Strategic Games and Truly Playable Effectivity Functions

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Outline

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Concrete and Abstract Models of Interaction

- "Concrete" game models: actions and transitions are represented explicitly $\leadsto$ normal form games
- Abstract models: "distill" an abstract representation of individual and coalitional powers $\leadsto$ effectivity functions

Concrete Models: Strategic Games

Definition (Strategic game)

A strategic game G is a tuple $(N, \{\Sigma_i | i \in N\}, o, W)$ that consists of a nonempty finite set of players N, a nonempty set of strategies Σ_i for each player $i \in N$, a nonempty set of outcomes W, and an outcome function $o : \prod_{i \in N} \Sigma_i \to W$ which associates an outcome with every strategy profile.

Example: Battle of Sexes

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Abstract Models: Coalitional Effectivity Models

How can we "distill" powers of agents and coalitions in a game?

Definition (Effectivity function)

An effectivity function is a function

$$E : 2^{\mathbb{A}\mathrm{gt}} \to 2^{2^W}$$

that associates a family of sets of states with each set of players.

Intuitively, elements of $E(C)$ are choices available to coalition C: if $X \in E(C)$ then by choosing X the coalition C can force the outcome of the game to be in X.

Example: Effectivity in Battle of Sexes

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E(\{2\}) &= \{\{w_1,w_2\},\{w_2,w_3\},\{w_1,w_2,w_3\}\}\\
E(\emptyset) &= \{\{w_1,w_2,w_3\}\}
\end{aligned}
$$

Outline

Correspondence Between Concrete and Abstract Models

Which effectivity patterns correspond to models of “ordinary” games (i.e., our concrete models)?

Playable Effectivity Functions

Definition (Playability (Pauly 2001))

An effectivity function E is playable iff the following conditions hold:

Outcome monotonicity: $X \in E(C)$ and $X \subseteq Y$ implies $Y \in E(C)$;

Liveness: $\emptyset \notin E(C)$;

Safety: $E(C) \neq \emptyset$;

$\mathbb{A}\mathrm{gt}$-maximality: $\overline{X} \notin E(\emptyset)$ implies $X \in E(\mathbb{A}\mathrm{gt})$;

Superadditivity: if $C \cap D = \emptyset$, $X \in E(C)$ and $Y \in E(D)$, then $X \cap Y \in E(C \cup D)$;

Theorem (Pauly 2001)

A coalitional effectivity function E corresponds to a strategic game if and only if E is playable.

Correspondence Between Concrete and Abstract Models

How to read the result?

- We characterize the limitations of concrete models
- Implementability: we characterize which abstract patterns of effectivity can be implemented by concrete models
- We characterize classes of models for which the semantics of strategic logics if fully equivalent

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Unfortunately, the result is wrong!

Counterexample to Representation Theorem

We start with the following observation:

Theorem

For every effectivity function E of a strategic game, $E(\emptyset)$ is the principal filter generated by $Z = \{w \in W \mid w = o(s_{\mathbb{A}\mathrm{gt}})$ for some strategy profile $s_{\mathbb{A}\mathrm{gt}}\}$.

The following function is playable but $E(\emptyset)$ is not a principal filter:

$$
\begin{aligned}
\mathbb{A}\mathrm{gt} &= \{a\} \\
W &= \mathbb{N} \\
E(\mathbb{A}\mathrm{gt}) &= \{X \mid X \text{ is infinite}\} \\
E(\emptyset) &= \{X \mid \overline{X} \text{ is finite}\}
\end{aligned}
$$

Correct Correspondence

Definition (True playability)

An effectivity function E is truly playable iff the following conditions hold:

Outcome monotonicity: $X \in E(C)$ and $X \subseteq Y$ implies $Y \in E(C)$;

Liveness: $\emptyset \notin E(C)$;

Safety: $E(C) \neq \emptyset$;

$\mathbb{A}\mathrm{gt}$-maximality: $\overline{X} \notin E(\emptyset)$ implies $X \in E(\mathbb{A}\mathrm{gt})$;

Superadditivity: if $C \cap D = \emptyset$, $X \in E(C)$ and $Y \in E(D)$, then $X \cap Y \in E(C \cup D)$;

Determinacy: if $X \in E(N)$ then $\{w\} \in E(N)$ for some $w \in X$.

Correct Correspondence

Theorem (Goranko, Jamroga and Turrini 2011)

A coalitional effectivity function E corresponds to a strategic game if and only if E is **truly** *playable.*

How Big Was the Damage?

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In finite domains playability implies true playability.

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- $\rightsquigarrow$ No disastrous consequences for axiomatizations as long as the logic has finite model property (basic ATL does)
- $\rightsquigarrow$ On the other hand: SL doesn't; open problem for ATL with imperfect information

How Big Was the Damage?

Theorem

In *finite domains* *playability implies true playability.*

In consequence, Pauly's characterization is **correct for effectivity functions over finite sets of outcomes**.

- $\rightsquigarrow$ No disastrous consequences for axiomatizations as long as the logic has finite model property (basic ATL does)
- $\rightsquigarrow$ On the other hand: SL doesn't; open problem for ATL with imperfect information

Anyway, **who cares about infinite domains?**

What else is in the paper?

- Alternative characterizations of truly playable functions
- Characterization and examples of non-truly playable effectivity functions
- Translation of playable to truly playable effectivity functions, preserving the power of most (but not all) coalitions
- Logical (and axiomatic) characterization of true playability

Valentin Goranko, Wojciech Jamroga and Paolo Turrini (2013), Strategic Games and Truly Playable Effectivity Functions. *Journal of Autonomous Agents and Multi-Agent Systems*, 26(2), pp. 288-314, Springer.

Thank you for your attention!