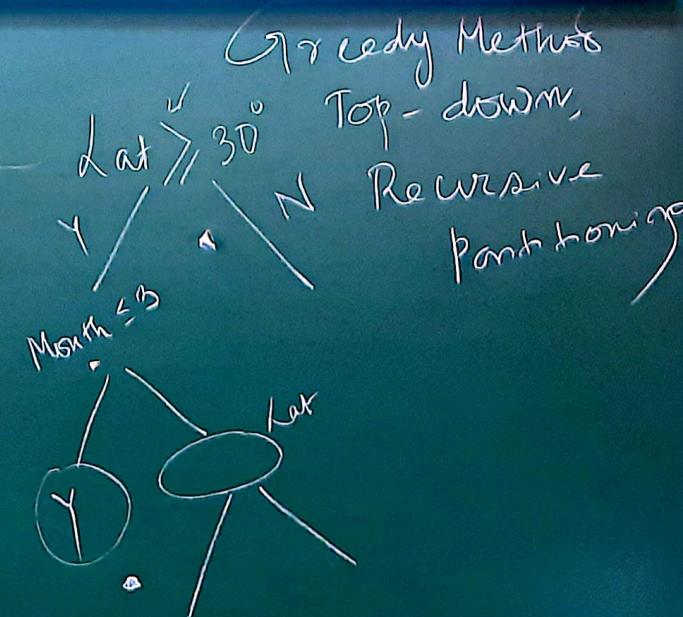
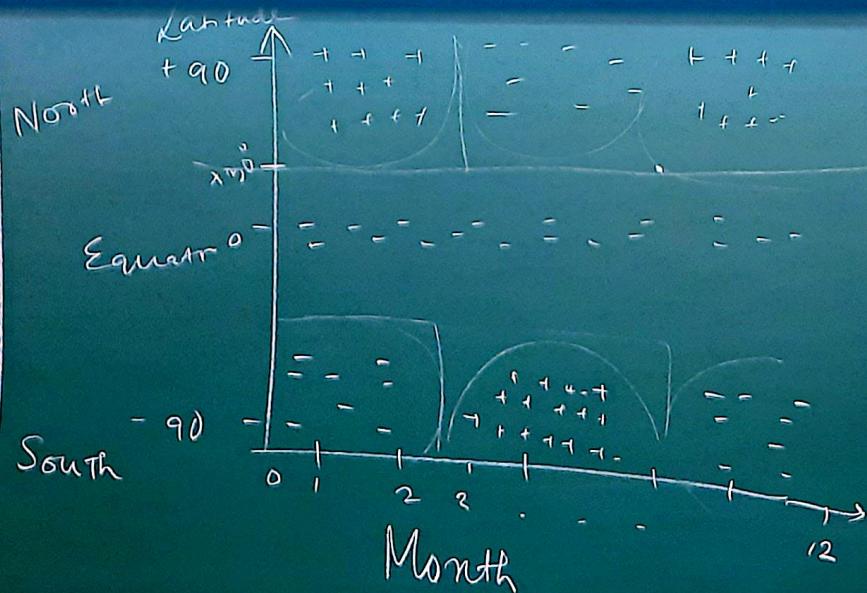


## Decision Trees

### 1. Non-linear Model

(Ch. 3 Tom Mitchell)

Binary Class



Greedy Method

Top-down,

Recursive

Partitioning

Region of Parent  
:  $R_P$

Looking for a split:  $S_P$

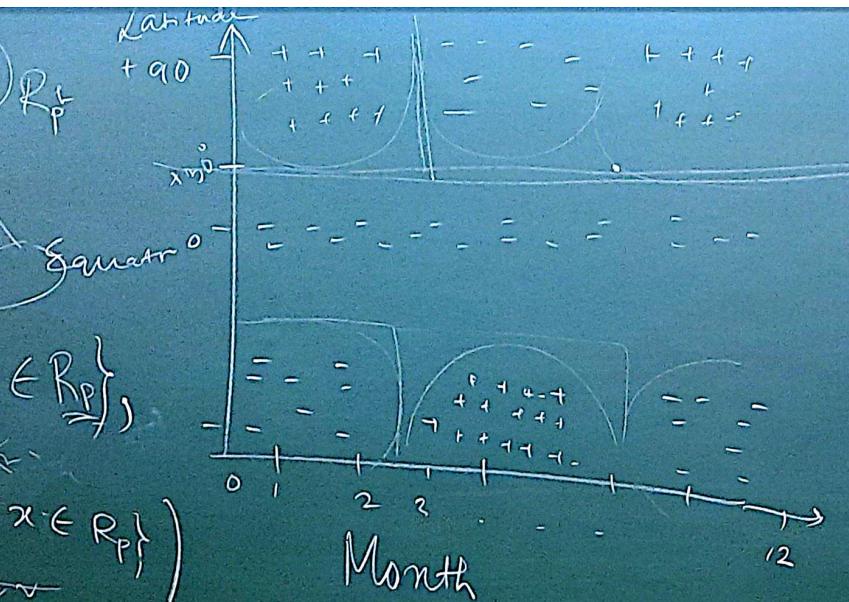
$$S_P(i, t) = \left( \begin{array}{l} \{x \mid x_i < t; x \in R_P\}, \\ \{x \mid x_i \geq t; x \in R_P\} \end{array} \right)$$

1<sup>st</sup> feature threshold

Loss  $L(R)$  = misclassification loss

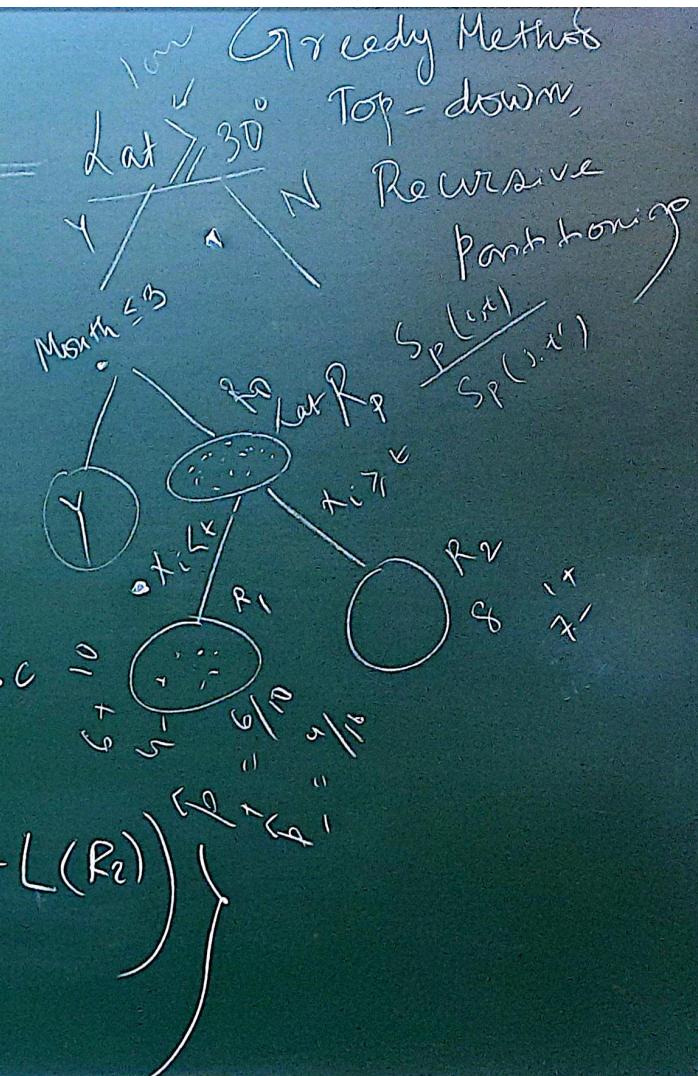
Given class  $C$ , define  $\hat{p}_C$  to be the proportion of examples in  $R$  belong to  $C$

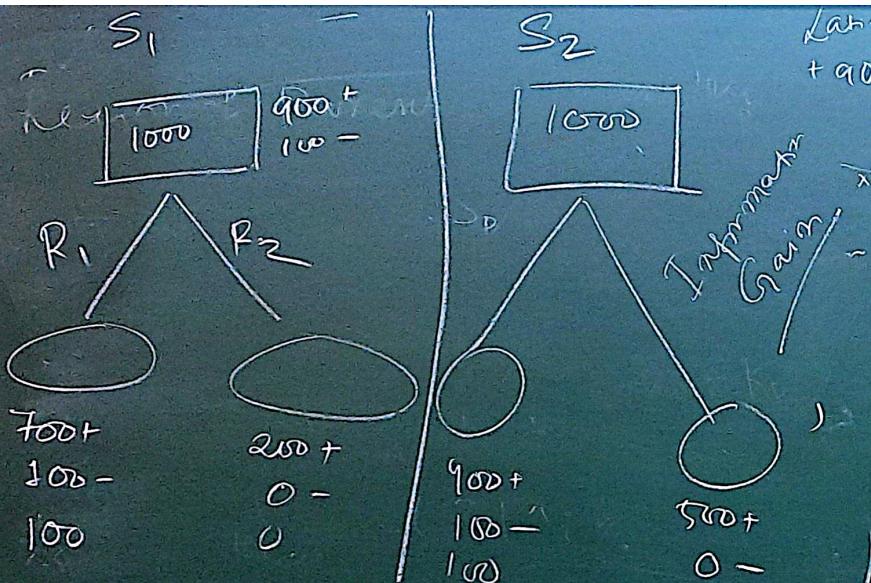
$$L_{R_1} = 1 - \max_C \hat{p}_C$$



Come up with a split s.t.

$$\max \left\{ L(R_P) - (L(R_1) + L(R_2)) \right\}$$





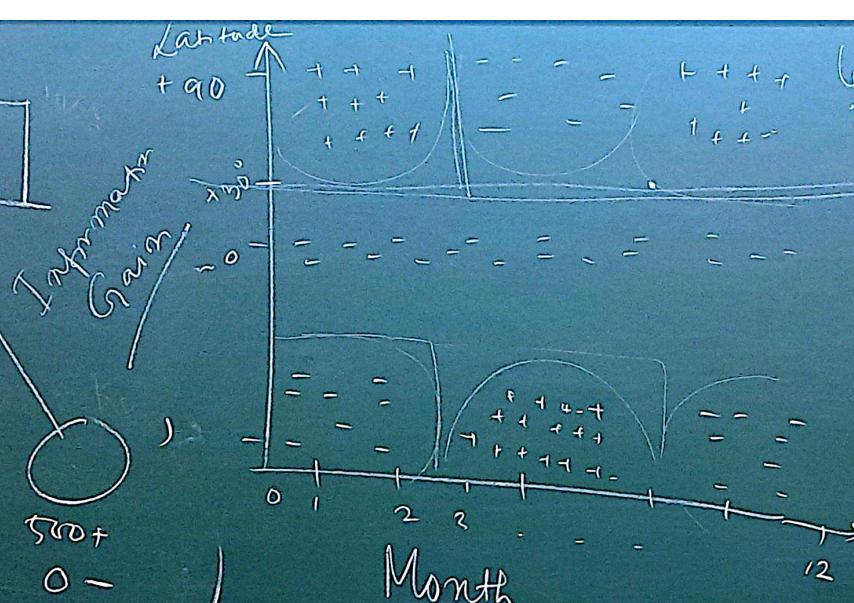
$\log L(R) = \text{misclassification loss}$

Given class  $C$ , define  $\hat{p}_C$  to be the proportion of examples in  $R$  belong to  $C$

$$L_R = 1 - \max_C \hat{p}_C$$

Come up with a split set

$$\max \left\{ \underline{L}(R_p) - \overline{L}(R_1) + L(R_2) \right\}$$



## 1 Information Gain

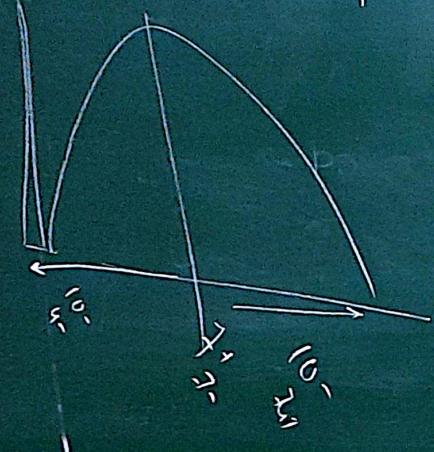
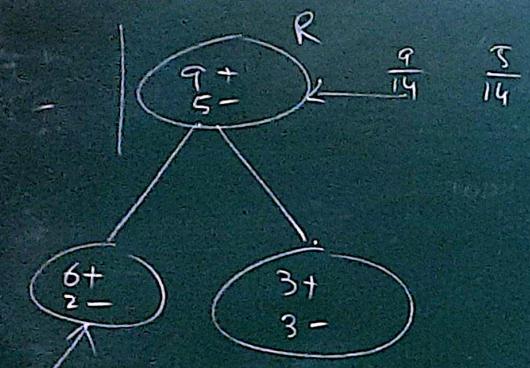
$$= P_H \log P_H - P_T \log P_T \quad X = \{ \cdot \}$$

$$= \sum P_i \log P_i$$

$P_i$  = prob of the occurrence of the outcome  $i$

$\log P_i$  = min. no of bits required to encode the output  $i$

$P_i \log P_i$  = expected no of bits reqd. to encode the outcomes of the event,  $X$



+ Example in Ruby

## 1 Information Gain

$$= P_H \log P_H - P_T \log P_T$$

$$\text{Gain}(R, A) = \text{Entropy}(R) -$$

$$- \sum P_i \log(P_i)$$

$P_i$  = prob of the outcome

$\log P_i$  = min no of

$P_i \log P_i$  = expected no

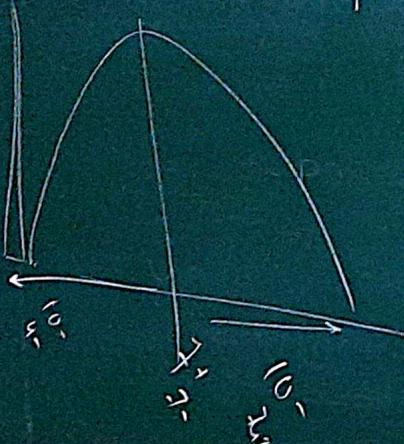
A = Humidity

$A = H$

$A = L$

$R_H = 8$

$R_L = 6$



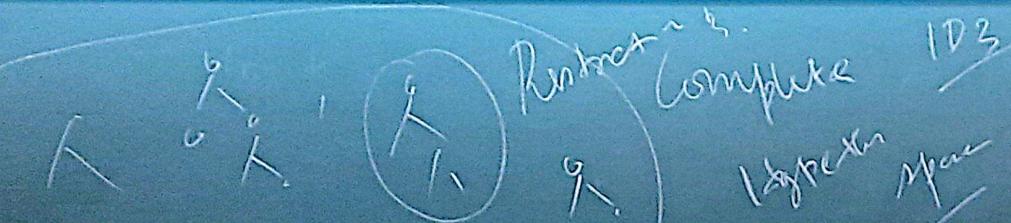
$$\sum_{V \in \text{Value}(A)} \frac{|R_V|}{|R|} \text{Entropy}(R_V)$$

$A_1, A_2, \dots, A_m$

Preference Rule

{ Shorter trees are preferred over longer trees. Trees that place high info gain attribute closer to the root will be preferred.

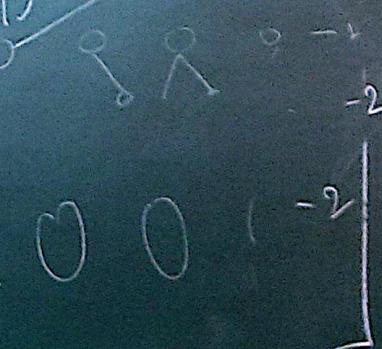
Candidate Elimination



Retract Complete ID3

Hypothesis space

BFS Tree



A<sub>2</sub> Occam's Razor

Overfitting

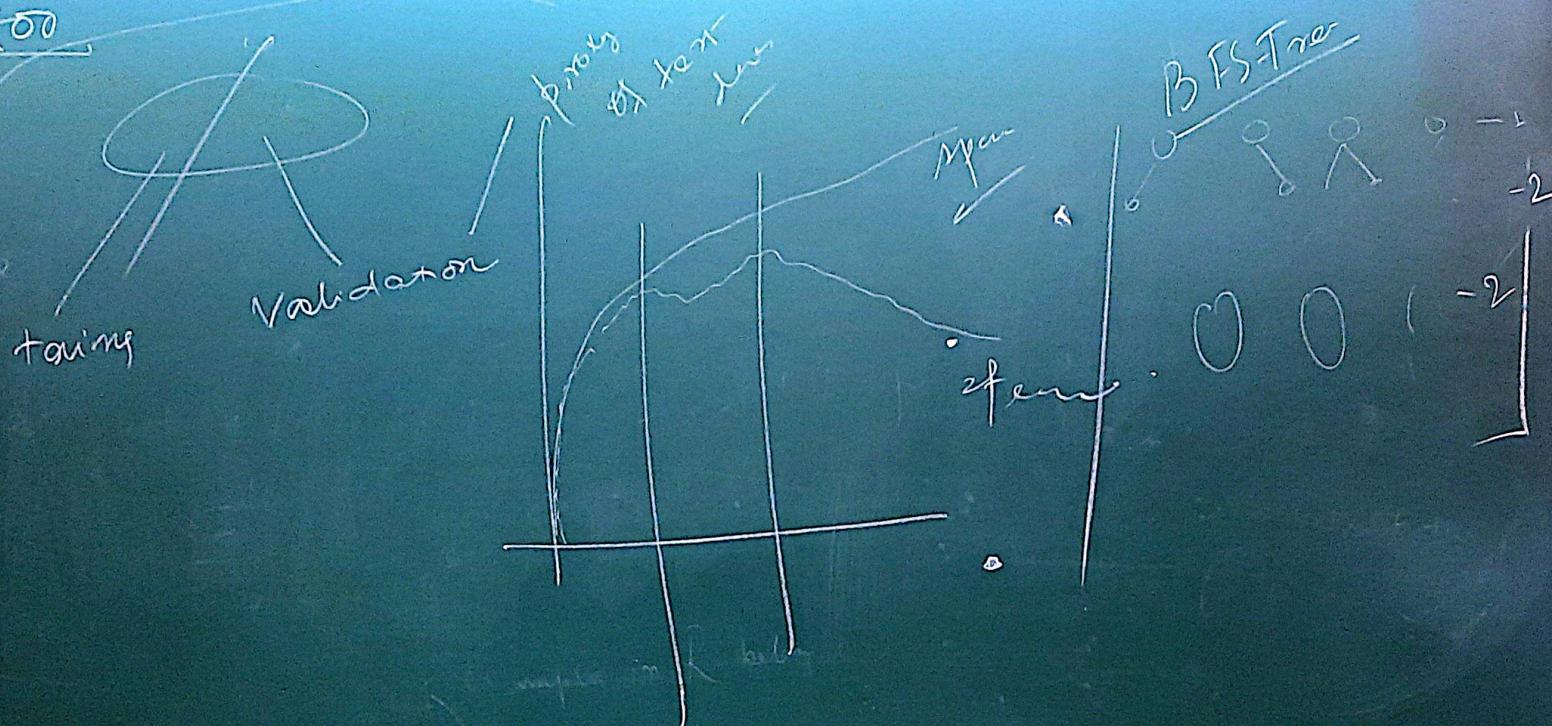
$$h_1 \in \mathcal{H}$$

$\text{Loss}_{\text{test}}(h_1)$

$$h_2 \in \mathcal{H}$$

$$\text{Loss}_{\text{train}}(h_1) < \text{Loss}_{\text{train}}(h_2)$$

$$\text{Loss}_{\text{test}}(h_1) > \text{Loss}_{\text{test}}(h_2)$$



BFS Tree