I worked with Jun Peng and Andrew Perry.

1. *You take out a 30-year, fixed-rate mortgage for $1,000,000. The interest rate on the mortgage loan is 5.25% (APR, compounded monthly), and it requires you to make equal payments at the end of each of the next 360 months (i.e., the first payment is 1 month from today and the last payment is 360 months from today).*

a. *What is the amount of each monthly payment?*

$$\$1,000,000 = \frac{c}{1+r} + \frac{c}{(1+r)^2} + \ldots + \frac{c}{(1+r)^{360}}$$
$$= \frac{c}{r}\left(1 - \frac{1}{(1+r)^{360}}\right)$$
$$\Rightarrow c = \$5522.04$$

b. *What is the remaining balance on the loan immediately after the 100th loan payment?*

$$c\sum_{i=1}^{260}\frac{1}{(1+r)^i} = \frac{c}{r}\left(1 - \frac{1}{(1+r)^{260}}\right)$$
$$= \$856495.04$$

c. *How much interest in total will you pay during year 10 of the mortgage (i.e., what is the total amount of interest included in the 12 payments ending with the payment 120 months from today)?*

The present value (in month 120 terms) of the remaining balances at month 108 (beginning of year 10) and month 120 (end of year 10) are

$$balance_{108} = (1+r)^{12} \times c\sum_{i=1}^{252}\frac{1}{(1+r)^i}$$
$$= (1+r)^{12} \times \frac{c}{r}\left(1 - \frac{1}{(1+r)^{252}}\right)$$
$$= \$842076.74$$
$$balance_{120} = c\sum_{i=1}^{240}\frac{1}{(1+r)^i}$$
$$= \frac{c}{r}\left(1 - \frac{1}{(1+r)^{240}}\right)$$
$$= \$819482.8$$

The difference in balances is $22593.94 and the total payments for the year are $66264.44 . Thus, total interest paid is $43670.5 .

2. *The annually compounded one-, two- and three-year spot rates, $r_1(0,1), r_1(0,2),$ and $r_1(0,3)$, are 5%, 6% and 7% respectively.*

a. *What are the first three annually compounded (one-year) forward rates for years 1,2 and 3, $f_1(0,0,1), f_1(0,1,2)$ and $f_1(0,2,3)$?*

$$f_1(0,0,1) = r_1(0,1) = 0.05$$
$$1 + f_1(0,1,2) = \frac{(1 + r_1(0,2))^2}{1 + r_1(0,1)}$$
$$\Rightarrow f_1(0,1,2) \approx 0.0701$$
$$1 + f_1(0,2,3) = \frac{(1 + r_1(0,3))^3}{(1 + r_1(0,2))^2}$$
$$\Rightarrow f_1(0,2,3) \approx 0.0903$$

b. *If no explicit forward-rate agreements existed, how would you use a combination of spot lending and borrowing to effectively borrow $1,000 a year from today, and pay it back 3 years from today (plus interest)?*

Notice that

$$1 + f_1(0,1,3) = (1 + f_1(0,1,2)) \cdot (1 + f_1(0,2,3))$$
$$= \left(\frac{(1 + r_1(0,2))^2}{1 + r_1(0,1)} \right) \cdot \left(\frac{(1 + r_1(0,3))^3}{(1 + r_1(0,2))^2} \right)$$
$$\Rightarrow f_1(0,1,3) = 0.1667$$

We borrow in one year

$$\frac{\$1,000}{1 + f_1(0,0,1)} = \frac{\$1,000}{1 + r_1(0,1)}$$
$$= \frac{\$1,000}{1.05}$$
$$= \$952.38$$

and we pay back two years after (three years from today)

$$\$952.38 \cdot (1 + f_1(0,1,3)) = \$952.38 \cdot (1 + 0.1667)$$
$$= \$1111.15$$

3. *The annually compounded one-, two- and three-year forward rates, $f_1(0,0,1)$, $f_1(0,1,2)$ and $f_1(0,2,3)$, are 11%, 13% and 17% respectively. What are the annually compounded one, two and three-year spot rates, $r_1(0,1)$, $r_1(0,2)$ and $r_1(0,3)$?*

$$r_1(0,1) = f_1(0,0,1) = 0.11$$

$$1 + f_1(0,1,2) = \frac{(1 + r_1(0,2))^2}{1 + r_1(0,1)}$$

$$\Rightarrow r_1(0,2) = 0.12$$

$$1 + f_1(0,2,3) = \frac{(1 + r_1(0,3))^3}{(1 + r_1(0,2))^2}$$

$$\Rightarrow r_1(0,3) = 0.1364$$

4. *We defined a bond's yield to maturity in class. An alternative measure that is sometimes quoted is a bonds current yield, defined as its coupon rate (as a percentage) divided by its current price (quoted per $100 face value). So a bond with a 5% annual coupon rate and a current price of $102 would have a current yield of*

$$\frac{5}{102} = 4.0902\%.$$

Prove that for a discount bond (i.e., price < $100), YTM > current yield > coupon rate, and for a premium bond (i.e., price > $100), YTM < current yield < coupon rate.

A bond's YTM is y which solves

$$P = \frac{C}{(1+y)} + \frac{C}{(1+y)^2} + \dots \frac{C}{(1+y)^T} + \frac{f}{(1+y)^T}.$$

If the bond is discounted at $P^* < \$100$, then it's current yield (y^*) is greater than the coupon rate since

$$y^* = \frac{c \cdot 100}{P^*} > c.$$

If the bond is at a premium at $P^* > \$100$, then it's current yield (y^*) is less than the coupon rate since

$$y^* = \frac{c \cdot 100}{P^*} < c.$$

$$P = \frac{C}{(1+y)} + \frac{C}{(1+y)^2} + \dots \frac{C}{(1+y)^T} + \frac{f}{(1+y)^T}$$
$$= \frac{C}{y}\left(1 - \frac{1}{(1+y)^T}\right) + \frac{f}{(1+y)^T}$$

This implies that

$$y = \frac{\overbrace{c \cdot 100/P}^{y^*}}{1 - \frac{100-P}{P} \cdot \underbrace{\frac{1}{(1+y)^T - 1}}_{>0}}$$

We can see that if the bond is discounted, then the denominator is less than one implying that YTM¿current yield. If the bond is at a premium, then the denominator is greater than one implying that YTM¡current yield.

5. *Finish the fitting example that we went over in class (slides 34–38) by calculating the third forward rate, f_3*

We first calculate the bond price

$$\begin{aligned}
P &= \frac{c/2}{\exp\{y \cdot (10-6)/12\}} + \frac{100 + c/2}{\exp\{y \cdot (10/12)\}} \\
&= \frac{3}{\exp\{0.053 \cdot 4/12\}} + \frac{103}{\exp\{0.053 \cdot (10/12)\}} \\
&= 101.4973
\end{aligned}$$

and solve the equation for f_3

$$\begin{aligned}
P &= \frac{c/2}{\exp\{f_1 \cdot (10-6)/12\}} \\
&+ \frac{100 + c/2}{\exp\{f_1 \cdot 5/12\} + \exp\{f_2 \cdot (8-5)/12\} + \exp\{f_3 \cdot (10-8)/12\}} \\
101.4973 &= \frac{3}{\exp\{0.05 \cdot (4)/12\}} \\
&+ \frac{103}{\exp\{0.05 \cdot 5/12\} + \exp\{0.053 \cdot 3/12\} + \exp\{f_3 \cdot 2/12\}}
\end{aligned}$$

to obtain $f_3 = 0.0451$.

6. *Fit a yield curve to the input data you just downloaded, assuming that*

   - *The simple interest rate r_s is a continuous function of maturity, $r_s(t)$.*

   - *$r_s(t)$ is piecewise linear, with kinks at the maturity dates of the instruments in the input data set.*

   - *$r_s(t)$ is constant outside the range of maturities in the input data. I.e., if t_{min} and t_{max} are, respectively, the shortest and longest maturities in the input sample,*

$$r_s(t) = \begin{cases} r_s(t_{min}) & \text{if } t \leq t_{min} \\ r_s(t_{max}) & \text{if } t \geq t_{max}. \end{cases}$$

*[**This is Bloombergs default method 1 (see Bloomberg, 2021, pp. 78). When using this method, assume simple rates are quoted using the Actual/360 day-count.**]*

Your goal is to match the zero rates and forward rates calculated by Bloomberg, which I've provided for you.

Using your fitted yield curve, generate a table showing, at 3-month intervals from 3/4/2019 to 12/4/2068 (use the dates in the output spreadsheet you just downloaded from Bloomberg): [1]

a. *Your fitted discount factor.*

b. *Your fitted zero rate.*

c. *Bloomberg's fitted zero rate*

d. *Your fitted 3-month forward rate*

e. *Bloomberg's fitted 3-month forward rate*

f. *The difference between your fitted zero rate and Bloomberg's*

g. *The difference between your fitted forward rate and Bloomberg's*

---

[1] Report all numbers to 9 decimal places, and report the numbers in columns (b)–(g) as percentages (e.g., 2.615130000%). Your numbers should be identical to Bloombergs to at least this degree of precision. In other words, the numbers in columns (f) and (g) should all be ±0.000000000%. To achieve this, you should be aware of the Bloomberg quoting conventions: 1) Zero rates: semi-annually compounded and use 30I/360 day-count 2) Forward rates: simple compounding and use Actual/360 day-count. Note that this is not easy! There are a lot of details to get right to get the numbers to match exactly. In particular, if you manage to match only to, say, 7 decimal places, e.g., a difference of 0.000000026%, your calculations are not 100% right... [**To make this a lot easier, read Stanton (2022b) (provided as a handout for this problem set) in great detail. This is a pretty detailed discussion of the data and the compounding and timing assumptions you will need to make in order to get this to work.**]

Date	Zero Rate	Forward Rate	rzero	DCF	forward
20190304	0.000000	2.615130	0.000000	1.000000	2.741781
20190604	2.682177	2.592484	0.026822	0.993192	2.595283
20190904	2.670525	2.607773	0.026188	0.986790	2.636333
20191204	2.662162	2.632146	0.026178	0.980329	2.661276
20200304	2.664183	2.578889	0.026235	0.973849	2.636145
20200604	2.660323	2.522048	0.026162	0.967473	2.578067
20200904	2.648001	2.487822	0.026017	0.961278	2.291481
20201204	2.630193	2.459395	0.026424	0.955802	2.525587
20210304	2.609781	2.467129	0.026424	0.949805	2.486770
20210604	2.600904	2.446780	0.026397	0.943937	2.466248
20210907	2.591225	2.426249	0.026369	0.937961	2.445538
20211206	2.579552	2.409906	0.026343	0.932324	2.425914
20220304	2.566094	2.484115	0.026316	0.926828	2.503034
20220606	2.564516	2.478241	0.026370	0.920937	2.496995
20220906	2.562863	2.472552	0.026423	0.915224	2.491152
20221205	2.559260	2.467048	0.026476	0.909622	2.487859
20230306	2.553914	2.565859	0.026531	0.903937	2.590765
20230605	2.558458	2.571125	0.026655	0.898184	2.596643
20230905	2.562812	2.576118	0.026780	0.892391	2.602118
20231204	2.565451	2.580918	0.026904	0.886687	2.607565
20240304	2.568084	2.673980	0.027029	0.880944	2.693095
20240604	2.576397	2.685708	0.027198	0.875053	2.704599
20240904	2.584504	2.696912	0.027366	0.869176	2.715750
20241204	2.591096	2.707643	0.027535	0.863315	2.726555
20250304	2.596332	2.751446	0.027704	0.857470	2.770483
20250604	2.605373	2.763948	0.027888	0.851572	2.782807
20250904	2.614213	2.775882	0.028072	0.845688	2.794745
20251204	2.621704	2.787302	0.028256	0.839820	2.806302
20260304	2.627963	2.854133	0.028441	0.833970	2.873076
20260604	2.638308	2.868117	0.028648	0.828022	2.886906
20260904	2.648444	2.881459	0.028855	0.822089	2.900294
20261204	2.657330	2.894216	0.029062	0.816171	2.913245
20270304	2.665055	2.928247	0.029270	0.810270	2.956267
20270604	2.675334	2.942009	0.029488	0.804325	2.970337
20270907	2.685674	2.954394	0.029714	0.798200	2.983011
20271206	2.694506	2.966405	0.029930	0.792357	2.996058
20280306	2.703274	3.009487	0.030149	0.786466	3.027199
20280605	2.713568	3.021969	0.030372	0.780624	3.039705
20280905	2.723752	3.033934	0.030597	0.774737	3.051509
20281204	2.732771	3.045385	0.030820	0.768936	3.062966
20290305	2.740954	3.023484	0.031048	0.763028	3.041171
20290604	2.749686	3.033094	0.031261	0.757334	3.050837
20290904	2.758318	3.042233	0.031475	0.751602	3.060039
20291204	2.765975	3.050885	0.031690	0.745895	3.068857
20300304	2.772710	3.070508	0.031905	0.740216	3.089110
20300604	2.781110	3.078987	0.032124	0.734544	3.097519
20300904	2.789335	3.086908	0.032342	0.728899	3.105553
20301204	2.796649	3.094372	0.032560	0.723284	3.113219
20310304	2.803096	3.018976	0.032779	0.717698	3.120926
20310604	2.809110	3.023875	0.032997	0.712141	3.127885

Date	Zero Rate	Forward Rate	rzero	DCF	forward
20310904	2.814984	3.028334	0.033216	0.706616	3.134495
20311204	2.820053	3.032616	0.033434	0.701122	3.140763
20320304	2.825012	3.036774	0.033653	0.695660	3.146694
20320604	2.830504	3.040998	0.033871	0.690230	3.152799
20320907	2.836000	3.044032	0.034097	0.684653	3.157580
20321206	2.840563	3.047208	0.034313	0.679350	3.162315
20330304	2.844368	3.050508	0.034527	0.674139	3.167518
20330606	2.849462	3.053028	0.034750	0.668726	3.171631
20330906	2.854346	3.055385	0.034969	0.663465	3.175514
20331205	2.858488	3.057598	0.035185	0.658297	3.179450
20340306	2.862035	2.988192	0.035406	0.653049	3.178329
20340605	2.865355	2.988739	0.035621	0.647957	3.181567
20340905	2.868603	2.988984	0.035838	0.642844	3.184255
20341204	2.871198	2.989172	0.036053	0.637823	3.187116
20350305	2.873235	2.989315	0.036273	0.632726	3.189155
20350604	2.876199	2.989165	0.036488	0.627776	3.191351
20350904	2.879093	2.988724	0.036705	0.622807	3.193170
20351204	2.881394	2.988247	0.036922	0.617875	3.194746
20360304	2.883621	2.987733	0.037140	0.612979	3.196084
20360604	2.886260	2.986935	0.037357	0.608120	3.197190
20360904	2.888812	2.985865	0.037575	0.603298	3.198068
20361204	2.890806	2.984661	0.037792	0.598513	3.198725
20370304	2.892263	2.983677	0.038009	0.593764	3.199165
20370604	2.894582	2.982291	0.038227	0.589053	3.199393
20370904	2.896819	2.980652	0.038444	0.584379	3.199414
20371204	2.898529	2.978897	0.038662	0.579742	3.199233
20380304	2.899733	2.977372	0.038879	0.575142	3.198855
20380604	2.901750	2.975788	0.039096	0.570579	3.198698
20380907	2.903725	2.973227	0.039321	0.565903	3.197361
20381206	2.905154	2.969101	0.039536	0.561464	3.196291
20390304	2.906093	2.908190	0.039749	0.557112	3.107661
20390606	2.907053	2.904922	0.039951	0.552722	3.105293
20390906	2.907949	2.901729	0.040149	0.548464	3.102958
20391205	2.908391	2.898617	0.040344	0.544289	3.100757
20400305	2.908786	2.895484	0.040542	0.540102	3.098162
20400604	2.909516	2.892169	0.040738	0.535997	3.095708
20400904	2.910189	2.888639	0.040936	0.531880	3.092983
20401204	2.910432	2.885069	0.041134	0.527799	3.090139
20410304	2.910262	2.881750	0.041332	0.523753	3.087178
20410604	2.910783	2.878085	0.041530	0.519742	3.084104
20410904	2.911251	2.874253	0.041727	0.515765	3.080919
20411204	2.911311	2.870392	0.041925	0.511823	3.077626
20420304	2.910978	2.866784	0.042123	0.507915	3.074229
20420604	2.911304	2.862839	0.042321	0.504041	3.070731
20420904	2.911579	2.858738	0.042519	0.500201	3.067133
20421204	2.911467	2.854618	0.042717	0.496395	3.063440
20430304	2.910981	2.850754	0.042915	0.492622	3.059653
20430604	2.911122	2.846560	0.043113	0.488882	3.055775
20430904	2.911217	2.842219	0.043311	0.485176	3.051810
20431204	2.910943	2.837958	0.043508	0.481502	3.047758

Date	Zero Rate	Forward Rate	rzero	DCF	forward
20440304	2.910632	2.870458	0.043706	0.477861	3.070377
20440606	2.910963	2.865963	0.043914	0.474141	3.065974
20440906	2.911251	2.861591	0.044118	0.470534	3.061685
20441205	2.911184	2.857347	0.044319	0.466999	3.057681
20450306	2.910759	2.853089	0.044524	0.463418	3.053079
20450605	2.910920	2.848689	0.044725	0.459946	3.048812
20450905	2.911030	2.844100	0.044929	0.456467	3.044214
20451204	2.910802	2.839648	0.045130	0.453057	3.039904
20460305	2.910230	2.835183	0.045336	0.449602	3.035011
20460604	2.910222	2.830585	0.045537	0.446254	3.030459
20460904	2.910165	2.825806	0.045740	0.442899	3.025688
20461204	2.909780	2.821034	0.045944	0.439573	3.020853
20470304	2.909076	2.816524	0.046147	0.436279	3.015957
20470604	2.908903	2.811700	0.046350	0.433014	3.011001
20470904	2.908689	2.806753	0.046554	0.429779	3.005987
20471204	2.908161	2.801904	0.046757	0.426573	3.000917
20480304	2.907600	2.797099	0.046960	0.423396	2.995793
20480604	2.907274	2.792122	0.047164	0.420249	2.990617
20480904	2.906910	2.787030	0.047367	0.417130	2.985391
20481204	2.906246	2.781960	0.047571	0.414040	2.980116
20490304	2.905291	2.775641	0.047774	0.410978	3.220938
20490604	2.904812	2.770764	0.048027	0.407695	3.216172
20490907	2.904264	2.765141	0.048288	0.404336	3.210348
20491206	2.903474	2.760082	0.048538	0.401152	3.205019
20500304	2.902418	2.755342	0.048785	0.398034	3.200236
20500606	2.901784	2.749851	0.049044	0.394805	3.194465
20500906	2.901141	2.744526	0.049297	0.391677	3.188855
20501205	2.900237	2.739368	0.049547	0.388613	3.183566
20510306	2.899039	2.734205	0.049803	0.385511	3.177649
20510605	2.898322	2.728945	0.050053	0.382506	3.172119
20510905	2.897560	2.723532	0.050306	0.379496	3.166248
20511204	2.896549	2.718291	0.050556	0.376549	3.160594
20520304	2.895498	2.713045	0.050809	0.373597	3.154712
20520604	2.894652	2.707649	0.051061	0.370674	3.148773
20520904	2.893777	2.702165	0.051314	0.367778	3.142781
20521204	2.892648	2.696723	0.051567	0.364911	3.136736
20530304	2.891274	2.691540	0.051820	0.362072	3.130642
20530604	2.890321	2.686081	0.052073	0.359260	3.124500
20530904	2.889342	2.680538	0.052326	0.356476	3.118312
20531204	2.888120	2.675042	0.052579	0.353718	3.112080
20540304	2.886661	2.669803	0.052832	0.350987	3.105806
20540604	2.885609	2.664293	0.053084	0.348283	3.099491
20540904	2.884532	2.658705	0.053337	0.345605	3.093138
20541204	2.883222	2.653168	0.053590	0.342953	3.086748
20550304	2.881685	2.647884	0.053843	0.340327	3.080323
20550604	2.880541	2.642536	0.054096	0.337726	3.074149
20550907	2.879321	2.636533	0.054357	0.335065	3.067062
20551206	2.877946	2.631091	0.054607	0.332544	3.060707
20560306	2.876535	2.625648	0.054860	0.330019	3.054063
20560605	2.875321	2.620133	0.055110	0.327545	3.047651

Date	Zero Rate	Forward Rate	rzero	DCF	forward
20560905	2.874068	2.614488	0.055363	0.325069	3.040955
20561204	2.872617	2.609024	0.055613	0.322643	3.034582
20570305	2.870918	2.603559	0.055869	0.320187	3.027676
20570604	2.869625	2.598027	0.056119	0.317808	3.021166
20570904	2.868293	2.592368	0.056372	0.315426	3.014471
20571204	2.866753	2.586769	0.056625	0.313066	3.007757
20580304	2.865011	2.581415	0.056878	0.310730	3.001025
20580604	2.863625	2.575813	0.057131	0.308416	2.994276
20580904	2.862219	2.570149	0.057383	0.306125	2.987511
20581204	2.860614	2.564547	0.057636	0.303855	2.980731
20590304	2.858814	2.572755	0.057889	0.301608	2.961503
20590604	2.857445	2.567145	0.058139	0.299391	2.954727
20590904	2.856058	2.561474	0.058390	0.297196	2.947939
20591204	2.854476	2.555926	0.058640	0.295021	2.941142
20600304	2.852878	2.550438	0.058890	0.292868	2.934337
20600604	2.851439	2.545008	0.059141	0.290735	2.927765
20600907	2.849923	2.538983	0.059399	0.288553	2.920394
20601206	2.848299	2.533502	0.059647	0.286484	2.913564
20610304	2.846512	2.528319	0.059892	0.284458	2.907201
20610606	2.844968	2.522484	0.060148	0.282361	2.900055
20610906	2.843449	2.516832	0.060398	0.280328	2.893137
20611205	2.841770	2.511363	0.060645	0.278337	2.886525
20620306	2.839872	2.505895	0.060899	0.276321	2.879450
20620605	2.838331	2.500374	0.061146	0.274368	2.872757
20620905	2.836754	2.494741	0.061396	0.272412	2.865834
20621204	2.835024	2.489291	0.061644	0.270495	2.859215
20630305	2.833080	2.483841	0.061897	0.268554	2.852143
20630604	2.831485	2.478341	0.062145	0.266674	2.845450
20630904	2.829854	2.472732	0.062395	0.264790	2.838608
20631204	2.828055	2.467245	0.062645	0.262924	2.831768
20640304	2.826244	2.461819	0.062896	0.261076	2.824932
20640604	2.824576	2.456285	0.063146	0.259245	2.818101
20640904	2.822894	2.450706	0.063396	0.257432	2.811273
20641204	2.821049	2.445194	0.063647	0.255635	2.804451
20650304	2.819045	2.439909	0.063897	0.253855	2.797635
20650604	2.817330	2.434407	0.064147	0.252092	2.790824
20650904	2.815602	2.428860	0.064397	0.250345	2.784021
20651204	2.813717	2.423383	0.064648	0.248615	2.777224
20660304	2.811678	2.418128	0.064898	0.246901	2.770436
20660604	2.809919	2.412814	0.065148	0.245202	2.763860
20660907	2.808080	2.406974	0.065407	0.243464	2.756590
20661206	2.806181	2.401644	0.065655	0.241816	2.749836
20670304	2.804155	2.396592	0.065899	0.240202	2.743501
20670606	2.802310	2.390944	0.066155	0.238529	2.736474
20670906	2.800501	2.385476	0.066406	0.236908	2.729675
20671205	2.798567	2.380185	0.066653	0.235320	2.723093
20680305	2.796603	2.374895	0.066903	0.233729	2.716317
20680604	2.794789	2.369566	0.067151	0.232170	2.709756
20680904	2.792943	2.364141	0.067401	0.230608	2.703068
20681204	2.790954	2.358786	0.067652	0.229060	NaN

Date	Zero Rate	Forward Rate	dateold	yrs_actual	X1	X2	yhat
20190304	0.000000	2.615130	03/04/2019	0.000000	0.000000	0.000000	NaN
20190604	2.682177	2.592484	06/04/2019	0.255556	0.918943	0.007485	2.730569
20190904	2.670525	2.607773	09/04/2019	0.511111	0.846525	0.014820	2.694674
20191204	2.662162	2.632146	12/04/2019	0.763889	0.782383	0.021931	2.665204
20200304	2.664183	2.578889	03/04/2020	1.016667	0.724815	0.028899	2.640930
20200604	2.660323	2.522048	06/04/2020	1.272222	0.672518	0.035803	2.620966
20200904	2.648001	2.487822	09/04/2020	1.527778	0.625472	0.042567	2.604984
20201204	2.630193	2.459395	12/04/2020	1.780556	0.583510	0.049121	2.592569
20210304	2.609781	2.467129	03/04/2021	2.030556	0.545979	0.055474	2.583164
20210604	2.600904	2.446780	06/04/2021	2.286111	0.511238	0.061835	2.576108
20210907	2.591225	2.426249	09/07/2021	2.550000	0.478780	0.068267	2.571167
20211206	2.579552	2.409906	12/06/2021	2.800000	0.450878	0.074234	2.568383
20220304	2.566094	2.484115	03/04/2022	3.044444	0.425984	0.079952	2.567202
20220606	2.564516	2.478241	06/06/2022	3.305556	0.401726	0.085934	2.567394
20220906	2.562863	2.472552	09/06/2022	3.561111	0.380070	0.091664	2.568833
20221205	2.559260	2.467048	12/05/2022	3.811111	0.360672	0.097154	2.571269
20230306	2.553914	2.565859	03/06/2023	4.063889	0.342670	0.102591	2.574617
20230605	2.558458	2.571125	06/05/2023	4.316667	0.326126	0.107913	2.578727
20230905	2.562812	2.576118	09/05/2023	4.572222	0.310732	0.113181	2.583541
20231204	2.565451	2.580918	12/04/2023	4.822222	0.296841	0.118225	2.588792
20240304	2.568084	2.673980	03/04/2024	5.075000	0.283857	0.123219	2.594561
20240604	2.576397	2.685708	06/04/2024	5.330556	0.271710	0.128159	2.600784
20240904	2.584504	2.696912	09/04/2024	5.586111	0.260454	0.132993	2.607333
20241204	2.591096	2.707643	12/04/2024	5.838889	0.250116	0.137671	2.614072
20250304	2.596332	2.751446	03/04/2025	6.088889	0.240597	0.142199	2.620941
20250604	2.605373	2.763948	06/04/2025	6.344444	0.231528	0.146728	2.628127
20250904	2.614213	2.775882	09/04/2025	6.600000	0.223067	0.151157	2.635440
20251204	2.621704	2.787302	12/04/2025	6.852778	0.215243	0.155441	2.642762
20260304	2.627963	2.854133	03/04/2026	7.102778	0.207993	0.159587	2.650061
20260604	2.638308	2.868117	06/04/2026	7.358333	0.201041	0.163731	2.657555
20260904	2.648444	2.881459	09/04/2026	7.613889	0.194514	0.167782	2.665056
20261204	2.657330	2.894216	12/04/2026	7.866667	0.188442	0.171700	2.672464
20270304	2.665055	2.928247	03/04/2027	8.116667	0.182782	0.175488	2.679760
20270604	2.675334	2.942009	06/04/2027	8.372222	0.177324	0.179273	2.687172
20270907	2.685674	2.954394	09/07/2027	8.636111	0.172007	0.183091	2.694762
20271206	2.694506	2.966405	12/06/2027	8.886111	0.167246	0.186625	2.701879
20280306	2.703274	3.009487	03/06/2028	9.138889	0.162686	0.190116	2.708994
20280605	2.713568	3.021969	06/05/2028	9.391667	0.158361	0.193527	2.716018
20280905	2.723752	3.033934	09/05/2028	9.647222	0.154210	0.196896	2.723018
20281204	2.732771	3.045385	12/04/2028	9.897222	0.150351	0.200115	2.729762
20290305	2.740954	3.023484	03/05/2029	10.150000	0.146636	0.203293	2.736470
20290604	2.749686	3.033094	06/04/2029	10.402778	0.143098	0.206397	2.743061
20290904	2.758318	3.042233	09/04/2029	10.658333	0.139687	0.209460	2.749602
20291204	2.765975	3.050885	12/04/2029	10.911111	0.136468	0.212417	2.755948
20300304	2.772710	3.070508	03/04/2030	11.161111	0.133425	0.215272	2.762099
20300604	2.781110	3.078987	06/04/2030	11.416667	0.130450	0.218120	2.768257
20300904	2.789335	3.086908	09/04/2030	11.672222	0.127603	0.220898	2.774280
20301204	2.796649	3.094372	12/04/2030	11.925000	0.124906	0.223578	2.780106
20310304	2.803096	3.018976	03/04/2031	12.175000	0.122347	0.226163	2.785736
20310604	2.809110	3.023875	06/04/2031	12.430556	0.119837	0.228741	2.791355

7. *Fit the 5-parameter model of Nelson and Siegel (1987) to your input data. Generate the same table as in Question 6, but including only columns (a), (b) and (d).*

Minimizing τ_1 and τ_2 are 1.39 and 15.6 , respectively. The betas $\beta_0, \beta_1, \beta_2$ are 2.000361, 0.773191, and 2.953684, respectively. The model produced a minimizing MSE of 0.0001596504901041256 .

Date	Zero Rate	Forward Rate	dateold	yrs_actual	X1	X2	yhat
20310904	2.814984	3.028334	09/04/2031	12.686111	0.117428	0.231253	2.796837
20311204	2.820053	3.032616	12/04/2031	12.938889	0.115137	0.233674	2.802124
20320304	2.825012	3.036774	03/04/2032	13.191667	0.112934	0.236034	2.807276
20320604	2.830504	3.040998	06/04/2032	13.447222	0.110790	0.238359	2.812349
20320907	2.836000	3.044032	09/07/2032	13.711111	0.108660	0.240696	2.817443
20321206	2.840563	3.047208	12/06/2032	13.961111	0.106716	0.242851	2.822136
20330304	2.844368	3.050508	03/04/2033	14.205556	0.104881	0.244904	2.826599
20330606	2.849462	3.053028	06/06/2033	14.466667	0.102989	0.247039	2.831230
20330906	2.854346	3.055385	09/06/2033	14.722222	0.101202	0.249071	2.835629
20331205	2.858488	3.057598	12/05/2033	14.972222	0.099513	0.251006	2.839804
20340306	2.862035	2.988192	03/06/2034	15.225000	0.097862	0.252909	2.843898
20340605	2.865355	2.988739	06/05/2034	15.477778	0.096264	0.254760	2.847865
20340905	2.868603	2.988984	09/05/2034	15.733333	0.094701	0.256579	2.851749
20341204	2.871198	2.989172	12/04/2034	15.983333	0.093220	0.258308	2.855425
20350305	2.873235	2.989315	03/05/2035	16.236111	0.091769	0.260007	2.859021
20350604	2.876199	2.989165	06/04/2035	16.488889	0.090362	0.261658	2.862496
20350904	2.879093	2.988724	09/04/2035	16.744444	0.088984	0.263278	2.865888
20351204	2.881394	2.988247	12/04/2035	16.997222	0.087660	0.264834	2.869125
20360304	2.883621	2.987733	03/04/2036	17.250000	0.086376	0.266343	2.872246
20360604	2.886260	2.986935	06/04/2036	17.505556	0.085115	0.267824	2.875285
20360904	2.888812	2.985865	09/04/2036	17.761111	0.083891	0.269258	2.878209
20361204	2.890806	2.984661	12/04/2036	18.013889	0.082713	0.270634	2.880989
20370304	2.892263	2.983677	03/04/2037	18.263889	0.081581	0.271952	2.883631
20370604	2.894582	2.982291	06/04/2037	18.519444	0.080456	0.273257	2.886224
20370904	2.896819	2.980652	09/04/2037	18.775000	0.079361	0.274520	2.888708
20371204	2.898529	2.978897	12/04/2037	19.027778	0.078306	0.275728	2.891061
20380304	2.899733	2.977372	03/04/2038	19.277778	0.077291	0.276885	2.893287
20380604	2.901750	2.975788	06/04/2038	19.533333	0.076280	0.278027	2.895460
20380907	2.903725	2.973227	09/07/2038	19.797222	0.075263	0.279166	2.897599
20381206	2.905154	2.969101	12/06/2038	20.047222	0.074324	0.280207	2.899527
20390304	2.906093	2.908190	03/04/2039	20.291667	0.073429	0.281190	2.901322
20390606	2.907053	2.904922	06/06/2039	20.552778	0.072496	0.282203	2.903142
20390906	2.907949	2.901729	09/06/2039	20.808333	0.071606	0.283158	2.904828
20391205	2.908391	2.898617	12/05/2039	21.058333	0.070756	0.284057	2.906388
20400305	2.908786	2.895484	03/05/2040	21.311111	0.069917	0.284933	2.907877
20400604	2.909516	2.892169	06/04/2040	21.563889	0.069097	0.285775	2.909278
20400904	2.910189	2.888639	09/04/2040	21.819444	0.068288	0.286593	2.910608
20401204	2.910432	2.885069	12/04/2040	22.072222	0.067506	0.287369	2.911839
20410304	2.910262	2.881750	03/04/2041	22.322222	0.066750	0.288106	2.912975
20410604	2.910783	2.878085	06/04/2041	22.577778	0.065994	0.288828	2.914055
20410904	2.911251	2.874253	09/04/2041	22.833333	0.065255	0.289519	2.915053
20411204	2.911311	2.870392	12/04/2041	23.086111	0.064541	0.290173	2.915963
20420304	2.910978	2.866784	03/04/2042	23.336111	0.063850	0.290790	2.916787
20420604	2.911304	2.862839	06/04/2042	23.591667	0.063158	0.291392	2.917554
20420904	2.911579	2.858738	09/04/2042	23.847222	0.062481	0.291965	2.918246
20421204	2.911467	2.854618	12/04/2042	24.100000	0.061826	0.292505	2.918858
20430304	2.910981	2.850754	03/04/2043	24.350000	0.061191	0.293011	2.919394
20430604	2.911122	2.846560	06/04/2043	24.605556	0.060555	0.293502	2.919873
20430904	2.911217	2.842219	09/04/2043	24.861111	0.059933	0.293967	2.920282
20431204	2.910943	2.837958	12/04/2043	25.113889	0.059330	0.294401	2.920620

Date	Zero Rate	Forward Rate	dateold	yrs_actual	X1	X2	yhat
20440304	2.910632	2.870458	03/04/2044	25.366667	0.058738	0.294809	2.920893
20440606	2.910963	2.865963	06/06/2044	25.627778	0.058140	0.295206	2.921109
20440906	2.911251	2.861591	09/06/2044	25.883333	0.057566	0.295569	2.921255
20441205	2.911184	2.857347	12/05/2044	26.133333	0.057015	0.295901	2.921338
20450306	2.910759	2.853089	03/06/2045	26.386111	0.056469	0.296213	2.921363
20450605	2.910920	2.848689	06/05/2045	26.638889	0.055933	0.296503	2.921330
20450905	2.911030	2.844100	09/05/2045	26.894444	0.055402	0.296774	2.921238
20451204	2.910802	2.839648	12/04/2045	27.144444	0.054892	0.297017	2.921093
20460305	2.910230	2.835183	03/05/2046	27.397222	0.054385	0.297242	2.920892
20460604	2.910222	2.830585	06/04/2046	27.650000	0.053888	0.297446	2.920638
20460904	2.910165	2.825806	09/04/2046	27.905556	0.053394	0.297631	2.920327
20461204	2.909780	2.821034	12/04/2046	28.158333	0.052915	0.297794	2.919969
20470304	2.909076	2.816524	03/04/2047	28.408333	0.052449	0.297936	2.919565
20470604	2.908903	2.811700	06/04/2047	28.663889	0.051982	0.298062	2.919103
20470904	2.908689	2.806753	09/04/2047	28.919444	0.051522	0.298168	2.918592
20471204	2.908161	2.801904	12/04/2047	29.172222	0.051076	0.298254	2.918040
20480304	2.907600	2.797099	03/04/2048	29.425000	0.050637	0.298323	2.917442
20480604	2.907274	2.792122	06/04/2048	29.680556	0.050201	0.298374	2.916792
20480904	2.906910	2.787030	09/04/2048	29.936111	0.049773	0.298408	2.916097
20481204	2.906246	2.781960	12/04/2048	30.188889	0.049356	0.298424	2.915368
20490304	2.905291	2.775641	03/04/2049	30.438889	0.048951	0.298423	2.914605
20490604	2.904812	2.770764	06/04/2049	30.694444	0.048543	0.298406	2.913785
20490907	2.904264	2.765141	09/07/2049	30.958333	0.048129	0.298372	2.912895
20491206	2.903474	2.760082	12/06/2049	31.208333	0.047744	0.298323	2.912013
20500304	2.902418	2.755342	03/04/2050	31.452778	0.047373	0.298262	2.911115
20500606	2.901784	2.749851	06/06/2050	31.713889	0.046983	0.298180	2.910118
20500906	2.901141	2.744526	09/06/2050	31.969444	0.046607	0.298085	2.909105
20501205	2.900237	2.739368	12/05/2050	32.219444	0.046245	0.297978	2.908080
20510306	2.899039	2.734205	03/06/2051	32.472222	0.045885	0.297856	2.907009
20510605	2.898322	2.728945	06/05/2051	32.725000	0.045531	0.297721	2.905905
20510905	2.897560	2.723532	09/05/2051	32.980556	0.045178	0.297570	2.904755
20511204	2.896549	2.718291	12/04/2051	33.230556	0.044838	0.297409	2.903599
20520304	2.895498	2.713045	03/04/2052	33.483333	0.044500	0.297234	2.902400
20520604	2.894652	2.707649	06/04/2052	33.738889	0.044163	0.297044	2.901157
20520904	2.893777	2.702165	09/04/2052	33.994444	0.043831	0.296842	2.899884
20521204	2.892648	2.696723	12/04/2052	34.247222	0.043507	0.296630	2.898596
20530304	2.891274	2.691540	03/04/2053	34.497222	0.043192	0.296409	2.897295
20530604	2.890321	2.686081	06/04/2053	34.752778	0.042874	0.296171	2.895937
20530904	2.889342	2.680538	09/04/2053	35.008333	0.042561	0.295921	2.894553
20531204	2.888120	2.675042	12/04/2053	35.261111	0.042256	0.295664	2.893158
20540304	2.886661	2.669803	03/04/2054	35.511111	0.041959	0.295399	2.891754
20540604	2.885609	2.664293	06/04/2054	35.766667	0.041659	0.295117	2.890294
20540904	2.884532	2.658705	09/04/2054	36.022222	0.041363	0.294825	2.888810
20541204	2.883222	2.653168	12/04/2054	36.275000	0.041075	0.294526	2.887319
20550304	2.881685	2.647884	03/04/2055	36.525000	0.040794	0.294221	2.885823
20550604	2.880541	2.642536	06/04/2055	36.780556	0.040511	0.293899	2.884271
20550907	2.879321	2.636533	09/07/2055	37.044444	0.040222	0.293557	2.882646
20551206	2.877946	2.631091	12/06/2055	37.294444	0.039952	0.293224	2.881087
20560306	2.876535	2.625648	03/06/2056	37.547222	0.039683	0.292878	2.879490
20560605	2.875321	2.620133	06/05/2056	37.800000	0.039418	0.292524	2.877874

Date	Zero Rate	Forward Rate	dateold	yrs_actual	X1	X2	yhat
20560905	2.874068	2.614488	09/05/2056	38.055556	0.039153	0.292157	2.876221
20561204	2.872617	2.609024	12/04/2056	38.305556	0.038898	0.291790	2.874587
20570305	2.870918	2.603559	03/05/2057	38.558333	0.038643	0.291411	2.872916
20570604	2.869625	2.598027	06/04/2057	38.811111	0.038391	0.291024	2.871229
20570904	2.868293	2.592368	09/04/2057	39.066667	0.038140	0.290625	2.869507
20571204	2.866753	2.586769	12/04/2057	39.319444	0.037895	0.290223	2.867787
20580304	2.865011	2.581415	03/04/2058	39.569444	0.037655	0.289819	2.866071
20580604	2.863625	2.575813	06/04/2058	39.825000	0.037414	0.289398	2.864302
20580904	2.862219	2.570149	09/04/2058	40.080556	0.037175	0.288970	2.862518
20581204	2.860614	2.564547	12/04/2058	40.333333	0.036942	0.288541	2.860739
20590304	2.858814	2.572755	03/04/2059	40.583333	0.036715	0.288109	2.858967
20590604	2.857445	2.567145	06/04/2059	40.838889	0.036485	0.287661	2.857142
20590904	2.856058	2.561474	09/04/2059	41.094444	0.036258	0.287208	2.855305
20591204	2.854476	2.555926	12/04/2059	41.347222	0.036036	0.286753	2.853475
20600304	2.852878	2.550438	03/04/2060	41.600000	0.035817	0.286292	2.851633
20600604	2.851439	2.545008	06/04/2060	41.855556	0.035599	0.285820	2.849760
20600907	2.849923	2.538983	09/07/2060	42.119444	0.035376	0.285327	2.847815
20601206	2.848299	2.533502	12/06/2060	42.369444	0.035167	0.284855	2.845961
20610304	2.846512	2.528319	03/04/2061	42.613889	0.034965	0.284388	2.844139
20610606	2.844968	2.522484	06/06/2061	42.875000	0.034752	0.283884	2.842182
20610906	2.843449	2.516832	09/06/2061	43.130556	0.034546	0.283385	2.840258
20611205	2.841770	2.511363	12/05/2061	43.380556	0.034347	0.282893	2.838367
20620306	2.839872	2.505895	03/06/2062	43.633333	0.034148	0.282391	2.836447
20620605	2.838331	2.500374	06/05/2062	43.886111	0.033952	0.281884	2.834518
20620905	2.836754	2.494741	09/05/2062	44.141667	0.033755	0.281367	2.832561
20621204	2.835024	2.489291	12/04/2062	44.391667	0.033565	0.280857	2.830639
20630305	2.833080	2.483841	03/05/2063	44.644444	0.033375	0.280337	2.828688
20630604	2.831485	2.478341	06/04/2063	44.897222	0.033187	0.279813	2.826731
20630904	2.829854	2.472732	09/04/2063	45.152778	0.032999	0.279280	2.824745
20631204	2.828055	2.467245	12/04/2063	45.405556	0.032815	0.278749	2.822775
20640304	2.826244	2.461819	03/04/2064	45.658333	0.032634	0.278214	2.820800
20640604	2.824576	2.456285	06/04/2064	45.913889	0.032452	0.277669	2.818797
20640904	2.822894	2.450706	09/04/2064	46.169444	0.032272	0.277121	2.816789
20641204	2.821049	2.445194	12/04/2064	46.422222	0.032097	0.276576	2.814797
20650304	2.819045	2.439909	03/04/2065	46.672222	0.031925	0.276034	2.812824
20650604	2.817330	2.434407	06/04/2065	46.927778	0.031751	0.275476	2.810802
20650904	2.815602	2.428860	09/04/2065	47.183333	0.031579	0.274916	2.808776
20651204	2.813717	2.423383	12/04/2065	47.436111	0.031411	0.274359	2.806768
20660304	2.811678	2.418128	03/04/2066	47.686111	0.031246	0.273805	2.804779
20660604	2.809919	2.412814	06/04/2066	47.941667	0.031079	0.273236	2.802742
20660907	2.808080	2.406974	09/07/2066	48.205556	0.030909	0.272646	2.800636
20661206	2.806181	2.401644	12/06/2066	48.455556	0.030750	0.272084	2.798638
20670304	2.804155	2.396592	03/04/2067	48.700000	0.030595	0.271533	2.796682
20670606	2.802310	2.390944	06/06/2067	48.961111	0.030432	0.270942	2.794591
20670906	2.800501	2.385476	09/06/2067	49.216667	0.030274	0.270361	2.792541
20671205	2.798567	2.380185	12/05/2067	49.466667	0.030121	0.269791	2.790535
20680305	2.796603	2.374895	03/05/2068	49.719444	0.029968	0.269212	2.788505
20680604	2.794789	2.369566	06/04/2068	49.972222	0.029817	0.268632	2.786473
20680904	2.792943	2.364141	09/04/2068	50.227778	0.029665	0.268043	2.784419
20681204	2.790954	2.358786	12/04/2068	50.480556	0.029516	0.267459	2.782385

Date	Zero Rate	Forward Rate	dateold	yrs_actual	X1	X2	X3	yhat
20190304	0.000000	2.615130	03/04/2019	0.000000	0.000000	0.000000	0.000000	NaN
20190604	2.682177	2.592484	06/04/2019	0.255556	0.835708	0.145229	0.007951	2.665082
20190904	2.670525	2.607773	09/04/2019	0.511111	0.706373	0.229613	0.015732	2.689476
20191204	2.662162	2.632146	12/04/2019	0.763889	0.604722	0.274201	0.023266	2.686525
20200304	2.664183	2.578889	03/04/2020	1.016667	0.523175	0.294037	0.030640	2.670813
20200604	2.660323	2.522048	06/04/2020	1.272222	0.456549	0.298334	0.037936	2.650161
20200904	2.648001	2.487822	09/04/2020	1.527778	0.402298	0.293054	0.045074	2.629236
20201204	2.630193	2.459395	12/04/2020	1.780556	0.358171	0.282436	0.051983	2.610531
20210304	2.609781	2.467129	03/04/2021	2.030556	0.321895	0.269179	0.058669	2.594948
20210604	2.600904	2.446780	06/04/2021	2.286111	0.290836	0.254437	0.065357	2.582302
20210907	2.591225	2.426249	09/07/2021	2.550000	0.263869	0.239038	0.072109	2.572630
20211206	2.579552	2.409906	12/06/2021	2.800000	0.242169	0.224885	0.078365	2.566366
20220304	2.566094	2.484115	03/04/2022	3.044444	0.223894	0.211766	0.084351	2.562644
20220606	2.564516	2.478241	06/06/2022	3.305556	0.207006	0.198699	0.090605	2.560930
20220906	2.562863	2.472552	09/06/2022	3.561111	0.192648	0.186913	0.096588	2.561170
20221205	2.559260	2.467048	12/05/2022	3.811111	0.180327	0.176334	0.102312	2.562937
20230306	2.553914	2.565859	03/06/2023	4.063889	0.169318	0.166550	0.107972	2.566005
20230605	2.558458	2.571125	06/05/2023	4.316667	0.159539	0.157620	0.113506	2.570135
20230905	2.562812	2.576118	09/05/2023	4.572222	0.150711	0.149386	0.118974	2.575194
20231204	2.565451	2.580918	12/04/2023	4.822222	0.142956	0.142033	0.124204	2.580842
20240304	2.568084	2.673980	03/04/2024	5.075000	0.135874	0.135234	0.129373	2.587118
20240604	2.576397	2.685708	06/04/2024	5.330556	0.129385	0.128944	0.134479	2.593926
20240904	2.584504	2.696912	09/04/2024	5.586111	0.123483	0.123178	0.139468	2.601100
20241204	2.591096	2.707643	12/04/2024	5.838889	0.118148	0.117937	0.144289	2.608474
20250304	2.596332	2.751446	03/04/2025	6.088889	0.113304	0.113157	0.148948	2.615973
20250604	2.605373	2.763948	06/04/2025	6.344444	0.108746	0.108644	0.153600	2.623793
20250904	2.614213	2.775882	09/04/2025	6.600000	0.104538	0.104468	0.158143	2.631718
20251204	2.621704	2.787302	12/04/2025	6.852778	0.100684	0.100636	0.162531	2.639622
20260304	2.627963	2.854133	03/04/2026	7.102778	0.097142	0.097108	0.166770	2.647467
20260604	2.638308	2.868117	06/04/2026	7.358333	0.093769	0.093746	0.171000	2.655487
20260904	2.648444	2.881459	09/04/2026	7.613889	0.090622	0.090606	0.175129	2.663482
20261204	2.657330	2.894216	12/04/2026	7.866667	0.087711	0.087700	0.179115	2.671345
20270304	2.665055	2.928247	03/04/2027	8.116667	0.085010	0.085002	0.182963	2.679060
20270604	2.675334	2.942009	06/04/2027	8.372222	0.082415	0.082410	0.186802	2.686868
20270907	2.685674	2.954394	09/07/2027	8.636111	0.079897	0.079893	0.190666	2.694833
20271206	2.694506	2.966405	12/06/2027	8.886111	0.077649	0.077647	0.194237	2.702278
20280306	2.703274	3.009487	03/06/2028	9.138889	0.075501	0.075500	0.197759	2.709696
20280605	2.713568	3.021969	06/05/2028	9.391667	0.073469	0.073468	0.201193	2.716996
20280905	2.723752	3.033934	09/05/2028	9.647222	0.071523	0.071522	0.204579	2.724250
20281204	2.732771	3.045385	12/04/2028	9.897222	0.069716	0.069716	0.207808	2.731218
20290305	2.740954	3.023484	03/05/2029	10.150000	0.067980	0.067980	0.210990	2.738132
20290604	2.749686	3.033094	06/04/2029	10.402778	0.066328	0.066328	0.214092	2.744908
20290904	2.758318	3.042233	09/04/2029	10.658333	0.064738	0.064738	0.217148	2.751616
20291204	2.765975	3.050885	12/04/2029	10.911111	0.063238	0.063238	0.220091	2.758109
20300304	2.772710	3.070508	03/04/2030	11.161111	0.061822	0.061822	0.222927	2.764389
20300604	2.781110	3.078987	06/04/2030	11.416667	0.060438	0.060438	0.225751	2.770662
20300904	2.789335	3.086908	09/04/2030	11.672222	0.059115	0.059115	0.228499	2.776786
20301204	2.796649	3.094372	12/04/2030	11.925000	0.057862	0.057862	0.231145	2.782696
20310304	2.803096	3.018976	03/04/2031	12.175000	0.056674	0.056673	0.233692	2.788398
20310604	2.809110	3.023875	06/04/2031	12.430556	0.055508	0.055508	0.236225	2.794079

8. Repeat Question 7, this time using the model of Svensson (1994).

Minimizing τ_1 and τ_2 are 0.57 and 15.3, respectively. The betas $\beta_0, \beta_1, \beta_2$ are 1.997775, 0.774489, and 2.961562, respectively. The model produced a minimizing MSE of 9.481207746983983e-05 .

Date	Zero Rate	Forward Rate	dateold	yrs_actual	X1	X2	X3	yhat
20310904	2.814984	3.028334	09/04/2031	12.686111	0.054390	0.054390	0.238688	2.799611
20311204	2.820053	3.032616	12/04/2031	12.938889	0.053328	0.053328	0.241058	2.804937
20320304	2.825012	3.036774	03/04/2032	13.191667	0.052306	0.052306	0.243361	2.810119
20320604	2.830504	3.040998	06/04/2032	13.447222	0.051312	0.051312	0.245625	2.815213
20320907	2.836000	3.044032	09/07/2032	13.711111	0.050324	0.050324	0.247895	2.820320
20321206	2.840563	3.047208	12/06/2032	13.961111	0.049423	0.049423	0.249983	2.825017
20330304	2.844368	3.050508	03/04/2033	14.205556	0.048573	0.048573	0.251967	2.829477
20330606	2.849462	3.053028	06/06/2033	14.466667	0.047696	0.047696	0.254024	2.834099
20330906	2.854346	3.055385	09/06/2033	14.722222	0.046868	0.046868	0.255978	2.838482
20331205	2.858488	3.057598	12/05/2033	14.972222	0.046085	0.046085	0.257832	2.842636
20340306	2.862035	2.988192	03/06/2034	15.225000	0.045320	0.045320	0.259651	2.846704
20340605	2.865355	2.988739	06/05/2034	15.477778	0.044580	0.044580	0.261415	2.850640
20340905	2.868603	2.988984	09/05/2034	15.733333	0.043856	0.043856	0.263143	2.854488
20341204	2.871198	2.989172	12/04/2034	15.983333	0.043170	0.043170	0.264781	2.858126
20350305	2.873235	2.989315	03/05/2035	16.236111	0.042498	0.042498	0.266386	2.861679
20350604	2.876199	2.989165	06/04/2035	16.488889	0.041846	0.041846	0.267940	2.865107
20350904	2.879093	2.988724	09/04/2035	16.744444	0.041208	0.041208	0.269459	2.868449
20351204	2.881394	2.988247	12/04/2035	16.997222	0.040595	0.040595	0.270913	2.871633
20360304	2.883621	2.987733	03/04/2036	17.250000	0.040000	0.040000	0.272319	2.874699
20360604	2.886260	2.986935	06/04/2036	17.505556	0.039416	0.039416	0.273692	2.877680
20360904	2.888812	2.985865	09/04/2036	17.761111	0.038849	0.038849	0.275018	2.880543
20361204	2.890806	2.984661	12/04/2036	18.013889	0.038304	0.038304	0.276284	2.883262
20370304	2.892263	2.983677	03/04/2037	18.263889	0.037779	0.037779	0.277492	2.885842
20370604	2.894582	2.982291	06/04/2037	18.519444	0.037258	0.037258	0.278683	2.888369
20370904	2.896819	2.980652	09/04/2037	18.775000	0.036751	0.036751	0.279831	2.890786
20371204	2.898529	2.978897	12/04/2037	19.027778	0.036263	0.036263	0.280923	2.893072
20380304	2.899733	2.977372	03/04/2038	19.277778	0.035793	0.035793	0.281964	2.895230
20380604	2.901750	2.975788	06/04/2038	19.533333	0.035324	0.035324	0.282987	2.897334
20380907	2.903725	2.973227	09/07/2038	19.797222	0.034853	0.034853	0.284001	2.899399
20381206	2.905154	2.969101	12/06/2038	20.047222	0.034419	0.034419	0.284922	2.901258
20390304	2.906093	2.908190	03/04/2039	20.291667	0.034004	0.034004	0.285788	2.902984
20390606	2.907053	2.904922	06/06/2039	20.552778	0.033572	0.033572	0.286674	2.904731
20390906	2.907949	2.901729	09/06/2039	20.808333	0.033160	0.033160	0.287504	2.906345
20391205	2.908391	2.898617	12/05/2039	21.058333	0.032766	0.032766	0.288280	2.907834
20400305	2.908786	2.895484	03/05/2040	21.311111	0.032377	0.032377	0.289031	2.909252
20400604	2.909516	2.892169	06/04/2040	21.563889	0.031998	0.031998	0.289747	2.910582
20400904	2.910189	2.888639	09/04/2040	21.819444	0.031623	0.031623	0.290437	2.911840
20401204	2.910432	2.885069	12/04/2040	22.072222	0.031261	0.031261	0.291086	2.912999
20410304	2.910262	2.881750	03/04/2041	22.322222	0.030911	0.030911	0.291697	2.914066
20410604	2.910783	2.878085	06/04/2041	22.577778	0.030561	0.030561	0.292290	2.915074
20410904	2.911251	2.874253	09/04/2041	22.833333	0.030219	0.030219	0.292851	2.916002
20411204	2.911311	2.870392	12/04/2041	23.086111	0.029888	0.029888	0.293375	2.916842
20420304	2.910978	2.866784	03/04/2042	23.336111	0.029568	0.029568	0.293865	2.917599
20420604	2.911304	2.862839	06/04/2042	23.591667	0.029248	0.029248	0.294336	2.918297
20420904	2.911579	2.858738	09/04/2042	23.847222	0.028934	0.028934	0.294778	2.918921
20421204	2.911467	2.854618	12/04/2042	24.100000	0.028631	0.028631	0.295188	2.919466
20430304	2.910981	2.850754	03/04/2043	24.350000	0.028337	0.028337	0.295566	2.919937
20430604	2.911122	2.846560	06/04/2043	24.605556	0.028042	0.028042	0.295926	2.920350
20430904	2.911217	2.842219	09/04/2043	24.861111	0.027754	0.027754	0.296259	2.920694
20431204	2.910943	2.837958	12/04/2043	25.113889	0.027475	0.027475	0.296563	2.920969

Date	Zero Rate	Forward Rate	dateold	yrs_actual	X1	X2	X3	yhat
20440304	2.910632	2.870458	03/04/2044	25.366667	0.027201	0.027201	0.296842	2.921180
20440606	2.910963	2.865963	06/06/2044	25.627778	0.026924	0.026924	0.297104	2.921332
20440906	2.911251	2.861591	09/06/2044	25.883333	0.026658	0.026658	0.297336	2.921418
20441205	2.911184	2.857347	12/05/2044	26.133333	0.026403	0.026403	0.297540	2.921443
20450306	2.910759	2.853089	03/06/2045	26.386111	0.026150	0.026150	0.297724	2.921410
20450605	2.910920	2.848689	06/05/2045	26.638889	0.025902	0.025902	0.297885	2.921320
20450905	2.911030	2.844100	09/05/2045	26.894444	0.025656	0.025656	0.298025	2.921172
20451204	2.910802	2.839648	12/04/2045	27.144444	0.025420	0.025420	0.298142	2.920973
20460305	2.910230	2.835183	03/05/2046	27.397222	0.025185	0.025185	0.298238	2.920718
20460604	2.910222	2.830585	06/04/2046	27.650000	0.024955	0.024955	0.298314	2.920411
20460904	2.910165	2.825806	09/04/2046	27.905556	0.024726	0.024726	0.298371	2.920049
20461204	2.909780	2.821034	12/04/2046	28.158333	0.024504	0.024504	0.298407	2.919641
20470304	2.909076	2.816524	03/04/2047	28.408333	0.024289	0.024289	0.298424	2.919189
20470604	2.908903	2.811700	06/04/2047	28.663889	0.024072	0.024072	0.298422	2.918679
20470904	2.908689	2.806753	09/04/2047	28.919444	0.023859	0.023859	0.298402	2.918122
20471204	2.908161	2.801904	12/04/2047	29.172222	0.023653	0.023653	0.298364	2.917525
20480304	2.907600	2.797099	03/04/2048	29.425000	0.023449	0.023449	0.298308	2.916883
20480604	2.907274	2.792122	06/04/2048	29.680556	0.023248	0.023248	0.298234	2.916190
20480904	2.906910	2.787030	09/04/2048	29.936111	0.023049	0.023049	0.298143	2.915455
20481204	2.906246	2.781960	12/04/2048	30.188889	0.022856	0.022856	0.298036	2.914685
20490304	2.905291	2.775641	03/04/2049	30.438889	0.022668	0.022668	0.297915	2.913885
20490604	2.904812	2.770764	06/04/2049	30.694444	0.022480	0.022480	0.297776	2.913027
20490907	2.904264	2.765141	09/07/2049	30.958333	0.022288	0.022288	0.297615	2.912100
20491206	2.903474	2.760082	12/06/2049	31.208333	0.022109	0.022109	0.297448	2.911184
20500304	2.902418	2.755342	03/04/2050	31.452778	0.021938	0.021938	0.297271	2.910254
20500606	2.901784	2.749851	06/06/2050	31.713889	0.021757	0.021757	0.297068	2.909224
20500906	2.901141	2.744526	09/06/2050	31.969444	0.021583	0.021583	0.296854	2.908180
20501205	2.900237	2.739368	12/05/2050	32.219444	0.021416	0.021416	0.296631	2.907126
20510306	2.899039	2.734205	03/06/2051	32.472222	0.021249	0.021249	0.296393	2.906027
20510605	2.898322	2.728945	06/05/2051	32.725000	0.021085	0.021085	0.296142	2.904896
20510905	2.897560	2.723532	09/05/2051	32.980556	0.020921	0.020921	0.295875	2.903721
20511204	2.896549	2.718291	12/04/2051	33.230556	0.020764	0.020764	0.295602	2.902541
20520304	2.895498	2.713045	03/04/2052	33.483333	0.020607	0.020607	0.295314	2.901319
20520604	2.894652	2.707649	06/04/2052	33.738889	0.020451	0.020451	0.295011	2.900055
20520904	2.893777	2.702165	09/04/2052	33.994444	0.020297	0.020297	0.294696	2.898761
20521204	2.892648	2.696723	12/04/2052	34.247222	0.020148	0.020148	0.294373	2.897454
20530304	2.891274	2.691540	03/04/2053	34.497222	0.020002	0.020002	0.294043	2.896136
20530604	2.890321	2.686081	06/04/2053	34.752778	0.019855	0.019855	0.293695	2.894762
20530904	2.889342	2.680538	09/04/2053	35.008333	0.019710	0.019710	0.293337	2.893362
20531204	2.888120	2.675042	12/04/2053	35.261111	0.019568	0.019568	0.292972	2.891953
20540304	2.886661	2.669803	03/04/2054	35.511111	0.019431	0.019431	0.292602	2.890537
20540604	2.885609	2.664293	06/04/2054	35.766667	0.019292	0.019292	0.292214	2.889065
20540904	2.884532	2.658705	09/04/2054	36.022222	0.019155	0.019155	0.291816	2.887571
20541204	2.883222	2.653168	12/04/2054	36.275000	0.019021	0.019021	0.291414	2.886071
20550304	2.881685	2.647884	03/04/2055	36.525000	0.018891	0.018891	0.291007	2.884567
20550604	2.880541	2.642536	06/04/2055	36.780556	0.018760	0.018760	0.290583	2.883009
20550907	2.879321	2.636533	09/07/2055	37.044444	0.018626	0.018626	0.290135	2.881378
20551206	2.877946	2.631091	12/06/2055	37.294444	0.018501	0.018501	0.289703	2.879815
20560306	2.876535	2.625648	03/06/2056	37.547222	0.018377	0.018377	0.289259	2.878215
20560605	2.875321	2.620133	06/05/2056	37.800000	0.018254	0.018254	0.288807	2.876597

Date	Zero Rate	Forward Rate	dateold	yrs_actual	X1	X2	X3	yhat
20560905	2.874068	2.614488	09/05/2056	38.055556	0.018131	0.018131	0.288342	2.874944
20561204	2.872617	2.609024	12/04/2056	38.305556	0.018013	0.018013	0.287880	2.873310
20570305	2.870918	2.603559	03/05/2057	38.558333	0.017895	0.017895	0.287406	2.871641
20570604	2.869625	2.598027	06/04/2057	38.811111	0.017778	0.017778	0.286925	2.869957
20570904	2.868293	2.592368	09/04/2057	39.066667	0.017662	0.017662	0.286432	2.868238
20571204	2.866753	2.586769	12/04/2057	39.319444	0.017549	0.017549	0.285937	2.866523
20580304	2.865011	2.581415	03/04/2058	39.569444	0.017438	0.017438	0.285442	2.864813
20580604	2.863625	2.575813	06/04/2058	39.825000	0.017326	0.017326	0.284930	2.863051
20580904	2.862219	2.570149	09/04/2058	40.080556	0.017215	0.017215	0.284412	2.861276
20581204	2.860614	2.564547	12/04/2058	40.333333	0.017107	0.017107	0.283893	2.859506
20590304	2.858814	2.572755	03/04/2059	40.583333	0.017002	0.017002	0.283375	2.857744
20590604	2.857445	2.567145	06/04/2059	40.838889	0.016896	0.016896	0.282840	2.855931
20590904	2.856058	2.561474	09/04/2059	41.094444	0.016791	0.016791	0.282299	2.854106
20591204	2.854476	2.555926	12/04/2059	41.347222	0.016688	0.016688	0.281760	2.852289
20600304	2.852878	2.550438	03/04/2060	41.600000	0.016587	0.016587	0.281215	2.850462
20600604	2.851439	2.545008	06/04/2060	41.855556	0.016485	0.016485	0.280659	2.848604
20600907	2.849923	2.538983	09/07/2060	42.119444	0.016382	0.016382	0.280081	2.846676
20601206	2.848299	2.533502	12/06/2060	42.369444	0.016285	0.016285	0.279528	2.844839
20610304	2.846512	2.528319	03/04/2061	42.613889	0.016192	0.016192	0.278984	2.843034
20610606	2.844968	2.522484	06/06/2061	42.875000	0.016093	0.016093	0.278398	2.841098
20610906	2.843449	2.516832	09/06/2061	43.130556	0.015998	0.015998	0.277820	2.839194
20611205	2.841770	2.511363	12/05/2061	43.380556	0.015906	0.015906	0.277252	2.837324
20620306	2.839872	2.505895	03/06/2062	43.633333	0.015814	0.015814	0.276673	2.835426
20620605	2.838331	2.500374	06/05/2062	43.886111	0.015723	0.015723	0.276090	2.833520
20620905	2.836754	2.494741	09/05/2062	44.141667	0.015631	0.015631	0.275498	2.831586
20621204	2.835024	2.489291	12/04/2062	44.391667	0.015543	0.015543	0.274915	2.829688
20630305	2.833080	2.483841	03/05/2063	44.644444	0.015455	0.015455	0.274323	2.827763
20630604	2.831485	2.478341	06/04/2063	44.897222	0.015368	0.015368	0.273727	2.825832
20630904	2.829854	2.472732	09/04/2063	45.152778	0.015281	0.015281	0.273122	2.823874
20631204	2.828055	2.467245	12/04/2063	45.405556	0.015196	0.015196	0.272521	2.821932
20640304	2.826244	2.461819	03/04/2064	45.658333	0.015112	0.015112	0.271917	2.819985
20640604	2.824576	2.456285	06/04/2064	45.913889	0.015028	0.015028	0.271303	2.818012
20640904	2.822894	2.450706	09/04/2064	46.169444	0.014945	0.014945	0.270687	2.816034
20641204	2.821049	2.445194	12/04/2064	46.422222	0.014864	0.014864	0.270076	2.814074
20650304	2.819045	2.439909	03/04/2065	46.672222	0.014784	0.014784	0.269468	2.812132
20650604	2.817330	2.434407	06/04/2065	46.927778	0.014703	0.014703	0.268845	2.810143
20650904	2.815602	2.428860	09/04/2065	47.183333	0.014624	0.014624	0.268220	2.808150
20651204	2.813717	2.423383	12/04/2065	47.436111	0.014546	0.014546	0.267600	2.806177
20660304	2.811678	2.418128	03/04/2066	47.686111	0.014470	0.014470	0.266985	2.804222
20660604	2.809919	2.412814	06/04/2066	47.941667	0.014392	0.014392	0.266354	2.802221
20660907	2.808080	2.406974	09/07/2066	48.205556	0.014314	0.014314	0.265701	2.800153
20661206	2.806181	2.401644	12/06/2066	48.455556	0.014240	0.014240	0.265080	2.798191
20670304	2.804155	2.396592	03/04/2067	48.700000	0.014168	0.014168	0.264472	2.796272
20670606	2.802310	2.390944	06/06/2067	48.961111	0.014093	0.014093	0.263821	2.794220
20670906	2.800501	2.385476	09/06/2067	49.216667	0.014020	0.014020	0.263183	2.792210
20671205	2.798567	2.380185	12/05/2067	49.466667	0.013949	0.013949	0.262557	2.790243
20680305	2.796603	2.374895	03/05/2068	49.719444	0.013878	0.013878	0.261923	2.788253
20680604	2.794789	2.369566	06/04/2068	49.972222	0.013808	0.013808	0.261288	2.786262
20680904	2.792943	2.364141	09/04/2068	50.227778	0.013737	0.013737	0.260645	2.784249
20681204	2.790954	2.358786	12/04/2068	50.480556	0.013669	0.013669	0.260008	2.782257