Stat 207 HW8

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March 11, 2015

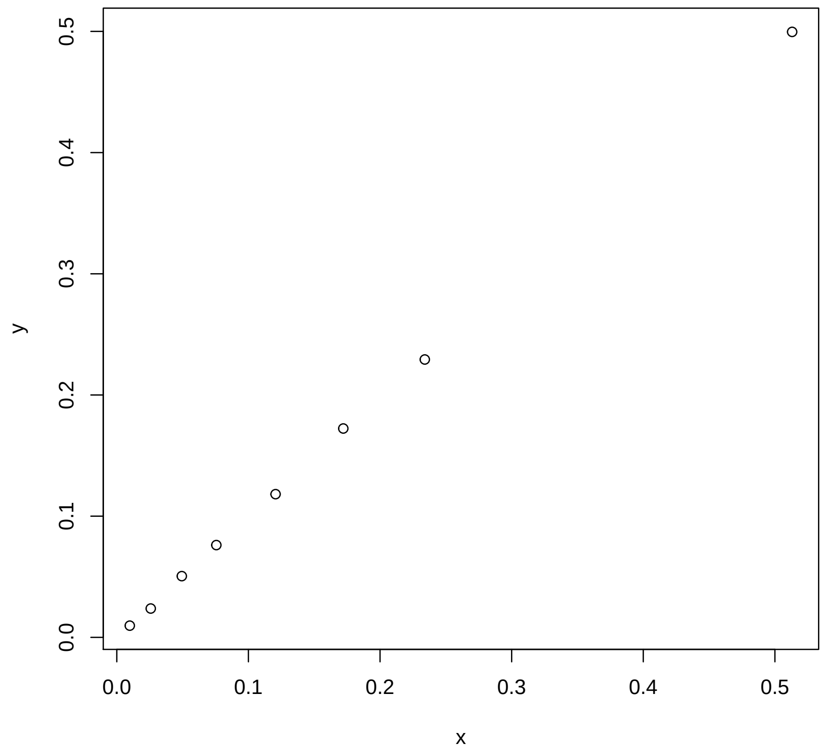
1 14.28

(a)

```
dat = read.table("CH14PR14.txt")
names(dat) = c("Y", "X1", "X2", "X3")
logit = glm(Y ~ X1 + X2 , data = dat, family = "binomial")
logitv = logit$fitted.values
dat = dat[order(logitv), ]
a = rep(1:8, each = 20)
a = a[-1]
b = split(dat, a)
Oj1 = sapply(b, function(x){sum(x[[1]])})
Ej1 = sapply(split(sort(logitv), a), sum)
Oj0 = sapply(b, function(x){length(x[[1]])-sum(x[[1]])})
Ej0 = sapply(b, function(x){length(x[[1]])})-Ej1
rbind(Oj1, Ej1, Oj0, Ej0)
```

```
##             1          2          3         4         5        6         7
## Oj1  0.000000  1.0000000  0.0000000  2.000000  1.000000  8.00000  2.000000
## Ej1  0.187472  0.5159059  0.9878718  1.512501  2.412695  3.44151  4.680125
## Oj0 19.000000 19.0000000 20.0000000 18.000000 19.000000 12.00000 18.000000
## Ej0 18.812528 19.4840941 19.0121282 18.487499 17.587305 16.55849 15.319875
##             8
## Oj1 10.000000
## Ej1 10.261919
## Oj0 10.000000
## Ej0  9.738081
```

```
y = sapply(split(sort(logitv), a), median)
x = Ej1/sapply(b, function(x){length(x[[1]])})
plot(x, y)
```



The plot seems to be linear, it's consistent with a response function of monotonic sigmoidal shape.

(b)

```
X.squ = sum((rbind(Oj1, Oj0)-rbind(Ej1, Ej0))^2/rbind(Ej1, Ej0));X.squ
```

```
## [1] 12.11578
```

$$H_0{:}E(Y) = [1 + exp(-\beta_0 - \beta_1 X1 - \beta_2 X2)]^{-1}$$

$$\text{VS. } H_1{:}E(Y) \neq [1 + exp(-\beta_0 - \beta_1 X1 - \beta_2 X2)]^{-1}$$

$$X^2 = \sum_j \sum_k \frac{(O_{jk} - E_{jk})^2}{E_{jk}} = 12.11578$$

we can reject H_0 if $X^2 > \chi^2(0.95, 8-2) = 12.5916$,otherwise reject$H_1$

so that reject H_1 because $X^2 < 12.5916$, Pvalue is 0.05943518.

(c)
```r
p = summary(logit)
dr = p$deviance.resid;dr
```

```
##           1           2           3           4           5           6
## -0.54602312 -0.51373259  1.15260237 -0.17517944 -0.18924467 -0.69185916
##           7           8           9          10          11          12
## -0.18367445 -0.73777334 -0.47288089 -0.35855218 -0.62785882 -0.33244658
##          13          14          15          16          17          18
## -0.49058859 -0.23345211 -0.11467682 -0.59485941 -0.85058862 -0.16214474
##          19          20          21          22          23          24
## -0.22270681 -0.13939895 -0.34822874 -0.28168748 -0.24470112 -0.09084127
##          25          26          27          28          29          30
## -0.50595282 -0.10798927 -0.69416101 -0.26781191 -0.83451488 -0.68109529
##          31          32          33          34          35          36
## -0.32519247 -0.45851859 -0.92893094 -0.53211206 -0.33897782 -0.16073331
##          37          38          39          40          41          42
##  0.74365705  1.38971984 -0.61999873 -0.41310085  1.89079347 -0.35679774
##          43          44          45          46          47          48
##  1.71627562 -0.33244658 -0.72647043 -0.32961191  2.84304938 -0.69729005
##          49          50          51          52          53          54
## -0.10990097 -0.18854516 -0.33530428 -0.21617318 -0.72171075 -0.59079687
##          55          56          57          58          59          60
##  2.23462510 -0.24806553 -0.42012390 -0.28029236 -1.30166964  2.00689554
##          61          62          63          64          65          66
##  1.10849910 -0.15933400 -0.68109529 -0.32124458 -0.10387671 -1.01249619
##          67          68          69          70          71          72
## -0.12087236 -0.36424502 -0.62497861  1.86302691 -0.23960963 -0.48311783
##          73          74          75          76          77          78
## -0.38006335  1.82187884 -1.44789883 -0.84697959 -0.83715068 -0.35248880
##          79          80          81          82          83          84
## -0.58403320 -0.45631940 -0.35855218  1.94481807 -0.72970910 -0.53462959
##          85          86          87          88          89          90
## -0.36293048 -1.15124676 -0.62286031 -0.65995923  2.34647993 -0.81345988
##          91          92          93          94          95          96
##  1.12311780 -0.17737218  1.27437628 -0.55762351 -0.49703364 -0.30029670
##          97          98          99         100         101         102
## -1.06493877 -0.71059309  0.95982139 -0.33201751 -0.67802259  1.90510957
##         103         104         105         106         107         108
## -0.45631940 -0.42162878 -0.12900059 -0.17648324 -0.19257415 -0.14876060
##         109         110         111         112         113         114
##  1.82969794 -0.32283373 -0.53091804 -0.13409993 -0.61013597 -0.72086770
##         115         116         117         118         119         120
## -0.50887859 -0.61087534 -0.16356841 -0.16073331 -0.12675915 -0.49293406
##         121         122         123         124         125         126
## -0.14931436 -0.21351074  1.84295971 -0.40421564  1.40875849 -0.30814469
```

```
##        127        128        129        130        131        132
## -0.40618549 -0.49528940 -0.51130056 -0.73777334 -0.28412789 -0.58877426
##        133        134        135        136        137        138
## -0.49703364 -1.22831729 -0.31850052  1.89079347 -0.29409388 -0.69416101
##        139        140        141        142        143        144
## -0.20982689 -0.48482421 -0.61925036 -0.28658859 -0.39176366 -0.48311783
##        145        146        147        148        149        150
## -1.11866815 -0.31307905 -0.35422338 -0.28065744 -0.72970910 -0.20982689
##        151        152        153        154        155        156
## -0.41110024 -0.42725579 -0.28554134 -0.38936035 -0.29263987 -0.25647527
##        157        158        159
##  0.42476809  0.86785144  1.67453806
```

```
lows = lowess(logitv, dr, .7, 0);lows
```

```
## $x
##   [1] 0.004117568 0.005380657 0.005813874 0.006020913 0.006553816
##   [6] 0.007278447 0.008001755 0.008286057 0.008951093 0.009668986
##  [11] 0.011003867 0.011085487 0.012613438 0.012834524 0.012834524
##  [16] 0.013059435 0.013288233 0.015226801 0.015452533 0.015607367
##  [21] 0.016726682 0.017571713 0.017617602 0.017747398 0.018371549
##  [26] 0.021773129 0.021773129 0.022535610 0.023094563 0.024494189
##  [31] 0.026882013 0.028298274 0.029495578 0.030299729 0.032354793
##  [36] 0.035226199 0.038520373 0.038618820 0.038897215 0.039560541
##  [41] 0.039947136 0.040234706 0.041915256 0.042323848 0.044087651
##  [46] 0.046367188 0.047827674 0.049456440 0.050290414 0.050776320
##  [51] 0.051501501 0.052872923 0.053626356 0.053761250 0.053761250
##  [56] 0.054663641 0.055833717 0.058830122 0.060233817 0.060809646
##  [61] 0.061668811 0.062257449 0.062257449 0.063737383 0.063737383
##  [66] 0.064184764 0.069677590 0.072999098 0.073868811 0.078447141
##  [71] 0.079182416 0.081029912 0.081787246 0.082350177 0.084469906
##  [76] 0.085049591 0.087232199 0.098877166 0.098877166 0.099783195
##  [81] 0.105784220 0.110149156 0.110149156 0.110883724 0.113379806
##  [86] 0.114401854 0.115431916 0.116197114 0.116197114 0.120141456
##  [91] 0.121446705 0.122531422 0.123624463 0.131455955 0.132006995
##  [96] 0.133171735 0.133478501 0.138491730 0.143988948 0.150896868
## [101] 0.156796870 0.159137868 0.160140341 0.162160612 0.162883307
## [106] 0.167369727 0.167369727 0.169836784 0.170211426 0.174474811
## [111] 0.174857529 0.176323549 0.176323549 0.177411440 0.178894230
## [116] 0.183003454 0.187514843 0.190211007 0.195692078 0.205353025
## [121] 0.207010578 0.207010578 0.212848500 0.214103167 0.214103167
## [126] 0.215812168 0.223121468 0.228813560 0.229282366 0.229282366
## [131] 0.231934023 0.233743020 0.233743020 0.238263628 0.238263628
## [136] 0.246094701 0.281693191 0.294047656 0.295601219 0.301407172
## [141] 0.303543886 0.350437581 0.370722956 0.380731597 0.401048267
```

```
## [146] 0.432802264 0.443961491 0.465118050 0.484534317 0.514661378
## [151] 0.529698682 0.532220820 0.540973420 0.571374597 0.630886977
## [156] 0.649433730 0.686202121 0.758423830 0.913735656
##
## $y
##   [1] -0.088172324 -0.091193892 -0.092230236 -0.092725513 -0.094000327
##   [6] -0.095733792 -0.097464093 -0.098144200 -0.099735105 -0.101452449
##  [11] -0.104645759 -0.104841011 -0.108496180 -0.109025064 -0.109025064
##  [16] -0.109563096 -0.110097898 -0.114629173 -0.115156807 -0.115518722
##  [21] -0.118135047 -0.120110252 -0.120217515 -0.120520906 -0.121979817
##  [26] -0.129930790 -0.129930790 -0.131674467 -0.132952707 -0.136153436
##  [31] -0.141614024 -0.144852796 -0.147590845 -0.149429815 -0.154063466
##  [36] -0.160537759 -0.167965287 -0.168187260 -0.168814970 -0.170298321
##  [41] -0.171162836 -0.171805908 -0.175564005 -0.176477711 -0.180421979
##  [46] -0.185519548 -0.188785532 -0.192398644 -0.194248659 -0.195326551
##  [51] -0.196935228 -0.199977472 -0.201648820 -0.201948059 -0.201948059
##  [56] -0.203949843 -0.206545438 -0.213049988 -0.216097110 -0.217347109
##  [61] -0.219212173 -0.220489979 -0.220489979 -0.223702599 -0.223702599
##  [66] -0.224673765 -0.235828329 -0.242573485 -0.244070632 -0.251951891
##  [71] -0.253217613 -0.256397943 -0.257701638 -0.258386084 -0.260963379
##  [76] -0.261668196 -0.264321944 -0.225045104 -0.225045104 -0.219128032
##  [81] -0.179936676 -0.154186042 -0.154186042 -0.149852507 -0.135127045
##  [86] -0.129097542 -0.124673161 -0.121386438 -0.121386438 -0.104444471
##  [91] -0.098838092 -0.094178954 -0.092681826 -0.081955110 -0.082230732
##  [96] -0.082813318 -0.082966758 -0.085474301 -0.094606794 -0.103042914
## [101] -0.106432963 -0.107778064 -0.107963004 -0.108335712 -0.108469038
## [106] -0.109296710 -0.109296710 -0.112443596 -0.112921475 -0.118359689
## [111] -0.118847870 -0.120717870 -0.120717870 -0.122933578 -0.125953576
## [116] -0.134322830 -0.144160594 -0.150039986 -0.164525881 -0.191343995
## [121] -0.195401210 -0.195401210 -0.209690769 -0.212761835 -0.212761835
## [126] -0.214943573 -0.224274748 -0.224766647 -0.224807160 -0.224807160
## [131] -0.225036311 -0.225576792 -0.225576792 -0.226927430 -0.226927430
## [136] -0.228381345 -0.216959869 -0.223662497 -0.223545473 -0.223108129
## [141] -0.222011054 -0.180446860 -0.167914434 -0.163179048 -0.152280685
## [146] -0.128270882 -0.116634223 -0.092479525 -0.069813508 -0.033587453
## [151] -0.015034380 -0.011922648 -0.001158003  0.035380766  0.102419493
## [156]  0.122606667  0.162324533  0.242116112  0.426978094
```

It shows that the model is adequate, because the plot shows approximately a horizontal line with zero intercept.

2 14.39

(a)
```
dat = read.table("CH14PR39.txt")
names(dat) = c("Y", "X1", "X2", "X3", "X4")
poi = glm(Y ~ . , data = dat, family = "poisson")
summary(poi)
```

```
##
## Call:
## glm(formula = Y ~ ., family = "poisson", data = dat)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -2.1854  -0.7819  -0.2564   0.5449   2.3626
##
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept)  0.489467   0.336869   1.453  0.14623
## X1          -1.069403   0.133154  -8.031 9.64e-16 ***
## X2          -0.046606   0.119970  -0.388  0.69766
## X3           0.009470   0.002953   3.207  0.00134 **
## X4           0.008566   0.004312   1.986  0.04698 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for poisson family taken to be 1)
##
##     Null deviance: 199.19  on 99  degrees of freedom
## Residual deviance: 108.79  on 95  degrees of freedom
## AIC: 377.29
##
## Number of Fisher Scoring iterations: 5
```

$$
\begin{aligned}
b_0 &= 0.489467 & s(b_0) &= 0.336869 \\
b_1 &= -1.069403 & s(b_1) &= 0.133154 \\
b_2 &= -0.046606 & s(b_2) &= 0.119970 \\
b_3 &= 0.009470 & s(b_3) &= 0.002953 \\
b_4 &= 0.008566 & s(b_4) &= 0.004312
\end{aligned}
$$

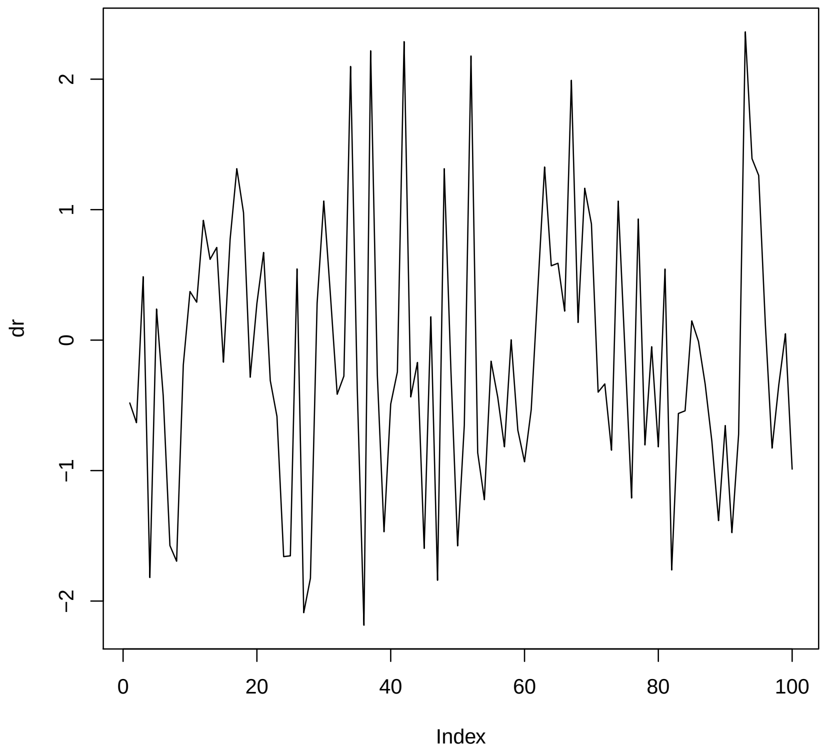
$$\mu = exp(0.489467165 - 1.069402551X1 - 0.046606063X2 + 0.009469987X3 + 0.008565829X4)$$

(b)
```
p = summary(poi)
dr = p$deviance.resid;dr
```

```
##           1            2            3            4            5
```

```
## -0.481563003 -0.632820229  0.485684782 -1.819828480  0.238302497
##           6           7           8           9          10
## -0.427206484 -1.574566470 -1.694831446 -0.190494237  0.372414202
##          11          12          13          14          15
##  0.290568448  0.917894609  0.618945304  0.710218879 -0.169117683
##          16          17          18          19          20
##  0.774212615  1.313966589  0.976533023 -0.284161510  0.281078819
##          21          22          23          24          25
##  0.671826155 -0.309823439 -0.585274974 -1.659501048 -1.653549229
##          26          27          28          29          30
##  0.545695411 -2.089197070 -1.825162331  0.283158353  1.066089357
##          31          32          33          34          35
##  0.338495236 -0.414912576 -0.276062096  2.097482057 -0.373770021
##          36          37          38          39          40
## -2.185378873  2.217373787 -0.269196968 -1.468917177 -0.490568471
##          41          42          43          44          45
## -0.243689445  2.287659410 -0.435460637 -0.171363655 -1.596370920
##          46          47          48          49          50
##  0.178588604 -1.840819930  1.313615961 -0.233624875 -1.576829772
##          51          52          53          54          55
## -0.652282150  2.177759158 -0.862317963 -1.223172388 -0.161565009
##          56          57          58          59          60
## -0.438457085 -0.817656201  0.002451800 -0.690070466 -0.932653968
##          61          62          63          64          65
## -0.532725622  0.400431092  1.326821516  0.569505372  0.589162479
##          66          67          68          69          70
##  0.222202292  1.991163395  0.134986579  1.163783804  0.890153115
##          71          72          73          74          75
## -0.398888037 -0.335860573 -0.843868576  1.065308951 -0.068529260
##          76          77          78          79          80
## -1.210403791  0.928316209 -0.803424449 -0.050707876 -0.817762385
##          81          82          83          84          85
##  0.544674549 -1.761754891 -0.562223893 -0.541853422  0.147418906
##          86          87          88          89          90
## -0.009840529 -0.337422359 -0.774688780 -1.383658148 -0.654757856
##          91          92          93          94          95
## -1.475851025 -0.722343228  2.362545161  1.391144309  1.262066676
##          96          97          98          99         100
##  0.117099736 -0.827717570 -0.345172951  0.048822205 -0.988857456
```

```r
plot(dr, type = "l")
```



(c)
```
logLik(poi)

## 'log Lik.' -183.6439 (df=5)

poiR = glm(Y ~ .-X2 , data = dat, family = "poisson")
logLik(poiR)

## 'log Lik.' -183.7194 (df=4)

qchisq(1-0.05, 5-4)

## [1] 3.841459

pchisq(0.151, 1, lower.tail = FALSE)

## [1] 0.6975815
```

$$H_0\text{:}\beta_2 = 0$$
$$\text{VS. } H_1\text{:}\beta_2 \neq 0$$
The full model: $\mu = exp(\beta_0 + \beta_1 X1 + \beta_2 X2 + \beta_3 X3 + \beta_4 X4)$

ln(L(F))= -183.6439

The reduced model: $\mu = exp(\beta_0 + \beta_1 X1 + \beta_3 X3 + \beta_4 X4)$

ln(L(R))= -183.7194

G^2 = -2(ln(L(R)-ln(L(F)))) = 0.151

we can reject H_0 if $G^2 > \chi^2(1 - 0.05, 5 - 4) = 3.8415$,otherwise reject$H_1$

so that reject H_1 because $G^2 < 3.8415$,

therefore, X2 can be dropped from the regression model, and the P-value is 0.6975815.And the result is the same as the result we get in (b).

(d)
```
summary(poiR)

##
## Call:
## glm(formula = Y ~ . - X2, family = "poisson", data = dat)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -2.2152  -0.7512  -0.2594   0.5830   2.2893
##
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept)  0.443890   0.317289   1.399  0.16181
## X1          -1.077770   0.131415  -8.201 2.38e-16 ***
## X3           0.009471   0.002957   3.203  0.00136 **
## X4           0.008979   0.004190   2.143  0.03209 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for poisson family taken to be 1)
##
##     Null deviance: 199.19  on 99  degrees of freedom
## Residual deviance: 108.94  on 96  degrees of freedom
## AIC: 375.44
##
## Number of Fisher Scoring iterations: 5

b1 = -1.077770
s1 = 0.131415
z = qnorm(1-0.05/2)
c(b1-s1*z, b1+s1*z)
```

```
## [1] -1.3353387 -0.8202013
```

From summary(poiR), we get $s(b_1) = 0.131415, b_1 = -1.077770$, based on $b_k \pm z(1-\alpha/2)sb_k$, we conclude that we are 95 % confident that β_1 is between -1.3353387 and -0.8202013. Because the confidence interval is smaller than 0, aerobic exercise reduce the frequency of falls when controlling for balance and strength.

3 14.44

$$lnL(\beta_0, \beta_1) = \sum_{i=1}^{n} y_i(\beta_0 + \beta_1 X_i) - \sum_{i=1}^{n}(1 + exp(\beta_0 + \beta_1 X_i))$$

$$\frac{\partial lnL}{\partial \beta_0} = -\sum_{i=1}^{n}(Y_i - \frac{exp(\beta_0 + \beta_1 X_i)}{[1 + exp(\beta_0 + \beta_1 X_i)]^2})$$

$$\frac{\partial lnL}{\partial \beta_1} = -\sum_{i=1}^{n}(Y_i X_i - \frac{exp(\beta_0 + \beta_1 X_i)}{[1 + exp(\beta_0 + \beta_1 X_i)]^2})$$

$$\frac{\partial^2 lnL}{\partial \beta_0^2} = -\sum_{i=1}^{n} \frac{exp(\beta_0 + \beta_1 X_i)}{[1 + exp(\beta_0 + \beta_1 X_i)]^2}$$

$$\frac{\partial^2 lnL}{\partial \beta_1^2} = -\sum_{i=1}^{n} \frac{X_i^2 exp(\beta_0 + \beta_1 X_i)}{[1 + exp(\beta_0 + \beta_1 X_i)]^2}$$

$$\frac{\partial^2 lnL}{\partial \beta_0 \partial \beta_1} = -\sum_{i=1}^{n} \frac{X_i exp(\beta_0 + \beta_1 X_i)}{[1 + exp(\beta_0 + \beta_1 X_i)]^2}$$

```
  dat = read.table("CH14TA01.txt")
  names(dat) = c("X", "Y", "pi")
  b0 = -3.0597
  b1 = 0.1615
  t = exp(b0 + b1*dat$X)
  t1 = sum(t/(1+t)^2);t1

## [1] 4.176239

  t2 = sum((t * dat$X)/(1 + t)^2);t2

## [1] 74.57466

  t3 = sum(((dat$X)^2*t)/(1 + t)^2);t3

## [1] 1568.482

  H = matrix(c(t, t1, t2, t3), 2, 2);H
```

```
##             [,1]      [,2]
## [1,] 0.4499135 0.1236006
## [2,] 5.0723286 2.6586009

  H_1 = solve(H);H_1

##             [,1]       [,2]
## [1,]  4.670787 -0.2171488
## [2,] -8.911367  0.7904346

  sqrt(1.586)

## [1] 1.259365

  sqrt(0.7904346)

## [1] 0.8890639
```

H^{-1} we get after rootsquare is the same as the estimated standard deviation in table 14.1(b)

4 14.45

$$
\begin{aligned}
Y_i &= \frac{\gamma_0}{1+\gamma_1 exp(\gamma_2 X_i)} + \epsilon_i \\
E(Y_i) &= \frac{\gamma_0}{1+\gamma_1 exp(\gamma_2 X_i)} \\
\text{if} \gamma_0 &= 1 \\
E(Y_i) &= \frac{1}{1+\gamma_1 exp(\gamma_2 X_i)} \\
&= \frac{1}{1+exp(ln\gamma_1+\gamma_2 X_i)} \\
&= \frac{1}{1+exp(-\beta_0-\beta_1 X_i)} \\
&= \frac{exp(\beta_0+\beta_1 X_i)}{1+exp(\beta_0+\beta_1 X_i)} \\
\text{if} \gamma_1 &= exp(-\beta_0), \gamma_2 = -\beta_1
\end{aligned}
$$

5 14.46

$$E(Y) = [1 + exp(-\beta_0 - \beta_1 X_1 - \beta_2 X_2 - \beta_3 X_1 X_2)]^{-1}$$
$$\pi^{'}(X_1) = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_1 X_2$$
$$\pi^{'}(X_1 + 1) = \beta_0 + \beta_1 (X_1 + 1) + \beta_2 X_2 + \beta_3 (X_1 + 1) X_2$$
$$\pi^{'}(X_1 + 1) - \pi^{'}(X_1) = ln(oddsratio) = \beta_1 X_1 + \beta_3 X_1 X_2$$

Hence the odds ratio for X1 is $exp(\beta_1 X_1 + \beta_3 X_1 X_2)$therefore, they are different.

6 14.47

$$\pi = 1 - exp[-exp(\frac{X_i - \gamma_0}{\gamma_1})]$$
$$1 - \pi = exp[-exp(\frac{X_i - \gamma_0}{\gamma_1})]$$
$$ln[-ln(1 - \pi)] = -\frac{\gamma_0}{\gamma_1} + \frac{1}{\gamma_1} X_i$$
$$= \beta_0 + \beta_1 X_i$$