

Hardness





Hardness Beyond 2×2 Games

Game Theory Course:

Game Theory

Game

Theory

entability Online

Jackson, Leyton-Brown & Shoham

Hardness beyond 2×2 games Algorithms

Two example algorithms for finding NE

 LCP (Linear Complementarity) formulation • [Lemke-Howson '64]

- Support Enumeration Method
- [Porter et al. '04]

Game Theory Course: Jackson, Leyton-Brown & Shoham

Hardness beyond 2×2 games Early History

- 1928 von Neumann: existence of Equilibrium in 2-player, zero-sum games proof uses Brouwer's fixed point theorem; • led directly to algorithms:
- Danzig '57: equivalent to LP duality
- Khachiyan'79: polynomial-time solvable

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Early History

Hardness beyond 2×2 games

• 1928 von Neumann: existence of Equilibrium in 2-player, zero-sum games

- led directly to algorithms: Danzig '57: equivalent to LP duality Khachiyan'79: polynomial-time solvable
- 1950 Nash: existence of Equilibrium in multiplayer,
 - general-sum games
- proof also uses Brouwer's fixed point theorem; • intense effort on equilibrium algorithms: • Kuhn '61, Mangasarian '64, Lemke-Howson '64, Rosenmüller '71,

proof uses Brouwer's fixed point theorem;

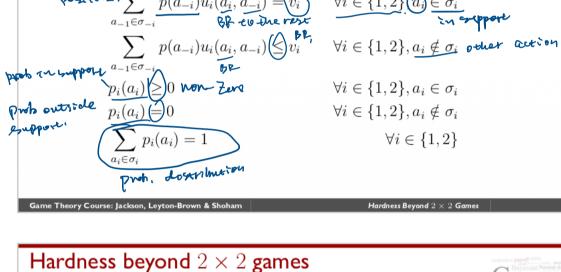
- Wilson '71, Scarf '67, Eaves '72, Laan-Talman '79, Porter et al. '04, all exponential in the worst case

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The Lemke-Howson Algorithm Normalize it - probiplager 2 - Strategy Os LCP (Linear Complementarity) formulation $\forall j \in A_1, \ \forall k \in A_2$

Support Enumeration Method: Porter et al. 2004 • Step 1: Finding a NE with a specific support

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support General Size es other 2 17 x

small slarge

Support Enumeration Method: Porter et al. 2004

Game

• Step 2: Smart heuristic search through all sets of support

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From Algorithms to Complexity Analysis

These algorithms have exponential worst-case time complexity.

Game

I found on Poly.

Game

hardest part of NP alass

Theory

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• Reminder of a (small part) of the complexity hierarchy.

From Algorithms to Complexity Analysis

So do all known others.

· Can we do better?

NP-complete

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NP dynamical time courtion Hardness beyond 2×2 games

equilibrium in G? 3. (Guaranteed payoff) Given a game G and a value v, does there exist an equilibrium in G in which some player i obtains an expected payoff of at least v? with integral i in the content of i in the content (Guaranteed social welfare) Given a game G, does there exist an equilibrium in which the sum of agents' utilities is at least k?

From Algorithms to Complexity Analysis

(Action inclusion) Given a game G and an action $a_i \in A_i$ for some player $i \in N$, does there exist an equilibrium of \widetilde{G} in which player i plays action a_i with strictly positive 6. (Action exclusion) Given a game G and an action $a_i \in A_i$ for some player $i \in N$, does there exist an equilibrium of G in which player i plays action a_i with zero probability?

So, is it NP-complete to find a Nash equilibrium?

 Strictly speaking, no, since a solution is guaranteed to exist... However, it is NP-complete to find a "tiny" bit more info than a

Nash equilibrium; e.g., the following are NP-complete: (Uniqueness) Given a game G, does there exist a unique equilibrium in G? 2. (Pareto optimality) Given a game G, does there exist a strictly Pareto efficient

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Still, finding even a single Nash equilibrium seems hard; how do we capture that?

• FNP problems are constructive versions of NP problems (F stands

• Enter PPAD ("Polynomial Parity Arguments on Directed

for "Functional") • TFNP is a subclass of FNP for problems for which a solution is guaranteed to exist (T stands for "Total")

item (Papadimitriou '94)

graphs")

At a high level:

 PPAD is a subclass of TFNP where the proofs are based on parity arguments in directed graphs

From Algorithms to Complexity Analysis Where is PPAD?

Polynomial time prothem

PPAD

Game

Theory

From Algorithms to Complexity Analysis

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The Complexity of the Nash Equilibrium **Theorem:** Computing a Nash equilibrium is PPAD-complete...

- for games with ≥4 players; [Daskalakis, Goldberg, Papadimitriou '05] • for games with 3 players; [Chen, Deng '05] &
- for games with 2 players. [Chen, Deng '06]

[Daskalakis, Papadimitriou '05]

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NP-complete NP

Hardness beyond 2×2 games From Algorithms to Complexity Analysis Online Where is PPAD? NP-complete