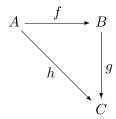
## Homework 2

1. Let A be a set. Define an A-monoid to be a monoid M equipped with a function  $m: A \to U(M)$  (to the underlying set of M). A morphism  $h: (M, m) \to (N, n)$  of A-monoids is to be a monoid homomorphism  $h: M \to N$  such that  $U(h) \circ m = n$  (a commutative triangle). Together with the evident identities and composites, this defines a category A-Mon of A-monoids.

Show that an initial object in A-Mon is the same thing as a free monoid M(A) on A. (Hint: compare their respective UMPs.)

- 2. Show that a function between sets is an epimorphism if it is surjective. Conclude that the isos in **Sets** are exactly the epi-monos.
- 3. With regard to a commutative triangle,



in any category C, show

- a. if f and g are isos (resp. monos, resp. epis), so is h;
- b. if h is monic, so is f;
- c. if h is epic, so is q;
- d. (by example) if h is monic, g need not be.
- 4. Show that a homomorphism  $h: G \to H$  of graphs is monic just if it is injective on both edges and vertices.
- 5. Show that in any category, any retract of a projective object is also projective.
- 6. In any category with binary products, show directly from the UMP that:

$$A \times (B \times C) \cong (A \times B) \times C.$$

7. \* Show that the epis among posets are the surjections (on elements), and that the one-element poset 1 is projective. Show that sets, regarded as discrete posets, are projective in the category of posets. Give an example of a poset that is not projective. Show that every projective poset is discrete, i.e. a set. Conclude that **Sets** is (isomorphic to) the "full subcategory" of projectives in **Pos**, consisting of all projective posets and all monotone maps between them.