# Bernoulli Trials Problems, 2009

For each of the following statements, the contestants must decide (or guess) whether the statement is true or false. They are eliminated after their second incorrect answer.

**1:** Some positive integral power of 3 ends with the digits 0001.

**2:** The numbers $1, 2, 3, \cdots, 2009$ can be rearranged and then written one after the other in this new order to produce a single number which is a perfect cube.

**3:** The sum $\sum_{n=4}^{\infty} \binom{n}{4}^{-1}$ is rational.

**4:** Let $f(x)$ be increasing, differentiable and bounded for $x \in [0, \infty)$. Then $\lim_{x \to \infty} f'(x) = 0$.

**5:** There exists an integer $p > 3$ such that $p$, $2p + 1$ and $4p + 1$ are all prime.

**6:** There exists a positive integer $n$ such that $P_n + 1$ is a perfect square, where $P_n$ is the product of the first $n$ primes.

**7:** If the positive integers are written out in order, then the $10^{10}$th digit in the resulting infinite string is equal to 1.

**8:** A slab of stone of length 3 is rolled along the positive $x$-axis on 4 cylindrical logs of radius $\frac{1}{4}$. As the stone moves forwards, the trailing log is left behind. When the front of the stone overhangs the leading log by 1 unit, the trailing log is placed under the front of the stone. Initially, the stone is between $x = 0$ and $x = 3$ and the centers of the 4 cylinders are at $x = 0, 1, 2$ and $3$. A curious, but somewhat ill-fated worm watches the proceedings from $x = \frac{9}{2}$. The unfortunate worm will be squashed twice.

**9:** The sum $\sum_{n=2}^{\infty} \binom{n}{2}^{-2}$ is rational.

**10:** A regular octadecagon (18-gon) with sides of length 1 fits inside a circle of radius 3.

**11:** A regular icosahedron (20 triangular faces) with edges of length 1 fits inside the unit sphere.

**12:** In any 11 month period, the Moon moves around the Sun in a simple convex path.