Questions for Thurston/Blor

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At page 2 of Thurston's expository article on the essence of mathematical proofs. He gave a recursive definition of mathematics.

- Mathematics includes the natural numbers and plane and solid geometry.

- Mathematics that which mathematicians study.

- Mathematicians are those humans who advance human understanding of mathematics.

How to understand this? Obviously, this definition renders mathematics a time-dependent concept. If none of the mathematicians is studying some undiscovered subfield of mathematics today, this subfield is not part of math, right?

On the other hand, we can make similar definition which is less dependent on the details of **human understandings**:

- Mathematics includes the natural numbers and plane and solid geometry.

- Mathematics that which mathematicians study.

- Mathematicians are those humans who extend to realm of mathematics.

Then, the question is **would mathematics be an infinite set under certain isomorphism?**, Is this definition independent of the universe we are in? (For example, we can consider a universe of only one integer 0, the 0 him/herself is a mathematician).

For the article, I would say his argument involving $\mathbb{Z}/3\mathbb{Z}$ does not make much sense way we can't define $2 + 2 = 4$ in it. On the other hand in the category of rings, we can say that $2 + 2 = 4$ is a **universal identity**, which means we can always find a unique ring morphism $\varphi : \mathbb{Z} \to R$ which maps $2 +_\mathbb{Z} 2 =_\mathbb{Z} 4$ to $\varphi(2) +_R \varphi(2) =_R \varphi(4)$, where $+_R$ and $=_R$ is the addition and equality in the ring R, this is true because $\mathbb{Z}$ is the initial object in the category of rings.