Berkeley, Netwon, Wallis

Historical background

In Cavalieri/Wallis introduction it is said that negative integers and fractional numbers are already being used. When were these two kinds of numbers established in the society of scientists and how strongly?
Negative numbers are in use in Europe since the 14th or 15th century. Fractions have a longer history. But the acceptance of some of these numbers was still contested until the mid 19th century.

What are the paradoxes mentioned on page 66?
That an infinite whole can be made equal to its part (e.g. whole numbers and even numbers); that an infinitely small quantity violates the law stating: "if a non-zero magnitude of some kind (e.g. length) is multiplied by a large enough finite number, it will exceed any other magnitude of the same kind"; and other principles that were considered as applicable to any magnitude.

page 89: How important was the "Arithmetica infinitorium" in mathematics? Did it have a big impact at the time?
This (together with algebra) was the main motor for new developments in the 17th-18th century.

Q54: (Can we replace infinitesimal methods by finitary ones?): isn't the answer an obvious "no"?
The answer is actually yes: whatever was done at the time by infinitesimals could be done by the finite method of exhaustion, which is a geometrical method that can be translated into the modern (Weierstrassian) formalism.

Why did one publish the works 'Opus geometricum' and 'Traite des indivisibles' so late?
Perhaps because print is expensive, and there was not too much demand for such high and suspect mathematics at the time of their creation. Perhaps because the authors thought they were not ready for publication.

Is it true to say that Wallis and Newton are more motivated by the usefulness of mathematics? In contrary to Berkeley, who is more interested in a coherent mathematics?
We could say that about Wallis but no really about Newton. Newton is interested in theoretical considerations and coherence, but believes that his fluxions are properly established.

"See here in what a confusion you are when you resist the truth". It's a really strong statement, something that it's very rare in modern scientific literature. Was it normal for the time? How were the scientific letters that different mathematicians sent each other? Same thing happens in Berkeley's essay, where the tone is even harsher.
The mathematical scene was often quite polemical, like the general intellectual scene.

"Since limits are definite, to determine them is a purely geometrical problem" (p.60). Why does the author not consider other areas of mathematics?

There are two issues here. First, geometry is the most established and secure part of mathematics at the time. Second, geometry is also used as the general name for mathematics at the time.

Mathematical clarifications

Why is there a difference between a parallelogram with no height, and a line?
This is one of the problems of the infinitesimal method. If it's just a line, then no matter how much you multiply it, it will not make a surface. If it's a parallelogram, then it has to have finite, non-zero width. Some mathematicians tried to conceptualize a hybrid: a parallelogram with no width or infinitesimal width.

The sum of indivisibly small lines led to the area under a parabola, and the volume of a hyperbolic solid. This resembles what we do today with integration, and is quite standard. Were the results he produced an approximation, or was the method applied by Torricelli equivalent to integration?
It was an intuitive version of precise integration.

Page 68: "for let ∞ denote an infinite number" One? Are there different type of ∞?
Infinity was a contextual magnitude. This means that mixing infinities from different contexts, without verifying their relations, may lead to error. But there was no proper quantitative theory of kinds of infinities.

Page 68: can one write $1/\infty$? Is that not the same as writing 0?
Not if you believe in infinitesimals.

So, was Wallis the first one who introduced the definition of infinity (p.60, last par.)?
No, just the symbol.

What is the difference between a solid and a hyperbolic solid?
A solid is a 3 dimensional body. A hyperbolic solid is the body obtained by rotating a hyperbola around its symmetry line.

Page 93: what is being calculated? What is the duplicate ratio?
Wallis calculates the ratio of the volume of a cone and a cylinder. Duplicate ratio is square ratio (so 1:9 is the duplicate ration of 1:3)

What was the meaning of "limit" at this time, in comparison with nowadays?
Limit was not defined precisely, it was used intuitively as something approached but not reached, or as the edge of geometric magnitude (line, area, etc.).

How did Newton define a fluxion? Did he do it by the differential quotient in the limit (was this Newton or Leibniz) or was it a geometrical definition?
In modern terms, Newton's term "fluxion" is something like momentary velocity. "Fluent" is the accumulated motion, or the integral of the fluxion.

I don't understand the velocity analogy of Newton. Surely the "limit" velocity of a stopping body is zero?

After it stops it's zero. But if a ball rolls down a slope and hits a wall, it reaches a certain velocity "just" as it stops.

Berkley refers to "moments" in relation to velocities, which reminds me of physics and the notion of momenta. But I can't connect the notion of mass to geometry at all. Are moments (these momentaneous increments or decrements) like the same as delta_x or a very small line?
Newton's moment has nothing to do with the notions of moment or mass in modern physics. It is more like an infinitesimal moment of time (so yes, delta_x or an infinitesimal line).

What's a nascent, and evanescent quantity? Also what's an evanescent divisibilia?
Nascent=emerging, evanescent=disappearing. Evanescent divisibilia are divisible (finite nonzero) magnitudes that are in the process of disappearing.

On page 59, Newton says that he considers "quantities as made up of particles" and he clarifies that they should be understood as "evanescent divisibilia" rather than "indivisibilia". Right on the next page (p. 60) he argues that "all quantities will consist of indivisibilia" in contrary to Euclid's Tenth Book.
Indivisible here means atomic (not divisible into smaller parts, as opposed to infinitesimal, which means infinitely small – some thinks these two are the same things, but most do not). In this quote from page 60 Newton argues by contradiction. If there had been indivisibilia, Euclid's theorems would be contradicted. Therefore, there can be no indivisibilia.

What are first and last ratios?
The speed of a moving object "just after" it begins to move and "just before" it stops.

On pages 68-69 of Berkeley/Newton, Berkeley makes a clear objection to one of the methods of Newton to find the derivative. I think it comes down to: Newton used something which has an effect, but later reduces that something to nothing which annihilates the effect and therefore the whole reasoning. How did Newton try to explain this hole in his reasoning (he must have thought about it himself)? Or doesn't he try to explain it?
Newton's explanation is an intuitive version of taking the limit as the increment o goes to zero. But it's not completely precise.

Pg. 60 §9-§10: How was the "contradiction" (f*g)'=f'g+fg'=f'g+fg'+f'g' explained or dealt with back then? In other words, what was the justification for the first argument (taking -½ and +½)? Did they check their answers to real world applications and see that the second formula was false and the first formula correct in their experiments?
Both formulas are correct, if you are allowed to ignore lower order terms, which was considered acceptable. Empirical verification did play a role in mathematics, but also other mathematical methods that verified the results (like the geometric method of exhaustion).

Why does Berkeley justify the expansion (x+o)^n = x^n + nx^{n-1} o + ... "by the method of infinte series" ? This is just the binomial theorem -- at least if n is a natural number. But if it is not a natural number, the binomial theorem doesn't work, the calculation would rely on the binomial series, which itself relies on derivatives. Was that already established?
Newton had already established the binomial theorem for non integer rational powers (but not with a proof that would be considered acceptable today).

Berkeley's critique

page 71: I don't understand the second part of paragraph 20.
Berkeley criticizes the mathematicians not for drawing false conclusions (their conclusions are true) but for drawing them incorrectly. He is criticizing the justification, not the result.

Why are religion and faith so much mentioned in the second text?
Berkeley is trying to argue that mathematics is based on mystery and faith no less than religion.

Pg 65 §7: Does Berkley argue that people who use differentials of higher order than 1 are essentially doing religion or working on blind faith?
Yes.

On page 62 he starts the chapter by saying something about being a judge. I am very confused and don't understand what he wants to say with this first paragraph.
Berkeley says that a person with mathematical knowledge should not consider himself an authority when it comes to other things, such as religion.

Does Berkeley approve of the idea of a curve consisting of indivisible line segments? Berkley talks about a ''least discernible quantity''. Did the notion of something like this really exist at that time and if yes what was it / the thought behind it?
Berkeley rejected indivisibles, but he tied perception to existence – what is beyond perception should not be considered as really existing (but this is only one part of a unique religious philosophy, which we may discuss in class). The critique of infinitesimals was common, but Berkeley's particular philosophy was not very common.

Berkeley: On page 64, I don't see the point of discussing infinitesimal of higher orders, i.e. "quantities infinitely less than the least discernible quantity; and others infinitely less than those infinitely small ones". Since the differential of differential can be seen as the differential of another function, what's the difference with respect to the previous derivatives of calculus?
A differential of second order was not considered as a differential of a function. In fact the notion of function at the time was not yet properly developed.

Berkeley: On page 66, he tries to critic Newton's method with a geometric example. But to me it seems like he is clearly (it becomes obvious if you make a sketch) calculating two different things and therefore it comes as no surprise that his results are different. Assuming I'm not just misunderstanding what he does, how did he miss such an obvious error in his argument?
It's clear that these are two different "things". The question is, why choose one (with the halves) and not the other (which seems more intuitive) to calculate the differential. It's seems like the method is chosen to fit the result.

How can Berkeley claim that the velocity of the velocity exceed all human understanding? He probably had an idea of acceleration.

He did have an idea of acceleration. But he didn't think of it as the velocity of velocity, because this combination made no sense to him.

Page 72: "If you had committed only one error, you would not have come at a true solution of the problem." I cannot imagine, that when one make one error, one get the false solution but for more than two errors one can get the true solution. Or did I false understand?
Two errors may cancel each other.

Q51 What are their difficulties?
The use of unsound logic and metaphysics

At several places Berkeley refers to the geometers as the exact mathematics. Is he referring to Greek mathematics a la Euclid, here?
He is referring to the classical standard set by Greek geometry, as understood at his time.

Miscellanea

About question 53 on page 91. What is the end of geometry? Is there even one? And if there is, why would practice be the end?
"End" is used here in the sense of goal, not in the sense of closure.

Does &c mean et cetera and was this notation commonly used?
Yes

Q48 I do not understand the question at all. What is unsound?
Non rigorous, incorrect

sum of lines=plane. How is sum understood?
Vaguely, as putting things together. The word for sum can also be translated as "totality".

p.89 "Wallis was unaware of them" How can one know that?
Historians can sometimes reconstruct the trajectory of manuscripts based on correspondences. It is always possible that someone made a secret copy and transmitted it to Wallis without any record, but it is not quite likely. Also, Wallis was in the habit of giving credit to his predecessors.

In Wallis Ch. 3.2.3, p.89 is a formula Rn / (n+1). Is this a printing error? Shouldn't it be Rn+1 / (n+1) ? I got at least for n=2 the latter result.
You're right

Page 89: what is the meaning of numerical interpolation?
Fitting a formula to the data

The mathematical statements (l+1)/3 l^2+(l+1)/6l l^2 don't make sense to me. I understand the mathematical argument by induction, but these symbols seem to just appear here randomly. Is it the term added in the induction step?

This is a formula that fits the results. It is not justified beyond the fact that it works. So this is not induction in the modern mathematical sense, just a generalization by guessing.

(P.93) I don't quite follow the first step of Wallis' induction. Is (0+1=1)/(1+1=2)=1/6 a typo?
Yes

On page 69 of Berkeley/Newton it says: "[...] such as would not be allowed of in Divinity." What is meant by this?
Divinity here means theology. Berkeley is saying that mathematicians allow themselves faulty arguments that would not be tolerated even by theologians.

Page 64: The writer use so often the word "infinitely". Is this a "coincidence" or is there a reason behind it? I got confused with so many same words!
It's irony…

Towards the end it looks like Berkeley's approach is monster barring in Lakatos' terms. Moreover, in point 12, he completely departs from Lakatos definition of proof and lemma. Was Lakatos aware of this text, do we know if he relied on it to formulate his thesis?
He was probably aware of it, because it is so famous in the history of mathematics. But I don't think he used it to construct his theory. His source materials are mostly from later periods.

Referring to page 58: what does it mean "giving a proof by reductio ad absurdum"?
Proof by contradiction.

Thakkar

Regarding Thakkars text: Were there any non-cleric mathematicians in the fourteenth century? If so, what was their stance on using Mathematics in Theology?
There weren't many non-religious scientists at the time – higher knowledge was more or less a monopoly of the church. But there was practical mathematics (accounting, engineering, etc.) at the time, and some of these practitioners may have been interested in theory as well. At any rate, the theoretical work of such people, if it was ever written, did not survive. The situation was very different in Muslim societies, where science and theory were not the monopoly of religious elites.

What's the point in using mathematical arguments in order to contradict something which is clearly beyond the reach of logic, such as faith? Or were these speculations just a subtle way to mock the religious institutions?
These arguments do not contradict faith! These are philosophical discussions by religious people about religious and mathematical issues.

What does the author mean by angelology and atomism?
Angelology is the theory of angels. Atomism is the theory according to which matter is composed of indivisible units (atoms).

Thakkar p.625: Why was it a problem for infinitely many things to exist at once? Wouldn't it be considered a manifestation of the omnipotence of God?

Most philosophers at the time (following Aristotle) acknowledged the infinity of God, but rejected the possibility of any actual infinity in the physical world.

Bardwardine argumes that there is a bijection between an infinite set and an infinite subset of another set (souls -> bodies). He considered it to be a "broken" idea. But trigonometric functions were known since antiquity and tan function does exactly that: it takes a finite length interval and maps it onto entire R. Why didn't he saw a contradiction?
Trigonometry was known, but not in terms of "functions", and the question is not about lengths, but about collections of discrete elements. A length was usually not considered as a collection of indivisible points, precisely because of problems involving infinities (you can place points *on* a continuous line, but a line is not a collection of point according to the prevalent classical understanding).

Why are angles of different kinds analogous to a man and an ass? This sounds just ridiculous.
The claim is that the difference between a man and an ass (meaning "donkey") is analogous to the difference between different kinds of angels, as both difference are infinite (note that the difference between 8 and 6 is analogous to the difference between 4 and 2 without 8 or 6 being analogous to 4 or 2).

Thakkar in his text cites Holcots idea of supertask: being sinful for a given amount of time, meritorious next half of predeceasing time, and so on and dying after completion. First question: how could he know, that such geometric series converge? Proof dates to Euler. Secondly a sum of such series, sinful marked as 1 and meritorious marked as -1, intuitivly converges to 1/2. Why no such interpretation (purgatory beeing 1/2) was considered?
The whole question of convergence is not relevant here. The question is what is the *last* state of the person, and this cannot be determined in this scenario. Catholic dogma stated that the final state of the soul determines its after-life destiny, not some average – this is due to the catholic understanding of repentance and forgiveness.

Thakkar p 622 - I don't understand their problem in the third paragraph. Indeed, we would have an infinite amount of sets starting from 2 pair, but in the end we only have 4 real world objects in total. Isn't that a problem with the notion of a set rather than infinity?
The question is whether the infinite collection of sets that can be constructed from the four elements is in the world or in the mind. If it is in the world, then we have an infinite number of real objects, which contradicts the accepted philosophical principles of the time. It may be acceptable to say that these objects are in the mind, because the mind could be consided infinite (like God), or because the mind could be understood as a potential infinity (we can potentially construct an infinite number of sets, but we have only actually constructed a finite number at any given time).

In thakkar, when they talk about an "indivisible", do they mean a theoretical indivisible (like a point) or a "real" one (like quarks or something like that)?
Usually they mean both (an indivisible part of a line is a point, of matter is an "atom").

Thakkar: p. 623, second half. The Author mentions, that Gregory erroneously argues that infinitely many indivisibles would yield an infinite magnitude. But why erroneously?

This is a matter of definition. If by infinite you mean “an infinite set”, then the whole is indeed infinite. But if you mean “an infinite length”, then an infinite number of points can easily be accommodated into a finite length.

Thakkar On p 623 what is meant by a line of n (6 and 15) points? Are n points drawn on a line segment? How can we deduce that the lines shorten "infinitely" so as to become "smaller than a point"?

This refers to the hypothesis that a line is equal to a string of 6 indivisible points. The argument draws a contradiction (generating something smaller than an indivisible point), thus refuting the hypothesis.

When distributing the infinite number of souls to the infinite number of bodies, where every n-th soul gets the k*n-th soul for any k, a contradiction arises when all souls have been distributed or some souls are left. But how can any soul be left? For every soul I can find a body and vice-versa. (The whole contradiction doesn't seem to be thought through.)

It is shown that both options lead to a contradiction: if all souls are assigned to bodies, then a collection is equivalent to its part, which contradicts Euclidean principles; if some souls are left, you get a contradiction because for every soul you can find a body. Since both alternatives lead to a contradiction, we simply can’t have infinitely many souls or bodies.

On p.629, after attempting to refute those who believe the whole need not be greater than its part, I don't understand the claim Bradwardine makes about comparing infinitudes - "which, consequently, they must also have to say about any two infinite amounts compared to one another". What do "they" have to say about any two infinite amounts?

The claim is that those who accept actual infinities must accept that any two infinities are the same (like the collection of all souls and the collection of all souls except one). Obviously, no one at the time had the idea of different powers of infinity. Note, however, that they allowed for different finite things to have an infinite relation to each other, as in a comparison between a linear angle and a horn angle (the angle between a circle and its tangent).