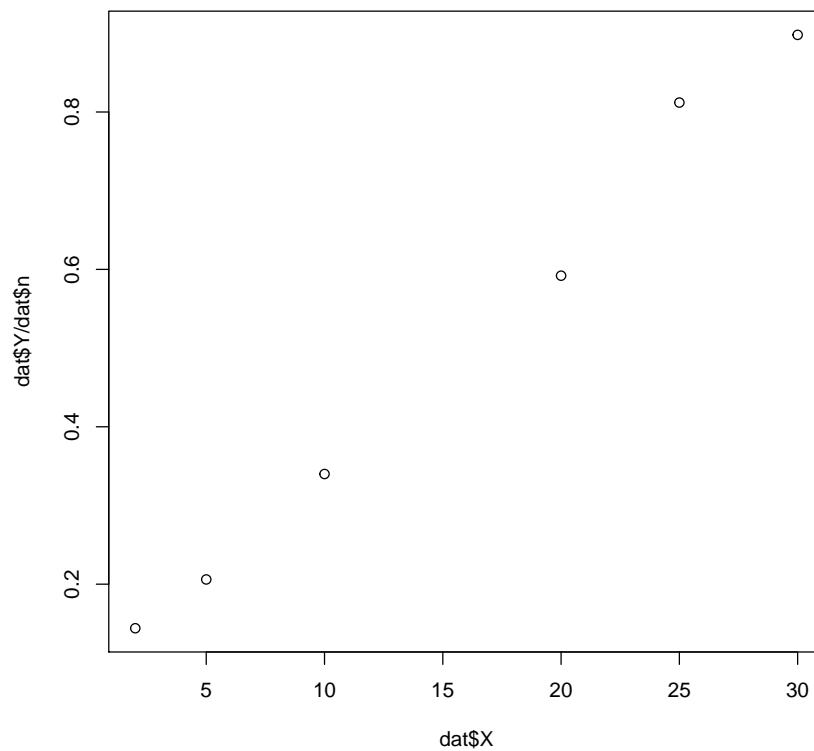# Stat 207 HW7

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1 14.11

(a)
```r
dat = read.table("CH14PR11.txt")
names(dat) = c("X", "n", "Y")
plot(dat$X, dat$Y/dat$n)
```


The plot support the analyst's belief that the logistic response functiion is appropriate.

(b)
```r
logit = glm(Y/n ~ X, data = dat, family = "binomial")
## Warning:  non-integer #successes in a binomial glm!

summary(logit)

##
## Call:
```

3

```
## glm(formula = Y/n ~ X, family = "binomial", data = dat)
##
## Deviance Residuals:
##         1         2         3         4         5         6
##  0.007846  0.019363  0.025865 -0.130556  0.056842  0.054601
##
## Coefficients:
##             Estimate Std. Error z value Pr(>|z|)
## (Intercept)  -2.0766     1.8970  -1.095    0.274
## X             0.1359     0.1067   1.273    0.203
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 2.216343  on 5  degrees of freedom
## Residual deviance: 0.024363  on 4  degrees of freedom
## AIC: 7.1154
##
## Number of Fisher Scoring iterations: 4
```

From the summary, the maximum likelihood estimates of $\hat{\beta}_0 = -2.0766$, $\hat{\beta}_1 = 0.1359$,

$$\hat{\pi} = \frac{exp(\beta_0 + \beta_1 X)}{1 + exp(\beta_0 + \beta_1 X)} = \frac{exp(-2.0766 + 0.1359X)}{1 + exp(-2.0766 + 0.1359X)}$$

(c)
```
plot(dat$X, fitted(logit), ylim = c(0, 1))
points(dat$X, dat$Y/dat$n, lwd = 1)
```

4
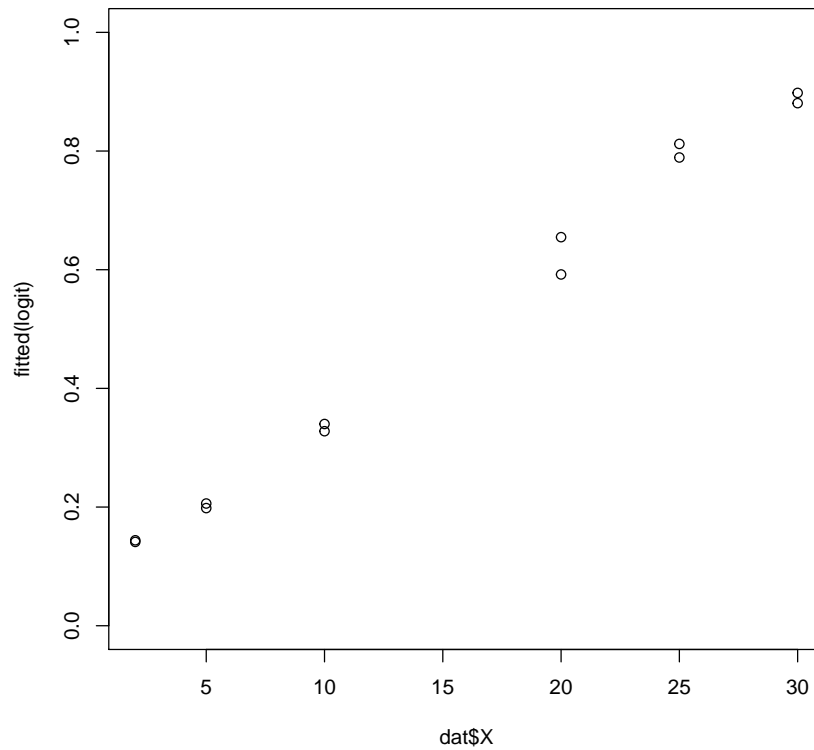
The fitted logistic response function appears to be well.

(d)
```r
exp(0.1359)
```
```
## [1] 1.145567
```

$exp(\beta_1) = 1.145567$, so that the odds of the bottles being returned is increased by 14.5567% with each one deposit level increased.

(e)
```r
newdat = data.frame(X = 15)
predict(logit, newdata = newdat, type = "response")
```
```
##         1
## 0.4903005
```

The estimated probability that a bottle will be returned when the deposit is 15 cents is 0.4903005.

(f)
```
newpi = 0.75
pi_2 = log(newpi/(1-newpi))
(pi_2 - (-2.0766))/0.1359
```

```
## [1] 23.36433
```

Estimate the amount of deposit for which 75% of the bottles are expected to be returned is 23.36433.

2   14.14

(a)
```
dat = read.table("CH14PR14.txt")
names(dat) = c("Y", "X1", "X2", "X3")
logit = glm(Y ~ X1 + X2 + X3, data = dat, family = "binomial")
summary(logit)
```

```
##
## Call:
## glm(formula = Y ~ X1 + X2 + X3, family = "binomial", data = dat)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -1.4037  -0.5637  -0.3352  -0.1542   2.9394
##
## Coefficients:
##             Estimate Std. Error z value Pr(>|z|)
## (Intercept) -1.17716    2.98242  -0.395  0.69307
## X1           0.07279    0.03038   2.396  0.01658 *
## X2          -0.09899    0.03348  -2.957  0.00311 **
## X3           0.43397    0.52179   0.832  0.40558
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 134.94  on 158  degrees of freedom
## Residual deviance: 105.09  on 155  degrees of freedom
## AIC: 113.09
##
## Number of Fisher Scoring iterations: 6
```

From the summary, the maximum likelihood estimates $\hat{\beta}_0 = -1.17716$,

6

$\hat{\beta}_1 = 0.07279,\ \hat{\beta}_2 = -0.09899,\ \hat{\beta}_4 = 0.43397$

$$\hat{\pi} = \frac{exp(\beta_0 + \beta_1 X1 + \beta_2 X2 + \beta_3 X3)}{1 + exp(\beta_0 + \beta_1 X1 + \beta_2 X2 + \beta_3 X3)} = \frac{exp(-1.17716 + 0.07279X1 - 0.09899X2 + 0.43397X3)}{1 + exp(-1.17716 + 0.07279X1 - 0.09899X2 + 0.43397X3)}$$

(b)
```
exp(0.07279)
```

```
## [1] 1.075505
```

```
exp(-0.09899 )
```

```
## [1] 0.9057518
```

```
exp(0.43397)
```

```
## [1] 1.543373
```

- $exp(\beta_1) = 1.075505$, so that the odds of getting a flu shot is increased by 7.5% with each one age increased.
- $exp(\beta_2) = 0.9057518$, so that the odds of getting a flu shot is decreased by 9.4% with each one health awareness index increased.
- $exp(\beta_3) = 1.543373$, so that the odds of getting a flu shot is increased by 54.3% from woman to man.

(c)
```
newdat = data.frame(X1 = 55, X2 = 60, X3 = 1)
predict(logit, newdata = newdat, type = "response")
```

```
##          1
## 0.06422197
```

The estimated probability with X1=55, X2=60 and X3=1 is 0.06422197
.

3 14.17

(a)
```
dat = read.table("CH14PR11.txt")
names(dat) = c("X", "n", "Y")
logit = glm(Y/n ~ X, data = dat, family = "binomial")
```
```
## Warning:  non-integer #successes in a binomial glm!
```
```
summary(logit)
```

```
## 
## Call:
## glm(formula = Y/n ~ X, family = "binomial", data = dat)
## 
## Deviance Residuals:
##         1         2         3         4         5         6
##  0.007846  0.019363  0.025865  -0.130556  0.056842  0.054601
## 
## Coefficients:
##             Estimate Std. Error z value Pr(>|z|)
## (Intercept)  -2.0766     1.8970  -1.095    0.274
## X             0.1359     0.1067   1.273    0.203
## 
## (Dispersion parameter for binomial family taken to be 1)
## 
##     Null deviance: 2.216343  on 5  degrees of freedom
## Residual deviance: 0.024363  on 4  degrees of freedom
## AIC: 7.1154
## 
## Number of Fisher Scoring iterations: 4

  b1 = 0.1359
  s1 = 0.1067
  z = qnorm(1-0.05/2)
  c(b1-s1*z, b1+s1*z)

## [1] -0.07322816  0.34502816

  c(exp(b1-s1*z), exp(b1+s1*z))

## [1] 0.9293888 1.4120297
```

From summary(logit), we get $s(b_1) = 0.1067, b_1 = 0.1359$, based on $b_k \pm z(1 - \alpha/2)sb_k$, we conclude that we are 95 % confident that β_1 is between -0.07322816 and 0.34502816, and corresponding confidence limits for the odds ratio $exp(\beta_1)$ is between 0.9293888 and 1.4120297.

(b)
```
  qnorm(1-0.05/2)

## [1] 1.959964
```

$$H_0{:}\beta_1 = 0$$
$$\text{VS. } H_1{:}\beta_1 \neq 0$$

$$z^* = \frac{b_1}{s(b_1)} = 0.1359/0.1067 = 1.273664$$

we can reject H_0 if $|z^*| > Z(1-0.05/2) = 1.959964$, otherwise reject H_1

so that reject H_1 because $|z^*| < 1.959964$,

therefore, X1 can be dropped from the regression model, and the P-value is 0.203

(c) `logLik(logit)`

```
## 'log Lik.' -1.557684 (df=2)
```

```
logitR = glm(Y/n ~ 1, data = dat, family = "binomial")
```

```
## Warning:  non-integer #successes in a binomial glm!
```

```
logLik(logitR)
```

```
## 'log Lik.' -4.158904 (df=1)
```

```
qchisq(1-0.05, 2-1)
```

```
## [1] 3.841459
```

```
pchisq(5.339856, 1, lower.tail = FALSE)
```

```
## [1] 0.02084319
```

$$H_0{:}\beta_1 = 0$$
$$\text{VS. } H_1{:}\beta_1 \neq 0$$

The full model: $\pi = [1 + exp(-(\beta_0 + \beta_1 X1))]^{-1}$

$\ln(L(F)) = -1.557684$

The reduced model: $\pi = [1 + exp(-(\beta_0))]^{-1}$

$\ln(L(R)) = -4.158904$

$$G^2 = -2(\ln(L(R))-\ln(L(F)))) = 5.20244$$

we can reject H_0 if $G^2 > \chi^2(1-0.05, 2-1) = 3.8415$, otherwise reject H_1

so that reject H_0 because $G^2 > 3.8415$,

therefore, X1 cannot be dropped from the regression model, and the P-value is 0.02084319.And the result is different from the result we get in (b).

4   14.20

(a)
```
dat = read.table("CH14PR14.txt")
names(dat) = c("Y", "X1", "X2", "X3")
logit = glm(Y ~ X1 + X2 + X3, data = dat, family = "binomial")
summary(logit)

##
## Call:
## glm(formula = Y ~ X1 + X2 + X3, family = "binomial", data = dat)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -1.4037  -0.5637  -0.3352  -0.1542   2.9394
##
## Coefficients:
##             Estimate Std. Error z value Pr(>|z|)
## (Intercept) -1.17716    2.98242  -0.395  0.69307
## X1           0.07279    0.03038   2.396  0.01658 *
## X2          -0.09899    0.03348  -2.957  0.00311 **
## X3           0.43397    0.52179   0.832  0.40558
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 134.94  on 158  degrees of freedom
## Residual deviance: 105.09  on 155  degrees of freedom
## AIC: 113.09
##
## Number of Fisher Scoring iterations: 6
```

$$z(1 - \tfrac{0.1}{2*2}) = 0.975, s(b_1) = 0.03038, s(b_2) = 0.03348$$

$$exp(30(0.07280.030381.96)) < exp(30\beta_1) < exp(30(0.0728 + 0.030381.96))$$

$$1.4878 < exp(30\beta_1) < 52.9837$$

$$exp(25(0.0990.033481.96)) < exp(25\beta_2) < exp(25(0.0728 + 0.033481.96))$$

$$0.0163 < exp(25\beta_2) < 31.824$$

(b)
```
qnorm(1-0.05/2)

## [1] 1.959964
```

$$H_0:\beta_3 = 0$$
$$\text{VS. } H_1:\beta_3 \neq 0$$
$$z^* = \frac{b_3}{s(b_3)} = 0.43397/0.52179 = 0.8316947$$
we can reject H_0 if $|z^*| > Z(1 - 0.05/2) = 1.959964$,otherwise reject$H_1$
so that reject H_1 because $|z^*| < 1.959964$,
therefore, X3 can be dropped from the regression model, and the P-value is 0.40558

(c)
```
logLik(logit)
```

```
## 'log Lik.' -52.54659 (df=4)
```

```
logitR = glm(Y ~ X1+X2, data = dat, family = "binomial")
logLik(logitR)
```

```
## 'log Lik.' -52.89769 (df=3)
```

```
qchisq(1-0.05, 4-3)
```

```
## [1] 3.841459
```

```
pchisq(0.70236, 1, lower.tail = FALSE)
```

```
## [1] 0.4019918
```

$$H_0:\beta_3 = 0$$
$$\text{VS. } H_1:\beta_3 \neq 0$$
The full model: $\pi = [1 + exp(-(\beta_0 + \beta_1 X1 + \beta_2 X2 + \beta_3 X3))]^{-1}$
ln(L(F))= -52.54659
The reduced model: $\pi = [1 + exp(-(\beta_0 + \beta_1 X1 + \beta_2 X2))]^{-1}$
ln(L(R))= -52.89769
$$G^2 = \text{-2}(\text{ln(L(R))-ln(L(F))})) = 0.70236$$
we can reject H_0 if $G^2 > \chi^2(1 - 0.05, 4 - 3) = 3.8415$,otherwise reject$H_1$
so that reject H_1 because $G^2 < 3.8415$,
therefore, X3 can be dropped from the regression model, and the P-value is 0.4019918.And the result is the same as the result we get in (b).

(d)
```
logitF = glm(Y ~ X1+X2+I(X1^2)+I(X2^2)+I(X1*X2), data = dat, family = "binomial")
logLik(logitF)
```

```
## 'log Lik.' -52.13072 (df=6)
```

```
logitR = glm(Y ~ X1+X2, data = dat, family = "binomial")
logLik(logitR)
```

```
## 'log Lik.' -52.89769 (df=3)
```

```
qchisq(1-0.05, 6-3)
```

```
## [1] 7.814728
```

```
pchisq(1.53394, 3, lower.tail = FALSE)
```

```
## [1] 0.6744594
```

$$H_0 : \beta_3 = \beta_4 = \beta_5 = 0$$

VS. $H_1 : not\, all\, \beta_3, \beta_4, \beta_5\, equal\, 0$

The full model:

$$\pi = [1 + exp(-(\beta_0 + \beta_1 X1 + \beta_2 X2 + \beta_3 X1^2 + \beta_4 X2^2 + \beta_5 X1 * X2))]^{-1}$$

$$\ln(L(F)) = -52.13072$$

The reduced model: $\pi = [1 + exp(-(\beta_0 + \beta_1 X1 + \beta_2 X2))]^{-1}$

$$\ln(L(R)) = -52.89769$$

$$G^2 = -2(\ln(L(R)) - \ln(L(F)))) = 1.53394$$

we can reject H_0 if $G^2 > \chi^2(1 - 0.05, 6 - 3) = 7.814728$, otherwise reject H_1

so that reject H_1 because $G^2 < 7.814728$,

therefore, $X1^2, X2^2, I(X1 * X2)$ can be dropped from the regression model, and the P-value is 0.6744594. And the result is the same as the result we get in (b).

5   14.22

(a)
```
dat = read.table("CH14PR14.txt")
names(dat) = c("y", "x1", "x2", "x3")
dat.new = scale(dat[, 2:4], scale = FALSE)
dat.new = as.data.frame(cbind(dat$y, dat.new))
names(dat.new) = c("y", "x1", "x2", "x3")
```

```r
  alpha = .1
  wrap.foo = function(formula, dat.new. = dat.new)
  {
    model = glm(formula, family = binomial, data = dat.new.)
    p.val = summary(model)$coef
    p.val[nrow(p.val), 4]
  }
  tmp = c(wrap.foo(y ~ x1),
          wrap.foo(y ~ x2),
          wrap.foo(y ~ I(x1*x2)),
          wrap.foo(y ~ I(x1^2)),
          wrap.foo(y ~ I(x2^2)),
          wrap.foo(y ~ I(x1^2):I(x2^2)))
  any(tmp < alpha)
```

```
## [1] TRUE
```

```r
  which.min(tmp)
```

```
## [1] 1
```

```r
  tmp = c(wrap.foo(y ~ x1 + x2),
          wrap.foo(y ~ x1 + I(x1*x2)),
          wrap.foo(y ~ x1 + I(x1^2)),
          wrap.foo(y ~ x1 + I(x2^2))
          )
  any(tmp < alpha)
```

```
## [1] TRUE
```

```r
  which.min(tmp)
```

```
## [1] 1
```

```r
  tmp = c(wrap.foo(y ~ x1 + x2 + I(x1*x2)),
          wrap.foo(y ~ x1 + x2 + I(x1^2)),
          wrap.foo(y ~ x1 + x2 + I(x2^2))
          )
  any(tmp < alpha)
```

```
## [1] FALSE
```

X1 enters in step 1 ; X2 enters in step 2; no variables satisfy criterion

for entry in step 3.

(b)
```r
wrap.foo = function(formula, dat.new. = dat.new)
{
  model = glm(formula, family = binomial, data = dat.new.)
  p.val = summary(model)$coef
  p.val = p.val[, 4]
  p.val[-1]
}

tmp = wrap.foo(y ~ x1 + x2 + I(x1*x2) + I(x1^2) + I(x2^2))
any(tmp > alpha)
```

```
## [1] TRUE
```

```r
which.max(tmp)
```

```
## I(x1^2)
##       4
```

```r
tmp = wrap.foo(y ~ x1 + x2 + I(x1*x2) + I(x2^2))
any(tmp > alpha)
```

```
## [1] TRUE
```

```r
which.max(tmp)
```

```
## I(x1 * x2)
##          3
```

```r
tmp = wrap.foo(y ~ x1 + x2 + I(x2^2))
any(tmp > alpha)
```

```
## [1] TRUE
```

```r
which.max(tmp)
```

```
## I(x2^2)
##       3
```

```r
tmp = wrap.foo(y ~ x1 + x2)
any(tmp > alpha)
```

```
## [1] FALSE
```

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X11 is deleted in step 1; X12 is deleted in step 2; X3 is deleted in step 3; X22 is deleted in step 4; X1 and X2 are retained in the model.

(c)
```r
model = glm(y ~ x1 + x2 + I(x1*x2) + I(x1^2) + I(x2^2) + I(x1^2*x2^2),
            family = binomial, data = dat.new)
step(model)
```

```
## Start:  AIC=116.67
## y ~ x1 + x2 + I(x1 * x2) + I(x1^2) + I(x2^2) + I(x1^2 * x2^2)
##
##                      Df Deviance    AIC
## - I(x2^2)             1   102.68 114.68
## - I(x1^2)             1   103.56 115.56
## - I(x1 * x2)          1   103.84 115.84
## - I(x1^2 * x2^2)      1   104.26 116.26
## <none>                    102.67 116.67
## - x2                  1   107.48 119.48
## - x1                  1   109.46 121.46
##
## Step:  AIC=114.68
## y ~ x1 + x2 + I(x1 * x2) + I(x1^2) + I(x1^2 * x2^2)
##
##                      Df Deviance    AIC
## - I(x1^2)             1   103.83 113.83
## - I(x1 * x2)          1   103.84 113.84
## <none>                    102.68 114.68
## - I(x1^2 * x2^2)      1   105.71 115.71
## - x2                  1   107.94 117.94
## - x1                  1   109.50 119.50
##
## Step:  AIC=113.83
## y ~ x1 + x2 + I(x1 * x2) + I(x1^2 * x2^2)
##
##                      Df Deviance    AIC
## - I(x1 * x2)          1   104.74 112.74
## - I(x1^2 * x2^2)      1   105.75 113.75
## <none>                    103.83 113.83
## - x1                  1   109.97 117.97
## - x2                  1   111.00 119.00
##
## Step:  AIC=112.74
## y ~ x1 + x2 + I(x1^2 * x2^2)
##
##                      Df Deviance    AIC
## - I(x1^2 * x2^2)      1   105.80 111.80
```

```
## <none>                    104.74 112.74
## - x1               1    109.97 115.97
## - x2               1    111.40 117.40
##
## Step:  AIC=111.8
## y ~ x1 + x2
##
##          Df Deviance    AIC
## <none>        105.80 111.80
## - x1     1    113.20 117.20
## - x2     1    116.27 120.27
##
## Call:  glm(formula = y ~ x1 + x2, family = binomial, data = dat.new)
##
## Coefficients:
## (Intercept)            x1            x2
##    -2.29705       0.07787      -0.09547
##
## Degrees of Freedom: 158 Total (i.e. Null);  156 Residual
## Null Deviance:      134.9
## Residual Deviance: 105.8  AIC: 111.8
```

The best model according to the AICp criterion is based on y   x1 + x2. AIC = 111.8.

(d)
```
  step(model, k = log(159))

## Start:  AIC=138.15
## y ~ x1 + x2 + I(x1 * x2) + I(x1^2) + I(x2^2) + I(x1^2 * x2^2)
##
##                    Df Deviance    AIC
## - I(x2^2)           1    102.68 133.10
## - I(x1^2)           1    103.56 133.97
## - I(x1 * x2)        1    103.84 134.25
## - I(x1^2 * x2^2)  1    104.26 134.68
## - x2               1    107.48 137.89
## <none>                  102.67 138.15
## - x1               1    109.46 139.87
##
## Step:  AIC=133.1
## y ~ x1 + x2 + I(x1 * x2) + I(x1^2) + I(x1^2 * x2^2)
##
##                    Df Deviance    AIC
## - I(x1^2)           1    103.83 129.17
## - I(x1 * x2)        1    103.84 129.19
```

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```
## - I(x1^2 * x2^2)  1    105.71 131.05
## <none>                  102.68 133.10
## - x2               1    107.94 133.28
## - x1               1    109.50 134.84
##
## Step:  AIC=129.17
## y ~ x1 + x2 + I(x1 * x2) + I(x1^2 * x2^2)
##
##                      Df Deviance    AIC
## - I(x1 * x2)          1    104.74 125.02
## - I(x1^2 * x2^2)      1    105.75 126.03
## <none>                    103.83 129.17
## - x1                  1    109.97 130.24
## - x2                  1    111.00 131.28
##
## Step:  AIC=125.02
## y ~ x1 + x2 + I(x1^2 * x2^2)
##
##                      Df Deviance    AIC
## - I(x1^2 * x2^2)      1    105.80 121.00
## <none>                    104.74 125.02
## - x1                  1    109.97 125.18
## - x2                  1    111.40 126.61
##
## Step:  AIC=121
## y ~ x1 + x2
##
##          Df Deviance    AIC
## <none>        105.80 121.00
## - x1      1   113.20 123.33
## - x2      1   116.27 126.41
##
## Call:  glm(formula = y ~ x1 + x2, family = binomial, data = dat.new)
##
## Coefficients:
## (Intercept)            x1             x2
##    -2.29705       0.07787       -0.09547
##
## Degrees of Freedom: 158 Total (i.e. Null);   156 Residual
## Null Deviance:      134.9
## Residual Deviance: 105.8   AIC: 111.8
```

The best model according to the SBCp criterion is based on y   x1 +
x2. SBC = 121.

6   14.23

```
dat = read.table("CH14PR11.txt")
names(dat) = c("X", "n", "Y")
logit = glm(Y/n ~ X, data = dat, family = "binomial")

## Warning:  non-integer #successes in a binomial glm!

Oj1 = dat$Y
Ej1 = round(dat$n*fitted(logit), 1)
Oj0 = dat$n-dat$Y
Ej0 = dat$n-Ej1
rbind(Oj1, Oj0)

##      [,1] [,2] [,3] [,4] [,5] [,6]
## Oj1   72  103  170  296  406  449
## Oj0  428  397  330  204   94   51

rbind(Ej1, Ej0)

##          1     2     3     4     5     6
## Ej1   70.6  99.1 163.9 327.4 394.6 440.3
## Ej0  429.4 400.9 336.1 172.6 105.4  59.7

X.squ = sum((rbind(Oj1, Oj0)-rbind(Ej1, Ej0))^2/rbind(Ej1, Ej0));X.squ

## [1] 12.28748
```

$$H_0 : E(Y) = [1 + exp(-\beta_0 - \beta_1 X1)]^{-1}$$
$$\text{VS. } H_1 : E(Y) \neq [1 + exp(-\beta_0 - \beta_1 X1)]^{-1}$$
$$X^2 = \sum_j \sum_k \frac{(O_{jk} - E_{jk})^2}{E_{jk}} = 12.287$$
we can reject H_0 if $X^2 > \chi^2(0.99, 3) = 13.2767$,otherwise reject$H_1$
so that reject H_1 because $X^2 < 13.2767$,

7   14.40

$$\frac{exp(\beta_0 + \beta_1 Xi)}{1 + exp(\beta_0 + \beta_1 Xi)} = \frac{1}{\frac{1}{exp(\beta_0 + \beta_1 Xi)} + 1}$$
$$= \frac{1}{exp(0 - \beta_0 - \beta_1 Xi) + 1}$$
$$= [1 + exp(-\beta_0 - \beta_1 Xi)]^{-1}$$

18

8   14.41

For given observations $Y_1, Y_2..., Y_n$, all terms with a given X value, X_j, we get

$$Y_{.j}(\beta_0 + \beta_1 X_j) - n_j ln(1 + exp(\beta_0 + \beta_1 Xj)$$

Because there are $\binom{n_j}{Y_{.j}}$ ways of getting these, we must add $ln\binom{n_j}{Y_{.j}}$, hence we get

$$ln\binom{n_j}{Y_{.j}} + Y_{.j}(\beta_0 + \beta_1 X_j) - n_j ln(1 + exp(\beta_0 + \beta_1 Xj)$$

9   14.42

$$\pi_i = \frac{exp(\pi_i')}{1 + exp(\pi_i')}$$

$$1 - \pi_i = \frac{1}{1 + exp(\pi_i')}$$

$$\frac{\pi_i}{1 - \pi_i} = exp(\pi_i')$$

$$F_L^{-1}(\pi_i) = \pi_i' = log_e(\frac{\pi_i}{1 - \pi_i})$$

10   14.43

$$lnL(\beta_0, \beta_1) = \sum_{i=1}^{n} y_i(\beta_0 + \beta_1 X_i) - \sum_{i=1}^{n}(1 + exp(\beta_0 + \beta_1 X_i))$$

$$\frac{\partial^2 lnL}{\partial \beta_0^2} = -\sum_{i=1}^{n} \frac{exp(\beta_0 + \beta_1 X_i)}{[1 + exp(\beta_0 + \beta_1 X_i)]^2}$$

$$\frac{\partial^2 lnL}{\partial \beta_1^2} = -\sum_{i=1}^{n} \frac{X_i^2 exp(\beta_0 + \beta_1 X_i)}{[1 + exp(\beta_0 + \beta_1 X_i)]^2}$$

$$\frac{\partial^2 lnL}{\partial \beta_0 \partial \beta_1} = -\sum_{i=1}^{n} \frac{X_i exp(\beta_0 + \beta_1 X_i)}{[1 + exp(\beta_0 + \beta_1 X_i)]^2}$$

$$-E\{\frac{\partial^2 lnL}{\partial \beta_0 \partial \beta_1}\} = -g_{01} = -g_{10} \text{which is reduced to (14.51)}$$