Economics 202A
Macroeconomics
Fall 2020

Problem Set 2
Due on November 10, 2020
Please upload solutions to Gradescope

Please write up your solutions as clearly as possible. Your grade will be reduced if your solution is unreasonably difficult to follow.

Consider the following household consumption-savings problem with uninsurable idiosyncratic income risk. The household's utility function is

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\gamma}}{1-\gamma},$$

where C_t denotes consumption, β is the household's subjective discount factor, and γ is the coefficient of relative risk aversion and also the reciprocal of the elasticity of intertemporal substitution. The household starts off with assets A_0 and each period receives labor income Y_t. Labor income is affected by two types of risks. First, the household may become unemployed and receive only unemployment benefits b. When the household is employed, its labor income follows an AR(1) process. These assumptions can be represented mathematically as

$$Y_t = \begin{cases} \tilde{Y}_t, \text{if employed} \\ b, \text{if unemployed} \end{cases}$$

where

$$\tilde{Y}_t = (1-\rho)\mu + \rho\tilde{Y}_{t-1} + \epsilon_t$$

and $\epsilon_t \sim \mathrm{N}(0, \sigma^2)$. The parameter μ denotes the unconditional mean of labor income when employed, ρ is the persistence of fluctuations in labor income when employed, and σ^2 is the variance of fluctuation in labor income when employed. When a household is employed, its probability of becoming unemployed next period is p. When a household is unemployed, its probability of becoming employed again next period is q.

The household has access to a risk-free savings technology that yields a net return on savings of r. However, the household cannot borrow. Each period, the household faces a choice of how much to consume and how much to save. Its budget constraint is therefore

$$C_t + \frac{A_{t+1}}{1+r} = Y_t + A_t.$$

and the borrowing contraint is

$$A_t \geq 0.$$

The rest of the problem set asks you to solve this problem using value function iteration. For those new to value function iteration, I recommend that you follow closely the steps laid out in Alisdair McKay's numerical analysis notes, which are available here:

https://alisdairmckay.com/Notes/NumericalCrashCourse/index.html#.

I have written the rest of the problem as a set of step-by-step instructions assuming that you are basing your work on McKay's notes and code. However, you are welcome to solve the problem without reference to these steps and this code. If you do, please describe the steps you take and report the relevant code.

You are welcome to interact with other students in the class as you solve the problem. But each student should write their own code and hand in results based on this code.

When solving the problem, please consider the following set of parameter values to be the baseline parameter values and use these unless otherwise instructed: $\gamma = 2$, $\beta = 0.98$, $\mu = 1$, $b = 0.4$, $\rho = 0.95$, $\sigma^2 = 0.01$, $p = 0.05$, $q = 0.25$, and $r = 0.01$.

A. Write the household's problem recursively. Be sure to state what variables are chosen and all the constraints. Hints: This problem has three state variables: assets brought into the period A, income if employed $\tilde{Y}$, and whether the household is employed. But one of these state variables only takes two values (whether the household is employed). The easiest way to solve the problem is therefore to think of the household has having two value functions V_e and V_u representing the value if employed and the value if unemployed. Each of these functions is then a function of two variables (A and $\tilde{Y}$).

B. Write a version of McKay's PolyBasis function for this problem. Use a second order polynomial basis.

C. Write a version of McKay's PolyGetCoeff function for this problem.

D. Start your main Matlab program by reading in the parameter values into a structure.

E. Create a grid on A and $\tilde{Y}$. Feel free to use McKay's tauchen function as needed. Use 7 grid points for $\tilde{Y}$ and 100 grid points for A. Choose reasonable values for the size of the grid (i.e., min and max points for each dimension).

F. Write a version of McKay's Bellman matlab function for this problem. Since we have two value functions (V_e and V_u), the best way to go here is to write two functions BellmanE and BellmanU, one for each value function.

G. Write a version of McKay's MaxBellman function for this problem. Again, you will have two functions MaxBellmanE and MaxBellmanU.

H. Write the value function iteration for-loop for this problem.

I. (Optional) Implement the Howard acceleration for this problem. Report the speed improvement that you are able to achieve.

J. Adapt McKay's Simulate function for this problem. Note that you need to simulate the evolution employment/unemployment in addition to A, $\tilde{Y}$, and C.

Now you can view some results:

K. Using your new Simulate function, produce a 10,000 period simulation of the evolution of A, $\tilde{Y}$, C, and employment status for a single household. Report a histogram of A, Y, and C. Report the mean and standard deviation of each variable. Plot the evolution of A, Y, and C over a 100 period stretch starting from period 1000. (Note that I am asking you to report Y, not $\tilde{Y}$.)

L. Plot savings (i.e., percentage change in A) when employed as a function A for several values of $\tilde{Y}$. Have the x-axis go from half a standard deviation below the mean value of A reported above to half a standard deviation above the mean value of A. On the same figure, also plot savings when unemployed as a function of A. Do this for the average value of $\tilde{Y}$. Plot the same thing again but with the x-axis going all the way down to zero.

M. Plot the marginal propensity to consume when employed as a function of A for several values of $\tilde{Y}$. On the same figure, also plot the marginal propensity to consume when unemployed as a function of A (again for the average value of $\tilde{Y}$). Plot this for the same two ranges of values for A as above. You can approximate the marginal propensity to consume as the extra consumption in the period that results from a windfall gain of 1% of average A.

N. (Optional) Explore how the accuracy of the solution depends on the number of grid points used. In particular, reproduce the figures above with only 20 grid points for A. Compare the results with those produced before and comment on any results that look "puzzling" in the less accurate case.

O. (Optional) Explore how changing the value of ρ changes the answer to parts K-M above (in particular part K). Deaton (1991) discusses how self-insurance is easier and more self-insurance is therefore optimal when income shocks are more transitory. Can you reproduce this result? In other words, how does the relative standard deviation of C_t and Y_t vary with ρ?

References

DEATON, A. (1991): "Saving and Liquidity Constraints," *Econometrica*, 59(5), 1221–1248.