University of Toronto

Department of Computer & Mathematical Sciences

STAB57: an Introduction to Statistics

Week 5 Assignment

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-email me

-website

-textbook

This week's list of problems is based on the material from:
Chapter 6, §1,2
You are expected to work on this list of problems prior to the upcoming tutorial.
Problems have the following tags:
: difficult, : Book exercise, : extra exercise

Terminology and Concepts to learn:

- likelihood function
- MLE
- statistic
- sufficiency
- minimality
- equivalence relation

Problem 1

Practice your skills on the sufficient and minimal statistics by doing problems 6.1.15 (this requires you to read through example 6.1.5 and 2.8.5 to refresh mulitnomial models), 6.1.21 (6.1.20 will help you), 6.1.23, 6.1.24

Problem 2

Practice your skills on the MLE by doing problems:6.2.4, 6.2.6, 6.2.11, 6.2.12

Problem 3 ☺

Give an example of a set X together with a relation that is

- reflexive, yet not symmetric or transitive
- not reflexive, is symmetric and not transitive
- item not reflexive, not symmetric, but transitive

Problem 4

Let $\sim$ denote an equivalence relation on a set X. Give a detailed argument describing why

$$\overline{x} \cap \overline{y} \neq \emptyset \iff x \sim y \iff \overline{x} = \overline{y}$$

Problem 5 ☺

Let $\Delta \subset S$ be a finite set of size n. Let $\overline{x} = \frac{1}{n}\sum_{x\in\Delta} x$ be the sample mean and $s^2 = \frac{1}{n-1}\sum x \in \Delta(x-\overline{x})^2$ the *corrected* variance. Show that the likelihood function of a normal distribution with mean μ and variance σ^2 can also be written as

$$L((\mu,\sigma^2)|\Delta) = (2\pi\sigma^2)^{\frac{-n}{2}} \cdot \exp\left(-n\frac{(\overline{x}-\mu)}{2\sigma^2}\right) \cdot \exp\left(-(n-1)\frac{s^2}{2\sigma^2}\right)$$

Problem 6 ☺

Argue in words why:

- a statistic could fail to be sufficient.
- a statistic could be sufficient yet fail to be minimal
- an MLE could fail to be a sufficient statistic