

Contextual Locking for Dynamic Pushdown Networks

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Motivation

- Decidable model for recursive concurrent programs with locks
- Concurrency
 - 2 processes, n processes, dynamic process creation
- Locking disciplines
 - Syntactically nested (`synchronized (l) { ... }`)
 - Bounded lock chains (`[l1 l2]1 [l3 l2] ... [lc lc-1]c`)
 - Contextual (`proc p { [l1 l2]1 [l2 ...]`)
 - Procedure releases all locks it acquires
 - Procedure releases no „foreign“ locks
- Join: Process waits until children have terminated
 - Subsumes parallel procedure calls

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- Single processes: Control state reachability
 - Can process reach control state p ?
 - Can express regular properties of stack
- Relation between processes:
 - n -state reachability:
 - Configuration containing distinct processes at p_1, \dots, p_n reachable?
 - all-state reachability
 - Configuration where all processes are at p reachable?
 - Can express regular reachability. E.g. $\exists p$ configuration where all processes are at p is configuration with a yield word some regular language reachable

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Regular reachability

- $\text{yield}(c)$ - Control-states in post-order traversal

$$\text{yield}((pw, X)\langle t_1 \dots t_n \rangle) = \text{yield}(t_1) \dots \text{yield}(t_n)p$$

- Let $A = (Q, \delta, q_0, F)$

- New control states: $\langle q, p, q' \rangle$

- Idea: Process evaluates to tree t with $q \xrightarrow{\text{yield}(t)}_{\delta} q'$

- Start with initial guess: $p_0 \xrightarrow{\tau} \langle q_0, p_0, q' \rangle$ for $q' \in F$

- Non-spawn rules preserve guess

- $pw \xrightarrow{\{\tau, \text{acq}, \text{rel}, \text{join}\}} p'w'$ becomes $\langle q, p, q' \rangle w \xrightarrow{\{\tau, \text{acq}, \text{rel}, \text{join}\}} \langle q, p', q' \rangle w'$
for all $q, q' \in Q$

- Spawn-rules distribute guess

- $p\gamma \xrightarrow{p_1\gamma_1} p_2\gamma_2$ becomes $\langle q, p, q_2 \rangle \gamma \xrightarrow{\langle q, p_1, q_1 \rangle \gamma_1} \langle q_1, p_2, q_2 \rangle \gamma_2$
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- Can we reach configuration where all control states have form

$$(\langle q, p, q' \rangle w, X) \text{ such that } (q, p, q') \in \delta$$

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Zoo of Results

	n-state			all-state/regular	
	2-proc.	n-proc.	dynamic	dynamic	join
nested	[KIG05]		[LMO08]	[LMOW09]	[GLMO ⁺ 11]
bounded lockchains	[Kah09]				
contextual	[CMV12]				
arbitrary	[KIG05]				

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nested	=NP				≥PSPACE
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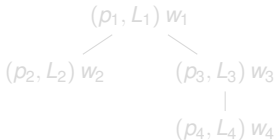
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Dynamic Pushdown Networks

- DPN = Pushdown automata + Dynamic thread creation
- Configurations: Trees
 - Nodes: Process configuration (control, locks, stack)
 - Edges: Genealogy of process creation



- Rules applied to nodes

(base) $p\gamma \xrightarrow{a} p'\gamma'$ where $a \in \{\tau, \text{acq } l, \text{rel } l, \text{join}\}$

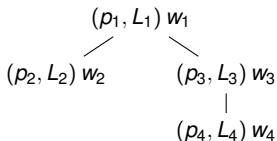
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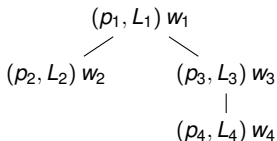
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n-state reachability for 3 processes with contextual locking is PSPACE-hard, even for a constant number of locks.

- Idea: Emulate linearly bounded Turing machine
 - Regard run $c_0 \overline{c_1} c_1 \overline{c_2} c_2 \dots \overline{c_n}$
 - Important: Configuration has constant length n
 - Process 1: Check that $c_i \overline{c_{i+1}}$ is valid step
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- Problem: How to synchronize the processes

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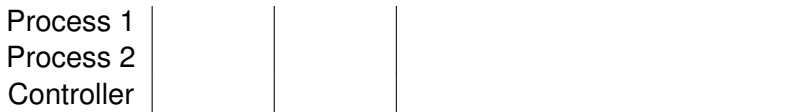
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Handshake-Synchronization via locks

Let $\text{use}(l) = \text{acq}(l)\text{rel}(l)$ and $\text{free}(l) = \text{rel}(l)\text{acq}(l)$



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Process 1	$\text{use}(l_1)$		
Process 2	$\text{use}(l_1)$		
Controller	$\text{free}(l_1)$		

- Controller frees l_1 to signal begin of handshake
- Another lock avoids multiple iterations
- Reverse pattern avoids not entering the synchronization
 - Problem: Only possible if process in initial context
 - Solution: Only check after constantly many handshakes
 - Processes 1 and 2 return to initial context after $2n$ handshakes

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Process 1	$\text{use}(l_1)$	$\text{use}(l_2)$	$\text{free}(l_3)\text{free}(l_4)$
Process 2	$\text{use}(l_1)$	$\text{use}(l_2)$	$\text{free}(l_5)\text{free}(l_6)$
Controller	$\text{free}(l_1)$	$\text{free}(l_2)$	$\text{use}(l_3)\text{use}(l_4) \text{ use}(l_5)\text{use}(l_6)$

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Process 1	$\text{use}(l_1)$	$\text{use}(l_2)$	$\text{free}(l_3)\text{free}(l_4)$
Process 2	$\text{use}(l_1)$	$\text{use}(l_2)$	$\text{free}(l_5)\text{free}(l_6)$
Controller	$\text{free}(l_1)$	$\text{free}(l_2)$	$\text{use}(l_3)\text{use}(l_4) \text{ use}(l_5)\text{use}(l_6)$

- Controller frees l_1 to signal begin of handshake
- Another lock avoids multiple iterations
- Reverse pattern avoids not entering the synchronization
 - Problem: Only possible if process in initial context
 - Solution: Only check after constantly many handshakes
 - Processes 1 and 2 return to initial context after $2n$ handshakes

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Upper Bound — n-state reachability

n-state reachability for DPN with contextual locking is in PSPACE.

- Sufficient to consider executions with polynomially bounded stack
 - Inline non-returning calls
 - Contract too deeply nested calls
 - If stack is deeper than $|V|^2|E|$, there are two nested calls that share the same context and in the same situation
 - Replace outer by inner one
- Extension to dynamic thread creation
 - Larger stack bound (must not drop important spawns)
 - Drop unimportant spawns
 - Contract too long chains of spawns (Similar to contracting calls)

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Upper Bound — all-state reachability

All-state reachability for DPN with contextual locking and join is decidable.

- Also bound stack
- Let \preceq be homeomorphic embedding on configurations
 - \preceq is a well-quasic ordering
 - All-state reachability is downward closed: $d \preceq c \wedge A_p(c) \implies A_p(d)$
 - $A_p(c)$ — All processes in c are at p
 - System (w/o join) is downward compatible:
 $d \preceq c \wedge c \rightarrow^* c' \implies \exists d' \preceq c'. d \rightarrow^* d'$
- Well structured transition system
 - Decide by saturating set of minimal reachable states

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Upper Bound — join

- Problem: Joins violate downward compatibility!
 - Children may have terminated, while grand-children still running
- Solution
 - Configurations are multisets of trees
 - On spawn, guess if process will be joined later
 - joined: Add as new child
 - not joined: Add as new root
 - Now: On join, all successors have terminated

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Conclusion

- Regular reachability for DPN + contextual locking + join is decidable
- PSPACE-hard even for 3 processes and constant number of locks
- Important sub-classes are PSPACE-complete

References



Rohit Chadha, P. Madhusudan, and Mahesh Viswanathan.

Reachability under contextual locking.

In *TACAS*, volume 7214 of *LNCS*, pages 437–450. Springer, 2012.



Thomas Martin Gawlitza, Peter Lammich, Markus Müller-Olm, Helmut Seidl, and Alexander Wenner.

Join-lock-sensitive forward reachability analysis for concurrent programs with dynamic process creation.

In *VMCAI*, volume 6538 of *LNCS*, pages 199–213. Springer, 2011.



Vineet Kahlon.

Boundedness vs. unboundedness of lock chains: Characterizing decidability of pairwise cfl-reachability for threads communicating via locks.

In *LICS*, pages 27–36. IEEE Computer Society, 2009.



Vineet Kahlon, Franjo Ivancic, and Aarti Gupta.

Reasoning about threads communicating via locks.

In *CAV*, volume 3576 of *LNCS*, pages 505–518. Springer, 2005.



Peter Lammich and Markus Müller-Olm.

Conflict analysis of programs with procedures, dynamic thread creation, and monitors.

In *SAS*, volume 5079 of *LNCS*, pages 205–220. Springer, 2008.



Peter Lammich, Markus Müller-Olm, Helmut Seidl, and Alexander Wenner.

Contextual locking for dynamic pushdown networks.

In *SAS*, volume 7935 of *LNCS*. Springer, 2013.



Peter Lammich, Markus Müller-Olm, and Alexander Wenner.

Predecessor sets of dynamic pushdown networks with tree-regular constraints.

In *CAV*, volume 5643 of *LNCS*, pages 525–539. Springer, 2009.