L7.notes

Binomial

Normal

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Lecture7print

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Math 661
Applied Statistics
Lecture 7
Chapter 1 and 5
Instructor: Padma Natarajan

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Normal Approximation to Binomial

- The normal distribution is used to approximate the binomial probabilities for cases in which n is large. It is difficult in such cases to calculate probabilities using binomial distribution.

n large… binomial prob → normal distribution

- If $np \geq 10$ and $nq \geq 10$, then the binomial random variable X is approximately normally distributed with
  - mean $\mu = np$
  - standard deviation $\sigma = \sqrt{npq}$

1 trial success
Prob success
n trials independent
outcomes success

$<$ $>$
$\leq$ $\geq$
$+0.5$ -0.5

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Normal approximation to binomial

- If X is a binomial random variable with parameters n and p,

$$Z = \frac{X - np}{\sqrt{np(1-p)}}$$

is approximately a standard normal random variable. To approximate a binomial probability with a normal distribution, a continuity correction is applied.

The correction factor is used to improve the approximation.

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Continuity Correction

$$P(X \leq x) = P(X \leq x + 0.5) \cong P\left(Z \leq \frac{x + 0.5 - np}{\sqrt{np(1-p)}}\right)$$

and

$$P(X \geq x) = P(X \geq x - 0.5) \cong P\left(Z \geq \frac{x - 0.5 - np}{\sqrt{np(1-p)}}\right)$$

The approximation is good for $np \geq 10$ and $n(1-p) \geq 10$

A way to remember the approximation is to write the probability in terms of $\leq$ and then add the 0.5 correction factor or $\geq$ and then subtract the 0.5 correction factor

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Example 20

- An antibiotic was 60% effective against a common bacteria. Suppose that the antibiotic is given to 25 patients with the bacteria. What is the probability that the antibiotic is effective in more than 15 patients?

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Solution

- p = 0.60 n = 25
- P(X > 15)
- np = 25(0.60) = 15 $\geq$ 10
- nq = 25(0.40) = 10 $\geq$ 10
- μ = 25(0.6)= 15
  σ=√np(1-p) = √25(0.6)(0.4)= √6 =2.45
- P(X ≥ 16) = P(X ≥ 15.5) = P(Z > $\frac{15.5-15}{2.45}$)
  = P(Z > 0.20)
  =1- 0.5793 = 0.4207

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EX 20:

$p = 6\% = 0.6 \quad n = 25 \quad Q\ P(X \geq 15)$

$X \sim Binomial(25, 0.6)$

$np = 25(0.6) = 15 \geq 10$

$nq = 25(0.4) = 10 \geq 10$

$\mu = np = 25(0.6) = 15$

$\sigma = \sqrt{npq} = \sqrt{25(0.6)(0.4)} = 2.45$

$X \sim N(15, 2.45)$

Q: $P(X > 15)$

Continuity correction
15 (16
$>$ $\geq$ -0.5

$P(X \geq 16) = P(X \geq 15.5)$

$= P\left(Z \geq \frac{15.5 - np}{\sqrt{np(1-p)}}\right) = P\left(Z \geq \frac{15.5 - 15}{\sqrt{6}}\right) = P(Z \geq 0.20)$

$= 1 - \Phi(0.20) = 1 - 0.5793$

$= 0.4207$

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Example 21

- An antibiotic was 60% effective against a common bacteria. Suppose that the antibiotic is given to 25 patients with the bacteria. What is the probability that the antibiotic is effective in at most 12 patients?

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Solution

- p = 0.60 n = 25
- P(X ≤ 12)
- np = 25(0.60) = 15 $\geq$ 10
- nq = 25(0.40) = 10 $\geq$ 10
- μ = 25(0.6)= 15
  σ=√np(1-p) = √25(0.6)(0.4)= √6 =2.45
- P(X ≤ 12) = P(X ≤ 12.5) = P(Z < $\frac{12.5-15}{2.45}$)
  = P(Z < -1.02)
  =0.1539

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Example 22

A certain manufacturing company produces 3% defective bulbs. Assume that the bulbs are independent and that a shipment contains 1000 bulbs.

a) Approximate the probability that more than 30 bulbs are defective

b) Approximate the probability that between 25 and 35 bulbs are defective

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Solution

Let X denote the number of defective bulbs in the shipment.

$n = 1000 \quad p = 0.03$

$\mu = np = 1000(0.03) = 30$

np and $n(1-p)$ are $\geq \boxed{10}$

Since np ≥ 10 and n(1- p) ≥ 10, you can use normal approximation to binomial

$\sigma = \sqrt{np(1-p)} = \sqrt{1000(0.03)(1-0.03)} = \sqrt{29.1} = 5.394$

$P(X > 30) = P(X \geq 31) = P(X \geq 30.5)$

$P\left(Z \geq \frac{30.5 - 30}{5.394}\right) = P(Z \geq 0.093) = 1 - P(Z \leq 0.093)$

$= 1 - 0.5359$

$= 0.4641$

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Solution

Given

$n = 1000 \quad p = 0.03$

$\mu = np = 1000(0.03) = 30$

$\sigma = \sqrt{np(1-p)} = \sqrt{1000(0.03)(1-0.03)} = \sqrt{29.1} = 5.394$

$P(25 < X < 35) = P(26 \leq X \leq 34) = P(25.5 \leq X \leq 34.5)$

$P\left(\frac{25.5-30}{5.394} \leq Z \leq \frac{34.5-30}{5.394}\right) = P(-0.834 \leq Z \leq 0.834) = 0.7967 - 0.2033$

$= 0.5934$

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EX 21

$p = 60\% = 0.6 \quad n = 25$

Q: $P(X \leq 12)$

$X \sim Binomial(25, 0.6)$

$np = 25(0.6) = 15 \geq 10$

$nq = 25(0.4) = 10 \geq 10$ → N

$\mu = np = 25(0.6) = 15$

$\sigma = \sqrt{npq} = \sqrt{25(0.6)(0.4)} = \sqrt{6} = 2.45$

$X \sim N(15, 2.45)$

continuity correction $<$ $\leq$ $+0.5$

$P(X \leq 12) = P(X \leq 12.5)$

$= P\left(Z \leq \frac{12.5 - np}{\sqrt{np(1-p)}}\right) = P\left(Z \leq \frac{12.5 - 15}{\sqrt{6}}\right)$

$= P(Z \leq -1.02) = \Phi(-1.02)$

$= 0.1539$

EX 22:

$p = 3\% = 0.03 \quad n = 1000$

Qa: $P(X > 30)$

Qb: $P(25 < X < 35)$

X: no. of bulbs defective in the next 1000 bulbs

$X \sim Binomial(1000, 0.03)$

$np = 1000(0.03) = 30 \geq 10$

$nq = 1000(0.97) = 970 \geq 10$ } N

$\mu = np = 1000(0.03) = 30$

$\sigma = \sqrt{npq} = \sqrt{1000(0.03)(0.97)} = 5.39$

$X \sim N(30, 5.39)$

Qa continuity correction
30 (31
$>$ $\geq$ -0.5

$P(X > 30) = P(X \geq 31) = P(X \geq 30.5)$

$P\left(Z \geq \frac{30.5 - np}{\sqrt{np(1-p)}}\right) = P\left(Z \geq \frac{30.5 - 30}{5.39}\right) = P(Z \geq 0.09)$

$1 - \Phi(0.09) = 1 - 0.5359 = 0.4641$

Qb: $P(25 < X < 35)$

continuity correction
34) 35
$<$ $\leq$ $+0.5$ $P(X \leq 34.5)$
25 (26
$>$ $\geq$ -0.5 $P(X \geq 25.5)$

$P(25.5 \leq X \leq 34.5)$

$= P\left(\frac{25.5 - np}{\sqrt{np(1-p)}} \leq Z \leq \frac{34.5 - np}{\sqrt{np(1-p)}}\right) = P\left(\frac{25.5 - 30}{5.39} \leq Z \leq \frac{34.5 - 30}{5.39}\right)$

$= P(-0.83 \leq Z \leq 0.83)$

$= \Phi(0.83) - \Phi(-0.83)$

$= 0.7967 - 0.2033$

$= 0.5934$