

1. (9 points) Write a program to simulate the result of Example 3.14 (page 168 of the textbook). *see HW6.py in submission*

Hint: Run 1000 trials and plot a histogram of the results in 2 minute intervals. Use the example as written with  $X$  uniformly distributed from 7:10 to 7:30.

2. (8 points) pg 190 problem 18  
3. (8 points) pg 190 problem 19 a, b

$$2) a) E[X] = \int_1^3 \frac{x^2}{4} dx = \left. \frac{x^3}{12} \right|_1^3 = \frac{27}{12} - \frac{1}{12} = \frac{13}{6}$$

$$b) P(A) = \int_2^3 \frac{x}{4} dx = \left. \frac{x^2}{8} \right|_2^3 = \frac{9}{8} - \frac{4}{8} = \frac{5}{8}$$

$$f_{X|A}(x) = \begin{cases} \frac{f_X(x)}{P(A)}, & \text{if } x \in A \\ 0, & \text{otherwise} \end{cases} = \begin{cases} \frac{2x}{5}, & \text{if } 2 \leq x \leq 3 \\ 0, & \text{otherwise} \end{cases}$$

$$E[X|A] = \int_2^3 f_{X|A}(x) \cdot x dx = \int_2^3 \frac{2x}{5} x dx = \int_2^3 \frac{2x^2}{5} dx$$

$$= \left. \frac{2x^3}{15} \right|_2^3 = \frac{54}{15} - \frac{16}{15} = \frac{38}{15}$$

$$b) E[Y] = E[X^2] = \int_1^3 \frac{x^3}{4} dx = \left. \frac{x^4}{16} \right|_1^3 = \frac{81}{16} - \frac{1}{16} = 5$$

$$E[Y^2] = E[X^4] = \int_1^3 \frac{x^3}{4} dx = \left. \frac{x^6}{24} \right|_1^3 = \frac{729}{24} - \frac{1}{24} = \frac{91}{3}$$

$$\text{Var}[Y] = E[Y^2] - (E[Y])^2 = \frac{91}{3} - 5^2 = \frac{16}{3}$$

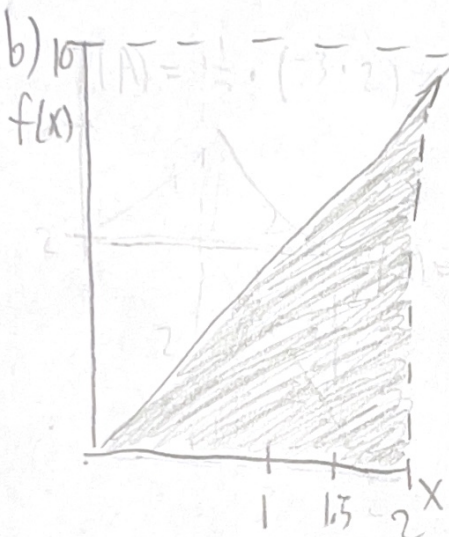
$$3) f(x) = \begin{cases} cx^{-2}, & \text{if } 1 \leq x \leq 2 \\ 0, & \text{otherwise} \end{cases}$$

$$\begin{aligned} a) 1 &= \int_1^2 cx dx + \int_{-\infty}^1 10 dx + \int_2^{\infty} 10 dx = \left[ \frac{c}{2} x^2 \right]_1^2 + [10x]_{-\infty}^1 + [10x]_2^{\infty} \\ &= \frac{c}{2} (2^2 - 1) + 10(1 - (-\infty)) + 10(\infty - 2) \\ &= c(4 - 1) + 10(1) + 10(\infty - 2) \\ &= 3c + 10 + 10(\infty - 2) \end{aligned}$$

$$3c + 10 = 1$$

$$3c = -9$$

$$c = -3$$



$$P(X \leq 1.5) = \frac{1}{2} \times (-3 \times 2) = -3$$

$$P(X \leq 1.5) = \int_1^{1.5} (-3x) dx$$

$$P(X \leq 1.5) = \left[ -\frac{3}{2} x^2 \right]_1^{1.5}$$

$$P(X \leq 1.5) = -\frac{3}{2} (2.25) + \frac{3}{2}$$

$$= -\frac{27}{8} + \frac{12}{8} = -\frac{15}{8}$$

$$P(A) = 1 - \left( -\frac{15}{8} \right) = 1 + \frac{15}{8} = \frac{23}{8}$$

$$f_{X|A}(x) = \frac{f_X(x)}{P(A)} = \frac{-3x}{\frac{23}{8}} = \frac{-24x}{23}$$

$$PDF = \begin{cases} \frac{-24x}{23}, & \text{if } 1.5 < x \leq 2 \\ 0, & \text{otherwise} \end{cases}$$