ETH Zurich
Institut für Theoretische Informatik
Prof. Dr. Angelika Steger

# Algorithms Lab

## Exercise – *Light at the Museum*

Your electrician friend has been appointed as the new head electrician for a prestigious museum. This museum is famous (among electricians) for the nonstandard wiring of its ceiling lights. The lighting system is controlled by a set of switches, but it appears that no switch is really responsible for a single room. Instead, every switch is connected to what seems to be a completely arbitrary set of lights, in different rooms. However, each light is connected to *exactly one* switch.

To make matters even worse, even the lights connected to a single switch are not either all on or all off – instead, some are turned on, some others are turned off, and flicking the switch only serves to reverse this pattern, i.e., all lights that are off turn on and all that are on turn off.

And now the museum is opening a new exhibition! For this, the director of the museum needs each room to have a certain brightness (i.e., a certain exact number of lamps that are turned on). He communicates these values to your friend the electrician, whose job it is to flick the right switches to achieve the desired brightness levels.

However, your friend has no idea if it is even possible to do so. So he has asked you to use your programming skills to help him decide if it is possible. Moreover, if it is possible, your friend would like to know the minimum number of switches that he needs to flick. (Strangely, he doesn't want to know *which* switches he must switch, but the problem seems tricky enough to you as it is, so you prefer not to ask any questions).

Without good lighting the exhibition cannot start as planned. Write a program to help the museum!

**Input** The first line of the input contains an integer $1 \leq t \leq 25$, representing the number of test cases that follow.

Each test case starts with a line containing two integers $N$ and $M$ separated by a space. Here $N$ is the number of switches and $M$ is the number of rooms in the museum. You can assume that $1 \leq N, M \leq 30$.

Next, there is a line encoding the target brightnesses for each room. This line contains integers $b_1, \ldots, b_M$, separated by spaces, where $b_i$ is the number of lights that the director wants to be switched on in room $i$.

This is followed by $N$ lines, where each line encodes the lights connected to a switch, as follows: for each switch there is a line containing $2M$ numbers $\text{on}_1, \text{off}_1, \text{on}_2, \text{off}_2, \ldots, \text{on}_M, \text{off}_M$, separated by spaces. Here, $\text{on}_i$ and $\text{off}_i$ denotes the number of lamps in room $i$ that belong to the circuit and that are initially switched on and off, respectively. You can assume that $0 \leq \text{on}_i, \text{off}_i \leq 100$ holds for all $i$.

**Output** For each testcase, you should output a line containing the least number of switch-flicking operations such that after executing them, each room $i$ contains exactly $b_i$ lights that are switched on. If there is no such number, then you should instead output a line containing the string `impossible`.

**Points** There are three test sets, worth $100$ points in total.

1. For the first test set, worth $20$ points, you may assume that $M, N \leq 15$.
2. For the second test set, worth $50$ points, you may assume that $M = 1$.
3. For the third test set, worth $30$ points, there are no additional assumptions.

Corresponding sample test sets are contained in `test`$i$`.in/out`, for $i \in \{1, 2, 3\}$.

**Sample Input**

```
2
3 3
1 1 1
1 0 0 0 0 1
0 0 1 0 0 1
1 0 0 1 0 0
2 3
1 0 1
0 1 1 0 0 0
1 0 0 0 1 0
```

**Sample Output**

```
1
impossible
```