Multiagent Systems (BE4M36MAS)

Introduction to Game Theory

Branislav Bošanský

Artificial Intelligence Center,
Department of Computer Science,
Faculty of Electrical Engineering,
Czech Technical University in Prague

branislav.bosansky@agents.fel.cvut.cz

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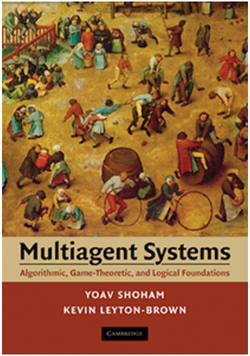
Previously ... on multi-agent systems.

1. Agent architectures
2. Belief Desire Intentions
3. Reactive agents

… and now for game theory.

Source for studying: `www.masfoundations.org`



Game Theory

What is game theory?

Real World Motivation

Game theoretic models are particularly useful for security applications in the physical world as well as in computer networks.

Game Theory

What is the main difference from the previous part of MAS course?

We **explicitly** consider that other agents pursue their goals (other agents are not ignored nor they are part of the environment).

If one agent chooses an action to play, other agent(s) can react accordingly to this choice.

The goals of the agents can be **conflicting**.

Games in Game Theory

What do we need to specify if we want to talk about (almost any) game:

- **Who?** – Which **agents (players)** are participating in the game?
- **What?** – What are the **actions** the agents can choose to play? What is the **outcome** of the game if agents choose their actions? What do the players **know** during the game?

Formal Representation of Games

There are many possible formal representations for games (we will see later). **Normal-form** (or matrix) representation is the most basic one.

Definition (Normal Form Game (NFG))

We call a triplet $\mathcal{G} = (\mathcal{N}, \mathcal{A}, u)$ *a normal-form game*, where

- $\mathcal{N}$ is a finite set of players, we use $n = |\mathcal{N}|$,
- $\mathcal{A}_i$ is a finite set of actions (pure strategies; hence, we also use $\mathcal{S}_i$ in some definitions) for player i,
- u_i is a utility function of player i that assigns the reward for joint action $a \in \mathcal{A}$, $a = (a_1, a_2, \ldots, a_{\mathcal{N}})$ to player i.

We assume that players are **rational** and they only maximize their expected utility value. (there are parts of game theory that deal with imperfectly rational players)

Normal-Form Game Examples

Rock Paper Scissors

	R	**P**	**S**
R	$(0,0)$	$(-1,1)$	$(1,-1)$
P	$(1,-1)$	$(0,0)$	$(-1,1)$
S	$(-1,1)$	$(1,-1)$	$(0,0)$

Prisoners' Dilemma

	C	**D**
C	$(-1,-1)$ NE	$(0,-5)$
D	$(-5,0)$	$(-4,-4)$ NE

Normal-Form Game Examples

Matching Pennies

	H	**T**
H	$(1, -1)$	$(-1, 1)$
T	$(-1, 1)$	$(1, -1)$

Battle of Sexes

	M	**F**
M	$(1, 2)^{NE}$	$(0, 0)$
F	$(0, 0)$	$(2, 1)^{NE}$

Classes of Games

Depending on players, actions, and outcomes, there are many different classes of games:

- Depending on the number of players, we can focus on 2, 3, or n-player games.
- Games can be **one-shot** or **dynamic** (sequential) with **finite** (or **infinite**) **horizon**.
- Games can be with **perfect** or **imperfect information**.
- Games can be **zero-sum** or **general-sum**.
- Games can be discrete or continuous (any of the set of players, actions, set of states can be infinitely large).

Strategies in Games

Choices players make in a game are called **strategies** (they do not necessarily to correspond to only a single action).

- We denote s_i to be a strategy of player $i \in \mathcal{N}$. $\mathcal{S}_i$ is a set of all strategies of player i.
- A set of strategies of all players is called a **strategy profile**

$$s = \langle s_1, s_2, \ldots, s_n \rangle$$

- Often, we need to refer to strategies all other player except player $i \in \mathcal{N}$:

$$s_{-i} = \langle s_1, s_2, \ldots, s_{i-1}, s_{i+1}, \ldots, s_n \rangle$$

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What Strategies Should an Agent Play?

What is a desirable outcome of a game?

- An outcome s, such that there is no other outcome s' where one player would be better of and all other players have at least the utility as in s – **pareto optimal outcome**
- An outcome that maximizes the sum of all players – **social welfare optimization**

	L	**C**	**R**
T	$(1,0)$	$(-1,1)$	$(1,-1)$
M	$(2,2)$	$(0,0)$	$(3,1)$
B	$(-1,1)$	$(1,-1)$	$(0,3)$

What Strategies Should an Agent Play?

Some strategies can be better than others.

	C	**D**
A	$(2, 1)$ x	$(3, 4)$ ✓ → NE
B	$(-1, 0)$ x	$(1, 1)$ x

Which strategy would you recommend to be played?

Strategy **A** yields a better outcome for player 1 than strategy **B** regardless of the action of player 2.

Dominance

Definition (Strong Dominance)

Let $\mathcal{G} = (\mathcal{N}, \mathcal{A}, u)$ be a normal-form game. We say that s_i *strongly dominates* s_i' if $\forall s_{-i} \in \mathcal{S}_{-i}, u_i(s_i, s_{-i}) > u_i(s_i', s_{-i})$.

Definition (Weak Dominance)

Let $\mathcal{G} = (\mathcal{N}, \mathcal{A}, u)$ be a normal-form game. We say that s_i *weakly dominates* s_i' if $\forall s_{-i} \in \mathcal{S}_{-i}, u_i(s_i, s_{-i}) \geq u_i(s_i', s_{-i})$ and $\exists s_{-i} \in \mathcal{S}_{-i}$ such that $u_i(s_i, s_{-i}) > u_i(s_i', s_{-i})$.

Definition (Very Weak Dominance)

Let $\mathcal{G} = (\mathcal{N}, \mathcal{A}, u)$ be a normal-form game. We say that s_i *very weakly dominates* s_i' if $\forall s_{-i} \in \mathcal{S}_{-i}, u_i(s_i, s_{-i}) \geq u_i(s_i', s_{-i})$.

Removal Dominated Strategies

Question

Would a rational agent play a (strongly/weakly/very weakly) dominated strategy?

Rational agent would never choose a strongly dominated strategy, hence we can remove those strategies from the game.

Iterative Removal of Dominated Strategies – a simple algorithm that iteratively removes (strongly) dominated strategies from a game.

Removal Dominated Strategies

Iterative Removal of Dominated Strategies

	L	**C**	**R**
T	$(1,0)$	$(-1,1)$	$(1,-1)$
M	$(2,2)$	$(0,0)$	$(3,1)$
B	$(-1,1)$	$(1,-1)$	$(0,3)$

- **T** is dominated by **M**
- **C** is then dominated by **R** (and not before)
- **B** is then dominated by **M** (and not before)
- **R** is then dominated by **L** (and not before)

What Strategies Should an Agent Play? – Deviations

While players are rational, they may not choose to play the *best outcome* (in pareto or social-welfare sense). Given a strategy of the opponents s_{-i}, if there is a better strategy for player i, he is going to deviate:

	C	**D**
A	$(5,5)$	$(0,6)$
B	$(6,0)$	$(1,1)$

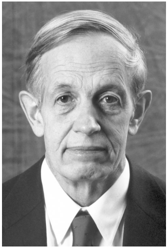
$$(A, C) \to (B, C) \to (B, D)$$

Definition (Best Response)

Let $\mathcal{G} = (\mathcal{N}, \mathcal{A}, u)$ be a normal-form game and let $BR_i(s_{-i}) \subseteq \mathcal{S}_i$ such that $s_i^* \in BR_i(s_{-i})$ iff $\forall s_i \in \mathcal{S}_i, u_i(s_i^*, s_{-i}) \geq u_i(s_i, s_{-i})$.

Nash Equilibrium

John Forbes Nash Jr. (1928 - 2015)
Nash (1950) "Equilibrium Points in N-person Games". Proceedings of the National Academy of Sciences of the United States of America.



Definition (Nash Equilibrium)

Let $\mathcal{G} = (\mathcal{N}, \mathcal{A}, u)$ be a normal-form game. Strategy profile $s = \langle s_1, \ldots, s_n \rangle$ is a Nash equilibrium iff $\forall i \in \mathcal{N}, s_i \in BR_i(s_{-i})$.

Nash Equilibrium

Question

How do we look for a Nash equilibrium?

So far, we considered only actions being played in a game. Hence, if all players choose a strategy, exactly one outcome is selected.

It is sufficient to check whether there is some agent that wants to deviate or not.

If not, this outcome is a Nash equilibrium.

Nash Equilibrium

What are Nash equilibria in these games?

	C	**D**
A	$(5, 5)$	$(0, 6)$
B	$(6, 0)$	$(1, 1)$

	L	**C**	**R**
T	$(1, 0)$	$(-1, 1)$	$(1, -1)$
M	$(2, 2)$	$(0, 0)$	$(3, 1)$
B	$(-1, 1)$	$(1, -1)$	$(0, 3)$

Nash Equilibrium

Characteristics of a Nash equilibrium (NE)

- NE is a descriptive solution concept – it *describes* which strategy profile is stable, it does not describe which strategies the players should be playing!
- NE is generally not unique and there may exist many NE. If one agent plays a strategy from a NE strategy profile, there are generally no guarantees on an (expected) outcome.
- NE is optimal in a sense of unilateral deviations. Strong NE is a variant that is optimal in a sense of group deviations.