his consequentiality, meaning he'd probably dismiss the applicability problem as just another in a long line of pseudo-problems, which philosophy has thrown at mathematics.

There is also the problem, or problems, of set theory. Franks remarks that "the trail of philosophy never crosses into mathematical terrain." (Page 62.) Set theory is not really mentioned by Franks, but it must be said that in that field, philosophy has managed to build not a trail, but a four lane highway. "Second order reflection," as Franks calls it, on the nature of the infinite, on the notion of maximality, on the bivalent—or not—nature of mathematical truth, has been going on in the set theory community for some time. It is one of the main sources of the discovery and further development of reflection principles and of large cardinal concepts. It simply cuts to the core of the subject—and always has.

This puts Franks in the position of having to become involved in the taxonomy of subjects—a rather pedestrian topic to come across in a book of this quality; in the position of classifying as mathematics or philosophy Woodin's argument, for example, against the generic multiverse view—an argument seemingly rife with second order considerations. On the other hand Franks could simply suggest, along with Quine, that (this area of) set theory should not be considered real mathematics. But this goes against the grain, presumably. The whole point of this book is to put an end to such prohibitions.

JULIETTE KENNEDY

Department of Mathematics and Statistics, P.O. Box 68, 00014-University of Helsinki, Finland. juliette.kennedy@helsinki.fi.

JACQUES SAKAROVITCH. ***Elements of automata theory.*** Cambridge University Press, Cambridge, 2009, xxiv + 758 pp.

Automata theory is one of the longest established areas in Computer Science and its applications include pattern matching, syntax analysis, software verification and linguistics.

Jacques Sakarovitch's ***Elements of automata theory***, published by Cambridge University Press in 2009, was originally published in French as ***Éléments de théorie des automates*** by Vuibert, Paris in 2003. It covers in great detail the classical theory of finite automata and the part of its mathematical foundations that rely on noncommutative algebra: semigroups, semirings and formal power series. It only treats finite automata on finite words, but includes automata with outputs, also called transducers, and automata with coefficients, also known as weighted automata or formal power series. It does not cover connections with logic, except for decidability results.

Reuben Thomas' remarkable translation succeeds in conveying the inimitable style of the author, including puns and citations. The reader should prepare himself for a deluge of footnotes and marginal notes. The original French version already contained a lot of them, but the English edition adds a number of *Notes added in translation*. Marginal notes are used to give references to related matter like exercises or propositions elsewhere in the book, which turns out to be very convenient. Footnotes are reserved for comments on terminology and notation, further references and entertaining personal thoughts of the author. I recommend the reading of the footnotes on pages xxiv, 24, 103 and 158 to catch a glimpse of Sakarovitch's humoristic style.

The author provides a great variety of exercises, ranging from straightforward results to more challenging ones taken from research papers. Following a practice that is common nowadays, part of these exercises form an integral part of the text and are sometimes used later in the book. But contrary to most authors, who simply throw the responsibility of providing intricate proofs onto the reader, Sakarovitch takes the pain to give detailed solutions to most of his exercises. Even for this reason alone, the book should be highly recommended as a reference textbook for students and researchers interested in automata theory.

Chapter 0 covers reminders on relations, words and languages, algebraic structures such as monoids, semirings and matrices, terminology from graph theory and basic results on complexity and decidability. The exercises also introduce a number of nontrivial notions and results: Fine and Wilf's theorem on periods of a word, conjugate words, equidivisible monoids. The native English speaker will also enjoy the fact (proved at the bottom of page 41) that the Robert & Collins Dictionary does not follow any lexicographic order: the problem arises when the hyphenation symbol is used.

According to the author, Chapter 1 presents the traditional theory of finite automata on finite words. It contains indeed the material expected to be found in any basic course on finite automata: recognisable languages (i.e., recognised by a finite automaton), rational (also called regular) expressions and rational languages, determinisation, minimisation, derivatives and Kleene's theorem. However, it actually contains much more. I recommend in particular Section 4, which gives an exhaustive survey on rational expressions and on derivatives and which compares, for the first time, the algorithms of Brzozowski and McCluskey (BMC), McNaughton and Yamada (MNY), Thompson, and Antimirov. Further, already in this first chapter, the author takes the opportunity to bring up some famous problems of automata theory. Rational identities are introduced on p. 128 and used to analyse the BMC and MNY algorithms. Conway's problem of finding a complete set of identities for rational expressions is clearly out of the scope of this book, but references to Krob's solution to this problem are given in the references of the chapter. The star height problems are illustrated by two early results on the (restricted) star height problem: Eggan's theorem relating star height and loop complexity and Dejean and Schützenberger's proof that the star height hierarchy is infinite. The author also presents a deep improvement over the usual pumping lemma, due to Ehrenfeucht, Parikh and Rozenberg, which permits him to *characterize* recognisable languages. Further, the chapter includes the Knuth–Morris–Pratt string matching algorithm (p. 156), Schützenberger's proof of Shepherdson's result on two-way automata (p. 173) and a number of classical results on rational languages (pp. 176–178). A very short section (p. 175) is devoted to Moore and Mealy machines, probably for historical reasons. These machines are indeed special cases of the sequential transducers studied in Chapter 4.

The first two sections of Chapter 2, entitled *The power of algebra*, cover automata over a monoid and their matrix representations, [unambiguous] rational and recognisable sets over arbitrary monoids, syntactic congruences and syntactic monoids. This material can also be found in Eilenberg's two volumes ***Automata, languages and machines*** or in Berstel's book ***Transductions and context-free languages***. But the topics presented in the remaining sections are rarely found in books. Section 3 on coverings and Section 4 on universal automata (p. 273) are some of the highlights of the book. They present, for the first time, a comprehensive survey on a notion frequently rediscovered since its introduction by J. H. Conway in 1971. I just have a slight regret: the author misses the opportunity to mention residual finite state automata, a notion introduced by Denis, Lemay and Terlutte [***STACS 2001***, LNCS 2010, p. 144–157], that is also strongly related to residuals. Section 5 is devoted to two results: Higman's theorem that the subword order is a well quasi-order and an extension of this result due to Ehrenfeucht, Haussler and Rozenberg. The last two sections describe the rational subsets of two important monoids: the free group and the free commutative monoids.

Chapter 3 is devoted to formal power series (in noncommutative variables) over a semiring. This topic is also covered in other books, notably in the book of Berstel and Reutenauer, ***Rational series and their languages***, Springer 1988. The chapter deals with series over a semiring, weighted automata and their matrix representations. Derivatives, Hankel matrices and Kleene–Schützenberger's theorem on the equivalence between rational and recognisable series are carefully explained. Algorithmic and decidability issues are also considered, notably the equality problems for two recognisable series (decidable) and for the supports of two $\mathbb{Z}$-rational series (undecidable). Another interesting question concerns the rationality of the support of a series. In particular, in the colourful terminology of the author, the *skimming theorem* states that if s is an $\mathbb{N}$-rational series, then $s - \text{support}(s)$ is also $\mathbb{N}$-rational. Sections 5 and 6 treat some more advanced matters: series on an arbitrary monoid with coefficients in a complete semiring and rational subsets in free products. The chapter ends with a primer on noncommutative linear algebra (modules over a ring and vector spaces over a skew field). As everywhere in the book, the numerous exercises and their solutions are an invaluable source of examples and research topics.

Chapter 4 on transducers deals with relations realised by finite automata. The first section covers rational and recognisable relations, their representations by real-time transducers and the Rabin–Scott model. It follows roughly the presentation given by Berstel [***Transduction and context-free languages***, Teubner 1979] or Eilenberg [***Automata, languages and machines***, vol. A, Academic Press 1974]. Section 2 introduces K-relations in a very general setting and Section 3 gives an exhaustive survey on rational K-relations and their representations. Section 4 and Section 7 form one of the most substantial and valuable parts of the book. It contains a proof of several important decidability and undecidability results, dealing with intersection, recognisability and equality of rational relations over various semirings. Section 5 is devoted to deterministic transducers and deterministic relations, their matrix representations and the related decidability questions. The highly valuable Section 6 presents for the first time a unified treatment of various variants of deterministic relations: letter-to-letter, bounded-length discrepancy and synchronising relations.

Chapter 5 deals with functions realised by finite automata. The chapter starts with a proof that deciding whether a finite transducer computes a function is a decidable property. It follows that the inclusion and the equivalence of rational functions is decidable. Then the author introduces a new terminology which, in my opinion, should be strongly supported since it is much better than the traditional one. He calls *sequential* a function that can be realised by a deterministic automaton with a possible initial output and possible final outputs. This definition, originally introduced by Schützenberger under the name of a sub-sequential function, turns out to be the really important notion. For instance, it allows a one-state representation for the simple functions $x \mapsto xu$ and $x \mapsto ux$, where u is a fixed word, which would be impossible with the old definition. The captivating Section 2 investigates the uniformisation problem from descriptive theory in the context of rational relations. It is proved that every rational relation from A^* to B^* can be uniformised by an unambiguous rational function. As a consequence, every rational function can be obtained as the composition of a sequential function with a co-sequential function. An extended version of the skimming theorem and various decidability results complete this nice section. Section 3 turns to a related question, namely cross-sections of rational functions. Section 4 covers two important topics: Choffrut's topological characterization of sequential functions and the minimisation of sequential transducers.

On the whole, the book is well written and pleasant to read, but given its length (758 p.), I would recommend reading it chapter by chapter. The book starts with a preface and a prologue on Pascal's division machine. Every chapter begins with a discussion of the motivations and ends with detailed notes and references. Many of the more elaborate proofs are preceeded by a discussion of the ideas of the proof, which greatly aids the understanding of the constructions. As mentioned earlier, many interesting results are presented as exercises, for most of which complete solutions are provided. ***Elements of automata theory*** will be a very valuable resource for students and researchers working in automata theory.

JEAN-ÉRIC PIN

LIAFA, Université Paris Diderot-Paris 7, Case 7014 F-75205 Paris Cedex 13, France. Jean-Eric.Pin@liafa.jussieu.fr.