FRIEDRICH-ALEXANDER
UNIVERSITÄT
ERLANGEN-NÜRNBERG
TECHNISCHE FAKULTÄT

Backtracking

Ausgewählte Kapitel aus "The Art of Computer Programming"

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Outline

Einführung

n Queens

Langford Pairs

Einführung

Anwendungsbereich

Sequenzen $x_1, x_2, x_3 \ldots x_n$ für welche die Bedingung $P_n(x_1, x_2, x_3 \ldots x_n)$ gilt.

P hat dabei folgende Eigenschaften:

- $P_l(x_1, x_2, x_3 \ldots x_l)$ gilt nur, wenn $P_{l-1}(x_1, x_2, x_3 \ldots x_{l-1})$ gilt
- wenn $P_l(x_1, x_2, x_3 \ldots x_l)$ gilt, ist $P_{l+1}(x_1, x_2, x_3 \ldots x_{l+1})$ einfach zu testen
- $P_0()$ gilt immer

Anwendungsbereich

Sequenzen $x_1, x_2, x_3 \ldots x_n$ für welche die Bedingung $P_n\left(x_1, x_2, x_3 \ldots x_n\right)$ gilt.

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- wenn $P_l\left(x_1, x_2, x_3 \ldots x_l\right)$ gilt, ist $P_{l+1}\left(x_1, x_2, x_3 \ldots x_{l+1}\right)$ einfach zu testen
- $P_0()$ gilt immer

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- $P_0()$ gilt immer

Algorithmus

```rust
/// A required set of methods needed for the generic backtracking algorithms.
pub trait Sequence {
    type Step;
    type Steps: IntoIterator<Item = Self::Step>;

    /// Checks if this sequence satisfy its condition.
    ///
    /// This function can assume that the  parent of `self` satisfied this condition.
    fn satisfies_condition(&self) -> bool;

    /// generates all possible next steps at this current state.
    fn next_steps(&self) -> Self::Steps;

    /// applies a `step` to `self`, returning the resulting sequence.
    ///
    /// this function will only be called if `self.satisfies_condition() == true`.
    fn apply_step(&self, step: Self::Step) -> Self;
}
```

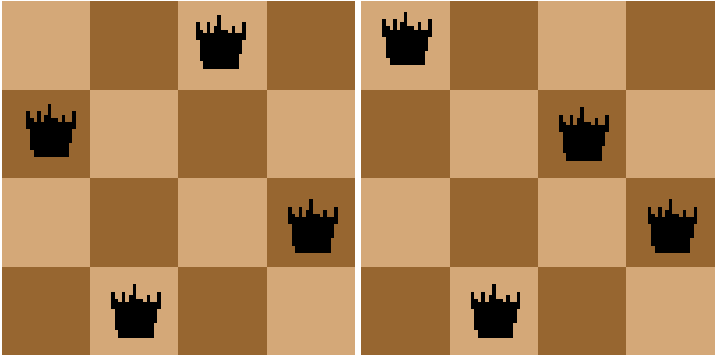
Algorithmus

```
pub fn b<T: Sequence>(initial: T, n: usize) -> Vec<T> {
    let mut results = Vec::new();
    let mut states = Vec::new();
    let steps = initial.next_steps().into_iter();
    states.push((initial, steps));

    while let Some((state, steps)) = states.last_mut() {
        if let Some(step) = steps.next() {
            let next_state = state.apply_step(step);
            if next_state.satisfies_condition() {
                if states.len() < n {
                    let next_steps = next_state.next_steps().into_iter();
                    states.push((next_state, next_steps));
                } else {
                    results.push(next_state);
                }
            }
        } else {
            states.pop();
        }
    }
    results
}
```

n Queens

Damenproblem

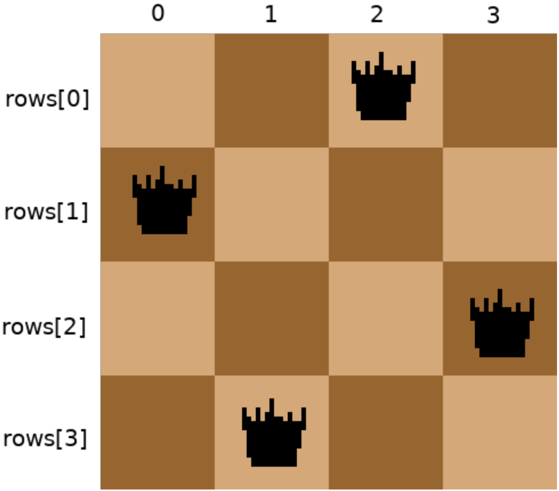
Wie kann man n Damen auf einem $n * n$ Schachbrett aufstellen, ohne dass sich zwei Damen schlagen können, also keine 2 Damen in der selben Zeile, Reihe oder Diagonalen stehen.



(a) Richtig

(b) Falsch

impl Sequence



impl Sequence

```rust
pub struct Queens {
    n: usize,
    rows: Vec<usize>,
}

impl Queens {
    pub fn new(n: usize) -> Self {
        Self {
            n,
            rows: Vec::new(),
        }
    }
}
```

impl Sequence

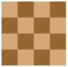
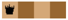
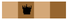
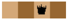
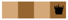
```
fn satisfies_condition(&self) -> bool {
    if self.rows.is_empty() {
        return true;
    }

    let k = self.rows.len() - 1;

    for j in 0..k {
        let k_col = self.rows[k] as isize;
        let j_col = self.rows[j] as isize;

        if k_col == j_col || (j_col - k_col).abs() as usize == k - j {
            return false;
        }
    }
    true
}
```

impl Sequence

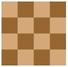
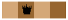
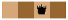
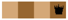
```rust
fn next_steps(&self) -> Self::Steps {
    0..self.n
}

fn apply_step(&self, step: Self::Step) -> Self {
    let mut rows = self.rows.clone();
    rows.push(step);
    Self { n: self.n, rows }
}
```

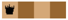
```
let results = b(Queens::new(4), 4);

pub fn b<T: Sequence>(initial: T, n: usize) -> Vec<T> {
    let mut results = Vec::new();
    let mut states = Vec::new();

    let steps = initial.next_steps().into_iter();
    states.push((initial, steps));
    // <- we are here
    // ...
}
```

$results = [\,]$

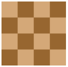
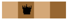
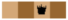
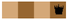
$states = [($  $, [$  $,$  $,$  $,$  $])]$

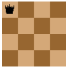
```
// ...
while let Some((state, steps)) = states.last_mut() {
    if let Some(step) = steps.next() {
        // <- we are here
        let next_state = state.apply_step(step);
        if next_state.satisfies_condition() {
            if states.len() < n {
                let next_steps = next_state.next_steps().into_iter();
                states.push((next_state, next_steps));
            } else {
                results.push(next_state);
            }
        }
    } else {
        states.pop();
    }
}
// ...
```

$states = [($  $, [$  $,$  $,$  $])]$

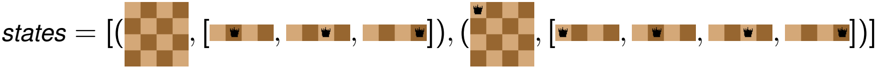
$step =$ 

```
// ...
while let Some((state, steps)) = states.last_mut() {
    if let Some(step) = steps.next() {
        let next_state = state.apply_step(step);
        // <- we are here
        if next_state.satisfies_condition() {
            if states.len() < n {
                let next_steps = next_state.next_steps().into_iter();
                states.push((next_state, next_steps));
            } else {
                results.push(next_state);
            }
        }
    } else {
        states.pop();
    }
}
// ...
```

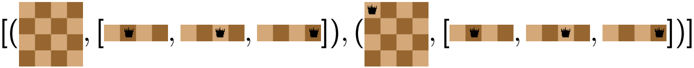
$states = [($  $, [$  $,$  $,$  $])]$

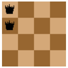
$next\_state =$ 

```
// ...
while let Some((state, steps)) = states.last_mut() {
    if let Some(step) = steps.next() {
        let next_state = state.apply_step(step);
        // our previous position
        if next_state.satisfies_condition() {
            if states.len() < n {
                let next_steps = next_state.next_steps().into_iter();
                states.push((next_state, next_steps));
                // <- we are here
            } else {
                results.push(next_state);
            }
        }
    } else {
        states.pop();
    }
}
// ...
```



```rust
// ...
while let Some((state, steps)) = states.last_mut() {
    if let Some(step) = steps.next() {
        let next_state = state.apply_step(step);
        // <- we are here
        if next_state.satisfies_condition() {
            if states.len() < n {
                let next_steps = next_state.next_steps().into_iter();
                states.push((next_state, next_steps));
            } else {
                results.push(next_state);
            }
        }
    } else {
        states.pop();
    }
}
// ...
```

$states =$ 

$next\_state =$ 

```
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        }
    } else {
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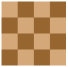
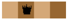
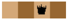
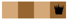
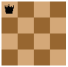
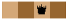
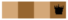
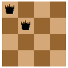
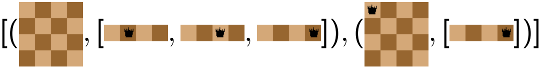
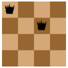
    }
}
// ...
```

states = [(, [, , ]), (, [, ])]

next_state = 

```
// ...
while let Some((state, steps)) = states.last_mut() {
    if let Some(step) = steps.next() {
        let next_state = state.apply_step(step);
        // <- we are here
        if next_state.satisfies_condition() {
            if states.len() < n {
                let next_steps = next_state.next_steps().into_iter();
                states.push((next_state, next_steps));
            } else {
                results.push(next_state);
            }
        }
    } else {
        states.pop();
    }
}
// ...
```

$states =$ 

$next\_state =$ 

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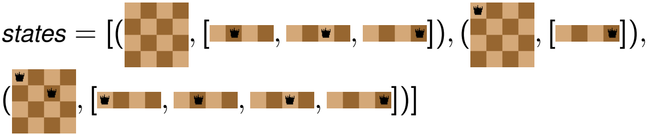
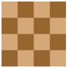
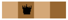
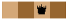
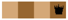
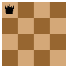
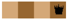
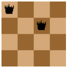
                let next_steps = next_state.next_steps().into_iter();
                states.push((next_state, next_steps));
                // <- we are here
            } else {
                results.push(next_state);
            }
        }
    } else {
        states.pop();
    }
}
```

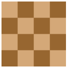
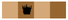
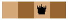
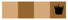
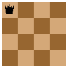
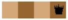
states =



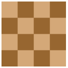
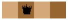
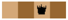
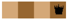
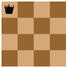
```
while let Some((state, steps)) = states.last_mut() {
    // <- we are here after discarding all 4 steps
    if let Some(step) = steps.next() {
        let next_state = state.apply_step(step);
        if next_state.satisfies_condition() {
            if states.len() < n {
                let next_steps = next_state.next_steps().into_iter();
                states.push((next_state, next_steps));
            } else {
                results.push(next_state);
            }
        }
    } else {
        states.pop();
    }
}
```

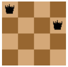
states = [(, [, , ]), (, []), (, [ ])]

```
while let Some((state, steps)) = states.last_mut() {
    if let Some(step) = steps.next() {
        let next_state = state.apply_step(step);
        if next_state.satisfies_condition() {
            if states.len() < n {
                let next_steps = next_state.next_steps().into_iter();
                states.push((next_state, next_steps));
            } else {
                results.push(next_state);
            }
        }
    } else {
        states.pop();
        // <- we are here
    }
}
```

$states = [($  $, [$  $,$  $,$  $]), ($  $, [$  $])]$

```
while let Some((state, steps)) = states.last_mut() {
    if let Some(step) = steps.next() {
        let next_state = state.apply_step(step);
        // <- we are here
        if next_state.satisfies_condition() {
            if states.len() < n {
                let next_steps = next_state.next_steps().into_iter();
                states.push((next_state, next_steps));
            } else {
                results.push(next_state);
            }
        }
    } else {
        states.pop();
    }
}
```

states = [(, [, , ]), (, [ ])]

next_state = 

```
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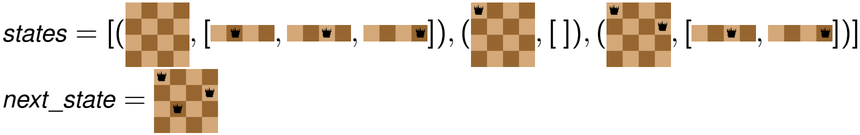
        // <- we are here
        if next_state.satisfies_condition() {
            if states.len() < n {
                let next_steps = next_state.next_steps().into_iter();
                states.push((next_state, next_steps));
            } else {
                results.push(next_state);
            }
        }
    } else {
        states.pop();
    }
}
```



```
while let Some((state, steps)) = states.last_mut() {
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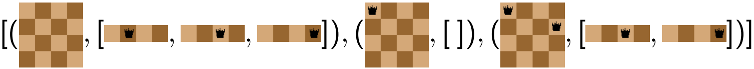
                states.push((next_state, next_steps));
            } else {
                results.push(next_state);
            }
        }
    } else {
        states.pop();
        // <- we are here
    }
}
```

$states =$



```
while let Some((state, steps)) = states.last_mut() {
    if let Some(step) = steps.next() {
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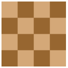
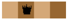
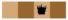
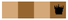
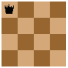
        if next_state.satisfies_condition() {
            if states.len() < n {
                let next_steps = next_state.next_steps().into_iter();
                states.push((next_state, next_steps));
            } else {
                results.push(next_state);
            }
        }
    } else {
        states.pop();
        // <- we are here
    }
}
```

$\mathit{states} = [($  $, [$  $,$  $,$  $]), ($  $, [\,]),]$

```
while let Some((state, steps)) = states.last_mut() {
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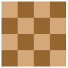
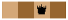
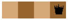
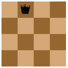
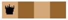
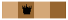
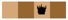
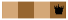
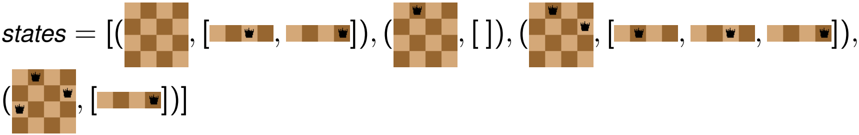
            if states.len() < n {
                let next_steps = next_state.next_steps().into_iter();
                states.push((next_state, next_steps));
                // <- we are here
            } else {
                results.push(next_state);
            }
        }
    } else {
        states.pop();
    }
}
```

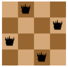
states = [( , [ , ]), ( , [ ,  ,  , ])]

```
while let Some((state, steps)) = states.last_mut() {
    if let Some(step) = steps.next() {
        let next_state = state.apply_step(step);
        if next_state.satisfies_condition() {
            // <- we are here
            if states.len() < n {
                let next_steps = next_state.next_steps().into_iter();
                states.push((next_state, next_steps));
            } else {
                results.push(next_state);
            }
        }
    } else {
        states.pop();
    }
}
```

states =



next_state =



```
while let Some((state, steps)) = states.last_mut() {
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        let next_state = state.apply_step(step);
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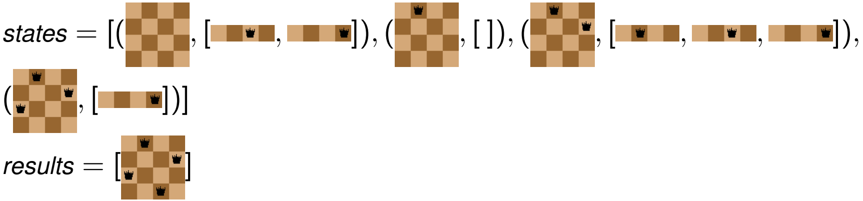
                states.push((next_state, next_steps));
            } else {
                results.push(next_state);
                // <- we are here
            }
        }
    } else {
        states.pop();
    }
}
```


states = [(, [,]), (, [ ]), (, [, ,]), (, [ ])]

results = [ ]

```
while let Some((state, steps)) = states.last_mut() {
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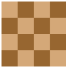
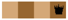
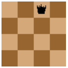
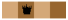
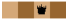
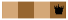
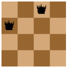
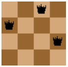
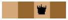
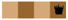
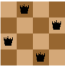
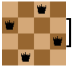
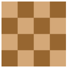
            } else {
                results.push(next_state);
                // <- we are here
            }
        }
    } else {
        states.pop();
    }
}
```

states = [(, []), (, [, , ]), (, [ ]),
(, [, ])]

results = [, ]

```
while let Some((state, steps)) = states.last_mut() {
    if let Some(step) = steps.next() {
        let next_state = state.apply_step(step);
        if next_state.satisfies_condition() {
            if states.len() < n {
                let next_steps = next_state.next_steps().into_iter();
                states.push((next_state, next_steps));
            } else {
                results.push(next_state);
            }
        }
    } else {
        states.pop();
        // <- we are here
    }
}
```

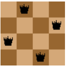
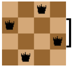
$states = [($  $, [\,])]$

```
while let Some((state, steps)) = states.last_mut() {
    if let Some(step) = steps.next() {
        let next_state = state.apply_step(step);
        if next_state.satisfies_condition() {
            if states.len() < n {
                let next_steps = next_state.next_steps().into_iter();
                states.push((next_state, next_steps));
            } else {
                results.push(next_state);
            }
        }
    } else {
        states.pop();
        // <- we are here
    }
}
```

$$states = [\,]$$

```
while let Some((state, steps)) = states.last_mut() {
    if let Some(step) = steps.next() {
        let next_state = state.apply_step(step);
        if next_state.satisfies_condition() {
            if states.len() < n {
                let next_steps = next_state.next_steps().into_iter();
                states.push((next_state, next_steps));
            } else {
                results.push(next_state);
            }
        }
    } else {
        states.pop();
    }
}
// <- we are here
results
```

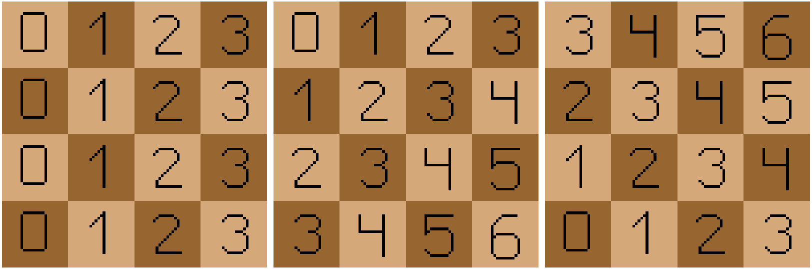
$$\mathit{states} = [\,]$$

results = [ , ]

B*

satisfies_condition vergleicht momentan die neue Reihe mit allen anderen Reihen.

Kann mit drei Bitsets ersetzt werden:



0	1	2	3
0	1	2	3
0	1	2	3
0	1	2	3

(a) Spalten

0	1	2	3
1	2	3	4
2	3	4	5
3	4	5	6

(b) Gegendiagonale

3	4	5	6
2	3	4	5
1	2	3	4
0	1	2	3

(c) Hauptdiagonale

Algorithm

```
pub fn b_star(n: usize) -> Vec<Queens> {
    // setup
    let mut results = Vec::new();

    let mut columns = BitVec::from_elem(n, false);
    let mut left_diagonals = BitVec::from_elem(2 * n - 1, false);
    let mut right_diagonals = BitVec::from_elem(2 * n - 1, false);

    let mut rows = Vec::new();
    let mut column = 0;

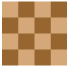
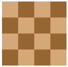
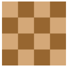
    loop {
        // ...
    }
}
```

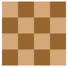
```
loop {
    while column < n {
        if !(columns[column]
            || left_diagonals[column + rows.len()]
            || right_diagonals[column + n - 1 - rows.len()])
        {
            if rows.len() + 1 < n {
                columns.set(column, true);
                left_diagonals.set(column + rows.len(), true);
                right_diagonals.set(column + n - 1 - rows.len(), true);
                rows.push(column);
                column = 0;
            } else {
                let mut q = rows.clone();
                q.push(column);
                results.push(Queens { n, rows: q });
                column += 1;
            }
        } else {
            column += 1;
        }
    }

    if let Some(prev) = rows.pop() {
        right_diagonals.set(prev + n - 1 - rows.len(), false);
        left_diagonals.set(prev + rows.len(), false);
        columns.set(prev, false);
        column = prev + 1;
    } else {
        return results;
    }
}
```

```rust
let results = b_star(4);

pub fn b_star(n: usize) -> Vec<Queens> {
    let mut results = Vec::new();

    let mut columns = BitVec::from_elem(n, false);
    let mut left_diagonals = BitVec::from_elem(2 * n - 1, false);
    let mut right_diagonals = BitVec::from_elem(2 * n - 1, false);

    let mut rows = Vec::new();
    let mut column = 0;

    // ...
}
```

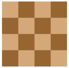
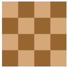
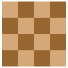
$\mathit{results} = [\,]$

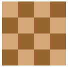
$\mathit{columns} =$  , $\mathit{left\_diagonals} =$  , $\mathit{right\_diagonals} =$ 

$\mathit{rows} =$ 

$\mathit{column} = 0$

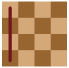
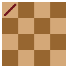
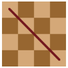
```
loop {
    while column < n {
        if !(columns[column]
            || left_diagonals[column + rows.len()]
            || right_diagonals[column + n - 1 - rows.len()])
        {
            if rows.len() + 1 < n {
                // <- we are here
                columns.set(column, true);
                left_diagonals.set(column + rows.len(), true);
                right_diagonals.set(column + n - 1 - rows.len(), true);
                rows.push(column);
                column = 0;
            } else {
                // add to results, removed for clarity
            }
        } else {
            column += 1;
        }
    }
    // backtracking, removed for clarity
}
```

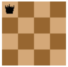
$columns =$  , $left\_diagonals =$  , $right\_diagonals =$ 

$rows =$ 

$column = 0$

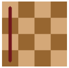
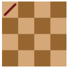
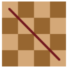
```
loop {
  while column < n {
    if !(columns[column]
      || left_diagonals[column + rows.len()]
      || right_diagonals[column + n - 1 - rows.len()])
    {
      if rows.len() + 1 < n {
        columns.set(column, true);
        left_diagonals.set(column + rows.len(), true);
        right_diagonals.set(column + n - 1 - rows.len(), true);
        rows.push(column);
        // <- we are here
        column = 0;
      } else {
        // add to results, removed for clarity
      }
    } else {
      column += 1;
    }
  }
  // backtracking, removed for clarity
}
```

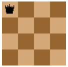
$columns =$  , $left\_diagonals =$  , $right\_diagonals =$ 

$rows =$ 

$column = 0$

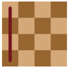
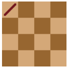
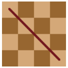
```
loop {
    while column < n {
        if !(columns[column]
            || left_diagonals[column + rows.len()]
            || right_diagonals[column + n - 1 - rows.len()])
        {
            if rows.len() + 1 < n {
                columns.set(column, true);
                left_diagonals.set(column + rows.len(), true);
                right_diagonals.set(column + n - 1 - rows.len(), true);
                rows.push(column);
                column = 0;
            } else {
                // add to results, removed for clarity
            }
        } else {
            column += 1;
            // <- we are here
        }
    }
    // backtracking, removed for clarity
}
```

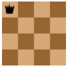
$columns =$  , $left\_diagonals =$ , $right\_diagonals =$ 

$rows =$ 

$column = 1$

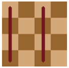
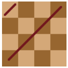
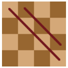
```
loop {
    while column < n {
        if !(columns[column]
            || left_diagonals[column + rows.len()]
            || right_diagonals[column + n - 1 - rows.len()])
        {
            if rows.len() + 1 < n {
                columns.set(column, true);
                left_diagonals.set(column + rows.len(), true);
                right_diagonals.set(column + n - 1 - rows.len(), true);
                rows.push(column);
                column = 0;
            } else {
                // add to results, removed for clarity
            }
        } else {
            column += 1;
            // <- we are here
        }
    }
    // backtracking, removed for clarity
}
```

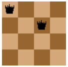
$columns =$ , $left\_diagonals =$ , $right\_diagonals =$ 

$rows =$ 

$column = 2$

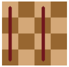
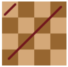
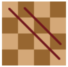
```
loop {
    while column < n {
        if !(columns[column]
            || left_diagonals[column + rows.len()]
            || right_diagonals[column + n - 1 - rows.len()])
        {
            if rows.len() + 1 < n {
                columns.set(column, true);
                left_diagonals.set(column + rows.len(), true);
                right_diagonals.set(column + n - 1 - rows.len(), true);
                rows.push(column);
                column = 0;
                // <- we are here
            } else {
                // add to results, removed for clarity
            }
        } else {
            column += 1;
        }
    }
    // backtracking, removed for clarity
}
```

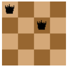
columns =

, *left_diagonals* =

, *right_diagonals* =


rows =


column = 0

```
loop {
    // while ...

    // <- we are here
    if let Some(prev) = rows.pop() {
        right_diagonals.set(prev + n - 1 - rows.len(), false);
        left_diagonals.set(prev + rows.len(), false);
        columns.set(prev, false);
        column = prev + 1;
    } else {
        break;
    }
}
```

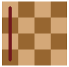
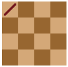
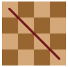
$columns =$  $, left\_diagonals =$  $, right\_diagonals =$ 

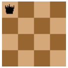
$rows =$ 

$column = 4$

```
loop {
    // while ...

    if let Some(prev) = rows.pop() {
        right_diagonals.set(prev + n - 1 - rows.len(), false);
        left_diagonals.set(prev + rows.len(), false);
        columns.set(prev, false);
        column = prev + 1;
        // <- we are here
    } else {
        return results;
    }
}
```

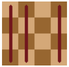
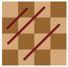
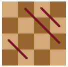
$prev = 2$

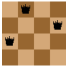
$columns =$  , $left\_diagonals =$  , $right\_diagonals =$ 

$rows =$ 

$column = 3$

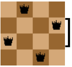
```
loop {
    while column < n {
        if !(columns[column]
            || left_diagonals[column + rows.len()]
            || right_diagonals[column + n - 1 - rows.len()])
        {
            if rows.len() + 1 < n {
                // update rows etc, removed for clarity
            } else {
                // <- we are here
                let mut q = rows.clone();
                q.push(column);
                results.push(Queens { n, rows: q });
                column += 1;
            }
        } else {
            column += 1;
        }
    }
    // backtracking, removed for clarity
}
```

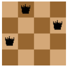
$columns =$ , $left\_diagonals =$ , $right\_diagonals =$ 

$rows =$ 

$column = 2$

```
loop {
    while column < n {
        if !(columns[column]
            || left_diagonals[column + rows.len()]
            || right_diagonals[column + n - 1 - rows.len()])
        {
            if rows.len() + 1 < n {
                // update rows etc, removed for clarity
            } else {
                let mut q = rows.clone();
                q.push(column);
                results.push(Queens { n, rows: q });
                column += 1;
                // <- we are here
            }
        } else {
            column += 1;
        }
    }
    // backtracking, removed for clarity
}
```

$results = [$  $]$

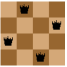
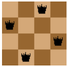
$rows =$ 

$column = 3$

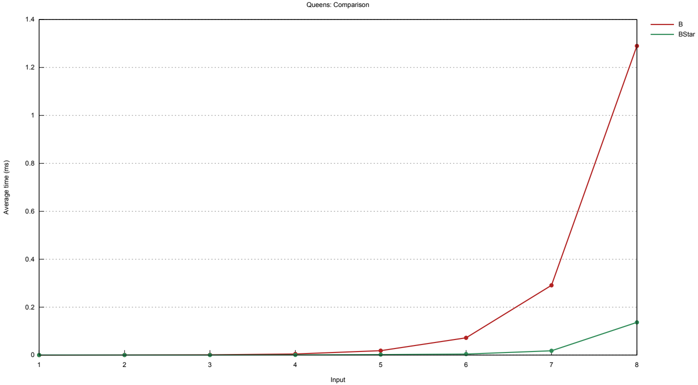
```
loop {
    // while ...

    // <- we are here
    if let Some(prev) = rows.pop() {
        right_diagonals.set(prev + n - 1 - rows.len(), false);
        left_diagonals.set(prev + rows.len(), false);
        columns.set(prev, false);
        column = prev + 1;
    } else {
        return results;
    }
}
```

columns = , *left_diagonals* = , *right_diagonals* =

rows =

column = 4 *results* = [ , ]

Performance



Langford Pairs

Langford Paare

Finde alle Permutationen der Menge $M = 1, -1, 2, -2, \ldots, n, -n$ für die gilt $x_l = p \Rightarrow x_{l+p+1} = -p$.

Für $n = 4$ sind $[2, 3, 4, -2, 1, -3, -1, -4]$ und $[4, 1, 3, -1, 2, -4, -3, -2]$ die einzigen Lösungen.

Implementation

```
pub fn l(n: usize) -> Vec<Vec<isize>> {
    let mut results = Vec::new();

    let mut sequence = vec![0; n * 2];
    let mut position = 0;

    let mut unused_values = (1..=n).collect::<Vec<_>>();
    unused_values.push(0);

    let mut undo = vec![0; n * 2];
    let mut ptr = 0;

    loop {
        // ...
    }
}
```

```
loop {
    while unused_values[ptr] != 0 && position + unused_values[ptr] + 1 < sequence.len() {
        if sequence[position + unused_values[ptr] + 1] == 0 {
            sequence[position] = unused_values[ptr] as isize;
            sequence[position + unused_values[ptr] + 1] = -(unused_values[ptr] as isize);
            undo[position] = ptr;
            unused_values[ptr] = unused_values[unused_values[ptr]];

            ptr = 0;
            position += 1;
            if unused_values[ptr] == 0 {
                results.push(sequence.clone());
            } else {
                while sequence[position] < 0 {
                    position += 1;
                }
            }
        } else {
            ptr = unused_values[ptr];
        }
    }
    if position != 0 {
        position -= 1;
        while sequence[position] < 0 {
            position -= 1;
        }

        let removed_value = sequence[position] as usize;
        sequence[position] = 0;
        sequence[position + removed_value + 1] = 0;
        unused_values[undo[position]] = removed_value;
        ptr = removed_value;
    } else {
        return results;
    }
}
```

```
pub fn l(n: usize) -> Vec<Vec<isize>> {
    let mut results = Vec::new();

    let mut sequence = vec![0; n * 2];
    let mut position = 0;

    let mut unused_values = (1..=n).collect::<Vec<_>>();
    unused_values.push(0);

    let mut undo = vec![0; n * 2];
    let mut ptr = 0;
    // <- we are here

    loop {
        // ...
    }
}
```

$$results = [\,]$$
$$sequence = [0, 0, 0, 0, 0, 0], position = 0$$
$$unused\_values = [1, 2, 3, 0], ptr = 0, unused\_values[ptr] = 1$$
$$undo = [0, 0, 0, 0, 0, 0]$$

```
loop {
    while unused_values[ptr] != 0 && position + unused_values[ptr] + 1 < sequence.len() {
        if sequence[position + unused_values[ptr] + 1] == 0 {
            // <- we are here
            sequence[position] = unused_values[ptr] as isize;
            sequence[position + unused_values[ptr] + 1] = -(unused_values[ptr] as isize);

            undo[position] = ptr;
            unused_values[ptr] = unused_values[unused_values[ptr]];

            // update position and reset ptr, removed for clarity
        } else {
            ptr = unused_values[ptr];
        }
    }
    // backtracking, removed for clarity
}
```

$sequence = [0, 0, 0, 0, 0, 0]$, $position = 0$
$unused\_values = [1, 2, 3, 0]$, $ptr = 0$, $unused\_values[ptr] = 1$
$undo = [0, 0, 0, 0, 0, 0]$

```
loop {
    while unused_values[ptr] != 0 && position + unused_values[ptr] + 1 < sequence.len() {
        if sequence[position + unused_values[ptr] + 1] == 0 {
            sequence[position + unused_values[ptr] + 1] = -(unused_values[ptr] as isize);
            sequence[position] = unused_values[ptr] as isize;
            // <- we are here
            undo[position] = ptr;
            unused_values[ptr] = unused_values[unused_values[ptr]];

            // update position and reset ptr, removed for clarity
        } else {
            ptr = unused_values[ptr];
        }
    }
    // backtracking, removed for clarity
}
```

$$sequence = [1, 0, -1, 0, 0, 0],\ position = 0$$
$$unused\_values = [1, 2, 3, 0],\ ptr = 0,\ unused\_values[ptr] = 1$$
$$undo = [0, 0, 0, 0, 0, 0]$$

```
loop {
    while unused_values[ptr] != 0 && position + unused_values[ptr] + 1 < sequence.len() {
        if sequence[position + unused_values[ptr] + 1] == 0 {
            sequence[position + unused_values[ptr] + 1] = -(unused_values[ptr] as isize);
            sequence[position] = unused_values[ptr] as isize;

            undo[position] = ptr;
            unused_values[ptr] = unused_values[unused_values[ptr]];
            // <- we are here
            // update position and reset ptr, removed for clarity
        } else {
            ptr = unused_values[ptr];
        }
    }
    // backtracking, removed for clarity
}
```

$sequence = [1, 0, -1, 0, 0, 0]$, $position = 0$
$unused\_values = [2, 2, 3, 0]$, $ptr = 0$, $unused\_values[ptr] = 2$
$undo = [0, 0, 0, 0, 0, 0]$

Thanks for listening.

Any questions?

References

References I