A *character* of a torus $T = \mathrm{Spec}(\mathbb{C}[x_1^{\pm 1}, \ldots, x_n^{\pm 1}])$ is a map of $\mathbb{C}$-algebras $\mathbb{C}[t^{\pm 1}] \to \mathbb{C}[x_1^{\pm 1}, \ldots, x_n^{\pm 1}]$ which induces a group homomorphism.

Start with $n = 1$.

(1) What are the maps of $\mathbb{C}$-algebras $\mathbb{C}[t^{\pm 1}] \to \mathbb{C}[x^{\pm 1}]$?

(2) Show which of your maps give group homomorphisms $\mathbb{C}^* \to \mathbb{C}^*$.

(3) Now show that if $(\mathbb{C}^*)^n \to \mathbb{C}^*$ is a character of T then composing with the inclusion $\mathbb{C}^* \to (\mathbb{C}^*)^n$ gives a character for each coordinate of $(\mathbb{C}^*)^n$.

(4) Construct a bijection between the characters of $(\mathbb{C}^*)^n$ and the elements of $\mathbb{Z}^n$.