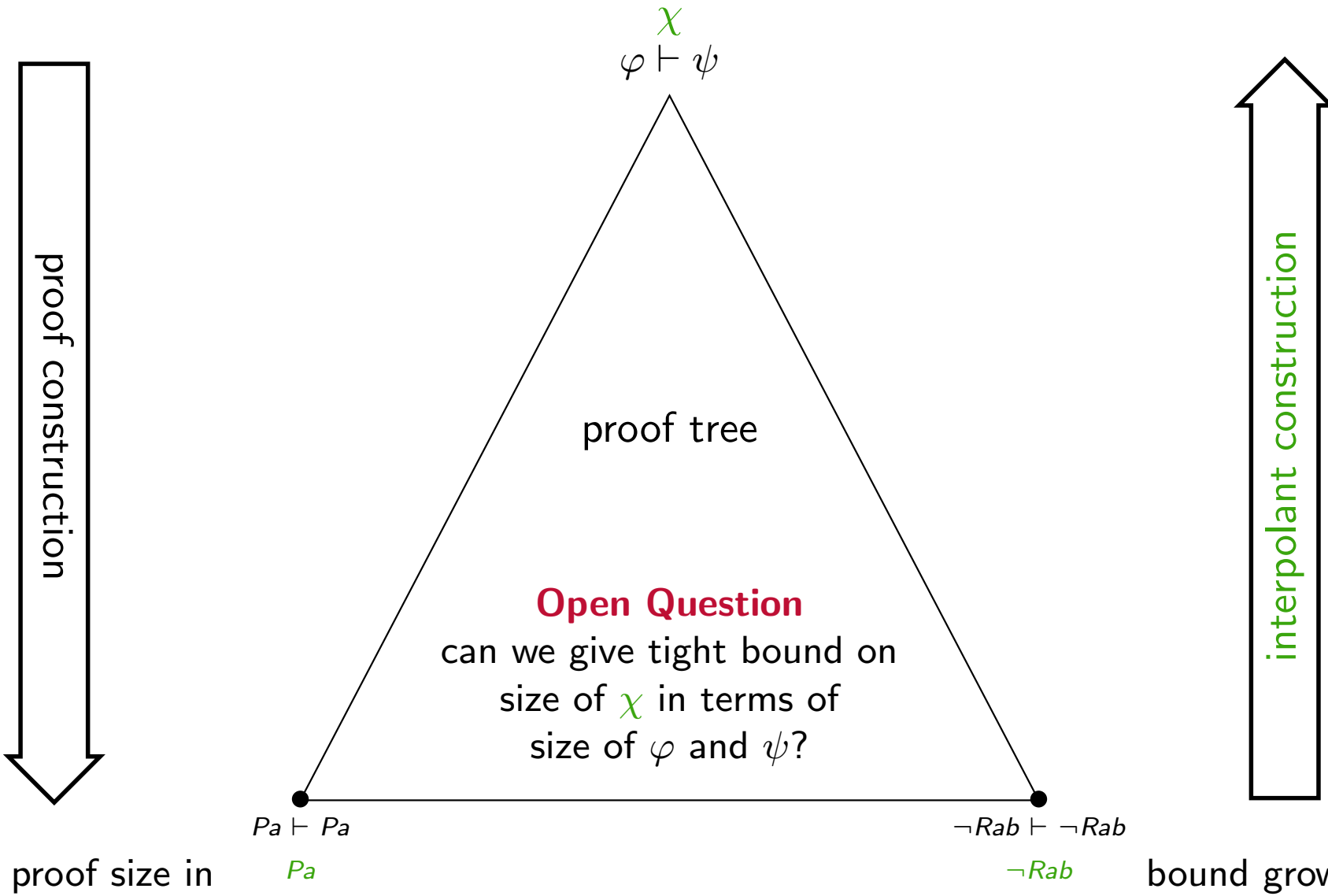


bound proof size in terms of size of φ and ψ

bound growth of interpolant at each stage

Constructive interpolation



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