HW 14

1

Let R be a ring with abelian group A. Define a map $\varphi : R \to End(A)$ which maps r to multiplication by r, i.e. let $\varphi(r) = \varphi_r$ where $\varphi_r(a) = ra$.

φ is a ring homomorphism. It preserves addition: this is equivalent to $\varphi_{r+s} = \varphi_r + \varphi_s$, or $\forall a \in A, (r+s)a = ra + sa$, which follows from the distributive law in R.

Similarly, φ preserves multiplication means $\varphi_r \varphi_s = \varphi_{rs}$ where the product on the left denotes function composition in $End(A)$. This means $\forall a \in A, r(sa) = (rs)a$ which follows from associativity of multiplication in R.

Now R acts on $A + \mathbb{Z}$ by multiplication. We can repeat the above proofs to get an injective homomorphism?