Solution Manual to Ravi Vakil's FOAG

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Definition 3.1. *The set Spec$\mathcal{A}$ is the set of prime ideals of $\mathcal{A}$. The prime ideals $\mathfrak{p}$ of $\mathcal{A}$ when considered as an element of Spec$\mathcal{A}$ will be denoted $[\mathfrak{p}]$, to avoid confusion. Element $a \in \mathcal{A}$ will be called **functions on** Spec$\mathcal{A}$, and their **value** at the point $[\mathfrak{p}]$ will be $a \mod \mathfrak{p}$.*

*If $\mathcal{A}$ is generated over a base field or base ring by element $x_1, ..., x_r$, the elements are often called **coordinates***

Afterwards, we will interpret functions on Spec$\mathcal{A}$ as global sections of the structure sheaf, i.e., as a function on a ringed space.

Example 3.2.

- *Spec $\mathbb{C}[x] = \mathbb{A}^1_{\mathbb{C}}$*
- *Spec $k[x] = \mathbb{A}^1_k$, where k is an algebraically closed field. This is called the affine line over k.*
- *Spec $\mathbb{Z}$. Isomorphic to the set of prime numbers union witn $\{(0)\}$*

Exercise 3.3. *aaa*

Exercise 3.4. *IMPORTANT EXERCISE FOR THOSE WITH A LITTLE EXPERIENCE WITH MANIFOLDS. Suppose that $\pi : X \longrightarrow Y$ is a continuous map of differentiable manifolds (as topological spaces not a priori* *differentiable). Show that is differentiable if differentiable functions pull back to differentiable functions, i.e., if pullback by gives a map $\mathcal{O}_Y \to \pi_*\mathcal{O}_X$. (Hint: check this on small patches. Once you figure out waht your trying to show you will find realize that the resuilt is immediate.)*

Proof. aaa $\square$