## Homework 9

- 1. Show that a functor category  $\mathbf{D}^{\mathbf{C}}$  has binary products if  $\mathbf{D}$  does (construct the product of two functors F and G "objectwise":  $(F \times G)(C) = F(C) \times G(C)$ ).
- 2. Show that the map of sets

$$\eta_A: A \longrightarrow PP(A)$$

$$a \longmapsto \{U \subseteq A | a \in U\}$$

is the component at A of a natural transformation  $\eta: 1_{\mathbf{Sets}} \to PP$ , where  $P: \mathbf{Sets}^{\mathrm{op}} \to \mathbf{Sets}$  is the (contravariant) power-set functor.

3. Let C be a locally small category. Show that there is a functor

$$\hom: \mathbf{C}^{\mathrm{op}} \times \mathbf{C} \to \mathbf{Sets}$$

such that for each object C of  $\mathbf{C}$ ,

$$hom(C, -) : \mathbf{C} \to \mathbf{Sets}$$

is the covariant representable functor and

$$hom(-, C) : \mathbf{C}^{op} \to \mathbf{Sets}$$

is the contravariant one. (Hint: use the Bifunctor Lemma)

4. (a) Complete the proof that, for any set I, the category of I-indexed families of sets, regarded as the functor category  $\mathbf{Sets}^{I}$ , is equivalent to the slice category  $\mathbf{Sets}/I$  of sets over I.

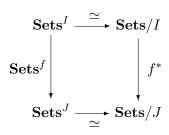
$$\mathbf{Sets}^I \ \simeq \ \mathbf{Sets}/I$$

(b) \* Show that reindexing of families along a function  $f: J \to I$ , given by precomposition,

$$\mathbf{Sets}^f((A_i)_{i\in I}) = (A_{f(i)})_{j\in J}$$

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is represented by pullback, in the sense that the following diagram of categories and functors commutes up to natural isomorphism.



Here  $f^*: \mathbf{Sets}/I \to \mathbf{Sets}/J$  is the pullback functor along the function  $f: J \to I$ .