

# Stat 207 HW7

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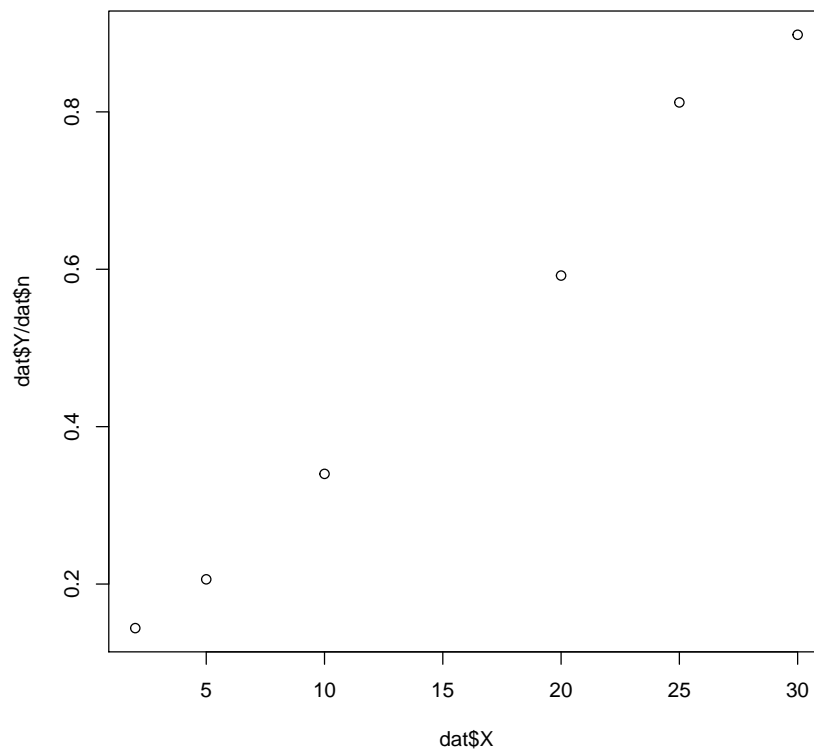
March 4, 2015



## 1 14.11

(a)

```
dat = read.table("CH14PR11.txt")
names(dat) = c("X", "n", "Y")
plot(dat$X, dat$Y/dat$n)
```



The plot support the analyst's belief that the logistic response function is appropriate.

(b)

```
logit = glm(Y/n ~ X, data = dat, weight = n, family = "binomial")
summary(logit)

##
## Call:
## glm(formula = Y/n ~ X, family = "binomial", data = dat, weights = n)
##
## Deviance Residuals:
```

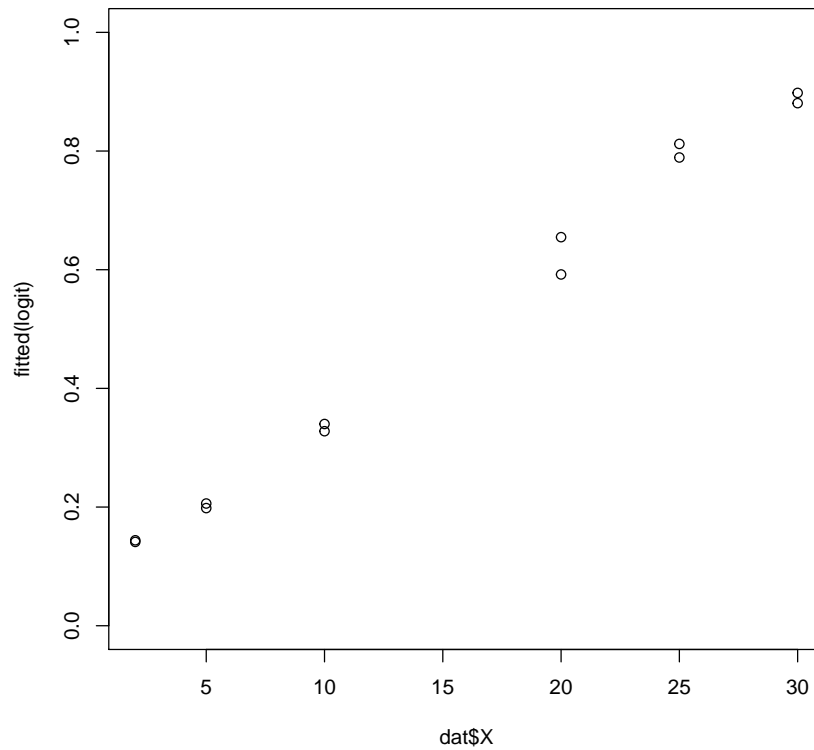
```
##          1          2          3          4          5          6
## 0.1754    0.4330    0.5784   -2.9193    1.2710    1.2209
##
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept) -2.076565   0.084839  -24.48  <2e-16 ***
## X           0.135851   0.004772   28.47  <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##    Null deviance: 1108.171  on 5  degrees of freedom
## Residual deviance:   12.181  on 4  degrees of freedom
## AIC: 53.419
##
## Number of Fisher Scoring iterations: 3
```

From the summary, the maximum likelihood estimates of  $\hat{\beta}_0 = -2.0766$ ,  $\hat{\beta}_1 = 0.1359$ ,

$$\hat{\pi} = \frac{\exp(\beta_0 + \beta_1 X)}{1 + \exp(\beta_0 + \beta_1 X)} = \frac{\exp(-2.0766 + 0.1359X)}{1 + \exp(-2.0766 + 0.1359X)}$$

(c) 

```
plot(dat$X, fitted(logit), ylim = c(0, 1))
points(dat$X, dat$Y/dat$n, lwd = 1)
```



The fitted logistic response function appears to be well.

(d) `exp(0.1359)`

```
## [1] 1.145567
```

$\exp(\beta_1) = 1.145567$ , so that the odds of the bottles being returned is increased by 14.5567% with each one deposit level increased.

(e) `newdat = data.frame(X = 15)`  
`predict(logit, newdata = newdat, type = "response")`

```
##      1
## 0.4903005
```

The estimated probability that a bottle will be returned when the deposit is 15 cents is 0.4903005.

```
(f) newpi = 0.75
    pi_2 = log(newpi/(1-newpi))
    (pi_2 - (-2.0766))/0.1359

## [1] 23.36433
```

Estimate the amount of deposit for which 75% of the bottles are expected to be returned is 23.36433.

## 2 14.14

```
(a) dat = read.table("CH14PR14.txt")
    names(dat) = c("Y", "X1", "X2", "X3")
    logit = glm(Y ~ X1 + X2 + X3, data = dat, family = "binomial")
    summary(logit)

##
## Call:
## glm(formula = Y ~ X1 + X2 + X3, family = "binomial", data = dat)
##
## Deviance Residuals:
##      Min       1Q   Median       3Q      Max
## -1.4037  -0.5637  -0.3352  -0.1542   2.9394
##
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept) -1.17716     2.98242  -0.395  0.69307
## X1           0.07279     0.03038   2.396  0.01658 *
## X2          -0.09899     0.03348  -2.957  0.00311 **
## X3           0.43397     0.52179   0.832  0.40558
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##      Null deviance: 134.94  on 158  degrees of freedom
## Residual deviance: 105.09  on 155  degrees of freedom
## AIC: 113.09
##
## Number of Fisher Scoring iterations: 6
```

From the summary, the maximum likelihood estimates  $\hat{\beta}_0 = -1.17716$ ,

$$\hat{\beta}_1 = 0.07279, \hat{\beta}_2 = -0.09899, \hat{\beta}_3 = 0.43397$$

$$\hat{\pi} = \frac{\exp(\beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3)}{1 + \exp(\beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3)} = \frac{\exp(-1.17716 + 0.07279X_1 - 0.09899X_2 + 0.43397X_3)}{1 + \exp(-1.17716 + 0.07279X_1 - 0.09899X_2 + 0.43397X_3)}$$

(b)

```
exp(0.07279)

## [1] 1.075505

exp(-0.09899)

## [1] 0.9057518

exp(0.43397)

## [1] 1.543373
```

- $\exp(\beta_1) = 1.075505$ , so that the odds of getting a flu shot is increased by 7.5% with each one age increased.
- $\exp(\beta_2) = 0.9057518$ , so that the odds of getting a flu shot is decreased by 9.4% with each one health awareness index increased.
- $\exp(\beta_3) = 1.543373$ , so that the odds of getting a flu shot is increased by 54.3% from woman to man.

(c)

```
newdat = data.frame(X1 = 55, X2 = 60, X3 = 1)
predict(logit, newdata = newdat, type = "response")

##          1
## 0.06422197
```

The estimated probability with X1=55, X2=60 and X3=1 is 0.06422197

### 3 14.17

(a)

```
dat = read.table("CH14PR11.txt")
names(dat) = c("X", "n", "Y")
logit = glm(Y/n ~ X, weight=n, data = dat, family = "binomial")
summary(logit)
```

```
##
## Call:
## glm(formula = Y/n ~ X, family = "binomial", data = dat, weights = n)
##
## Deviance Residuals:
##      1      2      3      4      5      6
## 0.1754  0.4330  0.5784 -2.9193  1.2710  1.2209
##
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept) -2.076565   0.084839  -24.48  <2e-16 ***
## X            0.135851   0.004772   28.47  <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##      Null deviance: 1108.171  on 5  degrees of freedom
## Residual deviance:   12.181  on 4  degrees of freedom
## AIC: 53.419
##
## Number of Fisher Scoring iterations: 3

b1 = 0.1359
s1 = 0.004772
z = qnorm(1-0.05/2)
c(b1-s1*z, b1+s1*z)

## [1] 0.1265471 0.1452529

c(exp(b1-s1*z), exp(b1+s1*z))

## [1] 1.134903 1.156332
```

From `summary(logit)`, we get  $s(b_1) = 0.004772$ ,  $b_1 = 0.1359$ , based on  $b_k \pm z(1-\alpha/2)sb_k$ , we conclude that we are 95 % confident that  $\beta_1$  is between 0.1265471 and 0.1452529, and corresponding confidence limits for the odds ratio  $\exp(\beta_1)$  is between 1.134903 and 1.156332.

(b) `qnorm(1-0.05/2)`

```
## [1] 1.959964
```

$$H_0: \beta_1 = 0$$



VS.  $H_1: \beta_1 \neq 0$

$$z^* = \frac{b_1}{s(b_1)} = 0.1359/0.004772 = 28.47863$$

we can reject  $H_0$  if  $|z^*| > Z(1 - 0.05/2) = 1.959964$ , otherwise reject  $H_1$   
 so that reject  $H_0$  because  $|z^*| > 1.959964$ ,  
 therefore,  $X_1$  can not be dropped from the regression model, and the  
 P-value is  $2e-16$

```
(c) logLik(logit)

## 'log Lik.' -24.70937 (df=2)

logitR = glm(Y/n ~ 1, weight = n, data = dat, family = "binomial")
logLik(logitR)

## 'log Lik.' -572.7044 (df=1)

qchisq(1-0.05, 2-1)

## [1] 3.841459

pchisq(1095.99, 1, lower.tail = FALSE)

## [1] 2.457179e-240
```

$H_0: \beta_1 = 0$

VS.  $H_1: \beta_1 \neq 0$

The full model:  $\pi = [1 + \exp(-(\beta_0 + \beta_1 X_1))]^{-1}$

$$\ln(L(F)) = -24.70937$$

The reduced model:  $\pi = [1 + \exp(-(\beta_0))]^{-1}$

$$\ln(L(R)) = -572.7044$$

$$G^2 = -2(\ln(L(R)) - \ln(L(F))) = 1095.99$$

we can reject  $H_0$  if  $G^2 > \chi^2(1 - 0.05, 2 - 1) = 3.8415$ , otherwise reject  $H_1$   
 so that reject  $H_0$  because  $G^2 > 3.8415$ ,

therefore,  $X_1$  cannot be dropped from the regression model, and the  
 P-value is  $2.457179e-240$ . And the result is different from the result we get  
 in (b).

## 4 14.20

```
(a) dat = read.table("CH14PR14.txt")
names(dat) = c("Y", "X1", "X2", "X3")
logit = glm(Y ~ X1 + X2 + X3, data = dat, family = "binomial")
summary(logit)

##
## Call:
## glm(formula = Y ~ X1 + X2 + X3, family = "binomial", data = dat)
##
## Deviance Residuals:
##      Min       1Q   Median       3Q      Max
## -1.4037  -0.5637  -0.3352  -0.1542   2.9394
##
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept) -1.17716     2.98242  -0.395  0.69307
## X1           0.07279     0.03038   2.396  0.01658 *
## X2          -0.09899     0.03348  -2.957  0.00311 **
## X3           0.43397     0.52179   0.832  0.40558
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##      Null deviance: 134.94  on 158  degrees of freedom
## Residual deviance: 105.09  on 155  degrees of freedom
## AIC: 113.09
##
## Number of Fisher Scoring iterations: 6
```

$$z(1 - \frac{0.1}{2*2}) = 0.975, s(b_1) = 0.03038, s(b_2) = 0.03348$$

$$\exp(30(0.07280.030381.96)) < \exp(30\beta_1) < \exp(30(0.0728 + 0.030381.96))$$

$$1.4878 < \exp(30\beta_1) < 52.9837$$

$$\exp(25(0.0990.033481.96)) < \exp(25\beta_2) < \exp(25(0.0728 + 0.033481.96))$$

$$0.0163 < \exp(25\beta_2) < 31.824$$

```
(b) qnorm(1-0.05/2)
```

```
## [1] 1.959964
```

$H_0: \beta_3 = 0$   
 VS.  $H_1: \beta_3 \neq 0$   
 $z^* = \frac{b_3}{s(b_3)} = 0.43397/0.52179 = 0.8316947$   
 we can reject  $H_0$  if  $|z^*| > Z(1 - 0.05/2) = 1.959964$ , otherwise reject  $H_1$   
 so that reject  $H_1$  because  $|z^*| < 1.959964$ ,  
 therefore, X3 can be dropped from the regression model, and the P-value  
 is 0.40558

```

(c)  logLik(logit)

## 'log Lik.' -52.54659 (df=4)

logitR = glm(Y ~ X1+X2, data = dat, family = "binomial")
logLik(logitR)

## 'log Lik.' -52.89769 (df=3)

qchisq(1-0.05, 4-3)

## [1] 3.841459

pchisq(0.70236, 1, lower.tail = FALSE)

## [1] 0.4019918

```

$H_0: \beta_3 = 0$   
 VS.  $H_1: \beta_3 \neq 0$   
 The full model:  $\pi = [1 + \exp(-(\beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3))]^{-1}$   
 $\ln(L(F)) = -52.54659$   
 The reduced model:  $\pi = [1 + \exp(-(\beta_0 + \beta_1 X_1 + \beta_2 X_2))]^{-1}$   
 $\ln(L(R)) = -52.89769$   
 $G^2 = -2(\ln(L(R)) - \ln(L(F))) = 0.70236$   
 we can reject  $H_0$  if  $G^2 > \chi^2(1 - 0.05, 4 - 3) = 3.8415$ , otherwise reject  $H_1$   
 so that reject  $H_1$  because  $G^2 < 3.8415$ ,  
 therefore, X3 can be dropped from the regression model, and the P-value  
 is 0.4019918. And the result is the same as the result we get in (b).

```
(d) logitF = glm(Y ~ X1+X2+I(X1^2)+I(X2^2)+I(X1*X2), data = dat, family = "binomial")
      logLik(logitF)

## 'log Lik.' -52.13072 (df=6)

      logitR = glm(Y ~ X1+X2, data = dat, family = "binomial")
      logLik(logitR)

## 'log Lik.' -52.89769 (df=3)

      qchisq(1-0.05, 6-3)

## [1] 7.814728

      pchisq(1.53394, 3, lower.tail = FALSE)

## [1] 0.6744594
```

$$H_0: \beta_3 = \beta_4 = \beta_5 = 0$$

$$\text{VS. } H_1: \text{not all } \beta_3, \beta_4, \beta_5 \text{ equal } 0$$

The full model:

$$\pi = [1 + \exp(-(\beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_1^2 + \beta_4 X_2^2 + \beta_5 X_1 * X_2))]^{-1}$$

$$\ln(L(F)) = -52.13072$$

$$\text{The reduced model: } \pi = [1 + \exp(-(\beta_0 + \beta_1 X_1 + \beta_2 X_2))]^{-1}$$

$$\ln(L(R)) = -52.89769$$

$$G^2 = -2(\ln(L(R)) - \ln(L(F))) = 1.53394$$

we can reject  $H_0$  if  $G^2 > \chi^2(1 - 0.05, 6 - 3) = 7.814728$ , otherwise reject  $H_1$

so that reject  $H_1$  because  $G^2 < 7.814728$ ,

therefore,  $X_1^2, X_2^2, I(X_1 * X_2)$  can be dropped from the regression model, and the P-value is 0.6744594. And the result is the same as the result we get in (b).

## 5 14.22

```
(a) dat = read.table("CH14PR14.txt")
      names(dat) = c("y", "x1", "x2", "x3")
      dat.new = scale(dat[, 2:4], scale = FALSE)
      dat.new = as.data.frame(cbind(dat$y, dat.new))
      names(dat.new) = c("y", "x1", "x2", "x3")
```

```

alpha = .1
wrap.foo = function(formula, dat.new. = dat.new)
{
  model = glm(formula, family = binomial, data = dat.new.)
  p.val = summary(model)$coef
  p.val[nrow(p.val), 4]
}
tmp = c(wrap.foo(y ~ x1),
        wrap.foo(y ~ x2),
        wrap.foo(y ~ I(x1*x2)),
        wrap.foo(y ~ I(x1^2)),
        wrap.foo(y ~ I(x2^2)),
        wrap.foo(y ~ I(x1^2):I(x2^2)))
any(tmp < alpha)

## [1] TRUE

which.min(tmp)

## [1] 1

tmp = c(wrap.foo(y ~ x1 + x2),
        wrap.foo(y ~ x1 + I(x1*x2)),
        wrap.foo(y ~ x1 + I(x1^2)),
        wrap.foo(y ~ x1 + I(x2^2))
        )
any(tmp < alpha)

## [1] TRUE

which.min(tmp)

## [1] 1

tmp = c(wrap.foo(y ~ x1 + x2 + I(x1*x2)),
        wrap.foo(y ~ x1 + x2 + I(x1^2)),
        wrap.foo(y ~ x1 + x2 + I(x2^2))
        )
any(tmp < alpha)

## [1] FALSE

```

X1 enters in step 1 ; X2 enters in step 2; no variables satisfy criterion

for entry in step 3.

```
(b) wrap.foo = function(formula, dat.new. = dat.new)
{
  model = glm(formula, family = binomial, data = dat.new.)
  p.val = summary(model)$coef
  p.val = p.val[, 4]
  p.val[-1]

}

tmp = wrap.foo(y ~ x1 + x2 + I(x1*x2) + I(x1^2) + I(x2^2))
any(tmp > alpha)

## [1] TRUE

which.max(tmp)

## I(x1^2)
##      4

tmp = wrap.foo(y ~ x1 + x2 + I(x1*x2) + I(x2^2))
any(tmp > alpha)

## [1] TRUE

which.max(tmp)

## I(x1 * x2)
##      3

tmp = wrap.foo(y ~ x1 + x2 + I(x2^2))
any(tmp > alpha)

## [1] TRUE

which.max(tmp)

## I(x2^2)
##      3

tmp = wrap.foo(y ~ x1 + x2)
any(tmp > alpha)

## [1] FALSE
```

X11 is deleted in step 1; X12 is deleted in step 2; X3 is deleted in step 3; X22 is deleted in step 4; X1 and X2 are retained in the model.

```
(c) model = glm(y ~ x1 + x2 + I(x1*x2) + I(x1^2) + I(x2^2) + I(x1^2*x2^2),
               family = binomial, data = dat.new)
step(model)

## Start:  AIC=116.67
## y ~ x1 + x2 + I(x1 * x2) + I(x1^2) + I(x2^2) + I(x1^2 * x2^2)
##
##               Df Deviance    AIC
## - I(x2^2)      1   102.68 114.68
## - I(x1^2)      1   103.56 115.56
## - I(x1 * x2)   1   103.84 115.84
## - I(x1^2 * x2^2) 1   104.26 116.26
## <none>                102.67 116.67
## - x2                1   107.48 119.48
## - x1                1   109.46 121.46
##
## Step:  AIC=114.68
## y ~ x1 + x2 + I(x1 * x2) + I(x1^2) + I(x1^2 * x2^2)
##
##               Df Deviance    AIC
## - I(x1^2)      1   103.83 113.83
## - I(x1 * x2)   1   103.84 113.84
## <none>                102.68 114.68
## - I(x1^2 * x2^2) 1   105.71 115.71
## - x2                1   107.94 117.94
## - x1                1   109.50 119.50
##
## Step:  AIC=113.83
## y ~ x1 + x2 + I(x1 * x2) + I(x1^2 * x2^2)
##
##               Df Deviance    AIC
## - I(x1 * x2)   1   104.74 112.74
## - I(x1^2 * x2^2) 1   105.75 113.75
## <none>                103.83 113.83
## - x1                1   109.97 117.97
## - x2                1   111.00 119.00
##
## Step:  AIC=112.74
## y ~ x1 + x2 + I(x1^2 * x2^2)
##
##               Df Deviance    AIC
## - I(x1^2 * x2^2) 1   105.80 111.80
```

```
## <none>          104.74 112.74
## - x1           1    109.97 115.97
## - x2           1    111.40 117.40
##
## Step:  AIC=111.8
## y ~ x1 + x2
##
##           Df Deviance    AIC
## <none>      105.80 111.80
## - x1       1    113.20 117.20
## - x2       1    116.27 120.27
##
## Call:  glm(formula = y ~ x1 + x2, family = binomial, data = dat.new)
##
## Coefficients:
## (Intercept)          x1          x2
##   -2.29705      0.07787     -0.09547
##
## Degrees of Freedom: 158 Total (i.e. Null);  156 Residual
## Null Deviance:      134.9
## Residual Deviance: 105.8  AIC: 111.8
```

The best model according to the AICp criterion is based on  $y \sim x1 + x2$ . AIC = 111.8.

(d) `step(model, k = log(159))`

```
## Start:  AIC=138.15
## y ~ x1 + x2 + I(x1 * x2) + I(x1^2) + I(x2^2) + I(x1^2 * x2^2)
##
##           Df Deviance    AIC
## - I(x2^2)    1    102.68 133.10
## - I(x1^2)    1    103.56 133.97
## - I(x1 * x2)  1    103.84 134.25
## - I(x1^2 * x2^2) 1    104.26 134.68
## - x2         1    107.48 137.89
## <none>       102.67 138.15
## - x1         1    109.46 139.87
##
## Step:  AIC=133.1
## y ~ x1 + x2 + I(x1 * x2) + I(x1^2) + I(x1^2 * x2^2)
##
##           Df Deviance    AIC
## - I(x1^2)    1    103.83 129.17
## - I(x1 * x2)  1    103.84 129.19
```



```

## - I(x1^2 * x2^2) 1 105.71 131.05
## <none> 102.68 133.10
## - x2 1 107.94 133.28
## - x1 1 109.50 134.84
##
## Step: AIC=129.17
## y ~ x1 + x2 + I(x1 * x2) + I(x1^2 * x2^2)
##
## Df Deviance AIC
## - I(x1 * x2) 1 104.74 125.02
## - I(x1^2 * x2^2) 1 105.75 126.03
## <none> 103.83 129.17
## - x1 1 109.97 130.24
## - x2 1 111.00 131.28
##
## Step: AIC=125.02
## y ~ x1 + x2 + I(x1^2 * x2^2)
##
## Df Deviance AIC
## - I(x1^2 * x2^2) 1 105.80 121.00
## <none> 104.74 125.02
## - x1 1 109.97 125.18
## - x2 1 111.40 126.61
##
## Step: AIC=121
## y ~ x1 + x2
##
## Df Deviance AIC
## <none> 105.80 121.00
## - x1 1 113.20 123.33
## - x2 1 116.27 126.41
##
## Call: glm(formula = y ~ x1 + x2, family = binomial, data = dat.new)
##
## Coefficients:
## (Intercept) x1 x2
## -2.29705 0.07787 -0.09547
##
## Degrees of Freedom: 158 Total (i.e. Null); 156 Residual
## Null Deviance: 134.9
## Residual Deviance: 105.8 AIC: 111.8

```

The best model according to the SBCp criterion is based on  $y \sim x_1 + x_2$ . SBC = 121.

## 6 14.23

```

dat = read.table("CH14PR11.txt")
names(dat) = c("X", "n", "Y")
logit = glm(Y/n ~ X, data = dat, family = "binomial")

## Warning: non-integer #successes in a binomial glm!

Oj1 = dat$Y
Ej1 = round(dat$n*fitted(logit), 1)
Oj0 = dat$n-dat$Y
Ej0 = dat$n-Ej1
rbind(Oj1, Oj0)

##      [,1] [,2] [,3] [,4] [,5] [,6]
## Oj1   72  103  170  296  406  449
## Oj0  428  397  330  204   94   51

rbind(Ej1, Ej0)

##      1      2      3      4      5      6
## Ej1  70.6  99.1 163.9 327.4 394.6 440.3
## Ej0 429.4 400.9 336.1 172.6 105.4  59.7

X.squ = sum((rbind(Oj1, Oj0)-rbind(Ej1, Ej0))^2/rbind(Ej1, Ej0));X.squ

## [1] 12.28748

```

$H_0: E(Y) = [1 + \exp(-\beta_0 - \beta_1 X)]^{-1}$   
 VS.  $H_1: E(Y) \neq [1 + \exp(-\beta_0 - \beta_1 X)]^{-1}$   
 $X^2 = \sum_j \sum_k \frac{(O_{jk} - E_{jk})^2}{E_{jk}} = 12.287$   
 we can reject  $H_0$  if  $X^2 > \chi^2(0.99, 3) = 13.2767$ , otherwise reject  $H_1$   
 so that reject  $H_1$  because  $X^2 < 13.2767$ ,

## 7 14.40

$$\begin{aligned}
 \frac{\exp(\beta_0 + \beta_1 X_i)}{1 + \exp(\beta_0 + \beta_1 X_i)} &= \frac{1}{\frac{1}{\exp(\beta_0 + \beta_1 X_i)} + 1} \\
 &= \frac{1}{\exp(0 - \beta_0 - \beta_1 X_i) + 1} \\
 &= [1 + \exp(-\beta_0 - \beta_1 X_i)]^{-1}
 \end{aligned}$$

## 8 14.41

$$\begin{aligned}
\ln g(Y_1, \dots, Y_n) &= \ln \prod_i \binom{n_i}{Y_i} \pi_i^{Y_i} (1 - \pi_i)^{n_i - Y_i} \\
&= \sum_i [\ln \binom{n_i}{Y_i} + Y_i \ln(\pi_i) + (n_i - Y_i) \ln(1 - \pi_i)] \\
&= \sum_i [\ln \binom{n_i}{Y_i} + Y_i \ln(\pi_i / (1 - \pi_i)) + (n_i) \ln(1 - \pi_i)] \\
&= \sum_i [\ln \binom{n_i}{Y_i} + Y_i \ln(\beta_0 + \beta_1 X_i) - (n_i) \ln(1 + \exp(-\beta_0 - \beta_1 X_i))]
\end{aligned}$$

## 9 14.42

$$\begin{aligned}
\pi_i &= \frac{\exp(\pi'_i)}{1 + \exp(\pi'_i)} \\
1 - \pi_i &= \frac{1}{1 + \exp(\pi'_i)} \\
\frac{\pi_i}{1 - \pi_i} &= \exp(\pi'_i) \\
F_L^{-1}(\pi_i) &= \pi'_i = \log_e \left( \frac{\pi_i}{1 - \pi_i} \right)
\end{aligned}$$

## 10 14.43

$$\begin{aligned}
\ln L(\beta_0, \beta_1) &= \sum_{i=1}^n y_i (\beta_0 + \beta_1 X_i) - \sum_{i=1}^n (1 + \exp(\beta_0 + \beta_1 X_i)) \\
\frac{\partial^2 \ln L}{\partial \beta_0^2} &= - \sum_{i=1}^n \frac{\exp(\beta_0 + \beta_1 X_i)}{[1 + \exp(\beta_0 + \beta_1 X_i)]^2} \\
\frac{\partial^2 \ln L}{\partial \beta_1^2} &= - \sum_{i=1}^n \frac{X_i^2 \exp(\beta_0 + \beta_1 X_i)}{[1 + \exp(\beta_0 + \beta_1 X_i)]^2} \\
\frac{\partial^2 \ln L}{\partial \beta_0 \partial \beta_1} &= - \sum_{i=1}^n \frac{X_i \exp(\beta_0 + \beta_1 X_i)}{[1 + \exp(\beta_0 + \beta_1 X_i)]^2} \\
-E \left\{ \frac{\partial^2 \ln L}{\partial \beta_0^2} \right\} &= -g_{00} \\
-E \left\{ \frac{\partial^2 \ln L}{\partial \beta_1^2} \right\} &= -g_{11} \\
-E \left\{ \frac{\partial^2 \ln L}{\partial \beta_0 \partial \beta_1} \right\} &= -g_{01} = -g_{10}, \text{ which is reduced to (14.51)}
\end{aligned}$$