HW7.Solutions

2020年3月28日 星期六 下午3:43

Binomial ~ Normal

1. 1 trial Success: toss : H $p = 0.5$

$n = 60$

X: no. of tosses are H

Q $P(X < \frac{1}{3}(60) = 20)$

$X \sim Binomial(60, 0.5)$ $n=60$ $p=0.5$

$np = 60(0.5) = 30 \geq 10$ ✓

$nq = 60(0.5) = 30 \geq 10$ ✓

$\mu = np = 60(0.5) = 30$

$\sigma = \sqrt{npq} = \sqrt{60(0.5)(0.5)} = \sqrt{15} = 3.8730$

$X \sim N(30, 3.8730)$

continuity correction $\leq$ $\leq$ $+0.5$

$P(X < 20) = P(X \leq 19) = P(X \leq 19.5)$

$P(Z \leq \frac{19.5 - \mu_X}{\sigma_X}) = (Z \leq \frac{19.5 - 30}{3.8730}) = -2.71$

$P(Z \leq -2.71) = \Phi(-2.71) =$ 0.00234

3. Sample distribution of sample proportion

$p = 40\% = 0.4$

$\sigma_{\hat{p}} \leq 0.01$

$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.4(0.6)}{n}} \leq 0.01$

$\frac{0.24}{n} \leq (0.01)^2$

$n \geq$ 2400

4. Sample distribution of sample proportion

$n = 150$

success: adults support increase $\hat{p}$

all adults in Ohio: $p = 40\% = 0.4$

Q $P(\hat{p} > 50\%)$

$\mu_{\hat{p}} = p = 0.4$

$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.4(0.6)}{150}} = 0.04$

$\hat{p} \sim N(0.4, 0.04)$

$P(\hat{p} > 0.5) = P(Z > \frac{0.5 - \mu_{\hat{p}}}{\sigma_{\hat{p}}}) = P(Z > \frac{0.5 - 0.4}{0.04})$

$P(Z > 2.5) = 1 - \Phi(2.5) = 1 - 0.9938 =$ 0.0062

5. Sample distribution of sample mean

population $\mu = 50$ $\sigma^2 = 225$ $\sigma = 15$

Sample $n = 16$

sample mean $\bar{x}$

Q: $P(48.35 \leq \bar{X} \leq 55.74)$

$X \sim N(50, 15)$ $\mu = 50$ $\sigma = 15$

$\mu_{\bar{X}} = \mu = 50$

$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{15}{\sqrt{16}} = 3.74$

$\bar{X} \sim N(50, 3.74)$

$P(48.35 \leq \bar{X} \leq 55.74) = P(\frac{48.35 - \mu_{\bar{X}}}{\sigma_{\bar{X}}} \leq Z \leq \frac{55.74 - \mu_{\bar{X}}}{\sigma_{\bar{X}}})$

$= P(\frac{48.35 - 50}{3.74} \leq Z \leq \frac{55.74 - 50}{3.74}) = P(-0.44 \leq Z \leq 1.53)$

$= \Phi(1.53) - \Phi(-0.44) = 0.9370 - 0.3300 =$ 0.607

6. Binomial -> Normal Continuity correction

Success: US live in northeast $p = 20\% = 0.2$

trials: $n = 200$

X: no. of US live in northeast for 200 residents

Q $P(X \geq 50)$

continuity correction $\geq$ $\geq$ -0.5

$P(X \geq 50) = P(X \geq 51) = P(X \geq 50.5)$

$X \sim Binomial(200, 0.2)$ $n=200$ $p=0.2$

$np = 200(0.2) = 40 \geq 10$

$nq = 200(0.8) = 160 \geq 10$

$\mu = np = 200(0.2) = 40$

$\sigma = \sqrt{npq} = \sqrt{200(0.2)(0.8)} = \sqrt{32} = 5.6569$

$X \sim N(40, 5.6569)$

$P(X \geq 50.5) = P(Z \geq \frac{50.5 - \mu_X}{\sigma_X}) = P(Z \geq \frac{50.5 - 40}{5.6569})$

$P(Z \geq 1.86) = 1 - \Phi(1.86) = 1 - 0.9686 = 0.0314$ ✗

at least 50 - $P(X \geq 50)$

continuity correction $\geq$ $\geq$ -0.5

$P(X \geq 50) = P(X \geq 49.5)$

$P(Z \geq \frac{49.5 - \mu_X}{\sigma_X}) = P(Z \geq \frac{49.5 - 40}{\sqrt{32}})$

$P(Z \geq 1.68) = 1 - \Phi(1.68) = 1 - 0.9535 =$ 0.0465