

CHAPTER VIII: Sequences; Induction; the Binomial Theorem

Exercise 1.

In Problems 13–24, determine whether the given sequence is arithmetic, geometric, or neither. If the sequence is arithmetic, find the common difference and the sum of the first n terms. If the sequence is geometric, find the common ratio and the sum of the first n terms.

13. $\{a_n\} = \{n + 5\}$

14. $\{b_n\} = \{4n + 3\}$

15. $\{c_n\} = \{2n^3\}$

16. $\{d_n\} = \{2n^2 - 1\}$

17. $\{s_n\} = \{2^{3n}\}$

18. $\{u_n\} = \{3^{2n}\}$

19. $0, 4, 8, 12, \dots$

20. $1, -3, -7, -11, \dots$

21. $3, \frac{3}{2}, \frac{3}{4}, \frac{3}{8}, \frac{3}{16}, \dots$

22. $5, -\frac{5}{3}, \frac{5}{9}, -\frac{5}{27}, \frac{5}{81}, \dots$

23. $\frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \dots$

24. $\frac{3}{2}, \frac{5}{4}, \frac{7}{6}, \frac{9}{8}, \frac{11}{10}, \dots$

Exercise 2.

In Problems 25–30, find each sum

25. $\sum_{k=1}^{50} (3k)$

26. $\sum_{k=1}^{30} k^2$

27. $\sum_{k=1}^{30} (3k - 9)$

28. $\sum_{k=1}^{40} (-2k + 8)$

29. $\sum_{k=1}^7 \left(\frac{1}{3}\right)^k$

30. $\sum_{k=1}^{10} (-2)^k$

Exercise 3.

In Problems 37–40, find a general formula for each arithmetic sequence.

37. 7th term is 31; 20th term is 96

38. 8th term is -20 ; 17th term is -47

39. 10th term is 0; 18th term is 8

40. 12th term is 30; 22nd term is 50

Exercise 4.

In Problems 49–54, use the Principle of Mathematical Induction to show that the given statement is true for all natural numbers

49. $3 + 6 + 9 + \dots + 3n = \frac{3n}{2}(n + 1)$

50. $2 + 6 + 10 + \dots + (4n - 2) = 2n^2$

51. $2 + 6 + 18 + \dots + 2 \cdot 3^{n-1} = 3^n - 1$

52. $3 + 6 + 12 + \dots + 3 \cdot 2^{n-1} = 3(2^n - 1)$

53. $1^2 + 4^2 + 7^2 + \dots + (3n - 2)^2 = \frac{1}{2}n(6n^2 - 3n - 1)$

54. $1 \cdot 3 + 2 \cdot 4 + 3 \cdot 5 + \dots + n(n + 2) = \frac{n}{6}(n + 1)(2n + 7)$

Exercise 5.

In Problems 57–60, expand each expression using the Binomial Theorem.

57. $(x + 2)^5$

58. $(x - 3)^4$

59. $(2x + 3)^5$

60. $(3x - 4)^4$

Exercise 6.

- 61. Find the coefficient of x^7 in the expansion of $(x + 2)^9$.
- 62. Find the coefficient of x^3 in the expansion of $(x - 3)^8$.
- 63. Find the coefficient of x^2 in the expansion of $(2x + 1)^7$.
- 64. Find the coefficient of x^6 in the expansion of $(2x + 1)^8$.

Exercise 7.

- 1) Find the coefficient of x^{10} in the expansion of $\left(3x + \frac{1}{x}\right)^{20}$
- 2) Find the coefficient of x^{10} in the expansion of $\left(x^2 - \frac{5}{x}\right)^{20}$

HOMEWORK

Exercise 1: 14, 16

Exercise 2: 26, 28, 30

Exercise 3: 38, 40

Exercise 4: 50, 52, 54

Exercise 5: 58, 60

Exercise 6: 62, 64

Exercise 7: 2