Stat 207 HW8

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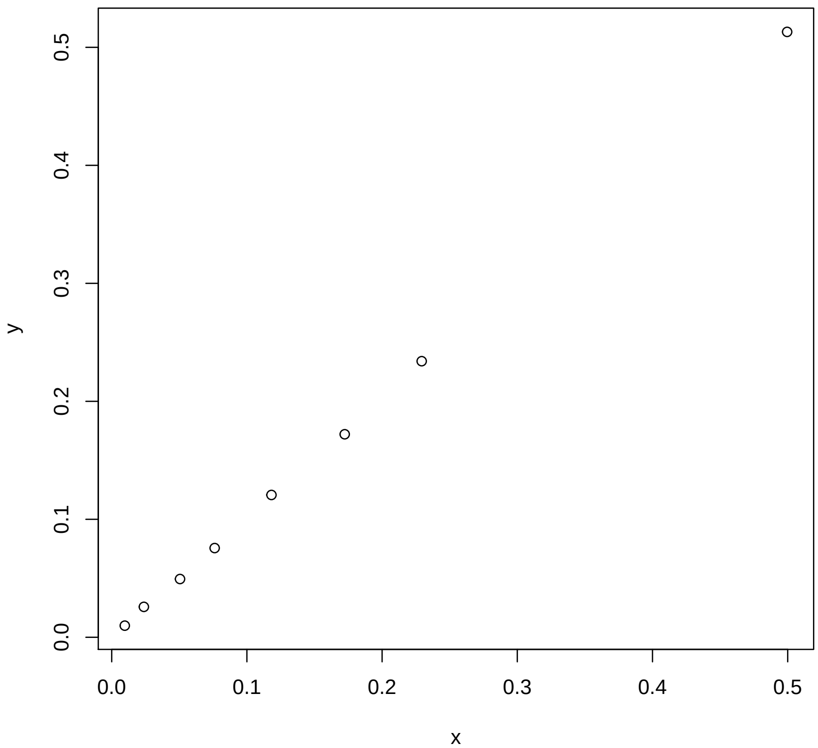
1 14.28

(a)

```
dat = read.table("CH14PR14.txt")
names(dat) = c("Y", "X1", "X2", "X3")
logit = glm(Y ~ X1 + X2 , data = dat, family = "binomial")
logitv = logit$fitted.values
dat = dat[order(logitv), ]
a = rep(1:8, each = 20)
a = a[-1]
b = split(dat, a)
Oj1 = sapply(b, function(x){sum(x[[1]])})
Ej1 = sapply(split(sort(logitv), a), sum)
Oj0 = sapply(b, function(x){length(x[[1]])-sum(x[[1]])})
Ej0 = sapply(b, function(x){length(x[[1]])})-Ej1
rbind(Oj1, Ej1, Oj0, Ej0)
```

```
##             1          2          3         4         5        6         7
## Oj1  0.000000  1.0000000  0.0000000  2.000000  1.000000  8.00000  2.000000
## Ej1  0.187472  0.5159059  0.9878718  1.512501  2.412695  3.44151  4.680125
## Oj0 19.000000 19.0000000 20.0000000 18.000000 19.000000 12.00000 18.000000
## Ej0 18.812528 19.4840941 19.0121282 18.487499 17.587305 16.55849 15.319875
##             8
## Oj1 10.000000
## Ej1 10.261919
## Oj0 10.000000
## Ej0  9.738081
```

```
x = sapply(split(sort(logitv), a), median)
y = Ej1/sapply(b, function(x){length(x[[1]])})
plot(x, y)
```



The plot seems to be linear, it's consistent with a response function of monotonic sigmoidal shape.

(b)

```r
X.squ = sum((rbind(Oj1, Oj0)-rbind(Ej1, Ej0))^2/rbind(Ej1, Ej0));X.squ
```

```
## [1] 12.11578
```

H_0:$E(Y) = [1 + exp(-\beta_0 - \beta_1 X1 - \beta_2 X2)]^{-1}$

VS. H_1:$E(Y) \neq [1 + exp(-\beta_0 - \beta_1 X1 - \beta_2 X2)]^{-1}$

$X^2 = \sum_j \sum_k \frac{(O_{jk} - E_{jk})^2}{E_{jk}} = 12.11578$

we can reject H_0 if $X^2 > \chi^2(0.95, 8-3) = 11.0705$,otherwise reject$H_1$

so that reject H_0 because $X^2 > 11.0705$, Pvalue is 0.03323531.

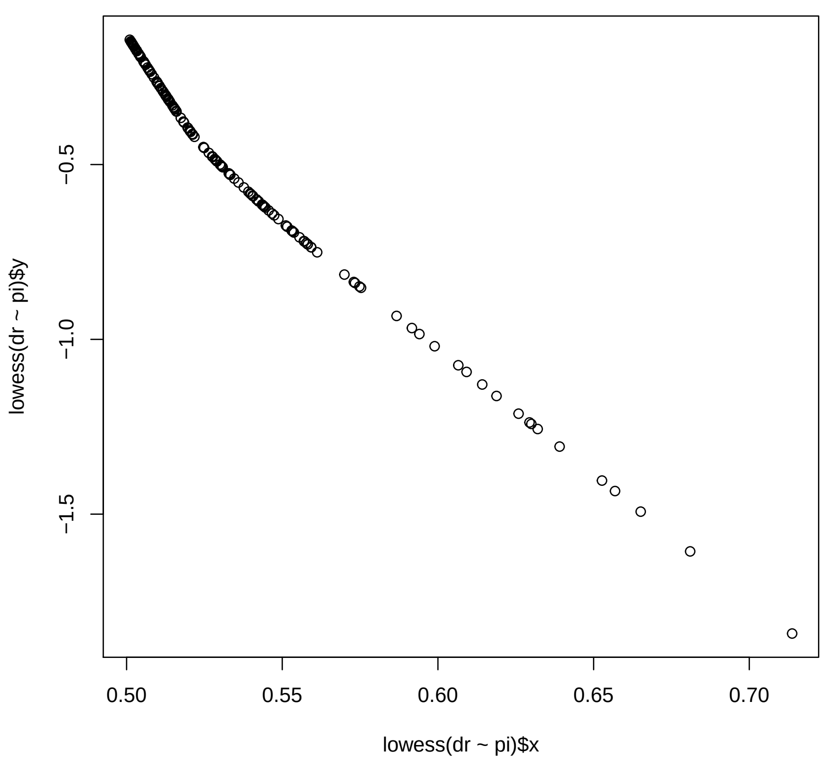
(c)

```
p = summary(logit)
dr = p$deviance.resid;dr
```

```
##           1           2           3           4           5           6
## -0.54602312 -0.51373259  1.15260237 -0.17517944 -0.18924467 -0.69185916
##           7           8           9          10          11          12
## -0.18367445 -0.73777334 -0.47288089 -0.35855218 -0.62785882 -0.33244658
##          13          14          15          16          17          18
## -0.49058859 -0.23345211 -0.11467682 -0.59485941 -0.85058862 -0.16214474
##          19          20          21          22          23          24
## -0.22270681 -0.13939895 -0.34822874 -0.28168748 -0.24470112 -0.09084127
##          25          26          27          28          29          30
## -0.50595282 -0.10798927 -0.69416101 -0.26781191 -0.83451488 -0.68109529
##          31          32          33          34          35          36
## -0.32519247 -0.45851859 -0.92893094 -0.53211206 -0.33897782 -0.16073331
##          37          38          39          40          41          42
##  0.74365705  1.38971984 -0.61999873 -0.41310085  1.89079347 -0.35679774
##          43          44          45          46          47          48
##  1.71627562 -0.33244658 -0.72647043 -0.32961191  2.84304938 -0.69729005
##          49          50          51          52          53          54
## -0.10990097 -0.18854516 -0.33530428 -0.21617318 -0.72171075 -0.59079687
##          55          56          57          58          59          60
##  2.23462510 -0.24806553 -0.42012390 -0.28029236 -1.30166964  2.00689554
##          61          62          63          64          65          66
##  1.10849910 -0.15933400 -0.68109529 -0.32124458 -0.10387671 -1.01249619
##          67          68          69          70          71          72
## -0.12087236 -0.36424502 -0.62497861  1.86302691 -0.23960963 -0.48311783
##          73          74          75          76          77          78
## -0.38006335  1.82187884 -1.44789883 -0.84697959 -0.83715068 -0.35248880
##          79          80          81          82          83          84
## -0.58403320 -0.45631940 -0.35855218  1.94481807 -0.72970910 -0.53462959
##          85          86          87          88          89          90
## -0.36293048 -1.15124676 -0.62286031 -0.65995923  2.34647993 -0.81345988
##          91          92          93          94          95          96
##  1.12311780 -0.17737218  1.27437628 -0.55762351 -0.49703364 -0.30029670
##          97          98          99         100         101         102
## -1.06493877 -0.71059309  0.95982139 -0.33201751 -0.67802259  1.90510957
##         103         104         105         106         107         108
## -0.45631940 -0.42162878 -0.12900059 -0.17648324 -0.19257415 -0.14876060
##         109         110         111         112         113         114
##  1.82969794 -0.32283373 -0.53091804 -0.13409993 -0.61013597 -0.72086770
##         115         116         117         118         119         120
## -0.50887859 -0.61087534 -0.16356841 -0.16073331 -0.12675915 -0.49293406
##         121         122         123         124         125         126
## -0.14931436 -0.21351074  1.84295971 -0.40421564  1.40875849 -0.30814469
```

```
##          127          128          129          130          131          132 
## -0.40618549 -0.49528940 -0.51130056 -0.73777334 -0.28412789 -0.58877426 
##          133          134          135          136          137          138 
## -0.49703364 -1.22831729 -0.31850052  1.89079347 -0.29409388 -0.69416101 
##          139          140          141          142          143          144 
## -0.20982689 -0.48482421 -0.61925036 -0.28658859 -0.39176366 -0.48311783 
##          145          146          147          148          149          150 
## -1.11866815 -0.31307905 -0.35422338 -0.28065744 -0.72970910 -0.20982689 
##          151          152          153          154          155          156 
## -0.41110024 -0.42725579 -0.28554134 -0.38936035 -0.29263987 -0.25647527 
##          157          158          159 
##  0.42476809  0.86785144  1.67453806

pi = exp(logitv)/(1+exp(logitv))
plot(lowess(dr ~ pi))
```



2 14.39

(a)
```r
dat = read.table("CH14PR39.txt")
names(dat) = c("Y", "X1", "X2", "X3", "X4")
poi = glm(Y ~ . , data = dat, family = "poisson")
summary(poi)
```

```
##
## Call:
## glm(formula = Y ~ ., family = "poisson", data = dat)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -2.1854  -0.7819  -0.2564   0.5449   2.3626
##
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept)  0.489467   0.336869   1.453  0.14623
## X1          -1.069403   0.133154  -8.031 9.64e-16 ***
## X2          -0.046606   0.119970  -0.388  0.69766
## X3           0.009470   0.002953   3.207  0.00134 **
## X4           0.008566   0.004312   1.986  0.04698 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for poisson family taken to be 1)
##
##     Null deviance: 199.19  on 99  degrees of freedom
## Residual deviance: 108.79  on 95  degrees of freedom
## AIC: 377.29
##
## Number of Fisher Scoring iterations: 5
```

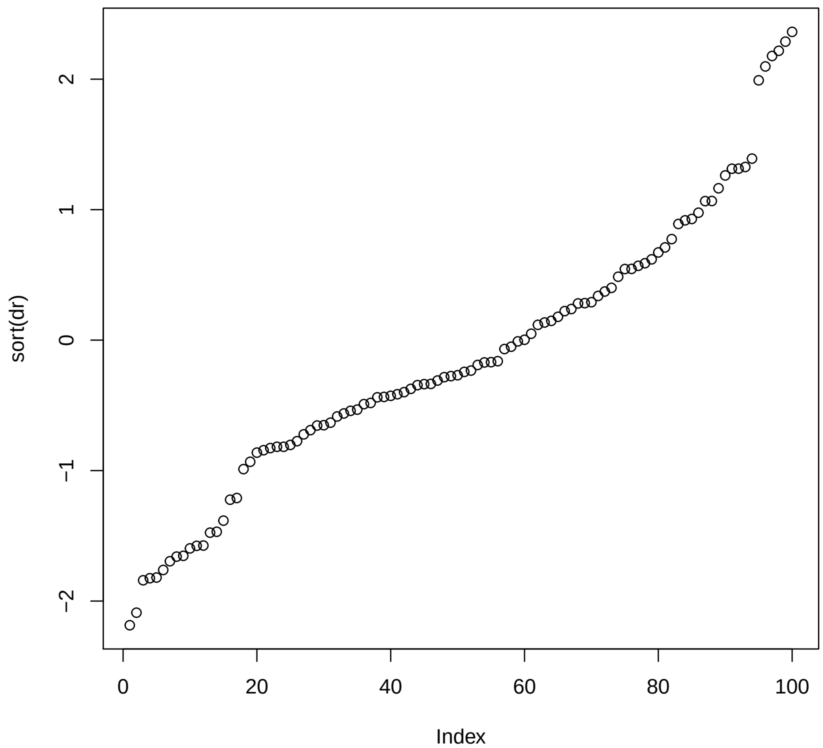
$$
\begin{aligned}
b_0 &= 0.489467 & s(b_0) &= 0.336869 \\
b_1 &= -1.069403 & s(b_1) &= 0.133154 \\
b_2 &= -0.046606 & s(b_2) &= 0.119970 \\
b_3 &= 0.009470 & s(b_3) &= 0.002953 \\
b_4 &= 0.008566 & s(b_4) &= 0.004312
\end{aligned}
$$

$$\mu = exp(0.489467165 - 1.069402551X1 - 0.046606063X2 + 0.009469987X3 + 0.008565829X4)$$

(b)
```r
p = summary(poi)
dr = p$deviance.resid;dr
```

```
##            1            2            3            4            5 
## -0.481563003 -0.632820229  0.485684782 -1.819828480  0.238302497 
##            6            7            8            9           10 
## -0.427206484 -1.574566470 -1.694831446 -0.190494237  0.372414202 
##           11           12           13           14           15 
##  0.290568448  0.917894609  0.618945304  0.710218879 -0.169117683 
##           16           17           18           19           20 
##  0.774212615  1.313966589  0.976533023 -0.284161510  0.281078819 
##           21           22           23           24           25 
##  0.671826155 -0.309823439 -0.585274974 -1.659501048 -1.653549229 
##           26           27           28           29           30 
##  0.545695411 -2.089197070 -1.825162331  0.283158353  1.066089357 
##           31           32           33           34           35 
##  0.338495236 -0.414912576 -0.276062096  2.097482057 -0.373770021 
##           36           37           38           39           40 
## -2.185378873  2.217373787 -0.269196968 -1.468917177 -0.490568471 
##           41           42           43           44           45 
## -0.243689445  2.287659410 -0.435460637 -0.171363655 -1.596370920 
##           46           47           48           49           50 
##  0.178588604 -1.840819930  1.313615961 -0.233624875 -1.576829772 
##           51           52           53           54           55 
## -0.652282150  2.177759158 -0.862317963 -1.223172388 -0.161565009 
##           56           57           58           59           60 
## -0.438457085 -0.817656201  0.002451800 -0.690070466 -0.932653968 
##           61           62           63           64           65 
## -0.532725622  0.400431092  1.326821516  0.569505372  0.589162479 
##           66           67           68           69           70 
##  0.222202292  1.991163395  0.134986579  1.163783804  0.890153115 
##           71           72           73           74           75 
## -0.398888037 -0.335860573 -0.843868576  1.065308951 -0.068529260 
##           76           77           78           79           80 
## -1.210403791  0.928316209 -0.803424449 -0.050707876 -0.817762385 
##           81           82           83           84           85 
##  0.544674549 -1.761754891 -0.562223893 -0.541853422  0.147418906 
##           86           87           88           89           90 
## -0.009840529 -0.337422359 -0.774688780 -1.383658148 -0.654757856 
##           91           92           93           94           95 
## -1.475851025 -0.722343228  2.362545161  1.391144309  1.262066676 
##           96           97           98           99          100 
##  0.117099736 -0.827717570 -0.345172951  0.048822205 -0.988857456
```

```r
plot(sort(dr))
```



The points whose devaiance residual near 100 seem to be outlyers.

(c)
```
  logLik(poi)

## 'log Lik.' -183.6439 (df=5)

  poiR = glm(Y ~ .-X2 , data = dat, family = "poisson")
  logLik(poiR)

## 'log Lik.' -183.7194 (df=4)

  qchisq(1-0.05, 5-4)

## [1] 3.841459

  pchisq(0.151, 1, lower.tail = FALSE)

## [1] 0.6975815
```

H_0:$\beta_2 = 0$

VS. H_1:$\beta_2 \neq 0$

The full model: $\mu = exp(\beta_0 + \beta_1 X1 + \beta_2 X2 + \beta_3 X3 + \beta_4 X4)$

ln(L(F))= -183.6439

The reduced model: $\mu = exp(\beta_0 + \beta_1 X1 + \beta_3 X3 + \beta_4 X4)$

ln(L(R))= -183.7194

G^2 = -2(ln(L(R)-ln(L(F)))) = 0.151

we can reject H_0 if $G^2 > \chi^2(1 - 0.05, 5 - 4) = 3.8415$,otherwise reject$H_1$

so that reject H_1 because $G^2 < 3.8415$,

therefore, X2 can be dropped from the regression model, and the P-value is 0.6975815.And the result is the same as the result we get in (b).

(d)

```
summary(poiR)

##
## Call:
## glm(formula = Y ~ . - X2, family = "poisson", data = dat)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -2.2152  -0.7512  -0.2594   0.5830   2.2893
##
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept)  0.443890   0.317289   1.399  0.16181
## X1          -1.077770   0.131415  -8.201 2.38e-16 ***
## X3           0.009471   0.002957   3.203  0.00136 **
## X4           0.008979   0.004190   2.143  0.03209 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for poisson family taken to be 1)
##
##     Null deviance: 199.19  on 99  degrees of freedom
## Residual deviance: 108.94  on 96  degrees of freedom
## AIC: 375.44
##
## Number of Fisher Scoring iterations: 5

b1 = -1.077770
s1 = 0.131415
z = qnorm(1-0.05/2)
c(b1-s1*z, b1+s1*z)
```

```
## [1] -1.3353387 -0.8202013
```

From summary(poiR), we get $s(b_1) = 0.131415, b_1 = -1.077770$, based on $b_k \pm z(1-\alpha/2)sb_k$, we conclude that we are 95 % confident that β_1 is between -1.3353387 and -0.8202013. Because the confidence interval is smaller than 0, aerobic exercise reduce the frequency of falls when controlling for balance and strength.

3 14.44

4 14.45

5 14.46

$$E(Y) = [1 + exp(-\beta_0 - \beta_1 X_1 - \beta_2 X_2 - \beta_3 X_1 X_2)]^{-1}$$
$$\pi^{'}(X_1) = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_1 X_2$$
$$\pi^{'}(X_1 + 1) = \beta_0 + \beta_1 (X_1 + 1) + \beta_2 X_2 + \beta_3 (X_1 + 1) X_2$$
$$\pi^{'}(X_1 + 1) - \pi^{'}(X_1) = ln(oddsratio) = \beta_1 X_1 + \beta_3 X_1 X_2$$

Hence the odds ratio for X1 is $exp(\beta_1 X_1 + \beta_3 X_1 X_2)$therefore, they are different.

6 14.47