(2b) Beginning with the equation for empirical risk for logistic regression:

$$R_{emp}(\theta) = \frac{1}{N}\sum_{i=1}^{N}(y_i - 1)\log(1 - f(x_i;\theta)) - y_i\log(f(x_i;\theta))$$

then factoring out y_i and negating:

$$= \frac{1}{N}\sum_{i=1}^{N} y_i\left(-\log(1-f(x_i;\theta)) + \log(f(x_i;\theta))\right) + \log(1-f(x_i;\theta))$$

$$= \frac{1}{N}\sum_{i=1}^{N} y_i\left(\log\left(\frac{f(x_i;\theta)}{1-f(x_i;\theta)}\right)\right) + \log(1-f(x_i;\theta))$$

raising to y_ith power

$$= \frac{1}{N}\sum_{i=1}^{N}\left(\log\left(\frac{f(x_i;\theta)^{y_i}}{1-f(x_i;\theta)^{y_i}}\right)\right) + \log(1-f(x_i;\theta))$$

$$= \frac{1}{N}\sum_{i=1}^{N}\log\left(\frac{f(x_i;\theta)^{y_i}}{1-f(x_i;\theta)^{y_i}}(1-f(x;\theta))\right)$$

$$= \frac{1}{N}\sum_{i=1}^{N}\log\left(f(x_i;\theta)^{y_i}(1-f(x_i;\theta))^{1-y_i}\right)$$

maximizing this value wrt. θ is equivalent to maximizing the likelihood function

$$P_{\theta,x}(y) = \prod_{i=1}^{N} f(x_i;\theta)^{y_i}(1-f(x_i;\theta))^{1-y_i}$$

Therefore logistic regression result maximizes $P_{\theta,x}(y)$.