Petri Nets

Stefan Ratschan

Katedra číslicového návrhu
Fakulta informačních technologií
České vysoké učení technické v Praze

Evropský sociální fond Praha & EU: Investujeme do vaší budoucnosti

Motivation

Example: Network protocol: merges two types of packets into one:

Two queues

Modeling as finite transition system?

States: $(0,0)$, $(0,1)$, $(1,0)$, $(0,1)$, $(0,2)$, $(1,1)$, $(2,0)$, $(0,3)$, $(1,2)$, $\ldots$

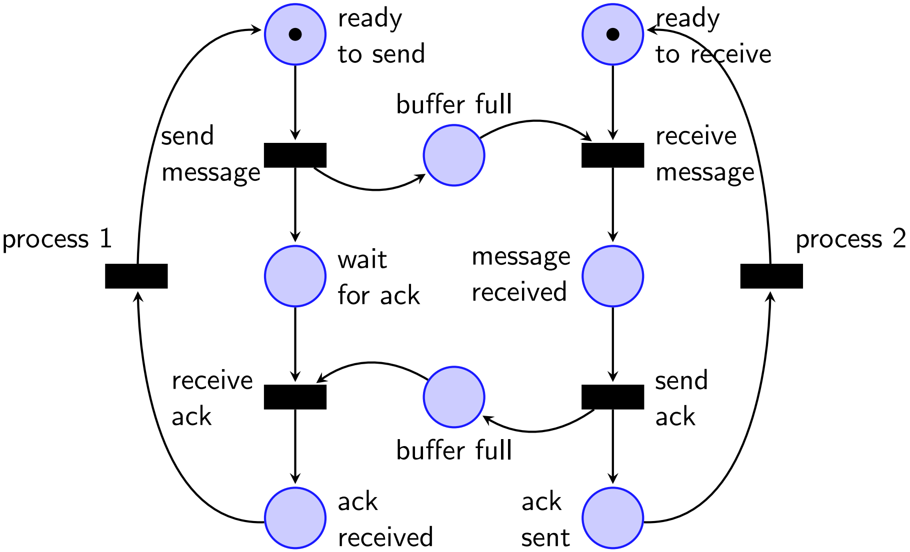
If unbounded, not possible

If bounded but big bound, tedious

Petri net

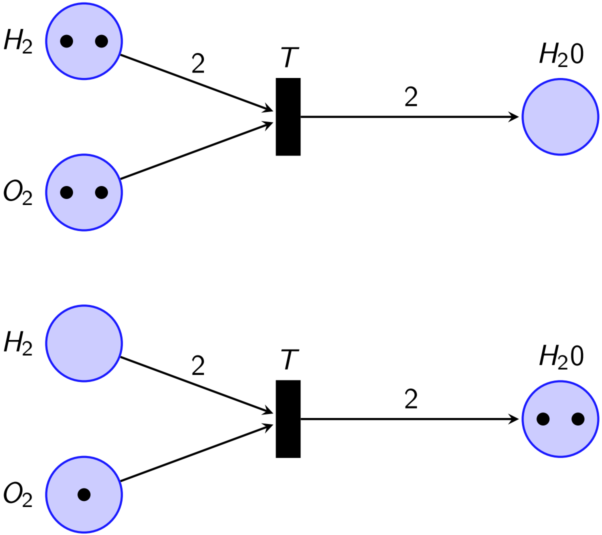
Examples of Applications

Protocol [Murata, 1989, Fig. 9]:



Examples of Applications

Chemical reaction: $2H_2 + O_2 \rightarrow 2H_2O$ [Murata, 1989, Fig. 1]



Examples of Applications

PIPE demo: File/Examples/Accident & Emergency Unit

Useful in modeling many other phenomena (not only in technical technical applications, for example, also organizational)

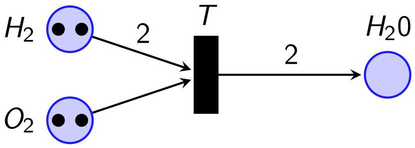
Input Places	Transition	Output Places
Preconditions	Event	Results
Input data	Process	Output Data
Input signals	Signal processor	Output signals
Conditions	Deduction step	Conclusions
Buffers	Processor	Buffers

Formal Definition

A *Petri net* is a 5-tuple (P, T, F, w, M_0) where

- P is a finite set whose elements we call *places*,
- T is a finite set whose elements we call *transitions*,
- $F \subseteq (P \times T) \cup (T \times P)$ whose elements we call *arcs*,
- $w : F \to \mathbb{N}$ called *weight function*,
- $M_0 : P \to \mathbb{N}_0$ called the *initial marking*, s.t.

$P \cap T = \emptyset$.



In the literature on can find a few different, but equivalent definitions.

Attention: transition of Petri net $\neq$ transition of transition system!

Petri Net Behavior

A *marking* (*state*) of a Petri net is a function $s : P \to \mathbb{N}_0$

Here, for a state s, and a place p,
$s(p) = n$ formalizes the intuition that p contains n tokens

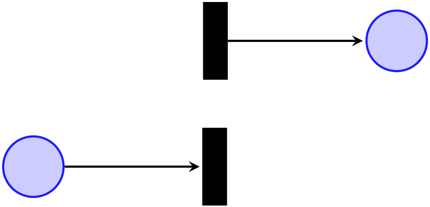
The state evolves, starting from the initial state,
according to the following transition (or firing) rules:

- We call a transition t *enabled*, if each input place p of t is marked with at least $w(p, t)$ tokens
- A transition t can *fire* iff it is enabled, firing
  removes $w(p, t)$ tokens from each input place p of t, and
  adds $w(t, p)$ tokens to each output place p of t.
- Ordering of firings is non-deterministic, that is, in case of more enabled transitions, arbitrary one is fired

firing of Petri net transition $\approx$ transition of transition system

Source and Sink Transitions

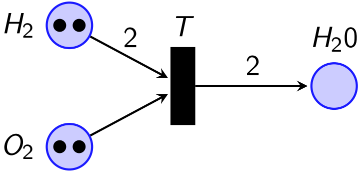
transition without inputs/outputs



Properties of Petri nets?

Reachability

Given a Petri net (P, T, F, w, M_0), e.g.,



A marking M' is *reachable from* a marking M iff
there exists a sequence of firings from M to M'.

A marking M is *reachable* iff it is reachable from the initial marking M_0.

For modeling reachability of more states
the literature on Petri nets does not use sets, but partial functions:

A *partial marking* is a partial function from $P \to \mathbb{N}_0$ (e.g., $\{H_20 \mapsto 2\}$)

A partial marking M is *reachable* iff
there exists a reachable marking M' that
coincides with M on its defined elements

Boundedness

A Petri net is *k-bounded* iff the number of tokens in each place
does not exceed k for any reachable marking

A Petri net is *bounded* iff
there is k s.t. the net is k-bounded.

`Ex8`, `Ex3`

Why is this useful?

Encoding to/from finite transition systems

Buffer overflows

Liveness

A transition is *live* iff from every reachable marking
we can reach a firing of this transition.

A Petri net is *live* iff every transition is live.

see also notion of "deadlock"

`Ex4`, `Ex1`

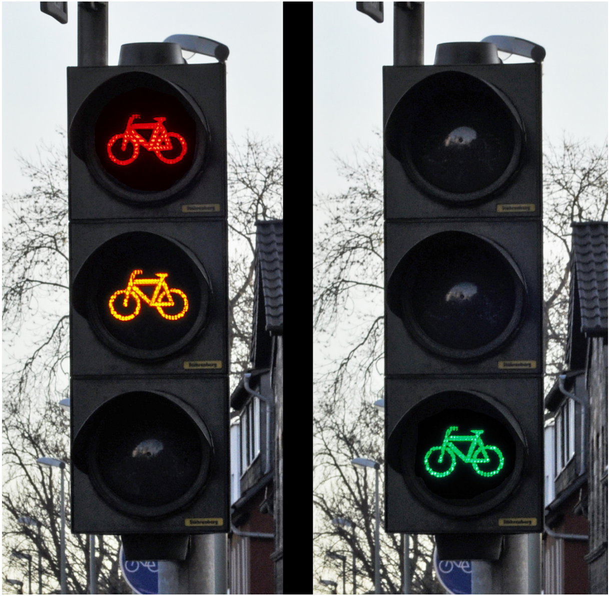
Automatic Analysis of Petri Net Properties

Problem: analyzing all reachable states is difficult

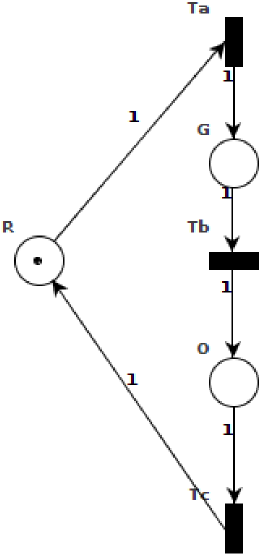
Structural properties:

only depend on Petri net structure, not on initial state

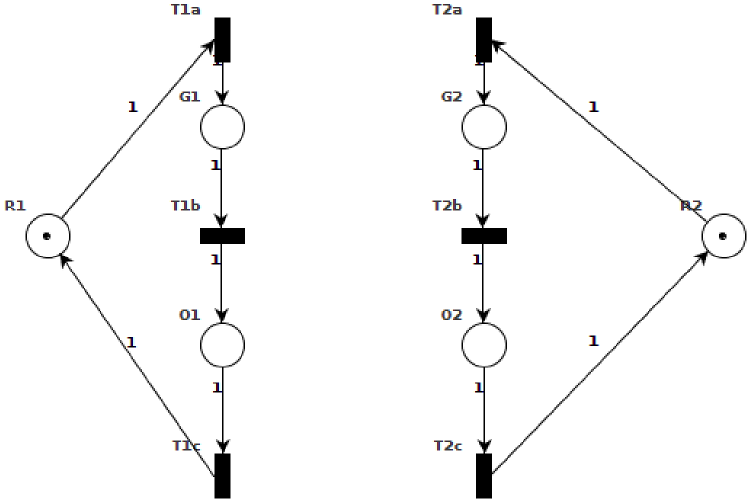
Example: Traffic Light



Example: Traffic Light



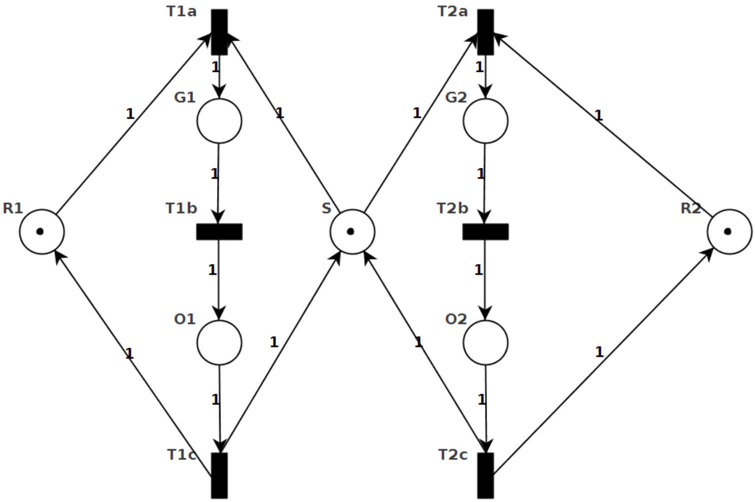
Junction



Result? Crash. How to avoid two green lights?

Additional place: switching to green allowed

Example: Junction



T-Invariants

Assumption: arbitrary initial marking

Which transitions lead any state back to itself?

`trafficlight2_TI.swf` (every traffic light has cycle of three firings)

- $\{T1a \mapsto 1, T1b \mapsto 1, T1c \mapsto 1, T2a \mapsto 0, T2b \mapsto 0, T2c \mapsto 0\}$
- $\{T1a \mapsto 0, T1b \mapsto 0, T1c \mapsto 0, T2a \mapsto 1, T2b \mapsto 1, T2c \mapsto 1\}$

T-invariant:
Function $f : T \to \mathbb{N}_0$ s.t.
for every sequence of firings from a marking M to a marking M' that
fires every transition $t \in T$ exactly $f(t)$ times,
$M = M'$.

Every linear combination of two T-invariants is again a T-invariant

trivial T-invariant

T-Invariants: Usage

- liveness verification (no deadlock)
- reversability (ability to return to the initial state M_0)

Further examples: `http://www.informatik.uni-hamburg.de/TGI/PetriNets/introductions/aalst`

computation of invariants: later (we will need more theory)

P-Invariants

What can we say about the numbers of tokens under arbitrary firings??

`trafficlight2_PI.swf`: $f : (\{R1, G1, O1, R2, G2, O2\} \to \mathbb{N}_0) \to \mathbb{Z}$

in both traffic lights constant number of tokens:

$f(M) = M(R1) + M(G1) + M(O1)$,
$f(M) = M(R2) + M(G2) + M(O2)$

tokens in coord. place S always added/removed with one red light:

$f(M) = M(R1) + M(R2) - M(S)$

P-invariant:

Function $f : (P \to \mathbb{N}_0) \to \mathbb{Z}$ s.t.
for every marking M and M' s.t.
M' is the result of a firing from M,
$f(M) = f(M')$.

Every linear combination of two P-invariants is again a P-invariant

trivial P-invariant

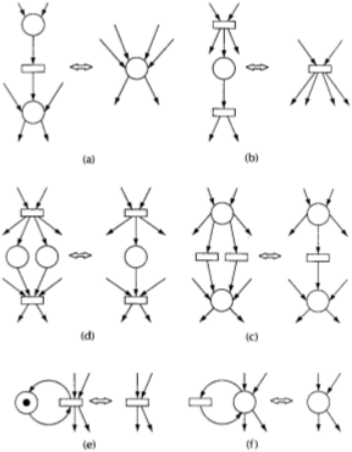
P-Invariants: Usage

- Liveness verification:
  Invariant can ensure that there is always an enabled transition.
- Boundedness verification (no token is lost or created).

Petri Net Transformations

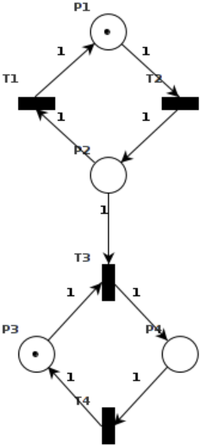
Certain simplifications preserve certain properties of Petri nets.

For example: [Murata, 1989, Fig. 22] preserve liveness, 1-boundedness, boundedness.



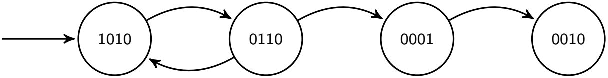
Translation to Transition System

Example:



Net is 1-bounded, so

we have a finite number of reachable states, that can be visualized together with transitions



Ex4:

Translation to Transition System: Formalization

States: markings, i.e., $S \subseteq \{P_1, \ldots, P_4\} \to \mathbb{N}_0$

We will represent them as vectors from $\mathbb{N}_0^4$

Set of initial states: $\{(1,0,1,0)^T\}$

Transitions? For a given Petri net transition, this

- transition has to be enabled, and
- firing changes the number of tokens.

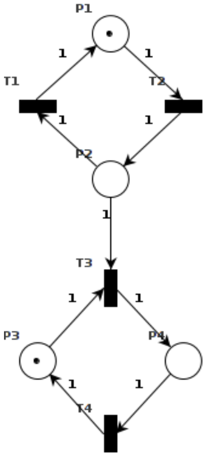
For example, transition T_1: if $s \geq (0,1,0,0)^T$, change

- $\Delta_1^- = (0,1,0,0)^T$
- $\Delta_1^+ = (1,0,0,0)^T$

Together: $\Delta_1 = \Delta_1^+ - \Delta_1^- = (1,-1,0,0)^T$

Corresponding transition of transition system:

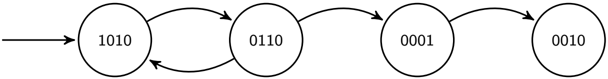
$$s' = s + (1,-1,0,0)^T, \text{ if } s \geq (0,1,0,0)^T$$

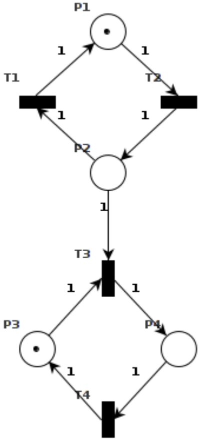


Translation to Transition System: Result

$(\mathbb{N}_0^4, \{(1,0,1,0)^T\}, T)$ where

$$T = \begin{array}{l} \{(s, s+(1,-1,0,0)^T \in \mathbb{N}_0^4 \times \mathbb{N}_0^4 \mid s \geq (0,1,0,0)^T\} \cup \\ \{(s, s+(-1,1,0,0)^T \in \mathbb{N}_0^4 \times \mathbb{N}_0^4 \mid s \geq (1,0,0,0)^T\} \cup \\ \{(s, s+(0,-1,-1,1)^T \in \mathbb{N}_0^4 \times \mathbb{N}_0^4 \mid s \geq (0,1,1,0)^T\} \cup \\ \{(s, s+(0,0,1,-1)^T \in \mathbb{N}_0^4 \times \mathbb{N}_0^4 \mid s \geq (0,0,0,1)^T\} \end{array}$$





Translation to (Infinite) Transition System: Definition

Given a Petri net $(\{P_1, \ldots, P_n\}, \{T_1, \ldots, T_m\}, F, w, M_0)$:

Set of states: $S = \mathbb{N}_0^n$

Set of initial states: $S_0 = \{(M_0(P_1), \ldots, M_0(P_n))^T\}$

Transition relation: For every $j \in \{1, \ldots, m\}$,

- let $\Delta_j^- := (\delta_1, \ldots, \delta_n)$, where
  $$\delta_i = \begin{cases} w(P_i, T_j), \text{ if}(P_i, T_j) \in F, and \\ 0, \text{ otherwise} \end{cases}$$
- let $\Delta_j^+ := (\delta_1, \ldots, \delta_n)$, where
  $$\delta_j = \begin{cases} w(T_j, P_i), \text{ if}(T_j, P_i) \in F, \text{ and} \\ 0, \text{ otherwise.} \end{cases}$$
- let $\Delta_j := \Delta_j^+ - \Delta_j^-$

Then: transition relation

$$\{(s, s + \Delta_j) \in S \times S \mid s \geq \Delta_j^-, j \in \{1, \ldots, m\}\}$$

Also works for unbounded Petri nets!

Finite Case

In the case of a k-bounded Petri net we can restrict ourselves to
a finite set of states $P \rightarrow \{0, \ldots, k\}$,
that we represent as vectors $\{0, \ldots k\}^{|P|}$.

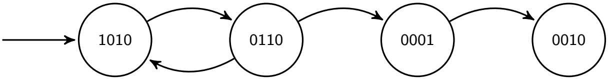
$$
\begin{aligned}
&\left\{(s, s + (1, -1, 0, 0)^T \in \{0,1\}^4 \times \{0,1\}^4 \mid s \geq (0,1,0,0)^T\right\} \cup \\
&\left\{(s, s + (-1, 1, 0, 0)^T \in \{0,1\}^4 \times \{0,1\}^4 \mid s \geq (1,0,0,0)^T\right\} \cup \\
&\left\{(s, s + (0, -1, -1, 1)^T \in \{0,1\}^4 \times \{0,1\}^4 \mid s \geq (0,1,1,0)^T\right\} \cup \\
&\left\{(s, s + (0, 0, 1, -1)^T \in \{0,1\}^4 \times \{0,1\}^4 \mid s \geq (0,0,0,1)^T\right\}
\end{aligned}
$$

Explicitely:

$$
\left\{
\begin{array}{c}
((0,1,0,0)^T, (1,0,0,0)^T), \\
((0,1,0,1)^T, (1,0,0,1)^T), \\
((0,1,1,0)^T, (1,0,1,0)^T), \\
((0,1,1,1)^T, (1,0,1,1)^T), \\
((1,0,0,0)^T, (0,1,0,0)^T), \\
((1,0,0,1)^T, (0,1,0,1)^T), \\
((1,0,1,0)^T, (0,1,1,0)^T), \\
((1,0,1,1)^T, (0,1,1,1)^T), \\
((0,1,1,0)^T, (0,0,0,1)^T), \\
((1,1,1,0)^T, (1,0,0,1)^T), \\
((0,0,0,1)^T, (0,0,1,0)^T), \\
((0,1,0,1)^T, (0,1,1,0)^T), \\
((1,0,0,1)^T, (1,0,1,0)^T), \\
((1,1,0,1)^T, (1,1,1,0)^T)
\end{array}
\right\}
$$

Transition Relation

Comparison with earlier figure:



$$\left\{\begin{array}{l}
((0,1,0,0)^T,(1,0,0,0)^T),\\
((0,1,0,1)^T,(1,0,0,1)^T),\\
\color{red}{((0,1,1,0)^T,(1,0,1,0)^T)},\\
((0,1,1,1)^T,(1,0,1,1)^T),\\
((1,0,0,0)^T,(0,1,0,0)^T),\\
((1,0,0,1)^T,(0,1,0,1)^T),\\
\color{red}{((1,0,1,0)^T,(0,1,1,0)^T)},\\
((1,0,1,1)^T,(0,1,1,1)^T),\\
\color{red}{((0,1,1,0)^T,(0,0,0,1)^T)},\\
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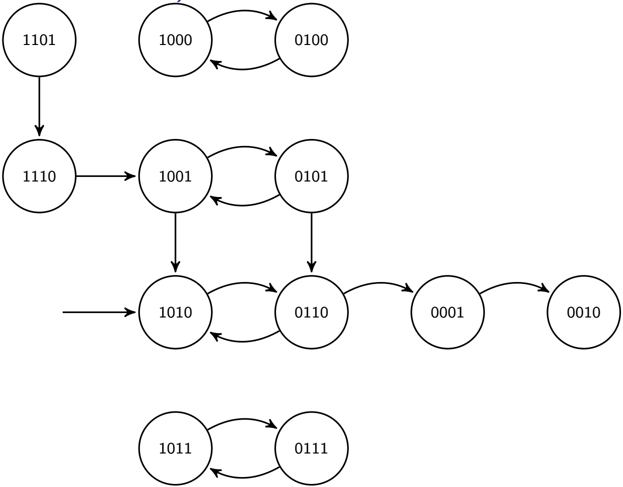
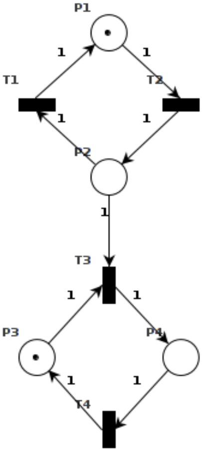
\color{red}{((0,0,0,1)^T,(0,0,1,0)^T)},\\
((0,1,0,1)^T,(0,1,1,0)^T),\\
((1,0,0,1)^T,(1,0,1,0)^T),\\
((1,1,0,1)^T,(1,1,1,0)^T)
\end{array}\right\}$$

Where do the additional states and transitions come frome?

Comparison with Original Petri Net (Only States With Transitions)

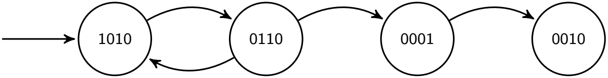


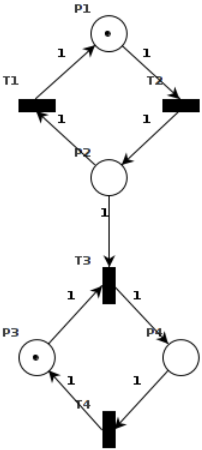


unreachable part corresponds to unreachable behavior of Petri net

In state space $\mathbb{N}_0^n$ even infinitely many transitions.

Comparison of Properties



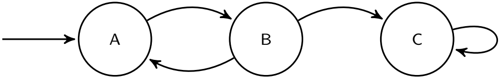


not left-total $\leftrightarrow$ state with no enabled transition

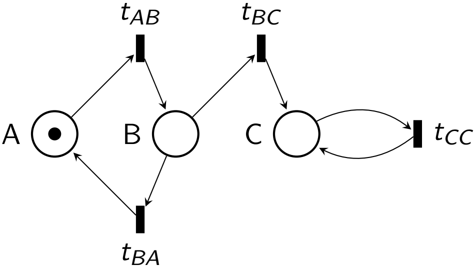
non-deterministic transition system $\leftrightarrow$ several enabled transitions

Translation from (Finite) Transition System: Example

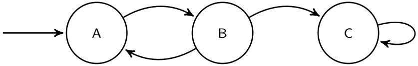
For the transition system

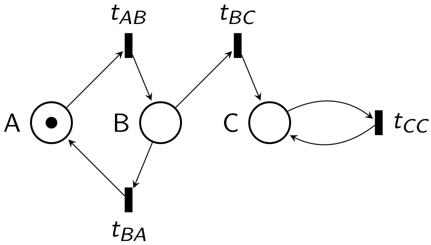


corresponding Petri net?



Translation from (Finite) Transition System: In General





Given a transition system $(S, \{s_0\}, R)$

Translation to *Petri net* (P, T, F, w, M_0), where

- Places P: S
- Transitions T: $\{t_r \mid r \in R\}$
- Arcs F: $\{(s, t_{(s,s')}) \mid (s, s') \in R\} \cup \{(t_{(s,s')}, s') \mid (s, s') \in R\}$
- Weight function w: $w(x, y) = 1$
- M_0: $M_0(s_0) = 1$, $M_0(p) = 0$, if $p \neq s_0$.

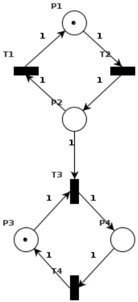
T-Invariant Computation

Function $f : T \to \mathbb{N}_0$ s.t.

for every sequence of firings from a marking M to a marking M' that fires every transition $t \in T$ exactly $f(t)$ times, $M = M'$.

Invariant with unknown number of firings $x_1, \ldots, x_4$ (ansatz):

$$\{T_1 \mapsto x_1, \ldots, T_4 \mapsto x_4\}$$



representation as transition system.

result of firing that many times from a state $s \in \mathbb{N}_0^4$,

$$s + \begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \end{pmatrix} x_1 + \begin{pmatrix} -1 \\ 1 \\ 0 \\ 0 \end{pmatrix} x_2 + \begin{pmatrix} 0 \\ -1 \\ -1 \\ 1 \end{pmatrix} x_3 + \begin{pmatrix} 0 \\ 0 \\ 1 \\ -1 \end{pmatrix} x_4$$

In general:

$$s + \Delta_1 x_1 + \cdots + \Delta_m x_m$$

T-Invariant Computation

For every state s,

state after x_1 firings of T_1, $\ldots$, x_4 firings of T_4 must be equal to s.

$$s + \begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \end{pmatrix} x_1 + \begin{pmatrix} -1 \\ 1 \\ 0 \\ 0 \end{pmatrix} x_2 + \begin{pmatrix} 0 \\ -1 \\ -1 \\ 1 \end{pmatrix} x_3 + \begin{pmatrix} 0 \\ 0 \\ 1 \\ -1 \end{pmatrix} x_4 = s$$

hence the expression

$$\begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \end{pmatrix} x_1 + \begin{pmatrix} -1 \\ 1 \\ 0 \\ 0 \end{pmatrix} x_2 + \begin{pmatrix} 0 \\ -1 \\ -1 \\ 1 \end{pmatrix} x_3 + \begin{pmatrix} 0 \\ 0 \\ 1 \\ -1 \end{pmatrix} x_4$$

must be zero.

T-Invariant Computation

$$\begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \end{pmatrix} x_1 + \begin{pmatrix} -1 \\ 1 \\ 0 \\ 0 \end{pmatrix} x_2 + \begin{pmatrix} 0 \\ -1 \\ -1 \\ 1 \end{pmatrix} x_3 + \begin{pmatrix} 0 \\ 0 \\ 1 \\ -1 \end{pmatrix} x_4 = 0$$

Linear system of equations $\Delta_1 x_1 + \cdots + \Delta_m x_m = \vec{0}$ (homogeneous)

Each solution $(x_1, \ldots, x_m)^T \in \mathbb{N}_0^m$ represents one T-invariant

In matrix form:

$$\begin{pmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & -1 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = 0$$

Incidence matrix: $A := (\Delta_1, \ldots, \Delta_m)$

P-Invariant Computation

Definition translated to corresponding transition system:

Function $f : \mathbb{N}_0^n \to \mathbb{Z}$ s.t. for every marking M and M' s.t.

M' is the result of a firing from M,

$f(M) = f(M')$.

Assumption: f linear with coefficients $y = (y_1, \dots, y_4)$, i.e.,

for a state $s = (s_1, \dots, s_4)^T$ of the corresponding transition system

$f(s_1, \dots, s_4) = y_1 s_1 + \dots + y_4 s_4$, that is

$f(s) = ys$

e.g.,

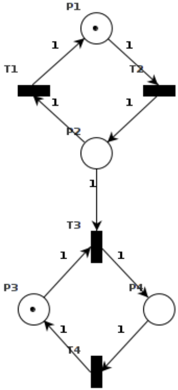
- $y = (1, 0, 0, 0)$: $f(s) = (1, 0, 0, 0) \begin{pmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{pmatrix} = s_1$:

  number of tokens at place M_1

- $y = (1, 1, 1, 1)$: $f(s) = (1, 1, 1, 1) \begin{pmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{pmatrix} = s_1 + s_2 + s_3 + s_4$:

  number of all tokens in net

P-Invariant Computation



Function $f : \mathbb{N}_0^n \to \mathbb{Z}$ s.t.
for every state s and s' s.t.
s' is the result of a transition from s,
$f(s) = f(s')$, where $f(s) = ys$

e.g., for the transition T_3:
$f(s) = f(s + \Delta_3)$, where $\Delta_3 = (0, -1, -1, 1)^T$
$ys = y(s + \Delta_3)$, which holds if $y\Delta_3 = 0$, i.e.,

$$(y_1, \ldots, y_4) \begin{pmatrix} 0 \\ -1 \\ -1 \\ 1 \end{pmatrix} = 0 \text{ i.e., } -y_2 - y_3 + y_4 = 0$$

Example solution: $y_2 = 1, y_3 = 1, y_4 = 2$, i.e.,
$s_2 + s_3 + 2s_4$ does not change for transition T_3 .

This should hold for every transition, i.e., for all $i \in \{1, \ldots, 4\}$:
$y(s + \Delta_i) = ys$

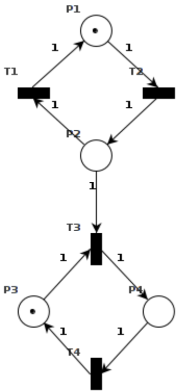
$y\Delta_1 = 0, \ldots, y\Delta_4 = 0$, again: homogeneous linear system of equations

$$(y_1, \ldots, y_4) \begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \end{pmatrix} = 0 \text{ i.e., } y_1 - y_2 = 0$$

$$(y_1, \ldots, y_4) \begin{pmatrix} -1 \\ 1 \\ 0 \\ 0 \end{pmatrix} = 0 \text{ i.e., } -y_1 + y_2 = 0$$

$$(y_1, \ldots, y_4) \begin{pmatrix} 0 \\ -1 \\ -1 \\ 1 \end{pmatrix} = 0 \text{ i.e., } -y_2 - y_3 + y_4 = 0$$

$$(y_1, \ldots, y_4) \begin{pmatrix} 0 \\ 0 \\ 1 \\ -1 \end{pmatrix} = 0 \text{ i.e., } y_3 - y_4 = 0$$



In matrix form:

$$\begin{pmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & -1 & 1 \\ 0 & 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix} = 0 \text{ (transposed incidence matrix)}$$

Summary: Invariant Computation

T-invariants: $\Delta_1 x_1 + \cdots + \Delta_m x_m = \vec{0}$ ($\Delta_1, \ldots, \Delta_m$: columns)

P-invariants: $y\Delta_1 = 0, \ldots, y\Delta_m = 0$ ($\Delta_1, \ldots, \Delta_m$: rows)

More compact notation:

Incidence matrix: $A := (\Delta_1, \ldots, \Delta_m)$

Then:

- Every solution $(x_1, \ldots, x_m)$ of $Ax = \vec{0}$ in the natural numbers represents
  a T-invariant $f : \{T_1, \ldots, T_m\} \to \mathbb{N}_0$ s.t.
  for all $i \in \{1, \ldots, m\}$, $f(T_i) = x_i$.
- Every solution $(y_1, \ldots, y_n)$ of $yA = \vec{0}$, i.e., $A^T y^T = \vec{0}$ represents
  a P-invariant $f : (\{P_1, \ldots, P_n\} \to \mathbb{N}_0) \to \mathbb{Z}$ s.t.
  for all $M : \{P_1, \ldots, P_n\} \to \mathbb{N}_0$, $f(M) = \sum_{i \in \{1,\ldots,n\}} y_i M(P_i)$.

Example: T-Invariant Computation

Let us try Gaussian elimination:

$$\begin{pmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & -1 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & -1 \end{pmatrix}$$

after adding rows 1 and 2, 3 and 4:

$$\begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

after subtracting row 2 from 3:

$$\begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

T-Invariant Computation: Result

$$\begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

which corresponds to the equations

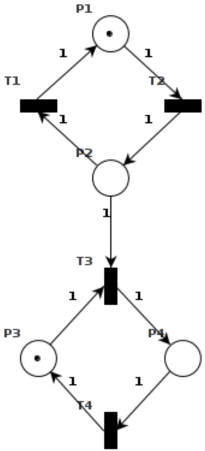
$$x_1 = x_2, x_3 = 0, x_4 = 0$$

set of T-invariants in vector representation:

$$\{(x, x, 0, 0) \mid x \in \mathbb{N}\}$$

As functions:

$$\{f : \{T_1, \ldots, T_4\} \to \mathbb{N}_0 \mid \\ f(T_1) = f(T_2), f(T_3) = 0, f(T_4) = 0\}$$



P-Invariant computation

transposed incidence matrix: $A^T =$

$$\begin{pmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & -1 & 1 \\ 0 & 0 & 1 & -1 \end{pmatrix}$$

add rows 1 and 2, the result is a row with nulls only:

$$\begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & -1 & -1 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

add rows 2 and 3:

$$\begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

P-Invariant Computation: Result

$$\begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

which corresponds to the equations

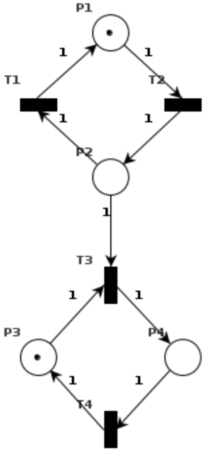
$$y_1 = y_2, y_2 = 0, y_3 = y_4$$

set of P-invariants in vector representation:

$$\{(0, 0, y, y) \mid y \in \mathbb{Z}\}$$

as functions:

$$\{f : (\{P_1, \dots, P_4\} \to \mathbb{N}_0) \to \mathbb{Z} \mid \\ f(M) = yM(P_3) + yM(P_4), y \in \mathbb{Z}\}$$



Solution of Equations

Gaussian elimination usually results in rational solutions.

If the solution is not integer:

From a rational solution

$$\frac{p_1}{q_1}, \dots, \frac{p_r}{q_r}$$

we obtain the integer solution:

$$lcm\{q_1, \dots, q_r\}p_1, \dots, lcm\{q_1, \dots, q_r\}p_r$$

In the case of T-invariants the solution must be natural numbers

There are algorithms for that, we will use our intelligence.

Keywords for software packages: "null space", "kernel", "echelon form"

Conclusion

Petri nets are a further possibility for modeling systems

Usage of Petri nets

- partially depends on the type of problem,
- but partially also on culture (certain people see the world more through Petri net glasses, other people do not use them at all).

This course: based on transition systems and automata

Analoguous models for Petri nets
(timed Petri net, stochastic Petri net etc.)

http://www.informatik.uni-hamburg.de/TGI/PetriNets/

Simulator PIPE2: http://pipe2.sourceforge.net

Literature

T. Murata. Petri nets: Properties, analysis and applications. *Proceedings of the IEEE*, 77(4):541 –580, April 1989. doi: 10.1109/5.24143.