$\pi :$
$X \longrightarrow$
Y
π
$X \longrightarrow$
$\pi_* X$
$p \in$
X
$(U, \varphi) \ni$
p
$(V, \phi) \ni$
$\pi(p)$
$\varphi :$
U^n
$\phi :$
V^m
π
$\phi \circ$
$\pi \circ$
$\varphi^{-1} :^n$
m
$\forall g$
$g \circ$
π
p
$g \circ$
$\phi^{-1} \circ$
$(\phi \circ$
$\pi \circ$
$\varphi^{-1})$
g
$g =$
ϕ_i
$\phi_i \circ$
$\phi^{-1} =$
$pr_i :^m$
i
$\phi \circ$
$\pi \varphi^{-1} =$
$(pr_1 \circ$
$\phi \circ$
$\pi \varphi^{-1}, ..., pr_m \circ$
$\phi \circ$
$\pi \varphi^{-1})$
$\pi :$
XY
$\pi(p) =$
q
$\pi^\# :_{Y,q}$
$_{X,p}$
$\pi^\#(_{Y,q}) \subset_{X,}$
q
p
$_{X,p}$
p
$\pi^\# :_{Y,q}$
$_{X,p}; [(f, U)] \longmapsto$
$[(f \circ$
$\pi, \pi^{-1} U)]$
$[(f, U)] \in_{Y,q}$
$(f, U) \sim$
$(0, V)$
$W \subseteq$
$U \cap$
V
$f|_W =$
0
$f \circ$
$\pi|_{\pi^{-1} W} =$
$0 [f \circ$
$\pi, \pi^{-1} W] \in_{X,p}$
$(k[\epsilon]/(\epsilon^2))$
$k[\epsilon]/(\epsilon^2)$
dual numbers
0
0
$k[x]_{(x)}$
$\{\in$
$(A), \supseteq\}$
$\{\in$
$(A/)\}$
$k[\epsilon]$
(ϵ^2)
(ϵ)
$(k[\epsilon]/(\epsilon^2))$
$=$
$\{[(\epsilon)]\}$

$$\{\in (A) : \cap S = \emptyset\} \{\in Spec(S^{-1} A)\}.$$

φ
$-a\varphi(x)+b = \varphi(x^2) = \varphi(x)\varphi(x).$

$\frac{1}{=}([X])$
$p(X) \in$
$[X]$
$[X]/(p(X)) \cong$
$[\alpha]$
$\underline{\alpha} \in$
Q
$p(\alpha) =$
0
$([X])$
(0)
$(x-$
$q)$
$q \in$
$p(X)$
$(p(X))$
k
$k[X]$
$k[X]$
$p_1, ..., p_n \in$
$k[X]$
$p =$
$p_1 \cdot$
$\cdot\cdot$
p_n+
1
p
p_i
$p_i|1$
p
$[x, y]$
$[x, y]$
(0)
$(x-$
$a, y-$
$b)(f(x, y))$
$f(x, y)$
$[x, y]$
$\subset$
$[x, y]$
$f(x, y), g(x, y)$
$f(x, y)$
$g(x, y)$
y
$c(x)$

$$f(x, y) = f_n(x)y^n+f_{n-1}(x)y^{n-1}+...+f_0$$

$$g(x, y) = g_m(x)y^m+g_{m-1}(x)y^{m-1}+...+g_0$$

$(x)[y]$
f, g
$(x)[y]$
R
$f, g \in$
$[y]$
$gcd(f, g) =$
1
$R[y]$
$gcd(f, g) =$
1
$K[y]$
K
R
$h \in$
$K[y]$
$h|f$
$h|g$
$K[y]$
$deg(h) =$
0
d
h
$k =$
$dh \in$
$R[y]$
$k|df$
$k|dg$
$K[y]$
$a, b \in$
$R\backslash\{0\}$
$k|(ad)f$
$k|bdg$

$\sqrt{I}$

$$\sqrt{\sqrt{I}} = \sqrt{I}.$$

$\sqrt{} =$

$x, y \in$
$\sqrt{I}$
$x^n \in$
I
$y^m \in$
I
$\forall a \in$
$A, (ax)^n \in$
I
$(ax+$
$by)^{n+m} \in$
I
$\sqrt{} \supseteq$
$x \in$
$\sqrt{\sqrt{I}}$
$x^n \in$
$\sqrt{I}$
$(x^n)^m \in$
$I x \in$
$\sqrt{I}$
$x^n \in$
$x \in$
$\sqrt{}^n =$

$$\subseteq \sqrt{}^n \subseteq \sqrt{} = .$$

$$\sqrt{} = \bigcap_{primes \supseteq}$$

$\frac{1}{k}$
$[(0)]$
$[(x-$
$a)], \forall a \in$
k
k^1
$[(0)]$
$[] \in^1_n$
$[]$
$[(0)]$
$\overline{}$
(0)
$\overline{}$
$(x-$
$a)$
$[] =$
$[(x-$
$a)]$
$\frac{1}{k}-$
$V(S)$
$[] \notin$
$V(S) S$
$\frac{1}{k}-$
$V(S) \ni$
$[(0)] S(0)$
S
$(0) \subset$
$k[x]$
(0)
(0)
$\phi :$
$B A$
$\pi =$
$\phi^* :$
$A B$
$Rings\, Top$
S
B
$V(S)$
B
$\pi^{-1} V(S)$

$$\pi^{-1} V(S) = \{[] \in A : \pi() \supset S\} = \{[] \in A : \phi^*() \supset S\}$$

$\pi^{-1} V(S) =$
$V(\phi(S))$
$\supset$
$\phi(S) \phi^{-1}() \supset$
S

$$\pi^{-1} V(S) = \{[] \in A : \phi^*() \supset S\} = \{[] \in A : \supset \phi(S)\} = V(\phi(S)).$$

π
$\mathfrak{p}.$