(Probabilistic) Context-Free Grammars

A phrase structure grammar

S → NP VP
VP → V NP
NP → N
VP → V NP PP
NP → NP NP
NP → NP PP
NP → *e*
PP → P NP

people fish tanks
people fish with rods

N → people
V → fish
N → fish
N → tanks
N → rods
V → people
V → tanks
P → with

Ambiguous: People people people, fish fish fish

Phrase structure grammars = context-free grammars (CFGs)

- G = (T, N, S, R)
  - T is a set of terminal symbols
  - N is a set of nonterminal symbols
  - S is a start symbol (S ∈ N)
  - R is a set of rules/productions of the form $X \rightarrow \gamma$
    - X ∈ N and $\gamma \in (N \cup T)^*$

- A grammar G generates a language L.

Phrase structure grammars in NLP

- G = (T, C, N, S, L, R)
  - T is a set of terminal symbols
  - C is a set of preterminal symbols
  - N is a set of nonterminal symbols
  - S is the start symbol ($S \in N$)
  - L is the lexicon, a set of items of the form $X \rightarrow x$
    - $X \in C$ and $x \in T$
  - R is the grammar, a set of items of the form $X \rightarrow \gamma$
    - $X \in N$ and $\gamma \in (N \cup C)^*$
- By usual convention, S is the start symbol, but in statistical NLP, we usually have an extra node at the top (ROOT, TOP)
- We usually write *e* for an empty sequence, rather than nothing

A phrase structure grammar (empty, unary, binary)

Grammar Rules

S → NP VP

VP → V NP

VP → V NP PP

NP → NP NP

NP → NP PP

NP → N

NP → *e*

PP → P NP

EMPTY fish tanks

people fish EMPTY

Lexicon

N → *people*

N → *fish*

N → *tanks*

N → *rods*

V → *people*

V → *fish*

V → *tanks*

P → *with*

Probabilistic/stochastic – context-free grammars (PCFGs)

- G = (T, N, S, R, P)
  - T is a set of terminal symbols
  - N is a set of nonterminal symbols
  - S is the start symbol (S ∈ N)
  - R is a set of rules/productions of the form $X \rightarrow \gamma$
  - P is a probability function
    - P: R $\rightarrow$ [0,1]
    - $$\forall X \in N, \sum_{X \rightarrow \gamma \in R} P(X \rightarrow \gamma) = 1$$

A PCFG

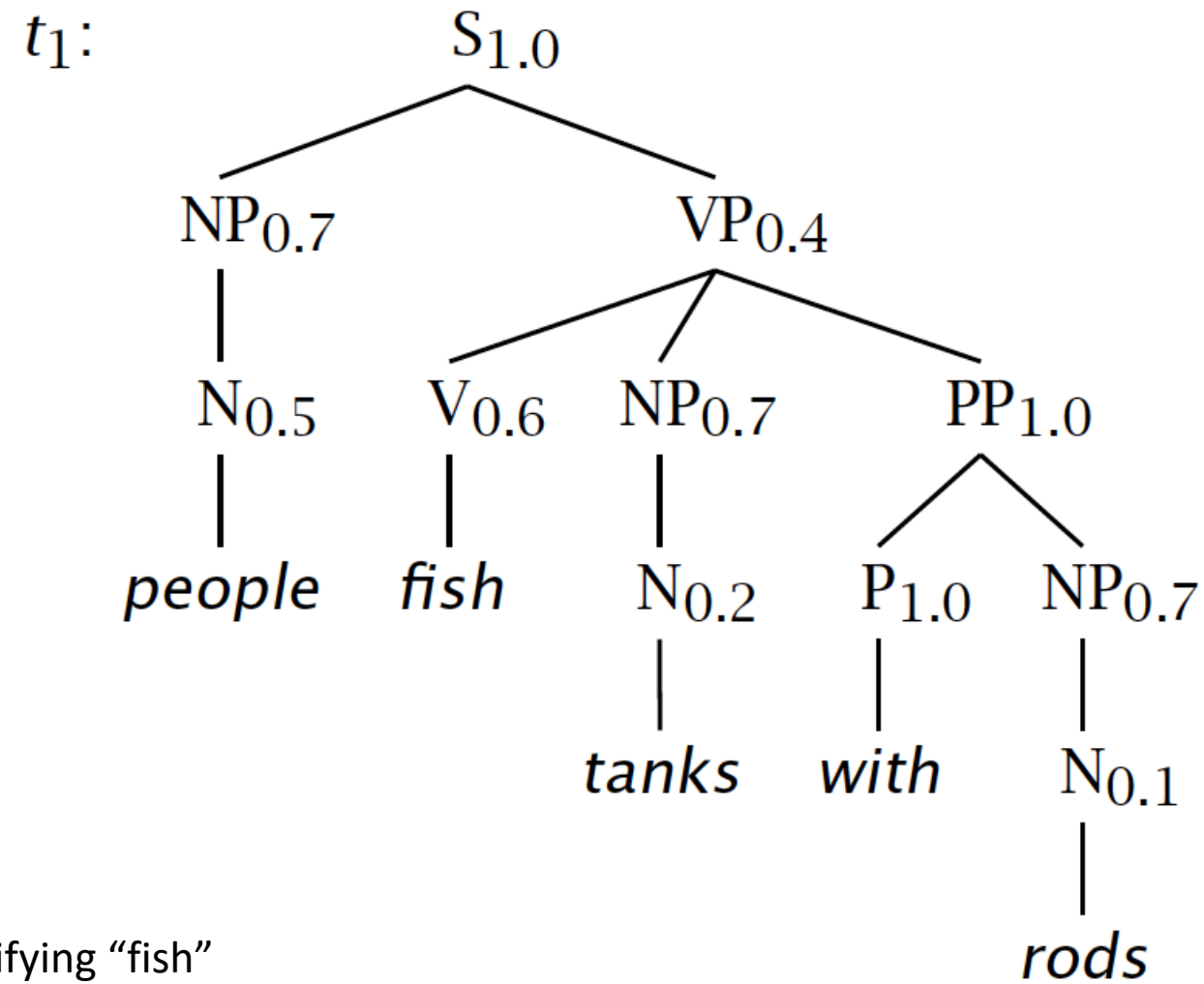
S → NP VP	1.0	
VP → V NP	0.6	
VP → V NP PP		0.4
NP → NP NP		0.1
NP → NP PP	0.2	
NP → N	0.7	
PP → P NP	1.0	

N → *people*	0.5
N → *fish*	0.2
N → *tanks*	0.2
N → *rods*	0.1
V → *people*	0.1
V → *fish*	0.6
V → *tanks*	0.3
P → *with*	1.0

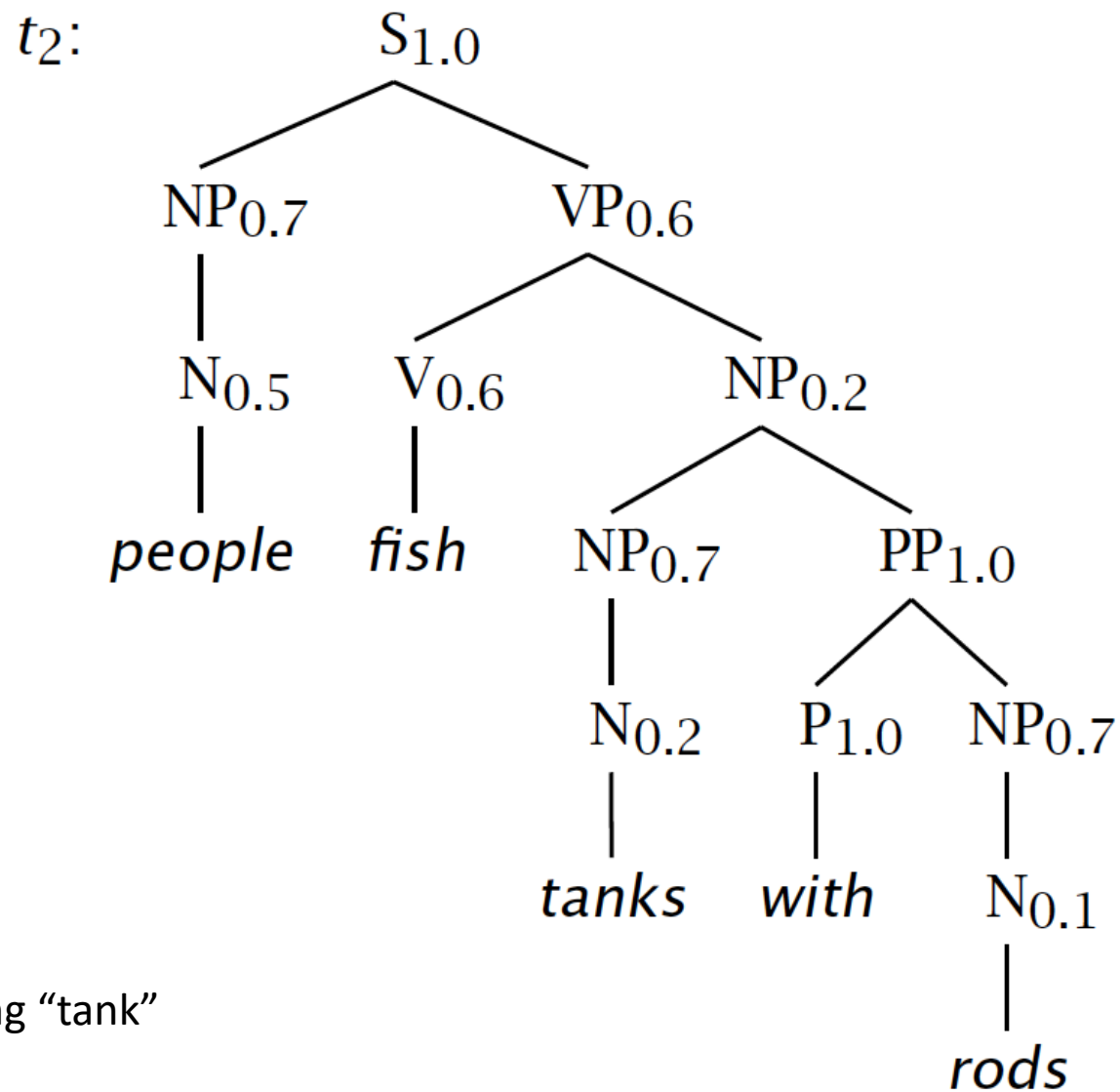
The probability of trees and strings

- P(t) – The probability of a tree t is the product of the probabilities of the rules used to generate it.
- P(s) – The probability of the string s is the sum of the probabilities of the trees which have that string as their yield

$$P(s) = \Sigma_t P(s, t) \text{ where } t \text{ is a parse of } s$$
$$= \Sigma_t P(t)$$



Preposition “with” modifying “fish”



Preposition “with” modifying “tank”

Tree and String Probabilities

- s = *people fish tanks with rods*
- $P(t_1)$ = $1.0 \times 0.7 \times 0.4 \times 0.5 \times 0.6 \times 0.7 \times 1.0 \times 0.2 \times 1.0 \times 0.7 \times 0.1$
  = 0.0008232
- $P(t_2)$ = $1.0 \times 0.7 \times 0.6 \times 0.5 \times 0.6 \times 0.2 \times 0.7 \times 1.0 \times 0.2 \times 1.0 \times 0.7 \times 0.1$
  = 0.00024696
- $P(s)$ = $P(t_1)$ + $P(t_2)$
  = $0.0008232 + 0.00024696$
  = 0.00107016

Verb attach

Noun attach

Grammar Transforms

Restricting the grammar form for efficient parsing

Chomsky Normal Form

- All rules are of the form $X \rightarrow Y\ Z$ or $X \rightarrow w$
  - $X, Y, Z \in N$ and $w \in T$
- A transformation to this form doesn't change the weak generative capacity of a CFG
  - That is, it recognizes the same language
    - But maybe with different trees
- Empties and unaries are removed recursively
- n-ary rules are divided by introducing new nonterminals ($n > 2$)

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VP → V NP

VP → V NP PP

NP → NP NP

NP → NP PP

NP → N

NP → *e*

PP → P NP

N → *people*

N → *fish*

N → *tanks*

N → *rods*

V → *people*

V → *fish*

V → *tanks*

P → *with*

Start discussing epsilon removal

Chomsky Normal Form steps

S → NP VP
S → VP
VP → V NP
VP → V
VP → V NP PP
VP → V PP
NP → NP NP
NP → NP
NP → NP PP
NP → PP
NP → N
PP → P NP
PP → P

N → *people*
N → *fish*
N → *tanks*
N → *rods*
V → *people*
V → *fish*
V → *tanks*
P → *with*

Start discussing unary removal downwards: remove S → VP

Chomsky Normal Form steps

S → NP VP

VP → V NP

S → V NP

VP → V

S → V

VP → V NP PP

S → V NP PP

VP → V PP

S → V PP

NP → NP NP

NP → NP

NP → NP PP

NP → PP

NP → N

PP → P NP

PP → P

N → *people*

N → *fish*

N → *tanks*

N → *rods*

V → *people*

V → *fish*

V → *tanks*

P → *with*

Remove more unaries, next S → V

Chomsky Normal Form steps

S → NP VP
VP → V NP
S → V NP
VP → V
VP → V NP PP
S → V NP PP
VP → V PP
S → V PP
NP → NP NP
NP → NP
NP → NP PP
NP → PP
NP → N
PP → P NP
PP → P

N → *people*
N → *fish*
N → *tanks*
N → *rods*
V → *people*
S → *people*
V → *fish*
S → *fish*
V → *tanks*
S → *tanks*
P → *with*

After remove S → V get this, and then do VP → V

Chomsky Normal Form steps

S → NP VP

VP → V NP

S → V NP

VP → V NP PP

S → V NP PP

VP → V PP

S → V PP

NP → NP NP

NP → NP

NP → NP PP

NP → PP

NP → N

PP → P NP

PP → P

N → *people*

N → *fish*

N → *tanks*

N → *rods*

V → *people*

S → *people*

VP → *people*

V → *fish*

S → *fish*

VP → *fish*

V → *tanks*

S → *tanks*

VP → *tanks*

P → *with*

Chomsky Normal Form steps

S → NP VP

VP → V NP

S → V NP

VP → V NP PP

S → V NP PP

VP → V PP

S → V PP

NP → NP NP

NP → NP PP

NP → P NP

PP → P NP

NP → *people*

NP → *fish*

NP → *tanks*

NP → *rods*

V → *people*

S → *people*

VP → *people*

V → *fish*

S → *fish*

VP → *fish*

V → *tanks*

S → *tanks*

VP → *tanks*

P → *with*

PP → *with*

And then binarize

Chomsky Normal Form steps

S → NP VP

VP → V NP

S → V NP

VP → V @VP_V

@VP_V → NP PP

S → V @S_V

@S_V → NP PP

VP → V PP

S → V PP

NP → NP NP

NP → NP PP

NP → P NP

PP → P NP

NP → *people*

NP → *fish*

NP → *tanks*

NP → *rods*

V → *people*

S → *people*

VP → *people*

V → *fish*

S → *fish*

VP → *fish*

V → *tanks*

S → *tanks*

VP → *tanks*

P → *with*

PP → *with*

A phrase structure grammar

S → NP VP

VP → V NP

VP → V NP PP

NP → NP NP

NP → NP PP

NP → N

NP → *e*

PP → P NP

N → *people*

N → *fish*

N → *tanks*

N → *rods*

V → *people*

V → *fish*

V → *tanks*

P → *with*

Chomsky Normal Form steps

S → NP VP

VP → V NP

S → V NP

VP → V @VP_V

@VP_V → NP PP

S → V @S_V

@S_V → NP PP

VP → V PP

S → V PP

NP → NP NP

NP → NP PP

NP → P NP

PP → P NP

NP → *people*

NP → *fish*

NP → *tanks*

NP → *rods*

V → *people*

S → *people*

VP → *people*

V → *fish*

S → *fish*

VP → *fish*

V → *tanks*

S → *tanks*

VP → *tanks*

P → *with*

PP → *with*

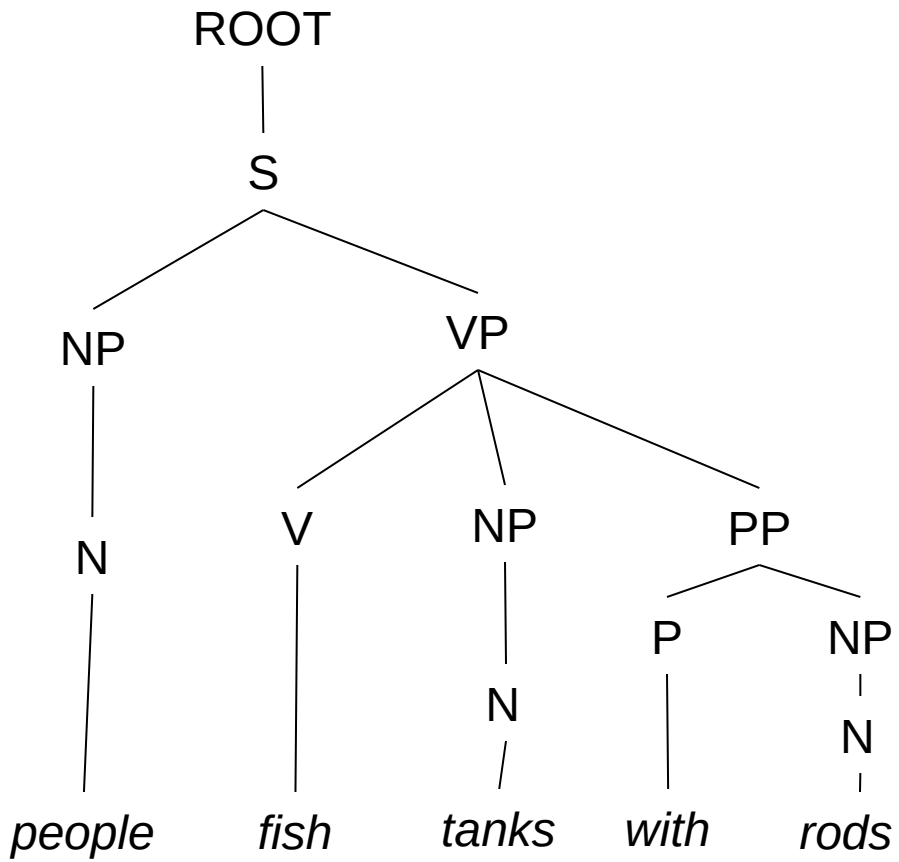
Chomsky Normal Form

- You should think of this as a transformation for efficient parsing
- With some extra book-keeping in symbol names, you can even reconstruct the same trees with a detransform
- In practice full Chomsky Normal Form is a pain
  - Reconstructing n-aries is easy
  - Reconstructing unaries/empties is trickier

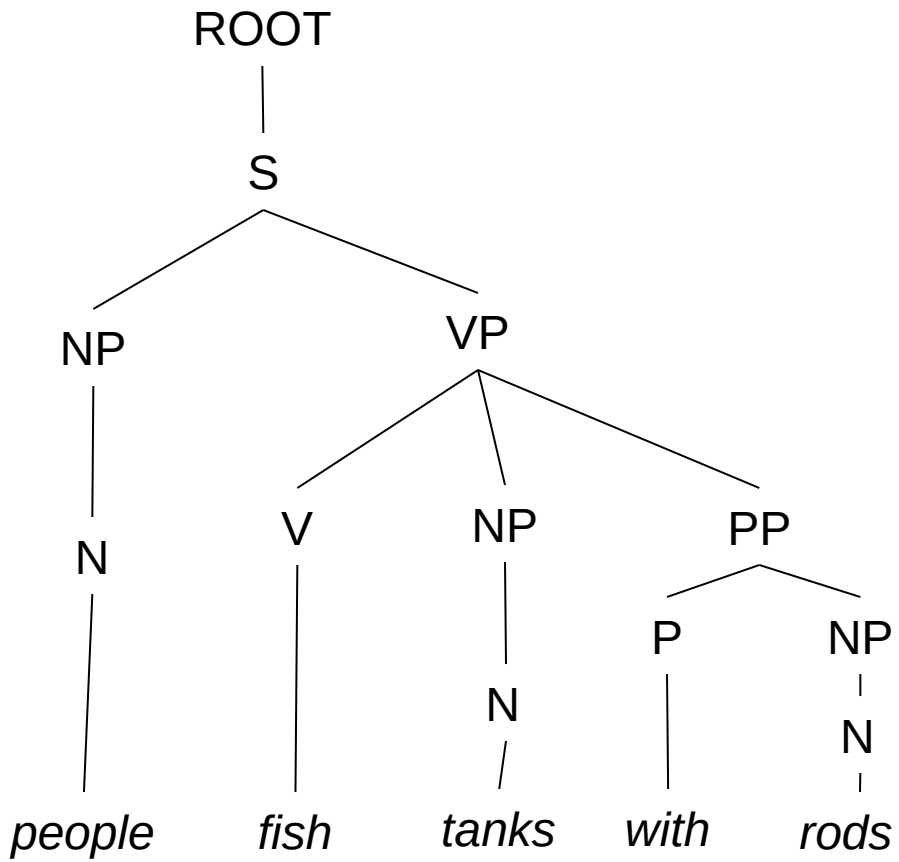
- **Binarization** is crucial for cubic time CFG parsing

- The rest isn't necessary; it just makes the algorithms cleaner and a bit quicker

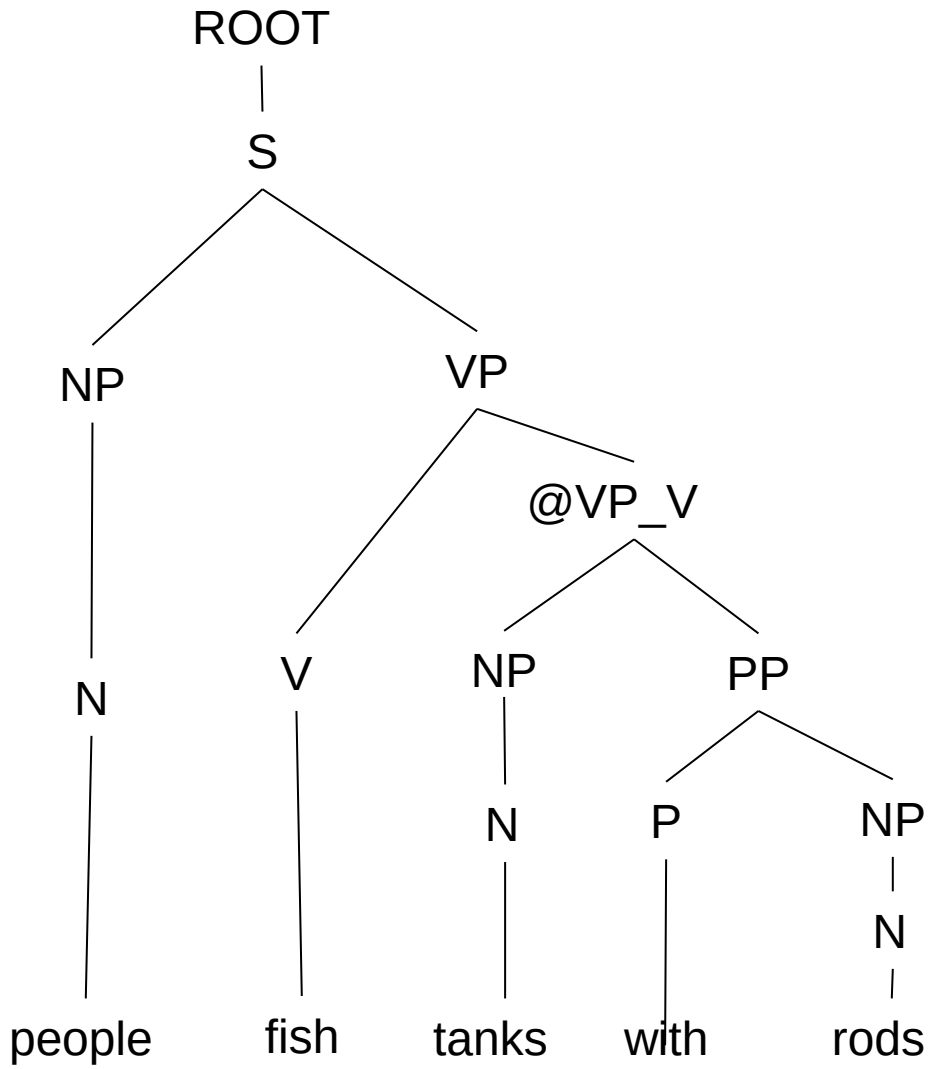
An example: before binarization…



An example: before binarization…

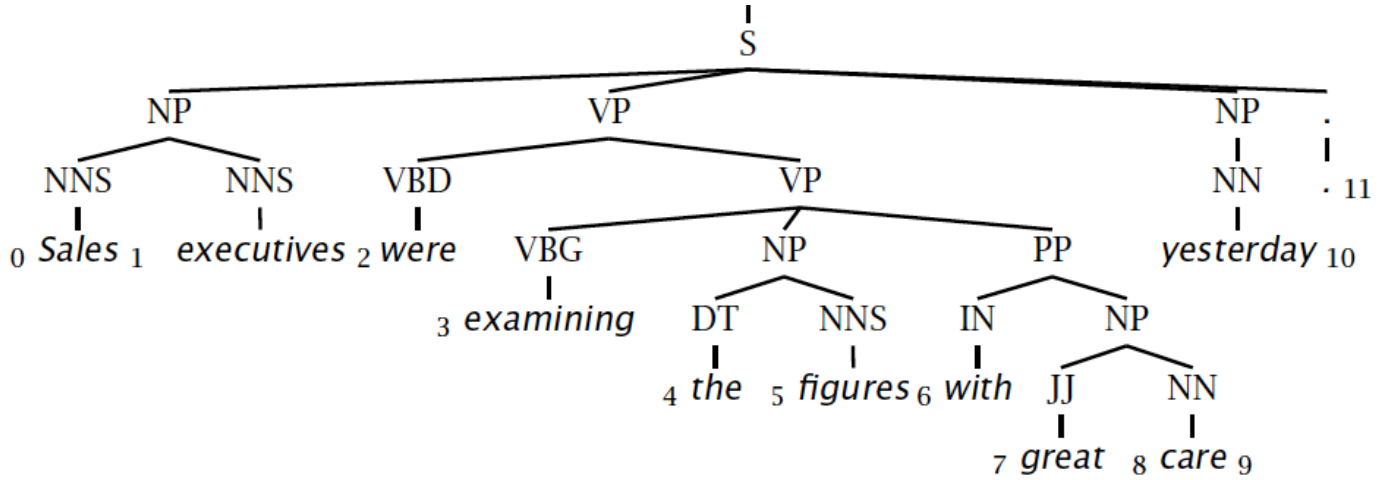


After binarization…

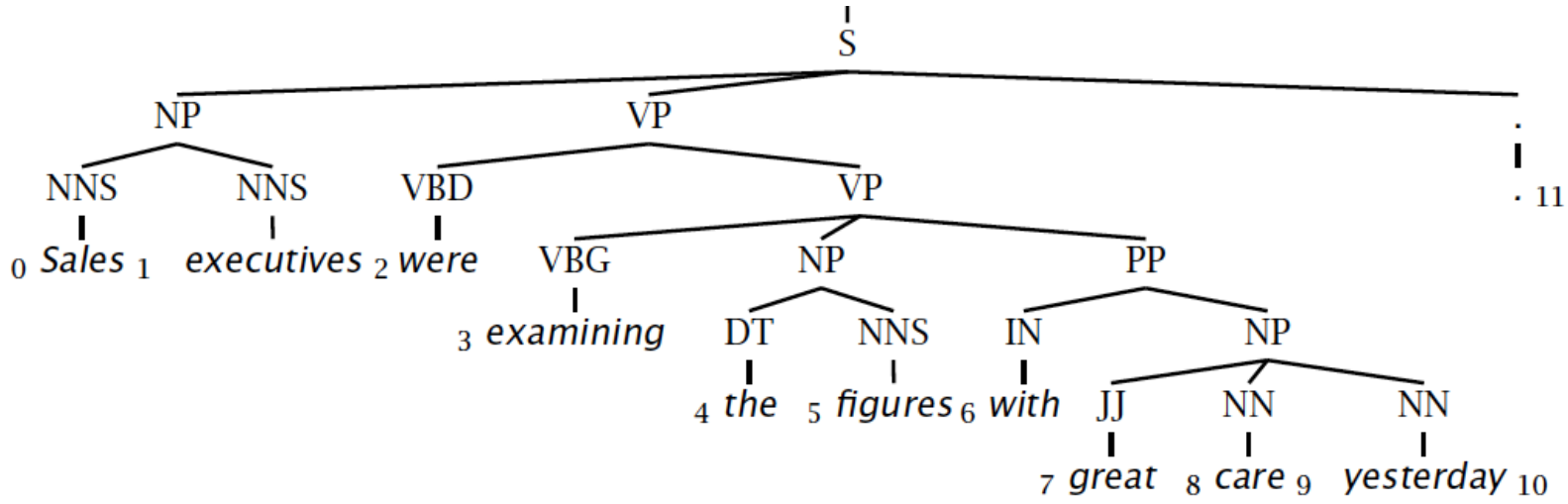


Evaluating constituency parsing

Gold standard brackets: **S-(0:11)**, **NP-(0:2)**, VP-(2:9), VP-(3:9), **NP-(4:6)**, PP-(6-9), NP-(7,9), NP-(9:10)



Candidate brackets: **S-(0:11)**, **NP-(0:2)**, VP-(2:10), VP-(3:10), **NP-(4:6)**, PP-(6-10), NP-(7,10)



Evaluating constituency parsing

Gold standard brackets:

S-(0:11), NP-(0:2), VP-(2:9), VP-(3:9), **NP-(4:6)**, PP-(6-9), NP-(7,9), NP-(9:10)

Candidate brackets:

S-(0:11), **NP-(0:2)**, VP-(2:10), VP-(3:10), **NP-(4:6)**, PP-(6-10), NP-(7,10)

Labeled Precision	3/7 = 42.9%
Labeled Recall	3/8 = 37.5%
LP/LR F1	40.0%
Tagging Accuracy	11/11 = 100.0%