2a) Starting with the equation for least squares,

$$\min_{\theta} \sum_{i=1}^{N} \left(y_i - f_{\theta,\sigma,x}(y_i)\right)^2 \qquad (1)$$

we want to show that this is equivalent to the maximum likelihood solution.

The likelihood function is given by

$$p(y|x,\theta,\sigma) = \prod_{i=1}^{N} \mathcal{N}(y_i | f(x_i,\theta), 1/\sigma) \qquad \text{where } \mathcal{N} \text{ Gaussian}$$

we maximize the above equation by maximizing the logarithm:

$$\ln p(y|x,\theta,\sigma) = \frac{-\sigma}{2} \sum_{i=1}^{N} \{f(x_i,\theta) - y_i\}^2 + \frac{N}{2}\ln\sigma - \frac{N}{2}\ln(2\pi)$$

we can ignore constants (no effect on maximizing) and minimize the negative log likelihood equivalently:

$$-\ln p(y|x,\theta,\sigma) \approx \sum_{i=1}^{N} \{y_i - f(x_i,\theta)\}^2$$

if we find the minimum of this equation, and change variables to the same as in question statement, we get

$$\min_{\theta} \sum_{i=1}^{N} \left(y_i - f_{\theta,\sigma,x}(y_i)\right)^2 \qquad (2)$$

(2) is equivalent to equation (1), so maximum likelihood solution = least squares solution.