L11 Problem Formative

avg — mean $\boxed{\mu}$

$\mu_0 = 1.5$

$H_0 = \mu = \mu_0 = 1.5$

$H_1 = \mu \neq \mu_0 = 1.5$　　2-sided test $Z_{\alpha/2}$

$\sigma = 0.01$

Sample $= n = 25$

$\bar{x} = 1.4975$

$\alpha = 0.01$

Q1 : test hypothesis

$\alpha = 0.01$　　$\alpha/2 = 0.005$　　$1-\alpha/2 = 0.995$

$Z_{\alpha/2} = 2.575$

$\phantom{Z_{\alpha/2}} = -2.575$

$Z_0 = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \dfrac{1.4975 - 1.5}{0.01/\sqrt{25}} = -1.25$

$-2.575 \quad -1.25$

$Z_0 > Z_{\alpha/2}$　　fail to reject H_0.

Q2 P value

$Z_0 = -1.25$

$\phi(-1.25) = 0.1056. \quad > 0.01$

P value $= 2(0.1056) = \boxed{0.2112}$

.0025

Q3 : power. β　　$\mu = 1.495$　　　$\delta = \mu - \mu_0$

$\beta = \phi\left(Z_{\alpha/2} - \dfrac{\delta\sqrt{n}}{\sigma}\right) - \phi\left(-Z_{\alpha/2} - \dfrac{\delta\sqrt{n}}{\sigma}\right)$

$ = 1.495 - 1.5$

$ = -0.005$

$Z_{\alpha/2} - \dfrac{\delta\sqrt{n}}{\sigma} = 2.575 - \dfrac{(-0.005)(\sqrt{25})}{0.01} = 5.075 \quad \phi(5.075) = 1$

$-Z_{\alpha/2} - \dfrac{\delta\sqrt{n}}{\sigma} = -2.575 - \dfrac{(-0.005)(\sqrt{25})}{0.01} = 0.075 \quad \phi(0.08) = 0.5319$

$\beta = \boxed{0.4681}$

~~**Q4**~~ : Sample Size.　　$\mu = 1.495$　power $= 0.9$　$\beta = 1 - 0.9 = 0.1$

$\delta = \mu - \mu_0 = 1.495 - 1.5 = -0.005$　　$Z_\beta = +1.28$

$n = \dfrac{(Z_{\alpha/2} + \boxed{Z_\beta})^2 \sigma^2}{\delta^2} = \dfrac{(2.575 + 1.28)^2 \cdot (0.01)^2}{(-0.005)^2} = \boxed{6.2081} \rightarrow \boxed{7}$

$59.46441 \quad \boxed{60}$

Q5 CI : μ

$m = \dfrac{Z_{\alpha/2} \cdot \sigma}{\sqrt{n}} = \dfrac{(2.575)(0.01)}{25} = 0.00515 \quad 1.4975 -$

$ 1.4924$

$\boxed{\begin{array}{l} 1.50 \; (1.4923, \; 1.5027) \quad \times \text{reject } H_0 \\ 1.3 \; \times \qquad\qquad\qquad\quad \checkmark \text{reject } H_0. \end{array}}$ explains CI

$\bar{x} - m = 1.4975 - 0.00525 = 1.4923$

$\bar{x} + m = 1.4975 + 0.00525 = 1.5027$