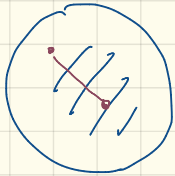
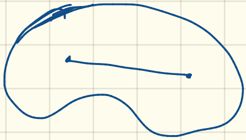
convex set



convex



non-convex

Let $x, y \in R^n$. The line segement between x and y in the set

$$\{(1-\lambda)x + y : \lambda \in [0,1]\}.$$

A set $C \subseteq R^n$ is convex if $\forall x, y \in C$, C contains the line segment between x and.

Consider the set

$$H = \{x \in R^n : a^T x \geq \beta\}$$

Halfspace, a set of points satisfies a single linear inequality with the conditions define the set coefficiensies are not all zero.

$$\boxed{\text{where } a \in R^n,\ a \neq 0 \quad \beta \in R}$$

Take $x, y \in H$,

consider $z = (1-\lambda)x + \lambda y$ for some $\lambda \in [0,1]$

Then $a^T z = a^T\big((1-\lambda)x + \lambda y\big)$

apply distribute law

You can treat the word polyhedron as a fancy name for the feasible region of a linear programming problem.

The set of feasible solutions are going to be convex.

$$a^T z = \underbrace{(1-\lambda)}_{\geq 0}\underbrace{a^T x}_{\geq \beta} + \underbrace{\lambda}_{\geq 0}\underbrace{a^T y}_{\geq \beta}$$

$$\geq (1-\lambda)\beta + \lambda\beta = \beta$$

$a^T z \geq \beta \implies z \in H$, so H is convex

Fact.

due to this fact

The intersection of two convex sets is convex.

A well studied problem.

A polyhedron is the intersection of finitely many halfspace. This set is called plyhedron.

e.g $\left\{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} : x_1 + 2x_2 \geq 5\right\} \cap \left\{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} : -2x_1 + x_2 \geq 0\right\}$

$$= \left\{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} : \begin{array}{l} x_1 + 2x_2 \geq 5 \\ -2x_1 + x_2 \geq 0 \end{array}\right\}$$

Basically. we can see a polyhedron is simply a set of feasible solutions of a linear programming problem.