Stat 207 HW3

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1 26.4

(a)

(b)

2 26.5

3 26.6

4 26.7

5 26.19

6 26.20

7 26.24

$$SSB + SSAB = na\sum_j(\bar{Y}_{\cdot j\cdot} - \bar{Y}_{\cdots})^2 + n\sum_i\sum_j(\bar{Y}_{ij\cdot} - \bar{Y}_{i\cdot\cdot} - \bar{Y}_{\cdot j\cdot} + \bar{Y}_{\cdots})^2$$

8 26.25

(a) Since

$$\bar{Y}_{ijk} = \mu_{ij} + \alpha_i + \beta_{j(i)} + \epsilon_{ijk}$$

then:

$$\begin{aligned}
\sigma^2\{\bar{Y}_{i\cdot\cdot}\} &= \sigma^2\{\mu_{i\cdot\cdot} + \alpha_i + \bar{\beta}_{\cdot(i)} + \bar{\epsilon}_{i\cdot\cdot}\} \\
&= \sigma^2\{\bar{\beta}_{\cdot(i)} + \bar{\epsilon}_{i\cdot\cdot}\} \\
&= \frac{\sigma_\beta^2}{b} + \frac{\sigma^2}{bn} \qquad \text{,since } \beta \text{ and } \epsilon \text{ are independent}
\end{aligned}$$

$$\begin{aligned}
\sigma^2\{\bar{Y}_{\cdots}\} &= \sigma^2\{\mu_{\cdots} + \bar{\beta}_{\cdot(\cdot)} + \bar{\epsilon}_{\cdots}\} \qquad \text{,since } \sum_i \alpha = 0 \\
&= \sigma^2\{\bar{\beta}_{\cdot(\cdot)} + \bar{\epsilon}_{\cdots}\} \\
&= \frac{\sigma_\beta^2}{ab} + \frac{\sigma^2}{abn} \qquad \text{,since } \beta \text{ and } \epsilon \text{ are independent}
\end{aligned}$$

(b)

$$\begin{aligned}
E(MSB(a)) &= \sigma^2 + n\sigma_\beta^2 \\
E(MSE) &= \sigma^2 \\
s_\beta^2 &= (MSB(A) - MSE)/n \\
\hat{\sigma}_\beta^2 &= max(0, s_\beta^2) = max(0, (MSB(A) - MSE)/n)
\end{aligned}$$

9 26.28