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T1 ________________	**1920838**	F1 ________________
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T3 ________________	Problem Chosen	F3 ________________
T4 ________________	**A**	F4 ________________

2019
MCM/ICM
Summary Sheet

A Song of Man and Dragons

Summary

As *a Song of Ice and Fire* depicts the figure of three dragons, questions about raising dragons also arise. To solve them, we establish a mathematical model named **Dragon-DOES**. At the beginning, we defined the figure of the dragons by the novel. Comparing with similar animals, we get the dragon's characteristics. Then, we assumed that the dragons live in an island without people's interruption.

Initially, we get inspiration from **Compartment Model** and decide to see the problems in the view of **energy flow.** So we analyse the intake and output energy of a dragon. **The Logistics Model** is chosen to fit the existing data to reflect the mass growing along with the time.

Next, the energy condition of the initial island without dragons is analysed. We suppose the island can provide enough energy for them and then create DOES. DOES means **Directly-Obtained-Energy System**, referred to the aggregate made up of the energy which can be used by our dragons. Then we add the dragons to DOES. Given a certain proportion of energy released from dragons go into DOES, **the circle of energy** forms, then the Dragon-DOES model also forms.

Finally, we analyse the area dragons need. It is estimated that the area they need is no more than $1.1 \times 10^{10} m^2$, equivalent to 100 times the area of Washington D.C. And the predation area is about $3 \times 10^7 m^2$, equal to 10 common university campus.We also analyse different climate impacts and get some conclusions. When moving to a temperate area, the predation area decreases. While moving to an Arctic area, it increases as the temperature becomes lower. If the temperature decrease to $-70°C$, the predation area grows to $3.6 \times 10^7 m^2$.Our model can be used in providing strategies about protecting animals and establishing the nature reserve. We have discussed the plan for stocking Père David's Deer, finding that the area needed is about $250 km^2$.

Keywords: Energy Flow; Logistic model;Directly-Obtained-Energy System

To George R.R. Martin

January 28, 2019

Dear George R.R. Martin,

It is our pleasure to have a chance to write a letter to you talking about the three amazing dragons in your great novel A Song of Ice and Fire.Our team attending the MCM have design a mathematical model to describe a ideal ecology system having the ability to support your three dragons basic living conditions on the basis of your description of them in your novel.

In our model, our main work is to analyse the whole ecology system in the respect of energy flow. Our conclusions consist of the area the dragons existing need and how much food they consume everyday in different stages of living.

Just as what you wrote in your novel,the dragon would continue to grow before it is killed. So it is quite rational to see that the area is increasing during their long life. We found that the maximum area of the mature dragon need is about , which is equal to the area of one tenth of Jiangsu Province in China. It makes it impossible for human beings nowadays to create such a specific area to raise the three dragons. And you may add more concrete details in your novel to describe the environment which the Daenerys Targaryen need to take care of the three ones.

On the basis of this model, we calculate that the quantity of food the three dragons need. Balerion the Black Dread, the 200-year-old dragon who have conquered the world in the novel, is a mature one whose everyday consumption is about 300,000 sheep. Towards a 6-year-old dragon, who weights about $1 \times 10^5 kg$, its daily consumption of energy is about 1.368×10^{13} , which is equal to $3.2 \times 10^9 kcal$. Based on these exact numbers, you may estimate how much food the dragon really need and write some attractive words to reflect the dieting habits of the dragon.

When it comes to the climate influence on the dragons, we analyse our model ulteriorly and make some useful conclusions. Clearly, when the dragon moves from an arid area to a temperate one, the food is much more richer and the number of preys increases. This change contributes to the dragons necessary area decreases.

Similarly, our team also analyse the condition where the dragons move to Arctic regions, which means that the temperature of the environment decreases and there is less food supported for their life . In our model, we find that the two factors both contribute to the area the dragon needs being enlarged. It means the three dear dragons are hunting for their preys in a much large land when they move to Arctic regions, which is also accord with science logically. In your novel A Song of Ice and Fire, the Mother of Dragons travel from arid regions to temperate regions taking the three dragons with herself, and then to Arctic regions. In this experience, the three dragons everyday habits and the area they need to support for their lives would change as what our model predict, which would help you add some more realistic information to your wonderful novel.

Thank you for your great novel which brings this interesting questions to us. Our model analyse the three dragons living habits from a respect of science. We really hope that our analysis may help you write more details about how dragons live in the land.

Sincerely,

Team 1920838

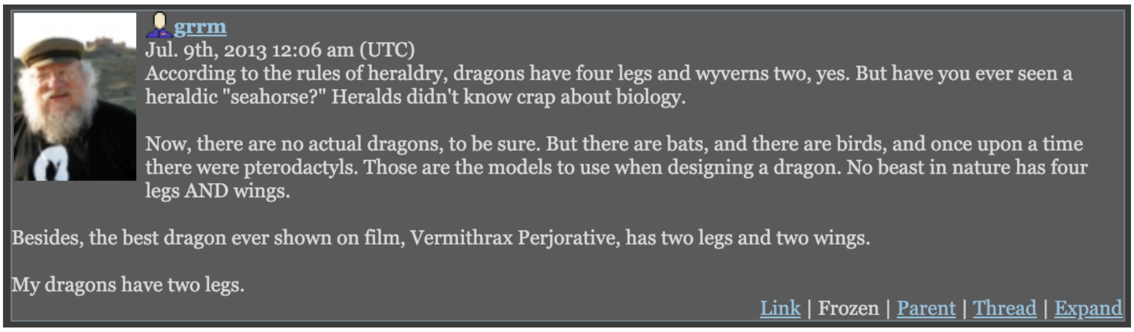
Contents

1 Introduction

1.1 Background

Before we discuss about anything else about dragon, we need to agree on the **definition of dragon**.

According to professor P. J. Hogarth [1] , even if dragons were real, they would be either *tetrapod* (two legs and two wings) or defined as a *myriapod* (four legs and two wings). By the **convergent evolution theory**, we use the former definition, which the writer of *A Song of Ice and Fire*, George R.R. Martin himself also agrees with: [2]


grrm
Jul. 9th, 2013 12:06 am (UTC)
According to the rules of heraldry, dragons have four legs and wyverns two, yes. But have you ever seen a heraldic "seahorse?" Heralds didn't know crap about biology.

Now, there are no actual dragons, to be sure. But there are bats, and there are birds, and once upon a time there were pterodactyls. Those are the models to use when designing a dragon. No beast in nature has four legs AND wings.

Besides, the best dragon ever shown on film, Vermithrax Perjorative, has two legs and two wings.

My dragons have two legs.

Link | Frozen | Parent | Thread | Expand

Figure 1: from G. R. R. M's blog

So we are actually talking about a vertebrata creature with 2 legs and a pair of wings like Figure 2 from the TV series, Game of Thornes. [3] Like George R. R.

(a) little dragon[3] (b) adult dragon[4]

Figure 2: Dragons

Martin said, this definition provides us with models (birds, bats, pterodactyls) to begin with ,which is proved to be very helpful in our problem solving process.

1.2 Restatement of the Problem

Consider that the three dragons are living today, do the following:
Analyze dragon characteristics, behaviour, habits, diet, and interaction with their

environment. To do that, we need to consider the following qualities of the dragon:

- ecological impact and requirements
- energy expenditures
- caloric intake requirements
- How large area is needed
- How large community is needed according to the varying level of external assistance

Additionally, dragons might travel to **different regions** due to their ability to fly. We are required to determine how the natural conditions will affect our analysis of the dragons.

Last but not least, we should give an example about ways of applying our mathematical modelling to a **realistic question**. And we have chosen to analyze how much area is required to build a nature reserve.

2 Assumptions

Dragon is a living organism, so there should be a lot of similarities between dragons and other creatures.

- Dragon matches the description of vertebrata (as said before, **two legs and two wings**), its skin is like dinosaurs and crocodiles. It has hard scale armour, claws.
- Dragon has metabolism. And it can only obtain energy by **eating and digesting**. Its temperature is related to temperature of the fire it breathes.
- Dragon has homoeostasis. And due to its high resistance, the surrounding conditions including both physical and chemical can do **no harm** to it.
- Dragon has infinity life. Moreover, they **continue to grow** throughout its life.

After specified information about dragon is given, we can start to crack the problem by some basic assumptions:

- Dragon **continue to grow** throughout its life. And it won't be attacked or killed by other creature due to its high niche.
- Dragon will only obtain energy by **eating meat**.
- Dragon's density keeps **constant** throughout its life.
- Dragon keeps eating every day.

3 Nomenclature

Notations	Meaning	Units
$P_{absorbed}$	the energy absorbed per time unit	W
P_m	mechanical power	W
$\bar{P}_{mechanical}$	mechanical	$24 \times 3600W$
$P_h(P_{heat})$	thermal power	W
P_{growth}	the energy gained through weight	W
m	weight	kg
V	volume	m^3
L	the length of the dragon's body	m
T_{ds}	the surface temperature of the dragon	$^{\circ}C$
T_{dc}	the core temperature of the dragon	$^{\circ}C$
T_{cs}	the surface temperature of the cow	$^{\circ}C$
T_{cc}	the core temperature of the cow	$^{\circ}C$
T_0	environmental temperature	$^{\circ}C$
v	dragon's flying velocity	m/s
k_f	air resistance coefficient	/
P_f	the power of friction	W
P_0	power of other movements	W
ω	surface roughness coefficient	/
Q	the loss/generated heat of the dragon	J
t	time	/
S_{gekko}	surface area of the gekko	m^2
m_{gekko}	weight of the gekko	kg
c	specific heat capacity	$J/^{\circ}C \cdot kg$
m_m	maximum weight	kg
m_0	integral constant	/
α_E	digestibility coefficient	/
γ	hunting rate	/
β_E	feedback rate	/
$k(t)$	the energy which can be directly obtained by dragon	J/m^2
ST	the total area we provided to raise the dragon	m^2
$S(t)$	the area where the dragon hunt for food	m^2
$t_{hunting}$	the time length the dragon use for hunting per day	d
d_{dragon}	the wing span of the dragon	m
r_{DOES}	increasing rate of DOSE	/
η	absorbable energy	J/kg
k_{max}	Maximal energy in DOES contained in the environment	m^2

4 Model Foundation

We hope to find inner relation between the dragon and ecology. To do this, we discussed about this problem in a pretty distinct aspect, which is energy. According to the **energy conservation law**, we have the following equation:

$$P_{absorbed} = \bar{P}_{\text{mechanical}} + P_{\text{growth}} + P_{\text{heat}} \tag{1}$$

where $P_{absorbed}$ is the energy absorbed by the dragon. P_{growth} is a part of the energy which is used to gain weight. $\bar{P}_{\text{mechanical}}$ is a part of the energy which is used to do mechanical work. P_{heat} is a part of the energy which is used to keep the dragon's temperature.

4.1 Basic arguments of the dragon

According to our assumptions we have proposed, we add some more details to our descriptions of the three dragons and then obtain several important parameters on the basis of a few articles about these fields.

- **Volume**(V)**:** From the pictures of the television series Game of Thrones, we can tell that the precise proportion of the dragon's body.Through those pictures we find that the height of the actress is approximately half of the width of the 6-year-old dragon's skull. We make an assumption that the height of the woman is $1.70m$, and then we could use a sketchy way to get the volume of the 6-year-old dragon and then determine the volume of the other ones.
  By measuring the bodies and comparing the results, the length of the head and long neck is equal to 5 times of the height of the actress, which means it is equal to $8.5m$.Similarly, the torso is about $39.5m$. And the wingspan of the dragon is about 6.67 times of the length of head and neck, equal to $56.7m$.
  Approximately, we consider the shape of the dragon without wings is a cylindrical, and the shape of the single wing is a cylinder with $0.1m$ as the thickness. As a result,the volume of the 6-year-old dragon is $566.8m^3$.

  Under our assumption, despite the increasing weight of the dragon during the whole period of its life, the density of its body is fixed. So if the length of its body is L ,the volume V of it is proportional to L^3.

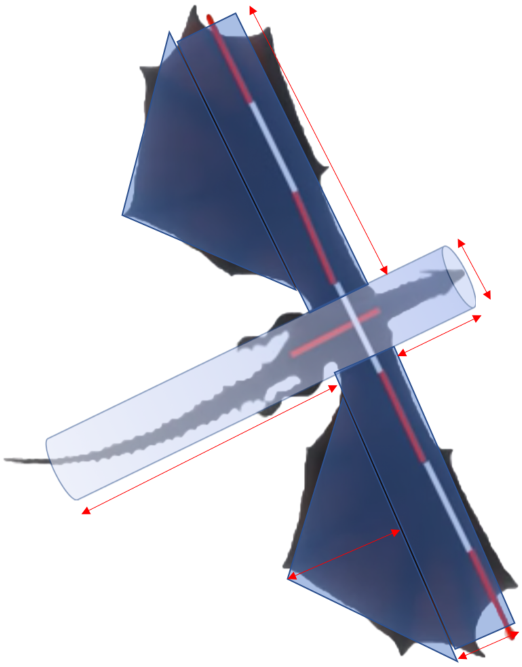


Figure 3: Estimate of the surface area

- **Surface area**(S_{dragon})**:** The method we used here is the same as the way we choose to calculate the volume of the dragon. Under the same simplifying model, we figured out that the surface area of the dragon is $1315.77m^2$.
- **Temperature of the dragon's body:** As what the novel describe, the dragon has the ability to breath fire. Eliminating the existence of supernatural factors, we think the fire is caused by flammable gas. Considering that most of these gases are poisonous, methane is the best choice.AS the ignition point of methane is about $530°$C, we attribute $500°$C to the dragon as its inner body temperature.
Through finding some articles, we obtain the way to get body surface temperature on the basis of the inner one [5].
Considering the properties of heat conduction and the analogy between the dragon and cow, we have:

$$\frac{T_{\mathrm{ds}} - T_0}{T_{\mathrm{dc}} - T_0} = \frac{T_{\mathrm{cs}} - T_0}{T_{\mathrm{cc}} - T_0} \tag{2}$$

where

  - T_{ds} :the surface temperature of dragon.
  - T_{dc} :the core temperature of dragon.
  - T_{cs} :the surface temperature of cow.
  - T_{cc} :the core temperature of cow.

By the analogy between the dragon and cow, we could get the surface temperature of the dragon is about $352.46°$C

- **Specific heat capacity of the muscle and the content of energy:** As the dragon is an ideal creature, we can't find exact data about it. As a result, we use the analogy method and utilize some related data of tuna as its content of protein is comparatively high. According to the database, the specific heat capacity of the dragon's muscle is $3J/(g \cdot K)$ and the content of energy is about $8.3 \cdot 10^6 J/kg$.

- **digestibility coefficient** (α):According a research[14], the digestibility coefficient of lamb is about $0.5 - 0.7$. Considering our basic assumption, we choose 0.6 as the digestibility coefficient of the dragon.

- **Hunting rate** (γ): Considering that dragon is excellence at hunting for food ,we let $\gamma = 0.3$.

- **Absorb-able energy in unit mass**(η):We use η of tuna during our calculation, which is $8.3 \times 10^6 J/kg$.

4.2 Basic argument of the DOES that can be contained in the environment

- **feedback rate** (β_E): Since the feedback energy decrease constantly as its passed to DOES, only a small part of energy can result in the feedback. Therefore, we let $\beta_E = 0.01$

- **Maximal energy per m^2 in DOES that can be contained in the environment**(k_{max}): In order to decide the value of K_{max}, we refer to a research[6] about the maximal number of rabbits in Australia, which is denoted as N_{rmax}. Using the statistics, we can calculate K_{max} as following:

$$k_{max} = \frac{\eta \cdot m_{rabbit} \cdot N_{rmax}}{S_{Australia}} = 7.6 \times 10^6 \quad J/m^2 \tag{3}$$

  Where m_{rabbit} is the mass of a single rabbit and $S_{Australia}$ is the total area of Australia.

- **Increasing rate of DOES**(r_{DOES}): We let the Increasing rate be $\frac{1}{300}$ according to the rate of rabbit.

4.3 Mechanical power

Since our dragon needs to feed on meat, it will have to hunt. And the dragon moves from here to there only by flying. So we need to its calculate energy consumed by flying. After research, we found that when something is flying very fast, there is no need to concern about the specific movement of its wings.[8]

So eventually, based on the assumption that a six-year-old dragon has the size of an Boeing 747.

Figure 4: settings from the TV series *Game of Thrones*

So we take several types of Boeing 747's [1] standard to have an estimate of the dragon's speed. So the formula of velocity is:

$$v = \frac{6m}{10^5} + 201 \tag{4}$$

Here v is the flying velocity of the dragon and m is the mass of the dragon.

The dragon has to keep balanced when flying. So here we use the free body diagram of bats to represent dragon's:

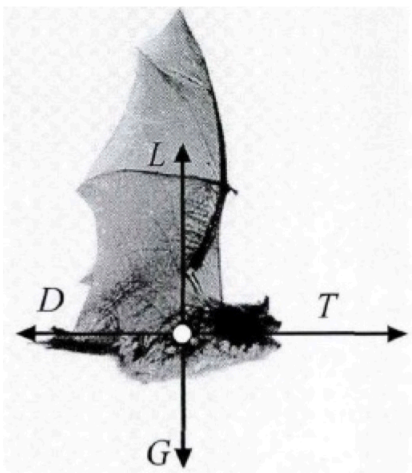


Figure 5: Free body diagram of bat

The flying process can be cut down to two parts-uplifting and horizontal flying. Because the uplifting height is relatively small compared to the flying distance, so we can neglect the influence of uplifting. When the dragon is flying in uniform rectilinear motion horizontally, the drag (D) of the dragon equals its thrust (T). And the gravity (G) equals the dragon's lift (L). Because the dragon is moving horizontally, so the gravity and thrust do zero work. According to

[1] Boeing 747-100, 747-200, 747-300, 747-400.

work-energy principle, we can have the following equation between drag and velocity:

$$D = k \cdot v \tag{5}$$

where k is around 0.08 for plane and 0.1-0.2 for birds. Due to the streamline design of the plane, we use the argument k of birds, roughly taking $k = 0.15$. We define the mechanical power of the dragon to be:

$$P_m = P_f + P_0 \tag{6}$$

where P_0 is some constant power of dragon doing other activities. And P_f is :

$$\begin{aligned} P_f &= T \cdot v \\ &= D \cdot v \\ &= k \cdot v^2 \\ &= k \cdot (\frac{6m}{10^5} + 201)^2 \end{aligned} \tag{7}$$

To simplify our model, we assume that the dragon flies only aiming to attain food, which means that $P_m = P_f$ and $P_0 = 0$.

$$S.t. \quad P_m = P_f = k \cdot (\frac{6m}{10^5} + 201)^2$$

4.4 Thermal power

Under our analysis, we attribute the energy used in containing the basic daily physiological activities to the heat which is produced every single day. On the basis of endotherm's characteristics, we consider that there is a constant balance between the heat produced and the heat lost. **The quantity of the heat production** is equal to that of **heat dissipation**. So we explore the **later one** to figure out the former one.

As what we have told before, the temperature of the dragon's surface is determined.According to the existing data about the study of gekko japonicus [7] and Newton Cooling Theory, it is easy for us to get the following formula:

$$\frac{\mathrm{dQ}}{\mathrm{dt}} = c \cdot m_{\text{gekko}} \frac{\mathrm{dT}}{\mathrm{dt}} = \omega S_{\text{gekko}} \left(T - T_0\right) \tag{8}$$

In this formula, ω is a constant which is only related to the extent of roughness of bodies surface. We suppose that the extent of dragon's surface roughness is the same as that of gekko japonicus. So ω of the two ones is fixed.

In this experiment, the temperature of the theromostat T_0 coincide with this formula:

$$T_0 = 28 - t \quad (^\circ C) \tag{9}$$

Through equation (1),(2), we define:

$$\epsilon = \frac{\omega \cdot S_{\text{gekko}}}{c \cdot m_{\text{gekko}}} = 1.31394 \tag{10}$$

According to the definition of specific heat capacity, we can also get:

$$P_h = \frac{\mathrm{dQ}}{\mathrm{dt}} = \omega \bigtriangleup \mathrm{T} S_{\mathrm{dr}} \tag{11}$$

Ultimately, the efficiency of heat production is:

$$P_h = \frac{\mathrm{dQ}}{\mathrm{dt}} = c \cdot \epsilon \cdot m_{\mathrm{gekko}} \cdot \frac{S_{\mathrm{dragon}}}{S_{\mathrm{gekko}}} \cdot \bigtriangleup \mathrm{T} \tag{12}$$

4.5 The growth curve model

As is quoted from the novels *A Song of Ice and Fire*[**?**],"A dragon never stops growing, Your Grace, so long as he has food and freedom". We manage to build a model under the assumption that the three dragon always has food and freedom.

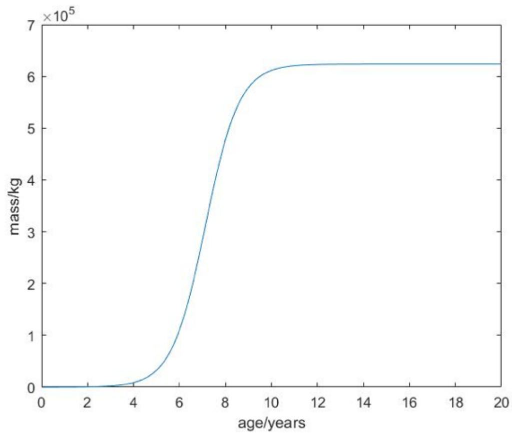
However, we are still interested in whether the dragon always grow at the same rate, which is obviously impossible on the Earth. According a study about A Song of Ice and Fire [13], we obtain the size of Drogon(6 years old) and Balerion the Black Dread(200 years old).

Using the statistics provided from the study ,we find out that the rate of growth decrease as the dragon become older. Out of three main methods used to obtain the growth curve, namely Logistic, Gompenz, Bertalan, we choose Logistic model, which is the most commonly used one, to fit the growth curve of the dragon:

$$\frac{\mathrm{dm}}{\mathrm{dt}} = r\left(1 - \frac{m}{m_m}\right)m \tag{13}$$

$$m(t) = \frac{m_m}{\left(\frac{m_m}{m_0} - 1\right)e^{-\mathrm{rt}} + 1} \tag{14}$$

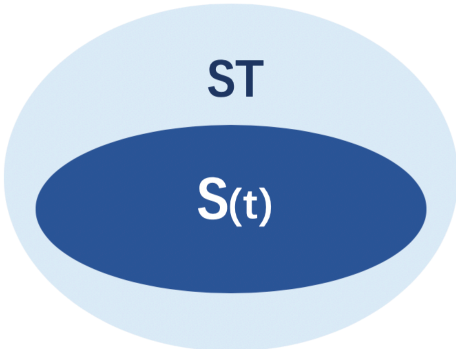
Where m_m, m_0, r are constant. Using the statistics about the age and corresponding size, we use MATLAB to fit the curve and get: $m_m = 1.6461 \times 10^5$,$\frac{m_m}{m_0} - 1 = 1.646 \times 10^4$, $r = \frac{1.362}{365}$, where we use day to measure time. The result of fitting is illustrated as follow:



When fitting the curve we use the statistics provided in the problem, namely the dragon is roughly $10kg$ when hatched. To verify the result, we let t=1 year, and get mass=$38kg$, which fits the statistics provided by the problem that they grow to roughly $30 - 40kg$ after a year.

5 Model Development

To simplify the problem, we assume the food in the area spread so fast that food always equally distributed in ST. As a matter of fact, since the center of S(t) will constantly change, this assumption is rational.



- **S(t)**: the area where the dragon hunts for food
- **ST**: The total area we provided to raise the dragon

We then let $k(t)$ stand for the energy that can directly obtained by dragon per unit area. However, when hunting, a dragon will not eat all the food in $S(t)$, we use γ to describe the coefficient. Also, not all the energy a dragon eat can be absorbed, we use α_E to describe the coefficient.

5.1 Simplification of the problem

In order to know the trend of S(t),when the dragons are still young, we firstly consider a simplified model, where we assume ST is large enough and $k(t)$ do not change over time, which means we assume $k(t) = k_c$ is a constant. Under such assumption, according to Energy Conservation Law, we can easily get:

$$P_{absorbed} = \alpha_E \gamma S(t) k_c = \bar{P}_{\text{mechanical}} + \bar{P}_{\text{growth}} + P_{\text{heat}} \tag{15}$$

Where we still use day to measure time, because the period of the hunting activity of dragon is $1day$. However, a dragon will not hunting for food all day long.

As a rough estimation, we have:

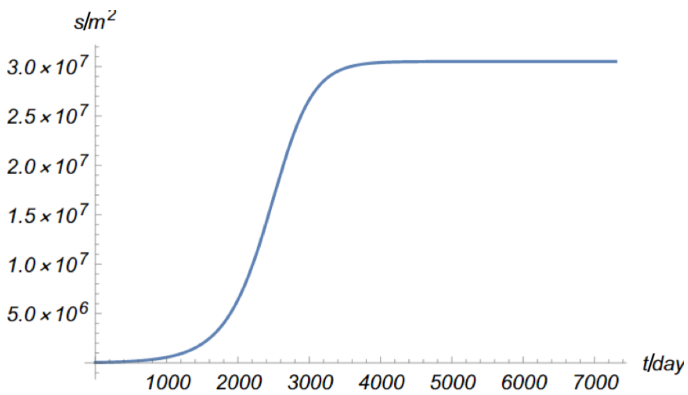
$$t_{\text{hunting}} = \frac{S(t)}{v \cdot \text{dragon}} \tag{16}$$

Where t_{hunting} stands for the time length the dragon use for hunting per day. By using such measurement, we calculate the average mechanical energy cost per day. And according to our assumption that a dragon only do work when it's hunting (flying), we have the following equation:

$$\bar{P}_{\text{mechanical}} = \bar{P}_{\text{hunting}} = P_{\text{hunting}} \frac{\text{hunting}}{1} \quad \text{J/day} \tag{17}$$

Using the result above, we can obtain:

$$\alpha_E \gamma S(t) k_c = (24 \times 60) \cdot k_Q \cdot m_{gecko} \cdot c_Q \cdot \frac{S_{dragon}}{S_{decko}} + k_f \cdot v^2 \cdot \frac{S(t)}{v \cdot d_{\text{dragon}}} + \eta \frac{\text{dm}}{\text{dt}} \tag{18}$$



5.2 Implementing the model

As has been mentioned above, $k(t) \cdot ST$ stand for the total energy that is directly obtained by the dragon from the area ST. Here we ignore the detailed structure of the ecosystem, which means we view all the energy, including aurochs, sheep and so on, that can be directly obtained by the dragon as a whole. To make it easy to write, we will call the system **DOES**(*Directly-Obtained-Energy System*) for short. Before we take the impact of the dragon into consideration, we firstly consider the DOES without dragons. Since the growing rate of DOES is proportional to its size and the rate will decrease when it growth beyond some specific size, we can also apply Logistic Model as follow:

$$\frac{\text{d}}{\text{dt}}(k(t)ST)|_{t=0} = k(t) r_{\text{DOES}}(1 - \frac{k(t)}{k_{\max}})|_{t=0} = 0 \tag{19}$$

Which means:

$$k(t)|_{t=0} = k_{\max} \tag{20}$$

5.3 Dragon's DOES

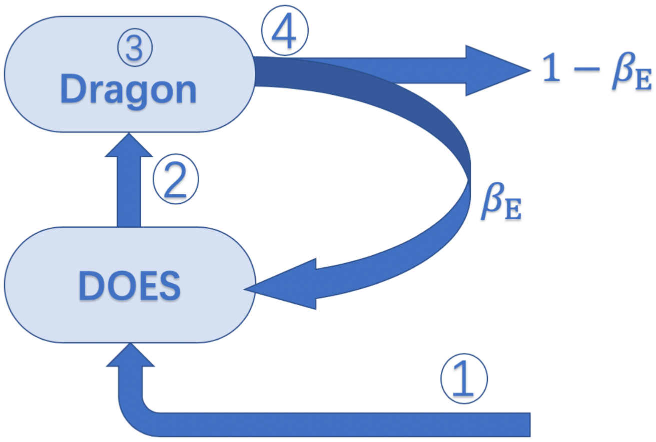


Figure 6: Dragon- DOES model

- ①: $r_{DOES} \cdot (1 - \frac{k(t)}{k_{max}}) \cdot k(t)$
- ②: $\gamma \cdot k(t) \cdot S(t)$
- ③: $P_{growth} = \eta \frac{dm}{dt}$
- ④: upper part:

$$(1-\beta)((1-\alpha_E) \cdot \gamma \cdot k(t) \cdot S(t) + \bar{P}_{mechanical} + P_h)$$

lower part:

$$\beta((1-\alpha_E) \cdot \gamma \cdot k(t) \cdot S(t) + \bar{P}_{mechanical} + P_h)$$

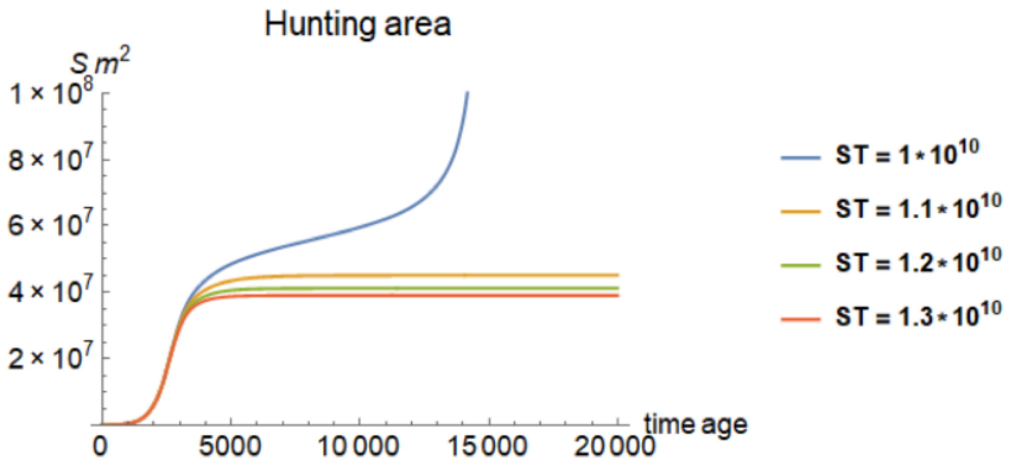
As is illustrated above, we manage to take the dragon into consideration. Firstly, DOES will increase correspond to its own size. The rate is given by Logistic Model. Secondly, part of the energy in DOES will pass to the dragon due to the hunting activities. However, not all this energy is absorbed by the dragon. We use α_E to denote the coefficient, which we call digestibility coefficient. Thirdly, part of the absorbed energy will stay in the body of the dragon, which cause the dragon to gain weight. Fourthly, the dragon will release some energy in several ways as is illustrated in the picture. However, part of the energy will be passed to DOES as feedback. This coefficient is denoted by β_E. Therefore, using day to measure time, we obtain:

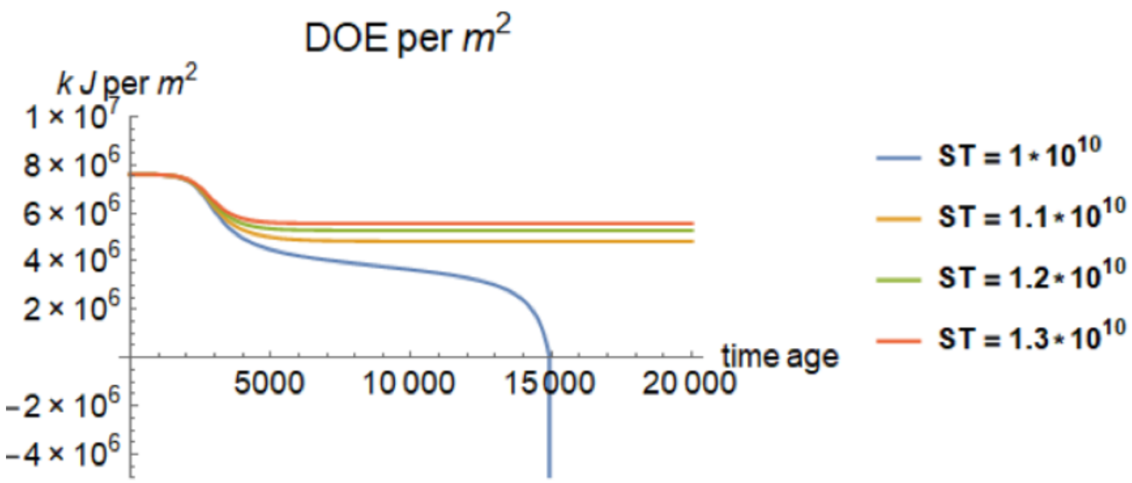
$$\alpha_E \cdot \gamma \cdot k_(t) \cdot S(t) = \eta \frac{dm}{dt} + k_f \cdot v^2 \cdot \frac{S(t)}{d_{\text{dragon}} \cdot v} + (24 \times 60) \cdot k_Q \cdot m_{\text{gecko}} \cdot c_Q \cdot \cdot \frac{S_{\text{dragon}}}{S_{\text{gecko}}} \cdot \Delta\text{T} \tag{21}$$

$$\frac{d}{dt}(k(t)ST) = \beta_E \left(\gamma k(t) S(t) - \eta \frac{dm}{dt} \right) + r_{\text{DOES}} \left(1 - \frac{k(t)}{k_{\max}} \right) k(t)ST - \gamma k(t) S(t) \tag{22}$$

$$k(t)|_{t=0} = k_{\max} \tag{23}$$

By solving the three simultaneous function above, we can obtain S(t) and k(t). Out of all the parameter, only ST can be changed. We choose different ST and plot S(t) and k(t) as following:

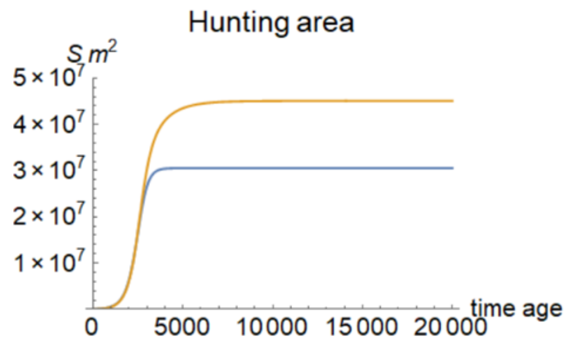




From the result above, we find that when $ST \leq 1 \times 10^{10}$, S(t) becomes diverging at some specific time and go beyond ST, which means the dragon grow too fast that the food in the area can't keep on supporting the dragon. To make full use of the area, we choose the smallest ST that satisfy the model.

Next we can take a look at the case that $ST \geq 1 \times 10^{10}$. For the former graph, we can see the area that the dragon hunting for food expand at first and gradually arrive a stable value. For the latter graph, we can see that before the dragon enter the ecosystem, DOE(Directly-Obtained Energy) has already achieve a stable value. Nevertheless, when the dragon enter the system, it break the balance and gradually form a new balance.

Lastly, we compare the result with the former model that view $k(t)$ as a constant. As is illustrate in the graph below, the dragon affect the ecosystem. As a result, the stable value of $S(t)$ become larger as we take $k(t)$ into consideration.



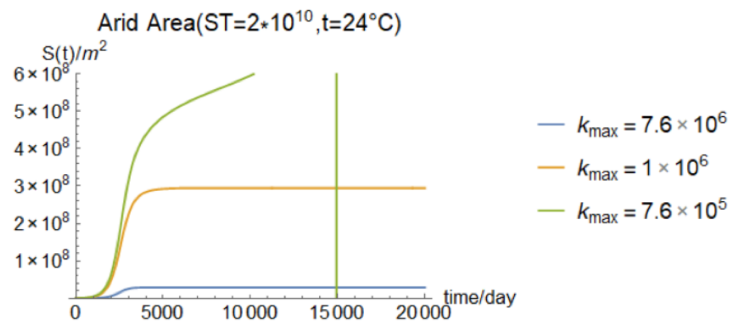
6 The Model Results

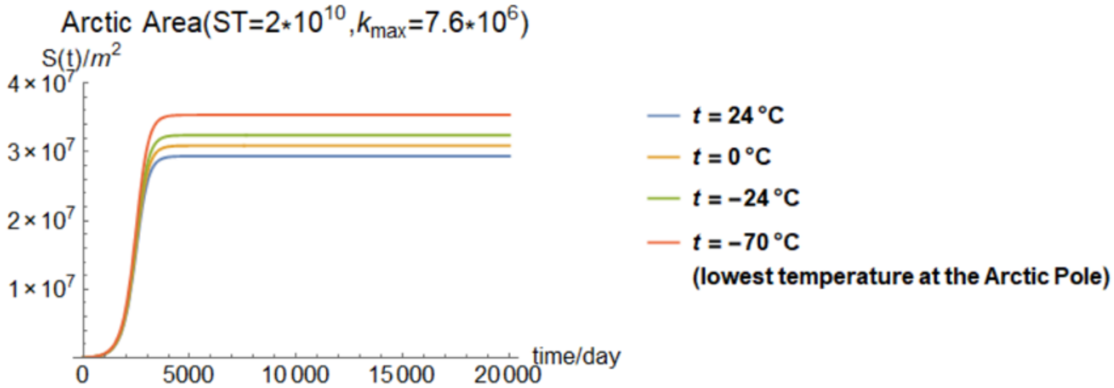
On the basis of our models towards this interesting questions, we get a series of exact conclusions. Now we have the relation among those factors we have utilized in our model:

$$P_{absorbed} = \bar{P}_{\text{mechanical}} + P_{\text{growth}} + P_{\text{heat}} \tag{24}$$

Before we get the ultimate outcomes, it is necessary to analyse the authenticity and accuracy of our models. Under our initial assumption, we just need to discuss the condition of a single dragon.

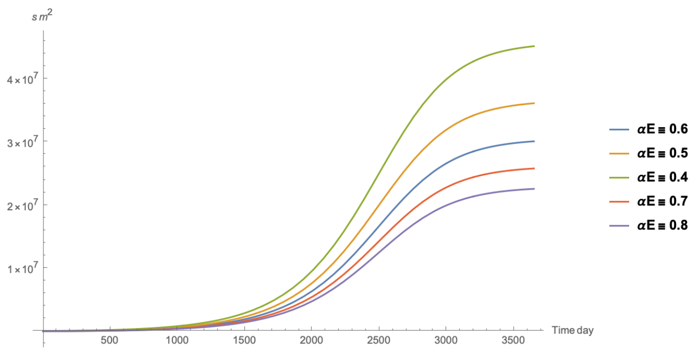
Then, we use to parameter to describe a environment, namely k_{max} and T_0. Roughly speaking, arid region has smaller k_{max} and arctic region has lower temperature T_0. Such impact can be illustrated as following:





Firstly, we reduce k_{max} to simulate arid area. As is shown in the former graph, S(t) increase as k_{max} decrease at a relatively high rate. When k_{max} decrease to 7.6×10^5, there exist a singular point, which means the environment can not support the dragon any more. Similarly, we also decrease the temperature in order to simulate Arctic area. It's obvious that the decrease of temperature also cause the rise of $S(t)$. Nevertheless, the impact of temperature is relatively small compared with the impact of k_{max}.

Secondly, we compare the impacts of different α_E with the other factors fixed. In this condition we get the following figure, which tells that the increase of α_E contributes to the decrease of the area a dragon needs. It means that when the ability of the dragon's absorption improved, the area supporting its daily life decrease. It is also logically correct.



Based on the above two tests which are comparatively accord with what we think about in advance, the model we put forward have a certain content of authenticity. It can describe the condition where the dragons live today approximately.

7 Insights provided

If the problem change to another endotherm animal community, our model still holds. All we need to do is input the arguments of average weight, the function of certain animals population changing with time, the energy consumed by mechanical work and the energy consumed by heating itself. By using our model, one can easily estimate that how much area is needed to raise a whole community of the animal, which is of huge importance in founding a nature reserve.

Now we are going to present an example of Elaphurus davidianus, which is also known as Père David's deer. It is a kind of deer which once went extinction in China. However, the restoration of this deer has made some effort since the importing of 38 deers in 1987. [10] In 2009, the population of the Père David's deer in the same area reached 500. In 2013, the number kept increasing to 1016.

So based on these three numbers and **logistic regressive model**, using the same method when we study the growth curve of the dragon, we can have the following graph:

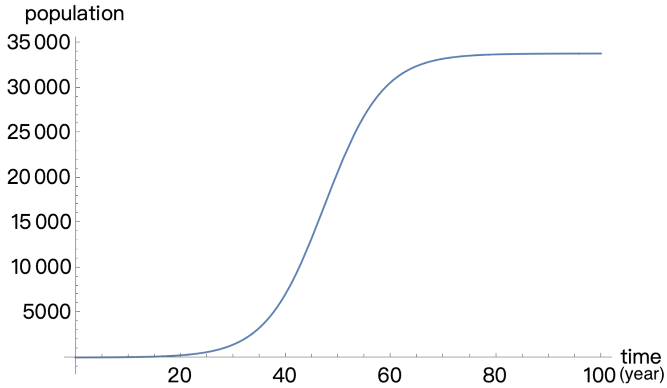


Figure 7: Population of the Père David's deer

And we take the average weight of the Père David's deer, which is around $150kg$. By using the average weight to multiply the population: we can have a graph which is very close to the growth curve of the dragon:

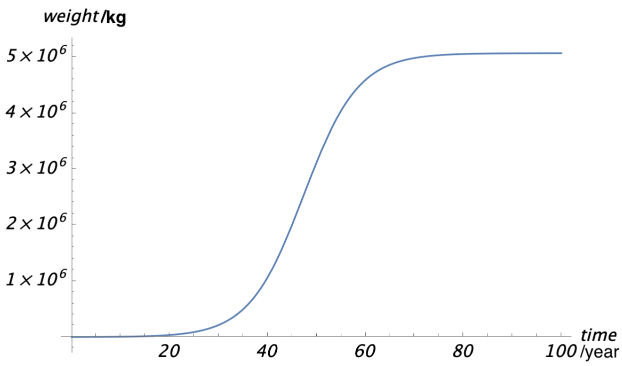


Figure 8: Total weight of the deers

Then, we are going to calculate the mechanical power of the deer. When the deer is walking in uniform rectilinear motion. The gravity(G) should equals the supporting force(N), and the thrust(T) equals the fraction(f).

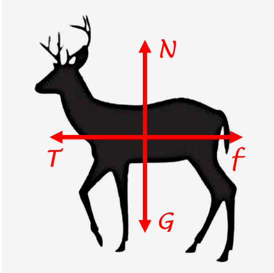


Figure 9: Free body diagram of the deer

According to **work-energy principle**, we consider the following formula:

$$P_m = \frac{W_{accelerating} + W_f + W_{deceleration}}{\triangle t} \tag{25}$$

where $W_{accelerating} = W_{deceleration} = E_k$, and E_k is the kinetic energy of the deer while walking.

However, the Père David's deer eats grass. So it doesn't need to run very fast, and it will walk a long distance through the day. So we can neglect the accelerating and deceleration process.
Therefore, the mechanical power of the deer becomes:

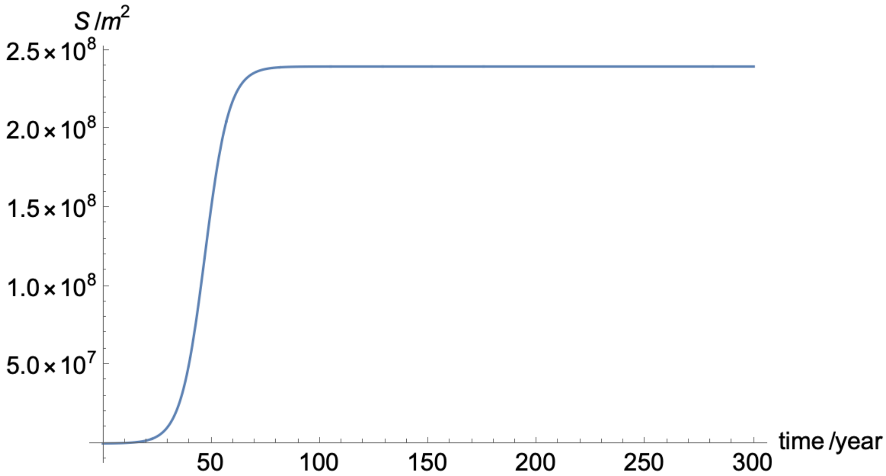
$$\begin{aligned} P_m &= \frac{W_f}{\triangle t} \\ &= f \cdot v \\ &= \mu m g v \end{aligned} \tag{26}$$

Here we take the $\mu = 0.4$, and $v = 3m/s$ (the velocity divergence between the adult and young deer is ignored due to their walking pace are both quite slow).

Then we can proceed to calculate the heating power of the deer. As Père David's deer is an concrete animal really existing in the world, it is comparatively easier for us to find a method to measure the heat production. According to the article we find[11], we get the data of the heat production of ochotona curzoniae, a kind of rabbit, on the basis of a empirical model measuring the daily energy budget of mammals.

The two animals are both mammals having similar habits of dieting, so we consider that their daily energy budget is a constant, equal to$1.2552KJ/(day{\cdot}Kg)$. The mean weight of Père David's deer is about $150Kg$, which means that the efficiency of the heat production of the Père David's deer is $1.883 \times 10^8 J/(day)$.

Now that all the arguments are set, along with the prepared coefficients of the ecosystem, we can calculate the living and hunting area of the Père David's deer. And the calculation process uses exactly our former model, which is omitted here. At the end, the result goes like this: You can see that if given k_c is about 7.8×10^7, one should need at least $250m^2$ to keep the population of Père David's deer prospering.



8 Conclusions

Through a series of assumptions based on the Question A, our model comes into being, having the ability to solve the concrete problems proposed and provide a few suggestions for the managers of the three dragons.

Based on our model, the dragons are at the top of food chain, absorbing nutrition by eating meat mostly and releasing heat as a reaction to the ecology system. Through eating meat, the dragons contribute to the stabilization of other animals population while satisfying the dragons own need of energy. The ecology system should be kept in a stable dynamic state, only in which the dragons would be able to continue to grow as the ideal velocity.

In our model, the energy expenditure consists of the heat production, the mechanical energy used in flying and the energy used in increasing the mass of the body. The heat production is related to the ratio of the surface area to the volume, which is also affected by the environment temperature. The mechanical energy is influenced by the mass and flying velocity, and the energy used in growing is proportional to the velocity of growth.

And the caloric intake requirements mostly referred to the energy from the rabbits, which is expressed as $\alpha_E \cdot \gamma \cdot k_c \cdot S(t)$ in our model. It is also affected by the dragon's ability of digesting and the predation ratio.

When we make an assumption that the area is sufficient, we get the the maximum area the dragons need is about $1.1 \times 10^{10} m^2$, which is equivalent to one tenth of the area of JiangSu province. In the real world, it is probable for people to find such an island and manage it well. At the same time, the predation area of dragons is about $3 \times 10^7 m^2$, which is equivalent to 10 times of the area of Nanjing University Xianlin Campus. This number is also rational.

On the basis of our model's analysis, we can find that the managers of the three dragons should create an ideal ecology system covering about $1.1 \times 10^{10} m^2$, having the potential to provide at least $7.6 \times 10^7 J$ energy in every square meter of the land. The managers most work is to help the island ecology system keep the dynamic balance, protecting it from interrupting of the external factors. The standards have been describe as the following formula:

$$\alpha_E \cdot \gamma \cdot k_(t) \cdot S(t) = \eta \frac{dm}{dt} + k_f \cdot v^2 \cdot \frac{S(t)}{d_{\text{dragon}} \cdot v} + (24 \times 60) \cdot k_Q \cdot m_{\text{gecko}} \cdot c_Q \cdot \frac{S_{\text{dragon}}}{S_{\text{gecko}}} \cdot \Delta \mathrm{T}$$

When some of the constants value differ from what we designed, the actual values used should satisfy this relation.

9 Strengths and Weaknesses

9.1 Strengths

- In over model, the ecosystem achieve a balance that is stable, which means when we apply a small interaction on the system, the system will go back to the former balance or achieve a new balance.
- Our model does well on practical model. The result fits the reality as is shown when we apply it to deer.

9.2 Weakness

- We didn't consider the dragon's other activities(movements) besides hunting and hibernation.Take hibernation as an example. When hibernation, a dragon doesn't consume mechanical energy, which is the same as when the dragon is not hunting. Therefore, we improve our model by adjust the coefficient when we calculate the average mechanical energy cost per day.
- We assumed that the dragon is powerful enough to get exactly how much energy it needs and only takes what it need. However, in reality, the environment will effect the growing rate of the dragon. We can add another differential equation to describe the effect.
- We also ignore some environmental factor that affect the dragon's energy consumption like weather. For example, raining will bring extra heat from the dragon. We need to rebuild a new model to describe the energy cost during raining day and use the raining frequency to adjust our model.

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