Sample Test

Instructions

Please answer the following questions to the best of your ability. For each question, 10 points are possible. If you encounter complicated computations, you are allowed to simply write down the formula without calculating the final number.

1. An unfair coin is constructed such that the probability of landing heads (outcome 1) is four times as likely as landing tails (outcome 0).
   (a) What is the probability mass function (PMF) for the outcome?
   (b) What is the expected value of the outcome?
2. We have two fair coins: one coin has heads on both sides, and the other coin has one head and one tail.
   (a) What is the probability of obtaining tails if a coin is chosen at random and tossed?
   (b) If the toss results in tails, what is the probability that the other side of the selected coin is a head?
3. Let X be a uniformly distributed discrete random variable in the range $[0, 8]$.
   (a) What is $E[X]$?
   (b) Calculate $E[X^{100}]$. Provide a pseudo-code to determine this value.
4. Let X and Y be continuous uniform random variables, both in the range $[0, 1]$. What is the probability $P(Y - X > \frac{1}{2})$?
5. Passwords are chosen from a set of symbols containing 26 lowercase letters and 26 uppercase letters.
   (a) How many possible passwords are there if the password has exactly 8 characters, with at least two uppercase letters and at least two lowercase letters, without repeating characters?
   (b) How many passwords are possible if the length of the password is 8 characters, must begin with an uppercase letter, end with a lowercase letter, with repeating characters allowed?
6. Among 40 students, including Tom and Jerry, are to be split into 2 groups of equal size, randomly. What is the probability that Tom and Jerry end up in the same group?