4.Introduction-to-Forecasting--Holt-Winters-for-Trend-and-Seasonality

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Exponential Smoothing with Trend and Seasonality

Here we go! The big enchilada! Triple Exponential Smoothing!

There is an obvious periodic or seasonal component to the *AirPassengers* data set introduced in the last lecture. This is true for many data sets (think back to the Mauna Loa CO2 data set). We have already seen how to take a smoothed average in order to use past values in a forecast, and we can also accommodate a rising and falling data set by making a guess on current trend via current system value and past trend values. We will do something similar to bring seasonality into our forecasting. We'll also allow ourselves to look h steps into the future, instead of just one step as we've been doing so far.

We start with the "additive form" of the equations (i.e. we start with seasonal differences which are additive.)

- For example, if you believe your seasonal differences are *additive*, you might adjust your projected sales figures for canoes for the month of June based on, say, your January sales figures by adding a constant amount to your January. That is, you might take January sales data and add an extra 10,000 canoes to obtain the June projection.

- If you feel the seasonal differences are *multiplicative*, you might instead adjust your projected sales figures for canoes for the month of June based on your January sales figures by multiplying your January data by some constant amount. That is, you might take January sales data and multiply by three to obtain the June projection.

Additive seasonality doesn't care about the overall levels, whereas multiplicative seasonality does. A nice website to explore these differences is available from Nikolaos Kourentzes:

http://www.forsoc.net/2014/11/11/can-you-identify-additive-and-multiplicative-seasonality/

It asks the germane question "Can you identify additive and multiplicative seasonality?" and gives you some practice. We will discuss this some more in the context of Air Passenger Miles.

Developing Triple Exponential Smoothing Formulas with Additive Seasonality

The equations for your forecast with ***additive seasonality*** will look like (don't worry about that strange looking last term just yet)

repeating pattern
variability - constant in time — √ noise
√ systematic change in variability

$$\hat{x}_{n+h} = \text{level}_n + h \cdot \text{trend}_n + \text{seasonal}_{n+h-m}$$

We hope you won't mind that, once we develop a one-step-ahead term called $trend_n$, we move into the future h steps in the simplest possible way by multiplying $trend_n$ by the number of time lags h we wish to look into the future (i.e. distance travelled is the size of your step multiplied by the number of steps).

variability grow

Multiplicative seasonality will look much the same, except that we will *multiply*

$$\hat{x}_{n+h} = (\text{level}_n + h \cdot \text{trend}_n) \cdot \text{seasonal}_{n+h-m}$$

Common Format

Our level, trend, and seasonal terms will all look like

$$\text{greek letter} \cdot \textit{this} + (1 - \text{greek letter}) \cdot \textit{that}$$

We will peer h steps into the future for a time series with seasonality of length m.

- h = number of steps into the future for forecast
- m = length of the season in your data

√ level, trend, seasonality

You can break the forecast down like this.

1. **Smooth the Level**

   To get started, develop the level term similarly to what we did before, but now take a weighted average of the seasonally adjusted level $x_n - s_{n-m}$ (we say we are *deseasonalizing*) with the non-seasonal forecast $level_{n-1} + trend_{n-1}$.

   $$level_n = \alpha(x_n - \text{seasonal}_{n-m}) + (1-\alpha)\,(level_{n-1} + trend_{n-1})$$

   cycle season

2. **Smooth the Trend**

   For the trend term, we keep it just as before

   $$trend_n = \beta \cdot (level_n - level_{n-1}) + (1-\beta) \cdot trend_{n-1}$$

   Smooth level - get rid of noise effect

3. **Smooth the Season**

   Finally, for the seasonal term we bring in a new parameter, γ and smooth over past seasons

   $$seasonal_n = \gamma \cdot (x_n - level_n) + (1-\gamma)\, seasonal_{n-m}$$

   stage of cycle

4. **Update the Forecast**

   For additive seasonality we use

   current smooth level
   seasonal contributions

   $$\hat{x}_{n+h} = level_n + h \, trend_n + seasonal_{n+h-m}$$

   h steps smooth trend step size
   trend contributions

   And for multiplicative seasonality we modify very slightly

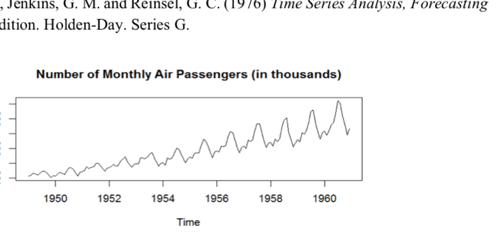
   $$\hat{x}_{n+h} = (level_n + h \cdot trend_n) \cdot seasonal_{n+h-m}$$

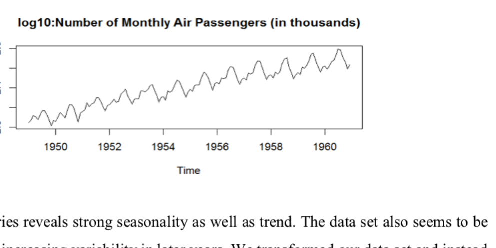
Your software will find optimal values of α and β and γ.

Airline Data

Recall the *AirPassengers* data set describing monthly totals of international airline passengers during the years 1949 to 1960.

Source: Box, G. E. P., Jenkins, G. M. and Reinsel, G. C. (1976) *Time Series Analysis, Forecasting and Control.* Third Edition. Holden-Day. Series G.





A plot of the time series reveals strong seasonality as well as trend. The data set also seems to be heteroscedastic, with increasing variability in later years. We transformed our data set and instead we considering $\log_{10}$(number of passengers). This gives us a data set with apparently additive seasonality and a calmer looking plot.

We saw that SES essentially led us to naïve forecasting.

$$\hat{x}_{n+1} = \alpha x_n$$

We obtained a quick quality measure,

AirPassengers.SES = HoltWinters(log10(AirPassengers), beta=FALSE, gamma=FALSE)

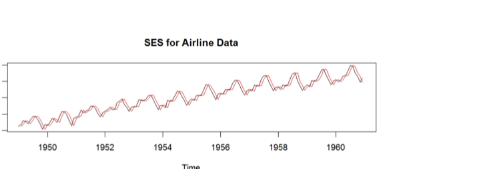
AirPassengers.SES$SSE # 0.3065102

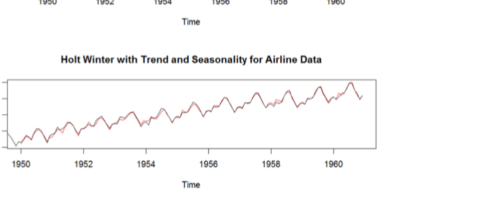
Let's see if we can reduce the error. The call will be very simple! Additive seasonality is the default. (Standard R documentation tells us that additive is the default since it is the first possibility listed for the seasonal argument: *seasonal = c("additive", "multiplicative")*

AirPassengers.HW = HoltWinters(log10(AirPassengers))

Our error certainly decreases and we have forecasts that hug the curve:

AirPassengers.HW$SSE # 0.0383026





The output gives us our parameters and we can build our forecast:

Results:

Smoothing parameters:

alpha:	*AirPassengers.hw$alpha*	#returns 0.326612
beta:	*AirPassengers.hw$beta*	#returns 0.005744246
gamma:	*AirPassengers.hw$gamma*	#returns 0.8207255

AirPassengers.hw$coefficients

Coefficients		s1 -0.031790733	s7 0.127820293
[,1]		s2 -0.061224237	s8 0.119893066
a	2.680599830	s3 -0.015941495	s9 0.038321003
b	0.003900787	s4 0.006307818	s10 -0.014181600
		s5 0.014138008	s11 -0.085995400
		s6 0.067260071	s12 -0.044672797

Forecast for January 1961:

$$\hat{x}_{n+h} = level_n + h \cdot trend_n + seasonal_{n+h-m}$$
$$\hat{x}_{144+1} = level_{144} + 1 \cdot trend_{144} + seasonal_{144+1-12}$$
$$\hat{x}_{144+1} = 2.680599 + 1 \cdot 0.003900787 + (-0.031790733)$$
$$\hat{x}_{144+1} = 2.652709$$

Forecast for August 1961:

$$\hat{x}_{n+h} = level_n + h \cdot trend_n + seasonal_{n+h-m}$$
$$\hat{x}_{144+8} = level_{144} + 8 \cdot trend_{144} + seasonal_{144+8-12}$$
$$\hat{x}_{144+8} = 2.680599 + 8 \cdot 0.003900787 + (0.119893066)$$
$$\hat{x}_{144+8} = 2.831698$$

Can you build the forecast for December 1961? How about March 1962?

We've been eliding an issue and implicitly assuming with our subscript notation that we'd only look into the future by $h \le m$ months. Using modular (clock) arithmetic we can set up a more complete (but less obvious) notation.

Since you are likely to use software for your forecasts this is not much of a problem. In a hand calculation example like we've been doing, if you are looking far into the future, just make sure you grab the correct seasonal coefficient for the month of interest. For March 1962:

$$\hat{x}_{144+15} = 2.680599 + 15 \cdot 0.003900787 + (-0.015941495) = 2.723169$$

Rather than build the explicit forecast ourselves from the return values, let's make a quick call to the routine *forecast.HoltWinters()* in the library *forecast*.

```
#install.packages("forecast")
library("forecast")
AirPassengers.hw <- HoltWinters(log10(AirPassengers))
AirPassengers.forecast <- forecast.HoltWinters(AirPassengers.hw)
```

You can obtain point estimates and confidence intervals (we have for 80% and 95%) for your forecasts. I'll include the forecasts for 1961 below. You can see the point estimates in our figure as a solid line, together with shading indicating the interval estimates. (Remember that these are transformed values!)

AirPassengers.forecast

	Point Forecast	Lo 80	Hi 80	Lo 95	Hi 95
Jan 1961	2.652709	[illegible]	[illegible]	[illegible]	[illegible]
Feb 1961	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Mar 1961	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Apr 1961	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
May 1961	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Jun 1961	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Jul 1961	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Aug 1961	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Sep 1961	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Oct 1961	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Nov 1961	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Dec 1961	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Jan 1962	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Feb 1962	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Mar 1962	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Apr 1962	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
May 1962	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Jun 1962	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Jul 1962	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Aug 1962	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Sep 1962	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Oct 1962	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Nov 1962	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Dec 1962	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

And, of course we should produce a plot.

plot(AirPassengers.forecast, xlim=c(1949, 1963))

The plot shows the point forecasts, together with shading regions indicating the appropriate confidence bands.