

CHAPTER 3 – FUNCTIONS AND THEIR GRAPHS

Exercise 1.

In problems 3-8, find the following of each functions

a) $f(2)$ b) $f(-x)$ c) $-f(x)$ d) $f(x - 2)$ e) $f(2x)$

3. $f(x) = \frac{3x}{x^2 - 1}$

4. $f(x) = \frac{x^2}{x + 1}$

5. $f(x) = \sqrt{x^2 - 4}$

6. $f(x) = |x^2 - 4|$

7. $f(x) = \frac{x^2 - 4}{x^2}$

8. $f(x) = \frac{x^3}{x^2 - 9}$

Exercise 2.

In problems 12-16, find the domain of each functions

12. $f(x) = \sqrt{x + 2}$

13. $h(x) = \frac{\sqrt{x}}{|x|}$

14. $g(x) = \frac{|x|}{x}$

15. $f(x) = \frac{x}{x^2 + 2x - 3}$

16. $F(x) = \frac{1}{x^2 - 3x - 4}$

Exercise 3.

In Problems 17–22, find $f + g$, $f - g$, $f \cdot g$, f/g for each pair of functions. State the domain of each of these functions.

17. $f(x) = 2 - x$; $g(x) = 3x + 1$

18. $f(x) = 2x - 1$; $g(x) = 2x + 1$

19. $f(x) = 3x^2 + x + 1$; $g(x) = 3x$

20. $f(x) = 3x$; $g(x) = 1 + x + x^2$

21. $f(x) = \frac{x + 1}{x - 1}$; $g(x) = \frac{1}{x}$

22. $f(x) = \frac{1}{x - 3}$; $g(x) = \frac{3}{x}$

Exercise 4.

In Problems 23 and 24, find the difference quotient of each function f ; that is, find

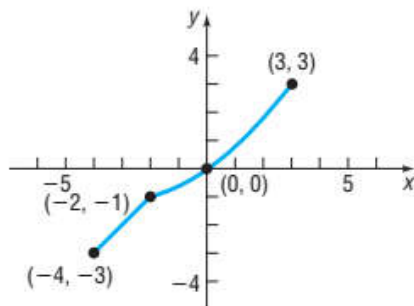
$$\frac{f(x + h) - f(x)}{h}, \quad h \neq 0$$

23. $f(x) = -2x^2 + x + 1$

24. $f(x) = 3x^2 - 2x + 4$

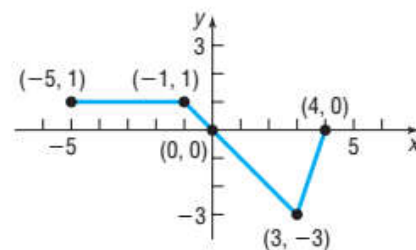
Exercise 5.

25. Using the graph of the function f shown:



- Find the domain and the range of f .
- List the intercepts.
- Find $f(-2)$.
- For what value of x does $f(x) = -3$?
- Solve $f(x) > 0$.
- Graph $y = f(x - 3)$.
- Graph $y = f\left(\frac{1}{2}x\right)$.
- Graph $y = -f(x)$.

26. Using the graph of the function g shown:



- Find the domain and the range of g .
- Find $g(-1)$.
- List the intercepts.
- For what value of x does $g(x) = -3$?
- Solve $g(x) > 0$.
- Graph $y = g(x - 2)$.
- Graph $y = g(x) + 1$.
- Graph $y = 2g(x)$.

Exercise 6.

In Problems 29–36, determine (algebraically) whether the given function is even, odd, or neither

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|--------------------------|--------------------------------------|--|----------------------------------|
| 29. $f(x) = x^3 - 4x$ | 30. $g(x) = \frac{4 + x^2}{1 + x^4}$ | 31. $h(x) = \frac{1}{x^4} + \frac{1}{x^2} + 1$ | 32. $F(x) = \sqrt{1 - x^3}$ |
| 33. $G(x) = 1 - x + x^3$ | 34. $H(x) = 1 + x + x^2$ | 35. $f(x) = \frac{x}{1 + x^2}$ | 36. $g(x) = \frac{1 + x^2}{x^3}$ |

Exercise 7.

In Problems 43–46, find the average rate of change from 2 to $2 + h$ for each function

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|---------------------|-----------------------|------------------------|---------------------------|
| 43. $f(x) = 2 - 5x$ | 44. $f(x) = 2x^2 + 7$ | 45. $f(x) = 3x - 4x^2$ | 46. $f(x) = x^2 - 3x + 2$ |
|---------------------|-----------------------|------------------------|---------------------------|

Exercise 8.

In Problems 55–66, graph each function using the techniques of shifting, compressing or stretching, and reflections. Identify any intercepts on the graph. State the domain and, based on the graph, find the range.

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|----------------------------|----------------------------|-----------------------------|------------------------------|
| 55. $F(x) = x - 4$ | 56. $f(x) = x + 4$ | 57. $g(x) = -2 x $ | 58. $g(x) = \frac{1}{2} x $ |
| 59. $h(x) = \sqrt{x - 1}$ | 60. $h(x) = \sqrt{x} - 1$ | 61. $f(x) = \sqrt{1 - x}$ | 62. $f(x) = -\sqrt{x + 3}$ |
| 63. $h(x) = (x - 1)^2 + 2$ | 64. $h(x) = (x + 2)^2 - 3$ | 65. $g(x) = 3(x - 1)^3 + 1$ | 66. $g(x) = -2(x + 2)^3 - 8$ |

Exercise 9.

In Problems 67–70,

- Find the domain of each function.
- Based on the graph, find the range.

- Locate any intercepts.
- Is f continuous on its domain?

(c) Graph each function.

$$67. f(x) = \begin{cases} 3x & \text{if } -2 < x \leq 1 \\ x + 1 & \text{if } x > 1 \end{cases}$$

$$68. f(x) = \begin{cases} x - 1 & \text{if } -3 < x < 0 \\ 3x - 1 & \text{if } x \geq 0 \end{cases}$$

$$69. f(x) = \begin{cases} x & \text{if } -4 \leq x < 0 \\ 1 & \text{if } x = 0 \\ 3x & \text{if } x > 0 \end{cases}$$

$$70. f(x) = \begin{cases} x^2 & \text{if } -2 \leq x \leq 2 \\ 2x - 1 & \text{if } x > 2 \end{cases}$$

Exercise 10.

71. A function f is defined by

$$f(x) = \frac{Ax + 5}{6x - 2}$$

If $f(1) = 4$, find A .

72. A function g is defined by

$$g(x) = \frac{A}{x} + \frac{8}{x^2}$$

If $g(-1) = 0$, find A .

Exercise 11.

A page with dimensions of $8\frac{1}{2}$ inches by 11 inches has a border of uniform width x surrounding the printed matter of the page, as shown in the figure

- (a) Develop a model that expresses the area A of the printed part of the page as a function of the width x of the border.
- (b) Give the domain and the range of A .
- (c) Find the area of the printed page for borders of widths 1 inch, 1.2 inches, and 1.5 inches.
- (d) Graph the function $A = A(x)$

Exercise 12.

A rectangle has one vertex in quadrant I on the graph of $y = 10 - x^2$, another at the origin, one on the positive x -axis, and one on the positive y -axis.

- (a) Express the area A of the rectangle as a function of x .
- (b) Find the largest area A that can be enclosed by the rectangle.

Exercise 4: 23

Exercise 5: 25

Exercise 6: 30, 32

Exercise 7: 44

Exercise 8: 61, 62

Exercise 9: 67, 68

Exercise 10: 71,

Exercise 11

Exercise 12