2-2-MixedNash

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2-2-MixedNa...

Mixed Strategies and Nash Equilibrium

Game Theory Course:
Jackson, Leyton-Brown & Shoham

Mixed Strategies

- It would be a pretty bad idea to play any deterministic strategy in matching pennies

确定性的

2 ↓ 　 ↓

	Heads	Tails
1 → Heads	1, −1	−1, 1
Tails	−1, 1	1, −1

start

cycle.
no pair of deterministic strategy
w both

Mixed Strategies

- It would be a pretty bad idea to play any deterministic strategy in matching pennies
- Idea: confuse the opponent by playing randomly
- Define a strategy s_i for agent i as any probability distribution over the actions A_i.
  - pure strategy: only one action is played with positive probability
  - mixed strategy: more than one action is played with positive probability
    set
    - these actions are called the support of the mixed strategy
- Let the set of all strategies for i be S_i
- Let the set of all strategy profiles be $S = S_1 \times \ldots \times S_n$.

prob distri
$i - s_i - A_i - \frac{a_i}{s_i}$
S_i
S
support

Utility under Mixed Strategies

- What is your payoff if all the players follow mixed strategy profile $s \in S$?
  - We can't just read this number from the game matrix anymore: we won't always end up in the same cell

Utility under Mixed Strategies

- What is your payoff if all the players follow mixed strategy profile $s \in S$?
  - We can't just read this number from the game matrix anymore: we won't always end up in the same cell
- Instead, use the idea of expected utility from decision theory:

take expectation over all action profile
support of strategy profile
weight.

sum of all cells
utility of each cell

$$u_i(s) = \sum_{a \in A} u_i(a) Pr(a|s)$$

prob - cell reach
given mixed strategy
action profile a - strategy profile s

	0.5	0.5
0.5	0.75	
0.5		

$$Pr(a|s) = \prod_{j \in N} s_j(a_j)$$

product - each player play his part of action profile

Best Response and Nash Equilibrium

Our definitions of best response and Nash equilibrium generalize from actions to strategies.

Definition (Best response)

other strategy

$s_i^* \in BR(s_{-i})$ iff $\forall s_i \in S_i,\ u_i(s_i^*, s_{-i}) \geq u_i(s_i, s_{-i})$

a set of BR to s_{-i} strategy profile

Definition (Nash equilibrium)

$s = \langle s_1, \ldots, s_n \rangle$ is a Nash equilibrium iff $\forall i,\ s_i \in BR(s_{-i})$

may be ≥1

Best Response and Nash Equilibrium

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Definition (Best response)

$s_i^* \in BR(s_{-i})$ iff $\forall s_i \in S_i,\ u_i(s_i^*, s_{-i}) \geq u_i(s_i, s_{-i})$

Definition (Nash equilibrium) Pure strategy Nash

$s = \langle s_1, \ldots, s_n \rangle$ is a Nash equilibrium iff $\forall i,\ s_i \in BR(s_{-i})$

a $\langle a_1 \cdots a_n \rangle$ 　 $a_i \in BR(a_{-i})$

Theorem (Nash, 1950)

Every finite game has a Nash equilibrium.

game take finite amount of space to write down
# of player,
# of actions / player), utility

stable thing, - know other do → x change

Example: Matching Pennies

	0.5 Heads	0.5 Tails
0.5 Heads	1, −1	−1, 1
0.5 Tails	−1, 1	1, −1

Example: Coordination

	0.5 Left	0.5 Right
0.5 Left	1, 1	0, 0
0.5 Right	0, 0	1, 1

Example: Prisoner's Dilemma

	C	D
C	−1, −1	−4, 0
D	0, −4	−3, −3