# Hierarchical Multiscale Recurrent Neural Networks

## Problem proposed in this paper

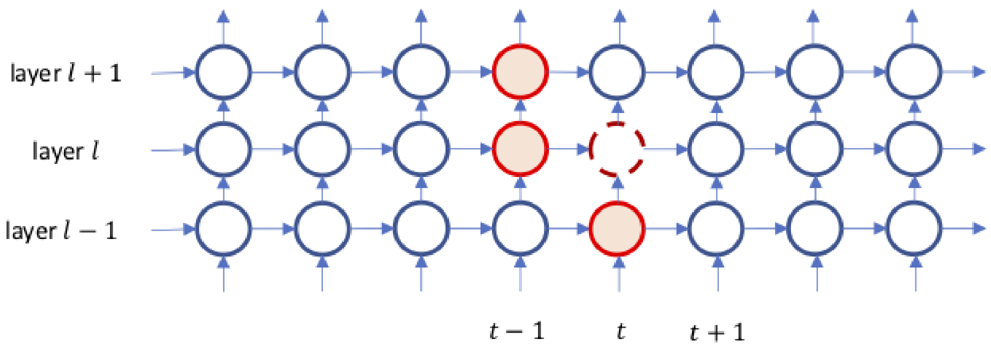
Learn the hierarchical multiscale structure from temporal data ***without explicit boundary information***.

## Key ideas

1. Introduce ***a parametrized binary boundary detector*** at each layer.

- turned on only at the time steps where a segment of the corresponding abstraction level is completely processed.
- avoids "soft" gating which leads to "curse of updating every timestep".

2. select one of the three operations below according to boundary state $z_t^{l-1}$ and $z_{t-1}^{l}$ depicted in the below picture.




- UPDATE: similar to update rule of the LSTM.
- COPY: *simply copies* cell and hidden states of the previous time step which is unlike the *leaky integration* in LSTM/GRU.
- FLUSH: executed when a boundary is detected, where it first ejects the summarized representation of the current segment to the upper layer and then reinitializes the states to start processing the next segment.

| $z_{t-1}^{l}$ **left** | $z_{t}^{l-1}$ **buttom** | **The Selected Operation** | |
|---|---|---|---|
| 0 | 0 | COPY | buttom and left states both do not reach to a boundary. |

| $z_{t-1}^{l}$ left | $z_{t}^{l-1}$ buttom | The Selected Operation | |
|---|---|---|---|
| 0 | 1 | UPDATE | left state does not reaches to a boundary but buttom does. UPDATE is executed sparsely |
| 1 | 0 | FLUSH | left state reaches to a boundary. |

| $z_{t-1}^{l}$ left | $z_{t}^{l-1}$ buttom | The Selected Operation | |
|---|---|---|---|
| 1 | 1 | FLUSH | left state reaches to a boundary. |

3. boundary state is a discrete variable.*straight-through estimator* is used to calculate its gradient.
    - the straight-through estimator is a very easy-to-implement solution to incorporate *a binary stochastic variable* into the neural network. It seems that some research work generalizes this method into a binary vector to quantize NN.

# HM-LSTM: computation details

# 0. Recap LSTM's equation first:

$$\begin{bmatrix} \mathbf{i}_t \\ \mathbf{f}_t \\ \mathbf{u}_t \\ \mathbf{o}_t \end{bmatrix} = W\mathbf{x}_t + U\mathbf{h}_{t-1} + \mathbf{b} \qquad \text{(gates and candidate)}$$

$$\mathbf{c}_t = \mathbf{c}_{t-1} \odot \sigma(\mathbf{f}_t) + \tanh(\mathbf{u}_t) \odot \sigma(\mathbf{i}_t) \qquad \text{(cell state)}$$

$$\mathbf{h}_t = \sigma(\mathbf{o}_t) \odot \tanh(\mathbf{c}_t) \qquad \text{(hidden state)}$$

HM-RNN is based on LSTM cell.

# 1. compute pre-activation

$$\begin{pmatrix} \mathbf{f}_t^\ell \\ \mathbf{i}_t^\ell \\ \mathbf{o}_t^\ell \\ \mathbf{g}_t^\ell \\ \tilde{z}_t^\ell \end{pmatrix} = \begin{pmatrix} \mathtt{sigm} \\ \mathtt{sigm} \\ \mathtt{sigm} \\ \mathtt{tanh} \\ \mathtt{hard\,sigm} \end{pmatrix} f_{\mathtt{slice}} \left( \mathbf{s}_t^{\text{recurrent}(\ell)} + \mathbf{s}_t^{\text{top-down}(\ell)} + \mathbf{s}_t^{\text{bottom-up}(\ell)} + \mathbf{b}^{(\ell)} \right),$$

where

$$\begin{aligned} \mathbf{s}_t^{\text{recurrent}(\ell)} &= U_\ell^\ell \mathbf{h}_{t-1}^\ell, \\ \mathbf{s}_t^{\text{top-down}(\ell)} &= z_{t-1}^\ell U_{\ell+1}^\ell \mathbf{h}_{t-1}^{\ell+1}, \\ \mathbf{s}_t^{\text{bottom-up}(\ell)} &= z_t^{\ell-1} W_{\ell-1}^\ell \mathbf{h}_t^{\ell-1}. \end{aligned}$$

# 2. cell update

$$\mathbf{c}_t^l = \begin{cases} f_t^{\,l} \odot \mathbf{c}_{t-1}^l + \mathbf{i}_t^l \odot \mathbf{g}_t^l, & \text{if UPDATE} \\ \mathbf{c_{t-1}^l}, & \text{if COPY} \\ \mathbf{i}_t^i \odot \mathbf{g}_t^l, & \text{if FLUSH} \end{cases}$$

## 3. hidden update

$$\mathbf{h}_t^l = \begin{cases} h_{t-1}^l, & \text{if COPY} \\ \mathbf{o}_t^l \odot \tanh(\mathbf{c}_t^l), & \text{otherwise} \end{cases}$$

- $\mathbf{g}$ is a cell proposal vector.
- $\mathbf{i}$, $\mathbf{f}$, $\mathbf{o}$ are the input/forget/output gate.

# Calculate the gradient for boundary detector

1. Straight–through estimator
   - forward pass uses the step function to activate $z_t^l$
   - backward pass uses hardsigmoid function as the biased estimator of the outgoing gradient.

$$\sigma(x) = \max(0, \min(1, (\alpha x + 1)/2))$$

2. Slope annealing
   - start from slope $\alpha = 1$.
   - slowly increase the slope until it reaches a threshold. In the paper, the annealing function task-specific.

# Some claims made (and learned) from this paper

1. However, because *non-stationarity is prevalent in temporal data*, and that many entities of abstraction such as words and sentences are in variable length, we claim that *it is important for an RNN to dynamically adapt its timescales* to the particulars of the input entities of various length.
2. It has been a challenge for an RNN to discover the latent hierarchical structure in temporal data without explicit boundary information.
3. Although the LSTM has a *self-loop for the gradients that helps to capture the long-term dependencies* by mitigating the vanishing gradient problem, in practice, it is still limited to a few hundred-time steps due to the leaky

integration by which the contents to memorize for a long-term is gradually diluted at every time step.

# References


1. Hierarchical Multiscale Recurrent Neural Networks
    - its slides
2. Revisiting the Hierarchical Multiscale LSTM
3. This paper uses the [*straight-through estimator*] to train neural networks with discrete variables. This paper points to two references using this methods:
    - Binarized neural networks: Training deep neural networks with weights and activations constrained to+ 1 or−1
    - Strategic attentive writer for learning macro-actions
    - Estimating or Propagating Gradients Through Stochastic Neurons for Conditional Computation
4. Notes on Hierarchical Multiscale Recurrent Neural Networks