# 1-3-Define-2

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## Game Theory Intro

Game Theory Course:
Jackson, Leyton-Brown & Shoham



## Defining Games - Key Ingredients

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  - People? Governments? Companies? Somebody employed by a Company?...



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  - Enter a bid in an auction? Decide whether to end a strike? Decide when to sell a stock? Decide how to vote?
- Payoffs: what motivates players?
  - Do they care about some profit? Do they care about other players?...



## Defining Games - Two Standard Representations

- Normal Form (a.k.a. Matrix Form, Strategic Form) List what payoffs get as a function of their actions
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- Normal Form (a.k.a. Matrix Form, Strategic Form) List what payoffs get as a function of their actions
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  - But strategies encode many things...
- Extensive Form Includes timing of moves (later in course)
  - Players move sequentially, represented as a tree
    - Chess: white player moves, then black player can see white's move and react...
  - Keeps track of what each player knows when he or she makes each decision
    - Poker: bet sequentially – what can a given player see when they bet?



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    - $a = (a_1, \ldots, a_n) \in A = A_1 \times \ldots \times A_n$ is an action profile



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  - Action set for player $i$ $A_i$
    - $a = \underbrace{(a_1, \ldots, a_n)}_{\text{every player is doing}} \in A = A_1 \times \ldots \times A_n$ is an action profile
  - Utility function or Payoff function for each player $i$: $u_i : A \to \mathbb{R}$ ← an action played; what's the payoff?
    - $u = (u_1, \ldots, u_n)$, is a profile of utility functions — evaluate outcome of game



## Normal Form Games - The Standard Matrix Representation

- Writing a 2-player game as a matrix:
  - "row" player is player 1, "column" player is player 2
  - rows correspond to actions $a_1 \in A_1$, columns correspond to actions $a_2 \in A_2$
  - cells listing utility or payoff values for each player: the row player first, then the column



## Games in Matrix Form

Here's the TCP Backoff Game written as a matrix

player 2 column

player 1 row

| | $C$ | $D$ |
|---|---|---|
| $C$ | $-1, -1$ | $-4, 0$ |
| $D$ | $0, -4$ | $-3, -3$ |



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binary choice



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- Players: $N = \{1, \ldots, 10,000,000\}$
- Action set for player $i$ $A_i = \{Revolt, Not\}$
- Utility function for player $i$:
  - $u_i(a) = 1$ if $\#\{j : a_j = Revolt\} \geq 2,000,000$ — regardless $a_j$ Revolt or not
  - $u_i(a) = -1$ if $\#\{j : a_j = Revolt\} < 2,000,000$ and $a_i = Revolt$ — $i$ participant in revolt - fails - payoff = −1 — punishment
  - $u_i(a) = 0$ if $\#\{j : a_j = Revolt\} < 2,000,000$ and $a_i = Not$ — $i$ × in revolt

no. of people revolt

Depend on: themselves; all players doing



## Summary: Defining Games

- For Now: Normal Form (Strategic Form, Matrix Representation...)
  - Players: $N$
  - Actions: $A_i$
  - Payoffs: $u_i$
- Later: Extensive Form
  - Timing: in what order do things happen?
  - Information: what do players know when they act