Auctions II - Combinatorial Auctions

Branislav Bošanský, Michal Jakob, Dominik Seitz, Michal Šustr

Czech Technical University in Prague

branislav.bosansky@agents.fel.cvut.cz
jakob@agents.fel.cvut.cz
dominik.seitz@aic.fel.cvut.cz
michal.sustr@aic.fel.cvut.cz

December 1, 2020

Single Item vs. Multi-Item Mechanisms

Single-item one-sided auctions:

- One item, one valuation
- In the multi item case, we face combination of possibly different goods
- Different bidders might value some combinations differently than others

Combinatorial Auctions

Solution concepts in Single Item Auctions are straightforward: Clear valuations of auctioned items by each bidders, allocate items such that revenue/social welfare is maximized. In the combinatorial setting we face multiple challenges:

- Multiple goods are auctioned simultaneously
- Each bid may claim any combination of goods

- Maximizing revenue becomes much harder
- Necessity of expressing bids for any possible combination of items.

An example of a combination: a bundle of the shape ("\$100 for TV + DVD") More complex combinations are possible

Complementarity

- Complementary goods have a superadditive utility function: $V(a, b) > V(a) + V(b)$
- In the extreme, $V(a, b) >> 0$ but $V(a) = V(b) = 0$

Substitutability

- Goods have a subadditive utility function:
- $V(a, b) < V(a) + V(b)$
- In the extreme, $V(a, b) = MAX[V(a), V(b)]$

Complementarity vs Substitutability: Task 1

Come up with examples for both scenarios.

Complementarity vs Substitutability: Task 1

Come up with examples for both scenarios.

- Complementarity: Imagine a couple of searching for a flight. Having seats next to eachother is worth more than just having one of them.
- Substitutability: Imagine one person bidding for flight tickets to the same destination. Having two tickets to the same destination from different companies is useless.

Expressing bids: Outline of Bidding languages

- Atomic
- OR
- XOR
- Examples

Bidding languages: Atomic

- One-shot bid which can express AND
- TV+DVD:$150
- Disadvantages: Cannot express any other valuations

Bidding languages: OR

- Can express any combination
- {TV,DVD}:150 OR {TV,Blueray}:160
- Disadvantages: No substitutability

Bidding languages: OR (continued)

Example bid: ({a}, 3) OR ({b, c}, 4) OR ({c, d}, 4) implies
Which valuations can this bid convey?
– A value of 3 for {a}
– A value of 4 for {b, c, d}
– A value of 7 for {a, b, c}
How about the valuation function $v(a, b) = v(a) = v(b) = 1$ using the OR bidding language? The OR language is able to express complementarity, it is however bad for expressing substitutability

Bidding languages: XOR

- Can express any combination
- {TV,DVD}:150 XOR {TV,Blueray}:160
- Disadvantages: Bids exponential in the number of items

XOR (continued)

If we use XOR instead of OR, that means that only one of the bundle-value pairs can be accepted. Now we can express any valuation function (simply XOR together all bundles).

Example from before:

({a}, 3) OR ({b, c}, 4) OR ({c, d}, 4) now implies

– A value of 3 for {a}
– A value of 4 for {b, c, d}
– A value of 4 for {a, b, c}

Combining OR and XOR

We can also combine ORs and XORs to get benefits of both.
Example bid: (({a}, 3) XOR ({b, c}, 4)) OR ({c, d}, 4) implies:
– A value of 4 for {a, b, c}
– A value of 4 for {b, c, d}
– A value of 7 for {a, c, d}

Converting XOR to OR with Dummies

Example bid: ({a}, 3) XOR ({b, c}, 4)
Can be converted to exclusively using OR by:
({a, dummy1 }, 3) OR ({b, c, dummy1 }, 4)

Bidding languages: Examples

- left-sock ? right-sock:10
- blue-shirt:8 ? red-shirt:7
- stamp-A:6 ? stamp-B:8

Task: OR vs XOR

OR → Union of distincts is sum of Bids
→ Union of some common → Max
→ Not in Bid → 0

Consider the following bids made using the OR- bidding language:
{a,b}:7 OR {d,e}:8 OR {a,c}:4
How can the following valuations be expressed using OR?
{a}=? 0, {a, b}=? 7, {a, c}=? 4, {a, b, c}=? 7, {a, b, d, e}=? 15
Now consider XOR. How do the valuations change?
{a}=?, {a, b}=?, {a, c}=?, {a, b, c}=?, {a, b, d, e}=?
0 7 4 7 8

Inefficiency of sequential auctions

Suppose an auction offers a PC and a monitor. Your valuation is 200 for the PC, 100 for the monitor, but 500 for both. Now, say there is a sequential auction, first for the PC and then the monitor. If you bid 200, you may lose to a bidder who bids 250, only to find out that you could have won for 200. In contrast, if you bid anything higher, you may pay more than 200, only to find out that sells for 1000.

Incentive Incompatibility

$V^* = v_2(x) + v_3(y)$ $V^*_{-1} = V^*$ $V^*_{-3} = v_1(x,y)$

$V^*_{-2} = v_1(x,y)$

In naive auction settings, bidders are not incentivised to bid their true valuations.

Bidder 1	Bidder 2	Bidder 3
$v_1(x,y) = 100$	$v_2(x) = 75$	$v_3(y) = 40$
$v_1(x) = v_1(y) = 0$	$v_2(x,y) = v_2(y) = 0$	$v_3(x,y) = v_3(x) = 0$

This example from the lecture makes it easy to show that the auction is not incentive compatible. If agents 1 and 2 bid truthfully, agent 3 is better off declaring $v3(y) = 26$.
There is a mechanism which ensures this.

$P_3 = 40 - (115 - 100) = 25$

$P_2 = 75 - (115 - 100) = 60$

Combinatorial Auctions: Solution concepts: Vickrey-Clarke-Groves Mechanism (VCG)

The VCG payments are calculated as follows

- Let $V^\star$ be the total value of the optimal allocation, and for each bidder i let $V^\star_{-i}$ be the total value of the optimal allocation when i does not participate.
- Let V_i be the value of i's winning subset. Losing bidders pay zero and the payment made by each winning bidder is $p_i = Vi - (V^\star - V^\star_{-i})$.
- The payment calculations are not trivial, the winner determination problem must be solved anew for each payment calculation.

VCG application

Think back to this example:

Bidder 1	Bidder 2	Bidder 3
$v_1(x,y) = 100$	$v_2(x) = 75$	$v_3(y) = 40$
$v_1(x) = v_1(y) = 0$	$v_2(x,y) = v_2(y) = 0$	$v_3(x,y) = v_3(x) = 0$

Apply VCG to it and calculate winners and payoffs.

Example VCG solution

VCG would award x to 2 and y to 3. Bidder 2 would pay 60; without him in the auction bidder 1 would have gotten both goods, gaining 100 in value, while with bidder 2 in the auction the other bids only net a total value of 40 (from good x assigned to 3). Similarly, bidder 3 would pay 25; the difference between 100 and 75.

VCG Example Task

$V^* = 16 \quad V^*_{-1} = 12 \quad V^*_{-2} = 15$

$P_1 = 10 - (16 - 12) = 6$

$P_2 = 6 - (16 - 15) = 5$

Apply VCG to the following example:

	Item 1	Item 2	Item 1 and 2
Bidder 1	$\boxed{10}$	5	15
Bidder 2	1	$\boxed{6}$	12

Table: Combinatorial Auction Example

Revenue = 11

- How does the optimal allocation look like?
- How do the payments look like?

VCG Example Task

$V^* = 20 \quad V^*_{-1} = 12 \quad V^*_{-2} = 15$

$\left.\begin{array}{l} P_1 = 10 - (20 - 12) = 2 \\ P_2 = 10 - (20 - 15) = 5 \end{array}\right\} R = 7$

Apply VCG to the same example with switched numbers: How does the revenue change?

	Item 1	Item 2	Item 1 and 2
Bidder 1	10	5	15
Bidder 2	1	10	12

Table: Combinatorial Auction Example

- How does the optimal allocation look like?
- How do the payments look like?

VCG Task Solution

The following is an example of the VCG with 2 bidders and two items.

	item 1	item 2	item 1 & 2
Bidder 1	10	5	15
Bidder 2	1	6	12

Clearly the optimal allocation has a value of 16, assigning item 1 to bidder 1 and item 2 to bidder 2. The optimal solution without bidder 1 is 12 and the optimal solution without bidder 2 is 15. We thus have:

$$V^* = 16 \;\Bigg|\; \begin{array}{c} V^*_{-1} = 12 \\ V^*_{-2} = 15 \end{array} \;\Bigg|\; \begin{array}{c} V_1 = 10 \\ V_2 = 6 \end{array}$$

The payments by bidder 1 and 2 are:

$$p_1 = V_1 - (V^* - V^*_{-1}) = 10 - (16 - 12) = 6\,,$$
$$p_2 = V_2 - (V^* - V^*_{-2}) = 6 - (16 - 15) = 5$$

29

Example Task Solution (continued)

Switching bidder 2's bid from 6 to 10 changes the optimal allocation to a total value of 20. The new payments will be 10 (10 + 10 12) = 2 for bidder 1, and for bidder 2, 10 (10 + 10 15) = 5. The result of bidder 2 submitting a higher bid illustrates the **non-monotonicity problem**, the new revenue, 7, is lower compared to before.

	1	2	{1,2}
1	10	0	0
2	0	10	0

$V^* = 20 \quad V^*_{-1} = 10 \quad V^*_{-2} = 10$

$$\left.\begin{array}{l} P_1 = 0 = 10 - (20 - 10) \\ P_2 = 0 = \end{array}\right\} R = 0$$

VCG shortcomings

A bidder who declares his valuation truthfully has two main reasons to worry—one is that the seller will examine his bid before the auction clears and submit a fake bid just below, thus increasing the amount that the agent would have shill bid to pay if he wins. (This is a so-called shill bid.) Another possibility is both his competitors and the seller will learn his true valuation and will be able to exploit this information in a future transaction. Indeed, these two reasons are often cited as reasons why VCG auctions are rarely seen in practice. Other issues include the fact that VCG is vulnerable to collusion among bidders, and, conversely, to one bidder pseudonymous masquerading as several different ones (so-called pseudonymous bidding or falsebidding name bidding)