L13.1 chi -square

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part1_ch...

MATH 661

Lecture 13 part 1

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Chi-square

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condition contingency table
O counts – random samples
E count ≥ 5

Chi-Square Test of Independence

In a Chi-square test of independence,
The null hypothesis is

H_0: The variables are independent (not related)

The alternative hypothesis is

H_1: The variables are not independent (related)

Chi-square test of independence are always ***right-tailed.***

Introductory Applied Biostatistics, D'Agostino, Sullivan, and Beiser

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State hypotheses.
variables – independent.
contingency table
2 categorical variables

Chi-square Test of independence

- For the test to be used, the following must be true:
  - The observed counts are obtained from random samples.
  - Each expected count must be greater than or equal to 5.
- If these conditions are satisfied, then the sampling distribution for the chi-square test of independence is approximated by a chi-square distribution with $(r-1)(c-1)$ degrees of freedom, where r and c are the number of rows and columns, respectively, of a contingency table.

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Chi-square test – condition
distribution df
statistics

Chi-square Test of independence

- The **test statistic** for the chi-square test of independence is

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

where
O represents the observed frequency (count) for a cell
E represents the expected frequency for a cell

Remember that a chi-square test of independence is always *right-tailed*.

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Chi-Square Test of Independence

1. State the null and alternative hypotheses.	State H_0 and H_1. H_0: The row and column variables are independent (not related) H_1: The variables are not independent (related)
2. Identify the level of significance.	Identify α.
3. Identify the degrees of freedom (r and c are the number of rows and columns, respectively, of a contingency table)	df $= (r-1)(c-1)$
4. Determine the critical value.	Use Table F

(chi-square table)

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Chi-Square Test of Independence

5. Determine the rejection region.	
6. Calculate the test statistic.	$\chi^2 = \sum \frac{(O-E)^2}{E}$, $E = \frac{(\text{Row total})(\text{Column total})}{\text{overall total (i.e., sample size } n)}$
7. Make a decision to reject or fail to reject the null hypothesis.	If χ^2 is in the rejection region, reject H_0. Else, fail to reject H_0.

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Chi-Square Test of Independence

Test using P-value:

The *P*-value is the area to the right of the value of the test statistic.

- If P-Value ≤ α
  Reject H_0
- If P-Value > α
  Fail to Reject H_0

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治疗方案

Example 1

- The data in the 3 x 3 contingency table below shows the treatment regimens of patients (Column variable) measured at baseline by site (Row variable).
  Test the hypothesis that the two variables (site and treatment regimen) are independent. α=0.05

Site	Treatment Regimen: Diet & Exercise	Oral Hypoglycemics	Insulin	Total
HMO	294	827	579	1700
UTH	132	288	352	772
IPA	189	516	404	1109
Total	615	1631	1335	3581

Cells 3 3

Introductory Applied Biostatistics, D'Agostino, Sullivan, and Beiser

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Solution

1) Set up hypotheses:
H_0: Site and Treatment regimen are independent
H_1: H_0 is false.
(Site and Treatment regimen are dependent)

Number of Rows (R)= 3, Number of columns (C)= 3.

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Solution

2) Select the appropriate test statistic

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

The test statistic follows a chi-square distribution and has degrees of freedom given by df=(R-1)(C-1) =(3-1)(3-1) =2(2)=4

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df=(r-1)(c-1) α 0.05 critical value

Solution

3) **Decision Rule**
df = (R-1)(C-1) = (3-1)(3-1) =2(2)=4
Critical Value: 9.49 (α=0.05; df=4)
Reject H_0 if $\chi^2 \geq 9.49$
Do not Reject H_0 if $\chi^2 < 9.49$

9.49

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Solution

4) **Test Statistic:**

Site	Diet & Exercise	Oral Hypoglycemics	Insulin	Total
HMO	O: 294 E:(1700*615)/3581 =291.9	O: 827 E: (1700*1631)/3581 =774.3	O: 579 E:(1700*1335)/3581 =633.8	1700
UTH	O:132 E:(772*615)/3581 =132.6	O: 288 E: (772*1631)/3581 =351.6	O: 352 E: (772*1335)/3581 =287.8	772
IPA	O: 189 E:(1109*615)/3581 =190.5	O: 516 E:(1109*1631)/3581 =505.1	O:404 E:(1109*1335)/3581 =413.4	1109
Total	615	1631	1335	3581

The marginal totals of the expected frequencies = marginal totals of the observed frequencies.

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Solution

Site	Diet & Exercise	Oral Hypoglycemics	Insulin	Total
HMO	O: 294 E:291.9	O: 827 E: 774.3	O: 579 E:633.8	1700
UTH	O:132 E:132.6	O: 288 E: 351.6	O: 352 E: 287.8	772
IPA	O: 189 E:190.5	O: 516 E:505.1	O:404 E:413.4	1109
Total	615	1631	1335	3581

= check condition

$$\chi^2 = \sum\frac{(O-E)^2}{E} = \frac{(294-291.9)^2}{291.9} + \frac{(827-774.3)^2}{774.3} + \frac{(579-633.8)^2}{633.8} + \frac{(132-132.6)^2}{132.6} + \frac{(288-351.6)^2}{351.6} + \frac{(352-287.8)^2}{287.8} + \frac{(189-190.5)^2}{190.5} + \frac{(516-505.1)^2}{505.1} + \frac{(404-413.4)^2}{413.4}$$

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Solution

$$\chi^2 = \sum\frac{(O-E)^2}{E} = \frac{(294-291.9)^2}{291.9} + \frac{(827-774.3)^2}{774.3} + \frac{(579-633.8)^2}{633.8} + \frac{(132-132.6)^2}{132.6} + \frac{(288-351.6)^2}{351.6} + \frac{(352-287.8)^2}{287.8} + \frac{(189-190.5)^2}{190.5} + \frac{(516-505.1)^2}{505.1} + \frac{(404-413.4)^2}{413.4}$$
$= 0.014+3.359+4.732$
$+0.003+11.509+14.320$
$+0.011+0.235+0.215$
$=34.629$

Reject Ho
X independent

9.49

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Solution

5) Conclusion:
Reject H_0 since 34.629 ≥ 9.49 i.e., $\chi^2 \geq 9.49$
We have significant evidence, α=0.05 to show that site and treatment regimen are not independent (i.e., they are related)

Test Using P-value:
df=4 χ^2 = 34.629 α=0.05
P-Value < 0.0005 P-value: 0 (From Minitab)
P-Value < α
Reject H_0

0.005 ... 0.0005
4 20.00 34.629

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Chi-Square Test for Association: Site, Treatment regimen
Choose Stat > Tables > Chi-Square Test for association
Rows: Site Columns: Treatment regimen

	Diet and Exercise	hypoglycemics	insulin	All
HMO	294 292.0 0.0143	827 774.3 3.5895	579 633.8 4.7318	1700
UTH	132 132.6 0.0026	288 351.6 11.5093	352 287.8 14.3201	772
IPA	189 190.5 0.0112	516 505.1 0.2350	404 413.4 0.2154	1109
All	615	1631	1335	3581

Cell Contents: Count
Expected count
Contribution to Chi-square

Pearson Chi-Square = 34.629, DF = 4, P-Value = 0.000

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Example 2

- Suppose an investigator wants to evaluate if the treatment regimens differ by gender in HMO patients. The following 2 X 3 contingency table shows the number of male and female HMO patients according to their treatment regimens. Is there evidence of a significant relationship between gender and treatment regimen among the HMO patients? Test at α=0.05

Gender	Treatment Regimen: Diet & Exercise	Oral Hypoglycemics	Insulin	Total
Female	147	435	256	838
Male	147	392	323	862
Total	294	827	579	1700

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Solution

1) Set up hypotheses:
H_0: Gender and Treatment regimen are independent
H_1: H_0 is false. α=0.05
(Gender and Treatment regimen are dependent)

Number of Rows (R)= 2, Number of columns (C)= 3.

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Solution

2) Select the appropriate test statistic

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

df = (R-1)(C-1) = (2-1)(3-1) =1(2)=2

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Solution

3) **Decision Rule**
df = (R-1)(C-1) = (2-1)(3-1) =1(2)=2
Critical Value: 5.99 (α=0.05; df=2)
Reject H_0 if $\chi^2 \geq 5.99$
Do not Reject H_0 if $\chi^2 < 5.99$

α 0.05 5.99 χ^2

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Solution

4) **Test Statistic:**

Gender	Diet & Exercise	Oral Hypoglycemics	Insulin	Total
Female	O: 147 E:(838*294)/1700 =144.9	O: 435 E: (838*827)/1700 =407.7	O: 256 E:(838*579)/1700 =285.4	838
Male	O:147 E:(862*294)/1700 =149.1	O: 392 E: (862*827)/1700 =419.3	O: 323 E: (862*579)/1700 =293.6	862
Total	294	827	579	1700

The marginal totals of the expected frequencies = marginal totals of the observed frequencies.

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Solution

Gender	Diet & Exercise	Oral Hypoglycemics	Insulin	Total
Female	O: 147 E:144.9	O: 435 E: 407.7	O: 256 E:285.4	838
Male	O:147 E:149.1	O: 392 E: 419.3	O: 323 E: 293.6	862
Total	294	827	579	1700

$$\chi^2 = \sum\frac{(O-E)^2}{E} = \frac{(147-144.9)^2}{144.9} + \frac{(435-407.7)^2}{407.7} + \frac{(256-285.4)^2}{285.4} + \frac{(147-149.1)^2}{149.1} + \frac{(392-419.3)^2}{419.3} + \frac{(323-293.6)^2}{293.6}$$
$=0.030+1.833+3.031+0.029+1.782+2.947$
$=9.652$

Reject Ho.

5.99

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Solution

5) Conclusion:
Reject H_0 since 9.652 ≥ 5.99 i.e., $\chi^2 \geq 5.99$
We have significant evidence, α=0.05 to show that gender and treatment regimen are not independent (i.e., they are related) in the sample of HMO patients.

Test Using P-value:
df=2 χ^2 = 9.652 α=0.05
0.005 < P-Value < 0.01 P-value: 0.008 (From Minitab)
P-Value < α
Reject H_0

P (0.01 0.005)
2 9.21 9.652 10.61
P (0.005, 0.01) < 0.05 reject Ho

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Choose Stat > Tables > Chi-Square Test for association
Chi-Square Test for Association: Gender, Treatment Regimen
Rows: Gender Columns: Treatment Regimen

	Diet and Exercise	Hypoglycemics	Insulin	All
Female	147 144.9 0.0297	435 407.7 1.8332	256 285.4 3.0311	838
Male	147 149.1 0.0289	392 419.3 1.7822	323 293.6 2.9467	862
All	294	827	579	1700

Cell Contents: Count
Expected count
Contribution to Chi-square

Pearson Chi-Square = 9.652, DF = 2, P-Value = 0.008

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Chi-Square Goodness of Fit Test

Used to test the hypothesis that an observed frequency distribution fits some claimed distribution.

The test statistic follows a chi-square distribution (χ^2)

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Chi-square Goodness of fit test

- For the test to be used, the following must be true:
  - The observed counts are obtained from random samples.
  - Each expected count must be greater than or equal to 5.
- If these conditions are satisfied, then the sampling distribution for the goodness-of-fit test is approximated by a chi-square distribution with $k-1$ degrees of freedom, where k is the number of categories.

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Chi-square Goodness of fit test

- The **test statistic** for the chi-square goodness-of-fit test is

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

where
O represents the observed count of each category (from the experiment or sample)
E represents the expected count of each category (counts that we expect to obtain if the null hypothesis were true)
Remember that a chi-square goodness-of-fit test is always *right-tailed*.

categories observed counts expected counts
sample Ho true

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Goodness-of-Fit Test

Degrees of Freedom for a Goodness-of-Fit Test
In a goodness-of-fit test, the ***degrees of freedom*** are

$$df = k-1$$

where k denotes the number of possible response categories.

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Example (from text)

Mars, Inc. makes milk chocolate candies. Here's what the company's Consumer Affairs Department says about the color distribution of its M&M's candies:

On average, the new mix of colors of M&M's milk chocolate candies will contain 13 percent of each of browns and reds, 14 percent yellows, 16 percent greens, 20 percent oranges, and 24 percent blues.

The **one-way table** below summarizes the data from a sample bag of M&M's

Color	Blue	Orange	Green	Yellow	Red	Brown	Total
Count	9	8	12	15	10	6	60

The sample proportion of blue M&M'S is $\hat{p} = \frac{9}{60} = 0.15$.

Introduction to the Practice of Statistics, Moore McCabe, and Craig

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The Chi-Square Test for Goodness of Fit

We can write the hypotheses in symbols as:

subscript – color

claim H_0: $p_{blue} = 0.24$, $p_{orange} = 0.20$, $p_{green} = 0.16$, $p_{yellow} = 0.14$, $p_{red} = 0.13$, $p_{brown} = 0.13$,

H_1: At least one of the proportions is different than claimed

where p_{color} = the true population proportion of M&M's of that color.

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The Chi-Square Test for Goodness of Fit

The idea of the chi-square test for goodness of fit is this:

We compare the **observed counts** from our sample with the counts that would be **expected** if H_0 is true.

The more the observed counts differ from the **expected counts,** the more evidence we have against the null hypothesis.

In general, the expected counts can be obtained by multiplying the proportion of the population distribution in each category by the sample size.

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Color distribution stated: 13 percent browns, 13 percent reds, 14 percent yellows, 16 percent greens, 20 percent oranges, and 24 percent blues.

Assuming that the color distribution stated by Mars, Inc. is true, 24% of all M&M's produced are blue.
For random samples of 60 candies, the average number of blue M&M's should be (0.24)(60) = 14.40.
This is our expected count of blue M&M's.
Using this same method, we can find the expected counts for the other color categories:

Orange: (0.20)(60) = 12.00
Green: (0.16)(60) = 9.60
Yellow: (0.14)(60) = 8.40
Red: (0.13)(60) = 7.80
Brown: (0.13)(60) = 7.80

Color	Observed	Expected
Blue	9	14.40
Orange	8	12.00
Green	12	9.60
Yellow	15	8.40
Red	10	7.80
Brown	6	7.80

in box

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The Chi-Square Test for Goodness of Fit

To calculate the chi-square statistic, use the formula

$$\chi^2 = \sum \frac{(\text{Observed} - \text{Expected})^2}{\text{Expected}}$$

Color	Observed	Expected
Blue	9	14.40
Orange	8	12.00
Green	12	9.60
Yellow	15	8.40
Red	10	7.80
Brown	6	7.80

$$\chi^2 = \frac{(9-14.40)^2}{14.40} + \frac{(8-12.00)^2}{12.00} + \frac{(12-9.60)^2}{9.60} + \frac{(15-8.40)^2}{8.40} + \frac{(10-7.80)^2}{7.80} + \frac{(6-7.80)^2}{7.80}$$
$\chi^2 = 2.025 + 1.333 + 0.600$
$+ 5.186 + 0.621 + 0.415$
$= 10.180$

df=k-1=5

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The Chi-Square Test for Goodness of Fit

We computed the chi-square statistic for our sample of 60 M&M's to be $\chi^2 = 10.180$. Because all of the expected counts are at least 5, the χ^2 statistic will follow a chi-square distribution with df=6-1=5 reasonably well when H_0 is true.

P-value: 0.07

df	.15	.10	.05
4	6.74	7.78	9.49
5	8.12	9.24	11.07
6	9.45	10.64	12.59

p right tail > α = 0.05

Since our *P*-value is between 0.05 and 0.10, it is greater than α = 0.05. Therefore, we fail to reject H_0. We don't have sufficient evidence to conclude that the company's claimed color distribution is incorrect.

34 $\chi^2_{0.05,5} = 11.07$
11.07
10.18 X reject Ho.

Stat > Tables > Chi-Square Goodness-of-Fit Test (One Variable).

Chi-Square Goodness-of-Fit Test for Observed Counts in Variable: Counts

```
Using category names in Color

                    Historical     Test             Contribution
Category  Observed    Counts   Proportion  Expected   to Chi-Sq
Blue          9         24        0.24       14.4      2.02500
Orange        8         20        0.20       12.0      1.33333
Green        12         16        0.16        9.6      0.60000
Yellow       15         14        0.14        8.4      5.18571
Red          10         13        0.13        7.8      0.62051
Brown         6         13        0.13        7.8      0.41538

 N  DF  Chi-Sq  P-Value
60   5  10.1799   0.070
```

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Chi-Square Goodness-of-Fit Test

1. State the null and alternative hypotheses.	State H_0 and H_1. H_0: p_1=…,p_2=…,p_3=…… H_1: H_0 is false.
2. Identify the level of significance.	Identify α.
3. Identify the degrees of freedom. (k is the number of categories)	df $= k-1$
4. Determine the critical value.	Use Table F

$\chi^2_{\alpha, df}$

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Chi-Square Goodness-of-Fit Test

5. Determine the rejection region.	
6. Calculate the test statistic.	$\chi^2 = \sum \frac{(O-E)^2}{E}$
7. Make a decision to reject or fail to reject the null hypothesis.	If χ^2 is in the rejection region, reject H_0. Else, do not reject H_0. **(or)** **Obtain the P-value.** Compare it with α. If P-value ≤ α Reject H_0. Else, Fail to reject H_0

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Chi-Square Test of Independence Using P-value

The *P*-value is the area under the χ^2 distribution (with k-1 degrees of freedom) to the right of the value of the test statistic.

- If P-Value ≤ α
  Reject H_0
- If P-Value > α
  Fail to Reject H_0

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Example

- A total if 100 patients are involved in a study that investigates which of the three time slots are the most popular or convenient in the follow up of a medical treatment.

Time Slot	Mon 6-7pm	Thurs 4 -5:30pm	Sat 8 -9:30pm
No. of Patients	47	32	21

observed

We want to test the hypothesis that the three day and time options are equally popular.

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Solution

1) Set up hypotheses:
H_0: $p_1=p_2=p_3$, α=0.05 (population proportions are equal)
H_1: H_0 is false. (population proportions are not equal)
(We do not write H_1: $p_1 \neq p_2 \neq p_3$, as we will reject H_0 if any of the three proportions are not equal)
H_0: $p_1=0.33, p_2=0.33, p_3=0.33$, α=0.05
H_1: H_0 is false.

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Solution

2) Select the appropriate test statistic
(The test statistic is no longer based on sample proportion. It is based on observed frequencies)

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

In this example, 47 patients are observed in the first category, 32 in the second and 21 in the third
If the null hypothesis were true i.e., if the proportion of patients in each of the three response categories were equal, then we would expect approximately 33 patients in each response category.

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Solution

3) **Decision Rule**
df = k-1 = 3-1 =2
(k is the number of categories)
Critical Value: 5.99 (α=0.05; df=2)

Reject H_0 if $\chi^2 \geq 5.99$
Do not Reject H_0 if $\chi^2 < 5.99$

α 0.05 df 2 5.99
5.99

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Solution

4) **Test Statistic:**

Time Slot	Mondays 6 to 7:30 pm	Thursdays 4 to 5:30 pm	Saturdays 8 to 9:30 am	Total
O (Observed Frequency)	47	32	21	100
E (Expected Frequency)	33.3	33.3	33.3	100
(O – E)	13.7	-1.3	-12.3	0
(O – E)²/E	13.7²/33.3 = 5.64	-1.3²/33.3 = 0.05	-12.3²/33.3 = 4.54	10.23

The sum of the expected frequencies is equal to the sum of the observed frequencies (n=100)
The test statistic χ^2 = 10.23

5.99

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Solution

5) Conclusion:
Reject H_0 since 10.23 ≥ 5.99 i.e., $\chi^2 \geq 5.99$

We have significant evidence, α=0.05 to show that the three time slots are not equally popular
Test Using P-value:
df=2 χ^2 = 10.23
0.005 < P-Value < 0.01 P-value: 0.006
P-Value < α
Reject H_0

(0.01 0.005) < α=0.05.
2 9.21 10.60
Reject Ho. should be there
+ add. conclusion

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Stat > Tables > Chi-Square Goodness-of-Fit Test (One Variable).

```
Using category names in Time Slot

                          Test              Contribution
Category     Observed  Proportion  Expected   to Chi-Sq
M(6 to 7)        47     0.333333   33.3333     5.60333
R(4 to 5:30)     32     0.333333   33.3333     0.05333
S(8 to 9:30)     21     0.333333   33.3333     4.56333

  N  DF  Chi-Sq  P-Value
100   2   10.22    0.006
```

45 everything show work