Week 10

TIME VALUE OF MONEY

Economic and management principles

Review of the Last Lecture

IN THE LAST LECTURE WE ANALYZED FINANCIAL STATEMENTS OF A COMPANY

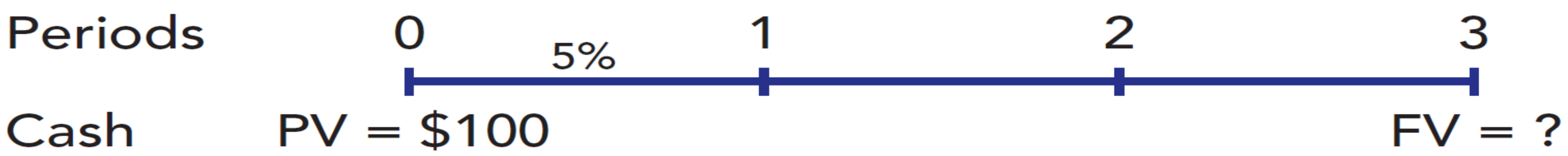
- We described the key financial statements and showed how they change as a firm's operations undergo change.
- We defined main ratios that allow us to compare performance of companies across a sector and over time.
- We explained that such information is important for
  - managers to improve performance;
  - lenders to evaluate the likelihood of collecting on loans;
  - stockholders to forecast earnings, dividends, and stock prices.

Time Value Of Money

- Today, we talk about time value of money and interest rates
- Financial statements help us to estimate future cash flows generated by the company.
- The question is, what is the value of these future cash flows at the moment when we evaluate them.
- We will show that this value depends on the length of the period separating the moment of evaluation and the moment of payment and on the way how we see the future.
- Time value analysis has many applications, including planning for retirement, valuing stocks and bonds, setting up loan payment schedules, and making corporate decisions regarding investing in new plant and equipment.

Present and Future Values

- The first step in time value analysis is to set up a time line of the analyzed problem
- Consider the following diagram, where PV represents $100 that is on hand today and FV is the value that will be in the account on a future date:



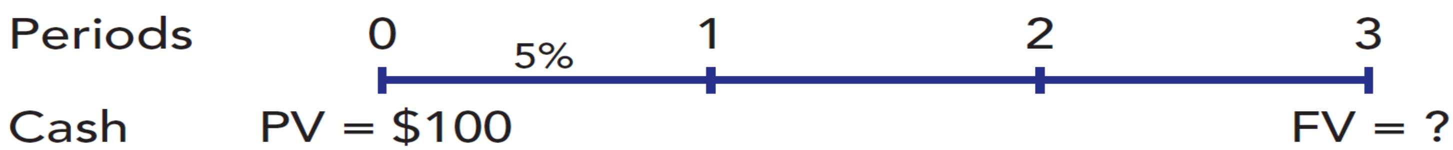
- The intervals from 0 to 1, 1 to 2, and 2 to 3 are time periods such as years or months.
- Cash flows are shown directly below the tick marks, and the relevant interest rate is shown just above the time line.

Present and Future Values

- Money today has greater value than money in future
- A dollar in hand today is worth more than a dollar to be received in the future because, if we had it now, we could invest it, earn interest, and end up with more than a dollar in the future.
- The value of the dollar in hand is called present value (PV).
  - Present value = the value today of a future cash flow or series of cash flows.
- The value of the dollar in future is called future value (FV).
  - Future value = the amount to which a cash flow or series of cash flows will grow over a given period of time when compounded at a given interest rate.
- The process of going to future values from present values is called compounding.

Present and Future Values

- Future value is derived from present value by the process of compounding
- The time line used to find the FV of \$100 compounded for three years at 5 percent:



- We multiply the initial amount, and each succeeding amount, by $(1 + i) = (1 + 0.05)$.
- Hence, we get

$$FV = (1 + i)^3 \cdot PV = (1 + 0.05)^3 \cdot 100 = 115.76$$

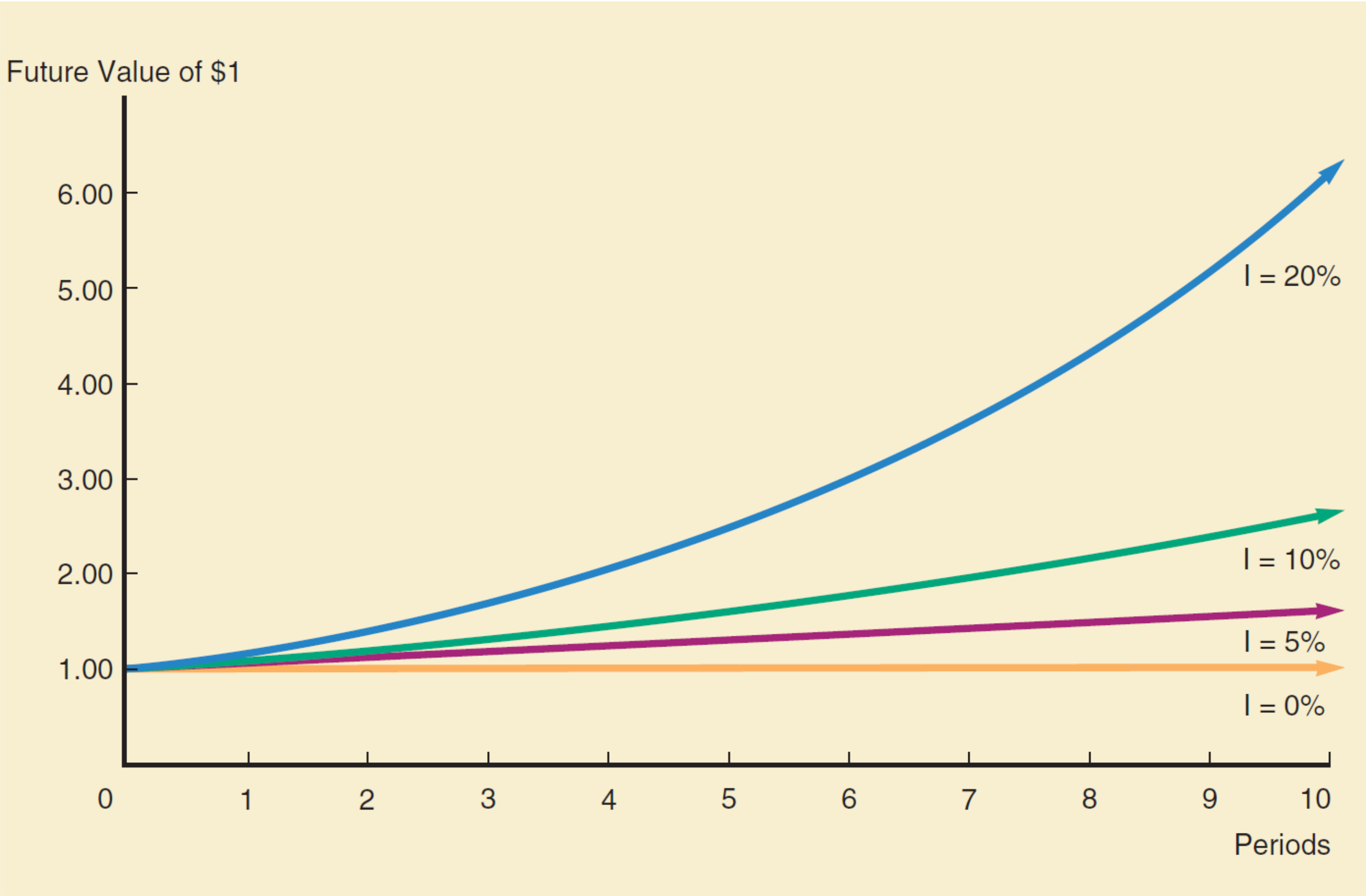
Present and Future Values

- Future value is derived from present value by the process of compounding
- In general, when compounding with the interest rate i over T periods, we get the following relationship between present value and future value:

$$FV = (1 + i)^T \cdot PV$$

- Such formula is not difficult to evaluate, we can also use financial calculators to find it.
- Future value depends on the interest rate and on the number of periods

Example of Future Values Based on Interest Rate

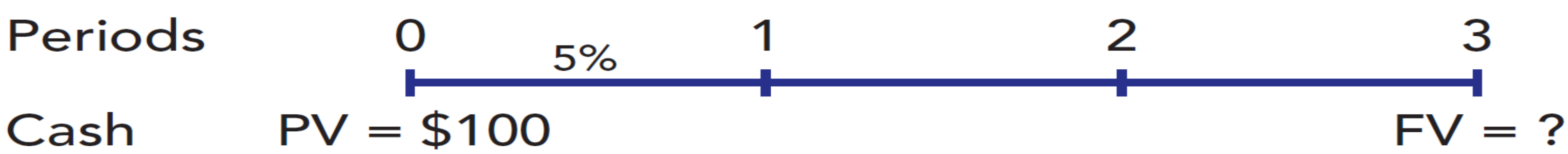


Present and Future Values

- Present value is derived from future value by the process of discounting.
- Finding a present value is the reverse of finding a future value.
- To evaluate future cash flows, we need to find their present value.
- The interest rate that we use for this purpose is defined in the sense of an opportunity cost (typically the interest that we could get if instead of investing money, we just saved it in a bank).
  - Opportunity cost = the rate of return we could earn on an alternative investment of similar risk.
- We find the present value through the process of discounting, which a reverse of compounding.

Present and Future Values

- Suppose a broker offers to sell you a Treasury bond that three years from now will pay $115.76.
- Banks are currently offering a guaranteed 5 percent interest on three-year certificates of deposit (CDs), and if you don't buy the bond you will buy a CD - the 5 percent rate paid on the CDs is defined as your opportunity cost.
- Recall from the future value example in the last section that if you invested $100 at 5 percent it would grow to $115.76 in three years.



Present and Future Values

- You would also have \$115.76 after three years if you bought the T-bond. Therefore, the most you should pay for the bond is \$100this is its "fair price".
- This price can be also found using the formula

$$p = PV = FV/(1 + i)^3 = 115.76/(1 + 0.05)^3 = 100 .$$

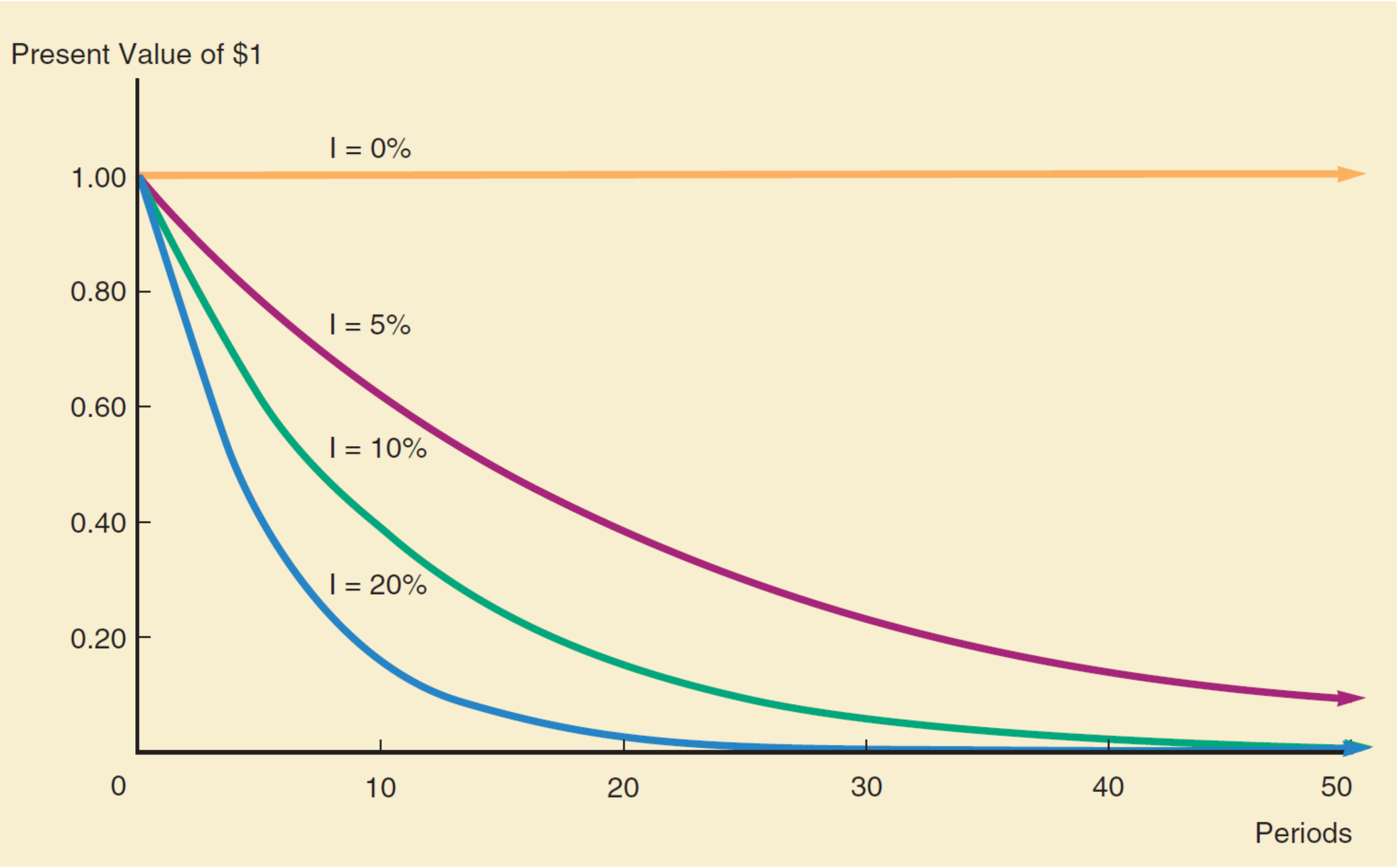
- In general, when discounting with the interest rate i over T periods, we get the following relationship between present value and future value:

$$PV = FV/(1 + i)^T .$$

Present and Future Values

- The present value lies at the heart of any valuation process
- The fundamental goal of financial management is to maximize the firms value, and the value of a business (or any asset, including stocks and bonds) is the present value of its expected future cash flows.
- The present value of a sum to be received in the future decreases and approaches zero as the payment date is extended further and further into the future and also that the present value falls faster the higher the interest rate.
- At relatively high rates, funds due in the future are worth very little today, and even at relatively low rates present values of sums due in the very distant future are quite small.

Example of Present Values Based on Interest Rate



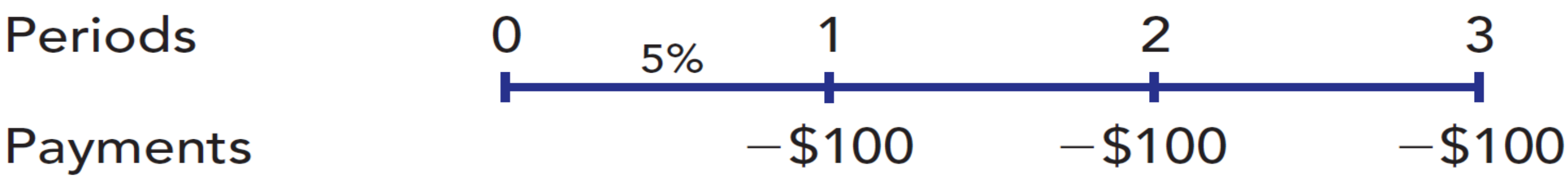
Annuities

- Payments that are constant and repeated over a fixed time period are called annuities
- Thus far we have dealt with single payments, or “lump sums”.
- However, many assets provide a series of cash inflows over time, and many obligations like auto, student, and mortgage loans require a series of payments.
- If the payments are equal and are made at fixed intervals for a specified number of period, then the series is an annuity.
- If the payments occur at the end of each year, then we have an ordinary (or deferred) annuity.
- If the payments are made at the beginning of each year, then we have an annuity due (less common).

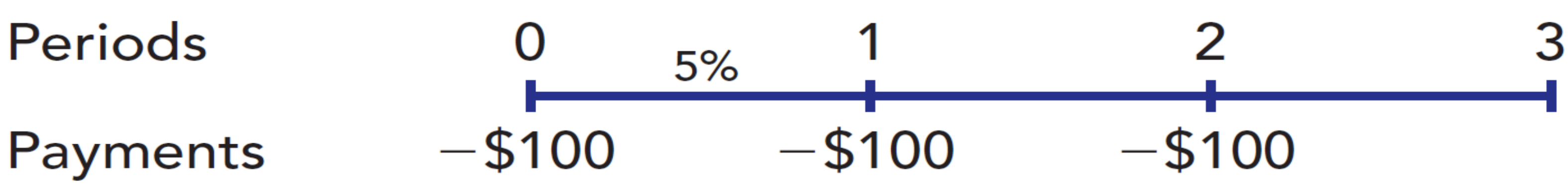
Annuities

- Payments that are constant and repeated over a fixed time period are called annuities

Ordinary Annuity:



Annuity Due:



Annuities

- Future value of an annuity is computed as some of future values of payments
- Future value of the depicted ordinary annuity can be found as

$$FV = 100 \times (1+0.5)^2 + 100 \times (1+0.5) + 100 \times (1+0.5)^0 = 315.25 .$$

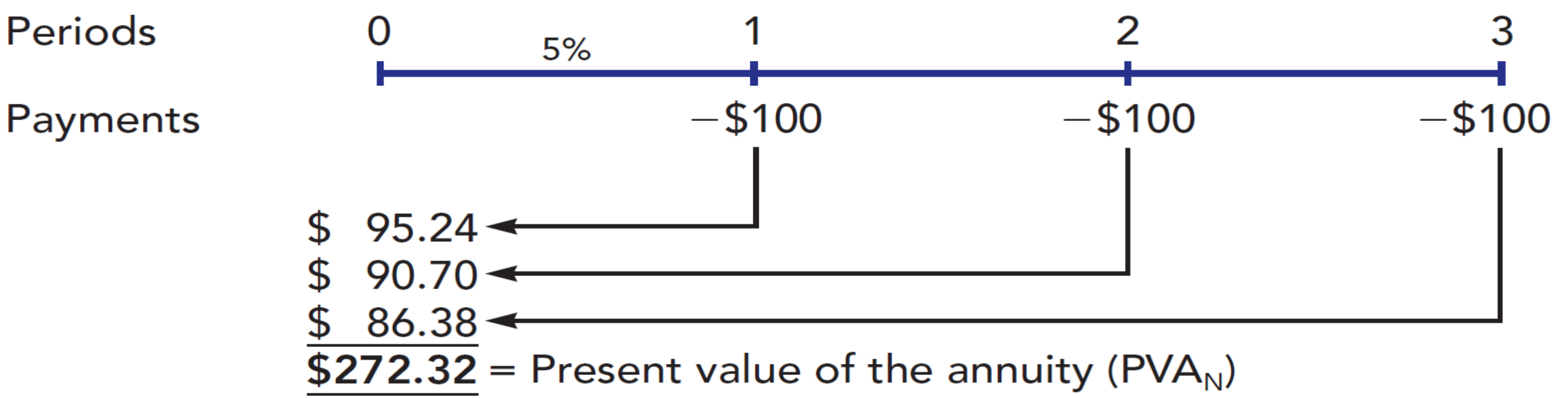
- Future value of a general ordinary annuity with fixed payment amount (PMT), interest rate i and T periods can be found as

$$FV = \sum_{t=1}^{T} PMT \cdot (1+i)^{T-t} = PMT \cdot \frac{((1+i))^T - 1}{i} .$$

Annuities

- Present value of an annuity is computed as some of present values of payments
- Present value of the depicted ordinary annuity can be found by summing discounted payment amounts:

$PV = 100/(1 + 0.05)^1 + 100/(1 + 0.05)^2 + 100/(1 + 0.5)^3 = 272.32$.



Annuities

- Present value of a general ordinary annuity with fixed payment amount (PMT), interest rate i and T periods can be found as

$$PV = \sum_{t=1}^{T} \frac{PMT}{(1+i)^t} = PMT \cdot \frac{1 - \frac{1}{(1+i)^T}}{i} \quad .$$

- These formulas are easy to evaluate, but we can use also financial calculators to do so.

Perpetuities

- An annuity lasting infinite number of periods is called perpetuity
- Annuities provide a constant stream of payments over a limited time periods.
- There are also securities that promise to make payments forever.
- We call such securities perpetuities.
- Since they are just annuities with extended life, their present value can be simply calculated as:

$$PV = \lim_{T \to +\infty} PMT \cdot \frac{1 - \frac{1}{(1+i)^T}}{i} = \frac{PMT}{i} \quad .$$

Uneven Cash Flows

- Most of the analyzed cash flows are not constant over time
- Annuities involve payments that are equal in every period.
- Although many financial decisions do involve constant payments, many others involve non-constant, or uneven, cash flows.
- There are two important classes of uneven cash flows:

  1. a stream that consists of a series of annuity payments plus an additional final lump sum

  2. all other uneven streams.
- Bonds represent the best example of the first type, while stocks and capital investments illustrate the other type.

Uneven Cash Flows

1. Annuity plus additional final payment:

Periods	0	1	2	3	4	5
	I = 12%					
Cash flows	$0	$100	$100	$100	$100	$ 100
						$ 1,000
						$1,100

2. Irregular cash flows:

Periods	0	1	2	3	4	5
	I = 12%					
Cash flows	$0	$100	$300	$300	$300	$500

Uneven Cash Flows

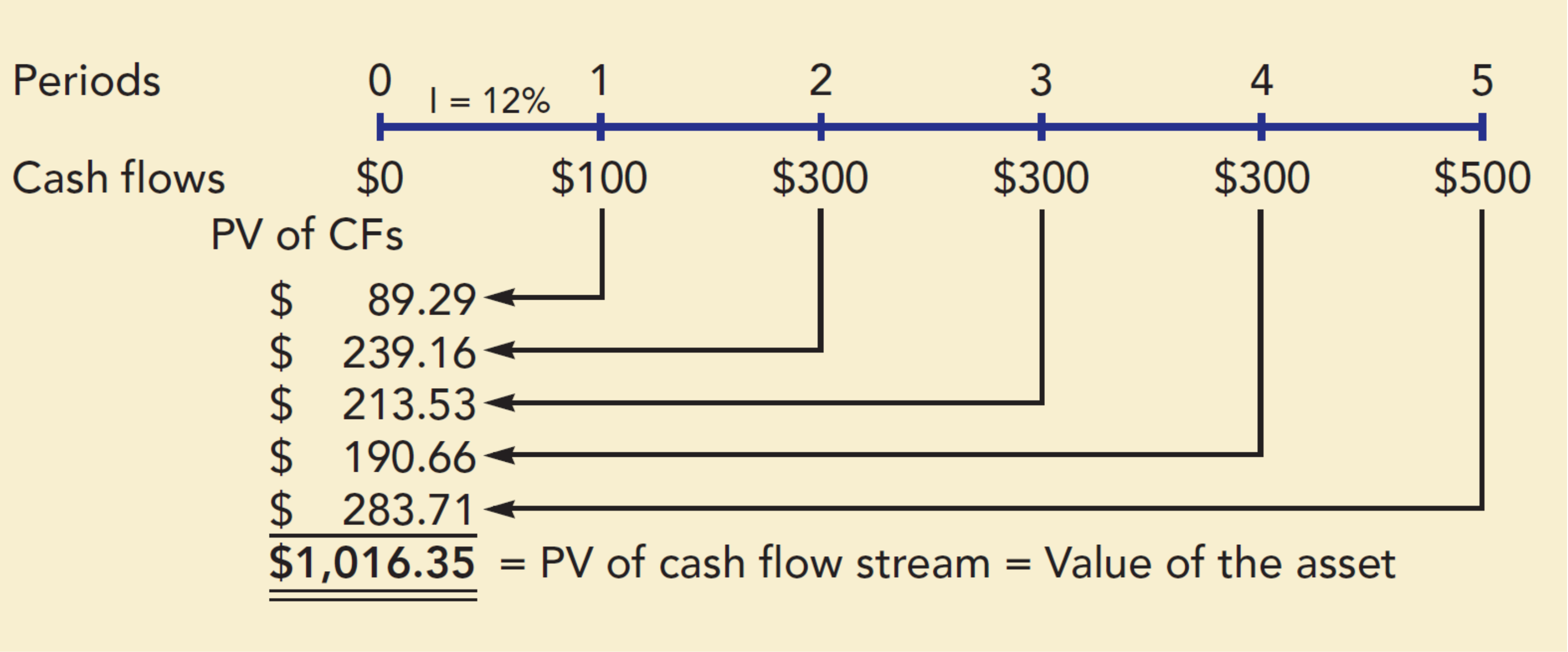
- Present values of uneven CFS are found by discounting, future values by compounding
- We can find the present value of uneven cash flow stream by discounting each cash flow and then summing them to find the PV of the stream:

$$PV = \sum_{t=1}^{T} \frac{CF_t}{(1+i)^t}$$

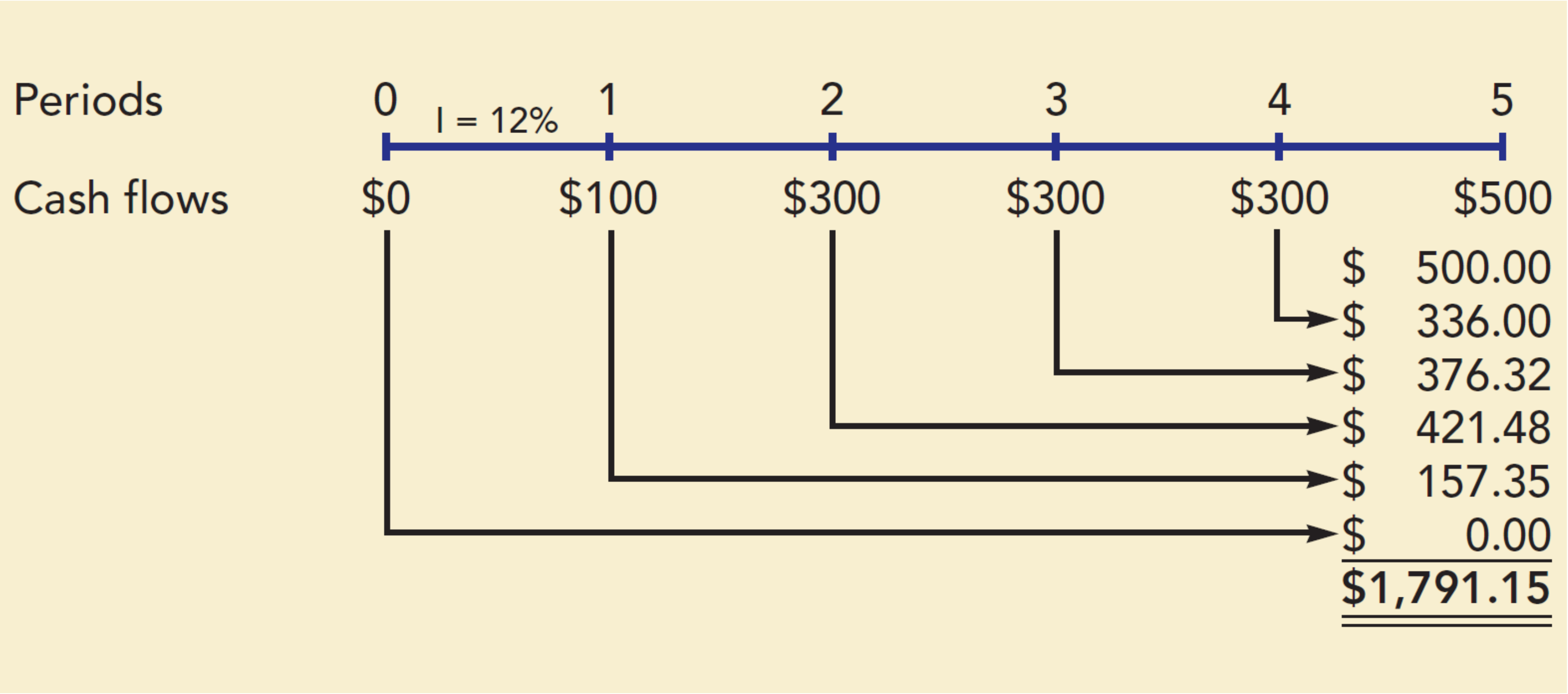
- We can find the future value of uneven cash flow stream by compounding rather than discounting:

$$FV = \sum_{t=1}^{T} CF_t \cdot (1+i)^{T-t}$$

Uneven Cash Flows



Uneven Cash Flows



Comparing interest rates

- Compounding can be performed on other than annual basis
- In all of our examples thus far we assumed that interest is compounded once a year, or annually.
- This is called annual compounding.
- However, we can also compound every six months (semiannual compounding), every month (monthly compounding) etc.
- Banks generally pay interest more than once a year, virtually all bonds pay interest semiannually, and most mortgages, student loans, and auto loans require monthly payments.
- Therefore, it is important to understand how to deal with non-annual compounding.

Comparing Interest Rates

- With other than annual compounding, we have to convert stated annual rate.
- Whenever payments occur more than once a year, we must make two conversions:
  1. convert the stated interest rate into a “periodic rate”,
  2. convert the number of years into “number of periods”.
- The conversions are done as follows, where i is the stated annual rate, M is the number of compounding periods per year, and N is the number of years:

Periodic rate = Stated annual rate/Number of payments per year =i/M

Number of periods = (Number of years)· (Periods per year) = N·M

Comparing Interest Rates

- Assume that we deposit $100 in an account that pays 5 percent and leave it there for 10 years.
- Future value under annual compounding would be:

$$FV = PV \cdot (1 + i)^N = 100 \cdot (1 + 0.05)^{10} = 162.89$$

- With a stated annual rate of 5 percent, compounded semiannually, the periodic rate is 5/2=2.5 percent.
- With 10 years and semiannual compounding, there are $10 \cdot 2 = 20$ periods.
- Hence, with semiannual compounding, the future value is:

$$FV = PV \cdot (1 + i_{per})^{N.M} = 100 \cdot (1 + 0.025)^{20} = 163.86$$

Comparing Interest Rates

- The compounding frequency influences the total interest earned on a deposit
- The following table shows the difference in interest with different compounding frequency.

	Interest on \$100 after first year (in \$)	
Compounding frequency	4%	18%
Annual	4.00	18.00
Semiannual	4.04	18.81
Quarterly	4.06	19.25
Daily	4.08	19.72

Comparing Interest Rates

- Different compounding periods are used for different types of investments
- Bank accounts often pay interest daily.
- Most bonds pay interest semiannually.
- Stocks pay dividends quarterly.
- Mortgages, auto loans, and other instruments require monthly payments.
- If we are to compare investments or loans with different compounding periods properly, we need to put them on a common basis.

Comparing Interest Rates

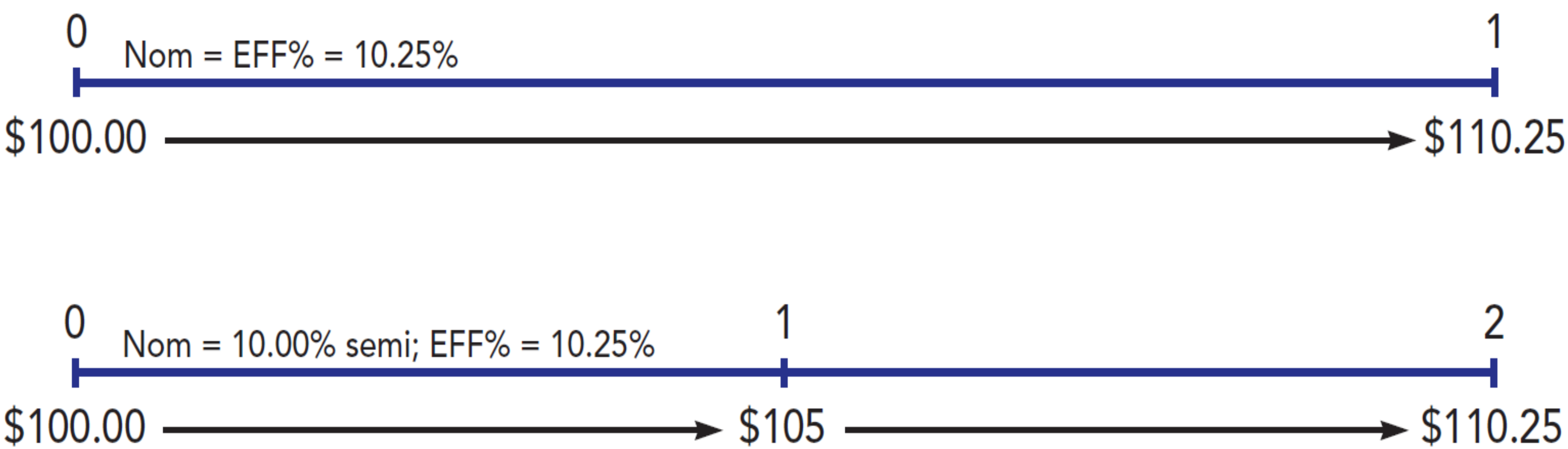
- When comparing two financial products, we have to consider rather effective than nominal rate.
- The nominal rate, also called the annual percentage rate, or stated, or quoted rate, is the rate that financial companies tell you they are charging on loans or paying on deposits.
- If two banks offer loans with a same stated rate but one requires monthly payments and the other quarterly payments, then they are not charging the same “true” rate (see table above).
- The bank that requires monthly payments is really charging more than the one with quarterly payments.

Comparing Interest Rates

- The effective annual rate, also called the equivalent annual rate is the rate that would produce the same future value under annual compounding as would more frequent compounding at a given nominal rate.
- If a loan or investment uses annual compounding, then its nominal rate is also its effective rate.
- However, if compounding occurs more than once a year, the effective annual rate is higher than the nominal rate.

Comparing Interest Rates

- To illustrate, a nominal rate of 10 percent, with semiannual compounding, is equivalent to a rate of 10.25 percent with annual compounding because both of those rates will cause $100 to grow to the same amount after one year:



Comparing Interest Rates

- We can find the effective annual rate, given the nominal rate and the number of compounding periods per year, with this equation:

$$\text{Effective annual rate} = \left(1 + \frac{i_{nom}}{M}\right)^{M} - 1 \ ,$$

where i_{nom} is the nominal rate and M is the number of compounding periods per year.

- For nominal rate 10 percent from the previous example with semiannual compounding, we have

$$\text{Effective annual rate} = \left(1 + \frac{10}{2}\right)^{2} - 1 = 0.1025 = 10.25\% \ .$$

Amortized Loans

- Amortized loans are paid off in equal installments over time
- An important application of compound interest involves loans that are paid off in installments over time.
- Included are automobile loans, home mortgage loans, student loans, and many business loans.
- A loan that is to be repaid in equal amounts on a monthly, quarterly, or annual basis is called an amortized loan.

Amortized Loans

- A homeowner borrows \$100,000 on a mortgage loan with 6 percent interest, and the loan is to be repaid in five equal payments at the end of each of the next five years:

- The task is to determine the payment the homeowner must make each year.
- The payments must be such that the sum of their present values equals \$100 000:

$$100000 = \sum_{t=1}^{5} \frac{PMT}{(1+0.06)^t},$$

which gives PMT= \$ 23 739,64.

Amortized Loans

- Amortized loans can be summarized in amortization schedule tables
- The amortization schedule is a table showing precisely how a loan will be repaid.
- It gives the required payment on each payment date and a breakdown of the payment, showing how much is interest and how much is repayment of principal.

Amount borrowed: $100,000
Years: 5
Rate: 6%
PMT: −$23,739.64

Year	Beginning Amount (1)	Payment (2)	Interest[a] (3)	Repayment of Principal[b] (4)	Ending Balance (5)
1	$100,000.00	$23,739.64	$6,000.00	$17,739.64	$82,260.36
2	82,260.36	23,739.64	4,935.62	18,804.02	63,456.34
3	63,456.34	23,739.64	3,807.38	19,932.26	43,524.08
4	43,524.08	23,739.64	2,611.44	21,128.20	22,395.89
5	22,395.89	23,739.64	1,343.75	22,395.89	0.00

Summary

- In this lecture, we explained why value of money differs in time
- We should understand the concept of future and present value
- We should be able to apply compounding and discounting.
- We should know how to deal with different interest rates.

- In the next lecture, we will talk about investment decisions.