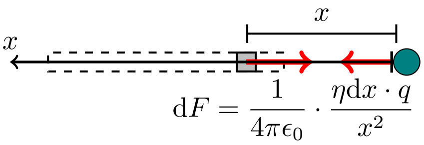
6-1

解：



电荷线密度$\eta = Q/L$.

如图取细棒微元，对牛顿第二定律积分

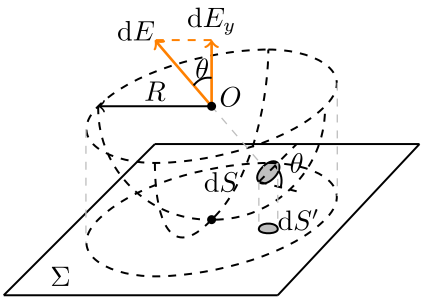
$$\int_0^F \mathrm{d}F = \int_{a-\frac{L}{2}}^{a+\frac{L}{2}} \frac{1}{4\pi\epsilon_0} \frac{\eta \mathrm{d}x \cdot q}{x^2}$$

得

$$F = \frac{Qq}{\pi\epsilon_0 \left(4a^2 - L^2\right)}.$$

6-3

解：



各物理量定义如上图. 取球面上面元$\mathrm{d}S$. $\mathrm{d}S$在O点处产生的场强大小为

$$\mathrm{d}E = \frac{1}{4\pi\epsilon_0} \cdot \frac{\sigma \mathrm{d}S}{R^2}.$$

由对称性得O点电场强度方向与平面Σ垂直，场强垂直Σ的分量

$$\begin{aligned}\mathrm{d}E_y &= \mathrm{d}E \cos\theta \\ &= \frac{1}{4\pi\epsilon_0} \cdot \frac{\sigma \mathrm{d}S}{R^2} \cdot \cos\theta \\ &= \frac{1}{4\pi\epsilon_0} \cdot \frac{\sigma \mathrm{d}S'}{R^2}.\end{aligned}$$

其中$\mathrm{d}S'$是$\mathrm{d}S$在平面Σ上的投影.

O点场强大小

$$\begin{aligned}E &= E_y \\ &= \iint_S \mathrm{d}E_y \\ &= \iint_{S'} \frac{1}{4\pi\epsilon_0} \cdot \frac{\sigma \mathrm{d}S'}{R^2} \\ &= \frac{1}{4\pi\epsilon_0} \cdot \frac{\sigma}{R^2} \cdot \pi R^2 \\ &= \frac{\sigma}{4\epsilon_0},\end{aligned}$$

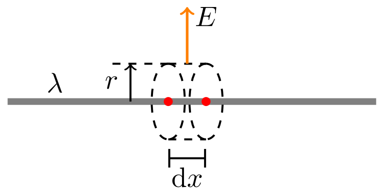
方向垂直开口圆面指向开口外侧.

6-5

解：

(1)

由对称性得平面上电场强度方向与两条导线垂直. 先求单条导线的场强大小$E_\lambda(r)$.



由高斯定理，

$$E \cdot 2\pi r \mathrm{d}x = \frac{1}{\epsilon_0} \lambda \mathrm{d}x$$

得

$$E_\lambda(r) = \frac{\lambda}{2\pi\epsilon_0 r}.$$

平面上的场强

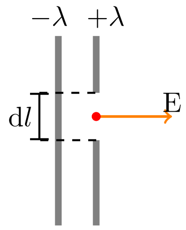
$$\begin{aligned}E(r) &= -E_{+\lambda}(\frac{a}{2} - x) + E_{-\lambda}(x + \frac{a}{2}) \\ &= \frac{2\lambda a}{\pi\epsilon_0 \left(4x^2 - a^2\right)}.\end{aligned}$$

方向沿$+x$为正.

(2)

先考虑电场力的大小F.

不妨在$+\lambda$上取线元$\mathrm{d}l$.



外场强度$E=E_{-\lambda}(a)=-\lambda/(2\pi\epsilon_0 a)$，线元所受电场力$\mathrm{d}F=\lambda\mathrm{d}l\cdot E_{-\lambda}(a)=-\lambda^2\mathrm{d}l/(2\pi\epsilon_0 a)$，符号表示方向向左. 单位长度线元所受电场力大小

$$\frac{\mathrm{d}F}{\mathrm{d}l}=\frac{\lambda^2}{2\pi\epsilon_0 a}.$$

方向均沿x轴指向对侧导线.

6-6

解：

使用叠加方法. 由高斯定理得半径为R的球体内部场强大小$E(r)$满足

$$4\pi r^2 E(r)=\frac{1}{\epsilon_0}\cdot\frac{4}{3}\pi r^3\rho,$$

整理得$E(r)=\rho r/(3\epsilon_0)$，写成矢量形式为

$$\vec{E}(\vec{r})=\frac{\rho}{3\epsilon_0}\vec{r}.$$

空腔内部点P处的场强为

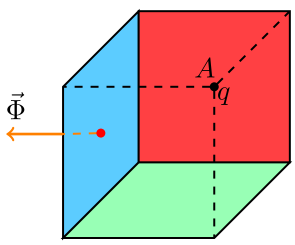
$$\begin{aligned}\vec{E}(\vec{O_2P})&=\frac{\rho}{3\epsilon_0}\left(\vec{O_1P}\right)-\frac{\rho}{3\epsilon_0}\left(\vec{O_2P}\right)\\&=\frac{\rho}{3\epsilon_0}\vec{O_1O_2}.\end{aligned}$$

代入$|O_1O_2|=a$得场强大小

$$E=\frac{\rho a}{3\epsilon_0}.$$

6-8

解：



如图，由对称性易得三个颜色的方形侧面的电场强度通量相等. 取8个这样的正方体按A点中心对称拼成一个大正方体，则24个上色小平面将大正方体完全覆盖. 设一个小平面的电场强度通量为Φ，由高斯定理

$$24\Phi=\frac{q}{\epsilon_0}$$

得

$$\Phi=\frac{q}{24\epsilon_0}.$$

6-10

解：

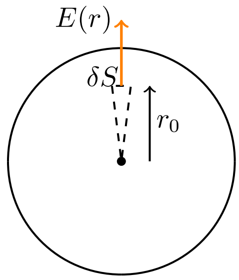
由对称性容易得到直线BP上电场强度沿线分量为0(交换正负电荷，由$\vec{E}=\int_L\frac{1}{4\pi\epsilon_0}\frac{\mathrm{d}q}{r^2}\cdot\hat{\mathbf{r}}$得交换前后电场强度反向，另有交换前后电场强度关于$BP$轴对称). 则$P$点电势$\phi(P)=\phi(B)=0$.

对O点电势使用积分处理. 已知B点和无穷远点电势均为0，则O点电势

$$\begin{aligned}\phi(O)&=\int_{-l}^{0}\frac{1}{4\pi\epsilon_0}\cdot\frac{-\lambda\mathrm{d}x}{x+2l}+\int_0^l\frac{1}{4\pi\epsilon_0}\cdot\frac{\lambda\mathrm{d}x}{x+2l}\\&=\frac{\lambda}{4\pi\epsilon_0}\left(\ln\left(\frac{3}{2}\right)-\ln 2\right)\\&=-\frac{\lambda}{4\pi\epsilon_0}\ln\frac{4}{3}.\end{aligned}$$

6-11

解：



在球体内部取沿径向的圆锥体高斯面，底面积为δS. 由对称性得电场强度沿径向，由高斯定理，

$$E(r_0)\cdot\delta S+o\left(E(r_0)\cdot\delta S\right)=\frac{1}{\epsilon_0}\int_0^{r_0}\rho\frac{\delta S r^2}{r_0^2}\mathrm{d}r,$$

其中$o(x)$表示x的高阶无穷小量. 解上式得$E(r)\cdot\delta S+o\left(E(r)\cdot\delta S\right)=k\delta S/(2\epsilon_0)$，令$\delta S\to 0$得

$$E(r)=\frac{k}{2\epsilon_0}.$$

方向沿径向向外.

在球外，由高斯定理

$$4\pi R^2 E(R) = 4\pi r^2 E(r)$$

得$E(r) = kR^2/(2\epsilon_0 r^2)$，综上，电场强度大小分布

$$E(r) = \begin{cases} \dfrac{k}{2\epsilon_0} & ,0 \le r \le R, \\ \dfrac{kR^2}{2\epsilon_0 r^2} & ,r > R. \end{cases}$$

方向沿径向向外.

使用积分法求电势. 当$r \le R$时，

$$\phi(r) = \int_r^{+\infty} E(r)\mathrm{d}r = \frac{kR^2}{2\epsilon_0 r}.$$

当$r < R$时，

$$\phi(r) = \int_r^R E(r)\mathrm{d}r + \phi(R) = \frac{k}{2\epsilon_0}(2R - r).$$

综上，球内外电势分布

$$\phi(r) = \begin{cases} \dfrac{k}{2\epsilon_0}(2R - r) & ,0 \le r < R, \\ \dfrac{kR^2}{2\epsilon_0 r} & ,r > R. \end{cases}$$

6-13

解:

初态体系中的电势能

$$\begin{aligned} E_0 &= \frac{1}{4\pi\epsilon_0}\sum_{i<j}\frac{q_i q_j}{r_{ij}} \\ &= \frac{1}{4\pi\epsilon_0}\left(\frac{2q\cdot q}{R} + \frac{2q\cdot(-3q)}{2R} + \frac{(-3q)\cdot q}{R}\right) \\ &= -\frac{q^2}{\pi\epsilon_0 R}. \end{aligned}$$

同理，末态体系中的电势能

$$E_1 = \frac{q^2}{4\pi\epsilon_0 R}\left(\frac{2\cdot 1}{3} + \frac{2\cdot(-3)}{2} + \frac{-3\cdot 1}{1}\right) = -\frac{4q^2}{3\pi\epsilon_0 R}.$$

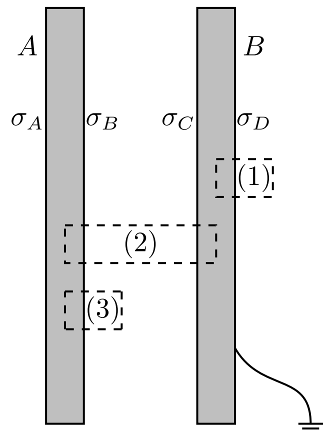
电场力做功

$$W = -\Delta E = E_0 - E_1 = \frac{q^2}{3\pi\epsilon_0 R}.$$

6-16

解:

对大平板A,B采用近似，A,B平板两侧的电荷面密度如图所示.



由于B板接地，B板右侧没有电场. 由高斯定理，

$$\sigma_D = 0 \tag{1}$$

$$\sigma_B + \sigma_C = 0 \tag{2}$$

由近似条件得BC间为匀强电场. 记BC间电场强度E向右为正，由高斯定理，

$$E = \frac{\sigma_B}{\epsilon_0} \tag{3}$$

由无穷大均匀带电平面的场强公式得

$$\frac{\sigma_A}{2\epsilon_0} + \frac{\sigma_B}{2\epsilon_0} - \frac{\sigma_C}{2\epsilon_0} = E \tag{4}$$

电荷守恒

$$\sigma_A + \sigma_B = \frac{Q_1}{S} \tag{5}$$

由(1)(2)(3)(4)(5)解得

$$\sigma_A = 0, \sigma_B = -\sigma_C = \frac{Q_1}{S}.$$

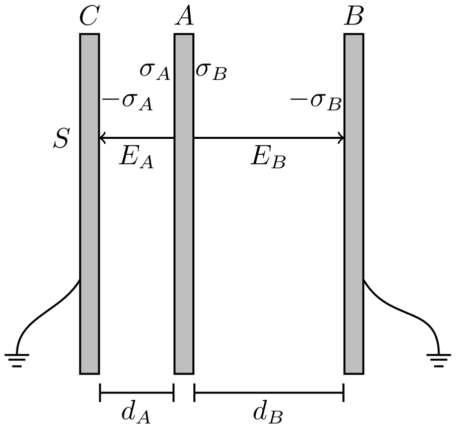
$$E = \frac{Q_1}{\epsilon_0 S},$$

方向由A板指向B板.

6-17

解:

(1)



B,C外侧无电场，由高斯定理可得B,C外侧不带电荷，且AC,AB之间相对的平面带电量之和为零.

由高斯定理可得平板间电场强度大小$E_A = \sigma_A/\epsilon_0$，$E_B = \sigma_B/\epsilon_0$. 又$\phi(C) = \phi(B) = 0$，故$\phi(A) = E_A d_A = E_B d_B$，即

$$\frac{E_A}{E_B} = \frac{\sigma_A}{\sigma_B} = \frac{d_B}{d_A}.$$

设平板面积为S，A板带电量为Q_A，由电荷守恒$(\sigma_A + \sigma_B)\, S = Q_A$得

$$\sigma_A S = \frac{d_B Q_A}{d_A + d_B}, \sigma_B S = \frac{d_A Q_A}{d_A + d_B}.$$

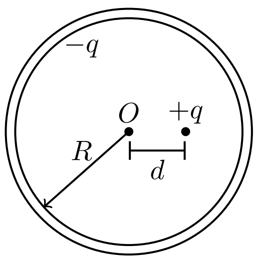
代入数值得C板带电量$Q_C = -\sigma_A S = -2.0 \times 10^{-7}\mathrm{C}$，$B$板带电量$Q_B = -\sigma_B S = -1.0 \times 10^{-7}\mathrm{C}$.

(2)

由(1)得$\phi(A) = E_A d_A = \sigma_A d_A/\epsilon_0 = -Q_C d_A/(\epsilon_0 S)$，代入数值得$\phi(A) = 2.26 \times 10^4\mathrm{V}$.

6-18

解:



由高斯定理得球壳接地后，球面外侧带电量为零，球面内侧不均匀带电，带电量$-q$，取消接地后球壳内侧的电荷分布及场强分布不变.取无穷远处为电势零点，O点电势

$$\begin{aligned}
\phi(O) &= \frac{q}{4\pi\epsilon_0 d} + \iint_S \frac{\mathrm{d}q}{4\pi\epsilon_0 R} \\
&= \frac{q}{4\pi\epsilon_0 d} + \frac{1}{4\pi\epsilon_0 R}\iint_S \mathrm{d}q \\
&= \frac{q}{4\pi\epsilon_0 d} - \frac{q}{4\pi\epsilon_0 R} \\
&= \frac{q}{4\pi\epsilon_0}\left(\frac{1}{d} - \frac{1}{R}\right).
\end{aligned}$$

6-21

解:

(1)

由电介质存在时的高斯定理,

① 当$r \geq R_3$时，有$4\pi\epsilon_0 r^2 E = Q_1 + Q_2$，即

$$E = \frac{Q_1 + Q_2}{4\pi\epsilon_0 r^2}(r > R_3).$$

② 当$0 \leq r < R_1$或$R_2 \leq r < R_3$时，有$E = 0$.

③ 当$R_1 \leq r < R_2$时，有$4\pi\epsilon_r\epsilon_0 r^2 E = Q_1$，即

$$E = \frac{Q_1}{4\pi\epsilon_r\epsilon_0 r^2}(R_1 \leq r < R_2).$$

综上，设径向单位矢量为$\hat{\mathbf{e}}_r$，空间电场强度分布

$$\mathbf{E}(\mathbf{r}) = \begin{cases} \dfrac{Q_1 + Q_2}{4\pi\epsilon_0 r^2} & , r \geq R_3, \\ 0 & , 0 \leq r < R_1 \text{ or } R_2 \leq r < R_3, \\ \dfrac{Q_1}{4\pi\epsilon_r\epsilon_0 r^2} & , R_1 \leq r < R_2. \end{cases}$$

(2)

电场能量密度

$$w = \frac{1}{2}\mathbf{D}\cdot\mathbf{E} = \frac{1}{2}\epsilon_r\epsilon_0 E^2 = \frac{Q_1^2}{32\pi^2\epsilon_r\epsilon_0 r^4}.$$

电场能量

$$\begin{aligned}
W_e &= \int_{R_1}^{R_2} w \cdot 4\pi r^2 \mathrm{d}r \\
&= \frac{Q_1^2}{8\pi\epsilon_r\epsilon_0}\int_{R_1}^{R_2}\frac{1}{r^2}\mathrm{d}r \\
&= \frac{Q_1^2}{8\pi\epsilon_r\epsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2}\right).
\end{aligned}$$

(3)

将数值代入(2)题答案，得

$$W_e = 5.99 \times 10^{-4}\ \mathrm{J}.$$

6-23

解：

先推导电容器的电容C.

设电容器带电Q，内极板带正电.由高斯定理得两极板之间距球心r处的电场强度

$$\mathbf{E} = \frac{Q}{4\pi\epsilon_0 r^2}\hat{\mathbf{e}_r}.$$

两极板间电势差

$$U = \int_{R_1}^{R_2} \frac{Q}{4\pi\epsilon_0 r^2}\mathrm{d}r = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2}\right).$$

电容

$$C = \frac{Q}{U} = \frac{4\pi\epsilon_0 R_1 R_2}{R_2 - R_1}.$$

电容器能量公式：

$$W_e = \frac{1}{2}C(\Delta U)^2 = \frac{2\pi\epsilon_0 R_1 R_2 (\Delta U)^2}{R_2 - R_1}.$$

电场能量公式：

电容带电量

$$Q = C\Delta U = \frac{4\pi\epsilon_0 R_1 R_2 \Delta U}{R_2 - R_1}.$$

电容器内部场强大小

$$E(r) = \frac{Q}{4\pi\epsilon_0 r^2}.$$

电场能量密度

$$w = \frac{1}{2}\mathbf{D}\cdot\mathbf{E} = \frac{1}{2}\epsilon_0\left[E(r)\right]^2 = \frac{Q^2}{32\pi^2\epsilon_0 r^4}.$$

电容器储存的电能

$$\begin{aligned} W_e &= \int_{R_1}^{R_2} w \cdot 4\pi r^2 \mathrm{d}r \\ &= \int_{R_1}^{R_2} \frac{Q^2}{8\pi\epsilon_0 r^2}\mathrm{d}r \\ &= \frac{Q^2}{8\pi\epsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \\ &= \frac{2\pi\epsilon_0 R_1 R_2 (\Delta U)^2}{R_2 - R_1}. \end{aligned}$$