Stat 207 HW3

Cheng Luo 912466499
Fan Wu 912538518

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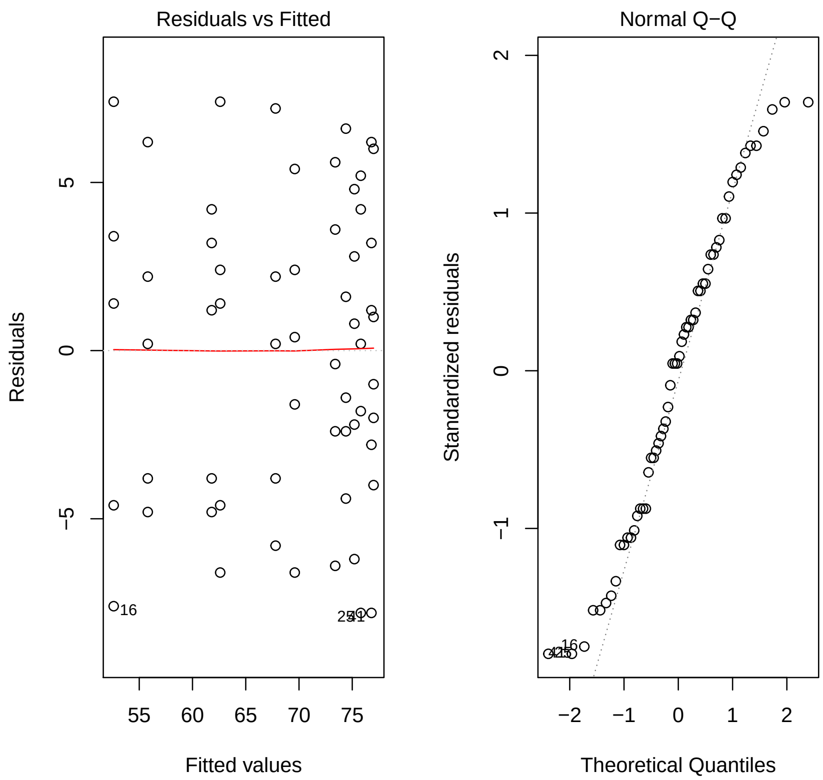
1 26.4

(a)

```
dat = read.table("CH26PR04.txt")
names(dat) = c("Y", "A", "B", "k")
dat$A = factor(dat$A)
dat$B = factor(dat$B)
a = length(unique(dat$A))
b = length(unique(dat$B))
n = length(unique(dat$k))
model = aov(Y ~ A + A/B, data = dat)
resid(model)
```

```
##    1    2    3    4    5    6    7    8    9   10   11   12   13   14   15
##  3.2 -3.8  1.2 -4.8  4.2  0.2 -5.8  7.2 -3.8  2.2 -6.6  2.4 -4.6  7.4  1.4
##   16   17   18   19   20   21   22   23   24   25   26   27   28   29   30
## -7.6  3.4  1.4 -4.6  7.4 -1.8  5.2  0.2  4.2 -7.8 -6.2  0.8  4.8  2.8 -2.2
##   31   32   33   34   35   36   37   38   39   40   41   42   43   44   45
## -3.8  0.2  6.2  2.2 -4.8 -4.0  1.0  6.0 -2.0 -1.0 -7.8  6.2 -2.8  1.2  3.2
##   46   47   48   49   50   51   52   53   54   55   56   57   58   59   60
## -6.6  0.4  2.4 -1.6  5.4  6.6 -2.4 -1.4  1.6 -4.4 -6.4  5.6 -0.4  3.6 -2.4
```

```
par( mfrow = c(1,2))
plot(model, which = 1)
plot(model, which = 2)
```

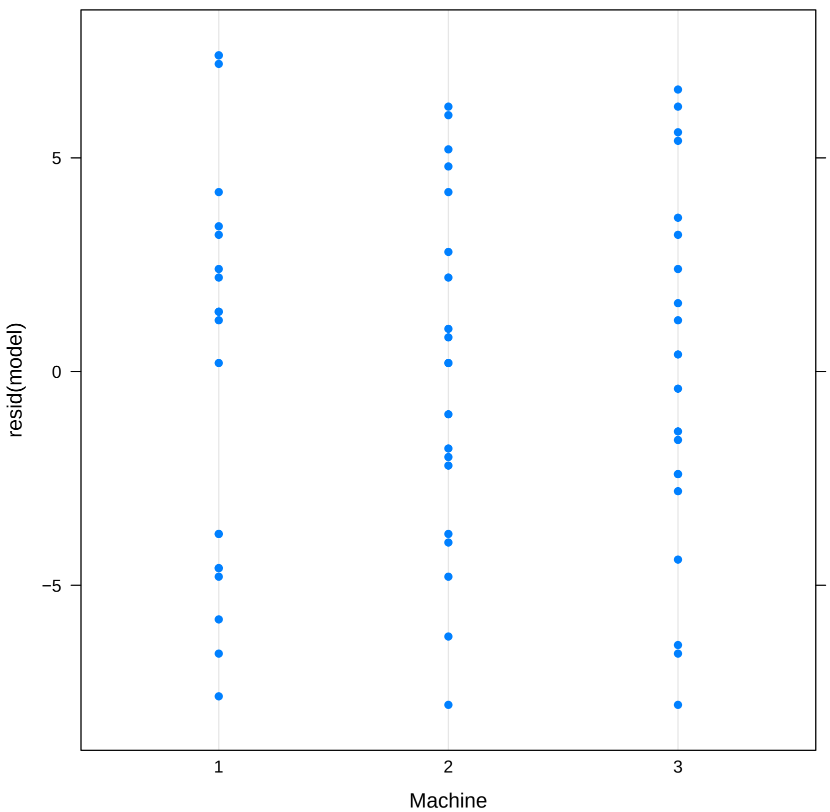


The residuals versus fitted values plots shows no sign for unequal variance.And the QQ-plot indicates approximately normal distribution with slightly light tail, so that normality assumption seems to be reasonable, we can use model(26.7) here.

(b)
```r
require("lattice")

## Loading required package:  lattice

dotplot(resid(model) ~ dat$A, xlab = "Machine" )
```



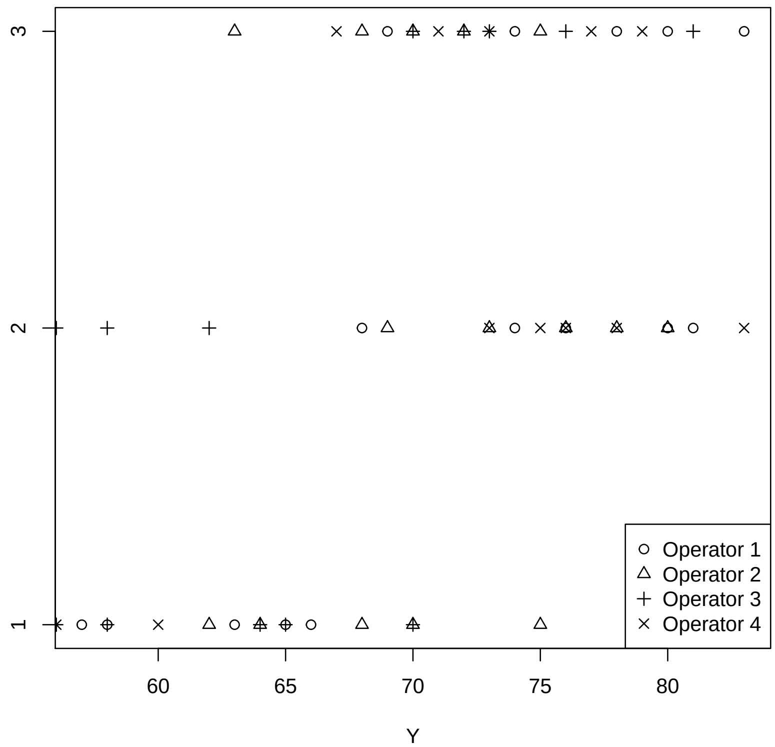
The plot shows no sign for unequal variance, so it support the assumption of constancy of the error variance.

2 26.5

(a) No. Since 4 operators worked 6-hour shifts each, the operator effects contains the effects of shift

(b)
```
stripchart(Y ~ A, data = dat, subset = B == '1', pch = 1)
stripchart(Y ~ A, data = dat, subset = B == '2', pch = 2,
           add = TRUE)
stripchart(Y ~ A, data = dat, subset = B == '3', pch = 3,
           add = TRUE)
stripchart(Y ~ A, data = dat, subset = B == '4', pch = 4,
           add = TRUE)
legend('bottomright', pch = 1:4,
       legend = c('Operator 1', 'Operator 2',
```

```
'Operator 3', 'Operator 4'))
```



It seems operator effect are present.

(c)
```
summary(model)
```

```
##              Df Sum Sq Mean Sq F value   Pr(>F)    
## A             2   1696   847.8   35.92 2.90e-10 ***
## A:B           9   2272   252.5   10.70 6.99e-09 ***
## Residuals    48   1133    23.6                     
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Test the mean outputs differ for three machine:

$$H_0\text{:all } \alpha_i \text{ equal zero(i=1,2,3)}$$

VS. H_1:not all α_i equal zero

$F^* = \frac{MSA}{MSE} = 847.8/23.6 = 35.92$

we can reject H_0 if $F^* > F(1-0.01; 2, 48) = 5.076664$,otherwise reject$H_1$

so that reject H_0 because $F^* > 5.076664$,

therefore, the mean outputs differ for three machine, and the P-value is 2.90e-10

(d) Test the mean outputs differ for the operator:

H_0:all $\beta_{j(i)}$ equal zero(i=1,2,3)

VS. H_1:not all $\beta_{j(i)}$ equal zero

$F^* = \frac{MSB(A)}{MSE} = 252.5/23.6 = 10.7$

we can reject H_0 if $F^* > F(1-0.01; 9, 48) = 2.801816$,otherwise reject$H_1$

so that reject H_0 because $F^* > 2.801816$,

therefore, our conclusion implies that operator within at least one machine differ in terms of mean shifts effects, and the P-value is 6.99e-09

(e)

```
means = with(dat, by(Y, list(A, B), mean))
means_A = with(dat, by(Y, list(A), mean))
sapply(1:3, function(x){
      n*sum((means[x,]-means_A[x])^2)
})

## [1]  599.20 1538.55  134.55
```

SS	df	MS
599.2	3	199.7333
1538.55	3	512.8500
134.55	3	44.8500

H_0:all $\beta_{j(1)}$ equal zero

VS. H_1:not all $\beta_{j(1)}$ equal zero

$F^* = \frac{MSB(A_i)}{MSE} = 199.7333/23.6 = 8.46$

we can reject H_0 if $F^* > F(1-0.01; 3, 48) = 4.22$,otherwise reject$H_1$

so that reject H_0 because $F^* > 4.22$,

therefore, our conclusion implies that operator within machine 1 differs in terms of mean shifts effects, and

H_0:all $\beta_{j(2)}$ equal zero
VS. H_1:not all $\beta_{j(2)}$ equal zero
$F^* = \frac{MSB(A_i)}{MSE} = 512.85/23.6 = 21.73$
we can reject H_0 if $F^* > F(1-0.01; 3, 48) = 4.22$,otherwise reject$H_1$
so that reject H_0 because $F^* > 4.22$,
therefore, our conclusion implies that operator within machine 2 differs in terms of mean shifts effects, and

H_0:all $\beta_{j(3)}$ equal zero
VS. H_1:not all $\beta_{j(3)}$ equal zero
$F^* = \frac{MSB(A_i)}{MSE} = 44.85/23.6 = 1.9$
we can reject H_0 if $F^* > F(1-0.01; 3, 48) = 4.22$,otherwise reject$H_1$
so that reject H_1 because $F^* < 4.22$,
therefore, our conclusion implies that operator within machine 3 does not differ in terms of mean shifts effects, and

(f)

$$\begin{aligned}\alpha &\leq 1-(1-\alpha_1)...(1-\alpha_5)\\ &= 1-(1-0.01)^5\\ &= 0.04900995\end{aligned}$$

We conclude that three machine differ in mean output, 4 operators in machine 1 have different mean output effect, 4 operators in machine 2 have different mean output effect, but 4 operators in machine 3 do not have different mean output effect.

3 26.6

(a)

```
means = with(dat, by(Y, A, mean))
D1 = means[1] - means[2]
D2 = means[1] - means[3]
D3 = means[2] - means[3]
tukey = 1/sqrt(2)*qtukey(0.95, 3, 48)
tukey

## [1] 2.418488

mse = 23.6
s = sqrt(2*mse/(b*n))
s
```

```
## [1] 1.536229

  c(D1-s*tukey, D1+s*tukey)

##          1          1
## -13.465351  -6.034649

  c(D2-s*tukey, D2+s*tukey)

##          1          1
## -16.065351  -8.634649

  c(D3-s*tukey, D3+s*tukey)

##         2         2
## -6.315351  1.115351
```

$$\bar{Y}_{1..} = 61.2, \bar{Y}_{2..} = 70.95, \bar{Y}_{3..} = 73.55$$

$$\hat{D}_1 = \bar{Y}_{1..} - \bar{Y}_{2..} = -9.75, \hat{D}_2 = \bar{Y}_{1..} - \bar{Y}_{3..} = -12.35, \hat{D}_3 = \bar{Y}_{2..} - \bar{Y}_{3..} = -2.6$$

$$S = \sqrt{\frac{MSE}{bn} * 2} = 1.536229, Tukey = \frac{1}{\sqrt{2}}\text{qtukey}(1 - alpha, a, ab(n-1)) = 2.418488$$

$$\text{base on}\hat{D}_i \pm S * T$$

$$-13.465351 \le D_1 \le -6.034649$$

$$-16.065351 \le D_2 \le -8.634649$$

$$-6.315351 \le D_3 \le 1.115351$$

We conclude that with 95% family confidence that the mean output is highest in machine 3, and the difference between machine 2 and machine 3 is not statistically significant.

(b)

```
  means = with(dat, by(Y, list(A,B) , mean))
  D1 = means[1,1] - means[1,2]
  D2 = means[1,1] - means[1,3]
  D3 = means[1,1] - means[1,4]
  D4 = means[1,2] - means[1,3]
  D5 = means[1,2] - means[1,4]
  D6 = means[1,3] - means[1,4]
  B = qt(1-0.05/(2*6), (n-1)*a*b)
  B

## [1] 2.752023
```

```r
  mse = 23.6
  s = sqrt(2*mse/(n))
  s

## [1] 3.072458

  c(D1-s*B, D1+s*B)

## [1] -14.455477   2.455477

  c(D2-s*B, D2+s*B)

## [1] -9.255477  7.655477

  c(D3-s*B, D3+s*B)

## [1]  0.7445233 17.6554767

  c(D4-s*B, D4+s*B)

## [1] -3.255477 13.655477

  c(D5-s*B, D5+s*B)

## [1]  6.744523 23.655477

  c(D6-s*B, D6+s*B)

## [1]  1.544523 18.455477
```

$$\bar{Y}_{11\cdot} = 61.8, \bar{Y}_{12\cdot} = 67.8, \bar{Y}_{13\cdot} = 62.6, \bar{Y}_{14\cdot} = 52.6$$

$$\hat{D}_1 = \bar{Y}_{11\cdot} - \bar{Y}_{12\cdot} = -6, \hat{D}_2 = \bar{Y}_{11\cdot} - \bar{Y}_{13\cdot} = -0.8, \hat{D}_3 = \bar{Y}_{11\cdot} - \bar{Y}_{14\cdot} = -9.2$$

$$\hat{D}_4 = \bar{Y}_{12\cdot} - \bar{Y}_{13\cdot} = 5.2, \hat{D}_5 = \bar{Y}_{12\cdot} - \bar{Y}_{14\cdot} = 15.2, \hat{D}_6 = \bar{Y}_{13\cdot} - \bar{Y}_{14\cdot} = 10$$

$$S = \sqrt{\frac{MSE}{n} * 2} = 3.072458, B = t(1 - \alpha/(2*6), ab(n-1)) = 2.752023$$

$$\text{base on} \hat{D}_i \pm S * B$$

$$-14.455477 \leq D_1 \leq 2.455477$$

$$-9.255477 \leq D_2 \leq 7.655477$$

$$0.7445233 \leq D_3 \leq 17.6554767$$

$$-3.255477 \leq D_4 \leq 13.655477$$

$$6.744523 \leq D_5 \leq 23.655477$$

$$1.544523 \leq D_6 \leq 18.455477$$

We conclude that with 95% family confidence in machine 1 the differences between operator 1 and operator 2, operator 1 and operator 3, operator 2 and operator 3 are not statistically significant.

(c)

```
L_hat = (means[1,1]+means[1,2]+means[1,3])/3 - means[1,4]
s = sqrt(mse/(n)*((1/3)^2*3+1))
s

## [1] 2.508652

t = qt(1-0.01/2, a*b*(n-1))
t

## [1] 2.682204

c(L_hat-s*t, L_hat+s*t)

## [1]  4.737951 18.195382
```

$$\hat{L} = \frac{\bar{Y}_{11.} + \bar{Y}_{12.} + \bar{Y}_{13.}}{3} + \bar{Y}_{14.} = 11.46667$$
$$c_1 = c_2 = c_3 = 1/3, c_4 = -1$$
$$S = \sqrt{\frac{MSE}{n} * \sum_i c_i^2} = 2.508652$$
$$t = t(1 - \alpha/2, ab(n-1)) = 2.682204$$
$$\text{base on } \hat{L} \pm S * t$$
$$4.737951 \leq D_1 \leq 18.195382$$

We are 99% confident that L is between 0.737951 and 18.195382.

4 26.7

(a) $\beta_{j(i)}$ are $i.i.d$ $N(0, \sigma_\beta^2$, and $\beta_{j(i)}$ and ϵ_{ijk} are independent.

(b)
```
model_new = aov(Y ~ A+ Error(A/B), data = dat)
summary(model_new)

##
## Error: A
##   Df Sum Sq Mean Sq
## A  2   1696   847.8
##
## Error: A:B
##           Df Sum Sq Mean Sq F value Pr(>F)
## Residuals  9   2272   252.5
##
## Error: Within
##           Df Sum Sq Mean Sq F value Pr(>F)
## Residuals 48   1133    23.6

s_square = (252.5-23.6)/n
s_square

## [1] 45.78
```

$$E(MSB(A)) = \sigma^2 + n\sigma_\beta^2$$
$$E(MSE) = \sigma^2$$
$$\hat{\sigma}_\beta^2 = s_\beta^2 = (MSB(A) - MSE)/n = 45.78$$

(c) Test:

$$H_0{:}\sigma^2_\beta = 0$$

$$\text{VS. } H_1{:}\sigma^2_\beta \neq 0$$

$$F^* = \frac{MSB(A)}{MSE} = 252.5/23.6 = 10.7$$

we can reject H_0 if $F^* > F(1-0.01; 9, 48) = 1.765318$,otherwise reject$H_1$

so that reject H_0 because $F^* > 1.765318$,

therefore, our conclusion implies that operator within at least one machine differ in terms of mean shifts effects, and the P-value is 6.976208e-09

```
c1=1/5
c2=-1/5
ms1=252.5
ms2=23.6
df1=9
df2=48
F1=qf(.95,df1,Inf)
F2=qf(.95,df2,Inf)
F3=qf(.95,Inf,df1)
F4=qf(.95,Inf,df2)
F5=qf(.95,df1,df2)
F6=qf(.95,df2,df1)
G1=1-1/F1
G2=1-1/F2
G3=((F5-1)^2-(G1*F5)^2-(F4-1)^2)/F5
G4=F6*( ((F6-1)/F6)^2 - ((F3-1)/F6)^2 - G2^2 )
Hl = sqrt( (G1*c1*ms1)^2 + ((F4-1)*c2*ms2)^2 - G3*c1*c2*ms1*ms2)
Hl

## [1] 23.77552

Hu = sqrt( (G2*c2*ms2)^2 + ((F3-1)*c1*ms1)^2 - G4*c1*c2*ms1*ms2)
Hu

## [1] 86.09976

sigma_mu =  (ms1-ms2)/(n)
c(max(0,sigma_mu-Hl), sigma_mu+Hu)

## [1]  22.00448 131.87976
```

E(MSB(A))= $nb\sigma_\beta^2+\sigma^2$ E(MSE)=σ^2
Base on $L=\sigma_\mu^2=c_1E(MSB(A))+c_2E(MSE)$
then $c_1=1/(n)=0.2$, $c_2=-1/(n)=-0.2$
and $MSB(A)=252.5, MSE=23.6, df1=9, df2=48$

According to MLS procedure, $H_l=22.00448$ $H_u=131.87976$ $\sigma_\beta^2=45.78$

so that 90% confident interval is $s_\beta^2-H_l\leq\sigma_\mu^2\leq s_\beta^2+H_u$,which means $22.00448\leq\sigma_\beta^2\leq 131.87976$

(d) Test the mean outputs differ for three machine:

H_0:all α_i equal zero(i=1,2,3)

VS. H_1:not all α_i equal zero

$$F^*=\frac{MSA}{MSB(A)}=847.8/252.5=3.357624$$

we can reject H_0 if $F^*>F(1-0.1;2,9)=3.006452$,otherwise reject$H_1$

so that reject H_0 because $F^*>3.006452$,

therefore, the mean outputs differ for three machine, and the P-value is 0.08140399

(e)
```
  means = with(dat, by(Y, A, mean))
  D1 = means[1] - means[2]
  D2 = means[1] - means[3]
  D3 = means[2] - means[3]
  tukey = 1/sqrt(2)*qtukey(0.9, 3, 9)
  tukey

## [1] 2.344595

  msb_a = 252.5
  s = sqrt(2*msb_a/(b*n))
  s

## [1] 5.024938

  c(D1-s*tukey, D1+s*tukey)

##          1          1
## -21.531444   2.031444

  c(D2-s*tukey, D2+s*tukey)
```

```
##          1          1
## -24.131444  -0.568556

  c(D3-s*tukey, D3+s*tukey)

##          2          2
## -14.381444   9.181444
```

$$\bar{Y}_{1..} = 61.2, \bar{Y}_{2..} = 70.95, \bar{Y}_{3..} = 73.55$$

$$\hat{D}_1 = \bar{Y}_{1..} - \bar{Y}_{2..} = -9.75, \hat{D}_2 = \bar{Y}_{1..} - \bar{Y}_{3..} = -12.35, \hat{D}_3 = \bar{Y}_{2..} - \bar{Y}_{3..} = -2.6$$

$$S = \sqrt{\frac{MSB(A)}{bn} * 2} = 5.024938, Tukey = \frac{1}{\sqrt{2}}\text{qtukey}(1 - alpha, a, a(b-1) = 2.344595$$

$$\text{base on} \hat{D}_i \pm S * T$$

$$-21.531444 \leq D_1 \leq 2.031444$$

$$-24.131444 \leq D_2 \leq -0.568556$$

$$-14.381444 \leq D_3 \leq 9.181444$$

We conclude that with 95% family confidence that the mean output is highest in machine 3, and the differences between machine 2 and machine 3, machine 1 and machine 2 are not statistically significant.

(f)

```
  means.a = as.numeric(with(dat, by(Y, A, mean)))
  means.ab = with(dat, by(Y, list(A, B), mean))
  means.ab.mx = matrix(means.ab, ncol = a, byrow =  TRUE)
  means.ab.mx

##      [,1] [,2] [,3]
## [1,] 61.8 75.8 76.8
## [2,] 67.8 75.2 69.6
## [3,] 62.6 55.8 74.4
## [4,] 52.6 77.0 73.4

  betas = sapply(1:a,
                 function(i)
                   means.ab.mx[,i] - median(means.ab.mx[,i]))
  betas = abs(as.numeric(betas))

  model_brown = aov(betas ~ factor(c(rep(1,b), rep(2,b), rep(3,b))))
  summary(model_brown)
```

```
##                                      Df Sum Sq Mean Sq F value
## factor(c(rep(1, b), rep(2, b), rep(3, b)))  2   23.3   11.64   0.306
## Residuals                             9  342.1   38.02
##                                      Pr(>F)
## factor(c(rep(1, b), rep(2, b), rep(3, b)))  0.744
## Residuals
```

Set $d_{ij} = |Y_{ij} - \tilde{Y}_i|$

$$H_0\text{:all } \sigma^2(\beta_{j(i)}) \text{ are equal(i=1,2,3)}$$
$$\text{VS. } H_1\text{:not all } \sigma^2(\beta_{j(i)}) \text{ are equal zero}$$
$$F^*_{BF} = \frac{MSTR(d)}{MSE(d)} = 11.64/38.02 = 0.306$$
$$\text{we can reject } H_0 \text{ if } F^* > F(1-0.01;2,9) = 8.021517\text{,otherwise reject} H_1$$
$$\text{so that reject } H_1 \text{ because } F^* < 8.021517,$$
$$\text{therefore, all } \sigma^2(\beta_{j(i)}) \text{ are equal(i=1,2,3)}$$

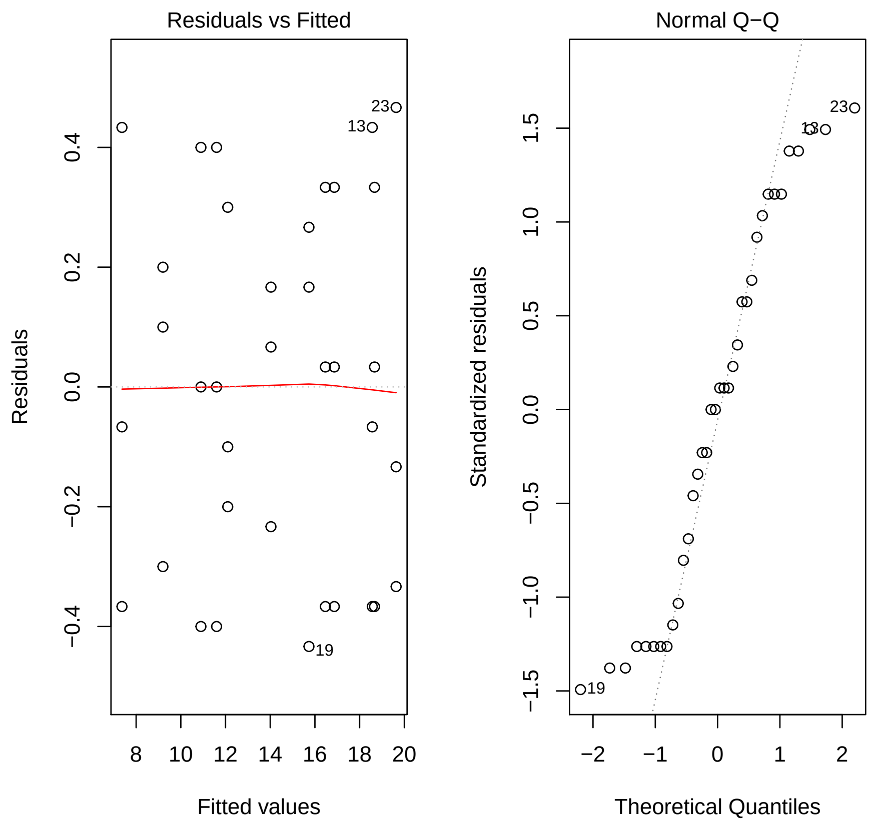
5 26.19

```
dat = read.table("CH26PR19.txt")
names(dat) = c("Y", "A", "B", "k")
dat$A = factor(dat$A)
dat$B = factor(dat$B)
a = length(unique(dat$A))
b = length(unique(dat$B))
n = length(unique(dat$k))
model = aov(Y ~ (A/B), data = dat)
resid(model)
```

```
##             1             2             3             4             5
## -4.000000e-01  7.382983e-15  4.000000e-01  3.333333e-02  3.333333e-01
##             6             7             8             9            10
## -3.666667e-01 -3.666667e-01  3.333333e-02  3.333333e-01  6.666667e-02
##            11            12            13            14            15
## -2.333333e-01  1.666667e-01  4.333333e-01 -6.666667e-02 -3.666667e-01
##            16            17            18            19            20
## -2.000000e-01  3.000000e-01 -1.000000e-01 -4.333333e-01  1.666667e-01
##            21            22            23            24            25
##  2.666667e-01 -1.333333e-01  4.666667e-01 -3.333333e-01 -3.666667e-01
##            26            27            28            29            30
##  3.333333e-01  3.333333e-02 -6.666667e-02  4.333333e-01 -3.666667e-01
##            31            32            33            34            35
## -3.000000e-01  2.000000e-01  1.000000e-01  4.000000e-01 -1.249001e-16
```

```
##              36
## -4.000000e-01
```

```r
par( mfrow = c(1,2))
plot(model, which = 1)
plot(model, which = 2)
```



The residuals versus fitted values plots shows no sign for unequal variance.And the QQ-plot indicates approximately normal distribution with slightly light tail, so that normality assumption seems to be reasonable, we can use subsample model here.

6 26.20

(a)
```r
model_final = aov(Y ~ Error(A/B), data = dat)
summary(model_final)
```

```
##
## Error: A
##           Df Sum Sq Mean Sq F value Pr(>F)
## Residuals  3  343.2   114.4
##
## Error: A:B
##           Df Sum Sq Mean Sq F value Pr(>F)
## Residuals  8  187.4   23.43
##
## Error: Within
##           Df Sum Sq Mean Sq F value Pr(>F)
## Residuals 24  3.033  0.1264
```

(b)
```r
114.4/23.43
```

```
## [1] 4.882629
```

```r
qf(1-0.05, 3, 8)
```

```
## [1] 4.066181
```

```r
pf(4.882629, 3, 8, lower.tail = FALSE)
```

```
## [1] 0.03242618
```

$$H_0{:}\sigma_\tau^2 = 0$$

$$\text{VS. } H_1{:}\sigma_\tau^2 \neq 0$$

$$F^* = \frac{MSTR}{MSEE} = 252.5/23.6 = 4.882629$$

we can reject H_0 if $F^* > F(1-0.05;3,8) = 4.066181$,otherwise reject$H_1$

so that reject H_0 because $F^* > 4.066181$,

therefore, our conclusion implies that there are variations in mean concentration levels between plants, and the P-value is 0.03242618

(c)
```r
23.43/0.1264
```

```
## [1] 185.3639
```

```r
qf(1-0.05, 8, 24)
```

```
## [1] 2.355081
```

```r
pf(185.3639, 8, 24, lower.tail = FALSE)
```

```
## [1] 1.15931e-19
```

$$H_0\text{:}\sigma^2 = 0$$

$$\text{VS. } H_1\text{:}\sigma^2 \neq 0$$

$$F^* = \frac{MSEE}{MSOE} = 23.43/0.1264 = 185.3639$$

we can reject H_0 if $F^* > F(1-0.05; 8, 24) = 2.355081$,otherwise reject$H_1$

so that reject H_0 because $F^* > 2.355081$,

therefore, our conclusion implies that there m'e variations in mean concentration levels between leaves of the same plant, and P-value is 1.15931e-19

(d)

```r
Y_mean = mean(dat$Y)
Y_mean
```

```
## [1] 14.26111
```

```r
t = qt(1-0.05/2, 3)
t
```

```
## [1] 3.182446
```

```r
MSTR = 114.4
s = sqrt(MSTR/(a*b*n))
s
```

```
## [1] 1.782632
```

```r
c(Y_mean-s*t, Y_mean+s*t)
```

```
## [1]  8.58798 19.93424
```

$$\bar{Y}_{\dots} = 14.26111$$

$$S = \sqrt{\frac{MSTR}{rmn}} = 1.782632$$

$$T = t(1-\alpha/2, r-1) = 3.182446$$

$$\text{base on } \bar{Y}_{\dots} \pm S * T$$

$$8.58798 \leq \mu_{\dots} \leq 19.93424$$

(e)

$$E(MSTR) = \sigma_\eta^2 + m\sigma^2 + nm\sigma_\tau^2$$

$$E(MSEE) = \sigma_\eta^2 + m\sigma^2$$

$$E(MSOE) = \sigma_\eta^2$$

$$s_\tau^2 = \frac{MSTR - MSEE}{nm} = (114.4 - 23.43)/(3*3) = 10.10778$$

$$s^2 = \frac{MSEE - MSOE}{m} = (23.43 - 0.1264)/(3) = 7.767867$$

$$s_\eta^2 = MSOE = 0.1264$$

Therefore, σ_τ^2 appears to be most important in the total variance

(f)

```
c1=1/9
c2=-1/9
ms1=114.4
ms2=23.43
df1=3
df2=8
F1=qf(.95,df1,Inf)
F2=qf(.95,df2,Inf)
F3=qf(.95,Inf,df1)
F4=qf(.95,Inf,df2)
F5=qf(.95,df1,df2)
F6=qf(.95,df2,df1)
G1=1-1/F1
G2=1-1/F2
G3=((F5-1)^2-(G1*F5)^2-(F4-1)^2)/F5
G4=F6*( ((F6-1)/F6)^2 - ((F3-1)/F6)^2 - G2^2 )
Hl = sqrt( (G1*c1*ms1)^2 + ((F4-1)*c2*ms2)^2 - G3*c1*c2*ms1*ms2)
Hl

## [1] 9.039359
```

```r
Hu = sqrt( (G2*c2*ms2)^2 + ((F3-1)*c1*ms1)^2 - G4*c1*c2*ms1*ms2)
Hu
```

```
## [1] 95.41479
```

```r
sigma_mu =  (ms1-ms2)/(b*n)
c(max(0,sigma_mu-Hl), sigma_mu+Hu)
```

```
## [1]   1.068419 105.522573
```

$s_\tau^2 = \frac{MSTR-MSEE}{nm}$ Base on $L = \sigma_\mu^2 = c_1E(MSTR) + c_2E(MSEE)$
then $c_1 = 1/(nm) = 0.11111$, $c_2 = -1/(nm) = -0.11111$
and $MSTR = 114.4, MSEE = 23.43, df1 = 3, df2 = 8$

According to MLS procedure, $H_l = 9.039359$ $H_u = 95.41479$ $\sigma_\tau^2 = 10.10778$

so that 90% confident interval is $s_\tau^2 - H_l \le \sigma_\tau^2 \le s_\tau^2 + H_u$,which means $1.068419 \le \sigma_\tau^2 \le 105.522573$

7 26.24

$$
\begin{aligned}
SSB + SSAB &= na\sum_j(\bar{Y}_{.j.} - \bar{Y}_{...})^2 + n\sum_i\sum_j(\bar{Y}_{ij.} - \bar{Y}_{i..} - \bar{Y}_{.j.} + \bar{Y}_{...})^2 \\
&= na\sum_j(\bar{Y}_{.j.} - \bar{Y}_{...})^2 + n\sum_i\sum_j((\bar{Y}_{ij.} - \bar{Y}_{i..})^2 + (\bar{Y}_{.j.} - \bar{Y}_{...})^2 - 2(\bar{Y}_{ij.} - \bar{Y}_{i..})(\bar{Y}_{.j.} - \bar{Y}_{...})) \\
&= na\sum_j(\bar{Y}_{.j.} - \bar{Y}_{...})^2 + n\sum_i\sum_j((\bar{Y}_{ij.} - \bar{Y}_{i..})^2) + na\sum_j((\bar{Y}_{.j.} - \bar{Y}_{...})^2) \\
&- 2n\sum_j((\bar{Y}_{.j.} - \bar{Y}_{...})\sum_i(\bar{Y}_{ij.} - \bar{Y}_{i..})) \\
&= na\sum_j(\bar{Y}_{.j.} - \bar{Y}_{...})^2 + n\sum_i\sum_j((\bar{Y}_{ij.} - \bar{Y}_{i..})^2) + na\sum_j((\bar{Y}_{.j.} - \bar{Y}_{...})^2) \\
&- 2na\sum_j(\bar{Y}_{.j.} - \bar{Y}_{...})^2 \\
&= n\sum_i\sum_j((\bar{Y}_{ij.} - \bar{Y}_{i..})^2) \\
&= SSB(A)
\end{aligned}
$$

8 26.25

(a) Since

$$\bar{Y}_{ijk} = \mu_{ij} + \alpha_i + \beta_{j(i)} + \epsilon_{ijk}$$

then:

$$\begin{aligned}
\sigma^2\{\bar{Y}_{i\cdot\cdot}\} &= \sigma^2\{\mu_{i\cdot\cdot} + \alpha_i + \bar{\beta}_{\cdot(i)} + \bar{\epsilon}_{i\cdot\cdot}\} \\
&= \sigma^2\{\bar{\beta}_{\cdot(i)} + \bar{\epsilon}_{i\cdot\cdot}\} \\
&= \frac{\sigma_\beta^2}{b} + \frac{\sigma^2}{bn} \qquad \text{,since } \beta \text{ and } \epsilon \text{ are independent}
\end{aligned}$$

$$\begin{aligned}
\sigma^2\{\bar{Y}_{\cdots}\} &= \sigma^2\{\mu_{\cdots} + \bar{\beta}_{\cdot(\cdot)} + \bar{\epsilon}_{\cdots}\} \qquad \text{,since } \sum_i \alpha = 0 \\
&= \sigma^2\{\bar{\beta}_{\cdot(\cdot)} + \bar{\epsilon}_{\cdots}\} \\
&= \frac{\sigma_\beta^2}{ab} + \frac{\sigma^2}{abn} \qquad \text{,since } \beta \text{ and } \epsilon \text{ are independent}
\end{aligned}$$

(b)

$$\begin{aligned}
E(MSB(A)) &= \sigma^2 + n\sigma_\beta^2 \\
E(MSE) &= \sigma^2 \\
s_\beta^2 &= (MSB(A) - MSE)/n \\
\hat{\sigma}_\beta^2 &= max(0, s_\beta^2) = max(0, (MSB(A) - MSE)/n)
\end{aligned}$$

9 26.28

$$\begin{aligned}
s^2\{\bar{Y}_{1j\cdot\cdot} - \bar{Y}_{2j\cdot\cdot}\} &= \frac{2}{cn}(MSBC(A) + \frac{MSC(A) - MSE}{b}) \\
&= \frac{2}{c}(\sigma_{\beta\gamma}^2 + \frac{\sigma^2}{n} + \sigma_\gamma^2)
\end{aligned}$$

$$\begin{aligned}
df &= \frac{(c_1 MS_1 + ... + c_h MS_h)^2}{\frac{(c_1 MS_1)^2}{df_1} + ... + \frac{(c_h MS_h)^2}{df_h}} \\
&= \frac{[bMSBC(A) + MSC(A) - MSE]^2}{\frac{(bMSBC(A))^2}{a(b-1)(c-1)} + \frac{(MSC(A))^2}{a(c-1)} + \frac{(MSE^2}{abc(n-1)}}
\end{aligned}$$