1) Let X denote the random variable representing the sum of three rolls

Let $n = 3$, the number of rolls

Let $P(x)$ denote the probability of rolling sum x

$$P(x) = \binom{n}{x} \times \left(\frac{1}{6}\right)^{x} \times \left(\frac{5}{6}\right)^{n-x}$$

combos to get sum x out of n rolls

probability of getting x successes in n trials

probability of getting n−x failures in n trials

$$P(11) = \binom{3}{11} \times \left(\frac{1}{6}\right)^{11} \times \left(\frac{5}{6}\right)^{3-11} = 0.097222$$

$$P(12) = \binom{3}{12} \times \left(\frac{1}{6}\right)^{12} \times \left(\frac{5}{6}\right)^{3-12} = 0.125$$

$P(11) < P(12)$

$\Rightarrow$ sum of 12 is more likely

3) Let X denote the random variable representing the total points earned over the weekend

Let Y_1 represent the outcome of the first game and Y_2 the second; 0 for loss, 1 for tie, 2 for win

$P(Y_1=0) = 1-0.4 = 0.6$

$P(Y_2=0) = 1-0.7 = 0.3$

$P(Y_1=1) = P(Y_1=2) = 1-0.5 = 0.5$

$P(Y_2=1) = P(Y_2=2) = 1-0.5 = 0.5$

Points (X)	PMF
0	0.18
1	0.3
2	0.45
3	0.5
4	0.25

$P(X=0) = P(Y_1=0) \times P(Y_2=0) = (0.6 \times 0.5) + (0.5 \times 0.3) = 0.18$

$P(X=1) = P(Y_1=0) \times P(Y_2=1) + P(Y_1=1) \times P(Y_2=0) = (0.6 \times 0.5) + (0.5 + 0.3) = 0.3$

$P(X=2) = P(Y_1=0) \times P(Y_2=2) + P(Y_1=2) \times P(Y_2=0) + P(Y_1=1) \times P(Y_2=1)$

$= (0.6 \times 0.5) + (0.5 \times 0.3) + (0.5 \times 0.5) = 0.45$

$P(X=3) = P(Y_1=2) \times P(Y_2=1) + P(Y_1=1) \times P(Y_2=2) = (0.5 \times 0.5) + (0.5 \times 0.5) = 0.5$

$P(X=4) = P(Y_1=2) \times P(Y_2=2) = 0.5 \times 0.5 = 0.25$

4) Let X be the random variable representing the number of girls out of 7 children

$X=0$: All children are boys

$P(X=0) = \frac{1}{2^7} = \frac{1}{128}$

$X=1$: 1 girl and 6 boys

$P(X=1) = \binom{5}{0} \times \binom{2}{1} \times \frac{1}{2^7} + \binom{5}{1} \times \binom{2}{0} \times \frac{1}{2^7} = \frac{10}{128}$

$X=2$: 2 girls and 5 boys

$P(X=2) = \binom{5}{0} \times \binom{2}{2} \times \frac{1}{2^7} + \binom{5}{1} \times \binom{2}{1} \times \frac{1}{2^7} + \binom{5}{2} \times \binom{2}{0} \times \frac{1}{2^7} = \frac{20}{128}$

$X=3$: 3 girls and 3 boys

$P(X=3) = \binom{5}{0} \times \binom{2}{3} \times \frac{1}{2^7} + \binom{5}{1} \times \binom{2}{2} \times \frac{1}{2^7} + \binom{5}{2} \times \binom{2}{1} \times \frac{1}{2^7} + \binom{5}{3} \times \binom{2}{0} \times \frac{1}{2^7} = \frac{20}{128}$

$X=4$: 4 girls and 2 boys

$P(X=4) = \binom{5}{0} \times \binom{2}{4} \times \frac{1}{2^7} + \binom{5}{1} \times \binom{2}{3} \times \frac{1}{2^7} + \binom{5}{2} \times \binom{2}{2} \times \frac{1}{2^7} + \binom{5}{3} \times \binom{2}{1} \times \frac{1}{2^7} + \binom{5}{4} \times \binom{2}{0} \times \frac{1}{2^7}$

$= \frac{10}{128}$

$X=5$: 5 girls and 2 boys; this can only happen if the natural and adopted children are girls

$P(X=5) = \frac{1}{2^7} = \frac{1}{128}$

$X=6$: 6 girls and 1 boy; can only occur if all children are girls

$P(X=6) = \frac{1}{2^7} = \frac{1}{128}$

$X=7$: All children are girls

$P(X=7) = \frac{1}{2^7} = \frac{1}{128}$

Num girls(x)	PMF
0	$\frac{1}{128}$
1	$\frac{10}{128}$
2	$\frac{20}{128}$
3	$\frac{20}{128}$
4	$\frac{10}{128}$
5	$\frac{1}{128}$
6	$\frac{1}{128}$
7	$\frac{1}{128}$

5) a) $X=0, Y=0 \Rightarrow \text{Mod } 3 = 0$
$X=1, Y=1 \Rightarrow \text{mod } 3 = 1$
$X=2, Y=2 \Rightarrow \text{mod } 3 = 2$
$X=3, Y=3 \Rightarrow \text{mod } 3 = 0$
$X=4, Y=4 \Rightarrow \text{mod } 3 = 1$
$X=5, Y=5 \Rightarrow \text{mod } 3 = 2$
$X=6, Y=6 \Rightarrow \text{mod } 3 = 0$
$X=7, Y=7 \Rightarrow \text{mod } 3 = 1$
$X=8, Y=8 \Rightarrow \text{mod } 3 = 2$
$X=9, Y=9 \Rightarrow \text{mod } 3 = 0$

Possible values: 0, 1, 2
Each has probability of $\frac{1}{3}$

Value of Y	PMF
0	$\frac{1}{3}$
1	$\frac{1}{3}$
2	$\frac{1}{3}$

b) $X=0, Y=5 \Rightarrow \text{Mod } (0+1) = 0$
$X=1, Y=5 \Rightarrow \text{mod } (1+1) = 1$
$X=2, Y=5 \Rightarrow \text{mod } (2+1) = 2$
$X=3, Y=5 \Rightarrow \text{mod } (3+1) = 1$
$X=4, Y=5 \Rightarrow \text{mod } (4+1) = 0$
$X=5, Y=5 \Rightarrow \text{mod } (5+1) = 5$
$X=6, Y=5 \Rightarrow \text{mod } (6+1) = 5$
$X=7, Y=5 \Rightarrow \text{mod } (7+1) = 5$
$X=8, Y=5 \Rightarrow \text{mod } (8+1) = 5$
$X=9, Y=5 \Rightarrow \text{mod } (9+1) = 5$

possible values: 0, 1, 2, 5

$Y=0$ when $X=0, 4$
$P(Y=0) = \frac{1}{10} + \frac{1}{10} = \frac{1}{5}$

$Y=1$ when $X=1, 3$
$P(Y=1) = \frac{1}{10} + \frac{1}{10} = \frac{1}{5}$

$Y=2$ when $X=2$
$P(Y=2) = \frac{1}{10}$

$Y=5$ when $X=5, 6, 7, 8, 9$
$P(Y=5) = \frac{5}{10} = \frac{1}{2}$

Value of Y	PMF
0	$\frac{1}{5}$
1	$\frac{1}{5}$
2	$\frac{1}{10}$
5	$\frac{1}{2}$