Game Tree Searching by Min/Max Approximation

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This paper introduces a new technique for searching game trees. According to the author "the key idea is to approximate the 'min' and 'max' operators with *generalized mean-value* operators". Ronald says that these are good approximations to min/max operators, which have continuous derivatives with respect to all arguments, and by taking the derivatives of the generalized mean value functions at each node and using the chain rule we can identify the leaf upon whose value the value of the root depends the most strongly, and then expand this leaf next.

Iterative heuristics

This is a technique that “grow” the search tree one step at time. This is how the process generally follows:

> *Step 1.* Initialize the partial game tree (*E*) and its current value at the root
> *Step 2.* While the tree is not completed or we have time:
>
> A. Pick an expandable tip (*c*) of *E*
> B. Expand *E* at tip *c*
> C. Update the value at c and back up the value to the root.

The main question here, and the purpose of this paper, is to define which expandable tip to pick.

Penalty-based search

This search defines the “penalty” *P(c)* of a tip ***c*** to be the sum of the penalties of all the edges between c and the root s. The idea is to expand the the tip node ***t*** which has the least penalty *P(t)*. The ***t***'s children is added to the tree, the estimate value for every ancestor of ***t*** is updated, along with the penalties on the edges between the ancestors and their children.

Searching by min/max approximation

The “min/max approximation” heuristic is a special case of the penalty-based search method, where the penalties are defined as the derivatives of the approximation functions.

Results

Finally, applying the min/max approximation (MM) to the game Connect-Four and playing against the alpha-beta pruning (AB) technique, it was found that the when resource bound per turn was based on time AB seemed superior than the MM approach. However, when the comparison was based on the moves bound per turn the MM showed better results.