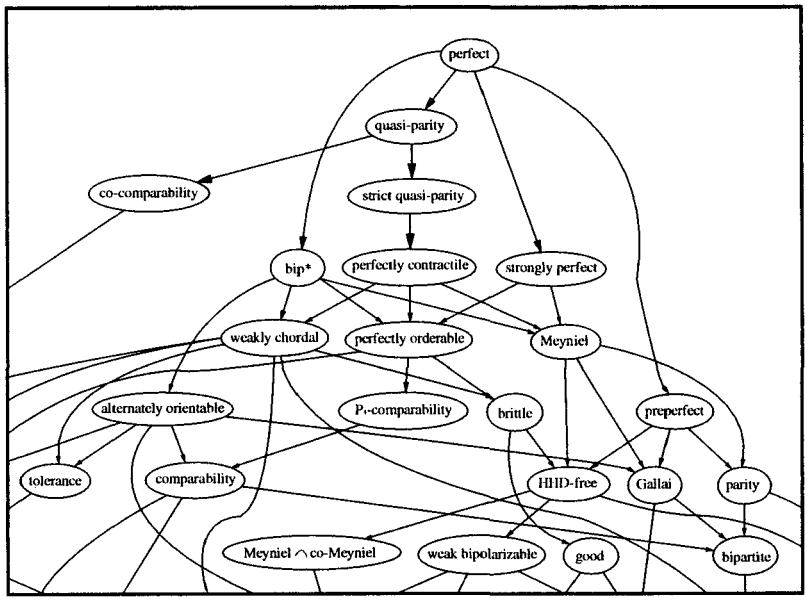
GRAPH CLASSES



A SURVEY

Andreas Brandstädt
University of Rostock
Rostock, Germany

Van Bang Le
University of Rostock
Rostock, Germany

Jeremy P. Spinrad
Vanderbilt University
Nashville, Tennessee

siam
Society for Industrial and Applied Mathematics Philadelphia

Contents

Preface ix

1 Basic Concepts **1**
1.1 Basic graph notions 1
1.2 Chordal graphs 6
1.3 Basic hypergraph notions and properties 7
1.4 Partial orders 11
1.5 Modular decomposition 13
1.6 Some basic algorithms and problems 15
1.7 Some special graphs 17

2 Perfection, Generalized Perfection, and Related Concepts **21**
2.1 Perfect graphs and perfect graph theorems 21
2.2 A semistrong perfect graph theorem 24
2.3 Sharpening perfection 28
2.4 Properties of perfect graphs 29
2.5 Generalized perfection 31
2.6 Related concepts 34

3 Cycles, Chords, and Bridges **39**
3.1 (k, l)-chordal graphs 39
3.2 Chordality conditions for G and $\overline{G}$ 40
3.3 Cycles and chords in bipartite graphs 41
3.4 Odd chords 43
3.5 Meyniel graphs, subclasses, and variants 44
3.6 Bridged graphs and isometric cycles 45

4 Models and Interactions **47**
4.1 Basic concepts 47
4.2 Line graphs and generalizations 48
4.3 Interval graphs and variants 50
4.4 Tree models and variants 52
4.5 Boxicity, intersection dimension, and dot product 54
4.6 Circular-arc graphs 55

4.7 Permutation, circle, and trapezoid graphs and similar concepts 56
4.8 Measured intersection 60
4.9 Other geometric objects 63
4.10 Other interactions—visibility 64

5 Vertex and Edge Orderings **67**
5.1 Perfect elimination and generalizations 67
5.2 Semiperfect elimination 70
5.3 Domination and distance-preserving elimination 72
5.4 Maximum neighborhood orderings and generalizations 74
5.5 Strong and simple orderings and generalizations 76
5.6 Perfect orderings 80
5.7 Special perfectly orderable graphs 81
5.8 Cop-win orderings 86
5.9 Edge elimination orderings 87

6 Posets **91**
6.1 Partial orders and their graphs 91
6.2 Poset dimension 92
6.3 Containment graphs 94
6.4 Series-parallel posets 96
6.5 Interval orders and semiorders 96
6.6 Arborescence orders and threshold orders 99
6.7 Comparability graphs with other restrictions 100
6.8 Posets and diagrams 101

7 Forbidden Subgraphs **105**
7.1 Finitely many forbidden induced subgraphs 105
7.1.1 One forbidden induced subgraph 105
7.1.2 $P_4, C_4, 2K_2$, and other subgraphs 106
7.1.3 $C_5, P_5, \overline{P_5}, C_6, \overline{C_6}, P_6, \overline{P_6}$, and other subgraphs 107
7.1.4 $K_{1,3}$ and other subgraphs 109
7.1.5 Other examples 111
7.2 Infinitely many forbidden induced subgraphs 112
7.2.1 Comparability graphs and variants 112
7.2.2 Chordality and suns 112
7.2.3 Hole-free graphs and variants 113
7.2.4 Asteroidal triples 114
7.3 Forbidden minors—planarity and variants 116
7.4 Forbidden induced ordered subgraphs 119

8 Hypergraphs and Graphs **123**
8.1 α-acyclicity of hypergraphs 123
8.2 Further acyclicity types; clique and neighborhood hypergraphs 125

8.3 Graphs with maximum neighborhood orderings and corresponding hypertrees . . . 127
8.4 Further classes with dual hypertree characterizations . . . 130
8.5 Disk-Helly, clique-Helly, and neighborhood-Helly graphs . . . 131
8.6 Perfect graphs and normal hypergraphs . . . 133
8.7 Interval hypergraphs . . . 134

9 Matrices and Polyhedra **135**
9.1 The consecutive 1s and the circular 1s properties . . . 135
9.2 Balanced and totally balanced matrices; doubly lexical orderings . . . 137
9.3 Perfect and totally unimodular matrices . . . 139
9.4 Birkhoff graphs and doubly stochastic matrices . . . 141
9.5 Forbidden sets of submatrices . . . 142
9.6 Eigenvalues and graphs . . . 143

10 Distance Properties **147**
10.1 Distance-hereditary and parity graphs . . . 147
10.2 Subclasses of distance-hereditary graphs . . . 149
10.2.1 Cographs . . . 149
10.2.2 Bipartite distance-hereditary graphs . . . 149
10.2.3 Chordal distance-hereditary graphs . . . 150
10.2.4 Block graphs and related classes . . . 151
10.3 Interval conditions . . . 152
10.4 Absolute retracts of reflexive and bipartite graphs . . . 155
10.5 Convexity . . . 157
10.6 Powers of graphs . . . 161

11 Algebraic Compositions and Recursive Definitions **167**
11.1 Trees, k-trees, and partial k-trees . . . 167
11.2 Series-parallel graphs . . . 172
11.3 Cographs and domination . . . 175
11.3.1 Cograph characterizations . . . 175
11.3.2 Domination properties . . . 176
11.4 Bounding the number of P_4s . . . 177
11.4.1 P_4-reducible and P_4-sparse graphs and variants . . . 177
11.4.2 p-trees . . . 179
11.4.3 (q,t)-graphs . . . 180
11.5 Tree-cographs and hookup classes . . . 181
11.6 Recursively defined perfect graphs . . . 182

12 Decompositions and Cutsets **187**
12.1 Modular decomposition—the poset aspect . . . 187
12.2 Homogeneous decomposition . . . 189
12.3 Split decomposition . . . 191
12.4 Other decompositions . . . 192

12.5 Minimal separators . . . 194
12.6 Classes with a polynomially bounded number of minimal separators . . . 194
12.7 Clique, biclique, and stable cutsets . . . 195
12.8 Small and balanced separators . . . 197

13 Threshold Graphs and Related Concepts **199**
13.1 The threshold dimension . . . 199
13.2 Constant-bounded Dilworth number . . . 201
13.3 Degree sequences . . . 203
13.4 Matroidal and matrogenic graphs . . . 204

14 The Strong Perfect Graph Conjecture **207**
14.1 Properties of minimal imperfect graphs . . . 208
14.2 Some equivalent versions of the SPGC . . . 213
14.3 Large classes of perfect graphs . . . 214
14.4 Graph classes satisfying the SPGC . . . 217
14.4.1 F-free graphs . . . 217
14.4.2 Graph-valued functions and intersection models . . . 218
14.4.3 Other graph classes . . . 219
14.5 Two semistrong perfect graph conjectures . . . 221
14.6 The weakened strong perfect graph conjecture . . . 222

Appendix A: Recognition **223**

Appendix B: Containment Relationships **229**

New References **251**

Bibliography **253**

Index **295**