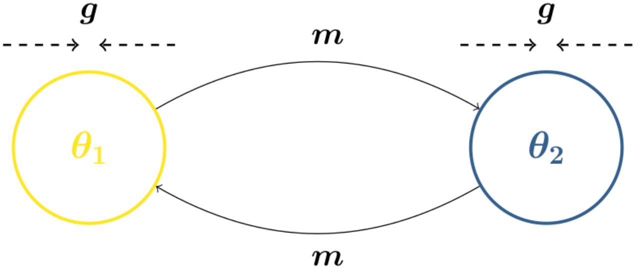
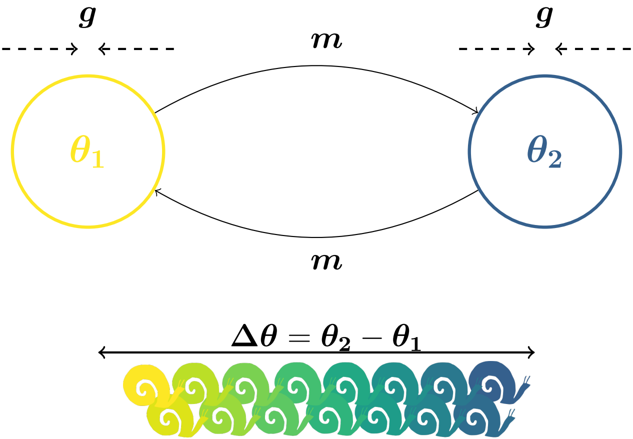
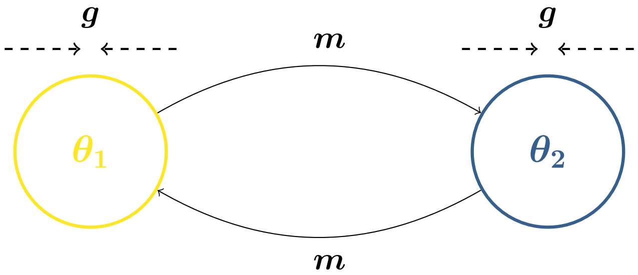
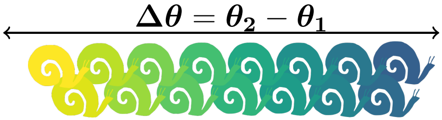
Quantitative trait in patchy environment: beneath the Gaussian approximation

Léonard Dekens




g
g
m
θ_1
θ_2
m
$\Delta\theta = \theta_2 - \theta_1$

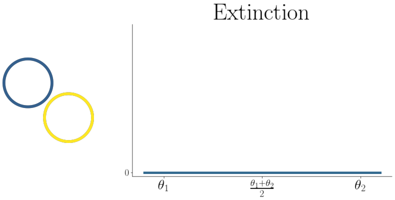




Main motivation

Given m (migration), g (selection), $n_1(t=0)$, $n_2(t=0)$ (initial state) $\rightsquigarrow$ Predict long term **evolutionary outcomes**?

What the outcomes might look like



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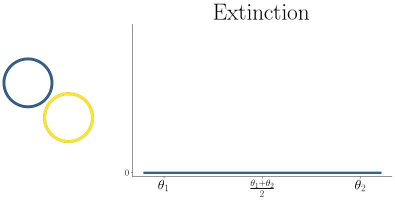

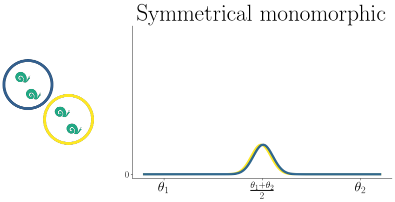

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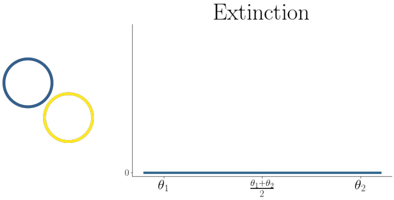

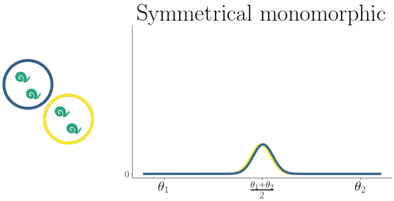

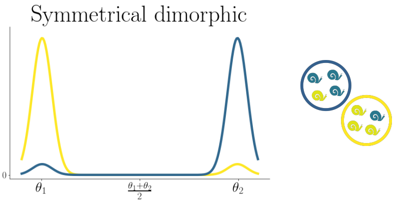

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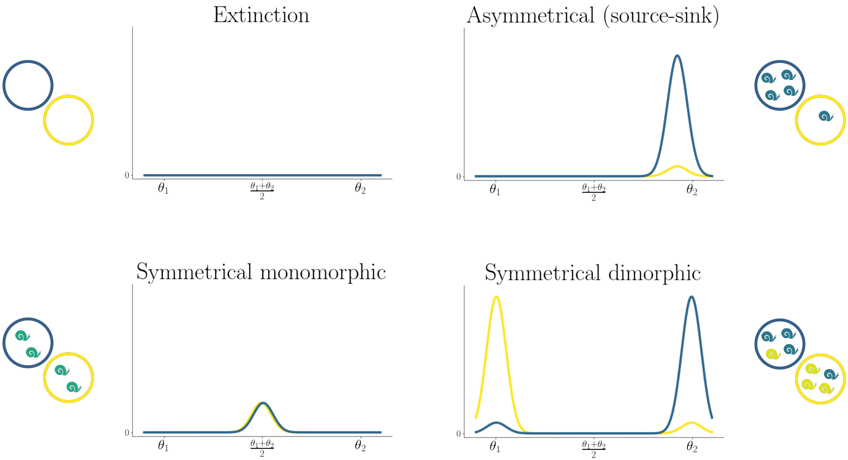
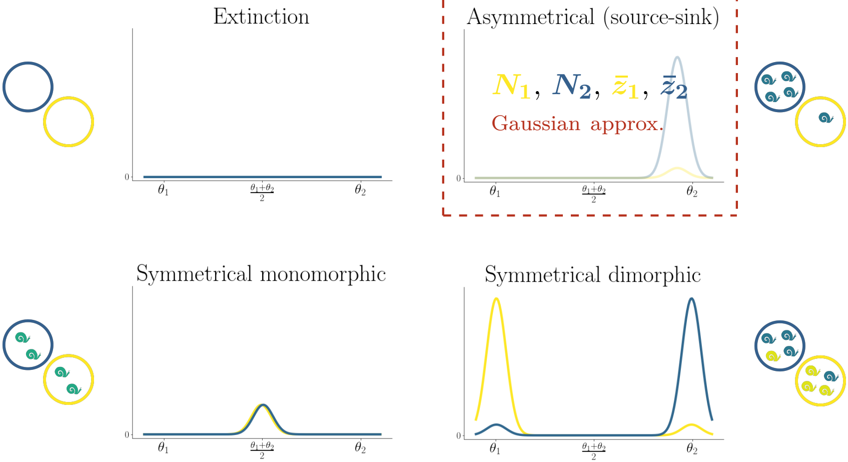


What the outcomes might look like

Num. observed in Ronce and Kirkpatrick 2001



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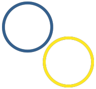
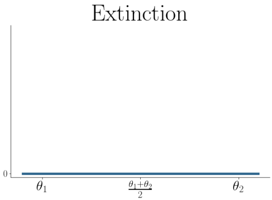
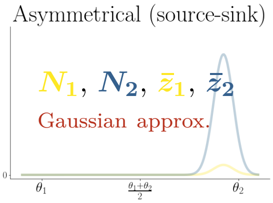
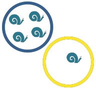


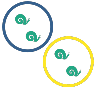


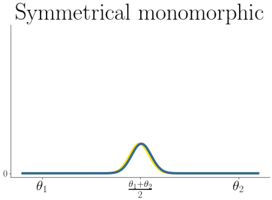


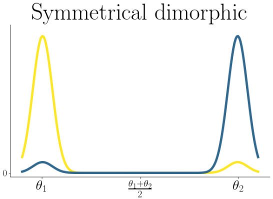
$\boldsymbol{N_1}$, $\boldsymbol{N_2}$, $\boldsymbol{\bar{z}_1}$, $\boldsymbol{\bar{z}_2}$

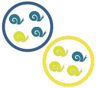
Gaussian approx.











Asexual models: Débarre, Ronce, and Gandon 2013; Mirrahimi and Gandon 2020

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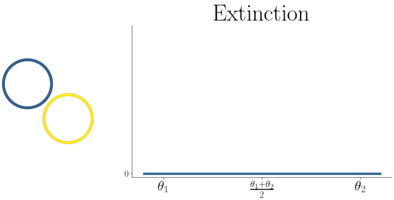
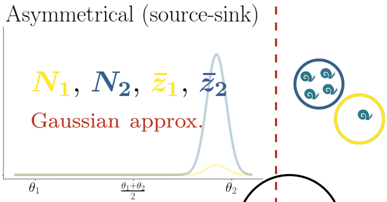
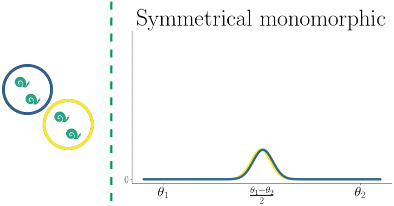
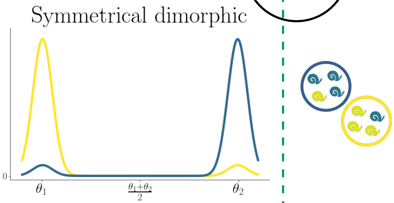




$\neq$?





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Description of the framework

Deterministic model on the **trait distributions*** $((i,j) \in \{1,2\})$:

$$\frac{\partial \boldsymbol{n_i}}{\partial \boldsymbol{t}} = \overbrace{\boldsymbol{\mathcal{B}}[\boldsymbol{n_i}]}^{\text{reproduction}} - \overbrace{\boldsymbol{N_i n_i}}^{\text{competition}} - \overbrace{\boldsymbol{g}\,(\boldsymbol{z} - \boldsymbol{\theta_i})^2\,\boldsymbol{n_i}}^{\text{selection}} + \overbrace{\boldsymbol{m}\,(\boldsymbol{n_j} - \boldsymbol{n_i})}^{\text{migration}},$$

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iii Exhaustive **analytical results**.

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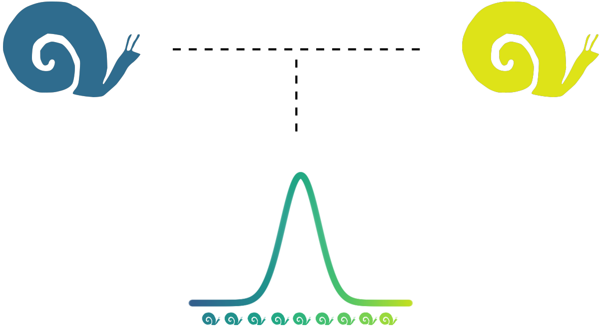
The infinitesimal model (Fisher 1919; Barton, Etheridge, and Véber 2017)

Within family trait distribution:



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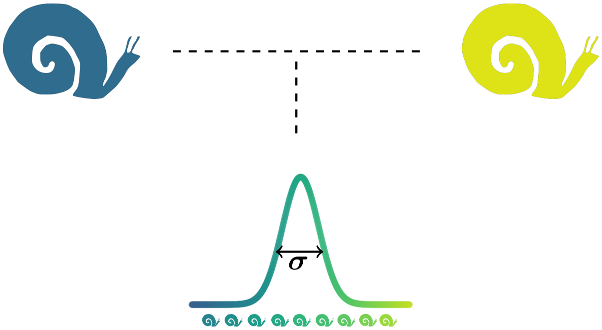

$\boldsymbol{\sigma^2}$: **segregational variance**,
parameter, constant across families

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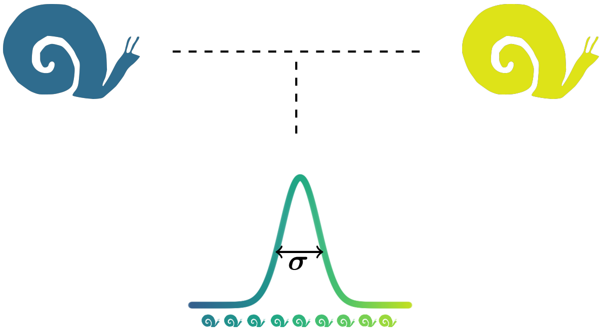

$$\boxed{\mathcal{B}[n](z) = \iint G_{0,\boldsymbol{\sigma}^2}\left(z - \frac{z_1 + z_2}{2}\right) n(z_1) \frac{n(z_2)}{N}\, dz_1\, dz_2}$$

Justify Gaussian approximation for (large) **population** trait distributions:

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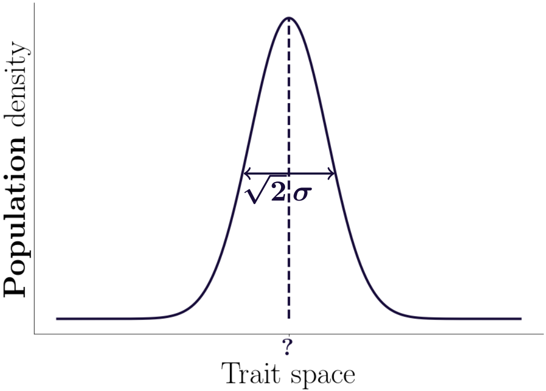
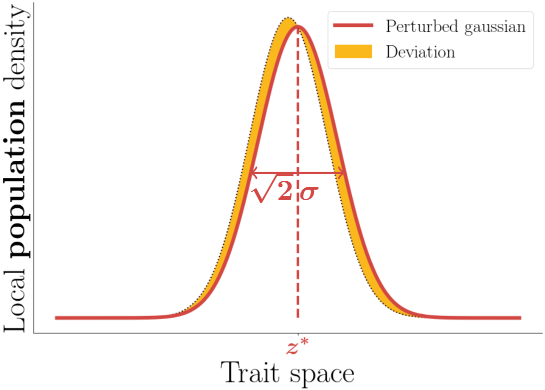
Homogeneous space, no selection:
blending effect of sexual reproduction



Justify Gaussian approximation for (large) **population** trait distributions: $\boldsymbol{\sigma \ll \Delta\theta}$

Patchy space, strong differentiation: only a perturbation, do an expansion[‡]

$\theta_1 \leftarrow$



$\rightarrow \theta_2$

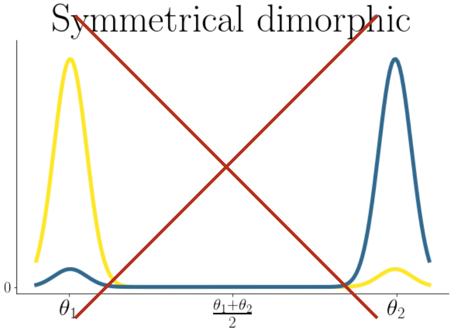
[‡]multiplicative: geometric optics tools adapted for concentration phenomena for quantitative genetics models in Diekmann et al. 2005

Justify Gaussian approximation for (large) **population** trait distributions: $\boldsymbol{\sigma \ll \Delta\theta}$

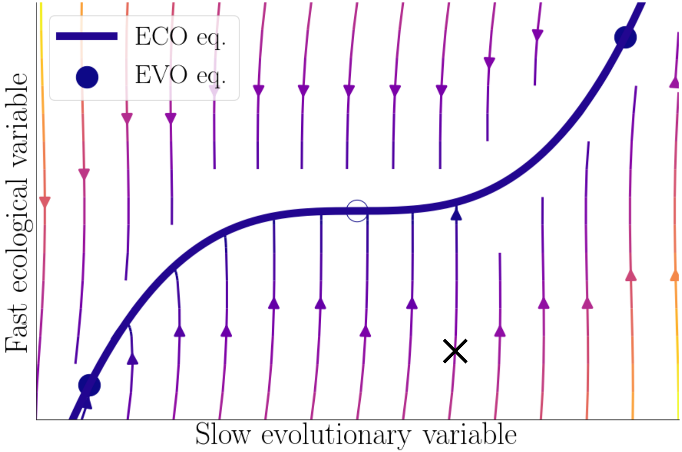
Implication: **monomorphism** in the meta population

$\approx$ Gaussian trait distribution in each patch and **migration**.

NOT GAUSSIAN



Separation of time scales ECO/EVO ($\boldsymbol{\sigma} \ll \boldsymbol{\Delta\theta}$)



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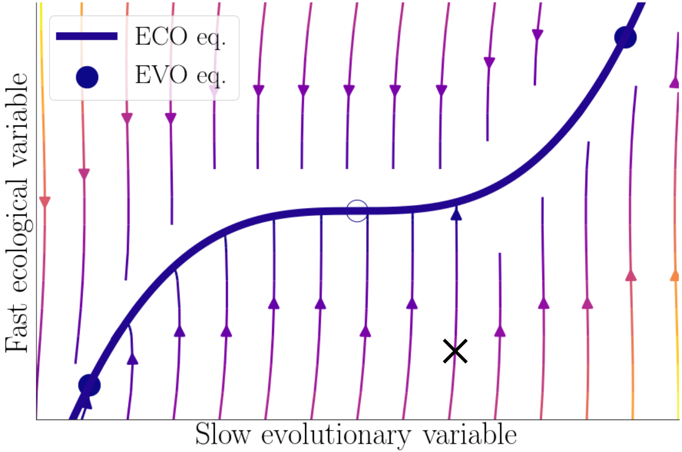
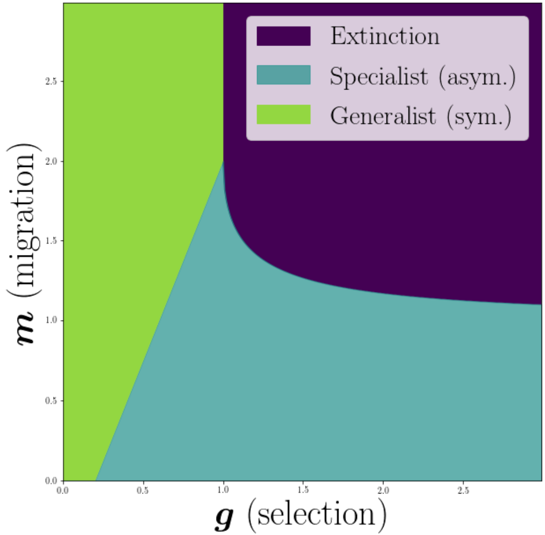


Reduction of complexity

Slow dynamics (ODE) on the line (algebraic equations).

Analytical long term evolutionary outcomes*



*Dekens (2021), https://arxiv.org/pdf/2012.10115.pdf

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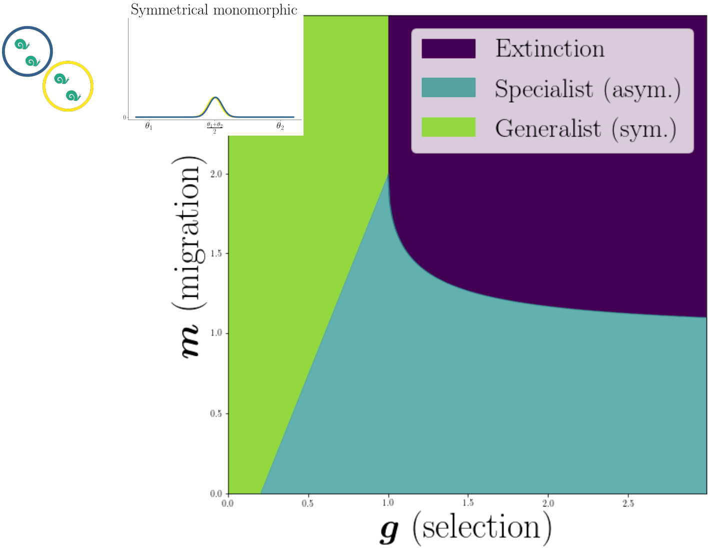

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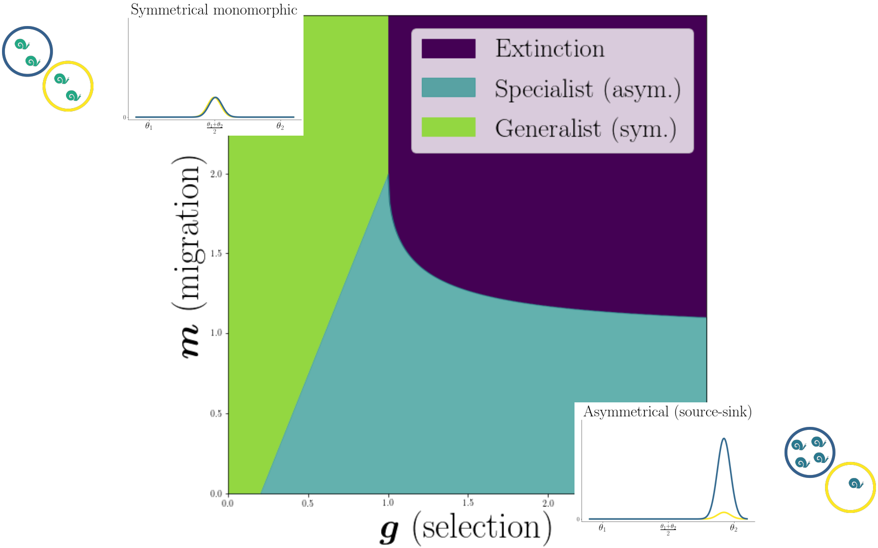

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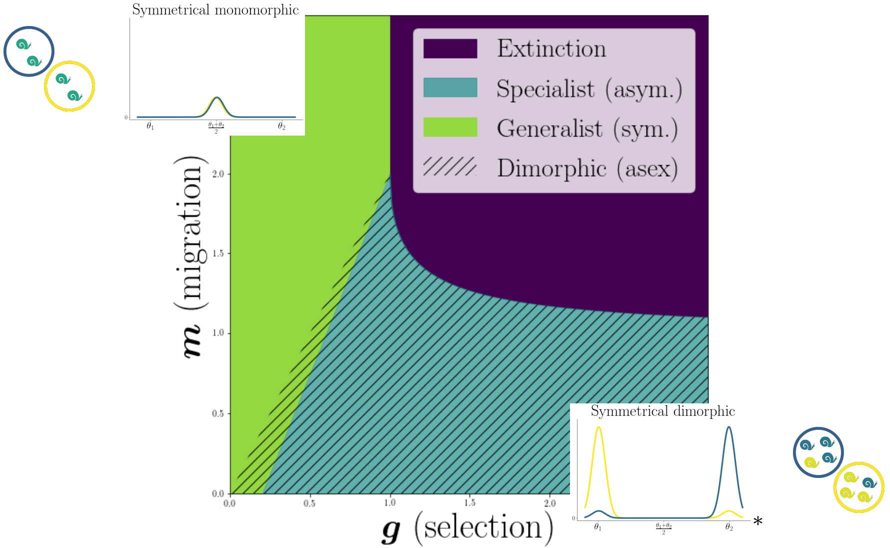

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Conclusion

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- Here, allows to **describe exhaustively the outcomes**. Substituting source-sink scenario to local adaptation by dimorphism due to **combined blending by sexual reproduction (within patches) and by migration (between patches)**.

- Can be used for **more complex genetic architecture** (major locus vs infinitesimal background, forthcoming) or **other biological contexts** (evolution of dispersion and range expansion, ...)

Acknowledgments

(a) Sepideh Mirrahimi†

(b) Vincent Calvez ‡

(c) Sarah Otto §

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† https://www.math.univ-toulouse.fr/ smirrahi/, credits: Vincent Moncorgé

‡ http://vcalvez.perso.math.cnrs.fr/

§ https://biodiversity.ubc.ca/people/faculty/sarah-otto