Notes for Differential Galois Theory by P. Jossen

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Contents

1 The Ring of Partial Differential Operators

Definition 1.1. *A Δ-ring $\mathcal{R}$ is a commutative ring with unit equipped with a set of commutative derivations $\Delta = \{\partial_1, ..., \partial_r\}$. A Δ-ideal $I \subset \mathcal{R}$ is an ideal of $\mathcal{R}$ s.t. $\partial_i I \subset I$ for all $i = 1, ..., r$. If $\mathcal{R}$ is a Δ-ring, the set of constants is $\{c \in \mathcal{R} | \partial_i(c) = 0, \forall i\}$. Through out the expository article, we will assume that for any Δ-ring, $\mathbb{Q} \subset \mathcal{R}$ and $\mathcal{C}$ is algebraically closed field.*

Example 1.2. *$\mathcal{C}$ be an algebraically closed with $t_1, ..., t_r$ indeterminates. And the derivations are define to by $\partial_i t_j = \delta_{ij}$*

Definition 1.3. *Let $\mathcal{K}$ be a Δ-field. The ring of partial differential operators $\mathcal{K}[\partial_1, ..., \partial_r]$ with coefficients in $\mathcal{K}$ is the noncommutative polynomial ring in the variables ∂_i, where $[\partial_i, \partial_j] = 0$ but $[\partial_i, a] = \partial_i(a)$*

Definition 1.4. *A $\mathcal{K}[\partial_1, ..., \partial_r]$-module $\mathcal{M}$ is a finite dimensional $\mathcal{K}$-vector space that is a left module for the ring $\mathcal{K}[\partial_1, ..., \partial_r]$.*

But in this case a quotient $\mathcal{R}/I$ of Δ