FUNCTIONAL CHARACTERIZATION OF SOME SEMANTIC EQUALITIES INSIDE λ-CALCULUS

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Abstract - Both (operational or denotational) semantics and type theories for λ-calculus induce in a natural way equivalence relations between terms. The aim of the present paper is to show that in some cases the semantic and functional equivalences coincide.

INTRODUCTION

It is well known that λ- calculus is a formalism to represent partial recursive functions which has played a central role in the development of recursion theory.

From the beginning λ-calculus has been used to study program properties and, moreover, it has also been proposed as a programming language in itself. In this last case the interest in building a semantics for it becomes clear. One of the first approachs was that of Morris [10] which gives an operational semantics concerned with the behaviour of terms inside arbitrary contextes. A deeper approach was found successively with the discovery of lattice - theoretic models [15]. These models, moreover, build a basis for Strachey's theory of programming language semantics which is now very satisfactory and still in development.

The problem of defining a semantic for λ-calculus (or combinatory logic) was also faced, in a completely different way, by Curry [6] , [7]. One of the original motivations of Curry for the study of λ-calculus, in fact, was to give a foundation for logic but he discovered that, in the pure unrestricted system, different kinds of paradoxes were possible. To avoid them he introduced its theory of functionality in which terms of λ-calculus are associated, by means of a formal set of axioms and deduction rules, with functional characters (as Curry call them) or types. The types give an explicit and consistent representation of the range-domain properties of terms when these last are considered as functions from terms to terms. It is then implicit that any interpretation of terms which is consistent with their functional characters is a good interpretation and no paradox can arise. Also in considering λ-calculus as a programming language, it is interesting to study functional characters of terms since they could be used as a tool to prove properties of programs like termination (every terms which possesses a functional character is strongly normalizable) or correctness. No substantial effort, however, has been done until now in this direction because of some limitations of Curry's theories. Functional characters, for example, are not preserved by convertibility and the set of terms to which it can be assigned a functional character is only a subset of normalizable terms. This is inacceptable since, as it is known, some foundamental primitives as recursion operators can be represented only by terms without normal form. Both the previous limitations, however, have been overcomed by the development of new functionality theories [4], [5] , [13] , [14], which extend in a natural way the notion of functional character. Many features of Curry's functionality, however, are preserved as its purely formal aspect and the fact that functional characters are still a consistent basis for the definition of a semantics of λ-calculus. In particular normalization properties still hold and have been extended in the sense that both the set of types that can be assigned only to terms which have normal form and head normal form can be characterized. In this paper we will deal with two extended functionality theories which we will introduce as theories G [5] and T [14].

It is now natural to ask if there are any relations, and which, between the notion of semantics suggested by theories G and T and that ones introduced by other approaches such as models or Morris' operational semantics. As it will be proved in this paper, in both the theories G and T the finite nature of functional characters can be explained with the notion of approximant as introduced by Wadsworth [17] and Hyland [8] . As a consequence of this we will be able to show that the theories G and T. induce the same equality relations between terms as that ones introduced, respectively, by the model P_ω and the extensional equality of Morris [10] . This fact gives a further motivation to consider functional characters as a basis for a semantics of λ-calculus. The importance of this fact is twofold. On one side it proposes the theories G and T as formal supports for proofs inside these semantics, and, on the other hand, allows to extend some known result about P_ω or extensional equalities to theories G and T. It follows, for example, that in both theories all fixed point combinators [2] have the same functional characterizations.

1 - Presentation of systems G and T.

Both systems G and T are functionality theories for λ-calculus. Let's recall that, according to the classical work of Curry [6] , a formula in a functionality theory is a statement τ X where X is a term and τ its functional character or type. So τ X means that τ is a type for X. Functional characters are build from a set of basic elements (which can be different in the various theories) and a composition operator F. If σ , τ are any types F σ τ is the type of a term which defines a function from the terms of type σ to the terms of type τ . We shall use here $[\sigma]\,\tau$ instead of $F\,\sigma\,\tau$.
Types are assigned to terms by means of axioms and deduction rules in a purely formal way. If we limit ourselves in the framework of natural deduction systems we must introduce the concept of basis for a deduction. A basis is a collection of statements of the shape τ X where τ is a type and X a term.
$B \vdash \tau X$ will mean that from the statements of B we can deduce type τ for X.
The fundamental axioms and rules that characterize functionality theories are the following:

(Axiom Ap) If B contains τ X then $B \vdash \tau X$.

(Rule F_e) $$\frac{B \vdash [\sigma]\,\tau\, X \quad B \vdash \sigma\, Y}{B \vdash \tau (XY)}$$

(Rule F_i) $$\frac{B, \sigma\, x \vdash \tau\, X}{B \vdash [\sigma]\,\tau\, \lambda x.X}$$ if x does not occur in B.

Rules F_e and F_i give a natural functional interpretation to the formation rules for term of λ-calculus. New objects and rules can be adjoined to the previous ones leading to different systems of functionality. Also in the theories G [5] and T [14] types are build from a set of basis elements $\{\omega, \varphi_1, \varphi_2, \ldots\}$. The meanings of $\varphi_1, \varphi_2, \ldots$ are different in the theories G and T (here, in particular, we have only two basic types except ω) while ω has, in both theories, the meaning of an universal category (as in [6 p.240]) i.e. each term has type ω. Types are build from basic elements by means of two operations, one of which is the classical composition (i.e. Curry's F). The other is the operations of "sequencing" which builds a new type as a collection of types. The meaning of σ X where σ is a sequence of types is then that X possesses all types of σ (with respect to a given basis). The syntax of types is then the following:

Definition 1 - The set of types (of G and T) is defined as follows:

i) each basic type is a type

ii) if $\sigma_1, \ldots, \sigma_n$ are types then $\sigma_1, \ldots, \sigma_n$ is a type

iii) if σ and τ are types then $[\sigma]\,\tau$ is a type.

In the following $\varphi_1, \varphi_2, \ldots$ are basic types different from ω and $\sigma, \tau, \rho, \mu, \ldots$ are metavariables which range over types.

Type ω is characterized by a particular axiom which defines its property of universal type.

(Axiom A_ω) for all terms X: $\vdash \omega X$.

In both theories G and T all properties of typed terms are proved to be invariant if we introduce the following equivalence relation between types:

$$(E_\omega) \qquad [\sigma]\,\omega = \omega \qquad \text{for all types } \sigma .$$

So we will consider here only the type with the minimum number of occurrences of basic types for each equivalence class induced by E_ω.

One can define the level of a type occurrence τ into a type σ as follows:

(i) if $\tau \equiv \sigma$ the level of τ in σ is 0

(ii) if $\sigma \equiv \sigma_1,\ldots,\sigma_n$ and τ occurs in σ_i the level of τ in σ is the level of τ in σ_i $(1 \leq i \leq n)$

(iii) if $\sigma \equiv [\rho]\mu$ the level of τ in σ is:

- one plus the level of τ in ρ if τ occurs in ρ

- the level of τ in μ if τ occurs in μ.

We will say that a type τ is proper if either τ does not contain ω or ω occurs in τ only at odd levels. A proper basis instead is a basis containing types in which either ω does not occur or the level of all occurrences of ω is even.

We introduce, lastly, some more technical definitions .

The lenght $\|\tau\|$ of a type τ is defined as the number of occurrences in it of basic types.

Let $\mathcal{D}$ be a deduction of $B \vdash \sigma X$ and $R \equiv (\lambda x.Y)Z$ a redex in X. The characteristic set of R in $\mathcal{D}$ is the set $C(R)$ of all types τ different from ω assigned to $\lambda x.Y$ in $\mathcal{D}$, i.e.

$$C(R) \equiv \{\tau \mid \tau \neq \omega \text{ and } \tau \text{ is a type of } \lambda x.Y \text{ in } \mathcal{D}\}.$$

The height h(R) of R is the maximum length of the types of $C(R)$, i.e. $h(R) = \max\{\|\tau\| \mid \tau \in C(R)\}$.

We say that a component Y of X is meaningful in a deduction $\mathcal{D}$ of $B \vdash \sigma X$ iff there is no component Z of X such that Y is a component of Z and $\mathcal{D}$ assigns to Z only type ω.

Let's associate to any deduction $\mathcal{D}$ a pair of non-negative integers $< m(\mathcal{D}), n(\mathcal{D}) >$ (measure of $\mathcal{D}$) defined as:

$m(\mathcal{D})$ = maximum height of meaningful redexes in $\mathcal{D}$

$n(\mathcal{D})$ = number of meaningful redexes with height $m(\mathcal{D})$ in $\mathcal{D}$

We intend $m(\mathcal{D}) = 0$ if there are no meaningful redexes in $\mathcal{D}$ and $n(\mathcal{D}) = 0$ if $m(\mathcal{D}) = 0$.

Integer pairs can be ordered by the usual lexicographic order relation ($\trianglelefteq$) defined as: $<h',k'> \trianglelefteq <h,k>$ iff $h' < h$ or $h' = h$ and $k' \leq k$.

The theory G.

In the theory G there is a numerable set of basic types different from ω (like basic objects of Curry's theory in [6 cap.9]).

The axioms and deductions rules are Ap, A_ω, Fe, Fi with the equivalence relation E ω . $\vdash_G$ will denote a deduction in G.

The principal results which can be proved in G are the following:

Theorem 1 - A term X has a β-normal (an head normal) form iff there are some proper basis B and proper type τ (some basis B and type τ different from ω) such that B $\vdash_G \tau$ X.

Theorem 2 - Any two β-convertible terms have in G the same set of types for any basis.

The following result will be used in the proof of Lemma 7 in section 3.

Lemma 1 [5] - Let D be any deduction of B $\vdash_G \tau$ X whose measure is not $< 0,0 >$. Then there is a term X' and a deduction D' of B $\vdash_G \tau$ X' such that X $\geq$ X' and the measure of D' is less than that one of D.

The theory T

In the theory T the basic types are 0 and 1 which have the following meaning (according to [4]):

– 0X means that X has a normal form
– 1X means that X $Y_1 .....Y_n$ has a normal form for all $n > 0$ and $Y_1 ,...,Y_n$ which, in their turn, have normal forms.

These meaningsof 0 and 1 justify the introduction of the following equivalence relations:

$$(E_0) \qquad [1]\ 0 = 0$$

$$(E_1) \qquad [0]\ 1 = 1 \quad .$$

In T then we will consider only types with the minimum number of occurrences of 0, 1 and ω for each equivalence class induced by E_0 , E_1 and E_ω . Therefore the lenght of a type must be computed modulo E_0 , E_1 and E_ω . $\vdash_T$ denotes a deduction in T.

The main results provable for theory T are the following:

Theorem 3 - A term X has a β- η- normal (and head normal) form iff there are some proper basis B and proper type τ (some basis B and type τ different from ω) such that $B \vdash_T \tau X$.

Theorem 4 - Any two β- η- convertible terms have in T the same set of types for any basis.

In analogy to Lemma 1 we have then:

Lemma 2 [14] - Let D be any deduction of $B \vdash_T \tau X$ whose measure is not $<0,0>$. Then there is a term X' and a deduction D' of $B \vdash_T \tau X'$ such that $X \geq X'$ and the measure of D' is less than that one of D.

2. Some equivalence relations on terms.

The theories G and T induce in a natural way the following equivalence relations between terms:

Definition 2. Let M and N be terms.

$M \sim_G N$ iff for any basis B and type τ :

$$B \vdash_G \tau M \quad \text{iff} \quad B \vdash_G \tau N$$

$M \sim_T N$ iff for any basis B and type τ :

$$B \vdash_T \tau M \quad \text{iff} \quad B \vdash_T \tau N .$$

In what follows, $M \sim_X N$ for $X = G$ or T will abbreviate $M \sim_G N$ or $M \sim_T N$.

Then $\sim_G$ and $\sim_T$ split the set of terms into equivalence classes such that all terms in the same class have the same functional characterization. It is obvious from Theorems 2 and 4 that both $\sim_G$ and $\sim_T$ extend the relation of β -convertibility. As said in the introduction, we are interested to study the relations between $\sim_G$, $\sim_T$ and other semantic equivalence relations on terms.

We give here a short review of some classical notions and results that will be used in section 3. First we introduce the definition of approximant (following Wadsworth [17] and Hyland [8], [9]) by adjoining a new constant Ω to the set of terms. Ω has the following reduction properties: for all terms M and variables x, ΩM and $\lambda x.\Omega$ are said Ω-redexes and both reduce to Ω (Ω -reductions). A term M is in β- Ω-normal form iff it contains no β-redexes and no Ω-redexes;

it is in β- Ω-η-normal form iff it is in β - Ω -normal form and it contains no η-redexes. A term A is said to be a direct approximant of a term M iff A and M are identical (modulo Ω-reductions) except at components which are occurrences of Ω in A and moreover A is in β -normal form.

For a given term M, we define its sets of approximants $\mathcal{A}$ (M) and $\mathcal{A}_e$ (M) as in [8] :

Definition 3 -

$\mathcal{A}(M) = \{A \mid \exists M', A'$ such that $M' =_\beta M$, $A' =_\Omega A$,

A' is a direct approximant of M' and A is a β- Ω - normal form $\}$

$\mathcal{A}_e(M) = \{A \mid \exists M', A'$ such that $M' =_{\beta\eta} M$, $A' =_{\Omega\eta} A$,

A' is a direct approximant of M' and A is a β - Ω - η -normal form $\}$.

As usual, a context C [ ] is a term in which one subterm is missing, C [M] denotes the result of filling the missing subterm with M.

$M =_{P\omega} N$ will mean that M , N have the same meaning in the model P_ω [12] [16] .
With $M =_e N$ we denote the extensional equivalence of [10] :

Definition 4 - If M, N are terms, $M =_e N$ iff for any context C [ ]:

either C [M] and C [N] reduce to the same β - η -normal form

or C [M] and C [N] do not possess any β- η - normal form.

As usual [17] we say that two head normal forms are similar iff they have the same head variable (after α-conversion, if necessary, so that bound variables agree) and the difference between the number of initial bound variables and the number of main arguments is the same for both. Two similar head normal forms are strongly similar iff they have the same number of main arguments (and therefore also the same number of initial bound variables).

Hyland [8] proofs that the equalities $=_{P\omega}$ and $=_e$ may be characterized by means of the sets of approximants:

Property 1 - $M =_{P\omega} N$ iff $\mathcal{A}(M) = \mathcal{A}(N)$.

Property 2 - $M =_e N$ iff $\mathcal{A}_e(M) = \mathcal{A}_e(N)$.

It follows immediately that M $=_{P\omega}$ N implies M $=_e$ N (but not viceversa).

Lastly we recall Böhm's Theorem and its extension to head normal forms.

Böhm's Theorem [3]. If M and N are distinct β-η-normal forms, then there exists a context C [ ] such that C [M] $=_\beta$ I and C [N] $=_\beta$ K.

Extension of Böhm's Theorem [9]. If M and N are non similar head normal forms then there exists a context C [ ] such that C [M] $=_\beta$ I and C [N] $=_\beta$ K.

3. Type theories vs models.

In this section we will prove that the equalities induced by theories G and T coincide respectively with the equalities $=_{P\omega}$ and $=_e$.

Let's prove, firstly, that $\sim_G$ and $\sim_T$ are invariant under any trasformation that can be defined by means of contexts.

Lemma 3 - If M, N are terms and M $\sim_X$ N then for any C [ ]; C [M] $\sim_X$ C [N] where X = G or T

Proof. Let B $\vdash_X$ τ C [M] and D any deduction of it. To obtain B $\vdash_X$ τ C [N] it is sufficient to replace in D each deduction σ M by the corresponding deduction σ N. □

The following two lemmas gives a first characterization of the equalities $\sim_G$ and $\sim_T$ with respect to the property of having head normal form or normal form.

Lemma 4 - If M $\sim_X$ N then either M and N are both unsolvable or they have similar head normal forms for X = G or T.

Proof - By Theorems 1 and 3 it is obvious that either M and N are both unsolvable or they have both head normal forms. Let ad absurdum M and N have non similar head normal forms. Then by the extension of Böhm's Theorem there is a context C [ ] such that C [M] $=_\beta$ I and C [N] $=_\beta$ K. It is clear that I $\not\sim_X$ K and this fact is in contradiction with Lemma 3 since types are invariant by β-conversion. □

Lemma 5 - If M, N have β- η-normal forms and $M \sim_X N$ for $X = G$ or T then M and N are β - η-convertible.

Proof. The proof is the same as that of Lemma 4, by replacing normal forms for similar head normal forms and Böhm's Theorem for its extension. □

A stronger result may be proved in the case of head normal forms with the same set of types in G.

Lemma 6 - If M and N are head normal forms, i.e.

$M \equiv \lambda x_1 \dots x_n . \zeta\ M_1 \dots M_m$, $N \equiv \lambda x_1 \dots x_{n'} . \zeta\ N_1 \dots N_{m'}$

and $M \sim_G N$ then M and N are strongly similar and $M_i \sim_G N_i$ $(1 \leq i \leq m)$.

Proof - Let's first observe that, for all terms M and N and variables x, $M \sim_G N$ iff $\lambda x.M \sim_G \lambda x.N$ (the proof follows trivially from Lemma 3).

By Lemma 4 M and N are similar. Now suppose, for example, $n' > n$. From above $\zeta\ M_1 \dots M_m \sim_G \lambda x_{n+1} \dots x_{n'} . \zeta\ N_1 \dots N_{m'}$. But this is impossible since we have $\underbrace{[\omega]\dots[\omega]}_{n} \varphi\zeta \vdash_G \varphi\ \zeta\ M_1 \dots M_m$ while, obviously, $\underbrace{[\omega]\dots[\omega]}_{n} \varphi\zeta \not\vdash_G \varphi\ \lambda x_{n+1} \dots x_{n'} . \zeta\ N_1 \dots N_m$, when φ is any basic type different from ω.

To prove $N_i \sim_G M_i$ $(1 \leq i \leq m)$ let's suppose that there exist B, τ such that $B \vdash_G \tau\ M_i$ and $B \not\vdash_G \tau\ N_i$. Let φ be any basic type which does not occur in B. Then

$B, \underbrace{[\omega]\dots[\omega]}_{i-1},[\tau]\underbrace{[\omega]\dots[\omega]}_{m-i}\varphi\zeta \vdash_G \varphi\ \zeta\ M_1 \dots M_m$ but

$B, \underbrace{[\omega]\dots[\omega]}_{i-1},[\tau]\underbrace{[\omega]\dots[\omega]}_{m-i}\varphi\zeta \not\vdash_G \varphi\ \zeta\ N_1 \dots N_m$. In fact, since φ does not occur in B, B cannot contain any statement $[\bar{\sigma}_1]\dots[\bar{\sigma}_m]\ \varphi\ \zeta$. Then we have $\zeta\ M_1 \dots M_m \not\sim_G \zeta\ N_1 \dots N_m$ against the hypothesis. □

Lastly let's consider the relations between the types of a term and those of its approximants.

Lemma 7 - For any term M, basis B and type τ, $B \vdash_G \tau\ M$ iff $B \vdash_G \tau\ A$ for some $A \in \mathcal{A}(M)$.

Proof.

If part. By induction on the measure of D, where D is any deduction of $B \vdash_G \tau\ M$.

First step. If the measure of D is $<0,0>$ then any redex of M is non meaningful, i.e. it occurs in a component to which only type ω is assigned in D . Then A can be obtained from M by replacing Ω for all non meaningful components. It is easy to verify that A is a β - Ω-normal form.

Inductive step. Immediate from Lemma 1.

Only if part. If $B \vdash_G \tau\ A$ and $A \in \mathcal{A}(M)$ then there exists M' such that $M' =_\beta M$ and A is a direct approximant of M', i.e. M' can be obtained from A by replacing each occurrence of Ω by suitable terms Z. Then a deduction of $B \vdash_G \tau\ M'$ can be obtained from any deduction of $B \vdash_G \tau\ A$ by replacing each $\vdash_G \omega\ \Omega$ by $\vdash_G \omega\ Z$. □

Lemma 8 - For any term M, basis B and type τ , $B \vdash_T \tau\ M$ iff $B \vdash_T \tau\ A$ for some $A \in \mathcal{A}_e(M)$.

Proof.

The proof succeds as that one of Lemma 7, if we replace $\vdash_G$ by $\vdash_T$, Lemma 1 by Lemma 2, $\mathcal{A}(M)$ by $\mathcal{A}_e(M)$ and $=_\beta$ by $=_{\beta\eta}$ □

Now we are able to prove the main results of the present paper.

Theorem 5. For any two terms M and N, $M \sim_G N$ iff $M =_{P\omega} N$.

Proof. By Property 1 it is sufficient to prove $M \sim_G$ iff $\mathcal{A}(M) = \mathcal{A}(N)$.

If part. If $B \vdash_G \tau\ M$ then by Lemma 7 there is $A \in \mathcal{A}(M)$ such that $B \vdash_G \tau\ A$. From $B \vdash_G \tau\ A$ and $A \in \mathcal{A}(N)$ it follows $B \vdash_G \tau\ N$ again by Lemma 7.

Only if part . Let $A \in \mathcal{A}(M)$. We prove by structural induction on A that $A \in \mathcal{A}(N)$. If $A \equiv \Omega$ or A is a single variable this is obvious. Else M must possess an head normal form strongly similar to A and by Lemma 6 also to that one of N. I.e. we have $A \equiv \lambda x_1 \dots x_n .\ \zeta\ A_1 \dots A_m$, $M = \lambda x_1 \dots x_n .\ \zeta\ M_1 \dots M_m$ and $N = \lambda x_1 \dots x_n .\ \zeta\ N_1 \dots N_m$. Then we have $A_i \in \mathcal{A}(M_i)$ $(1 \le i \le m)$ and $M_i \sim_G N_i$ by Lemma 6. By inductive hypothesis, then, $A_i \in \mathcal{A}(N_i)$ $(1 \le i \le m)$ and so $A \in \mathcal{A}(N)$. □

Theorem 6. For any two terms M and N, $M \sim_T N$ iff $M =_e N$.

Proof.

If part. The proof succeds as that one of the only if part of Theorem 5, if we recall that $M =_e N$ implies $A_e(M) = A_e(N)$ (Property 2), by replacing $\vdash_G$ by $\vdash_T$, Lemma 7 by Lemma 8, A by A_e.

Only if part. Let C [ ] by any context. There are two possible cases:

- C [M] has a proper type for a proper basis. By Lemma 3 also C [N] has this type for the same basis. This means that C [M] and C [N] possesthe same β - η -normal form by Theorem 3 and Lemma 5.
- C [M] does not have any proper type for any proper basis. By Lemma 3 this is true also for C [N] and so C [M] and C [N] do not possess β - η-normal form by Theorem 3. □

As immediate consequence of Theorems 5 and 6 we obtain that $\sim_T$ is an extension of $\sim_G$, i.e. for any two terms M and N : $M \sim_G N$ implies $M \sim_T N$.

Conclusion

The interest of the given functional characterization lies also in the fact that equality in P_ω coincides with equality in T_ω [12] [1] and in Levy's syntactic model. Further researches will be done to define constructively for each term M of λ -calculus an unique principal type scheme in the theory G, i.e. (according to [7 p. 296]) a type τ build from basic types and type variables such that each type of M is a specialization of τ . Then the semantic properties of terms in the above models can be proved in a system which is totally formalizable.

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