

## CHAPTER VIII: Sequences; Induction; the Binomial Theorem

### Exercise 1.

In Problems 13–24, determine whether the given sequence is arithmetic, geometric, or neither. If the sequence is arithmetic, find the common difference and the sum of the first  $n$  terms. If the sequence is geometric, find the common ratio and the sum of the first  $n$  terms.

**13.**  $\{a_n\} = \{n + 5\}$

**14.**  $\{b_n\} = \{4n + 3\}$

**15.**  $\{c_n\} = \{2n^3\}$

**16.**  $\{d_n\} = \{2n^2 - 1\}$

**17.**  $\{s_n\} = \{2^{3n}\}$

**18.**  $\{u_n\} = \{3^{2n}\}$

**19.**  $0, 4, 8, 12, \dots$

**20.**  $1, -3, -7, -11, \dots$

**21.**  $\frac{3}{2}, \frac{3}{4}, \frac{3}{8}, \frac{3}{16}, \dots$

**22.**  $5, -\frac{5}{3}, \frac{5}{9}, -\frac{5}{27}, \frac{5}{81}, \dots$

**23.**  $\frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \dots$

**24.**  $\frac{3}{2}, \frac{5}{4}, \frac{7}{6}, \frac{9}{8}, \frac{11}{10}, \dots$

### Exercise 2.

In Problems 25–30, find each sum

**25.**  $\sum_{k=1}^{50} (3k)$

**26.**  $\sum_{k=1}^{30} k^2$

**27.**  $\sum_{k=1}^{30} (3k - 9)$

**28.**  $\sum_{k=1}^{40} (-2k + 8)$

**29.**  $\sum_{k=1}^7 \left(\frac{1}{3}\right)^k$

**30.**  $\sum_{k=1}^{10} (-2)^k$

### Exercise 3.

In Problems 37–40, find a general formula for each arithmetic sequence.

**37.** 7th term is 31; 20th term is 96

**38.** 8th term is  $-20$ ; 17th term is  $-47$

**39.** 10th term is 0; 18th term is 8

**40.** 12th term is 30; 22nd term is 50

### Exercise 4.

In Problems 49–54, use the Principle of Mathematical Induction to show that the given statement is true for all natural numbers

**49.**  $3 + 6 + 9 + \dots + 3n = \frac{3n}{2}(n + 1)$

**50.**  $2 + 6 + 10 + \dots + (4n - 2) = 2n^2$

**51.**  $2 + 6 + 18 + \dots + 2 \cdot 3^{n-1} = 3^n - 1$

**52.**  $3 + 6 + 12 + \dots + 3 \cdot 2^{n-1} = 3(2^n - 1)$

**53.**  $1^2 + 4^2 + 7^2 + \dots + (3n - 2)^2 = \frac{1}{2}n(6n^2 - 3n - 1)$

**54.**  $1 \cdot 3 + 2 \cdot 4 + 3 \cdot 5 + \dots + n(n + 2) = \frac{n}{6}(n + 1)(2n + 7)$

### Exercise 5.

In Problems 57–60, expand each expression using the Binomial Theorem.

**57.**  $(x + 2)^5$

**58.**  $(x - 3)^4$

**59.**  $(2x + 3)^5$

**60.**  $(3x - 4)^4$

**Exercise 6.**

61. Find the coefficient of  $x^7$  in the expansion of  $(x + 2)^9$ .
62. Find the coefficient of  $x^3$  in the expansion of  $(x - 3)^8$ .
63. Find the coefficient of  $x^2$  in the expansion of  $(2x + 1)^7$ .
64. Find the coefficient of  $x^6$  in the expansion of  $(2x + 1)^8$ .

**Exercise 7.**

- 1) Find the coefficient of  $x^{10}$  in the expansion of  $\left(3x + \frac{1}{x}\right)^{20}$
- 2) Find the coefficient of  $x^{10}$  in the expansion of  $\left(x^2 - \frac{5}{x}\right)^{20}$

**HOMEWORK****Exercise 1: 14, 16****Exercise 2: 26, 28, 30****Exercise 3: 38, 40****Exercise 4: 50, 52, 54****Exercise 5: 58, 60****Exercise 6: 62, 64****Exercise 7: 2**