Problem Set 2

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Problem Set 2

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Graded Quiz • 10 min

Due May 4, 2:59 PM +08

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TO PASS 70% or higher

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Problem Set 2

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1. 1 / 1 point

1 \ 2	Left	Right
Left	4,2	5,1
Right	6,0	3,3

Find a mixed strategy Nash equilibrium where player 1 randomizes over the pure strategy Left and Right with probability p for Left. What is p?

- a) 1/4
- b) 3/4
- c) 1/2
- d) 2/3

Correct
(b) is true.

- In a mixed strategy equilibrium in this game both players must mix and so 2 must be indifferent between Left and Right.
- Left gives 2 an expected payoff: $2p + 0(1-p)$
- Right gives 2 an expected payoff: $1p + 3(1-p)$
- Setting these two payoffs to be equal leads to $p = 3/4$.

2. 1 / 1 point

1 \ 2	Left	Right
Left	X,2	0,0
Right	0,0	2,2

In a mixed strategy Nash equilibrium where player 1 plays Left with probability p and player 2 plays Left with probability q. How do p and q change as X is increased ($X > 1$)?

- a) p is the same, q decreases.
- b) p increases, q increases.
- c) p decreases, q decreases.
- d) p is the same, q increases.

Correct
(a) is true.

- In a mixed strategy equilibrium, 1 and 2 are each indifferent between Left and Right.
- For p:
- Left gives 2 an expected payoff: $2p$
- Right gives 2 an expected payoff: $2(1-p)$
- These two payoffs are equal, thus we have $p = 1/2$.
- For q: setting the Left expected payoff equal to the Right leads to $Xq = 2(1-q)$, thus $q = 2/(X+2)$, which decreases in X.

3. 1 / 1 point

- There are 2 firms, each advertising an available job opening.
- Firms offer different wages: Firm 1 offers $w_1 = 4$ and 2 offers $w_2 = 6$.
- There are two unemployed workers looking for jobs. They simultaneously apply to either of the firms.
- If only one worker applies to a firm, then he/she gets the job
- If both workers apply to the same firm, the firm hires a worker at random and the other worker remains unemployed (and receives a payoff of 0).

Find a mixed strategy Nash Equilibrium where p is the probability that worker 1 applies to firm 1 and q is the probability that worker 2 applies to firm 1.

- c) $p = q = 1/4$;
- b) $p = q = 1/3$;
- d) $p = q = 1/5$.
- a) $p = q = 1/2$;

Correct
(d) is correct.

- In a mixed strategy equilibrium, worker 1 and 2 must be indifferent between applying to firm 1 and 2.
- For a given p, worker 2's indifference condition is given by $2p + 4(1-p) = 6p + 3(1-p)$.
- Similarly, for a given q, worker 1's indifference condition is given by $2q + 4(1-q) = 6q + 3(1-q)$.
- Both conditions are satisfied when $p = q = 1/5$.

4. 1 / 1 point

- A king is deciding where to hide his treasure, while a pirate is deciding where to look for the treasure.
- The payoff to the king from successfully hiding the treasure is 5 and from having it found is 2.
- The payoff to the pirate from finding the treasure is 9 and from not finding it is 4.
- The king can hide it in location X, Y or Z.

Suppose the pirate has two pure strategies: inspect both X and Y (they are close together), or just inspect Z (it is far away). Find a mixed strategy Nash equilibrium where p is the probability the treasure is hidden in X or Y and $1-p$ that it is hidden in Z (treat the king as having two strategies) and q is the probability that the pirate inspects X and Y:

- a) $p = 1/2, q = 1/2$;
- b) $p = 4/9, q = 2/5$;
- c) $p = 5/9, q = 3/5$;
- d) $p = 2/5, q = 4/9$;

Correct
(a) is true.

- There is no pure strategy equilibrium, so in a mixed strategy equilibrium, both players are indifferent among their strategies.
- For p:
- Inspecting X \& Y gives pirate a payoff: $9p + 4(1-p)$
- Inspecting Z gives pirate a payoff: $4p + 9(1-p)$
- These two payoffs are equal, thus we have $p = 1/2$.
- For q: indifference for the king requires that $5q + 2(1-q) = 2q + 5(1-q)$, thus $q = 1/2$.

5. 1 / 1 point

- A king is deciding where to hide his treasure, while a pirate is deciding where to look for the treasure.
- The payoff to the king from successfully hiding the treasure is 5 and from having it found is 2.
- The payoff to the pirate from finding the treasure is 9 and from not finding it is 4.
- The king can hide it in location X, Y or Z.

Suppose that the pirate can investigate any two locations, so has three pure strategies: inspect XY or YZ or XZ. Find a mixed strategy Nash equilibrium where the king mixes over three locations (X, Y, Z) and the pirate mixes over (XY, YZ, XZ). The following probabilities (king), (pirate) form an equilibrium:

- a) (1/3, 1/3, 1/3), (4/9, 4/9, 1/9);
- b) (4/9, 4/9, 1/9), (1/3, 1/3, 1/3);
- c) (1/3, 1/3, 1/3), (2/5, 2/5, 1/5);
- d) (1/3, 1/3, 1/3), (1/3, 1/3, 1/3);

Correct
(d) is true.

- Check (a):
- Pirate inspects (XY, YZ, XZ) with prob (4/9, 4/9, 1/9);
- Y is inspected with prob 8/9 while X (or Z) is inspected with prob 5/9;
- King prefers to hide in X or Z, which contradicts the fact that in a mixed strategy equilibrium, king should be indifferent.
- Similarly, you can verify that (b) and (c) are not equilibria in the same way.
- In (d), every place is chosen by king and inspected by pirate with equal probability and they are indifferent between all strategies.