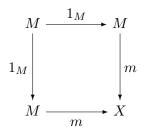
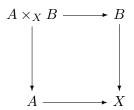
## Homework 5

- 1. Prove that every monoid in the category of groups is an internal group. (Hint: use the Eckmann-Hilton argument.)
- 2. Let C be a category with pullbacks.
  - (a) Show that an arrow  $m: M \to X$  in  ${\bf C}$  is monic if and only if the diagram below is a pullback.



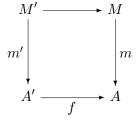
Thus as an object in  $\mathbb{C}/X$ , m is monic iff  $m \times m \cong m$ .

(b) Show that the pullback along an arrow  $f:Y\to X$  of a pullback square over X,



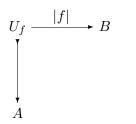
is again a pullback square over Y. (Hint: draw a cube and use the 2-pullbacks Lemma). Conclude that the pullback functor  $f^*$  preserves products.

(c) Conclude from the foregoing that in a pullback square

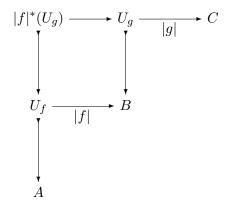


if m is monic, then so is m'.

- 3. (Pushouts)
  - (a) Dualize the definition of a pullback to define the "copullback" (called the "pushout") of two arrows with common domain.
  - (b) Indicate how to construct pushouts using coproducts and coequalizers (proof "by duality").
  - (c) What is the pushout in posets of the two maps  $0, 1 : \{*\} \rightarrow [0, 1]$ , where  $\{*\}$  is a one-element poset, and [0, 1] is the unit interval.
- 4. \*(Partial maps) For any category **C** with pullbacks, define the category  $\mathbf{Par}(\mathbf{C})$  of partial maps in **C** as follows: the objects are the same as those of **C**, but an arrow  $f: A \to B$  is a pair  $(|f|, U_f)$  where  $U_f \rightarrowtail A$  is a subobject (an equivalence class of monomorphisms) and  $|f|: U_f \to B$  (take a suitably-defined equivalence class of arrows), as indicated in the diagram:



Composition of  $(|f|, U_f): A \to B$  and  $(|g|, U_g): B \to C$  is given by taking a pullback and then composing to get  $(|g \circ f|, |f|^*(U_g), as$  suggested by the follow diagram.



Check to see that this really does define a category.