② Pistachio Problem

ⓐ X = time until combustion

$P(X > x+\Delta t \mid X > x) = P(X > x+\Delta t)$ [memoryless property]

(exponential distribution)

[Poisson process]

$P(X > x+\Delta t \mid X > x)$

$= P(X > \Delta t) = \underline{e^{-\lambda \Delta t}}$ since $\Delta t \cdot \lambda$ = prob. at Δt

$P(x < X \leq x+\Delta t \mid X > t) = \lambda \Delta t$

ⓑ $F(x) = P(X < x)$ (continuous r.v: $P(X=x)=0$)

$= 1 - P(X \geq x)$ (& using the above: $x = \Delta t$)

$= 1 - P(X > x)$

$\boxed{= 1 - \exp(-\lambda x)}$ ⓑ

$x \geq 0$

ⓒ pdf of $X = \frac{d}{dx} F(x) = \frac{d}{dx}(1 - e^{-\lambda x})$

$= -(-\lambda) e^{-\lambda x} = \boxed{\lambda e^{-\lambda x}} = f(x)$ ⓒ

ⓓ N pistachio piles, at times $\{t_i\}$ I.I.D.

Evaluate λ by computing ML parameter.

$f(x_i) = \lambda e^{-\lambda x_i}$ $x_i \geq 0$

$L(\lambda; x_1 \ldots x_N) = \prod_{i=1}^{N} f(x_i) = \prod_{i=1}^{N} \lambda e^{-\lambda x_i} = \lambda^N e^{\left(-\lambda \sum_{i=1}^{N} x_i\right)}$ likelihood function

→ use log-likelihood

$l = \ln(L) = \ln\left(\lambda^N e^{-\lambda \sum_{i=1}^{N} x_i}\right) = \ln(\lambda^N) + \cancel{\ln}\,\cancel{\exp}\left(-\lambda \sum_{i=1}^{N} x_i\right) = n \ln(\lambda) - \lambda \sum_{i=1}^{N} x_i$