

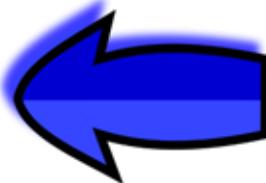


# **Chapter 2: The Language of Bits**

## **Basic Computer Architecture**

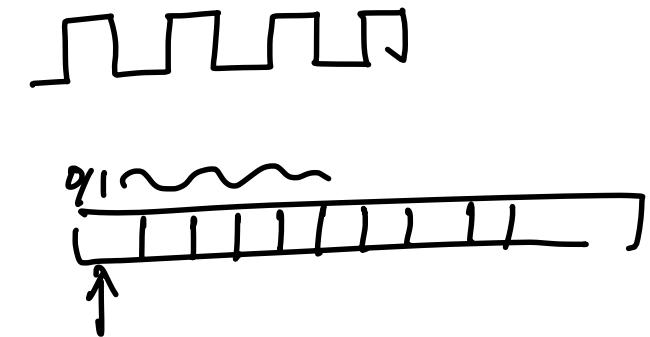
# Outline

- \* Boolean Algebra
- \* Positive Integers
- \* Negative Integers
- \* Floating-Point Numbers
- \* Strings



# What does a Computer Understand ?

- \* Computers do not understand natural human languages, nor programming languages
- \* They only understand the language of **bits**



Bit	0 or 1
Byte	8 bits
Word	4 bytes
kiloByte	1024 bytes
megaByte	$10^6$ bytes

$$1 \text{ KA} = 10^3 \text{ gm}$$
$$1 \text{ KB} = 2^{10} \text{ bite}$$
$$\downarrow 1024$$
$$\downarrow 10^3$$
$$\text{bit} \downarrow 10^3$$

$$\Rightarrow 2^{10} \times 2^{10} \\ 10^3 \times 10^3 = 10^6 \\ 2^{10} \times 2^{10}$$

# Review of Logical Operations

OR

$$* A + B \text{ (A or B)}$$

$A \text{ OR } B$   
 $A \text{ AND } B$

O/I

A	B	A + B	O/I
0	0	0	
1	0	1	
0	1	1	
1	1	1	

= Truth Table

AND

$$* A.B \text{ (A and B)}$$

$A \text{ AND } B$

NOT

A	B	A.B
0	0	0
1	0	0
0	1	0
1	1	1

# Review of Logical Operations - II

A	B	A NAND B
0	0	1
1	0	1
0	1	1
1	1	0

A	B	A NOR B
0	0	1
1	0	0
0	1	0
1	1	0

$\bar{A}$   
 $\sim A$   
 $A!$

- \* NAND and NOR operations
- \* These are **universal operations**. They can be used to implement any Boolean function.

# Review of Logical Operations

\* XOR Operation :  $(A \oplus B)$

A	B	A XOR B
0	0	0
1	0	1
0	1	1
1	1	0

How many truth tables we can build?

# Review of Logical Operations

## ✓ \* NOT operator

- \* Definition:  $\overline{0} = 1$ , and  $\overline{1} = 0$
- \* Double negation:  $\overline{\overline{A}} = A$ , NOT of (NOT of A) is equal to A itself

## \* OR and AND operators

- \* Identity:  $A + \underline{0} = A$ , and  $A.\underline{1} = A$
- \* Annulment:  $A + 1 = 1$ ,  $A.0 = 0$

- \* **Idempotence**:  $A + A = A$ ,  $A \cdot A = A$ , The result of computing the OR and AND of A with itself is A.
- \* **Complementarity**:  $\underline{A} + \underline{\overline{A}} = 1$ ,  $A \cdot \overline{A} = 0$
- \* **Commutativity**:  $A + B = B + A$ ,  $A \cdot B = B \cdot A$ , the order of Boolean variables does not matter
- \* **Associativity**:  $A + (B + C) = (A + B) + C$ ,  $A \cdot (B \cdot C) = (A \cdot B) \cdot C$ , similar to addition and multiplication.
- \* **Distributivity**:  $A \cdot (B + C) = \underline{A} \cdot B + \underline{A} \cdot C$ ,  $A + (B \cdot C) = (A + B) \cdot (A + C)$  → Use this law to open up parentheses and simplify expressions

$$x(y+z) = xy + xz$$

$$A + B$$

$$A + (B + C)$$

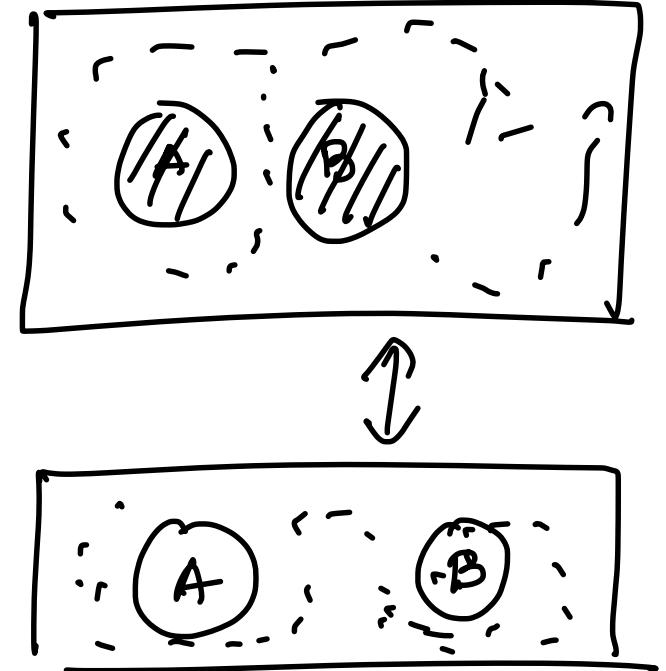
$$(A+B) + C$$

# De Morgan's Laws

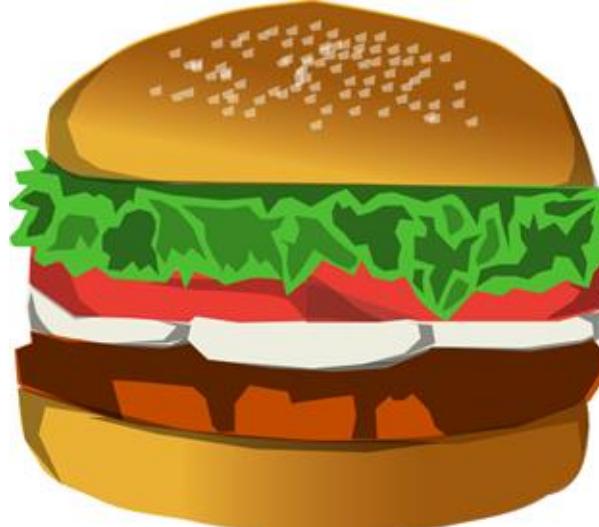
- \* Two very useful rules

$$\neg \overline{A + B} = \overline{\overline{A} \cdot \overline{B}}$$

$$\neg \overline{A \cdot B} = \overline{\overline{A}} + \overline{\overline{B}}$$



# Consensus Theorem



\* Prove :

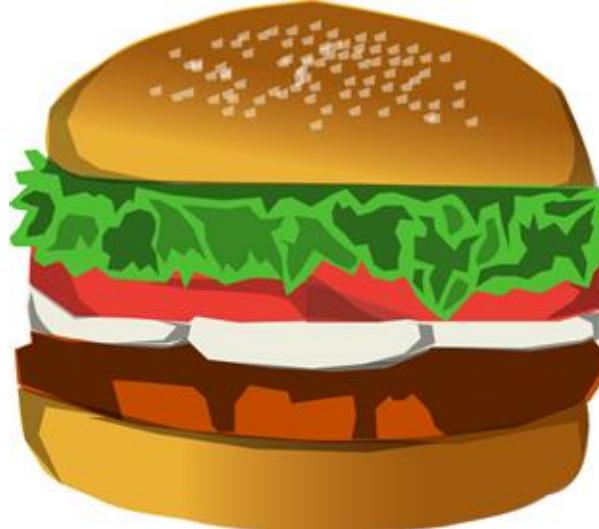
$$* \quad \boxed{X.Y + \bar{X}.Z + Y.Z = X.Y + \bar{X}.Z} \quad -$$

$$X.Y + \bar{X}Z + YZ (X + \bar{X})$$

$$\overbrace{XY}^{\leftarrow} + \overbrace{\bar{X}Z}^{\leftarrow} + \overbrace{YZ}^{\leftarrow} + \overbrace{\bar{X}YZ}^{\leftarrow}$$

$$XY(1+Z) + \bar{X}Z(1+\bar{Y}) = XY + \bar{X}Z$$

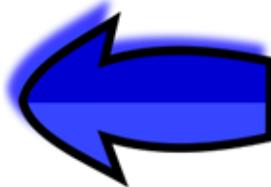
# Consensus Theorem



- \* Prove :
  - \*  $X.Y + \bar{X}.Z + Y.Z = X.Y + \bar{X}.Z$

# Outline

- \* Boolean Algebra
- \* Positive Integers
- \* Negative Integers
- \* Floating Point Numbers
- \* Strings



# Representing Positive Integers

## \* Ancient Roman System

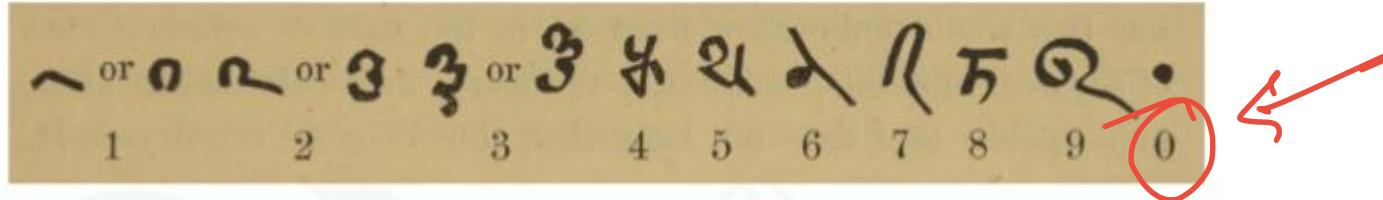
Symbol	I	V	X	L	C	D	M
Value	1	5	10	50	100	500	1000

place-value representation

## \* Issues :

- \* There was no notion of 0
- \* Very difficult to represent large numbers
- \* Addition, and subtraction (**very difficult**)

# Indian System (place -value system)



Bakshali numerals, 7<sup>th</sup> century AD

- \* Uses the place value system

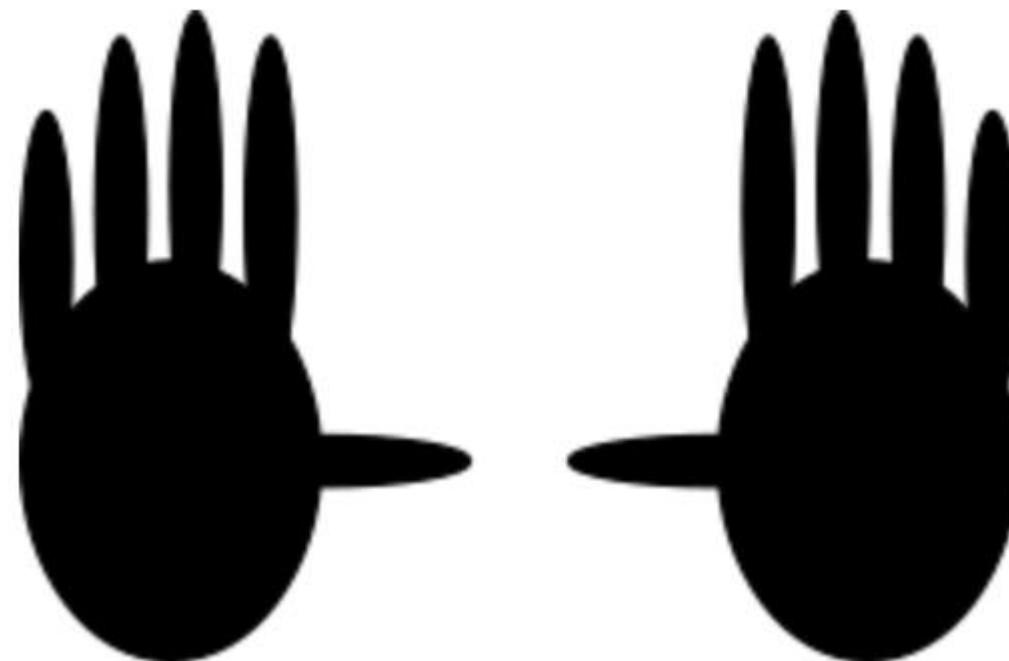
$$5301 = \underline{5} * \underline{10^3} + 3 * 10^2 + 0 * 10^1 + 1 * 10^0$$

Example in base 10

5301  
= place value.  
base value

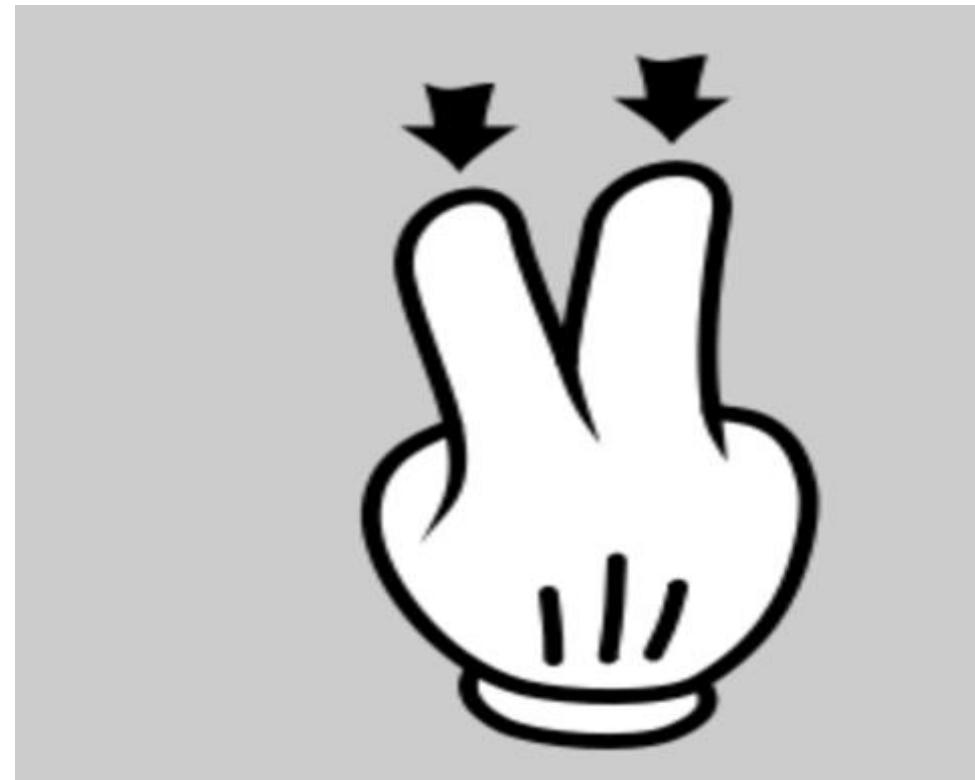
# Number Systems in Other Bases

- \* Why do we use base 10 ?
  - \* because ...



# What if we had a world in which ...

- \* People had only two fingers.



# Binary Number System

- \* They would use a number system with base 2.

Number in decimal	Number in binary
5	101
100	1100100
500	111110100
1024	10000000000

19  
= 10011  
↑  
MSB      LSB  
least significant bit

$$\begin{array}{l} \begin{array}{r} 2^4 \\ 2^3 \\ 2^2 \\ 2^1 \\ 2^0 \end{array} \end{array} \begin{array}{l} 1 \\ 1 \\ 0 \\ 1 \\ 0 \end{array} = \underline{\underline{101}}$$
$$= 10^0 \times 5 + 10^1 \times 1 + 10^2 \times 0 + 10^3 \times 1 + 10^4 \times 1$$

base value = 2  
place value starts from 0

# MSB and LSB

- \* **MSB (Most Significant Bit)** → The leftmost bit of a binary number. E.g., MSB of 1110 is 1
- \* **LSB (Least Significant Bit)** → The rightmost bit of a binary number. E.g., LSB of 1110 is 0

# Hexadecimal and Octal Numbers

- \* Hexadecimal numbers

- \* Base 16 numbers – 0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F
- \* Start with 0x

- \* Octal Numbers = base=8

- \* Base 8 numbers – 0,1,2,3,4,5,6,7
- \* Start with 0

1 Why Octal given that we have Hex a ?

0	Base = 16
1	
2	
3	
4	
5	
6	
7	
8	
9	
- A -	10
- B -	11
- C -	12
- D -	13
- E -	14
- F -	15

# Examples

Convert 110010111 to the octal format :  $\frac{110}{\underline{\quad}} \frac{010}{\underline{\quad}} \frac{111}{\underline{\quad}} = 0627$

Convert 11100010111 to the hex format :  $\frac{1110}{\underline{\quad}} \frac{0010}{\underline{\quad}} \frac{1111}{\underline{\quad}} = 0xE2F$

$$\begin{aligned} N &= a_0 a_1 a_2 a_3 a_4 a_5 a_6 a_7 \\ &= \underbrace{(a_0 \times 2^0 + a_1 \times 2^1 + a_2 \times 2^2)}_{2^0} + 2^3 (a_3 \times 2^0 + a_4 \times 2^1 + a_5 \times 2^2) + 2^6 (a_6 \times 2^0 + a_7 \times 2^1 + a_8 \times 2^2) \\ &= 2^0 (\underbrace{a_0}_{b_0} - \overline{f}) + 2^3 (\underbrace{a_3}_{b_1} - \overline{f}) + (2^3)^2 (\underbrace{a_6}_{b_2} - \overline{f}) + \dots \\ &= 8^0 b_0 + 8^1 b_1 + 8^2 b_2 8 + \dots \end{aligned}$$

# Examples

$$(x)_2 \rightarrow (y)_5$$

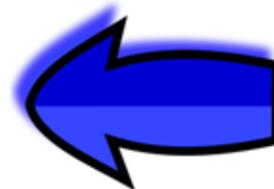
Convert 110010111 to the octal format :  $\underbrace{110}_{} \underbrace{010}_{} \underbrace{111}_{} = 0627$

Convert 11100010111 to the hex format :  $\underbrace{1110}_{} \underbrace{0010}_{} \underbrace{1111}_{} = \text{0xE2F}$

2. Can we convert a  $2^n$ -base system to a base system which can't be represented by  $2^m$ ; e.g.; binary  $\rightarrow (y)_{\cancel{2}}$  without converting to decimal

# Outline

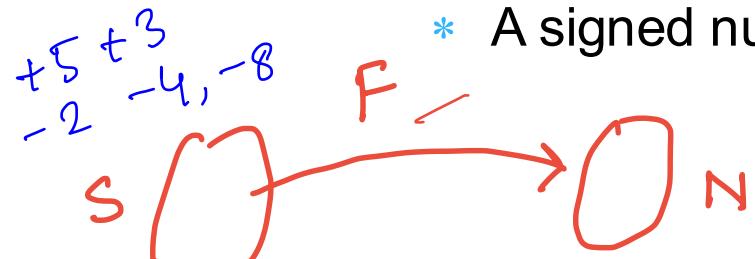
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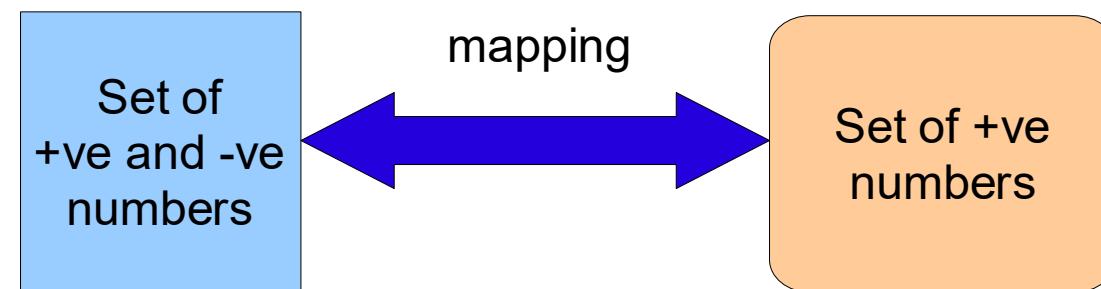
# Representing Negative Integers

## \* Problem

- \* Assign a **binary representation** to a **negative integer**
- \* Consider a negative integer, S
- \* Let its binary representation be :  $x_nx_{n-1}\dots x_2x_1$   
 $(x_i=0/1)$
- \* We can also expand it to represent an unsigned,  
+ve, number, N
- \* If we interpret the binary sequence as :
  - \* An unsigned number, **we get N**
  - \* A signed number, **we get S**



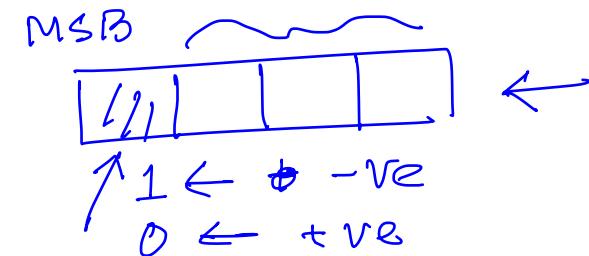
- \* We need a mapping :
  - \*  $F : S \rightarrow N$  (mapping function)
  - \*  $S \rightarrow$  set of numbers (both positive and negative – signed)
  - \*  $N \rightarrow$  set of positive numbers (unsigned)



# Properties of the Mapping Function

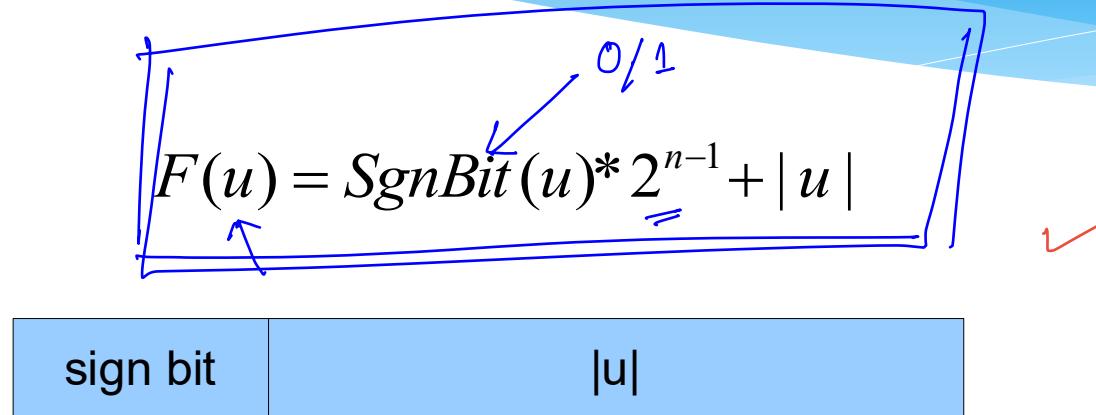
- \* Preferably, needs to be a one to one mapping
- \* All the entries in the set, S, need to be mapped
- \* It should be easy to perform addition and subtraction operations on the representation of signed numbers
- \* Assume an n bit number system

$$[-5] = 5$$



$$\text{SgnBit}(u) = \begin{cases} 1, & u < 0 \\ 0, & u \geq 0 \end{cases}$$

# Sign-Magnitude Base Representation



✓

$$\begin{array}{c} -5 \\ \hline \boxed{1|1|0|1} \\ \uparrow \quad \text{---} \quad |5| \\ 1 \times 2^3 \end{array} \quad n=4 \quad n-1=3 \Rightarrow 8 \quad \rightarrow 13$$

$$+4 \quad \boxed{0|1|0|0} = 4$$

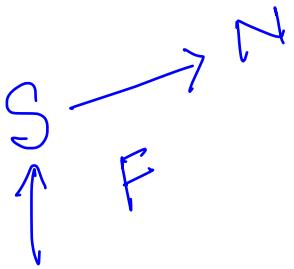
- \* Examples :
  - \* -5 in a 4 bit number system : 1101
  - \* 5 in a 4 bit number system : 0101
  - \* -3 in a 4 bit number system : 1011

# Problems

- \* There are two representations for 0
  - \* 000000
  - \* 100000
- \* Addition and subtraction are difficult ↴
- \* The most important takeaway point :
  - \* Notion of the sign bit



# 1's Complement Representation



$$F(u) = \begin{cases} u, & u \geq 0 \\ \sim(|u|) \text{ or } (2^n - 1 - |u|), & u < 0 \end{cases}$$

$n = 4$   
 $2^4 = 16$   
 $2^4 - 1 = 15$

Notion of sign bit also exists

$$\begin{aligned} f(-3) &= 2^4 - 1 - |-3| \\ &= 15 - 3 \\ &= 12 \\ &= 1100 \end{aligned}$$

- \*  $3 \rightarrow 0011$
- \*  $-3 \rightarrow \cancel{1100} \Rightarrow 12$
- \*  $5 \rightarrow 0101$
- \*  $-5 \rightarrow 1010$
- Annotations in blue:
- $x^3 = 0011$
  - $\sim 3 = \cancel{1100}$
  - $\sim 12 = 1010$

# Problems

- \* Two representations for 0

$$\begin{array}{r} 0 \\ 3 \\ + 2 \\ \hline 5 \end{array}$$

$0 = 000000$   
 $3 = 001111$   
 $+ 2 = 001010$   
 $\hline 5 = 0101$

- \*  $0000000$  {  
\*  $1111111$

- \* Easy to add +ve numbers

- \* Hard to add -ve numbers

- \* Point to note :

- \* The idea of a complement



$$\begin{array}{r} 3 + (-2) = 1 \\ 0011 \\ + 1101 \\ \hline 10000 \\ \xrightarrow{-1} 0001 \end{array}$$

# Bias Based Approach

$$F(u) = u + \text{bias}$$

- \* Consider a 4 bit number system with bias equal to 7

- \*  $-3 \rightarrow 0100$

$$-3 = -3 + 7 = +4$$

- \*  $3 \rightarrow 1010$

$$+2 = +2 + 7 = +9$$

- \*  $F(u+v) = F(u) + F(v) - \text{bias}$

- \* Add and Sub are also easy

$$\begin{array}{r} +3 \\ -3 \\ \hline \end{array} = \begin{array}{r} +3 + 7 = 10 \\ -3 + 7 = 4 \\ \hline \end{array} \quad \begin{array}{r} 1010 \\ -0100 \\ \hline 1110 \end{array}$$

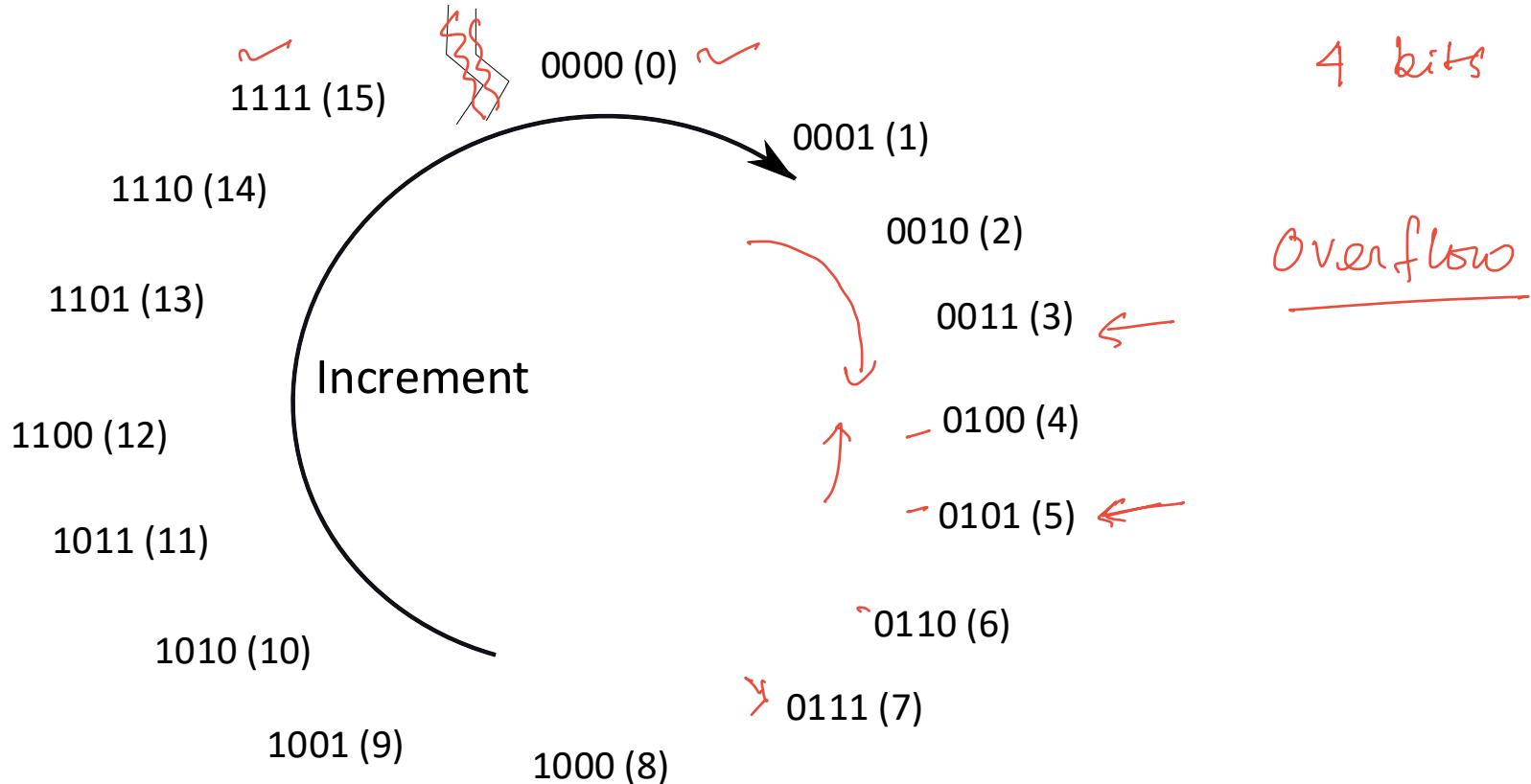
- \* Multiplication is difficult

$$F(u-v) = F(u) - F(v) + \text{bias}.$$

$$F(3-3) = F(3) - F(3) + 7 = 7 = 0$$

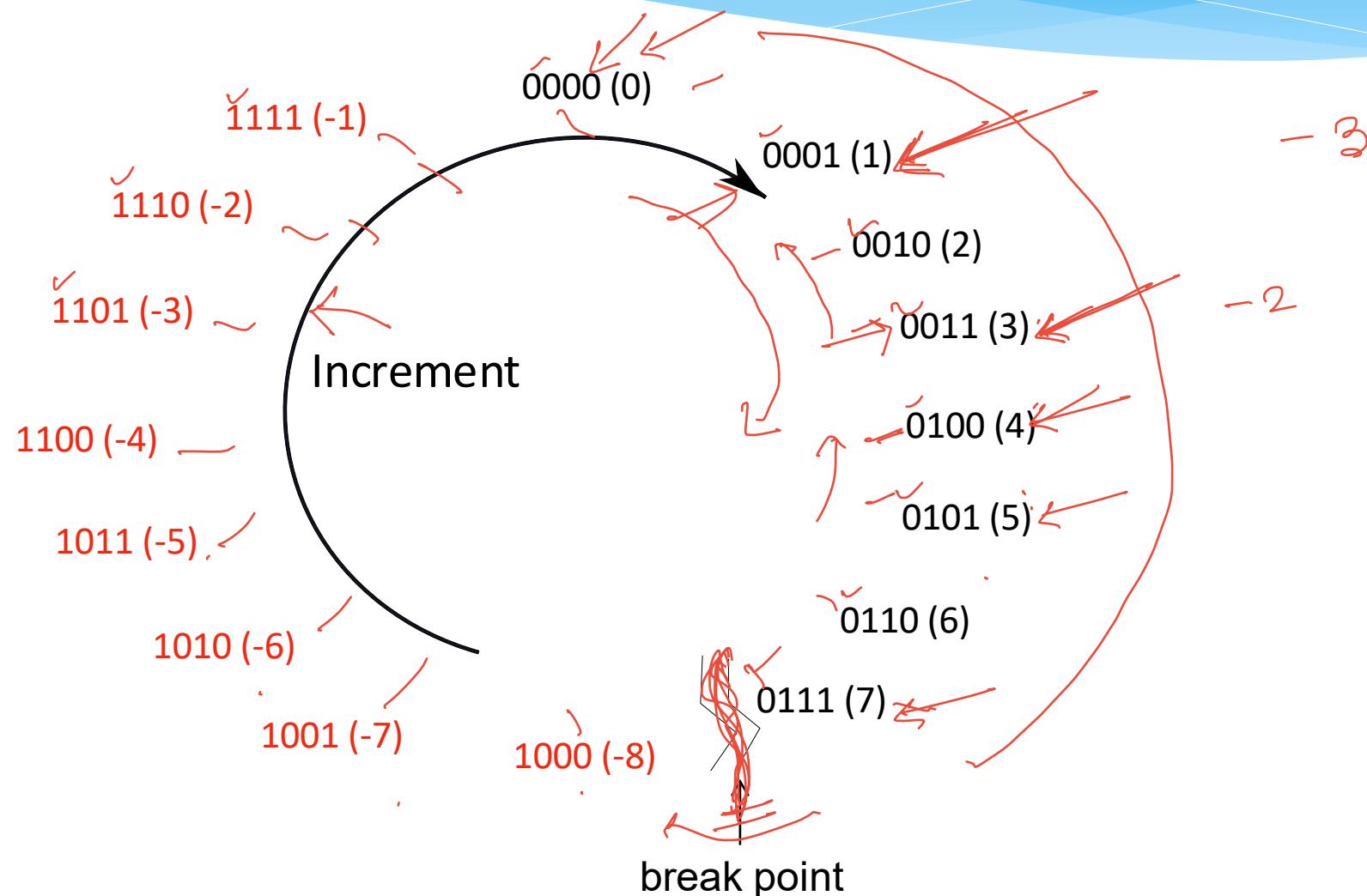
$$\begin{array}{r} 14 \\ -7 \\ \hline 7 \end{array}$$

# The Number Circle



Clockwise: increment  
Anti-clockwise: decrement

# Number Circle with Negative Numbers



# Using the Number Circle

- \* To add  $\underline{M}$  to a number,  $\underline{N}$

- \* locate  $\underline{N}$  on the number circle

- \* If  $\underline{M}$  is +ve

- \* Move M steps clockwise

- \* If  $\underline{M}$  is -ve

- \* Move M steps anti-clockwise, or  $2^n - \underline{M}$  steps clockwise

- \* If we cross the break-point

- \* We have an overflow / underflow

- \* The number is too large/ too small to be represented

2  
 $n = 16$

# 2's Complement Notation

$$F(u) = \begin{cases} u, & 0 \leq u \leq 2^{n-1} - 1 \\ 2^n - |u|, & -2^{n-1} \leq u < 0 \end{cases}$$

$$\begin{aligned} n &= 4 \\ 2^{3-1} &= 2^{n-1} = 8 \\ -2^{n-1} &= -8 \end{aligned}$$

- \*  $F(u)$  is the index of a point on the **number circle**. It varies from 0 to  $2^n - 1$
- \* Examples

$$* \rightarrow 4 \rightarrow \underline{0100}$$

$$* \rightarrow -4 \rightarrow \underline{1100}$$

$$* \rightarrow 5 \rightarrow \underline{0101}$$

$$* \rightarrow -3 \rightarrow \underline{1101}$$

$$\begin{aligned} -8 &\rightarrow \underline{\textcircled{0}} \\ -7 &\rightarrow \underline{1101} \\ -6 &\rightarrow \underline{1110} \\ -5 &\rightarrow \underline{1111} \\ -4 &\equiv 14 \\ -3 &\equiv 13 \end{aligned}$$

# Properties of the 2's Complement Notation

- \* Range of the number system :
  - \*  $-2^{(n-1)}$  to  $2^{n-1} - 1$  ✓  
- 8      + 7
- \* There is a unique representation for 0 → 000000
- \* msb of F(u) is equal to SgnBit(u)
  - \* Refer to the number circle
  - \* For a +ve number,  $F(u) < 2^{(n-1)}$ . MSB = 0
  - \* For a -ve number,  $F(u) \geq 2^{(n-1)}$ . MSB = 1

# Properties - II

- \* Every number in the range  $[-2^{(n-1)}, 2^{(n-1)} - 1]$

- \* Has a unique mapping

- \* Unique point in the number circle

$$a \equiv b \rightarrow (a = b \bmod 2^n)$$

$a - b \bmod 2^n = 0 \quad n = 4$

$+2 \equiv +18$

\*  $\equiv$  means same point on the number circle

- \*  $F(-u) \equiv 2^n - F(u)$

- \* Moving  $F(u)$  steps counter clock wise is the same as moving  $2^n - F(u)$  steps clockwise from 0

# Prove : $F(u+v) \equiv F(u) + F(v)$

\* Start at point  $u$

- \* Its index is  $F(u)$
- \* If  $v$  is +ve,
  - \* move  $v$  points clockwise. We arrive at  $F(u+v)$ .
  - \* Its index is equal to  $(F(u) + v) \bmod 2^n$ .
  - \* Since  $v = F(v)$ , we have  $F(u+v) = (F(u) + F(v)) \bmod 2^n$

$$u = 2$$

$$v = 3$$

$$\begin{aligned}F(2+3) &= F(2) + F(3) \\F(5) &= F(2) + F(3) \\&= 0010 + 0011 \\&= 0101\end{aligned}$$

# Prove : $F(u+v) \equiv F(u) + F(v)$



- \* If v is -ve,
  - \* move  $|v|$  points anti-clockwise.
  - \* Same as moving  $2^n - |v|$  points clockwise.
  - \* We arrive at  $F(u+v)$ . ✓
  - \*  $F(v) = 2^n - |v|$
  - \* The index –  $F(u+v)$  – is equal to:
    - \*  $(F(u) + 2^n - |v|) \text{ mod } 2^n = (F(u) + F(v)) \text{ mod } 2^n$

$$\begin{aligned} u &= 2 \\ v &= -4 \\ F(2+(-4)) &= F(2) + F(-4) \\ &= 0010 + 1100 \\ &= 1110 \\ &= F(-2) \end{aligned}$$

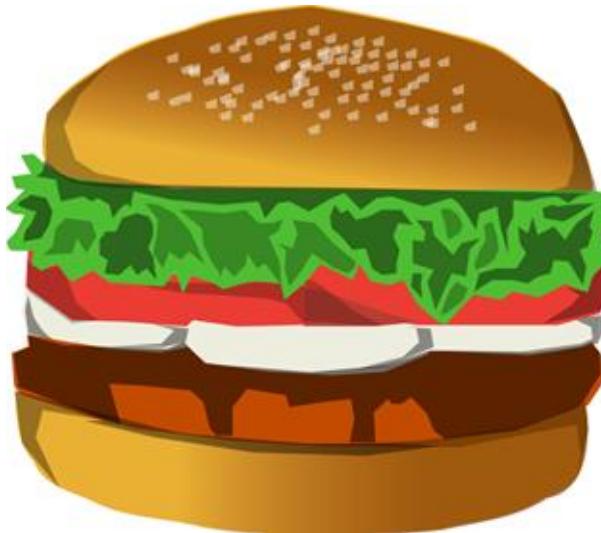
# Subtraction

- \*  $F(u-v) \equiv F(u) + F(-v)$   
 $\equiv F(u) + 2^n - F(v)$

$$\begin{aligned}F(3-5) &= F(3) + F(-5) \\&= 3 + 16 - 5 \\&= 14 = F(-2)\end{aligned}$$

- \* Subtraction is the same as addition
- \* Compute the 2's complement of  $F(v)$

# Prove that :



\* Prove that :

$$F(u^*v) \equiv F(u) * F(v)$$

$$u = 2, v = -3$$

$$\begin{aligned} & F(2 \times (-3)) \\ & \equiv F(2) \times F(-3) \\ & = 2 \times 13 \\ & = 26 \bmod 16 \\ & \Rightarrow 10 = F(-6) \end{aligned}$$

① if  $u$  and  $v$  are +ve, it is trivial.

② if  $u$  and  $v$  are -ve,  $u = -|u|$   $v = -|v|$

$$F(u) \times F(v) = \frac{(2^n - F(|u|)) \times (2^n - F(|v|))}{(2^n - F(|u|) + F(|v|))}$$

$$= \cancel{2^{2n}} - \cancel{2^n} (F(|u|) + F(|v|)) + F(|u|) \times F(|v|)$$

$$\approx \emptyset F(|u|) \times F(|v|)$$

$$= F(|u| \times |v|) = F(u \times v)$$

③  $u$  is +ve

$v$  is -ve

$$u = |u|$$

$$v = -|v|$$

$$F(u) \times F(v) = F(u) \times (2^n - F(|v|))$$

$$= 2^n \cancel{F(u)} - F(u) \times F(|v|)$$

$$= -F(u) \times F(|v|)$$

$$= -F(u \times |v|) \quad u > 0 \quad |v| > 0$$

$$F(-u) = 2^n - F(|u|)$$

negative rule

$$= 2^n - F(u \times |v|)$$

$$= F(-(u \times |v|))$$

$$= F(u \times \underline{(-|v|)}) = F(u \times v)$$

# Computing the 2's Complement

\*  $2^n - u$

$$= \cancel{2^n} - 1 - u + 1$$

$$= \cancel{-u} + 1$$

\*  $\sim u$  (1's complement)

$$2^m - 1$$

\* 1's complement of 0100

$$\begin{array}{r} & 1111 \\ - & 0100 \\ \hline & 1011 \end{array}$$

2's complement of  
0100

$$\begin{array}{r} 1011 \\ + 0001 \\ \hline 1100 \end{array}$$

# Sign Extension

- \* Convert a n bit number to a m bit 2's complement number ( $m > n$ )

$$\begin{array}{r} +4 = 0100 \\ \swarrow \\ 0000\ 0100 \end{array}$$

- \* +ve

- \* Add  $(m-n)$  0s in the msb positions
- \* Example, convert 0100 to 8 bits  $\rightarrow$  0000 0100

$$\begin{array}{r} 1100 \\ \swarrow \\ [1111\ 1100] = -282 \end{array}$$

- \* -ve

- \*  $F(u) = \underline{2^n - |u|}$  ( $n$  bit number) system
- \* Need to calculate  $F'(u) = \underline{\underline{2^m - |u|}}$

# Sign Extension - II

$m > n$

$$= 2^m + 2^{n+1} + \dots + 2^{m-1}$$

$$2^m - 2^n$$

$$= 2^m (2^{m-n-1})$$

$$2^{m-n-1} = 0 + 2 + 2^2 + \dots + 2^{n-1}$$

$$= 2^m (1 + 2 + 2^2 + \dots + 2^{m-n-1})$$

$$= 2^m (2 + 2^{n+1} + \dots + 2^{m-1})$$

$$\begin{aligned} * & \quad \underline{2^m - u - (2^n - u)} \\ &= \boxed{2^m - 2^n} \\ &= 2^n + 2^{(n+1)} + \dots + 2^{(m-1)} \\ &= \boxed{11110000} + (-2) \end{aligned}$$

$m-n$        $n$

$m = 4 \quad n = 2$

$$2^2 + 2^3$$

$$2^4 - 2^2$$

$$= 16 - 4$$

$$= 12$$

$$= 2^2 + 2^3$$

$$4 - 8$$

$$\begin{array}{r} 1111\ 0000 \\ + 1100 \\ \hline 1111\ 0100 \end{array}$$

$F'(u) = F(u) + 2^m - 2^n$

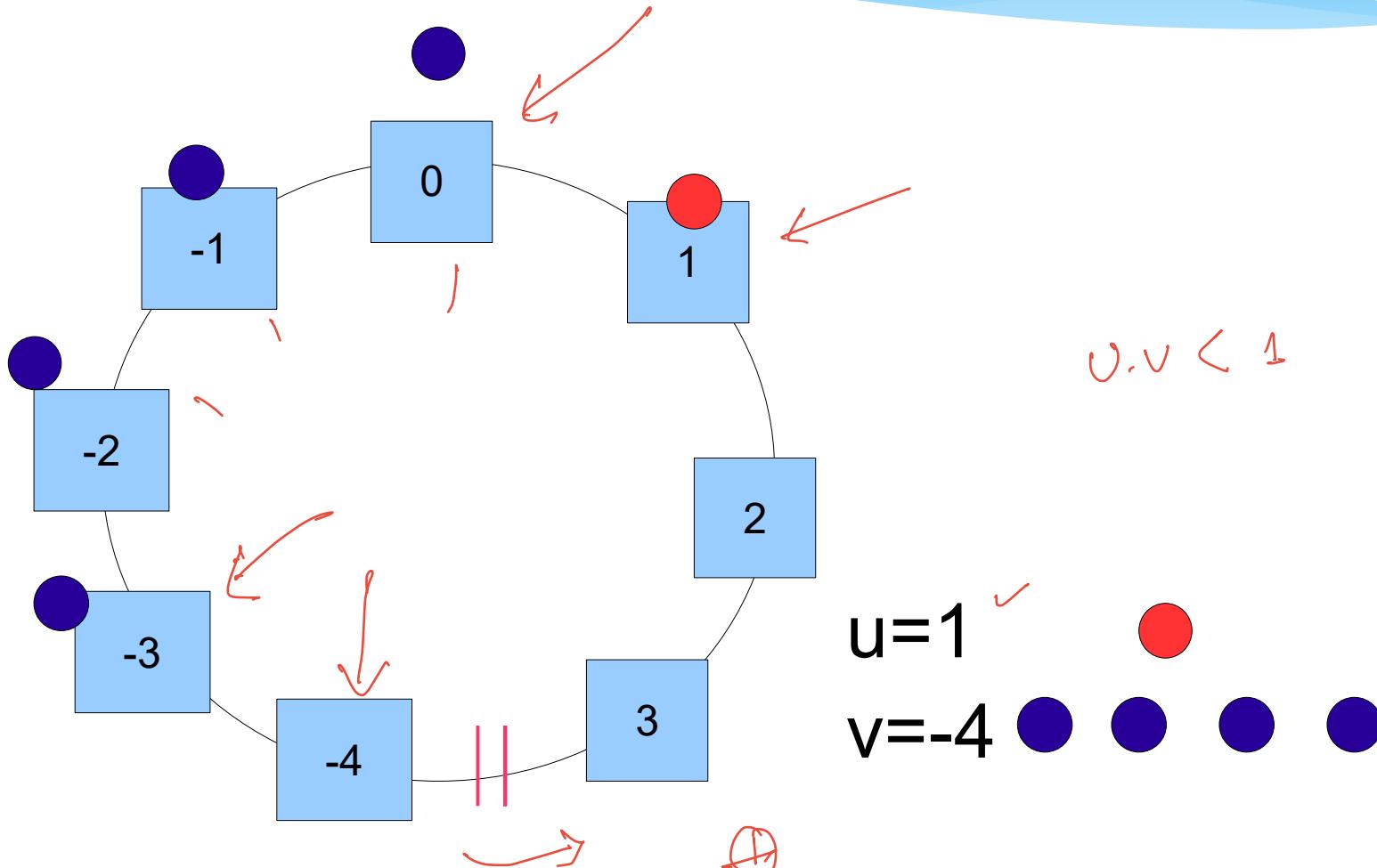
# Sign Extension - III

- \* To convert a negative number :
  - \* Add  $(m-n)$  1s in the msb positions
- \* In both cases, extend the sign bit by :
  - \*  $(m-n)$  positions

# The Overflow Theorem

- \* Add :  $u + v$
- \* If  $\underline{uv < 0}$ , there will **never be an overflow**
- \* Let us go back to the number circle
  - \* There is an overflow only when we cross the break-point
  - \* If  $\underline{uv = 0}$ , one of the numbers is 0 (no overflow)
  - \* If  $\underline{uv > 0}$ , an **overflow is possible**

# Number Circle: $uv < 0$

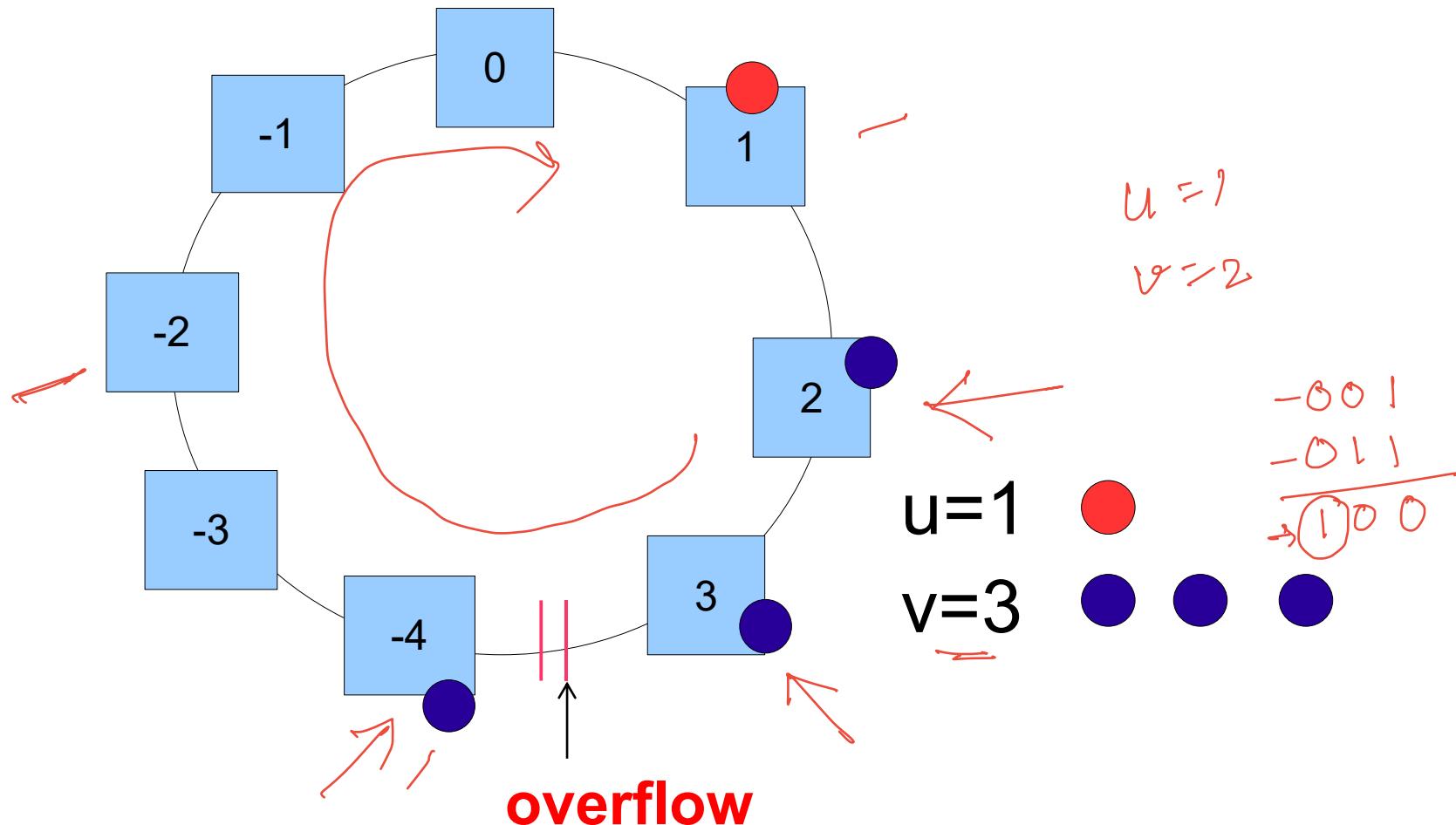


$$u=1$$

$$v=-4$$

( $\textcircled{1}$ ) both the numbers are positive,  
( $\textcircled{2}$ ) and the resulting number is.  
-ve and there is no overflow

# Number Circle: $uv > 0$

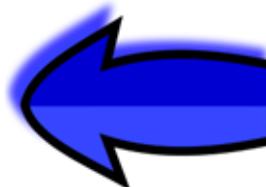


# Conditions for an Overflow

- \*  $uv \leq 0$ 
  - \* Never
- \*  $\underline{uv > 0}$  ( u and v have the same sign)
  - \* The sign of the result is different from the sign of u 

# Outline

- \* Boolean Algebra
- \* Positive Integers
- \* Negative Integers
- \* Floating-Point Numbers
- \* Strings



# Floating-Point Numbers

\* What is a floating-point number ?

- \* 2.356
- \* 1.3e-10
- \* -2.3e+5

\* What is a fixed-point number ?

- \* Number of digits after the decimal point is fixed
- \* 3.29, -1.83

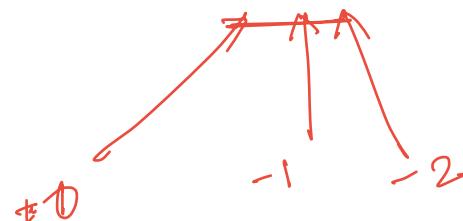
# Generic Form for Positive Numbers

- \* Generic form of a number in base 10

$$A = \sum_{i=-n}^n x_i 10^i$$

- \* Example :

- \*  $3.29 = 3 * 10^0 + 2 * 10^{-1} + 9 * 10^{-2}$



# Generic Form in Base 2

- \* Generic form of a number in base 2

$$A = \sum_{i=-n}^n x_i 2^i$$

Number	Expansion
0.375	$2^{-2} + 2^{-3}$
1	$2^0$
1.5	$2^0 + 2^{-1}$
2.75	$2^1 + 2^{-1} + 2^{-2}$
17.625	$2^4 + 2^0 + 2^{-1} + 2^{-3}$

$$\begin{aligned} 0.375 \times 2 &= 0.750 \\ 0.750 \times 2 &= 1.5 \\ 1.5 \times 2 &= 1.0 \quad \checkmark \\ &\quad \text{↓} \\ &\quad \underline{0.11} \end{aligned}$$

# Binary Representation

- \* Take the base 2 representation of a floating-point (FP) number
- \* Each coefficient is a binary digit

Number	Expansion	Binary Representation
0.375	$2^{-2} + 2^{-3}$	0.011
1	$2^0$	1.0
1.5	$2^0 + 2^{-1}$	1.1
2.75	$2^1 + 2^{-1} + 2^{-2}$	10.11
17.625	$2^4 + 2^0 + 2^{-1} + 2^{-3}$	10001.101

# Normalized Form

- \* Let us create a standard form of all floating point numbers

$$A = (-1)^S * P * 2^X, (P = 1 + M, 0 \leq M < 1, X \in Z)$$


\*  $\text{S} \rightarrow \text{sign bit}$ ,  $P \rightarrow \text{significand}$        $1 \leq P < 2$

- \*  $M \rightarrow \text{mantissa}$ ,  $X \rightarrow \text{exponent}$ ,  $Z \rightarrow \text{set of integers}$

$$A = 2 \cdot 6$$

$$(-1)^0 \times 1 \cdot 3 \times 2^1$$
$$P = 1 \cdot 3 \quad Z = 1$$
$$M = -3$$

# Examples (in decimal)

\*  $1.3827 * 1e-23$

\* Significand (P) = 1.3827

\* Mantissa (M) = 0.3827

\* Exponent (X) = -23

\* Sign (S) = 0 ✓

\*  $-1.2 * 1e+5$

\* P = 1.2 , M = 0.2

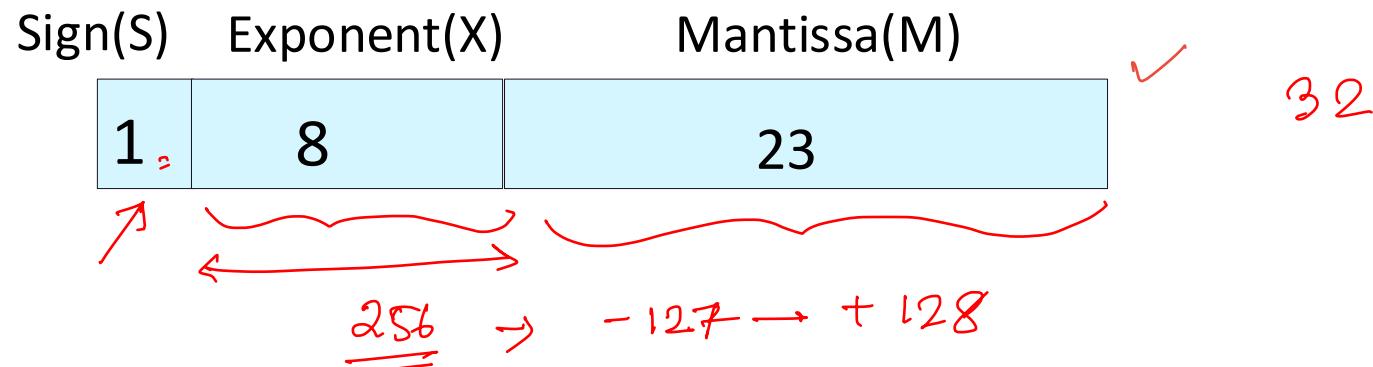
\* S = 1, X = 5

# IEEE 754 Format

## \* General Principles

- \* The **significand** is of the form : 1.xxxxx
- \* No need to waste 1 bit representing (1.) in the significand
- \* We can just save the **mantissa** bits
- \* Need to also store the sign bit (S), exponent (X)

# IEEE 754 Format - II



- \* sign bit – 0 (+ve), 1 (-ve)
  - \* exponent, 8 bits
  - \* mantissa, 23 bits

# Representation of the Exponent

- \* Biased representation

- \* bias = 127

- \*  $E = X + \underline{\text{bias}}$

- \* Range of the exponent

- \*  $0 - 255 \leftrightarrow -127 \text{ to } +128$

- \* Examples :

- \*  $X = 0, E = 127$

- \*  $X = -23, E = 104$

- \*  $X = 30, E = 157$

E

$$-127 \Rightarrow 0$$

$$-23 \Rightarrow -23 + 127$$

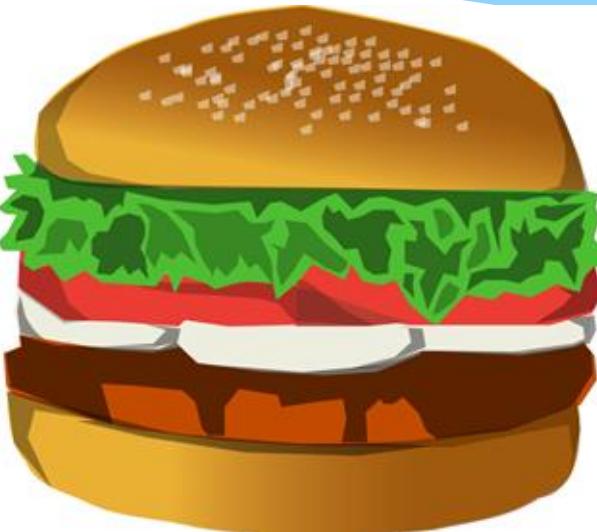
$$+128 \Rightarrow +128 + 127 = 255$$

# Normal FP Numbers

- \* Have an exponent between -126 and +127
- \* Let us leave the exponents : -127, and +128 for special purposes.

$-127 \leftrightarrow +128$

$$A = \underbrace{(-1)^S * P * 2^{\frac{E}{-bias}}}_{(P=1+M, 0 \leq M < 1, X \in Z, 1 \leq E \leq 254)}$$



- \* What is the largest +ve normal FP number ?
- \* What is the smallest -ve normal FP number ?

$$\checkmark \boxed{\pm (2 - 2^{-23}) \times 2^{127}}$$

Diagram illustrating the floating-point representation of a number:

S	8	23
0		111111

Annotations:

- Exp**: Exponent field (8 bits)
- M**: Mantissa field (23 bits)
- A green bracket under the mantissa is labeled  $2^{-23}$ .
- The mantissa is shown as  $(1 \cdot 111\dots1) \times 2^{127}$ .
- The formula for the largest positive normal FP number is derived as follows:

$$\begin{aligned}
 &= \sum_{i=0}^{-23} 2^i \\
 &= 2^0 - 2^{-23} \\
 &= (2 - 2^{-23})
 \end{aligned}$$

# Special Floating Point Numbers

$E$	$M$	Value
255	0	$\infty$ if $S = 0$
255	0	$-\infty$ if $S = 1$
255	$\neq 0$	NAN(Not a number)
0	0	0
0	$\neq 0$	Denormal number

- \*  $\text{NAN} + x = \text{NAN}$        $1/0 = \infty$
  - \*  $0/0 = \text{NAN}$        $-1/0 = -\infty$
  - \*  $\sin^{-1}(5) = \text{NAN}$

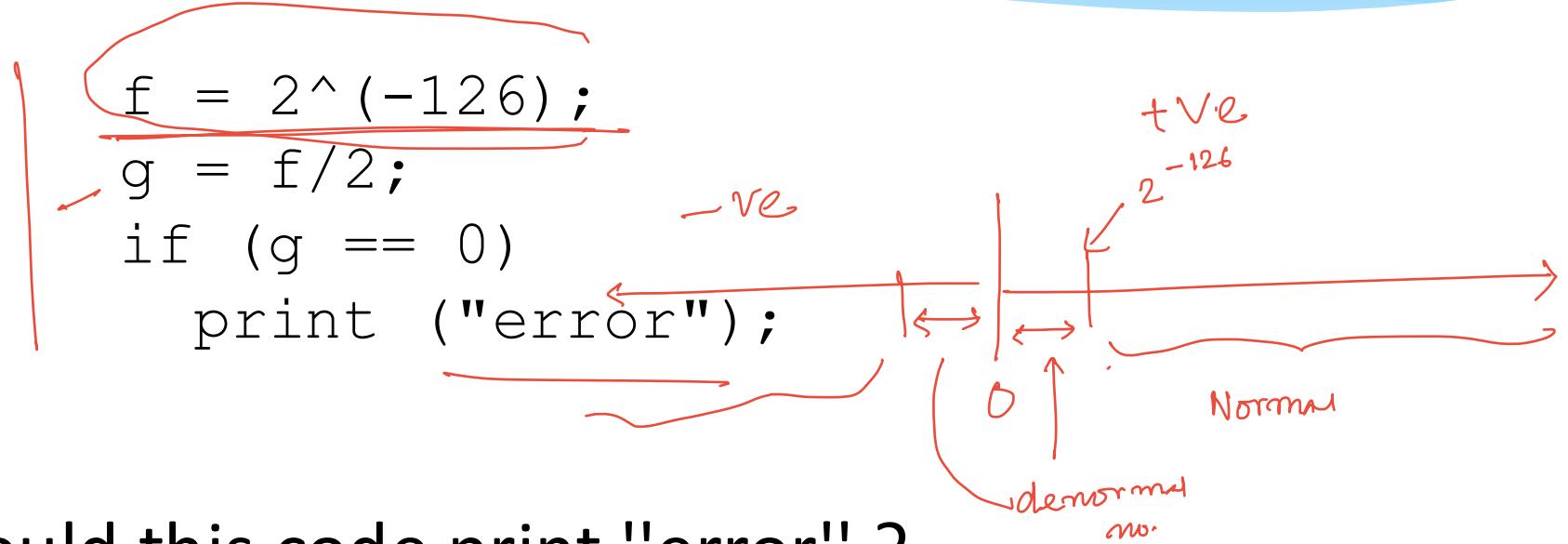
$$M = D$$

$$\underline{E = -126}$$

1/0 5/0

$$\begin{array}{r} F \\ - 127 = \emptyset \\ + 128 = 255 \end{array}$$

# Denormal Numbers



- \* Should this code print "error" ?
- \* How to stop this behaviour ?

# Denormal Numbers - II

smallest denormal no:  
 $0.000\ldots 1 \times 2^{-126}$   
 $= 2^{-23} + 2^{-149}$

$$A = (-1)^S * P * 2^{-126}$$

$(P = 0.M, 0 \leq M < 1)$

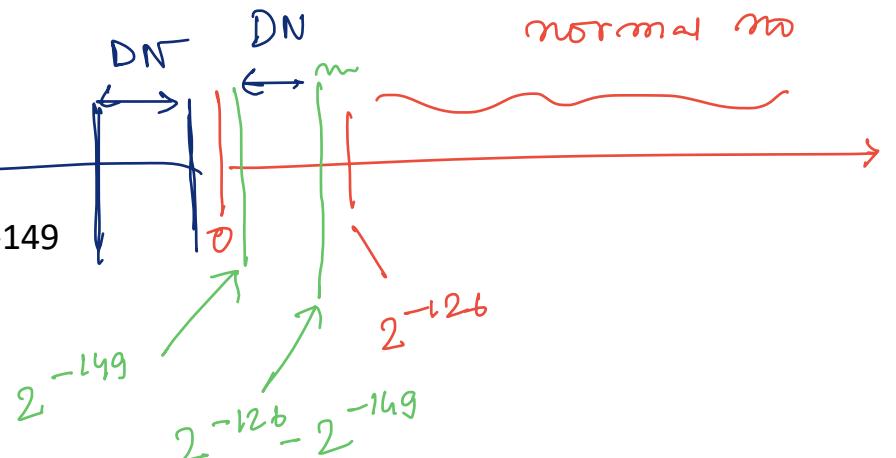
largest denormal no:  
 $(1 - 2^{-23}) \times 2^{-126}$   
 $= (2^{-126} - 2^{-149})$

- \* Significand is of the form : 0.xxxx
- \* E = 0, X = -126 (why not -127?)
- \* Smallest +ve normal number :  $2^{-126}$
- \* Largest denormal number :

$$0.11\ldots 11 * 2^{-126} = (1 - 2^{-23}) * 2^{-126}$$

$* = 2^{-126} - 2^{-149}$

Normal no.



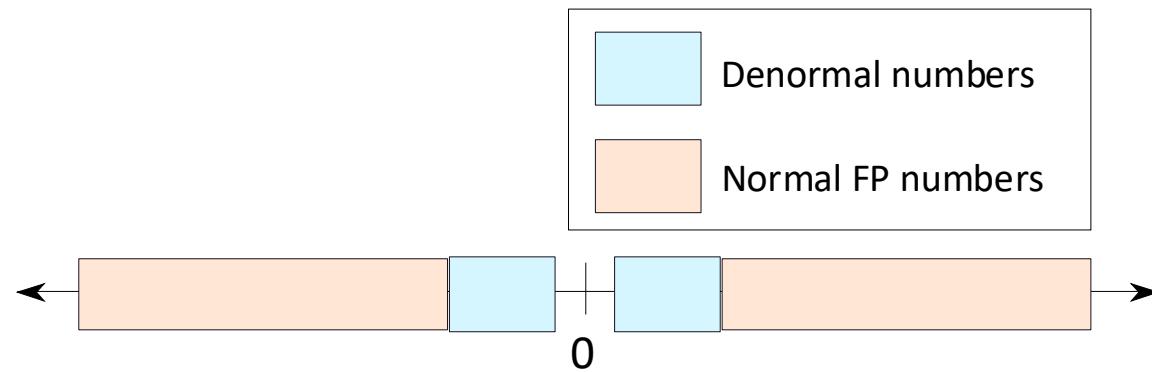
# Example

Find the ranges of denormal numbers.

**Answer**

- For positive denormal numbers, the range is  $[2^{-149}, 2^{-126} - 2^{-149}]$  ✓
- For negative denormal numbers, the range is  $[-2^{-149}, -2^{-126} + 2^{-149}]$  ✓

# Denormal Numbers in the Number Line



Extend the range of normal floating point numbers.

# Double Precision Numbers

Field	Size(bits)
$S$	1
$E$	11 = $10^{23}$
$M$	52

32 bit machine:  
int:  $2^{31}$  numbers  
 $\approx (10^9)$

32 bit fp :  $2^{128}$  number  
 $(10^{38})$

64 bits double =  $2^{1023}$   
 $\approx (10^{308})$

- Approximate range of **doubles**
  - $\pm 2^{1023} = \pm 10^{308}$
  - This is a lot !!!



Range of the normal +ve nos.

# Floating Point Mathematics

$$\begin{aligned} A &= 2^{50}; \checkmark \\ B &= 2^{10}; \checkmark \\ C &= (B+A) - A; \\ &\quad \text{---} \\ &= \end{aligned}$$

32 bits

$$\begin{aligned} A+B &= 2^{50} + 2^{10} \\ 2^{50} &= (-1)^0 \times 2^{50} \times (1+2^{-40}) \\ &= (-1)^0 \times 2^{50} \times (1+2^{-40}) \end{aligned}$$

$$m = 2^{-40}$$

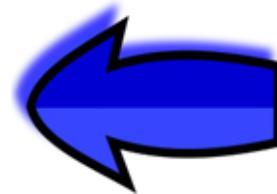
$$m = 2^{-23}$$

- \* C will be computed to be 0
  - \* There is no way of representing A+B in the IEEE 754 format
- \* A **smart compiler** can reorder the operations to increase precision
- \* Floating point math is **approximate**

*FLOP*  
*Quantization*

# Outline

- \* Boolean Algebra
- \* Positive Integers
- \* Negative Integers
- \* Floating Point Numbers
- \* Strings



# ASCII Character Set

- \* ASCII – American Standard Code for Information Interchange
- \* It has 128 characters
- \* First 32 characters (control operations)
  - \* backspace (8)
  - \* line feed (10)
  - \* escape (27)
- \* Each character is encoded using 7 bits

a 18 "

# ASCII Character Set

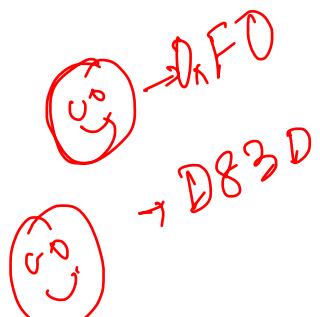
Character	Code	Character	Code	Character	Code
a	97	A	65	0	48
b	98	B	66	1	49
c	99	C	67	2	50
d	100	D	68	3	51
e	101	E	69	4	52
f	102	F	70	5	53
g	103	G	71	6	54
h	104	H	72	7	55
i	105	I	73	8	56
j	106	J	74	9	57
k	107	K	75	!	33
l	108	L	76	#	35
m	109	M	77	\$	36
n	110	N	78	%	37
o	111	O	79	&	38
p	112	P	80	(	40
q	113	Q	81	)	41
r	114	R	82	*	42
s	115	S	83	+	43
t	116	T	84	,	44
u	117	U	85	.	46
v	118	V	86	;	59
w	119	W	87	=	61
x	120	X	88	?	63
y	121	Y	89	@	64
z	122	Z	90	^	94

# Unicode Format

- \* UTF-8 (Universal character set Transformation Format)
  - \* **UTF-8 encodes 1,112,064 characters** defined in the Unicode character set. It uses 1-6 bytes for this purpose.  
E.g. ਅ ਆ ਕ ਖ, ਮ ਪ ਬ ਲ
    - ੀ
    - ੁ
    - ੍
    - ੰ
    - ੁ
    - ੰ
  - \* UTF-8 is **compatible** with ASCII. The first 128 characters in UTF-8 correspond to the ASCII characters. When using ASCII characters, UTF-8 requires just one byte. It has a leading 0.
  - \* Most of the languages that use variants of the Roman script such as French, German, and Spanish require 2 bytes in UTF-8. Greek, Russian (Cyrillic), Hebrew, and Arabic, also require 2 bytes.

# UTF-16 and 32

- \* **Unicode** is a standard across all browsers and operating systems
- \* **UTF-8** has been superseded by UTF-16, and UTF-32
- \* **UTF-16** uses 2 byte or 4 byte encodings (Java and Windows)
- \* **UTF-32** uses 4 bytes for every character (rarely used)





**THE END**