Notes for HoTT

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Orginary set theory has two layers:

1. deductive system of first order logic
2. axioms of a particular theory?

By contrast, type theory is its own deductive system. Instead of set and proposition, there is only one basic notion :**type** in type theory. Propositions are also types. Thus, proving a theorem is equivalent to constructing a type that represents the proposition.

We use deductive system to refer to a collection of rules for deriving thing called **judgment.** Inside the deductive system of first order logic, there is only one kind of judgment:"A proposition A has a proof". Notice that "A has a proof" exists at a different level from the proposition itself.

Analogous to "A has proof", in type theory, a basic judgment is written $a : A$ and read as "the term a has type A". When A is a type that represents a proposition, a may be called a witness or **evidence** of A. In this case, $a : A$ is derivable iff "A has proof" is derivable in first logic.

One important feature of type theory is that $x : A$ is the atomic statement. we cannot introduce a variable without specifying its type.

Treatment of equality. Equality in math are propositions, then we have to treat equality as types as well. $a =_A b$ is a **propositional equality**

We also need a equality judgment at the same level as $x : A$. It is called **judgmental equality** (Tautology?)