L10.P1.slides

2020年4月9日 星期四 下午5:29

Lecture10_ch6_ch7_...

4/5/2020

MATH 661

Lecture 10 part 1

Chapter 6
Chapter 7
Chapter 8

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Hypothesis Testing

Hypothesis testing is a procedure used to test claims (or statements) and make decisions regarding the value of the population parameter.

We test these types of claims (or statements) using sample data...

To test the claims (or statements), a pair of hypotheses are stated – one that represents the claim (or the statement) and the other, its complement.

Note that hypotheses are always statements about the population, not statements about the sample

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test claims / statements
population parameter μ, p
sample claim
pair of hypotheses
claim
complement

Stating a Hypothesis

A **null hypothesis** H_0 is a statistical hypothesis that contains a statement of equality such as $\leq$, $=$, or $\geq$. We would use just use $=$ format.

An **alternative hypothesis** (H_1 or H_a) is the complement of the null hypothesis. It contains a statement of inequality such as $>$, $\neq$, or $<$.

To write the null and alternative hypotheses, translate the claim made about the population parameter from a verbal statement to a mathematical statement.

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Stating a hypothesis: Example 1

For example, suppose we are attempting to decide whether or not the mean burning rate of a solid propellant used to power aircrew escape systems is 50 centimeters per second.

$H_0: \mu = 50$ null hypothesis
$H_1: \mu \neq 50$ alternative hypothesis

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claim
population
sample $\bar{x}$

Notes

A null hypothesis (H_0) concerning a population parameter will always be stated so as to specify an exact value of the parameter, whereas the alternative hypothesis (H_1) allows for the possibility of several values.

$H_0: \mu = 50$ — Two sided test
$H_1: \mu \neq 50$

One sided test $H_0: \mu = 50$ $H_1: \mu > 50$ or $H_0: \mu = 50$ $H_1: \mu < 50$ One sided test

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Stating a Hypothesis Example 2:

A manufacturer advertises that the mean mileage of its new subcompact models is greater than 47 miles per gallon.

$H_0: \mu = 47$ mpg
$H_1: \mu > 47$ mpg (Claim)

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Notes

No matter which hypothesis represents the claim, always begin the hypothesis test **assuming that the null hypothesis is true**.

At the end of the test, one of two decisions will be made:

1. reject the null hypothesis, or
2. fail to reject the null hypothesis.

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based on test statistic

null hypothesis: H0 reject V X

Hypothesis Testing

Hypothesis tests could be either two sided (two tailed) or one sided (left tailed or right tailed)

One-Sided and Two-Sided Hypotheses

Two-Sided Test: $H_0: \mu = \mu_0$ $H_1: \mu \neq \mu_0$

One-Sided Tests: $H_0: \mu = \mu_0$ $H_1: \mu > \mu_0$ or $H_0: \mu = \mu_0$ $H_1: \mu < \mu_0$

The type of test depends on the values of the sampling distribution that forms a rejection of H_0. This region is indicated by the alternative hypothesis.

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types of hypothesis

Tests on Mean of a normal distribution, variance known – Z

Suppose we wish to test the hypothesis

$H_0: \mu = \mu_0$ $H_1: \mu \neq \mu_0$ where μ_0 is a constant.

After stating the null and alternative hypotheses and specifying the level of significance, a random sample is taken from the population and sample statistic (such as $\bar{X}$) are calculated.

Since population distribution is normal, $\bar{X}$ has a normal distribution (with mean μ_0 and standard deviation $\sigma/\sqrt{n}$). The test procedure for $H_0: \mu = \mu_0$ uses the test statistic Z_0

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types of hypothesis
Z
random sample - sample statistic

Tests on Mean of a normal distribution, variance known

If the null hypothesis $H_0: \mu = \mu_0$ is true, then

$$\mu_{\bar{X}} = E(\bar{X}) = \mu_0$$

$$Z_0 = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}}$$

Distribution of Z_0 is the standard normal distribution ~N(0,1).

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Methods for making decisions

1) Based on Rejection Regions
2) Based on P-values

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Using Rejection Regions and Critical Values

A **rejection region** (or **critical region**) of the sampling distribution is the range of values for which the null hypothesis (H_0) is not probable.

If the test statistic (Z_0) falls in this region, the null hypothesis H_0 is rejected.

A critical value z_α or $z_{\alpha/2}$ separates the rejection region from the non rejection region.

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Using Rejection Region Method

1. Identify the null and alternative hypotheses. — State H_0 and H_1
2. Specify the level of significance. — Identify α.
3. Sketch the sampling distribution.
4. Determine the critical value(s) of z_α ; — Use Table A in Appendix.
5. Determine the rejection region (s).

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Rejection Regions for a Z-Test

6. Find the standardized test statistic Z_0.
7. Make a decision to reject or fail to reject the null hypothesis.
8. Interpret the decision in the context of the original claim.

If Z_0 is in the rejection region, reject H_0. Otherwise, fail to reject H_0.

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Testing hypothesis on mean (two sided)

Suppose that we specify the hypotheses as

$H_0: \mu = \mu_0$
$H_1: \mu \neq \mu_0$

We would reject H_0 if the observed value of the test statistic z_0 is either

$$z_0 > z_{\alpha/2} \quad \text{or} \quad z_0 < -z_{\alpha/2}$$

(define rejection region or critical region)

And we would fail to reject H_0 if

$$-z_{\alpha/2} \leq z_0 \leq z_{\alpha/2}$$

(define acceptance region)

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Testing hypothesis on mean (one sided)

Testing hypotheses on mean μ where the alternative hypothesis is one-sided. Suppose that we specify the hypotheses as

$H_0: \mu \geq \mu_0$
$H_1: \mu < \mu_0$

We would reject H_0 if the observed value of the test statistic z_0 is too small.

$$z_0 < -z_{\alpha}$$

(define rejection region or critical region)

And we would fail to reject H_0 if

$$z_0 \geq -z_{\alpha}$$

(define acceptance region)

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Tests on Mean of a normal distribution, variance known

Testing hypotheses on mean μ where the alternative hypothesis is one-sided. Suppose that we specify the hypotheses as

$H_0: \mu \geq \mu_0$
$H_1: \mu < \mu_0$

We would calculate Z_0 and reject H_0 if the value of z_0 is too small. The critical region is in the left tail of the standard normal distribution.

We would reject H_0 if $z_0 < -z_\alpha$

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Example 1

Suppose we are interested in testing a claim made by an investigator that the mean systolic blood pressure for males aged 50 or greater is 130 Hg when they are under stress at work. A sample of 108 males aged 50 is taken and the mean systolic blood pressure is found to be 135. Test at $\alpha = 0.05$. Given $\sigma = 15$

$H_0: \mu = 130$

$H_1: \mu > 130$ (claim) Right tailed test

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Solution

1) The parameter of interest is μ, the mean SBP

2) Null Hypothesis: $H_0: \mu = 130$

3) Alternative Hypothesis: $H_1: \mu > 130$

4) Test Statistic: $Z_0 = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$

5) Identify critical values: $\alpha = 0.05$, Hence $1-\alpha = 0.95$, $z_{0.05} = 1.645$

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Solution

6) Since

$$\bar{x} = 135 \text{ and } \sigma = 15$$

$$z_0 = \frac{135 - 130}{15/\sqrt{108}} = 3.46$$

7) Conclusion: Since $z_0 > z_{0.05}$, we reject H_0

We conclude that the mean SBP for males aged 50 is greater than 130, at $\alpha = 0.05$ based on a sample of 108 subjects.

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Problem 1

Suppose we want to test whether or not the average length of a phone call, for an average sample of 64 phone calls, the sample mean was 8 minutes. Test at $\alpha = 0.05$.

$H_0: \mu = 8$ $H_1: \mu \neq 8$

The standardized test statistic is

$$z_0 = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{7.5 - 8}{1.5/\sqrt{64}}$$

$$= -3.05.$$

Critical values $\pm z_{\alpha/2} = \pm 1.96$ z_α falls in the rejection region, so we **reject H_0**

At the 5% level of significance we reject the null hypothesis.

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Problem 2

Teachers at a high school claim that the mean salary of teachers at their school is 85,000 with a standard deviation of 15,000. A random sample of 100 high school teachers has a mean salary of 83,500. Let $\alpha = 0.05$, test the claim.

$H_0: \mu = \mu_0 = 85000$

$H_1: \mu \neq \mu_0 = 85000$ (claim)

1 tailed test

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Solution 2

$H_0: \mu = \$45000$

$H_1: \mu < \$45000$ (claim)

Because the test is a left tailed test and $\alpha = 0.05$. The critical value is $z_\alpha = -1.645$ and the rejection region is to the left of $z_\alpha = -1.645$. The standardized test statistic is

$$z_0 = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{43000 - 45000}{\frac{5000}{\sqrt{50}}} = -1.58$$

Because z_0 is not in the rejection region, we fail to reject the null hypothesis.

There is not enough evidence at the 5% level of significance to support the claim that the mean salary of teachers is less than $45,000

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explain values