Equivalence of Deterministic One-Counter Automata is NL-complete

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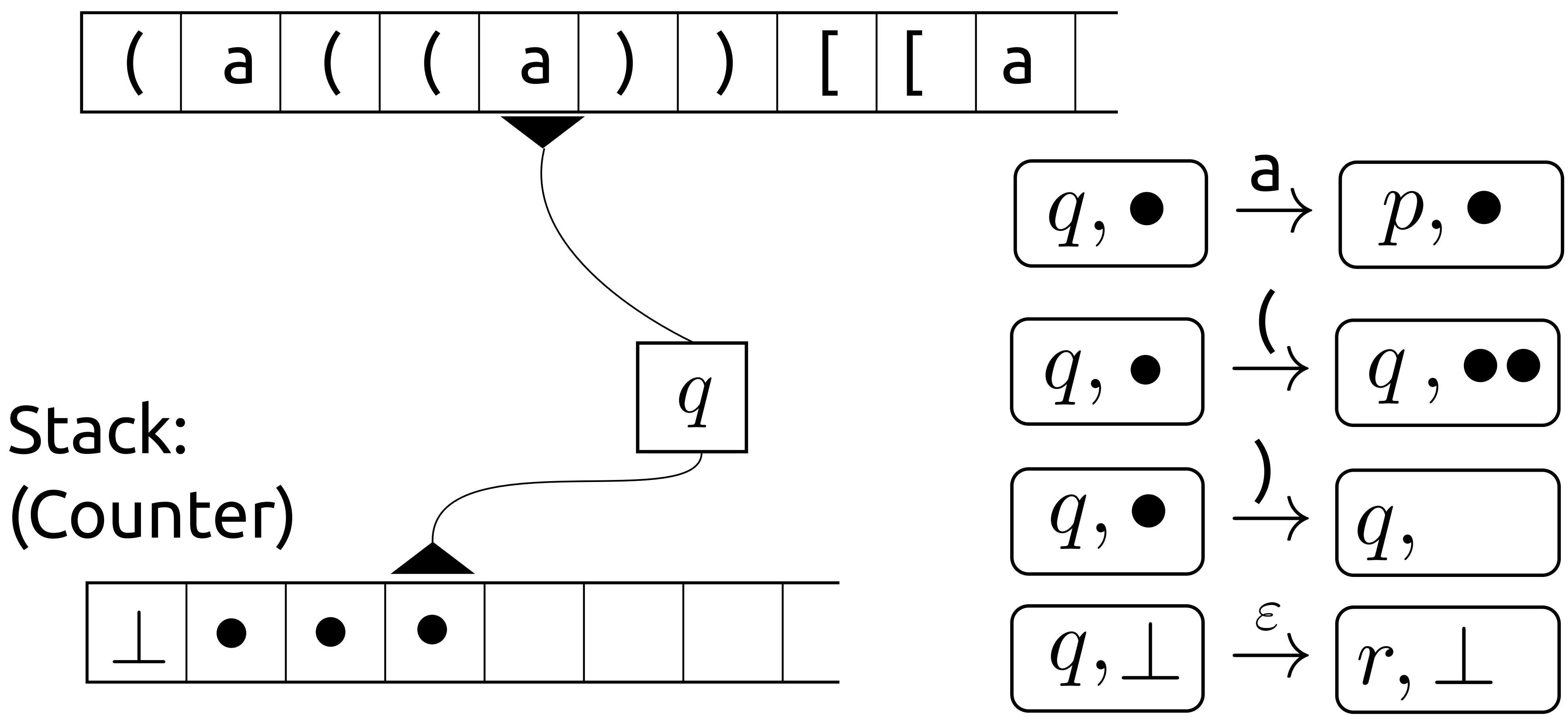
Deterministic One-counter automaton (DOCA)

$$A = (Q, \Sigma, \delta)$$

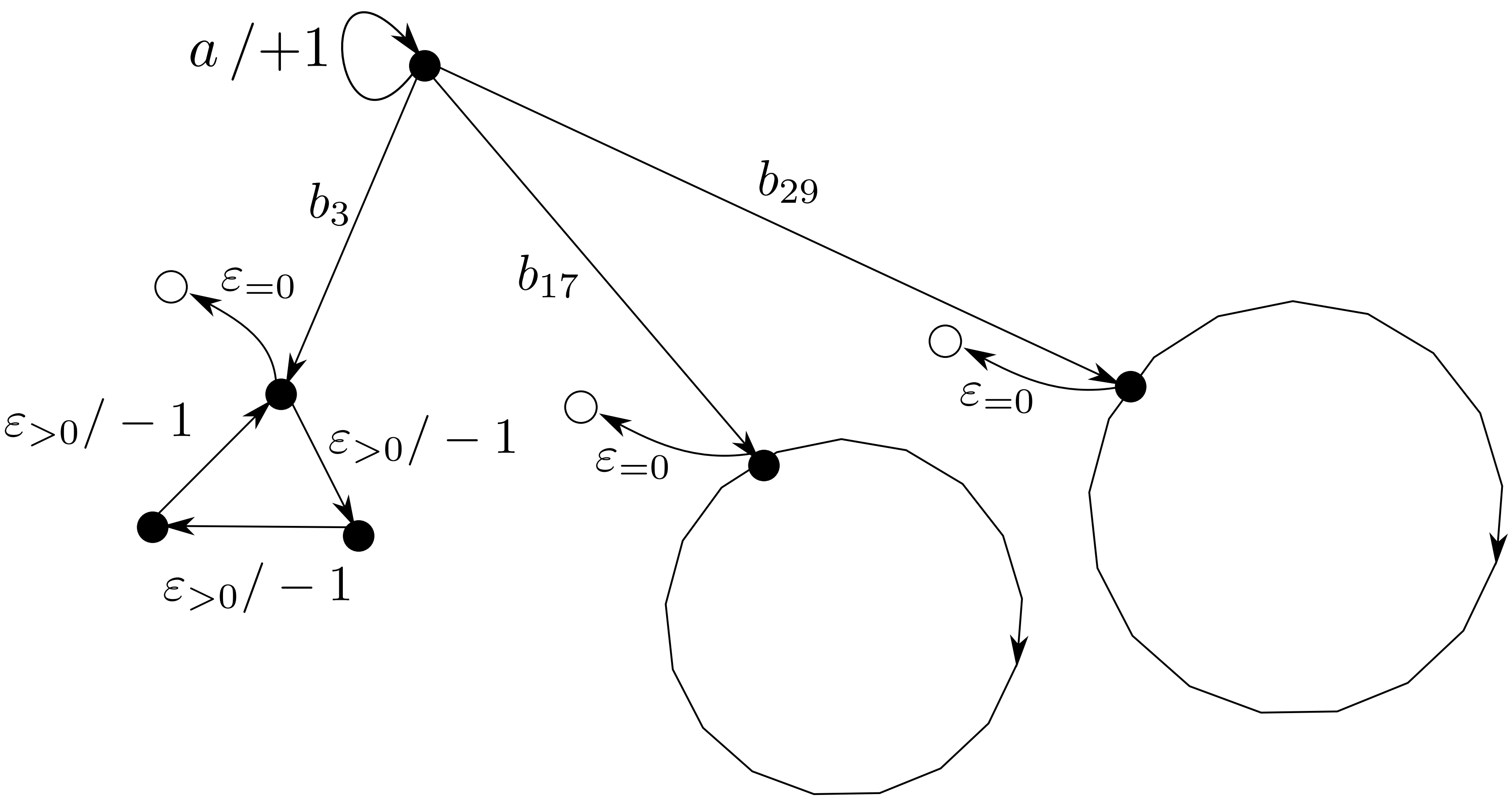
$$\delta : Q \times \{=_0, >_0\} \times (\Sigma \cup \{\varepsilon\}) \to Q \times \{-1, 0, 1\}$$

Input:



A doca language

$$P = \{b_3, b_{17}, b_{29}\}$$
$$L = \{a^n x \mid x \in P \wedge n \equiv 0 \pmod{x}\}$$

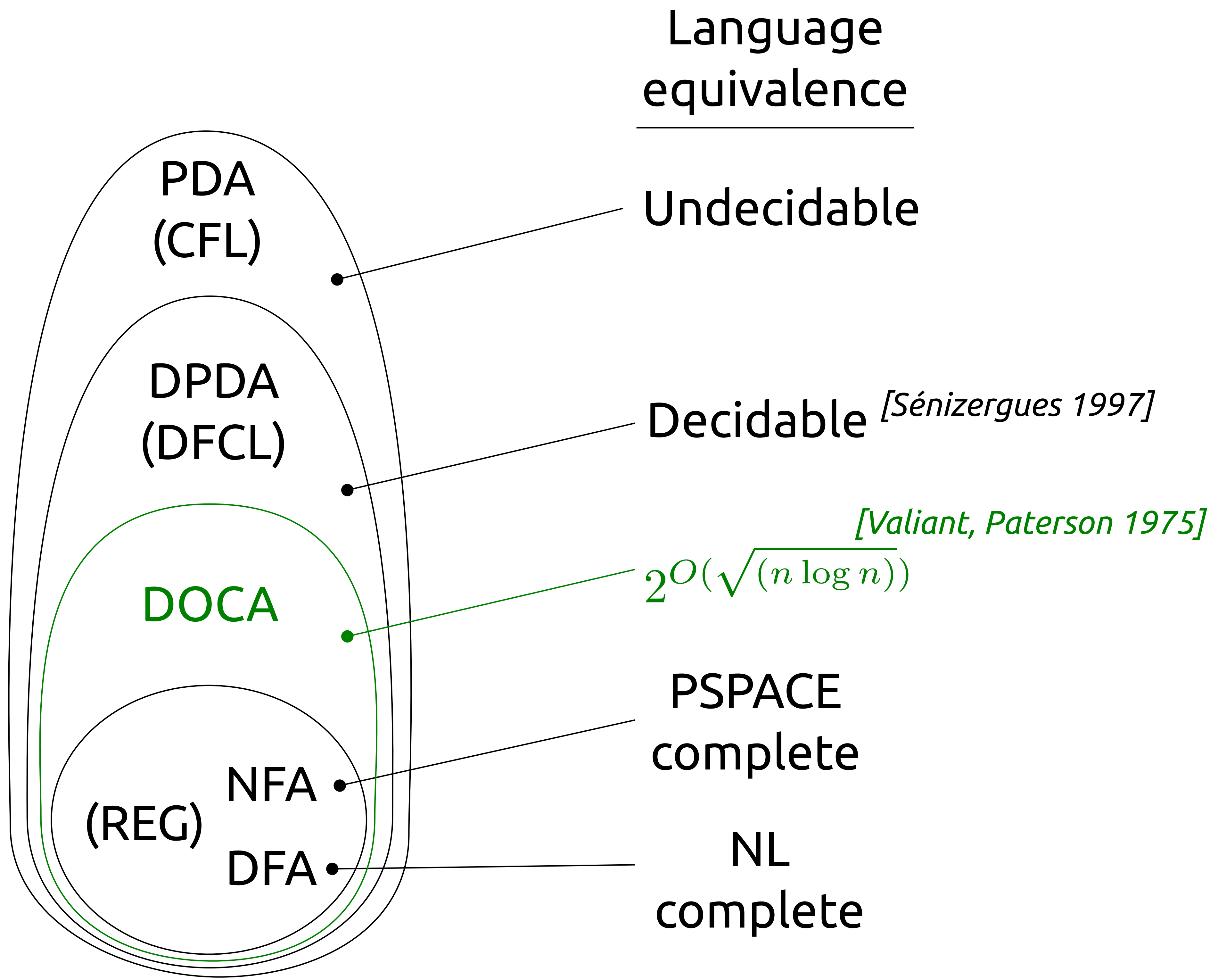


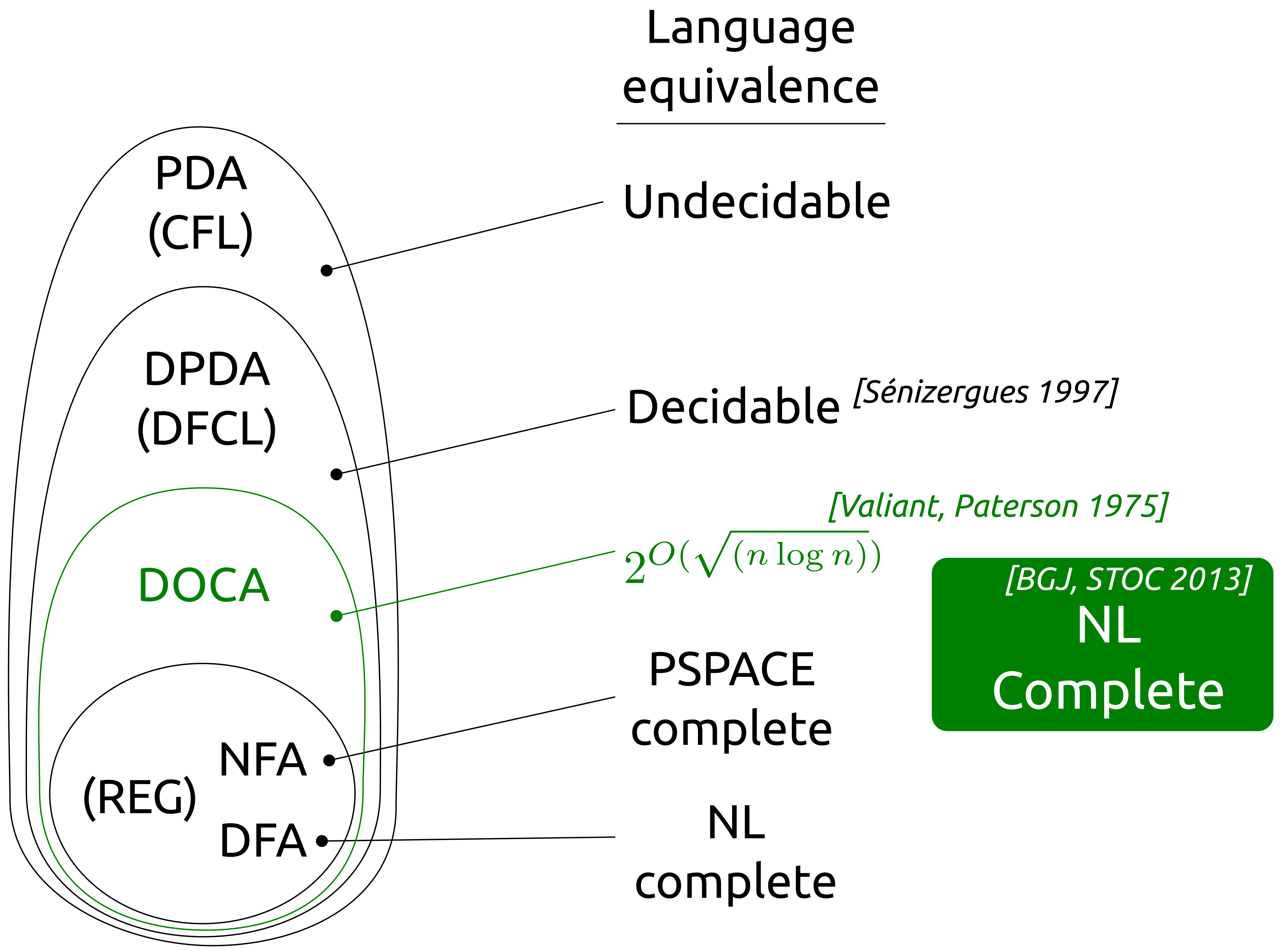
Note: $L = L(A)$ for DFA A with $3 \cdot 17 \cdot 29 + 1$ states. (Exponentiality!)

Decision problem

Input: $A = (Q, \Sigma, \delta, F), p, q \in Q$

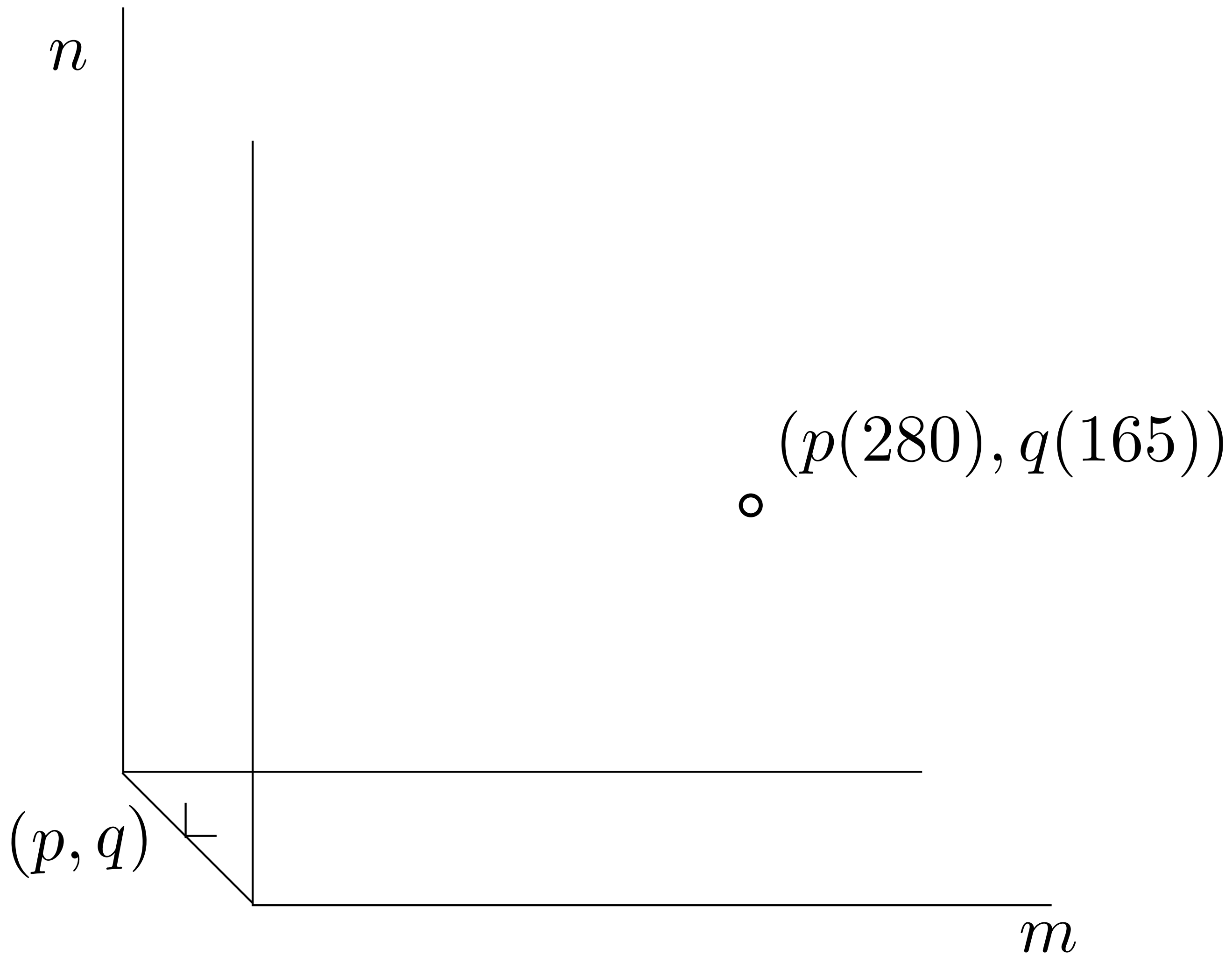
Question: $L(p(0)) \stackrel{?}{=} L(q(0))$


Language equivalence
PDA
(CFL)
Undecidable
DPDA
(DFCL)
Decidable [Sénizergues 1997]
DOCA
[Valiant, Paterson 1975]
$2^{O(\sqrt{(n \log n)})}$
PSPACE complete
(REG)
NFA
DFA
NL complete

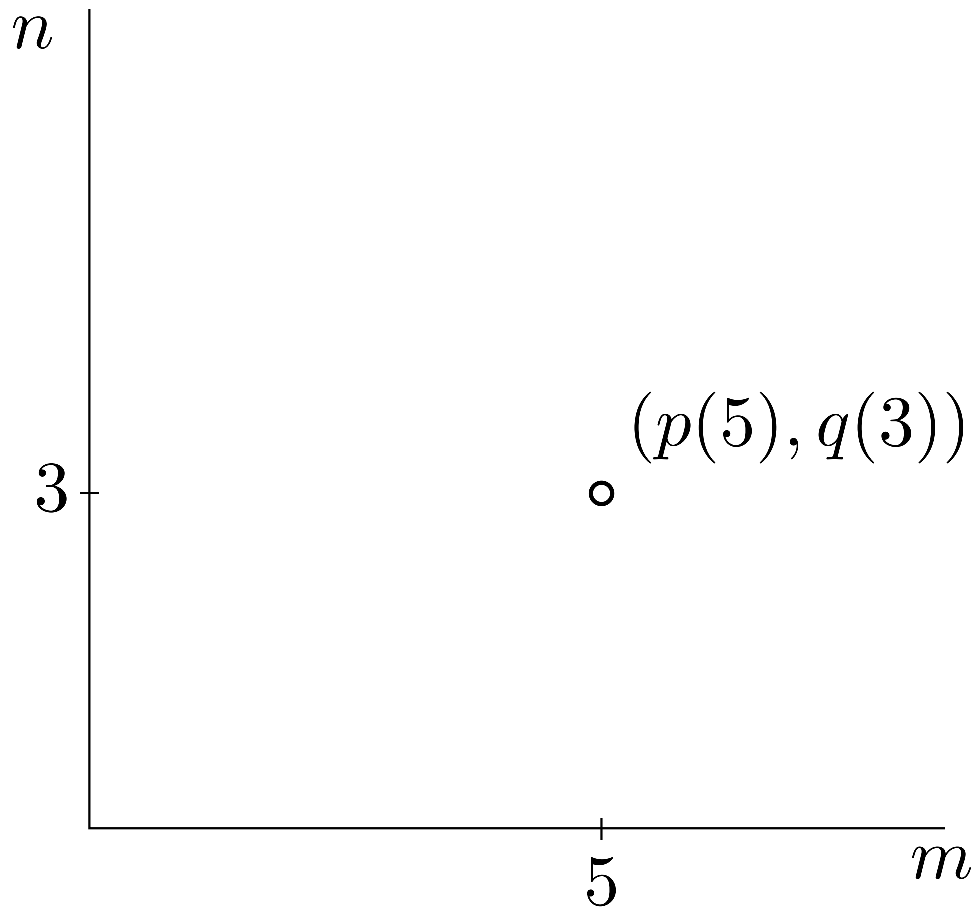


Language equivalence
PDA (CFL)
Undecidable
DPDA (DFCL)
Decidable [Sénizergues 1997]
DOCA
[Valiant, Paterson 1975]
$2^{O(\sqrt{(n \log n)})}$
[BGJ, STOC 2013]
NL Complete
NFA
(REG)
DFA
PSPACE complete
NL complete

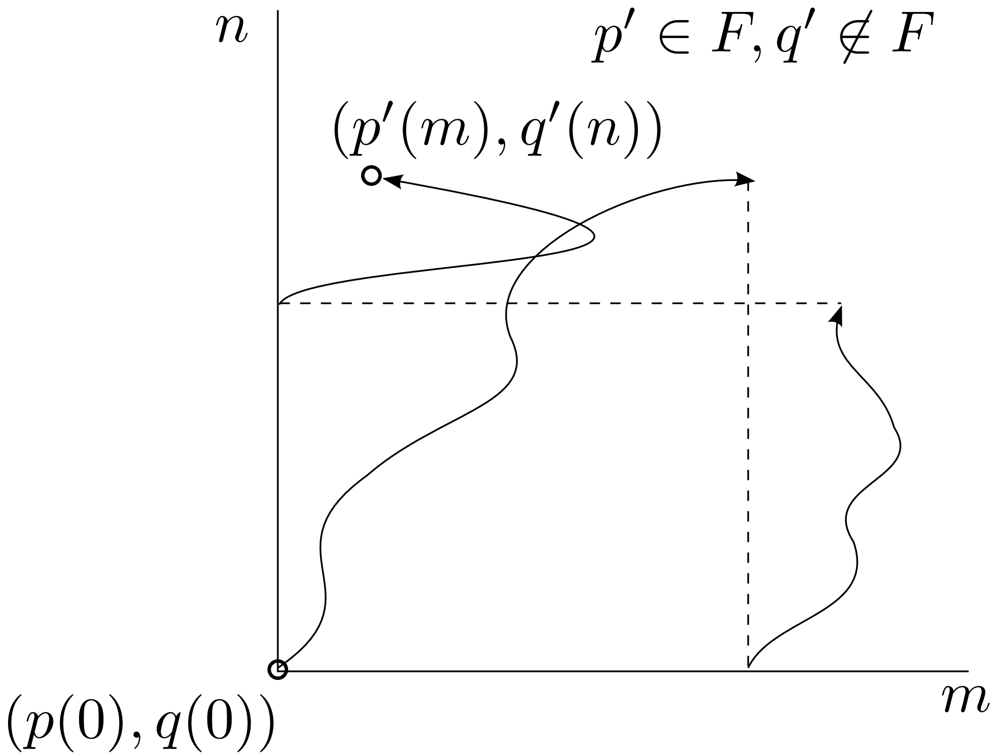
Configuration pairs in 3D space



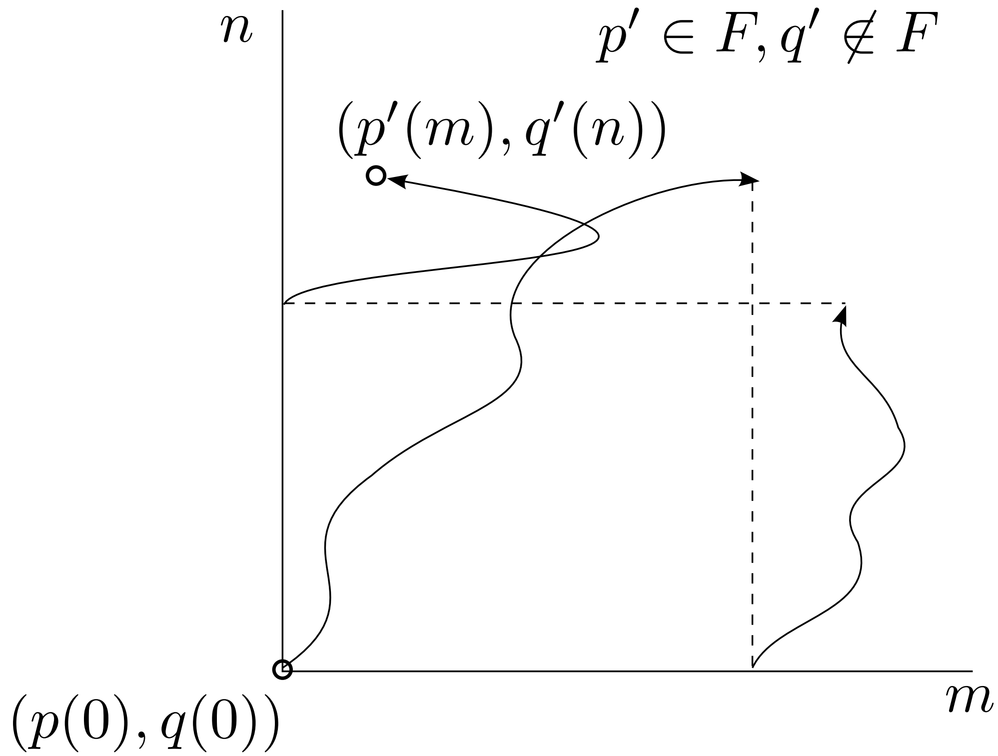
2D projection



Nonequivalence witness paths and equivalence levels



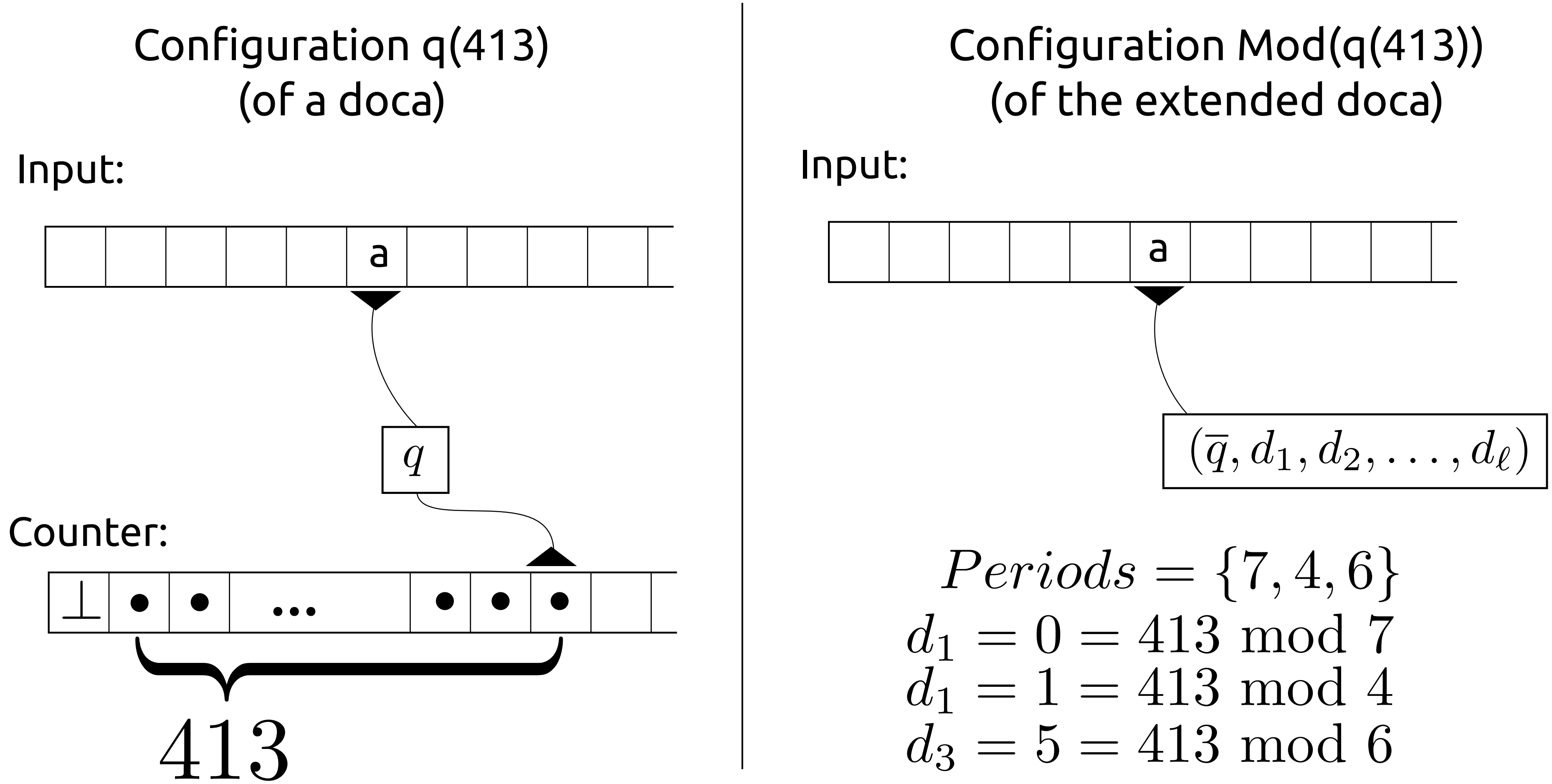
Nonequivalence witness paths and equivalence levels



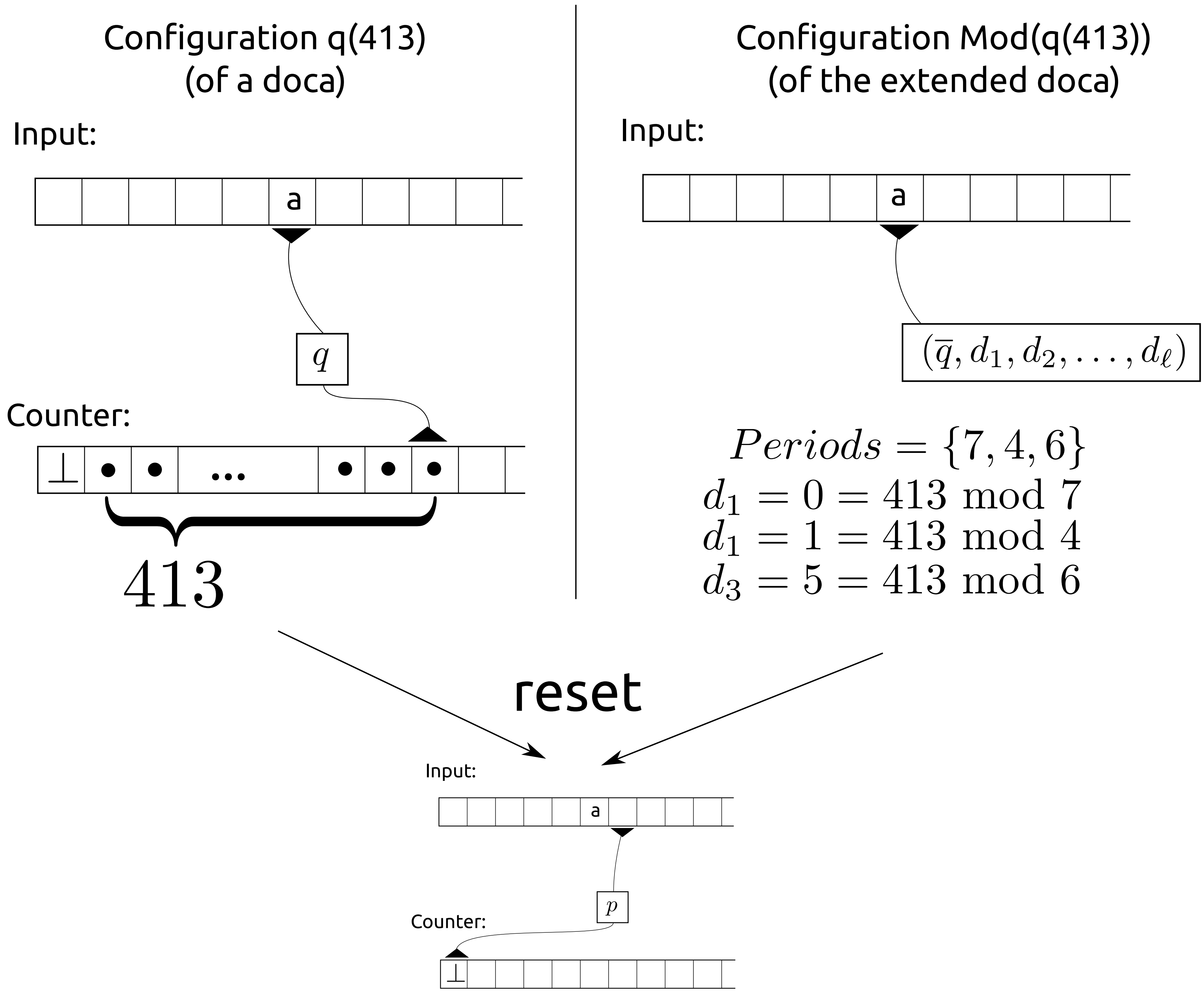
$p(m) \xleftrightarrow{e} q(n)$ ... e is the length of a shortest witness for $(p(m), q(n))$

$p(m) \xleftrightarrow{\omega} q(n)$ ... $L(p(m)) = L(q(n))$

Claim: Finite eq-levels of pairs of zero configurations are small (i.e. polynomial)



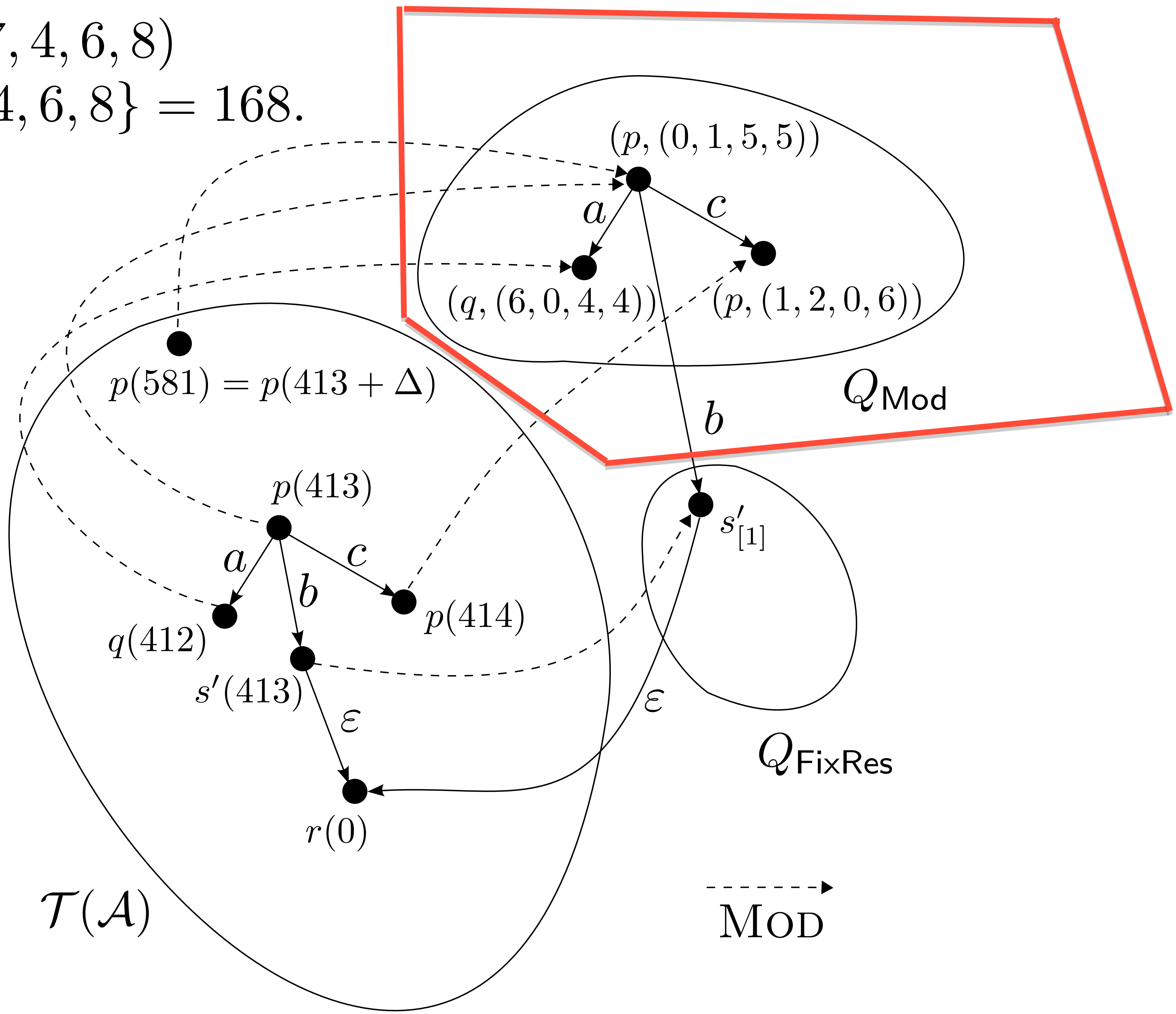
Configuration q(413)
(of a doca)
Input:
a
q
Counter:
$\perp$
413
Configuration Mod(q(413))
(of the extended doca)
Input:
a
$(\overline{q}, d_1, d_2, \ldots, d_\ell)$
$Periods = \{7, 4, 6\}$
$d_1 = 0 = 413 \bmod 7$
$d_1 = 1 = 413 \bmod 4$
$d_3 = 5 = 413 \bmod 6$



Configuration q(413)
(of a doca)
Input:
a
q
Counter:
$\perp$
...
413
Configuration Mod(q(413))
(of the extended doca)
Input:
a
$(\overline{q}, d_1, d_2, \ldots, d_\ell)$
$Periods = \{7, 4, 6\}$
$d_1 = 0 = 413 \bmod 7$
$d_1 = 1 = 413 \bmod 4$
$d_3 = 5 = 413 \bmod 6$
reset
Input:
a
p
Counter:
$\perp$

Transition system of extended doca

$$\text{Periods} = (7, 4, 6, 8)$$
$$\Delta = lcm\{7, 4, 6, 8\} = 168.$$



Linearity of "independence levels"

$$p(m) \xleftrightarrow{\;l\;} \mathsf{Mod}(p(m)) = (\overline{p}, d_1, d_2, \ldots, d_\ell)$$

$$\searrow w_1$$

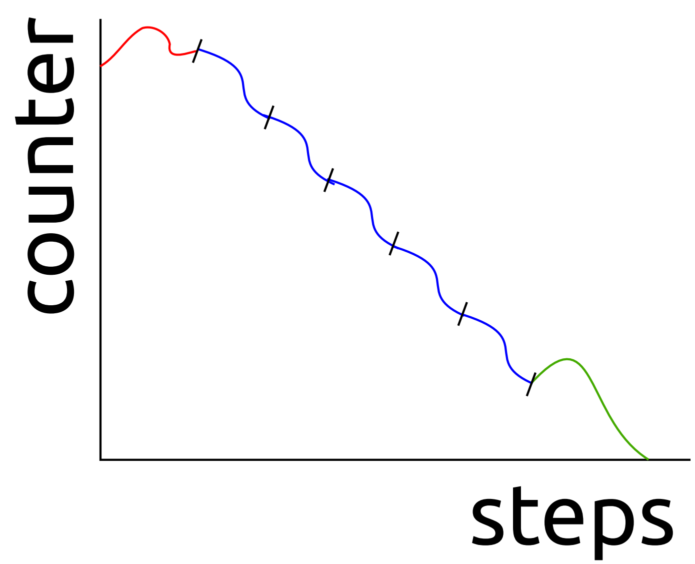
$$z(0) \xleftrightarrow{\;f\;} \mathsf{Mod}(z(0)) = (\overline{z}, 0, 0, \ldots, 0)$$

$$\searrow w_2$$

$$C_1 \xleftrightarrow{\;0\;} C_2$$

$$l = |w_1| + f$$

$$l = \frac{\alpha}{\beta} m + \frac{\gamma}{\delta} + f$$



Quadruples (b, l, r, o) of eq-levels

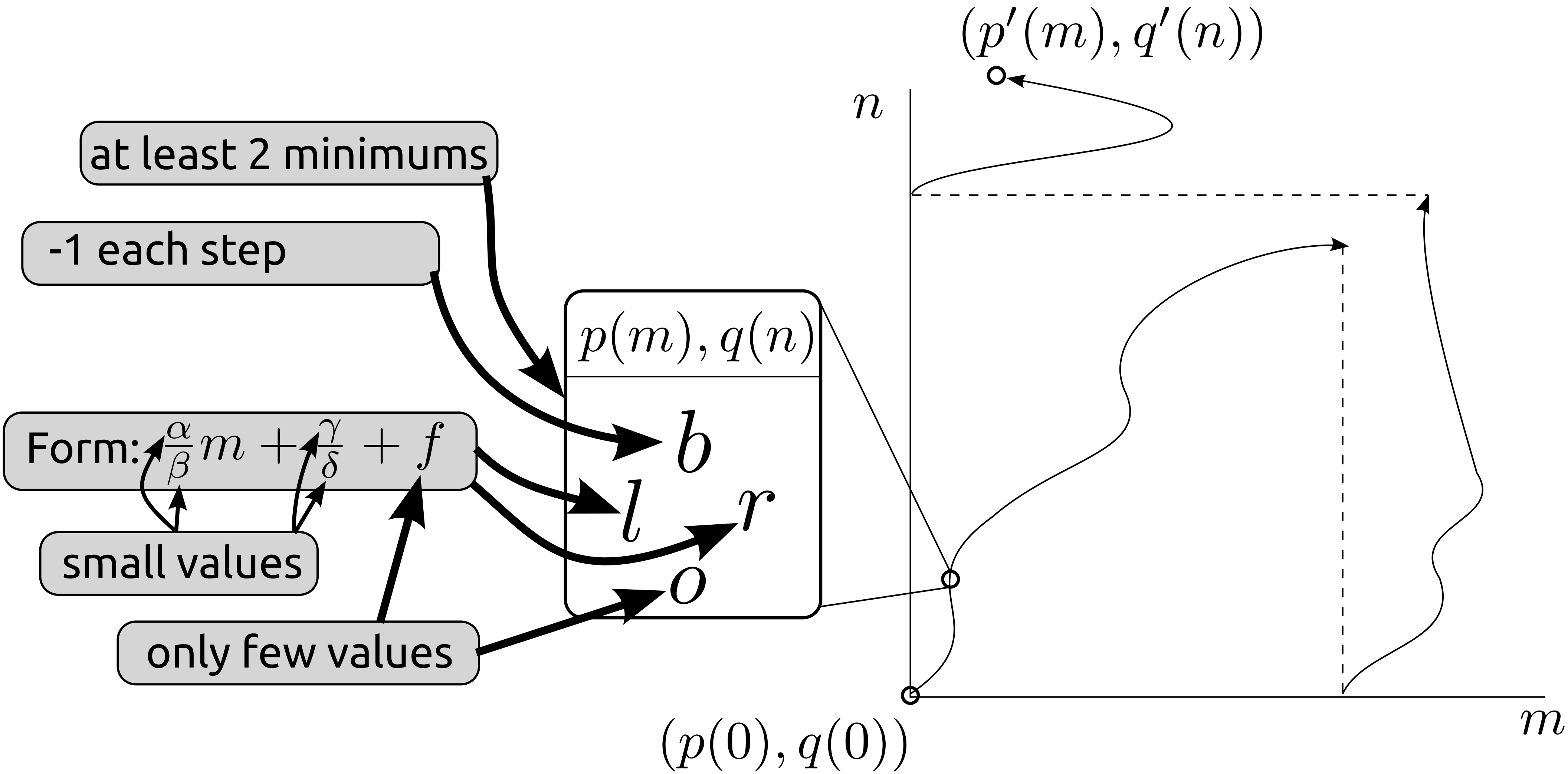
$$\begin{tikzcd} p(m) \arrow[r, leftrightarrow, "b"] \arrow[d, leftrightarrow, "l"'] & q(n) \arrow[d, leftrightarrow, "r"] \\ \mathsf{Mod}(p(m)) \arrow[r, leftrightarrow, "o"] & \mathsf{Mod}(q(n)) \end{tikzcd}$$

Observation:
At least two of b, l, r, o are the minimum in $\{b, l, r, o\}$

First step of the proof

($l \neq r$ only few times)

$$\begin{tikzcd}
p(m) \arrow[r, leftrightarrow, "b"] \arrow[d, leftrightarrow, "l"'] & q(n) \arrow[d, leftrightarrow, "r"] \\
\mathsf{Mod}(p(m)) \arrow[r, leftrightarrow, "o"] & \mathsf{Mod}(q(n))
\end{tikzcd}$$



Second step of the proof

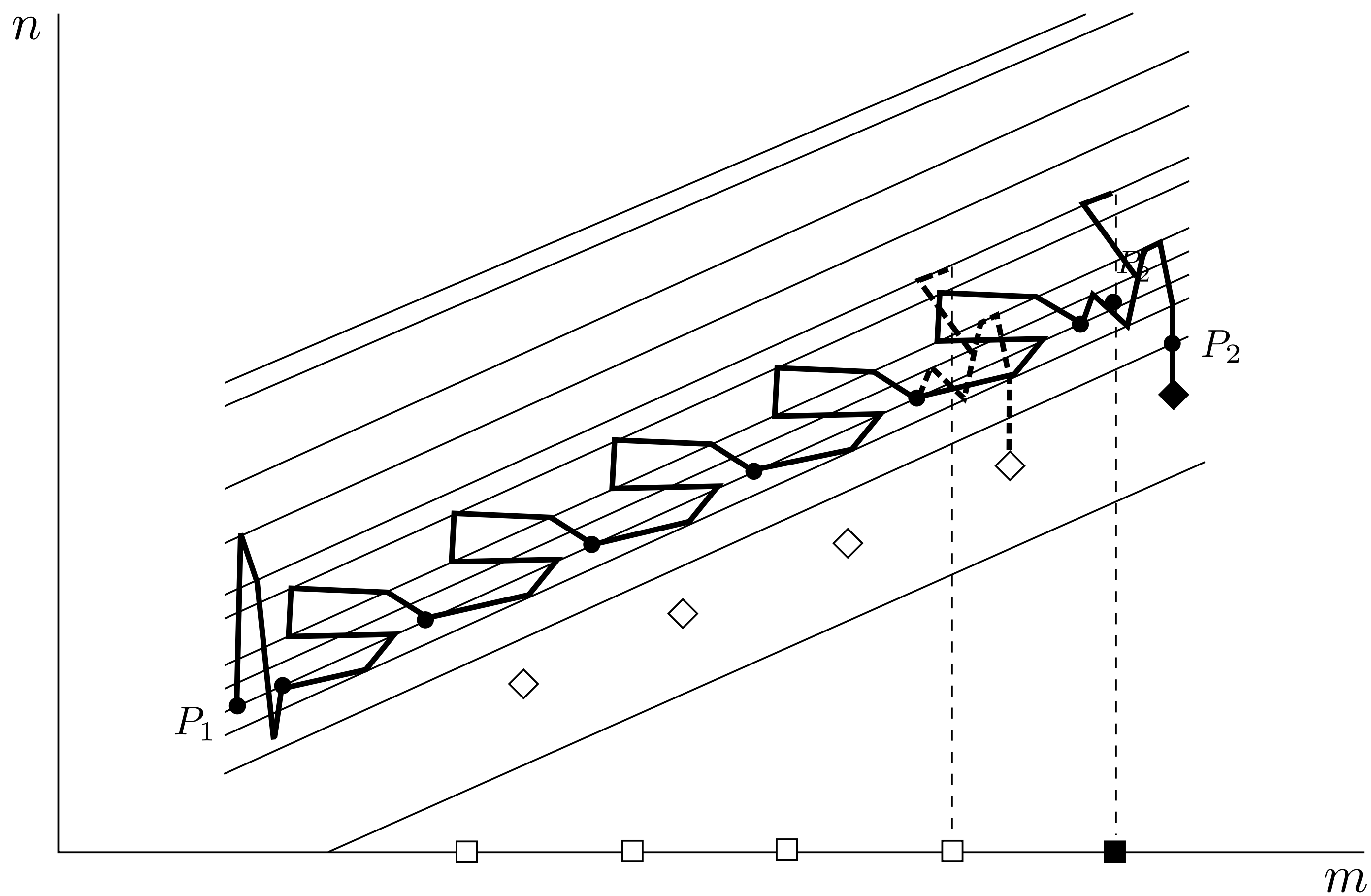
(linear belt climbing is short)

$$l = r$$

$$\frac{\alpha}{\beta} m + \underbrace{\frac{\gamma}{\delta} + f} = \frac{\alpha'}{\beta'} n + \underbrace{\frac{\gamma'}{\delta'} + f'}$$

slope

offset



Thank you for your attention!