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Models of Neural Systems, WS 2019/20
Project 2: Action potential propagation
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Background

The aim of this project is to model the propagation of an action potential along an axon. The relationship between the membrane current i_m and the voltage V along an axon is given by the equation

$$c_m \frac{\partial V}{\partial t} = \frac{1}{2ar_L} \frac{\partial}{\partial x}\left(a^2 \frac{\partial V}{\partial x}\right) - i_m + i_e, \tag{1}$$

where a is the radius of the axon, r_L is the intracellular resistivity and c_m the specific membrane capacitance.

The ionic current i_m flowing through a patch of axonal membrane is well-described by the Hodgkin-Huxley model:

$$i_m = \bar{g}_{\mathrm{L}}(V - E_{\mathrm{L}}) + \bar{g}_{\mathrm{Na}} m^3 h (V - E_{\mathrm{Na}}) + \bar{g}_{\mathrm{K}} n^4 (V - E_{\mathrm{K}}), \tag{2}$$

where m, h and n are Hodgkin-Huxley-type gating variables.

Combining the two equations leads to a partial differential equation that can be solved numerically by a multi-compartmental approximation. In a nonbranching cable, each compartment μ is coupled to two neighbors, and the equations for the membrane potentials of the compartments are

$$c_m \frac{dV_\mu}{dt} = -i_m^\mu + \frac{i_e^\mu}{A_\mu} + g_{\mu,\mu+1}(V_{\mu+1} - V_\mu) + g_{\mu,\mu-1}(V_{\mu-1} - V_\mu), \tag{3}$$

where μ labels the compartments, i_e^μ is the total electrode current flowing into the compartment μ, and A_μ is its surface area. The constant $g_{\mu,\mu-1}$ determines the resistive coupling from neighboring compartment $\mu - 1$ to compartment μ and, for nonbranching cables, can be shown to be equal to $g_{\mu,\mu-1} = a/(2r_L L^2)$ for a length L of each compartment. This defines a system of ordinary differential equations, which can be solved by generalized Euler methods.

Problems

1. Numerically solve the cable equation (equation 1) when injecting a steady depolarizing current halfway along the cable. Consider that the membrane is passive, i.e. $i_m = (V - E_L)/r_m$, where r_m is the specific membrane resistance. Take $r_m = 20\,\mathrm{k}\Omega\,\mathrm{cm}^2$, $r_L = 200\,\Omega\,\mathrm{cm}$, $c_m = 1\,\mu\mathrm{F}/\mathrm{cm}^2$, and cable radius $a = 2\,\mu\mathrm{m}$. Compare the solution to the analytical solution of the infinite cable.

2. Implement the Hodgkin-Huxley model of action potential propagation in the squid giant axon. Solve the partial differential equation using the Crank-Nicholson method (see e.g. Chapters 5.5, 5.6 and 6.6B from Dayan and Abbott, 2001). Take $a = 238\,\mu\mathrm{m}$ and $r_L = 35.4\ \Omega\,\mathrm{cm}$. Note that you will need to include your code with the project report!

3. Initiate an action potential on one end of the axon by injecting a current in the terminal compartment.

4. Determine the action potential propagation velocity as a function of the axon radius.

5. Initiate action potentials at both ends of the axon. Show that they annihilate when they collide.

6. Simulate action potential propagation in a myelinated axon (see e.g. Chapter 6.4 from Dayan and Abbott, 2001). Consider that the nodes of Ranvier (the unmyelinated spaces) are 2 μm long and are located at 1 mm intervals along the axon. Also consider that myelin increases the resistance across the cell membrane by a factor of 5,000 and decreases the capacitance by a factor of 50.

References

P Dayan, LF Abbott. Theoretical neuroscience, MIT Press 2001. Note the following if you have the first edition of the book. On page 226, equation 6.45: the symbol A_μ in the equation for D_μ denotes the surface area of compartment μ of the cable, it is not the coefficient denoted by A_μ that appears in the upper left equation and in equation 6.44.

Contact

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