HW 2

1.2

5.

Let $x = s^k r^i$ be an element which commutes with every element of D_{2n} with $k \leq 1, k < n$. We have $s^k s = ss^k = s^{1-k}$ since s has order 2. Since x commutes with s,

$$\begin{aligned} ss^k r^i &= s^k r^i s \\ s^{1-k} r^i &= s^k s r^{-i} \\ &= s^{k+1} r^{-i} \end{aligned}$$

By equating exponents of s (which we can do because the representation is unique), $1 - k = k + 1$ hence $k = 1$. By equating exponents of r, we have $i = -i \pmod{n}$ hence $2i = 0 \pmod{n}$ hence $i = 0 \pmod{n}$ because n is odd. Hence x is the identity.

7.

$s^2 = a^2 = 1$

$r^n = (s^2 r)^n = (ab)^n = 1$

$rsr = s(sr)(sr) = ab^2 = a = s$. Hence $rs = sr^{-1}$

Conversely,

$a^2 = s^2 = 1$

$b^2 = (sr)^2 = srsr = ss = 1$

$(ab)^n = (ssr)^n = r^n = 1$

1.3

11.

15.

17.