Schooled by Trade?
Retraining and Import Competition [*]

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Abstract

Retraining is often hailed as key policy tool for aiding displaced workers and smoothing the impact of sectoral shocks. We study the interaction of retraining and international trade in Germany, a highly open economy with extensive government-subsidized retraining programs. Using rich administrative data we provide evidence that workers routinely retrain in response to import competition and that the labour market effects of import competition are more muted for workers who do retrain. We introduce retraining into a model of workers from heterogeneous occupations who sort across sectors within a Ricardian trade framework. In our model, whenever retraining serves to broaden worker skills, it shrinks occupations' trade exposure and compresses the distribution of trade-induced welfare effects. Calibrated to match our empirical results, the model reveals that retraining has little effect on Germany's aggregate gains from rising imports from China and Eastern Europe but, in line with its skill-broadening function, reduces inequality among workers in the effects of import competition.

1   Introduction

International trade, like many sector-specific economic shocks, creates winners and losers. A substantial literature shows that whether individual workers are hurt or helped by trade depends on their labour market exposure to declining or growing sectors.[1] But, despite enormous interest from policymakers, relatively little is known about how specific policies can alter workers' exposure to different sectors and ultimately blunt the uneven effects of trade. The relevance of such policies will only increase as economies confront new sectoral shocks like the spread of automation, the transition towards greener technologies, and the adoption of artificial intelligence.

We fill this gap by studying the role of government-subsidised retraining in Germany, an open economy at the front lines of the integration of China and Eastern Europe into the global economy.[2] With the help of rich administrative data, we provide new evidence that workers use retraining as a margin of adjustment when faced with rising import competition from China and Eastern Europe. We then characterize its aggregate importance with a quantitative general equilibrium model of labour supply and international trade in which retraining allows workers to adjust their skillsets in response to shocks. We find that retraining reduces the dispersion of trade shocks' welfare effects by around a fifth. Retraining thus effectively shields the losers from import competition — without diminishing Germany's ability to benefit from international trade in the aggregate.

We first document salient features of retraining in Germany using administrative data from the Sample of Integrated Labour Market Biographies (SIAB). German government-subsidized retraining operates via vouchers cashed in with private providers, and retraining courses provide a mix of sector-specific skills and job-finding assistance. Retraining is common, with roughly one-quarter of workers retraining over their careers, and retraining courses vary widely in duration: one third of courses last less than twenty days while another third last more than a hundred days.

We next investigate the interaction of retraining with rising import competition from Eastern Europe and China. After documenting adverse effects of import competition on workers' employment and earnings, we show that import competition causes workers to retrain. A one standard deviation increase in the import exposure of a worker's occupation leads to a 7.6% increase in the probability of retraining over a seven year period. We also provide suggestive evidence that retraining shields exposed workers: for the subset of workers who retrain, we detect no significant effect of import competition on employment or earnings.

To map these individual effects into aggregate welfare and inequality, we build a quantitative general equilibrium model of labour supply and international trade with a retraining technology. Our economy consists of multiple occupations and sectors. Workers' occupations determine their

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[1]To give just two examples, Autor, Dorn, and Hanson (2013) shows empirically that American workers in local labor markets which specialise in import-competing sectors suffer experience lower employment and earnings, while Traiberman (2019) uses a structural model to show that Danish workers in occupations which specialise in import-competing sectors see similar adverse effects.

[2]We use 'retraining' to refer to a collection of policies that attempt to provide existing workers with training in specific skills as well as more general labour market assistance. Section 2 contains a detailed definition.

patterns of comparative advantage across sectors and ultimately their exposure to different sectoral shocks, such as fast productivity growth in China and Eastern Europe. We enrich this basic framework with a parsimonious model of retraining, which alters workers' comparative advantage across sectors. Retraining may either reinforce a worker's existing comparative advantage, which we refer to as skill deepening, or may do just the opposite, which we refer to as skill broadening. We show that the interaction of import competition with retraining in our model depends crucially on whether retraining broadens or deepens a worker's skillset. If retraining deepens a worker's skillset, it primarily benefits the winners from import competition and widens inequality in the effects of trade. If instead retraining broadens a worker's skillset, it primarily benefits the losers and narrows inequality.

We calibrate the parameters which govern this retraining technology, as well as key labour supply elasticities, by matching the effects of import competition in the data. We find that retraining has a strong skill-broadening effect. As a result, our model rationalises both the increase in retraining in response to import competition and the diminished effects of import competition on workers who retrain.

Having calibrated the model, we return to the central question of our paper: how does retraining alter the impact of trade shocks? We first use the model to study the equilibrium effects of rapid growth in Chinese and Eastern European productivity, and thus import competition, on German workers. While welfare rises by roughly 1% in the average occupation, import competition results in modest welfare losses for occupations specialised in the most exposed sectors. Next, we shut down retraining and simulate the same set of shocks. Without the option of retraining — which, according to our calibration, broadens a worker's skillset — occupations specialise more intensely in a few sectors in which they have a comparative advantage. Relative to the baseline economy, the effect of import competition on average welfare is essentially unchanged, but the standard deviation of welfare changes across occupations rises by one-fifth. We finally study the effects of a full move to autarky on Germany. In both counterfactuals, because retraining broadens workers' skillsets, it blunts the distributional effects of these sectoral shocks while leaving their aggregate effects largely unchanged.

Related Literature

An ever-growing set of papers, starting with Autor, Dorn, and Hanson (2013) in the US and W. Dauth, Findeisen, and Suedekum (2014) in Germany, studies inequality in worker outcomes following trade shocks. The welfare effects of trade vary widely across workers' sectors (W. Dauth, Findeisen, and Suedekum 2021), skill levels (Lee 2020; Traiberman 2019) and other characteristics of their firms (Schott, Pierce, and Tello-Trillo 2020), partly due to sector- or occupational-specific human capital and mobility barriers (Dix-Carneiro 2014; Traiberman 2019; Caliendo, Dvorkin, and Parro 2019). Losses are spatially concentrated (Galle, Rodriguez-Clare, and Yi 2021) and become substantially more dispersed when correctly accounting for spatial linkages in general equilibrium (Adão, Arkolakis, and Esposito 2021). In high-income countries, trade opening has historically

increased inequality and the skill premium (Antràs, Gortari, and Itskhoki 2017; Lee 2020; Galle, Rodriguez-Clare, and Yi 2021). Our paper highlights a policy solution to moderate the unequal effects of trade outlined in this literature, namely retraining to help affected workers switch to more lucrative sectors.

Simultaneously, a large empirical literature in labour economics evaluates the causal effects of worker training programs. These papers typically employ matching estimators which compare workers with similar observable characteristics (Fitzenberger and Speckesser 2007; Lechner and Wunsch 2009; Lechner, Miquel, and Wunsch 2011; Fitzenberger, Orlanski, et al. 2013) or estimate models of dynamic selection into retraining (Osikominu 2013; Biewen et al. 2014; Fitzenberger, Osikominu, and Paul 2021). Kluve (2010) and Card, Kluve, and Weber (2010), in a pair of meta-analyses, report generally modest but positive employment effects, particularly in the medium to long-run. Closest to our work, Hyman (2018) studies Trade Adjustment Assistance (TAA) retraining subsidies in the US and exploits quasi-random variation in case examiner assignment to identify positive effects of TAA benefits on earnings. We provide direct empirical evidence that workers retrain in response to trade and then, with the help of our structural model, go on to quantify the aggregate and distributional effects of retraining in the wake of trade shocks.

Outline

The remainder of the paper proceeds as follows. Section 2 describes the data and the institutional context of retraining in Germany. Section 3 presents our main empirical results on the interaction of import competition and retraining. Section 4 develops a theory of trade and retraining which highlights a key distinction between retraining which broadens versus deepens a worker's skillset. Section 5 calibrates the model's key parameters, and Section 6 uses the model to perform counterfactual experiments. Finally Section 7 concludes.

2 Retraining in Germany

German workers benefit from comprehensive government-sponsored retraining programs. In this section, we introduce the administrative labour market data with which we measure retraining and then discuss the institutional details of retraining in Germany.

2.1 Data

We observe workers' labour market outcomes and characteristics in the weakly anonymous version of the German Sample of Integrated Labour Market Biographies (SIAB) 1975-2021 (Graf et al. 2023).[3] The SIAB draws a 2% sample from Germany's primary administrative employment records, known as the Integrated Employment Biographies (IEB). The resulting worker-level

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[3]Data access was provided via on-site use at the Research Data Centre (FDZ) of the German Federal Employment Agency (BA) at the Institute for Employment Research (IAB) and subsequently via remote data access.

panel records, for each worker and month between 1975 and 2021, all active employment spells, unemployment benefits, and participation in "active labour market policies" such as retraining (Schmucker, Seth, and Berge 2023). Thanks to the unusually long panel dimension of these data, we can quantify long-run effects on individual workers, and a detailed occupational classification facilitates accurate characterization of worker trade exposure. Furthermore, the large sample size allows us to observe sufficient instances of relatively rare events such as retraining.

To compute workers' trade exposure, we combine this labour market data with trade flows from the UN's Comtrade Database.[4] The two major external shocks to the German economy in the period we study came from China and the former Eastern block countries. Following W. Dauth, Findeisen, and Suedekum (2021), we therefore focus on trade flows between Germany and a composite "East" which combines China and twenty former Eastern bloc countries.[5] We later follow common practice in the trade literature and additionally make use of trade flows between each of eight other high-income countries and this same "East."[6] Appendices A.1-A.2 further discuss the details of each data source as well as our cleaning and preparation procedures.

2.2 Institutional Context

The German Federal Employment Agency (BA) manages a variety of active labour market policies to support workers' employment prospects. These policies target workers across all career stages and regardless of their current employment status (C. Dauth 2020; Fitzenberger, Osikominu, and Paul 2021).

We focus on a three categories of programs which we collectively define as *retraining* and which comprise two-thirds of active labour market policy episodes during our sample period. First, *activation and vocational integration* measures match workers with jobs and provide necessary skills, from job application training to foreign language classes, to re-enter the labour market. Second, *career choice and vocational training* programs offer career advising and tutoring as well as government-sponsored internships (Bundesagentur für Arbeit 2021a; Roesler et al. 2021). Third, *vocational retraining and further education* programs range from short job search or computer skills courses to year-long training in more general occupational skills, e.g. marketing (Osikominu 2013; Biewen et al. 2014).[7] Table 1 summarizes the curricula of two such courses and highlights the extent to which retraining touches on both basic job search and substantive, job-specific skills.

These two courses also exemplify the modern German retraining system in their privatized

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[4]https://comtradeplus.un.org.

[5]The former Eastern bloc countries include Hungary, Poland, Romania, Bulgaria, Russia, Belarus, Estonia, Latvia, Lithuania, Moldova, Ukraine, Azerbaijan, Georgia, Kazakhstan, Kyrgyzstan, Tajikistan, Turkmenistan, Uzbekistan, Czechia, and Slovakia. Unlike W. Dauth, Findeisen, and Suedekum (2021), we exclude Slovenia due to missing 1990 data.

[6]The "other" high-income countries, again following W. Dauth, Findeisen, and Suedekum (2021) and their many precedents, include the UK, Sweden, Norway, Canada, Australia, New Zealand, Japan, and Singapore.

[7]These three program types correspond to the *erwstat* codes 10001 (activation and vocational integration), 10002 (career choice and vocational training), and 10003 (vocational retraining and further education) in the Participants-In-Measures History File (MTH/XMTH) portion of the SIAB. Note that the SIAB includes only retraining episodes starting in 2000 or later, so we restrict our worker panel to the post-2000 period.

Table 1: Retraining Curriculum Examples

Topic	*Wind turbine training, compact*	*Solar energy training module 3: photovoltaic systems*
Length	1 week	2 weeks
Prerequisites	Vocational training in sales or technical area, computer skills	Vocational training, 2 years' experience, and basic knowledge of solar technology.
Degree Obtained	Completion certificate	Completion certificate
Cost with Subsidy	0	0
Contents	- components of wind turbines - wind as energy source - yield and performance - examples of large wind parks - assembly - environmental effects - permitting process - profitability	- electrical fundamentals - grid-connected systems - stand-alone plants - planning and design - sizing - assembly - operation, maintenance - workplace safety

Notes: Table reproduces summary curricula for two courses for which workers could use government-provided retraining vouchers. Both example courses are drawn from among the offerings of the Institut für Berufliche Bildung AG, a nationwide private provider of worker training courses.

provision. After a series of market-based reforms in the 1990s, German retraining programs typically operate via vouchers distributed by employment agency caseworkers. The latter may restrict the range of retraining courses covered, but within these parameters, recipients can redeem vouchers at any certified training provider (C. Dauth 2020; Fitzenberger, Osikominu, and Paul 2021). Vouchers cover the direct costs of training as well as ancillary expenses such as transportation, and workers receive unemployment benefits for the duration of the program (Bundesagentur für Arbeit 2021b).

2.3 Descriptive Statistics

Finally, we quantify the prevalence and characteristics of retraining episodes. We leave the details of the underlying data to Appendices A.1 and A.3 and instead highlight three key facts.

Retaining is frequent: On average, 3.3% of German workers in our sample retrain each year, and, over a lifetime, a full 23% will have completed a retraining course at some point. Roughly 13% of workers in our sample retrain more than once.

Retraining is widespread: A broad swath of the German labour force make use of retraining. Table 2 shows annual retraining rates for workers grouped into broad sector categories. Workers in each sectoral group retrain at comparable rates, which range from 1.9% in capital and consumer goods to 4.1% in construction. Figure 1a shows the distribution of retraining episodes across age

Table 2: Retraining Rates by Sector

Sector	Yearly Retraining Rate
Agriculture, forestry and fishing	0.032
Food and beverages	0.033
Consumables	0.029
Production goods	0.026
Capital and consumer goods	0.019
Construction	0.041
Hospitality	0.035
Transport and storage	0.039
Education	0.031

Notes: Table displays workers' yearly rate of participation in retraining by their last observed sector of employment, where 3-digit sectors are grouped into nine aggregate sectors delineated by W. Dauth and Eppelsheimer (2020), which correspond to those employed in the IAB Establishment Panel. Appendix A.3.1 details the associated calculations.

Figure 1: Characteristics of Retraining Episodes

(a) Age Distribution   (b) Length Distribution
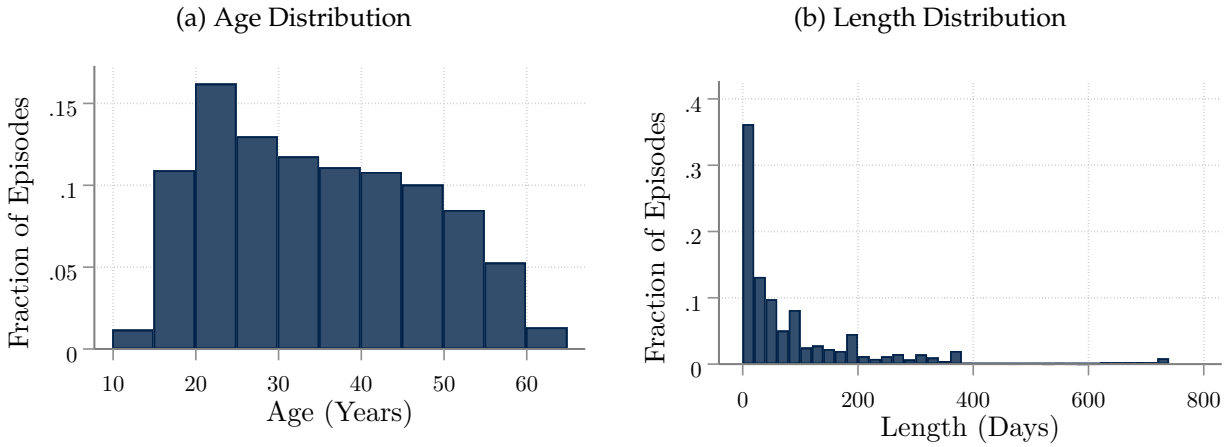
Notes: Panel (a) displays ages at which workers start retraining episodes in the SIAB (2000-2019). Panel (b) displays the distribution of lengths of retraining episodes, measured in days. Appendix A.1.4 details the associated data and calculations.

groups. While workers early in their careers naturally account for a larger share of episodes, retraining remains important for workers in every age group between twenty and sixty.

Retraining courses vary widely in their duration: Figure 1b plots the distribution of retraining episode lengths, in days. The mean retraining episode lasts one month, with substantial variation around this average. While around a third of retraining episodes last less than 20 days, the distribution in Figure 1b features a long right tail: a full 29% of episodes exceed 100 days.

3   Retraining, Import Competition, and the Eastern Shock

In this section we show empirically that German workers use retraining to adjust to trade shocks, in particular the rise in imports from China and Eastern Europe between 2001 and 2007. Paralleling the large literature on the "China shock" in the US (Autor, Dorn, and Hanson 2013), we henceforth refer to these developments as the "Eastern shock." We begin by documenting the adverse labour market consequences of exposure to the Eastern shock and then explore the role of retraining as a margin of adjustment for workers. As is standard in the trade literature, our empirical specifications identify relative effects of trade exposure on workers, as compared to their less-exposed peers. We later pin down aggregate effects using the model to be introduced in Section 4.

3.1   Measuring Exposure to the Eastern Shock

A worker's labour market exposure to any trade shock depends on the extent to which affected sectors demand that worker's skillset. In our context, occupations provide a natural measure of workers' skills and thus exposure to the Eastern shock.[8] We therefore measure trade exposure at the occupation level and define the import exposure of a worker ω initially in occupation t as a weighted average of changes in sectoral net imports,

$$\text{occupational import exposure}_t \equiv \sum_s l_{ts} \times \left(\frac{\Delta \text{net imports}_s}{\text{wage bill}_s} \right).$$

The weights l_{ts} equal the share of base-year-2000 workers of occupation t employed in sector s. Our Δ*net imports$_s$* variable equals the 2001-2007 change in net imports from the East to Germany, as discussed in Section 2. We focus not on a sector's imports in isolation but on its net imports, i.e., imports minus exports, to more completely measure the impact of the Eastern shock on demand. As in W. Dauth, Findeisen, and Suedekum (2021), we normalize net imports by the total sectoral base-year wage bill.[9]

With our exposure measure in hand, we estimate specifications for a series of labour market outcomes $y_{\omega t}$ for workers ω employed in the base year in occupation t as a function of occupational import exposure,

$$y_{\omega t} = \beta \times \text{occupational import exposure}_t + X_{\omega t} \gamma + \varepsilon_{\omega t}. \tag{1}$$

Controls in $X_{\omega t}$ include demographics, such as worker age, gender, education, and nationality, as well as characteristics of their primary base-year job: tenure, firm size, and a dummy for employ-

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[8]Using a structural model that allows for worker transitions across both sectors and occupations, Traiberman (2019) shows that occupational transitions are more costly than sectoral transitions, and that the majority of variation in exposure to trade shocks is accounted for by a worker's occupation.

[9]We delineate occupations t using the 3-digit version of the German Classification of Occupations 2010 (KldB 2010) and sectors s using the 3-digit German Classification of Economic Activities, Edition 1993 (WZ 93).

Table 3: Labour Market Effects of Eastern Import Exposure

	(1) Share days employed	(2) Share days in manufacturing	(3) Earnings growth
IV (2SLS)			
Occupational import exposure	-0.117***	-0.129***	-19.250***
	(0.021)	(0.036)	(7.261)
Demographic Controls	✓	✓	✓
Initial Job Controls	✓	✓	✓
Observations	275,850	275,850	275,850
Mean of Dependent Variable	0.765	0.255	79.168
First-Stage F Statistic:		73.107	

Notes: Robust standard errors in parentheses, clustered at the occupation level; *** indicates p<0.01, ** p<0.05, and * p<0.1. Columns 1–3 present instrumental variables estimates of (1) for workers employed in 2000 in the SIAB data. We measure the share of days employed and share of days employed in manufacturing (unconditional on employment) in 2007, and our earnings growth measure equals 100 times total 2007 earnings divided by those in the year 2000. We instrument for 2001-2007 occupational import exposure in Germany with an analogous measure calculated using net imports to other high-income countries. Demographic controls include age bins, gender, education, and foreign nationality, measured in 2000. Initial, year-2000 job controls include a dummy for manufacturing, tenure and firm size bins, and log earnings, the latter summed across all jobs. Following W. Dauth, Findeisen, and Suedekum (2021), all specifications omit workers initially in manufacture of knitted and crocheted articles.

ment in manufacturing. We relegate further details regarding the construction of our outcomes, import exposure measure, and controls to Appendix A.3.

A natural concern in estimating (1) is that changes in occupational import exposure in (1) may reflect domestic supply and demand shocks rather than our Eastern shock. We therefore follow Autor, Dorn, and Hanson (2013) and instrument for *occupational import exposure$_t$* with an analogous measure which instead uses imports from the East to the aforementioned other high-income countries.

3.2 Labour Market Effects of Eastern Import Exposure

Table 3 presents the labour market effects of exposure to import competition from China and Eastern Europe, as estimated from (1) by instrumental variables. In Column (1), we consider the effect of import exposure on the subsequent share of days during which a worker remains employed, measured over the course of the year 2007. Our highly-significant estimate of the coefficient β corresponds to a one-percentage-point decrease in this employment share in response to a one-standard-deviation increase in occupational import exposure. In Column (2), we find a slightly larger effect on the share of days employed in a manufacturing sector, unconditional on employment. This figure represents a relatively large decline given that the mean manufacturing employment probability in our sample is 0.255. These employment declines translate directly into lower earnings growth, which we measure analogously to W. Dauth, Findeisen, and Suedekum

Table 4: Eastern Import Exposure and Retraining

Outcome variable	(1) $\mathbb{P}(\text{Retrain})$	(2) $\mathbb{P}(\text{Retrain})$	(3) $\mathbb{P}(\text{Retrain})$	(4) Days retraining
Specification	Linear Probability Model (2SLS)		Logit	2SLS
Occupational import exposure	0.141***	0.100***	0.937***	6.127
	(0.044)	(0.033)	(0.118)	(4.424)
Demographic Controls		✓	✓	✓
Initial Job Controls		✓	✓	✓
Observations	277,822	275,850	275,850	275,850
Mean of Dependent Variable	0.079	0.079	0.079	9.705
First-Stage F Statistic:	77.709	73.107		73.107

Notes: Robust standard errors in parentheses, clustered at the occupation level, in Columns 1-2 and 4; standard errors in Column 3 bootstrapped. *** indicates p<0.01, ** p<0.05, and * p<0.1. Columns 1–2 and 4 present linear instrumental variables estimates of (1) for workers employed in 2000 in the SIAB data and an outcome equal to either a dummy variable for participation in retraining during 2001-2007 or the number of days of retraining in 2001-2007. Column 3 estimates a logit control function specification for the retraining dummy. In each case, we instrument for 2001-2007 occupational import exposure in Germany with an analogous measure calculated using net imports to other high-income countries. Specifications include controls as noted. Demographic controls include age bins, gender, education, and foreign nationality, measured in 2000. Initial, year-2000 job controls include a dummy for manufacturing, tenure and firm size bins, and log earnings, the latter summed across all jobs. Following W. Dauth, Findeisen, and Suedekum (2021), all specifications omit workers initially in manufacture of knitted and crocheted articles.

(2021) as (100 times) earnings in 2007 divided by those in 2000. In particular, the specification in Column (3) implies that a one-standard deviation increase in occupational import exposure decreases earnings growth by the equivalent of 1.2 percentage points. Together, these significant adverse effects of the import exposure of a worker's *occupation* echo the established employment and earnings effects of *sectoral* exposure in Germany (W. Dauth, Findeisen, and Suedekum 2021).

3.3 Eastern Import Exposure and Retraining

We now provide direct evidence of a new margin of adjustment to trade shocks: retraining. Due to its function as a long-run investment, we measure retraining over multiple years. Specifically, in Column (1) of Table 4, we estimate equation (1) with a dummy variable $y_{\omega t}$ equal to one for workers who retrain at any point during the full period 2001-2007, initially without controls. In Column (2), we add controls for demographic and initial-job characteristics. The coefficient in our preferred specification in Column (2), significant at the 1% level, implies that a one-standard-deviation increase in occupational import exposure increases the probability that a worker retrains by 0.60 percentage points. Given a mean 7-year retraining probability of 7.9%, import exposure thus causes an economically meaningful increase in retraining rates. The logit control function specification estimated in Column (3) yields an approximately equivalent effect: a one standard-deviation increase in occupational import exposure increases a worker's relative odds of retrain-

Table 5: Labour Market Effects by Retraining Status

IV (2SLS)	(1) Non-retrained Share days employed	(2) Share days employed in manufac.	(3) Earnings growth	(4) Retrained Share days employed	(5) Share days employed in manufac.	(6) Earnings growth
Occup. import exposure	-0.100***	-0.091**	-16.536**	-0.018	0.056	-8.577
	(0.020)	(0.039)	(8.143)	(0.064)	(0.059)	(8.220)
Demographic Controls	✓	✓	✓	✓	✓	✓
Initial Job Controls	✓	✓	✓	✓	✓	✓
Observations	254,102	254,102	254,102	21,748	21,748	21,748
Mean of Dep. Variable	0.787	0.267	81.898	0.504	0.119	47.445
First-Stage F Statistic:	72.002	72.002	72.002	81.752	81.752	81.752

Notes: Robust standard errors in parentheses, clustered by occupation; *** indicates p<0.01, ** p<0.05, and * p<0.1. Table presents split-sample instrumental variables estimates of (1), with sample split by retraining participation during 2001-2007, for workers employed in 2000 in the SIAB data. Columns 1-3 present estimates for the sample of workers who do not participate in retraining, while Columns 4-6 present estimates for workers who do participate in retraining. We measure the share of days employed and share of days employed in manufacturing (unconditional on employment) in 2007, and our earnings growth measure equals 100 times total 2007 earnings divided by those in the year 2000. We instrument for 2001-2007 occupational import exposure in Germany with an analogous measure calculated using net imports to other high-income countries. Demographic controls include age bins, gender, education, and foreign nationality, measured in 2000. Initial, year-2000 job controls include a dummy for manufacturing, tenure and firm size bins, and log earnings, the latter summed across all jobs. Following W. Dauth, Findeisen, and Suedekum (2021), all specifications omit workers initially in manufacture of knitted and crocheted articles.

ing by around 6%. Finally, Column (4) considers a different outcome, namely the number of days spent retraining. We again find a positive, but in this case statistically insignificant, effect of import exposure on retraining.

Why do exposed workers retrain? In Table 5, we present evidence on the differential effects of import exposure on the trajectories of workers who retrain versus those who do not. Columns (1) – (3) report results for a range of outcomes for workers who do not retrain, while Columns (4) – (6) report analogous results for workers who do retrain. Import exposure's adverse effects virtually disappear among workers who retrain. In particular, the effects of imports on these workers' labour market outcomes remain small and statistically insignificant, in contrast to the negative and significant effects on workers who do not retrain.

Workers, however, certainly select into retraining based on unobservable characteristics. This selection potentially complicates a causal interpretation of the differences between import exposure's effects on retrained and non-retrained workers in Table 5. Though the intercepts in each specification would absorb selection based on the level of human capital, workers who retrain might also possess other unobserved skills which allow them to better cope with import exposure. Without random assignment into retraining, we cannot entirely rule out this possibility. Nevertheless, the fact that, as documented in Table 4, workers do retrain in response to this shock does

support the interpretation of the results in Table 5 as capturing the effects of retraining. The model developed below will offer an explanation for this observed complementarity between exposure to import competition and retraining.

4   A Theory of Trade and Retraining

The empirical results in Section 3 establish a link between exposure to import competition and retraining. However, they do not quantify the extent to which retraining can help an economy weather trade shocks and boost its gains from international integration more broadly. To answer these aggregate questions, we now build a quantitative general equilibrium model of trade and retraining.

Our static model consists of $i = 1, ..., N$ countries, where $i = 1$ represents the domestic economy, i.e. Germany, and $s = 1, ..., S$ productive sectors, as well as a non-employment option, $s = 0$. Labour supply across sectors follows Galle, Rodriguez-Clare, and Yi (2021): workers from occupations $t = 1, ..., T$ differ in their skillsets and sort across sectors accordingly. We augment this labour supply framework with a retraining option, which allows workers to alter their skillsets in response to shocks. An international trade module along the lines of Eaton and Kortum (2002) and Caliendo and Parro (2015) determines wages, which workers take as given.

4.1   Labour Supply and Retraining in the Domestic Economy

We model a rich labour supply decision which incorporates the option of retraining. In this section, we drop the country subscript i and index individual workers by ω. A worker's occupation t is exogenous and fixed, and we denote the fraction of workers in occupation t by μ_t. A worker makes two decisions: whether or not to retrain, indexed by $j \in \{0, 1\}$, and in which sector s to supply labour or, in the case of $s = 0$, to enjoy leisure.

Human Capital and Worker Choices

Workers possess sector-specific stocks of human capital $x_s^j(\omega)$, which comprise deterministic components and idiosyncratic shocks and may depend on retraining status. In non-employment, denoted $s = 0$, a worker with retraining status j has a human capital endowment,

$$\log x_0^j(\omega) = \log b_t^0 + \left(\frac{1}{\gamma}\right) \eta^j(\omega) + \left(\frac{1}{\kappa}\right) \epsilon_0^j(\omega), \tag{2}$$

while for productive sectors with $s > 0$,

$$\log x_s^j(\omega) = \log z_{st}^j + \left(\frac{1}{\gamma}\right) \eta^j(\omega) + \left(\frac{1}{\kappa}\right) \varphi^j(\omega) + \left(\frac{1}{\nu}\right) \epsilon_s^j(\omega). \tag{3}$$

The parameter b_t^0 in the first term in (2) captures the deterministic value a worker in occupation t attaches to leisure. For employed workers in (3), z_{st}^j represents the deterministic human capital a particular occupation brings to sector s. Workers' human capital additionally depends on mean-zero, retraining-status-specific idiosyncratic shocks to the desirability of each option, indexed by ω. A retraining shock η^j enters human capital in all sectors, an employment shock φ^j augments the human capital stocks in each productive sector, and ϵ_s^j incorporates sector-specific idiosyncratic human capital. The parameters γ, κ, and ν control the scale of each respective idiosyncratic shock. As we will see below, the nested structure of these shocks will ultimately yield a flexible model of workers' employment and retraining decisions.

Importantly, the deterministic human capital component z_{st}^j varies by retraining status. In particular, for non-retrained workers, $j = 0$,

$$\log z_{st}^0 \equiv \log Z_t + \log z_{st}. \tag{4}$$

The component Z_t captures the average ability of workers in an occupation t: they may simply be more or less productive in every sector. Meanwhile, z_{st} captures the suitability of workers from occupation t for sector s. Workers who retrain, i.e. $j = 1$, instead have a deterministic human capital,

$$\log z_{st}^1 \equiv \log Z_t + (1 - \beta) \log z_{st} + \beta \log Z_s + \log \Delta_t. \tag{5}$$

Retraining thus has two distinct effects. First, retraining raises human capital available for use in every sector, an effect which we refer to as the *vertical component* of retraining. The parameter $\Delta_t > 0$ captures this across-the-board increase, net of the opportunity cost of time spent retraining. Second, retraining may alter the worker's pattern of comparative advantage across sectors. We refer to this mechanism as the *horizontal component* of retraining, and its effects are controlled by the parameter β and the sector-specific terms Z_s. If β is positive, retraining pulls a worker's human capital endowment in sector s towards the average in that sector, as captured by Z_s, whereas negative β induces the opposite effect. Intuitively, β determines whether retraining helps workers broaden ($\beta > 0$) or deepen ($\beta < 0$) their skillsets. The term Z_s in (5) captures 'average' ability in sector s; we assume that Z_s aggregates the occupation-specific z_{st}. In our context, a convenient functional form for this aggregator is,

$$Z_s = \left(\frac{\sum_t \mu_t z_{st}}{\sum_t \mu_t z_{st}^{1-\beta}} \right)^{\frac{1}{\beta}}. \tag{6}$$

Based in part on these human capital components, the worker then chooses whether to retrain and a sector in which to work, or enjoy leisure. If she chooses sector s, her utility $u_s^j(\omega)$, synonymous with real income $y_s^j(\omega)$, satisfies $u_s^j(\omega) = y_s^j(\omega) = \left(\frac{w_s}{P} \right) x_s^j(\omega)$. Here, w_s denotes the nominal wage in sector s and P is the aggregate price level, both determined in equilibrium by the trade side of our model. If the worker instead chooses non-employment, $s = 0$, her real income is zero but she nevertheless enjoys utility $u_0^j(\omega) = x_0^j(\omega)$, which we interpret as worker- and retraining-

status-specific enjoyment from leisure. The worker chooses the retraining-by-sector tuple which yields her the highest utility $u_s^j(\omega)$. We denote this maximum by $v(\omega)$.

Aggregation

We now shift from the problem of an individual worker ω to the characterization of aggregate outcomes. The following distributional assumptions on $\eta^j(\omega)$, $\varphi^j(\omega)$, and $\epsilon_s^j(\omega)$ make this aggregation feasible.

Assumption 1. *The shocks $\epsilon_s^j(\omega)$ are drawn from independent Type 1 Extreme Value Distributions with scale parameter one. The shocks $\varphi^j(\omega)$ and $\eta^j(\omega)$ are drawn from independent* Cardell (1997) *distributions with scale parameters $v^{-1}\kappa$ and $\kappa^{-1}\gamma$, respectively.*

This assumption immediately yields the following aggregation result.

Theorem 1. *In each occupation, human capital endowments across different sectors and retraining statuses are distributed multivariate γ-Fréchet (*Lind and Ramondo 2023*). Given wages w_s, the welfare of workers from occupation t then follows a Fréchet distribution with scale parameter V_t and shape parameter γ, where*

$$V_t = \left(\sum_j \left(V_t^j \right)^\gamma \right)^{\frac{1}{\gamma}}, \tag{7}$$

$$V_t^j = \left(\left(b_t^j \right)^\kappa + \left(v_t^j \right)^\kappa \right)^{\frac{1}{\kappa}}, \tag{8}$$

$$v_t^j = \left(\sum_s \left(z_{st}^j w_s \right)^\nu \right)^{\frac{1}{\nu}} P^{-1}. \tag{9}$$

A worker chooses to retrain with probability

$$R_t = \left(\frac{V_t^1}{V_t} \right)^\gamma. \tag{10}$$

Conditional on retraining status, j, workers' employment probability satisfies

$$E_t^j = \left(\frac{v_t^j}{V_t^j} \right)^\kappa. \tag{11}$$

Conditional on retraining status, j, and employment, the share of labour employed in sector s satisfies

$$\ell_{st}^j = \left(\frac{z_{st}^j w_s}{P v_t^j} \right)^\nu. \tag{12}$$

Finally, the average human capital supplied by workers from occupation t to sector s equals

$$h_{st} = \left(\frac{V_t P}{w_s} \right) \left(R_t E_t^1 \ell_{st}^1 + (1 - R_t) E_t^0 \ell_{st}^0 \right). \tag{13}$$

Proof. See Appendix B.1. ∎

Theorem 1 tells us how to aggregate over heterogeneous workers within an occupation. Thanks to the distributional choices in Assumption 1, workers' decisions take a convenient nested form, with distinct elasticities in each nest. The *retraining elasticity* γ controls workers' sensitivity to the costs and benefits of retraining. The *employment elasticity* κ varies inversely with the importance of idiosyncratic shocks to the employment decision, and the *intersectoral labour supply elasticity* ν plays an equivalent role across sectors.

Sectoral and occupational aggregates also take intuitive forms. Given human capital supplies in (13), it is straightforward to aggregate over occupations to obtain total human capital supplied to sector s,

$$H_s = \sum_t \mu_t h_{st}. \tag{14}$$

In the absence of a non-employment option, the average nominal income of workers from occupation t, denoted Y_t, would simply equal the product of the occupation-specific utility aggregator and the price level, $V_t P$. The introduction of a non-employment option simply scales this value by a weighted average of employment rates, so that,

$$Y_t = V_t P \left(R_t E_t^1 + (1 - R_t) E_t^0 \right). \tag{15}$$

4.2 Labour Supply in Other Countries

Labour supply in other countries $i > 1$ is simple. A homogeneous mass L_i of workers are perfectly mobile across sectors, so that wages w_i are equalised across sectors within each country. There is no non-employment option. Labour supplies in each sector L_{is} are pinned down by wage equalization across sectors and the requirement that they sum to L_i. We normalize the quantity of human capital supplied by each worker to one, so that $H_{is} = L_{is}$.

4.3 International Trade

The international trade side of the model maps endogenous shifts in human capital supply and exogenous productivity shocks into equilibrium wages and prices. We closely follow the Ricardian framework of Eaton and Kortum (2002), augmented with an input-output (IO) structure and exogenous trade deficits as in Caliendo and Parro (2015).[10]

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[10]IO linkages and deficits do not play a central role in our model, but are important for making quantitative sense of the data on trade flows we exploit in our calibration in Section 5.

Production in a sector $s > 0$ in country i operates under constant returns to scale and uses human capital and a bundle of intermediate inputs from all sectors.[11] These inputs are combined in a Cobb-Douglas fashion with a weight Γ_{is} on human capital and weights $(1 - \Gamma_{is})\Omega_{isr}$ on inputs from sector r. As in Eaton and Kortum (2002), we assume that there is a continuum of constant elasticity of substitution (CES) varieties within each sector; that producers are perfectly competitive; that producers' productivities are drawn from a Fréchet distribution with scale A_{is} and shape parameter θ; and that transporting goods from i to j in sector s entails an iceberg cost d_{ijs}. Finally, consumers in each country have Cobb-Douglas preferences across sectors with weight α_{is} on sector s in country i.

Under these assumptions, trade flows and sectoral aggregates take familiar forms. The bundle of inputs needed to produce one unit of output costs

$$x_{is} = \left(\frac{w_{is}}{A_{is}} \right)^{\Gamma_{is}} \left(\Pi_r P_{ir}^{\Omega_{isr}} \right)^{1-\Gamma_{is}}, \tag{16}$$

where P_{ir} is the price index for sector r in country i. The share of country j's expenditure in sector s devoted to varieties produced in i is,

$$\pi_{ijs} = \left(\frac{d_{ijs} x_{is}}{P_{js}} \right)^{-\theta}, \tag{17}$$

and the price index in j is given by[12]

$$P_{js} = \left(\sum_j \left(d_{ijs} x_{is} \right)^{-\theta} \right)^{-\frac{1}{\theta}}. \tag{18}$$

Cobb-Douglas preferences then imply the aggregate price index in i is given by,

$$P_i = \Pi_s P_{is}^{\alpha_{is}}. \tag{19}$$

Finally, we turn to market clearing. Total expenditure on sector s in country i equals

$$X_{is} = \alpha_{is} D_i + \sum_r (1 - \Gamma_{ir})\Omega_{irs} \left(\sum_j \pi_{ijs} X_{jr} \right). \tag{20}$$

Here, the first term captures demand for sector s coming from consumption and the second captures demand coming from the use of sector s as an intermediate good in production. Final demand in country i is given by

$$D_i = \sum_s w_s H_{is} + d_i, \tag{21}$$

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[11]In this section all sums over sectors run from $s = 1$ to S, ignoring the non-employment option.

[12]Relative to Eaton and Kortum (2002), this expression for prices is missing a constant of integration. This constant can be dropped given an appropriate assumption on the scale parameter of the idiosyncratic productivity distribution.

where d_i represents an exogenous trade deficit. Wages in each sector must clear the labour market,

$$w_{is} H_{is} = \Gamma_{is} \sum_j \pi_{ijs} X_{js}. \tag{22}$$

4.4 Equilibrium

An equilibrium of this economy is a set of wages w, prices P, occupation-by-sector labour and human capital supplies $\{l, h\}$, aggregate sectoral labour and human capital supplies $\{L, H\}$, and expenditures X such that:

1. Given wages w_{1s} in the domestic economy, occupations' sectoral labour supplies, ℓ_{st}, human capital supplies, h_{st}, and retraining probabilities R_t are those given by Theorem 1, and aggregate sectoral human capital supplies H_{1s} are given by (14).

2. In countries $i > 1$, wages w_{is} are equalised across sectors and aggregate sectoral labour supplies L_{is} sum to L_i.

3. Given wages w_{is} in every country and sector, input costs x_{is}, trade shares π_{ijs}, and prices P_{is} are given by (16) – (18).

4. Given aggregate sectoral human capital supplies H_{is} and trade shares π_{ijs}, wages w_{is} and expenditures X_{is} satisfy (20) – (22).

4.5 Discussion

To build intuition, we now study why workers gain from retraining and how these gains interact with import competition. In this subsection, we assume, for analytical tractability, that there is no non-employment option and work with first order approximations around $\beta = 0$, which corresponds to the case in which retraining has a symmetric effect on human capital across sectors. Proofs of the results in this section can be found in Appendix B.2.

Why Retrain?

Under the assumptions above, workers' individual benefits of retraining take a simple form which highlights two key sources of gains. In particular, the difference in the values of retraining versus not satisfies

$$\log V_t^1 - \log V_t^0 = \log \Delta_t + \beta \underbrace{\left(\sum_s \ell_{st} \left(\log Z_s - \log z_{st} \right) \right)}_{\equiv \mathcal{M}_t}, \tag{23}$$

where we refer to $\mathcal{M}_t$ as the 'mismatch' facing workers in occupation t. These expressions tell us that workers gain from retraining in two ways. First, retraining simply scales human capital

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in all sectors up or down by a factor Δ_t. This factor determines the level of retraining and its distribution across occupations, but, as a fixed parameter, Δ_t plays no role in the response of retraining to shocks such as import competition. Second, the decision to retrain interacts with the mismatch term $\mathcal{M}_t$. To gain intuition, consider an occupation with "unlucky" workers — who possess relatively high z_{st} in sectors with low wages. These workers will nonetheless work in sectors where they do not have a comparative advantage, and the employment-weighted sum in (23) will be small or negative—in other words, for these workers, mismatch $\mathcal{M}_t$ will be high. If $\beta > 0$, so that retraining helps workers broaden their skillsets, then retraining benefits these workers by mitigating mismatch. By contrast, if $\beta < 0$, so that retraining instead deepens workers' skillsets, then retraining is of little use to our proverbial unlucky workers.

These motives for retraining translate in a straightforward way into variation in retraining rates across occupations. The log-odds ratio for retraining in occupation t is given by

$$\log \left(\frac{R_t}{1 - R_t} \right) = \gamma \left(\log \Delta_t + \beta \mathcal{M}_t \right). \tag{24}$$

Workers naturally retrain at higher rates when retraining offers a larger increase Δ_t in their skills across all sectors. Increases in the mismatch term $\mathcal{M}_t$ instead have ambiguous effects on retraining rates. Higher mismatch raises retraining rates only in the skill-broadening, $\beta > 0$, case. The magnitude of this effect, in turn, depends on the retraining elasticity γ.

Retraining and the Eastern Shock

In order to study the effects of the Eastern productivity shock introduced in Section 3 analytically, we consider a model with exogenous wages. In line with our later quantitative results, which indicate a tight link between foreign productivity shocks and wages, we model the Eastern shock as heterogeneous wage changes across sectors. For simplicity, we assume that these $\Delta \log w_s$ have mean zero and are independent of baseline wages w_s as well as average human capital endowments Z_s.[13]

How does the Eastern shock affect workers? Let us first consider an economy without retraining. To a first-order approximation, the change in average earnings in occupation t due to the shock is

$$\Delta \log Y_t = \sum_s \ell_{st}^0 \Delta \log w_s \equiv -x_t^0, \tag{25}$$

where we refer to x_t^0 as the *exposure* of occupation t. Unsurprisingly, occupations whose employment (shares) ℓ_{st}^0 are concentrated in sectors with declining wages experience lower earnings, while the opposite is true for occupations where employment is concentrated in sectors with rising wages.

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[13]Of course, this is a substantial simplification of the model described above, in which the relationship between productivity shocks and wage changes depends on labour supply decisions, input-output linkages and the trade network. Reassuringly, however, our quantitative results in Section 6 do imply a tight link between foreign productivity shocks and domestic wages — see Figure 2 below. We therefore proceed taking wage changes as exogenous.

How does retraining change this picture? In the full model with retraining, the change in earnings is instead given by

$$\Delta \log Y_t = -x_t^0 + R_t \beta \Delta \mathcal{M}_t = -(1 - \beta R_t)\, x_t^0, \tag{26}$$

where the second equality makes use of the assumption that employment shares are not too dispersed to express the earnings effect in terms of our exposure measure.[14] Similarly, we can express the resulting change in retraining rates as,

$$\Delta \log R_t = \gamma \beta (1 - R_t)\Delta \mathcal{M}_t = \gamma \beta (1 - R_t)x_t^0. \tag{27}$$

Retraining thus alters the effect of the Eastern shock whenever the latter causes a change in the mismatch facing workers in occupation t. If the shock moves workers into sectors in which their occupation t does not have a comparative advantage, mismatch rises. Then, whenever $\beta > 0$, retraining reduces mismatch and effectively shields exposed workers from the shock; as a consequence, retraining rates rise. Exactly the opposite is true if $\beta < 0$.

Aggregate Effects

These worker-level gains, however, do not necessarily translate into aggregate effects. We define aggregate income as $Y \equiv \sum \mu_t Y_t$ and aggregate (26) over occupations to obtain an expression for the change in aggregate income, $\Delta \log Y = -\sum_t \mu_t (1 - \beta R_t)x_t^0$. The first order impact of the Eastern shock will thus generally depend on the joint distribution of trade exposure and baseline retraining rates across occupations. In the natural special case with a constant baseline retraining rate $\bar{R}$ across occupations, however, we have

$$\Delta \log Y = -(1 - \beta \bar{R}) \sum_t \mu_t x_t^0 = 0, \tag{28}$$

where the last equality follows from the fact that the wage shocks $\Delta \log w_s$ are mean zero and independent of baseline sector characteristics. In this case, the availability of retraining has no effect on the aggregate gains or losses from the Eastern shock. Retraining raises the exposure of some workers and lowers the exposure of others — but does so in a symmetric way that leaves the exposure of the aggregate economy unchanged.

However, as the occupation-level results suggest, retraining can have important distributional effects, which we summarize using the standard deviation of the income changes caused by the Eastern shock. Again under constant baseline retraining rates, this standard deviation $\sigma(\cdot)$ across occupations satisfies

$$\sigma(\Delta \log Y_t) = (1 - \beta \bar{R})\, \sigma(x_t^0). \tag{29}$$

If $\beta > 0$, higher retraining rates tend to compress the distribution of income changes caused by

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[14]Formally, we make use of the approximation that $\log(S\ell_{st}^j) \simeq S\ell_{st}^j - 1$.

the trade shock, while the opposite is true if $\beta < 0$.

Summary

Our analytical results produce three key takeaways. First, the decision to retrain is driven by trade-induced mismatch between the skills demanded by the labour market and the skills with which each occupation is endowed. The sign of this effect depends crucially on the parameter β, which determines whether retraining broadens or deepens a worker's skillset. Second, the option of retraining will typically not greatly alter the average welfare effects of trade shocks. Third, retraining does – potentially – have an important effect on the distributional consequences of such shocks. The direction of this effect is theoretically ambiguous and again hinges on the sign of β. We next assess the quantitative relevance of these lessons by calibrating the full model and performing counterfactual experiments.

5 Calibration

The model developed in Section 4 has five key parameters, $\{\beta, \gamma, \kappa, \nu, \theta\}$; an input-output structure summarized by the matrices Γ and Ω; and a large set of unknown fundamentals — productivities, trade costs, and so on. We difference out the model's fundamentals and input-output parameters using the standard 'hat algebra' approach (Dekle, Eaton, and Kortum 2007). We then outline and implement an indirect inference-style strategy that identifies the model's remaining parameters using the empirical facts from Section 3.

5.1 Hat Algebra

Our model contains a large number of exogenous constants which we collectively refer to as its fundamentals: productivities A_{is}, trade costs d_{ijs}, human capital endowments z_{st}, as well as the occupation-specific vertical components of retraining Δ_t. Theorem 2 below shows that, given a baseline equilibrium and values for the other parameters, the model can be used to perform counterfactual experiments without explicitly recovering these fundamentals.

Theorem 2. *Let $\mathcal{E} = \{\ell^0, \ell^1, Y, R, \Pi, X\}$ denote (data on) a baseline equilibrium, where ℓ^0 and ℓ^1 contain the sectoral labour supply choices of each occupation conditional on retraining status, Y and R contain the average earnings and retraining rates in each occupation, Π contains trade shares for every country pair and sector and X contains the total expenditure of each country on each sector. Let A' be some set of counterfactual productivities for each country and sector, and let a hat ($\hat{}$) over a variable denote the ratio of that variable between the counterfactual equilibrium and the baseline equilibrium. Then the following two systems of equations characterize the counterfactual equilibrium. The first system describes the trade side*

of the model,

$$\widehat{x}_{is} = \left(\frac{\widehat{w}_{is}}{\widehat{A}_{is}}\right)^{\Gamma_{is}} \left(\Pi_r \widehat{P}_{ir}^{\Omega_{isr}}\right)^{1-\Gamma_{is}}, \tag{30}$$

$$\widehat{P}_{js}^{-\theta} = \sum_i \pi_{ijs}\widehat{x}_{is}^{-\theta} \tag{31}$$

$$\widehat{\pi}_{ijs} = \left(\frac{\widehat{x}_{is}}{\widehat{P}_{js}}\right)^{-\theta} \tag{32}$$

$$\widehat{w}_{is}\widehat{H}_{is} = \sum_j \left(\frac{\pi_{ijs}X_{js}}{\sum_h \pi_{ihs}X_{hs}}\right) \widehat{\pi}_{ijs}\widehat{X}_{js}, \tag{33}$$

$$\widehat{X}_{js} = \sum_r \tilde{\alpha}_{jsr}\widehat{w}_{jr}\widehat{H}_{jr}, \tag{34}$$

where $\tilde{\alpha}_{jsr}$ is a composite parameter which depends only on the baseline equilibrium and the input-output parameters Γ and Ω. The second system of equations describes labour supply and retraining decisions in the domestic economy,

$$\widehat{V}_t = \left((1-R_t)\left(\widehat{V}_t^0\right)^\gamma + R_t\left(\widehat{V}_t^1\right)^\gamma\right)^{\frac{1}{\gamma}}, \tag{35}$$

$$\widehat{V}_t^j = \left((1-E_t^j) + E_t^j\left(\widehat{v}_t^j\right)^\kappa\right)^{\frac{1}{\kappa}}, \quad for \quad j \in \{0,1\}, \tag{36}$$

$$\widehat{v}_t^j = \left(\sum_s \ell_{st}^j \widehat{w}_s^\nu\right)^{\frac{1}{\nu}} \widehat{P}^{-1} \quad for \quad j \in \{0,1\}, \tag{37}$$

$$\widehat{R}_t = \left(\frac{\widehat{V}_t^1}{\widehat{V}_t}\right)^\gamma, \tag{38}$$

$$\widehat{E}_t^j = \left(\frac{\widehat{v}_t^j}{\widehat{V}_t^j}\right)^\kappa, \quad for \quad j \in \{0,1\}, \tag{39}$$

$$\widehat{\ell}_{st}^j = \left(\frac{\widehat{w}_s}{\widehat{v}_t^j \widehat{P}}\right)^\nu, \quad for \quad j \in \{0,1\}, \tag{40}$$

$$\widehat{h}_{st} = \left(\frac{\widehat{V}_t \widehat{P}}{\widehat{w}_s}\right) \left((1-\rho_{st})\left(\widehat{(1-R_t)}\widehat{E}_t^0\widehat{\ell}_t^0\right) + \rho_{st}\left(\widehat{R}_t\widehat{E}_t^1\widehat{\ell}_t^1\right)\right), \tag{41}$$

$$\widehat{H}_s = \sum_t \eta_{st}\widehat{h}_{st}, \tag{42}$$

where we suppress the i subscripts, ρ_{st} is the share of occupation t workers in sector s who have chosen to retrain in the baseline economy, and η_{st} is the share of human capital in sector s that is supplied by workers in occupation t in the baseline economy.

Proof. See Appendix B.3. ∎

Notice that two of the key parameters in our model of retraining, the vertical component Δ_t and the horizontal component β, do not appear directly in Theorem 2. The absence of Δ_t is intuitive: this vertical component shifts the overall attractiveness of retraining and thus the share R_t of workers who retrain, but once we condition on this share, Δ_t has no implications for counterfactuals. The role of β is more subtle. Although β does not appear explicitly in Theorem 2, it nonetheless enters the systems of equations through the baseline distributions of labour, ℓ_0 and ℓ_1, across sectors.

Measuring these baseline distributions of workers across occupations, sectors, and retraining statuses presents a challenge, however. We observe sectoral choices by occupation and retraining status in our data, but selection naturally contaminates naive comparisons of workers who have retrained against those who have not. Since we wish to exclude such selection effects from our model, we leverage observed retraining rates in combination with Theorem 1 to generate model-consistent occupation-by-sector distributions, separately by retraining status. Given data on the distribution of labour across sectors and occupations as well as values for the parameters $\{\beta, \gamma, \kappa, \nu, \theta\}$, we first solve for the values of ℓ^0 and ℓ^1 which are consistent with our model of retraining, as captured by Theorem 1 and observed retraining rates R_t, and then conduct counterfactuals using Theorem 2.

Implementation

Implementing Theorem 2 requires data on Germany's labour market and international trade flows in our base year, namely the year 2000. From the Sample of Integrated Labour Market Biographies (SIAB), we take data on employment counts, earnings, and retraining rates, the latter measured over multiple years to capture real-world dynamics.[15] The SIAB's disclosure requirements prevent an export of detailed occupation-by-sector employment counts. We therefore extract from SIAB employment counts at a more aggregated occupation-by-sector classification, then impute employment counts to our detailed occupations and sectors which match these as closely as possible while remaining consistent with total employment counts by sector and by occupation, as well as the measure of occupational trade exposure defined in Section 3.

On the international trade side, we calibrate the model to the three-digit manufacturing sectors used in Section 3 as well as a composite nontradable sector which absorbs all other employed workers. We again take data on trade flows from the UN Comtrade database. Values for the input-output coefficients Γ and Ω at this level of sectoral disaggregation do not exist. To deal with this challenge, for each three-digit manufacturing sectors we impute values for Γ and Ω based on the

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[15]To address SIAB's disclosure requirements, we use slightly aggregated versions of the 3-digit German occupational classifications and sectors introduced in Section 3, leaving us with 96 occupations and 215 (productive) sectors. Furthermore, our model is static, whereas in the real world the decision to retrain is likely a dynamic one, shaped by expectations about the future and having persistent effects. To capture this dimension, we aggregate retraining rates in the data over a span of seven years – so that our model maps cleanly to the empirical results in Section 3 – before feeding them into the model. Retraining rates before 2001, however, are not reliably reported in our data: we therefore assume that prior to 2001 retraining rates were constant and multiply their 2001 values by seven when feeding them into the model.

broader sector groupings in the World Input Output Database (WIOD), which provides data on trade flows and input-output linkages between 43 countries and 56 sectors (Timmer et al. 2015). We aggregate the data into four "countries:" Germany (G), the East (E), other developed countries (D), and the rest of the world (ROW). Appendices A.1-A.2 provide further details regarding our data sources, and Appendices A.3.4-A.3.5 describe how we construct the inputs into our hat algebra procedure.

5.2 Calibration Strategy

Having dealt with the model's fundamentals and input-output coefficients, we require values for five parameters: the horizontal component of retraining β, the retraining elasticity γ, the intersectoral labour supply elasticity ν, the employment elasticity κ, and the trade elasticity θ. For the trade elasticity, we choose a standard value of $\theta = 5$ based on estimates for comparable models in the literature (Costinot and Rodríguez-Clare 2014). To pin down the remaining four parameters, we first invert the model to identify the Eastern shock whose effects we measured empirically in Section 3. We then exploit this shock as a source of exogenous variation with which to calibrate the model.

Measuring the Eastern Shock

We now use our quantitative model to measure the actual productivity shocks in a composite East, $\hat{A}_E = \{\hat{A}_{E,s}\}$, which drove our empirical results.[16] In the spirit of Autor, Dorn, and Hanson (2013) and Caliendo, Dvorkin, and Parro (2019), we infer $\hat{A}_E$ from net imports from the East to other developed countries. For a given vector of productivity shocks $\hat{A}_E$, we solve the system of equations in Theorem 2 for the changes in trade shares $\hat{\pi}$ and expenditures $\hat{X}$ that would have occurred between 2001 and 2007 had these been the only shocks to occur. Using $\hat{\pi}$ and $\hat{X}$, we then construct the change in net imports in each sector from the East to other developed countries. Finally, we choose $\hat{A}_E$ to match the data as closely as possible.[17]

Calibration Targets

To identify $\{\beta, \gamma, \kappa, \nu\}$, we target four estimates from Section 3: the effects of the Eastern shock on the probability that a worker retrains, on the employment rates of workers who do and do not retrain, and on manufacturing employment.[18] Under the identifying assumptions discussed in Section 3, these estimates capture the causal effect of an increase in Eastern net imports to

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[16]Formally, these shocks are calibrated jointly alongside the parameters $(\gamma, \nu, \kappa, \beta)$. Quantitatively, however, these parameters, which relate to Germany's labour market, have a negligible effect on the calibrated values for $\hat{A}_E$.

[17]Formally, this procedure thus defines a function $f(\hat{A}_E)$ which maps the Eastern shock into changes in net imports. Define the change in net imports from the East (E) to other developed countries (D) in sector s as $\Delta NM_s = NM_s^{2007} - NM_s^{2001}$, where $NM_s^{2001} = \pi_{EDs}^{2001} X_{Ds}^{2001} - \pi_{DEs}^{2001} X_{Es}^{2001}$ and $NM_s^{2007} = \pi_{EDs}^{2007} X_{Ds}^{2007} - \pi_{DEs}^{2007} X_{Es}^{2007}$. Our estimate of the Eastern shock then solves $\hat{A}_E = argmin_x\{||\Delta NM - f(x)||\}$.

[18]More precisely, we use the estimates from Columns 2 and 4 of Table 3 and from Columns 1-2 of Table 5.

Table 6: Moments in Model and Data

Moment	Data	Model
Effect of import exposure on...		
... employment given retraining	-0.018	-0.018
... employment given no retraining	-0.100	-0.100
... retraining rate	0.100	0.100
... manufacturing employment	-0.129	-0.129

Notes: Table displays values of moments matched in our calibration, both in the data, as estimated in Section 3, and in the model, as estimated by running equivalent specifications in an equilibrium under the final calibrated parameters.

Germany driven by changes in Eastern productivity. In order to replicate these causal effects in the model, we set the $\hat{A}_E$ in Theorem 2 equal to the shocks calibrated above and set $\hat{A}_{is} = 1$ for all other countries and sectors.[19] We then solve the equations in Theorem 2 to obtain counterfactual values for trade flows and the distributions of earnings, employment and retraining decisions across occupations and sectors in 2007. We use these counterfactual values as data in a series of regressions that mimic those we ran in Section 3. Our calibration procedure then selects values for $\{\beta, \gamma, \kappa, \nu\}$ so that the resulting regression coefficients match their empirical counterparts in Table 6.[20]

Identification

Though all four targeted moments jointly identify the four parameters of interest, we are nonetheless able to provide a heuristic argument for identification. In line with the analytical results in Subsection 4.5, the horizontal retraining component β determines whether retraining mitigates or amplifies an occupation's import exposure. Thus, β is tightly connected to the differential effect of import exposure on the employment of workers who retrain versus those who do not, i.e, the first two rows of Table 6. Given β, the retraining elasticity γ then determines the sensitivity of the decision to retrain to labour market shocks and so is pinned down by the response of retraining rates to import exposure reported in the third row of Table 6. The employment elasticity κ determines the substitutability of employment versus non-employment and so determines the scale of the employment responses reported in the first two rows of Table 6. Finally, the intersectoral labour supply elasticity ν determines the willingness of workers to substitute across

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[19]Theorem 2 only explicitly deals with shocks to productivities but extends in a straightforward way to shocks to any other fundamental. In our calibration exercise we set all of these shocks to other fundamentals equal to 1.

[20]Specifically, we run the regressions

$$\tilde{R}_t = a_1 + b_1 \tilde{X}_t + c_1 C_t + e_{1t}, \tag{43}$$

$$\tilde{E}_t^0 = a_2 + b_2 \tilde{X}_t + c_2 C_t + e_{2t}, \tag{44}$$

$$\tilde{E}_t^1 = a_3 + b_3 \tilde{X}_t + c_3 C_t + e_{3t}, \tag{45}$$

$$\tilde{M}_t = a_4 + b_4 \tilde{X}_t + c_4 C_t + e_{4t} \tag{46}$$

where, for a variable x, we denote the corresponding counterfactual value by $\tilde{x}$, and $\tilde{X}_t$ is the model-implied change in the trade exposure of occupation t, calculated just as in Section 3. The coefficients of interest here are (b_1, b_2, b_3, b_4).

Table 7: Calibrated Parameters

Parameter	Description	Value	Source
θ	Trade elasticity	5.00	Costinot and Rodríguez-Clare (2014)
β	Horizontal component of retraining	0.68	Internally calibrated
γ	Retraining elasticity	11.23	Internally calibrated
κ	Employment elasticity	3.83	Internally calibrated
ν	Intersectoral elasticity	2.39	Internally calibrated

Notes: Table displays calibrated values of key trade, labour supply, and retraining parameters either externally calibrated from the literature or internally calibrated.

sectors within employment and therefore is pinned down by the effect of import exposure on manufacturing employment reported in the fourth row of Table 6.

5.3 Results

With four parameters targeting four moments, the model is able to exactly match the data, as evidenced by Table 6. Our calibrated parameter values, listed in Table 7, imply that retraining substantially broadens a worker's skillset ($\beta = 0.68$) and that retraining choices respond strongly to the associated returns ($\gamma = 11.23$). As a result, the model matches the large response of retraining to import exposure as well as the large difference in its effects across workers who retrain versus those who do not. Our calibration also implies that labour supply responds fairly strongly to changes in wages, both across sectors within employment ($\nu = 2.39$) and between employment and non-employment ($\kappa = 3.83$). These parameter values allow the model to replicate the substantial employment effects of the Eastern shock.

6 Counterfactuals

We finally employ the calibrated model to quantify the welfare effects of trade and retraining in Germany. In particular, we consider how the Eastern shock affected the labour market outcomes of German workers – and how the option of retraining shaped these effects. We then ask, more broadly, how retraining alters the gains from international trade.

6.1 The Effects of the Eastern Shock

We first investigate the effects of rising Eastern European and Chinese trade on Germany. To do so, we take as our baseline the economy in 2001 and simulate the shocks to Eastern productivities calibrated in Section 5. We then use Theorem 2 to solve for the resulting changes in employment, wages, retraining rates, and welfare across sectors and occupations.

Our calibrated model reveals that rapid productivity growth in the East substantially shifted wages as well as the distribution of workers across sectors. Figure 2 shows the relationship be-

tween the Eastern productivity shock, in percentage terms, and model-implied sectoral changes in real wages and employment. In the most-exposed sectors, Eastern productivity more than doubles. The corresponding sectors in Germany face lower demand and, as Panel (a) shows, lower wages. As a result of our relatively large calibrated employment and intersectoral elasticities, κ and ν, these trade-exposed sectors, in Panel (b) of Figure 2, ultimately shed a large fraction of their workers. The Eastern shock turns out to have strengthened Germany's historically-dominant sectors. Germany's famous auto industry, for example, experiences the third-largest employment gain in percentage terms. By contrast, consumer goods manufacturing, e.g., apparel, office machinery, and computers, as well as animal farming suffer large relative employment declines.

In equilibrium, the Eastern shock caused large increases in the average worker's employment, wages, and welfare. Column (1) of Table 8 reports summary statistics for four key labour market outcomes at the occupation level. Higher Eastern productivity lowers the aggregate price index in Germany and raises the payoff to working relative to non-employment. As a result, the average occupation's employment rate rises by 2.20%. Manufacturing employment rates, earnings, and welfare also rise for the typical worker, the latter by almost 1%.[21] By comparison, Caliendo, Dvorkin, and Parro (2019) and Galle, Rodriguez-Clare, and Yi (2021) both find aggregate welfare effects of the "China shock" on the US of only around 0.20%. Our estimates thus suggest that Germany, likely due to greater openness to international trade, benefited substantially more than the US from globalization during this period.[22]

These averages mask substantial heterogeneity across occupations. Column (1) of Table 8 also reports the standard deviation of each outcome across occupations; the standard deviation of welfare changes, for example, equals 0.35%. The blue bars in the two panels of Figure 4, which show the biggest winners and losers across occupations from the Eastern shock, underscore these unequal effects. Leather- and fur-making, textile-making, and clothing production experience modest declines in welfare of at most 1%. In contrast, occupations specialised in the growing auto-manufacturing sector, most notably metal-making and treatment, experience significant welfare gains that exceed 4%. This dispersion in outcomes around the mean echoes the US case (Caliendo, Dvorkin, and Parro 2019; Adão, Arkolakis, and Esposito 2021; Galle, Rodriguez-Clare, and Yi 2021).

6.2 Retraining and the Eastern Shock

How did the option of retraining change the employment and welfare effects of the Eastern shock on Germany? We answer this question with the following experiment. We consider an alternative baseline economy where the effectiveness of retraining, Δ_t, equals zero for all occupations. We then simulate the same productivity shocks as in the previous section to quantify the

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[21]In principle the average change in these aggregates across occupations might be different from the aggregate change. In practice, these two sets of numbers are very similar. For example, aggregate welfare rises by 1%, while the average welfare change is 0.96%.

[22]Of course, there are important differences between these studies and ours, not least the fact that we consider a composite East rather than China alone. A full comparison with the US case lies beyond the scope of our paper.

Figure 2: Labour Market Effects of the Eastern Shock
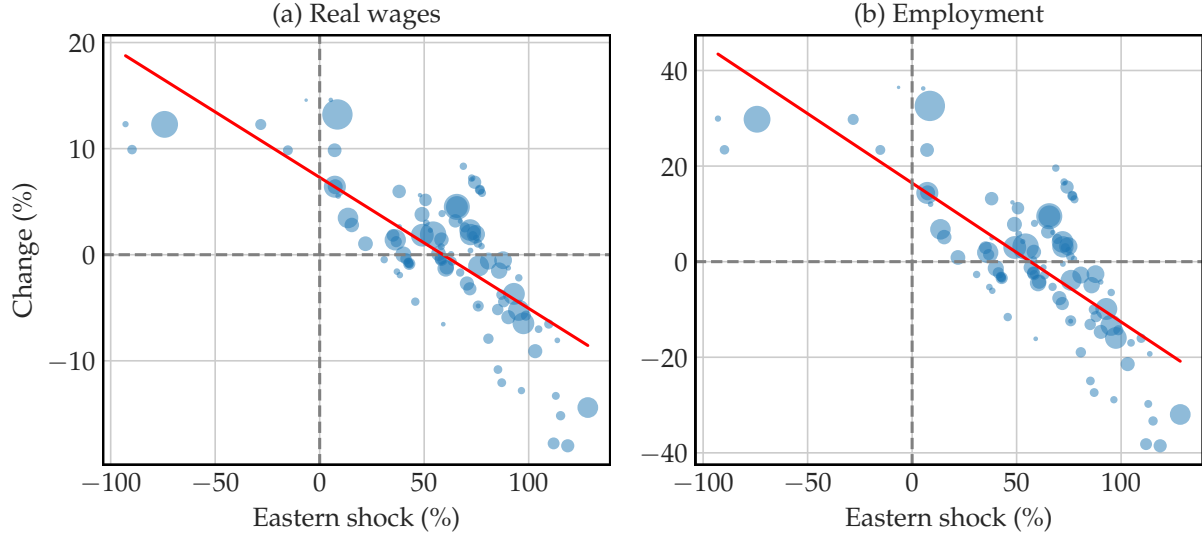
Notes: Scatterplots show relationship, at the sector level, between the Eastern (productivity) shock in percentage terms and the % change in real wages (Panel a) and employment (Panel b) from the baseline economy to the economy with the Eastern shock. Values of the Eastern shock are solved following Theorem 2. Marker sizes proportional to baseline employment. Three sectors with employment losses larger than 40% are excluded from the figure.

Table 8: Occupation-Level Effects of the Eastern Shock

| | (1) Baseline Economy | | (2) No Retraining | |
	Mean	*Standard Dev.*	*Mean*	*Standard Dev.*
Employment	2.20	0.91	2.03	1.22
Manufacturing Employment	2.62	3.96	2.17	7.18
Earnings	3.18	1.17	3.00	1.51
Welfare	0.96	0.35	0.95	0.42

Notes: Table displays summary statistics of the percentage changes in occupation-level outcomes between the baseline economy and the economy with the Eastern shock calibrated in Section 5. Column (1) corresponds to the model with parameters as estimated; Column (2) shows the same effects, but in an economy in which the retraining effectiveness, i.e., vertical component Δ_t, is counterfactually set to zero. In each case the two subcolumns show the mean and standard deviation of these percent changes across occupations.

effects of the Eastern shock in the absence of retraining.

In line with the analytical results in Section 4, retraining does not substantially alter the average welfare effect of the Eastern shock. Column (2) of Table 8 shows the Eastern-shock-induced changes in employment, earnings, and welfare in a world without retraining. Employment rises by 2.03%, and average earnings rise by 3.00%, in each case around 0.2% less than in the model with retraining. Similarly, the no-retraining economy experiences only a 0.01% smaller increase in welfare than the baseline economy.

However, retraining does substantially change the pattern of employment impacts across oc-

Figure 3: Manufacturing Employment Changes
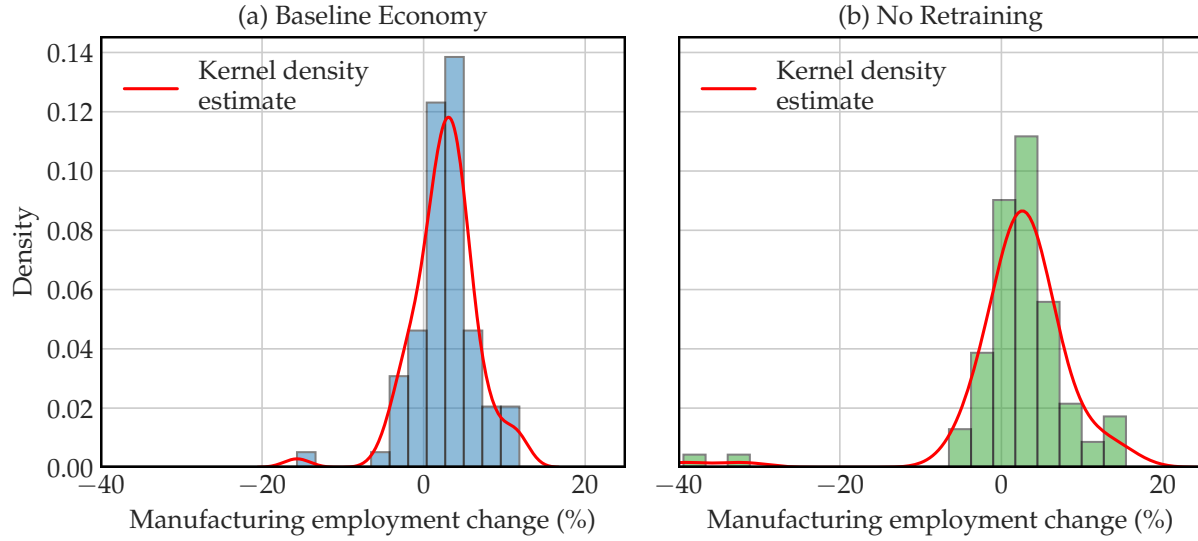
Notes: Figure shows the density of the percentages changes in manufacturing employment caused by the Eastern shock, as implied by the calibrated model. Panel (a) shows results for the baseline economy, while (b) shows results for an economy without retraining, i.e., where the vertical component Δ_t is set to zero. Red lines show kernel density estimates.

cupations. As Column (2) shows, the standard deviation of changes in employment across occupations is roughly one third larger in the absence of retraining. Most strikingly, the standard deviation of changes in occupations' manufacturing employment almost doubles. Evidently, occupations with a comparative advantage in hard-hit sectors, deprived of the option of adjusting their skillset by retraining, instead leave manufacturing entirely. Figure 3 visualizes this difference by plotting the distribution of changes in manufacturing employment across occupations with and without retraining. Changes in manufacturing employment are much less evenly distributed across occupations when retraining is impossible.

The final two rows of Table 8 show that retraining also compresses the distribution of earnings and, ultimately, welfare changes that result from the Eastern shock. The green bars in Figure 4 show the effects of removing retraining at the extremes of the distribution of earnings effects. The earnings of the highly-exposed leather-making, textile-making, and clothing production occupations all fall more sharply than in the baseline economy, while earnings gains at the top of the distribution scarcely change. Thus, retraining significantly reduces the dispersion of the Eastern shock's effects precisely because it enables the worst-affected occupations to limit their own exposure.

Figure 4: Largest Occupational Earnings Gains and Losses
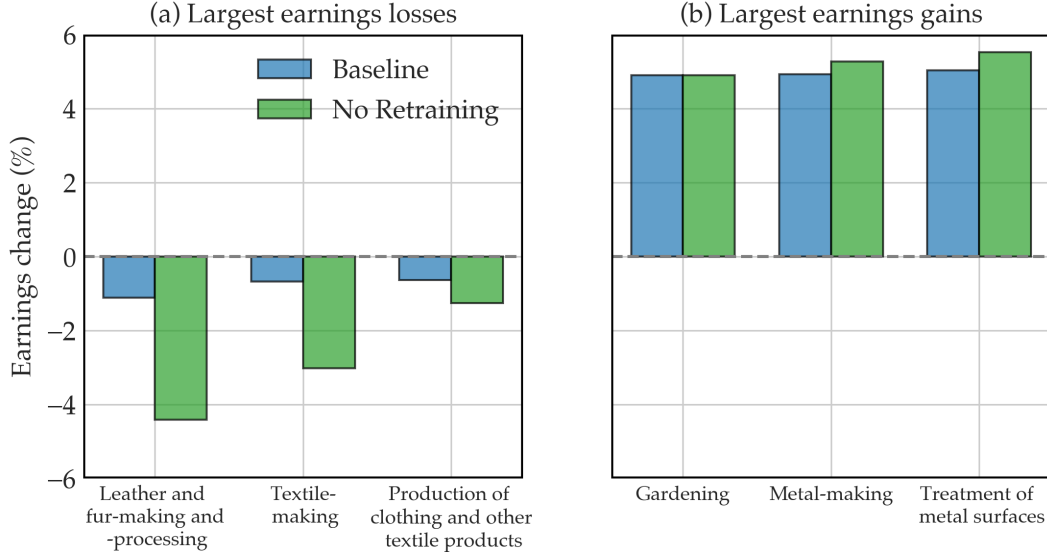
Notes: Figure shows earnings changes between the baseline economy and the economy with the Eastern shock, in percentage terms. The blue bars correspond to a counterfactual which uses the model parameters as estimated and show the three occupations that experienced the largest percentage earnings losses in (a) and the largest gains in (b) with all parameters as calibrated. The green bars show the same figures for the economy in which the retraining effectiveness, i.e., vertical component Δ_t, in every occupation is set to zero.

6.3 Retraining and the Gains from Trade

We now turn to a classic question in trade: how much do German workers gain from international integration — and how does retraining alter those gains? We again exploit Theorem 2 to answer these questions: we move Germany to autarky and measure the resulting changes in employment, earnings and welfare.[23] We then repeat this exercise in an economy without retraining. Table 9 reports the results.

A shift to autarky raises the German price level, lowers real wages, and ultimately causes large declines in employment and earnings, as reported in Column (1). On average, welfare falls by roughly 3.5%. This estimate of the gains from trade lies around the middle of the range of comparable estimates for other countries (Costinot and Rodríguez-Clare 2014). As in the case of the Eastern shock, these losses are unevenly distributed across occupations, but, in contrast, autarky leaves every occupation worse off.

Column (2) reports analogous figures for the economy without retraining which, when compared with Column (1), echo the Eastern-shock results in Table 8. The aggregate effects of autarky respond little to the availability of retraining: for example, while average earnings and employment fall less in the economy without retraining, average welfare actually declines slightly more.

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[23]Formally we move Germany to autarky by solving for a counterfactual equilibrium in which trade costs d_{ijs} are equal to infinity for every sector and the exogenous deficits d_i are set equal to zero.

Table 9: Occupation-Level Effects of Moving to Autarky

| | (1) Baseline Economy | | (2) No Retraining | |
	Mean	*Standard Dev.*	*Mean*	*Standard Dev.*
Employment	-9.09	3.19	-8.55	3.79
Manufacturing	-11.28	6.32	-8.11	8.57
Earnings	-12.27	3.97	-11.75	4.80
Welfare	-3.53	1.38	-3.55	1.78

Notes: Table displays summary statistics of the percentage changes in occupation-level outcomes between the baseline economy and the economy under autarky. Column (1) corresponds to the model with parameters as estimated; Column (2) shows the same effects, but in an economy in which the retraining effectiveness, i.e., vertical component Δ_t, is counterfactually set to zero. In each case the two subcolumns show the mean and standard deviation of these percent changes across occupations.

As in the case of the Eastern shock, retraining primarily compresses the distribution of gains from trade across occupations. In other words, the standard deviations of autarky-induced declines in employment, earnings, and welfare are all approximately 25% higher in the economy without retraining. We thus conclude that retraining is a powerful policy tool to blunt the distributional effects of international trade without curtailing aggregate gains.

7   Conclusion

In this paper, we evaluate retraining as a policy solution to help workers adjust to import competition. We show empirically that German workers use retraining to adjust to increases in import competition and that workers who retrain are less affected by these shocks. We build a rich quantitative model of trade and labour markets in which workers from different occupations sort across sectors according to their comparative advantage and, within this framework, develop a novel model of retraining. Our model demonstrates that retraining can, to the extent that it broadens workers' skills, shrink occupations' losses from trade shocks. We then calibrate the model's key parameters by matching our empirical results. Finally, we simulate the effects of realised increases in Eastern European and Chinese productivity on the German economy, with and without the option of retraining. We find that retraining has little effect on the aggregate gains from rising Eastern imports. However, its availability renders the effects of this import competition — and the gains from trade — substantially less unequal. Thus, retraining has the potential to not only moderate trade-induced inequality but also build political support for further global integration.

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A Data Appendix

A.1 German Sample of Integrated Labor Market Biographies (SIAB)

We measure labor market outcomes using standard German administrative labor market data, the Sample of Integrated Labor Market Biographies (SIAB) 1975-2021 (Graf et al. 2023), which we accessed on-site at the Research Data Centre (FDZ) of the German Federal Employment Agency (BA) at the Institute for Employment Research (IAB) and subsequently via remote data access. The SIAB draws a 2% sample from Germany's primary administrative worker database, known as the Integrated Employment Biographies (IEB). This panel records, for each worker and month between 1975 and 2021, any active employment spells, unemployment benefits, and participation in active labor market programs such as retraining (Schmucker, Seth, and Berge 2023).

A.1.1 General Cleaning and Preparation

We clean and prepare the SIAB for all further analysis using the excellent suite of Stata routines provided by W. Dauth, Eppelsheimer, and Stüber (2023). In particular, we implement their codes which take the raw worker-by-employment or benefit receipt episode panel and then split episodes which span multiple calendar years, correctly account for lump-sum payments, calculate job tenure, merge in firm characteristics, aggregate education levels, merge in values of the contribution assessment ceiling and marginal part-time income threshold for tax and social insurance payments, and deflate monetary values. We then restrict the panel to workers between the ages of 18 and 65, drop employment spells with zero income, and drop employment statuses not observed in all years: marginal employment spells, internships, and a catchall "other" category. Next, we use W. Dauth, Eppelsheimer, and Stüber (2023)'s imputation procedure to correct for top-coding of wages and later apply their final cleaning routine.

A.1.2 Sector and Occupation Aggregates

Having thus conducted basic cleaning, we compute aggregates for the 3-digit occupations in the German Classification of Occupations 2010 (KldB 2010) and 3-digit sectors in the German Classification of Economic Activities, Edition 1993 (WZ 93). Specifically, we use reported earnings per day and episode length of all observed employment episodes to compute the annual wage bill for each year at occupation-by-sector level. We also take an annual snapshot on 30 June to compute employment counts. To do so, we must define a primary job for each worker; we define the latter as the job held on 30 June with the highest daily earnings and break ties by selecting the job where the worker has the longest tenure in the firm and, if any ties remain, randomly selecting one episode. Based on these yearly primary jobs, we then compute employment counts at the occupation-by-sector level for each year. In computing all aggregates, we account for the fact that our data represents a 2% sample of the economy.

A.1.3 Worker Sample

We now prepare a worker-level dataset of labor market outcomes. We start with the data prepared as in Appendix A.1.1 and again build on the Stata routine provided by W. Dauth, Eppelsheimer, and Stüber (2023) to correct for parallel employment episodes. Whenever workers concurrently hold multiple jobs, we keep only a single "primary" job. In particular, we select the job where the worker has the longest tenure in the firm and break ties based on (in order) daily earnings, total length of the episode, and then randomly. We later define sector and occupation based on this primary job but also record the sum of earnings across all jobs held in parallel.

Our sample of workers then comes from a cross-sectional snapshot in the year 2000. Loosely following W. Dauth, Findeisen, and Suedekum (2021), we include all workers who, in their primary job, were full-time employed in manufacturing on 30 June 2000, had at least two years of tenure in their current job, earned above the marginal earnings threshold, and who were aged 24-65 during our whole sample period of 2001-2007.[24]

A.1.4 Retraining Episode Sample

The facts regarding episode length and age in Section 2.3 are based on a dataset of retraining episodes derived from the raw SIAB panel. As in the rest of the paper, we define retraining as the employment status (*erwstat*) codes 10001 (activation and vocational integration), 10002 (career choice and vocational training), and 10003 (vocational retraining and further education) in the Participants-In-Measures History File (MTH/XMTH) module of the SIAB. Since the SIAB only includes such episodes starting in 2000, we restrict our sample to retraining spells between 2000 and 2019. We then calculate the length of retraining episodes in days and workers' ages when they start each spell.

A.1.5 Quarterly Panel

To characterize retraining rates and calculate aggregates for calibration, we also build a quarterly panel of workers. We begin with the SIAB, cleaned as described in Appendix A.1.1, and correct for parallel employment episodes as described in Appendix A.1.3. We then adapt existing code for a yearly panel from W. Dauth, Eppelsheimer, and Stüber (2023) to instead generate a quarterly panel of workers from 1992 through 2019. In this panel, we measure workers' employment status, earnings, sector, and occupation on the midpoint date of each quarter; job characteristics are those of their primary job, as previously defined, on this date. We also define a quarterly retraining dummy which equals one if the worker participated in subsidized retraining at any point during that quarter, not necessarily only on the "sample" date. We classify worker-quarters missing from the data as non-employment. We associate non-employed workers with their last observed occupation observed on the sample date of a quarter.

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[24]Manufacturing sectors in the 3-digit German Classification of Economic Activities, Edition 1993 include sectors 151-212 and 241-366.

A.2   UN Comtrade Database

Our trade data comes from the UN Comtrade Database.[25] Comtrade provides primary values by importing and exporting country and SITC Rev. 3 sector; we map these values to the 3-digit sectors in the German Classification of Economic Activities, Edition 1993 (WZ 93) with a crosswalk provided by W. Dauth, Findeisen, and Suedekum (2021). We then convert all values to 2005 USD using the FRED Consumer Price Index for All Urban Consumers: All Items in US City Average (CPIAUCSL), seasonally adjusted, indexed to January 2005 and finally convert to 2005 Euros using December 2005 exchange rates.[26]

We follow W. Dauth, Findeisen, and Suedekum (2021) and focus on trade flows between Germany and a composite "East" which combines China and twenty former Eastern bloc countries. The former Eastern bloc countries include Hungary, Poland, Romania, Bulgaria, Russia, Belarus, Estonia, Latvia, Lithuania, Moldova, Ukraine, Azerbaijan, Georgia, Kazakhstan, Kyrgyzstan, Tajikistan, Turkmenistan, Uzbekistan, Czechia, and Slovakia. Note that we exclude Slovenia due to missing 1990 data. We also follow common practice in the trade literature and calculate trade flows between each of eight other high-income countries and this same "East;" our "other" high-income countries include the UK, Sweden, Norway, Canada, Australia, New Zealand, Japan, and Singapore.

A.3   Variable Definitions

A.3.1   *Retraining Rates*

The retraining rates cited in Section 2.3 are based on our SIAB-derived quarterly panel. We restrict the data to the period 2000-2019, as in the case of our retraining episode sample, and calculate the yearly retraining rate as the share of person-years during which a worker participates in subsidized retraining (employment status *erwstat* 10001-10003). We calculate both aggregate rates and rates by nine aggregate sectors delineated by W. Dauth and Eppelsheimer (2020), which correspond to those employed in the IAB Establishment Panel. Finally, we arrive at the lifetime retraining rate by computing the share of workers in this quarterly panel who complete retraining at some point during 1992-2019.

A.3.2   *Labor Market Outcomes*

We calculate earnings, employment, and retraining in the worker sample discussed in Appendix A.1.3; in doing so, we build on replication code provided by W. Dauth, Findeisen, and Suedekum (2021). First, we calculate earnings across all jobs during the year 2007, divide by total earnings across all jobs during the year 2000, and multiply by 100 to obtain our measure of *earnings growth*. Then, we calculate the share of calendar days in 2007 during which a worker is employed

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[25]Available at https://comtradeplus.un.org.

[26]CPI available at https://fred.stlouisfed.org/series/CPIAUCSL#0 and USD-Euro exchange rates at https://www.europarl.europa.eu/RegData/etudes/note/join/2007/379231/IPOL-TRAN_NT(2007)379231_EN.pdf.

in any job to obtain the *share of days employed* and the *share of* (calendar) *days employed in manufacturing,* defined, for the purposes of employment shares, as 3-digit sectors 151-366. Finally, for our linear probability model and logit specifications for the *Pr(retrain)*, we define a subsidized retraining dummy equal to one if we observe a worker participating in subsidized retraining, which we define as the employment status (*erwstat*) codes 10001 (activation and vocational integration), 10002 (career choice and vocational training), and 10003 (vocational retraining and further education) in the Participants-In-Measures History File (MTH/XMTH) module of the SIAB. We use this same measure to define workers' retraining status in our split-sample specifications. We finally define the *number days retraining* as the sum of the lengths (in days) of all episodes of subsidized retraining during 2001-2007.

A.3.3 Controls

Using the SIAB, we also calculate demographic and initial-job-related controls for our worker sample discussed in Appendix A.1.3. We define a female dummy as well as one for foreign nationality. We also define dummy variables for education dummies for three education levels: low (neither vocational training nor degree from university), medium (vocational training), and high (degree from a university or university of applied science). The age bins for which we include dummies include 24-34, 35-44, 45-54, and 55+. We also define controls related to workers' "initial" primary job in the year 2000. These controls include dummies for <1, 2-4, 5-9, and 10+ years of tenure in the initial job, as defined by W. Dauth, Eppelsheimer, and Stüber (2023), dummies for firm (i.e. plant) size bins of 1-9, 10-99, 100-499, and 500+ workers, and a dummy for initial jobs in manufacturing (3-digit sectors 151-212 and 241-366). Finally, we record total year-2000 earnings, summed across all jobs.

A.3.4 Trade Exposure

We then compute sector and occupation-level trade exposure. We convert Comtrade trade flow values to 2015 Euros and then calculate the change in net imports, Δnet imports$_s$, as the raw change in imports minus exports between Germany or the aforementioned other high-income countries and the East in a sector from 2001-2007.

To aggregate up to the occupation level, we define the import exposure of a worker initially in occupation t as a weighted average of changes in sectoral net imports from the East, either to Germany or to the set of other high income countries:

$$\text{occupational import exposure}_t \equiv \sum_s l_{ts} \times \left(\frac{\Delta \text{net imports}_s}{\text{wage bill}_s} \right).$$

The weights l_{ts} equal the share of base year (2000) workers of occupation t employed in sector s, computed as in Appendix A.1.2. As in W. Dauth, Findeisen, and Suedekum (2021), we normalize net imports by the sector's total year-2000 wage bill in Germany, again calculated as in Appendix

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. Note also that, in line with common practice, we focus only on manufacturing trade, which we define, for the purposes of calculating trade flows, as 3-digit sectors 151-212 and 241-366. Thus, Δnet imports$_s$ equals zero for all sectors s outside of manufacturing.

A.3.5   *Sectoral and Occupation Aggregates for Calibration*

Finally, we calculate sectoral and occupation aggregates for our hat algebra procedure, as described in Section 5.1. Due to IAB confidentiality requirements, we slightly aggregate the 3-digit German Classification of Occupations 2010 (KldB 2010) and the 3-digit sectors in the German Classification of Economic Activities, Edition 1993 (WZ 93) by combining 49 occupations and 11 sectors with small numbers of workers into groups of adjacent occupations or sectors. We refer to these slightly-aggregated versions as "modified" 3-digit occupations and sectors.

We then calculate vectors of occupational and sectoral aggregates, based on the sectors and occupations of workers observed in a sample quarter, namely the 2nd quarter of the year 2000. In particular, for the sample quarter, we calculate the number of employed and non-employed workers by modified 3-digit occupation, where the non-employed, as discussed above, include workers observed prior to or after but not on the sample date. We similarly calculate non-manufacturing employment as well as mean yearly earnings, imputed based on the sample date, by modified 3-digit occupation. For the full year 2000, we calculate a (yearly) retraining rate by occupation; we record a worker's occupation in the sample quarter but take into account retraining which occurs at any point during the year. For each of the modified 3-digit sectors, we calculate, for the sample quarter, employment and the total wage bill, the latter scaled up to an annual amount.

To display a sector-by-occupation distribution with sufficient observation counts in each cell for disclosure, we further aggregate both occupations and sectors. We aggregate the aforementioned modified 3-digit occupations into 8 employment-weighted quantiles of net import exposure, calculated for these modified 3-digit occupations as described in Appendix A.3.4; we refer to these quantiles as *occupation groups*. As for sectors, we first associate workers with ISIC Rev. 4 sectors and then aggregate these slightly into *sector groups* which simply involve, as before, the combination of 14 of the ISIC Rev. 4 sectors into sufficiently-large groups of adjacent sectors. We also combine all non-tradable sectors. For the sample quarter, we then calculate a matrix of the number of workers in each occupation group-by-sector group cell, where the latter include non-employment. We finally adjust all aggregates for the fact that we observe a 2% sample of the labor market.

B   Theory Appendix

B.1   Proof of Theorem 1

Let $j \in \{0,1\}$ index retraining status, $e \in \{0,1\}$ index employment status, and $s \in \{1,...,S\}$ index productive sectors. Let *jes* denote a particular retraining-employment-sector tuple. Then the log utility from choosing option *jes* for worker ω can be written,

$$\log(u_{jes}(\omega)) = \log(x_{jes}) + \left(\frac{1}{\gamma}\right)\lambda_{jes}(\omega) \tag{47}$$

for some deterministic component x_{jes} and a stochastic component $\lambda_{jes}(\omega)$. Under Assumption 1, this problem has the nested logit form defined in Cardell (1997), equation (3). Therefore by Lemma 4.1 in that paper λ has the following cumulative distribution function:

$$F_\lambda(\lambda) = \exp(-G(\exp(-\lambda))), \tag{48}$$

where the function G is given by

$$G(y) = \left(\sum_j \left(y_{j0}^\kappa + \left(\sum_{s>0} y_{j1s}^\nu\right)^{\frac{\kappa}{\nu}}\right)^{\frac{\gamma}{\kappa}}\right)^{\frac{1}{\gamma}}. \tag{49}$$

By Definition 2 in Lind and Ramondo (2023), this G is a correlation function. Now, pick some positive vector of utilities $\bar{u}$. The probability that an individual worker has $u \leq \bar{u}$ is,

$$
\begin{aligned}
\mathbb{P}\left(u_{jes} \leq \bar{u}_{jes} \quad \forall \quad j,e,s\right) &= \mathbb{P}\left(\log(u_{jes}) \leq \log(\bar{u}_{jes}) \quad \forall \quad j,e,s\right) \\
&= \mathbb{P}\left(\lambda_{jes} \leq \gamma\left(\log(\bar{u}_{jes}) - \log(\bar{x}_{jes})\right) \quad \forall \quad j,e,s\right) \\
&= F_\lambda(\gamma(\log x - \log \bar{u})), \\
&= \exp(-G(x^\gamma \bar{u}^{-\gamma})).
\end{aligned}
$$

The final line above implies that, by Lemma 1 in Lind and Ramondo (2023), utilities have a multivariate γ-Fréchet distribution, with the scale parameter for option *jes* equal to x_{jes}^γ.

From the results in that paper, it follows that the maximum among the options *jes* has a Fréchet distribution with shape parameter γ and scale parameter $G(x^\gamma)$. Moreover, from Lemma A.3 in Lind and Ramondo (2023), the probability any particular option *jes* is chosen is given by,

$$\mathbb{P}(jes = argmax \quad u_{j'e's'}) = \frac{G_{jes}(x^\gamma)}{G(x^\gamma)},$$

where G_{jes} is the partial derivative of G with respect to argument *jes*. From this observation one can derive all the choice probabilities stated in Theorem 2.

B.2 Derivation of Analytical Results

Derivation of (23) *and* (24)

First, to formalise the idea of removing the non-employment option, set $b_t^j = 0$ for all t and j. Then we have,

$$V_t^j = \left(\sum_s \left(z_{st}^j w_s \right)^\nu \right)^{\frac{1}{\nu}} P^{-1},$$

and thus,

$$\log V_t^1 - \log V_t^0 = \nu^{-1} \left(\log \left(\sum_s \left(z_{st}^1 w_s \right)^\nu \right) - \log \left(\sum_s (z_{st}^0 w_s)^\nu \right) \right).$$

Let us define

$$f(\beta) \equiv \log V_t^1 - \log V_t^0,$$

where β enters through the post-retraining human capital stocks z_{st}^1. Our goal is to approximate $f(\beta)$ by,

$$f(\beta) \simeq f(0) + \beta f'(0).$$

First note that at $\beta = 0$ we have,

$$z_{st}^1 = \Delta_t z_{st}^0 \implies f(0) = \log \Delta_t$$

Now, fix some β and take the derivative of f. We obtain,

$$f'(\beta) = \frac{\sum_s z_{st}^1 w_s \frac{d \log z_{st}^1}{d\beta}}{\sum_s z_{st}^1 w_s} = \sum_s \ell_{st}^1 \frac{d \log z_{st}^1}{d\beta}$$

Focus on a particular z_{st}^1 and take the derivative with respect to β. We obtain,

$$\frac{d \log z_{st}^1}{d\beta} = -\log z_{st} + \frac{\sum_s \mu_t z_{st}^{1-\beta} \log z_{st}}{\sum \mu_t z_{st}^{1-\beta}}.$$

After some algebra, we have

$$\lim_{\beta \to 0} \frac{d \log z_{st}^1}{d\beta} = \log Z_t - \log z_{st}.$$

Substituting, we have

$$f'(0) = \sum_s \ell_{st}^1 \left(\log Z_t - \log z_{st} \right) = \sum_s \ell_{st} \left(\log Z_t - \log z_{st} \right)$$

where the last equality follows because at $\beta = 0$ employment shares are the same across retraining status. (23) follows immediately, and (24) is obtained by substituting (23) into (10).

Derivation of (26) *and* (27)

We will start by making precise the assumption that shocks to wages are mean zero and independent of baseline sector characteristics. The first statement is straightforward: we assume,

$$\sum_s \Delta \log w_s = 0.$$

For the second statement, the only characteristics of sectors in our economy that are (i) mean productivity Z_s, and (ii) the baseline level of wages, w_s. We therefore assume that,

$$\sum_s \log Z_s \Delta \log w_s = 0,$$

$$\sum_s \log w_s \Delta \log w_s = 0.$$

Now, let us compare the exposure of workers who retrain versus those who do not. Define.

$$x_t^0 = -\sum_s \ell_{st}^0 \Delta \log w_s,$$

$$x_t^1 = -\sum_s \ell_{st}^1 \Delta \log w_s,$$

as the exposure of these two sets of workers. We will now derive an approximate expression for ℓ_{st}^1 and so relate these two measures. After some algebra, we can write ℓ_{st}^1 as

$$\log \ell_{st}^1 = (1 - \beta) \log \ell_{st}^0 + \nu \left(\beta \log Z_t + \beta \log Z_s + \beta \log w_s + (1 - \beta) \log V_t^0 - \log V_t^1 \right)$$

Now, multiply by $\Delta \log w_s$ and sum over s. By the mean-zero and independence assumptions above, we have

$$\sum_s \log \ell_{st}^1 \Delta \log w_s = (1 - \beta) \log \ell_{st}^0 \Delta \log w_s.$$

Now we will use an approximation: for ϵ sufficiently small,

$$\log(1 + \epsilon) \simeq \epsilon.$$

We will assume that employment shares ℓ_{st}^j are not too dispersed, which implies that they are close to S^{-1}. This in turn implies that $S\ell_{st}^j - 1$ is small, and thus that,

$$\log(S\ell_{st}^j) = \log(1 + (S\ell_{st}^j - 1)) \simeq S\ell_{st}^j - 1.$$

Applying this approximation to both sides of the equality above implies,

$$\sum_s \ell_{st}^1 \Delta \log w_s = (1 - \beta)\ell_{st}^0 \Delta \log w_s,$$

which is equivalent to

$$x_{st}^1 = (1 - \beta)x_{st}^0.$$

Now, to a first order approximation the change in average income in occupation t due to the import competition shock is,

$$\Delta \log Y_t = -R_t x_t^1 - (1 - R_t)x_t^0.$$

Substituting in the expression above gives (26). Noting that $\Delta \log V_t^1 - \Delta \log V_t^0 = x_t^0 - x_t^1$ gives (27).

B.3 Proof of Theorem 2

The first block of equations can be derived from the international trade part of the model in a straightforward way. (30), (31), and (32) all follow immediately from taking ratios of (16), (17), and (18) between baseline and counterfactual equilibria. (33) is simply the ratio of (20) between baseline and counterfactual equilibria. To obtain the final equation in this block, consider substituting (21) and (22) into (20) to obtain an expression involving only expenditures, trade shares, and exogenous deficits,

$$X_{is} = \alpha_{is} \left(\sum_r \Gamma_{ir} \sum_j \pi_{ijr} X_{jr} + d_i \right) + \sum_r (1 - \Gamma_{ir}) \Omega_{irs} \sum_j \pi_{ijr} X_{jr}.$$

Now, collect all terms in front of each $\pi_{ijr} X_{jr}$ into a constant $\tilde{\alpha}_{ijsr}$ to write,

$$X_{is} = \sum_j \sum_r \tilde{\alpha}_{ijsr} \pi_{ijr} X_{jr} + \alpha_{is} d_i.$$

To obtain (34), just evaluate this equation at the counterfactual equilibrium and note that by definition,

$$\pi'_{ijr} X'_{jr} = \hat{\pi}_{ijr} \hat{X}_{jr} \pi_{ijr} X_{jr}.$$

The second block of equations follows from the results in Theorem 1. In particular, (35), (36), and (37) all follow from taking ratios of (7), (8), and (9) between baseline and counterfactual equilibria and substituting out fundamentals using the relevant choice probabilities in the baseline equilibrium. (38), (39), and (40) are similarly obtained by taking ratios of (10), (11), and (12) between baseline and counterfactual equilibria. (41) follows from taking the ratio of (13) between baseline and counterfactual equilibria and defining,

$$\rho_{st} = \frac{R_t E_t^1 \ell_{st}^1}{R_t E_t^1 \ell_{st}^1 + (1 - R_t) E_t^0 \ell_{st}^0}.$$

Finally, (42) follows from taking the ratio of (14) between baseline and counterfactual equilibria and defining

$$\eta_{st} = \frac{\mu_t h_{st}}{\sum_o \mu_0 h_{so}}.$$