L10.P2.slides

2020年4月9日 星期四 下午5:29

Lecture10_ch6_ch8_...

P-value > α → fail to reject H0

P-value ≤ α reject H0

z0 z*

reject region

zα z0

4/5/2020

MATH 661

Lecture 10 part 2

Chapter 8
Chapter 6

Instructor: Padma Natarajan

1

P-values

The *P-value* is the smallest level of significance that would lead to the rejection of the null hypothesis with the given data.

The *P-value* of a hypothesis test depends on the nature of the test.

1 is the tail area beyond the value of the test statistic z_0 for a one sided test or twice this area for a two sided test.

P-value - smallest α → reject H0

2

Using P-values for a z-Test

Assumption: Population is normal and σ^2 is known, or any population when the sample size n is at least 30.

The **test statistic** is Z_0

$$Z_0 = \frac{\bar{X} - \mu_0}{\sigma / \sqrt{n}}$$

$\bar{X}$ sample mean

Population μ0

σ known

n ≥ 30

3

Finding P-values

P-value of a hypothesis test is the probability that the test statistic will take on a value that is at least as extreme as the observed value of the statistic when the null hypothesis H_0 is true.

a. For a left-tailed test, P= Area in left tail: $\Phi(z_0)$
b. For a right-tailed test, P= Area in right tail: $1-\Phi(z_0)$
c. For a two-tailed test, P= Area in left tail or the right tail:

4

Using P-values to Make a Decision

Decision Rule Based on *P-value*
To use a *P-value* to make a conclusion in a hypothesis test, compare the *P-value* with α.
1. If $P \le \alpha$ then reject H_0
2. If $P > \alpha$ then fail to reject H_0

If the *P-value* is as small or smaller than α we say the data are statistically significant at level α.
Significant indicates that there is evidence against the null hypothesis.

P ≤ α → reject H0

P > α → fail to reject H0

5

Examples

The standardized test statistic for a right-tailed test is $z_0 = 1.56$. Find the *P-value*.

The area to the right of $z_0 = 1.56$ is $1 - \Phi(1.56) = 0.0606$ = P-value

The standardized test statistic for a two-tailed test is $z_0 = -1.96$. Find the *P-value*.

The area to the left of $z_0 = -0.61$ is $\Phi(-0.61)$
The *P-value* is $2\Phi(-0.61) = 0.5416$

P-value → compare with α

6

Instructor: Padma Natarajan

1

4/5/2020

Using P-values for a z-Test

1. State the claim mathematically and verbally. Identify the null and alternative hypotheses.
2. Specify the level of significance.
3. Determine the standardized test statistic.
4. Find the P-value corresponding to z_0.
5. State the conclusion.

Step 1: H_0 and H_a

Step 2: Select α

Step 3: $z_0 = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$

Step 4: Use the Table A in Appendix.

Step 5: P-value ≤ α → reject H0; P-value > α → fail to reject H0

7

Using P-values for a z-Test

1. Find the P-value.
   a. For a left-tailed test, P= Area in left tail:
   b. For a right-tailed test, P= Area in right tail:
   c. For a two-tailed test, P= Area in left tail or the right tail:
2. Make a decision to reject or fail to reject the null hypothesis.

Step 1: $\Phi(z_0)$

Step 2: $1-\Phi(z_0)$

Step 3: $2\Phi(-|z_0|)$ if $z_0 \le 0$; $2\Phi(|z_0|)$ if $z_0 > 0$

Step 4: P-value ≤ α → reject H0; P-value > α → fail to reject H0

8

Example 1

A manufacturer claims that its rechargeable batteries are good for an average of more than 1000 hours. A random sample of 36 batteries has a mean life of 1050 charges. Given a 5% chance enough evidence to support the claim?

claim: average exceeds 1000

H_0: $\mu = 1000$ H_a: $\mu > 1000$ (Claim)

The level of significance is $\alpha = 0.05$.

The standardized test statistic is

$$z_0 = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} = \frac{1050 - 1000}{50/\sqrt{36}} = 6$$

H_0: μ = 1000

H_a: μ > 1000

$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$

z-value > α → fail to reject H0

P-value > α → fail to reject H0

9

Solution 1

A manufacturer claims that its rechargeable batteries are good for an average of more than 1000 hours. A random sample of 36 batteries has a mean life of 1050 charges. Given a 5% chance enough evidence to support the claim?

H_0: $\mu = 1000$ H_a: $\mu > 1000$ (Claim)

$\alpha = 0.05$

The area to the right of $z_0 = 1.45$ is $1 - \Phi(1.45) = 0.0735$

z-value > α → fail to reject H0

At the 5% level of significance, there is not enough evidence to support the claim that the rechargeable battery has an average life of more than 1000 charges.

10

Example 2

Administration at a hospital claims that the mean waiting time for patients at their hospital is less than 30 minutes. A random selection of 50 waiting times has a sample mean of 25 minutes. Given a 5% chance enough evidence to support the claim at $\alpha = 0.05$? Use a P-value.

claim: average < 30

H_0: $\mu \ge 30$

H_a: $\mu < 30$ (claim)

$z_0 = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$

$z_0 = \frac{25 - 30}{\sigma/\sqrt{50}}$

$z_0 = -2.57$

11

Solution

H_0: $\mu = 30$ minutes
H_a: $\mu < 30$ minutes (claim)

$\alpha = 0.01$

Left Tailed Test

$$z_0 = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{25 - 30}{\sqrt{50}} = -2.57$$

$\bar{x} = 25$, $\mu_0 = 30$, $\sigma = 5$, $n = 50$

The area corresponding to z_0= 0.57 is 0.0051. Since this test is left tailed $P = 0.0051$

Since P-value is less than α, reject the null hypothesis. At 1% level of significance, there is sufficient evidence to conclude that the mean waiting time is less than 30 minutes.

12

Instructor: Padma Natarajan

2

4/5/2020

Hypothesis testing for the mean, variance unknown

Suppose we wish to test the hypotheses

H_0: $\mu = \mu_0$

H_a: $\mu \ne \mu_0$

We will use the test statistic

$$T_0 = \frac{\bar{X} - \mu}{S/\sqrt{n}}$$

If the null hypothesis is true, T_0 has a t distribution with $n-1$ degrees of freedom.

13

Hypothesis testing for the mean, variance unknown

The t percentages points $-t_{\alpha/2,n-1}$ and $t_{\alpha/2,n-1}$ are the boundaries of the critical region for a two sided test(I am here would reject H_0 if $t_0 \le -t_{\alpha/2,n-1}$ or if $t_0 > t_{\alpha/2,n-1}$

where t_0 is the observed value of the test statistic T_0.

The procedure for hypothesis testing with t is very similar to hypothesis testing with Z, except that T_0 is used as a test statistic instead of Z_0 and the t distribution is used to define the critical/rejection region instead of the z distribution

14

Critical Values in a t Distribution

1. Identify the level of significance α.
2. Identify the degrees of freedom $\nu = n - 1$.
3. Find the critical value(s) using Table D in Appendix in the row with $\nu - 1$ degrees of freedom. If the hypothesis test:
   a. is a left test, use "upper tail probability p" or the "α" column and use a negative sign for t value, $-t_{\alpha,\nu-1}$
   b. is a right test, use "upper tail probability p" or the "α" column and a positive sign for t value, $t_{\alpha,\nu-1}$
   c. is a two-tailed test, use the "upper tail probability p" or the "α" column and use $\alpha/2$ and use t values with a negative and a positive sign, $-t_{\alpha/2,\nu-1}$ and $t_{\alpha/2,\nu-1}$

$-t_{\alpha,\nu-1}$

$t_{\alpha,\nu-1}$

$-t_{\alpha/2,\nu-1}$

$t_{\alpha/2,\nu-1}$

15

Example 1

Find the critical value for a left-tailed test given $\alpha = 0.025$ and $n = 15$.

$t_{0.025,14} = t_{\alpha,\nu-1} = 2.145$

The degrees of freedom are $df = n - 1 = 15 - 1 = 14$.

To find the critical value, use Table D with $df = 14$ and 0.01 in the "upper tail probability p" or the "α" column. Because the test is a left-tailed test, the critical value is negative.

$t_{\alpha,\nu-1} = t_{0.025,14} = 2.145$

16

Example 2

Find the critical value for a right-tailed test given $\alpha = 0.05$ and $n = 19$.

$-t_{\alpha,\nu-1} = -t_{0.05,18} = -1.734$

The degrees of freedom are $df = n - 1 = 19 - 1 = 18$.

To find the critical value, use Table D with $df = 18$ and $\alpha/2 = 0.025$ = 0.05 in the "upper tail probability p" or the "α" column. Because the test is a right-tailed test, one critical value is negative and one is positive.

$-t_{\alpha/2,\nu-1} = -t_{0.05,18} = -1.734$ and $t_{\alpha/2,\nu-1} = 1.734$

17

Example 3

Find the critical value for a two-tailed test given $\alpha = 0.05$ and $n = 15$.

$-t_{\alpha/2,\nu-1} = -t_{0.025,14} = -2.145$

$t_{\alpha/2,\nu-1} = t_{0.025,14} = 2.145$

The degrees of freedom are $df = n - 1 = 15 - 1 = 14$.

To find the critical value, use Table D with $df = 14$ and $\alpha/2 = 0.025$ = 0.05 in the "upper tail probability p" or the "α" column. Because the test is a two-tailed test, one critical value is negative and one is positive.

$-t_{\alpha/2,\nu-1} = -t_{0.05,18} = -1.734$ and $t_{\alpha/2,\nu-1} = 1.734$

18

Instructor: Padma Natarajan

3

4/5/2020

t'Test for Mean μ (σ Unknown)

1. Identify the null and alternative hypotheses.
2. Specify the level of significance.
3. Identify the degrees of freedom $\nu = n - 1$.
4. Determine any critical values.
5. Determine any rejection region(s).
6. Find the standardized test statistic.
7. Make a decision to reject or fail to reject the null hypothesis.

Step 1: H_0 and H_a

Step 2: Select α

Step 3: $\nu = n - 1$

Step 4: Use Table D in Appendix

Step 5: $t_0 \le -t_{\alpha,\nu-1}$ or $t_0 > t_{\alpha,\nu-1}$

Step 6: $t_0 = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$

Step 7: If t_0 is in the rejection region, reject H_0. Otherwise, fail to reject H_0.

19

Example 1

Suppose we want to test whether or not the average length of a phone call is 9 minutes. In a random sample of 19 phone calls, the sample mean was 8.75 minutes and the standard deviation was 0.5 minutes. Test at $\alpha = 0.05$.

H_0: $\mu = 9$ H_a: $\mu \ne 9$

The standardized test statistic is

$$t_0 = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{8.75 - 9}{0.5/\sqrt{19}} = -1.75$$

The standardized test statistic falls in the acceptance region, so we fail to reject H_0.

At the 5% level of significance, there is not enough evidence to reject the null hypothesis that the average length of a phone call is 9 minutes.

20

Example 1

Suppose we want to test whether or not the average length of a phone call is 9 minutes. In a random sample of 19 phone calls, the sample mean was 8.75 minutes and the standard deviation was 0.5 minutes. Test at $\alpha = 0.05$.

H_0: $\mu = 9$ H_a: $\mu > 9.000$ (Claim)

The level of significance is $\alpha = 0.05$.

The test is a two-tailed test.

Degrees of freedom are $df = 18 - 1 = 17$.

The critical values are $-t_{\alpha/2,\nu-1} = 2.110$ and $t_{\alpha/2,\nu-1} = 2.110$

$t_{\alpha/2,\nu-1} = t_{0.025,18} = 2.110$

$-t_{\alpha/2,\nu-1} = -t_{0.025,18} = -2.110$

$t_0 = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{8.75 - 9}{0.5/\sqrt{19}} = -1.75$

$-t_{\alpha/2,\nu-1} < t_0 < t_{\alpha/2,\nu-1}$

$-2.110 < -1.75 < 2.110$

fail to reject H0

21

Solution 1

Suppose we want to test whether or not the average length of a phone call is 9 minutes. In a random sample of 19 phone calls, the sample mean was 8.75 minutes and the standard deviation was 0.5 minutes. Test at $\alpha = 0.05$.

H_0: $\mu = 9$ H_a: $\mu \ne 9$

At the 5% level of significance, there is not enough evidence to reject the claim that the average length of a phone call is 9 minutes.

22

P-value for a t test

The P-value of a t-test is just the smallest level of significance at which the null hypothesis would be rejected.

Computation of exact P value directly from the table D is usually not possible.

However, it is possible to find the upper and lower bounds on the P value from this table

23

Obtaining P values

Suppose we calculate $t = 2.20$ for a **one-tailed** test from a sample size of 6. $df = n - 1 = 5$

$0.025 < P\text{-value} < 0.05$

Suppose we calculate $t = 2.20$ for a **two-tailed** test from a sample size of 6. $df = n - 1 = 5$

$0.05 < P\text{-value} < 0.02$

24

Example 1

H_0: $\mu = 9$ H_a: $\mu > 9.000$ (Claim)

The level of significance is $\alpha = 0.05$.

The test is a two-tailed test.

Degrees of freedom are $df = 18 - 1 = 17$

The standardized test statistic is

$$t_0 = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{8.75 - 9}{0.5/\sqrt{19}} = -1.75$$

$t_0 < t_{\alpha,\nu-1}$

$-1.75 < 1.740$

$0.025 < P < 0.05$

$P < \alpha$

reject H0

25

Example 2a

A manufacturer claims that the bulbs manufactured by the company have an average life greater than 1000 hours. A random sample of 36 bulbs has a mean life of 1050 hours and a standard deviation of 50. Is there enough evidence to support this claim at $\alpha = 0.05$? Test using P-value.

claim: average > 1000

H_0: $\mu = 1000$ H_a: $\mu > 1000$ (Claim)

The level of significance is $\alpha = 0.05$.

The degrees of freedom are $df = n - 1 = 10 - 1 = 9$.

The standardized test statistic is

$$t_0 = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{1050 - 1000}{50/\sqrt{36}} = 6$$

$t_0 = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$

$t_0 > t_{\alpha,\nu-1}$

$6 > 1.833$

reject H0

26

Example 2a

Using the $df = 9$ row from Table D, you can determine that P is greater 0.25 and is therefore also greater than 0.01 significance level (α). So we fail to reject H_0.

At the 1% level of significance, there is not enough evidence to support the manufacturer's claim.

27

Example 2b

A manufacturer claims that the bulbs manufactured by the company have an average life greater than 1000 hours. A random sample of 36 bulbs has a mean life of 1050 hours and a standard deviation of 50. Is there enough evidence to support this claim at $\alpha = 0.05$? Test using P-value.

H_0: $\mu = 1000$ H_a: $\mu > 1000$ (Claim)

The level of significance is $\alpha = 0.05$. $t_{0.025,9} = 2.262$

The degrees of freedom are $df = n - 1 = 36 - 1 = 35$.

The standardized test statistic is $t_0 = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{1050 - 1000}{50/\sqrt{36}} = 6.45$

$t_{0.025,35} = 2.030$ and $t_0 = 6.45$. Since the test statistic t_0 falls in the acceptance region, we do not fail to reject H_0.

At the 1% level of significance, there is not enough evidence to support the claim that the rechargeable battery has an average life of greater than 1000 charges.

28

Problem 1

- Suppose that the cholesterol levels for males aged 50 to 64 are investigated. Suppose also that whether cholesterol levels are significantly changed by modifying the diet only slightly.
- A random sample of 10 patients is chosen and they follow the modified diet for 3 months. After 3 months their cholesterol levels are measured and their mean cholesterol level is found to be 240 with standard deviation 15.
- Based on this data, is there statistical evidence that the modified diet reduces cholesterol? Test at the level of significance 5%.

H_0: $\mu = 240$

H_a: $\mu < 240$ (claim)

$t_0 = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$

29

Solution

The level of significance is $\alpha = 0.05$

H_0: $\mu = 241$ H_a: $\mu < 241$ (Claim)

The degrees of freedom are $df = n - 1 = 12 - 1 = 11$.

$t_{0.05,11} = t_{\alpha,\nu-1} = 1.796$

The standardized test statistic is

$$t_0 = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{235 - 241}{12/\sqrt{12}} = -1.66$$

$\bar{x} = 235$, $\mu_0 = 241$, $s = 12$, $n = 12$

Since $t_0 > -1.796$, We do not reject H_0 fail to reject H_0.

We do not have significant evidence at $\alpha = 0.05$, to show that the diet reduces cholesterol level.

30

Instructor: Padma Natarajan

5