

CHAPTER 6 – EXPONENTIAL AND LOGARITHMIC FUNCTIONS

Exercise 1.

In Problems 1–6, for the given functions f and g find:

- (a) $(f \circ g)(2)$ (b) $(g \circ f)(-2)$ (c) $(f \circ f)(4)$ (d) $(g \circ g)(-1)$
1. $f(x) = 3x - 5$; $g(x) = 1 - 2x^2$ 2. $f(x) = 4 - x$; $g(x) = 1 + x^2$ 3. $f(x) = \sqrt{x+2}$; $g(x) = 2x^2 + 1$
4. $f(x) = 1 - 3x^2$; $g(x) = \sqrt{4-x}$ 5. $f(x) = e^x$; $g(x) = 3x - 2$ 6. $f(x) = \frac{2}{1+2x^2}$; $g(x) = 3x$

Exercise 2.

In Problems 7–12, find $f \circ g$, $g \circ f$, $f \circ f$, $g \circ g$ for each pair of functions. State the domain of each composite function.

7. $f(x) = 2 - x$; $g(x) = 3x + 1$ 8. $f(x) = 2x - 1$; $g(x) = 2x + 1$ 9. $f(x) = 3x^2 + x + 1$; $g(x) = |3x|$
10. $f(x) = \sqrt{3x}$; $g(x) = 1 + x + x^2$ 11. $f(x) = \frac{x+1}{x-1}$; $g(x) = \frac{1}{x}$ 12. $f(x) = \sqrt{x-3}$; $g(x) = \frac{3}{x}$

Exercise 3.

In Problems 13 and 14, (a) verify that the function is one-to-one, and (b) find the inverse of the given function.

13. $\{(1, 2), (3, 5), (5, 8), (6, 10)\}$ 14. $\{(-1, 4), (0, 2), (1, 5), (3, 7)\}$

Exercise 4.

In Problems 17–22, the function is one-to-one. Find the inverse of each function and check your answer.

17. $f(x) = \frac{2x+3}{5x-2}$ 18. $f(x) = \frac{2-x}{3+x}$ 19. $f(x) = \frac{1}{x-1}$
20. $f(x) = \sqrt{x-2}$ 21. $f(x) = \frac{3}{x^{1/3}}$ 22. $f(x) = x^{1/3} + 1$

Exercise 5.

In Problems 29–32, find the domain of each logarithmic function.

29. $f(x) = \log(3x - 2)$ 30. $F(x) = \log_5(2x + 1)$ 31. $H(x) = \log_2(x^2 - 3x + 2)$ 32. $F(x) = \ln(x^2 - 9)$

Exercise 6.

In Problems 33–38, evaluate each expression. Do not use a calculator.

33. $\log_2\left(\frac{1}{8}\right)$ 34. $\log_3 81$ 35. $\ln e^{\sqrt{2}}$ 36. $e^{\ln 0.1}$ 37. $2^{\log_2 0.4}$ 38. $\log_2 2^{\sqrt{3}}$

Exercise 7.

In Problems 39–44, write each expression as the sum and/or difference of logarithms. Express powers as factors

39. $\log_3\left(\frac{uv^2}{w}\right)$, $u > 0, v > 0, w > 0$ 40. $\log_2(a^2\sqrt{b})^4$, $a > 0, b > 0$ 41. $\log(x^2\sqrt{x^3+1})$, $x > 0$
42. $\log_5\left(\frac{x^2+2x+1}{x^2}\right)$, $x > 0$ 43. $\ln\left(\frac{x\sqrt[3]{x^2+1}}{x-3}\right)$, $x > 3$ 44. $\ln\left(\frac{2x+3}{x^2-3x+2}\right)^2$, $x > 2$

Exercise 8.

In Problems 45–50, write each expression as a single logarithm.

$$45. 3 \log_4 x^2 + \frac{1}{2} \log_4 \sqrt{x}$$

$$46. -2 \log_3 \left(\frac{1}{x} \right) + \frac{1}{3} \log_3 \sqrt{x}$$

$$47. \ln \left(\frac{x-1}{x} \right) + \ln \left(\frac{x}{x+1} \right) - \ln(x^2 - 1)$$

$$48. \log(x^2 - 9) - \log(x^2 + 7x + 12)$$

$$49. 2 \log 2 + 3 \log x - \frac{1}{2} [\log(x+3) + \log(x-2)]$$

$$50. \frac{1}{2} \ln(x^2 + 1) - 4 \ln \frac{1}{2} - \frac{1}{2} [\ln(x-4) + \ln x]$$

Exercise 9.

In Problems 75–82, solve each equation

$$75. 8 = 4^{x^2} \cdot 2^{5x}$$

$$76. 2^x \cdot 5 = 10^x$$

$$77. \log_6(x+3) + \log_6(x+4) = 1$$

$$78. \log(7x - 12) = 2 \log x$$

$$79. e^{1-x} = 5$$

$$80. e^{1-2x} = 4$$

$$81. 9^x + 4 \cdot 3^x - 3 = 0$$

$$82. 4^x - 14 \cdot 4^{-x} = 5$$

Exercise 10.

A child's grandparents purchase a \$10,000 bond fund that matures in 18 years to be used for her college education. The bond fund pays 4% interest compounded semiannually. How much will the bond fund be worth at maturity? What is the effective rate of interest? How long will it take the bond to double in value under these terms?

Exercise 11.

Growth of Bacteria The number N of bacteria present in a culture at time t (in hours) obeys the law of uninhibited growth $N(t) = 1000e^{0.01t}$

- Determine the number of bacteria at $t = 0$ hours
- What is the growth rate of the bacteria?
- What is the population after 4 hours?
- When will the number of bacteria reach 1700?
- When will the number of bacteria double?

Exercise 1. 2; 4

Exercise 2. 8; 10

Exercise 3. 14

Exercise 4. 18; 20

Exercise 5. 30; 32

Exercise 6. 34; 36

Exercise 7. 40; 42;

Exercise 8. 46; 48

Exercise 9. 76; 78; 80; 82

Exercise 10

Exercise 11.