Homework 1

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Problem 1

(a) integrate.m:

```
function i = integrate(p, dx, dy)
  % Leaving both dx and dy unspecified should cause an error.
  if nargin < 2
    error('dx and/or dy must be specified')
  end


  if isvector(p)
    % approximate over p by dx
    if (nargin == 3 && ~isempty(dy))
      error('dy is specified but p is one-dimensional')
    end
    m = size(p);
    a = p(1);
    b = p(end);
    total = (a + b) + sum(p(2:end-1));
    i = dx * total;
    return
  else

  end

  % produces a column vector
  if ~isvector(p) && (nargin == 2 || isempty(dy))
    % todo: don't use an explicit loop
    [nrows, ncols] = size(p)
    cols = zeros(nrows, 1)
    for row=1:nrows
      cols(row, 1) = integrate(p(row, :), dx);
    end
%     cols = integrate(p(, :), repmat(dx, nrows, 1))
    i = cols
    return
  end
```

```
  % returns a row vector
  if isempty(dx) && ~isempty(dy)
    i = transpose(integrate(transpose(p), dy))
    return
  end

  % returns a scalar
  if nargin==3 % && size(size(p))==3
    % use Fubini's thm

    i = integrate( integrate(p, dx),  dy)
    return
  end

end
```

The marginal distribution $p(x)$, computed both analytically and numerically, is shown in Figure 1 below.

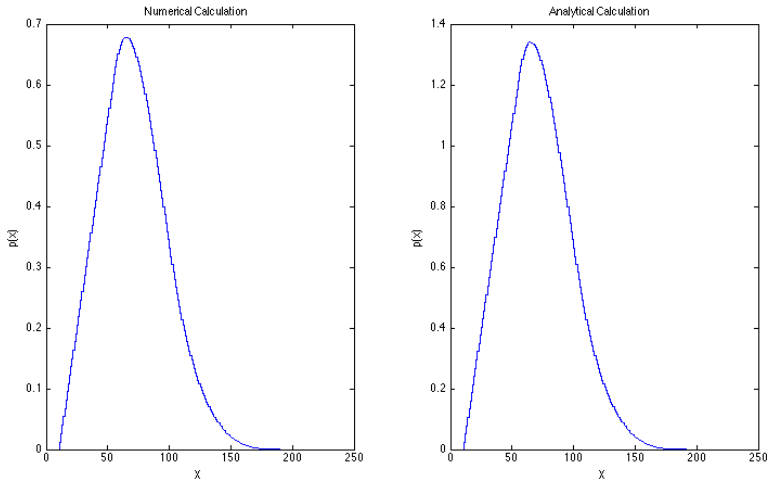

Figure 1: Marginal Distribution of x

(b) The root-mean-square discrepancy between the numerical and the analytical calcuations is 0.0078.

(c)

```
x = linspace(-0.1, 2.1, 201);
```

```
y = linspace(0, 1, 101);
P = pXYa(x, y)
dy = 0.01
pxn = integrate(P, [], dy)

[nrows, ncols] = size(P)
pygxn = P ./ (ones(nrows, 1) * pxn)
pygxn(isnan(pygxn)) = 0
contour(x, y, pygxn, 20, 'Color', 'k')
```

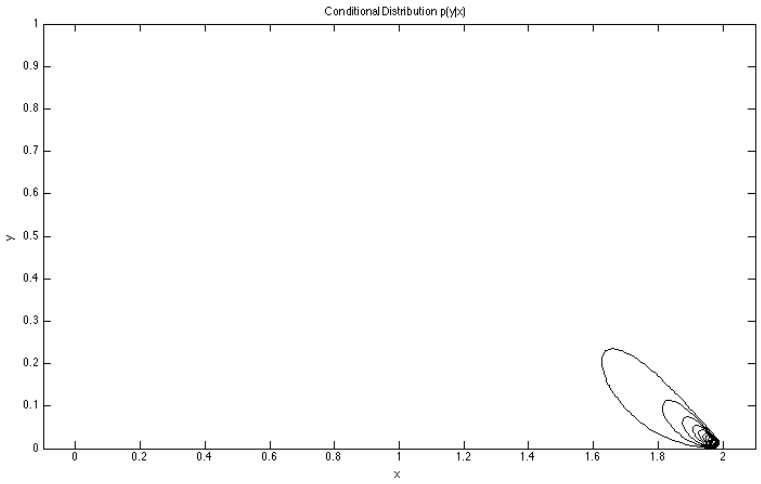

Figure 2: Conditional Distribution of $p(y|x)$

(d) Figure 2 above shows that X and Y are not independent because the probability of low values of Y is highly concentrated on the area where $1.6 < X < 2$. If the two variables were independent, the contours in the x direction would be horizontal lines, indicating that values of x do not give us information about the probability of y values.

(e) To compute samples of a joint probability distribution $p'(x, y)$ with the same marginals as above, I computed the marginals and multiplied them together. This is consistent with the independence assumption. The code snippet and contour plot are shown below.

```
pxn = integrate(pxya, [], dy)
pyn = integrate(pxya, dx)
pxy_indep = pyn * pxn
```

```
ppygx = pxy_indep ./ (ones(nrows, 1) * pxn)
ppygx(isnan(ppygx)) = 0
contour(x, y, ppygx, 20, 'Color', 'k')
xlabel('x')
ylabel('y')
title('Conditional Distribution p(y|x) When Independent')
```

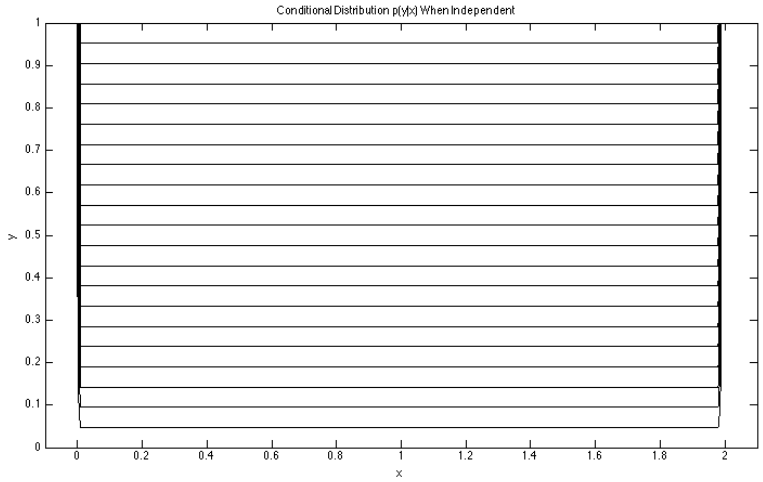

Figure 3: Conditional Distribution of $p(y|x)$ Under Independence$

(f) If there are $m+1$ samples in each of x and y, calculating the two-dimensional integral using Fubini's theorem and the trapezoidal rule takes m^2 sums.

(g) In general, computing a d-dimensional integral using Fubini's theorem and the trapezoidal rule takes m^d steps for $m+1$ samples.

(h) Since the complexity is exponential, using Fubini's theorem and the trapezoidal rule to compute high-dimensional integrals is very time consuming.

Problem 2

(a) The vectors in `cimg` are 12.25 ($\frac{28^2}{64}$) times shorter than the original images strung into vectors.

(b) A pair of images (original on the left, reconstructed on the right) is shown for each digit below.

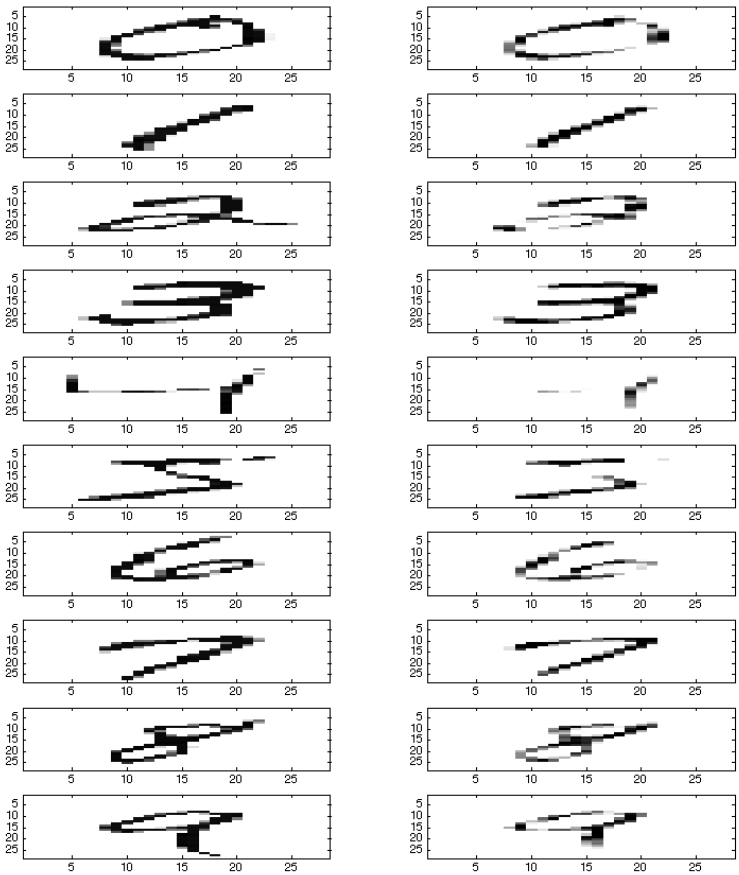

Figure 4: Original and Reconstructed Digits

(c) normalModel.m

```
function [likelihood, prior] = normalModel(X, L)
  X = double(X);
  [d, n] = size(X)
  labels = unique(L);
```

```
  l = size(labels, 2)
  m = zeros(d, l);
  s = zeros(d, l);
  prior = zeros(l, 1);
  size(prior)
  for i=1:l
    x = X(:, L==labels(i));
    k = size(x, 2);
    m(:, i) = mean(transpose(x));
    s(:, i) = std(transpose(x)).^2;
    prior(i, :) = k/n
  end
  likelihood = struct('M', m, 'S', s)
end
```

(d) The figure below shows samples drawn at random from the generative model, along with their labels.

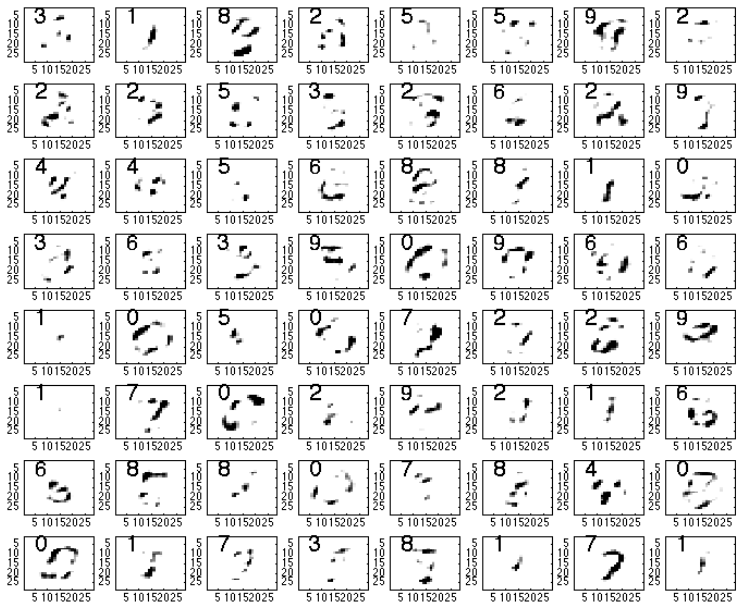

Figure 5: Sampled Images

(e) normalValue.m

```
function [v, delta]  = normalValue(X, m, s)
  [d, n] = size(X)
  mu = m * ones(1, n);
  diffsq = (X - mu).^2;
  quot = diffsq ./ (s * ones(1,n));
  delta = sqrt(sum(quot, 1));
  Sigma = diag(s);
  v = mvnpdf(transpose(X), transpose(m), Sigma);
end
```

(f) distances.m

```
function D = distances(L)
  [l, d] = size(L.M);
  D = zeros(l);
  for i = 1:l
    for j = 1:l
      s = diag( L.S(j, :) );
      D(i, j) = mahal( L.M(i, :), L.M(j, :),  s) ;
    end
  end
end

function m = mahal(mu1, mu2, s2)
  m = sqrt( sum(( (mu1-mu2).^2 ) / s2 ));
end
```

```
D =
  Columns 1 through 8

         0    7.18    4.86    7.19    5.26    4.35    4.93    7.52
    5.72         0    3.49    5.65    5.46    4.48    5.46    6.06
    6.05    4.53         0    4.53    3.30    2.88    3.57    3.83
    7.24    5.26    3.81         0    4.39    3.59    3.75    3.80
    8.01    6.00    2.86    4.35         0    3.56    3.32    3.62
    6.86    4.80    2.13    3.62    3.35         0    3.26    3.35
    6.95    5.05    2.48    3.06    2.82    2.32         0    3.15
    7.26    5.04    2.64    3.08    3.08    2.88    3.03         0
    5.77    4.52    1.78    2.80    2.99    1.81    1.96    2.69
    6.02    4.17    1.49    3.08    2.84    2.58    2.53    2.66
    6.50    4.72    1.73    2.70    2.88    1.94    2.39    2.06
    6.72    4.52    1.83    2.58    2.99    2.30    2.98    2.23
    6.98    4.63    1.88    2.85    2.58    2.39    2.05    1.89
    6.32    4.62    1.85    3.05    2.89    2.12    2.13    1.90
    5.38    4.31    1.61    3.16    2.93    2.35    2.29    2.98
```

5.93	4.42	1.76	2.88	2.77	2.35	2.69	2.24
5.92	4.26	1.86	2.77	2.74	2.43	2.08	2.13
6.72	4.23	1.76	2.67	2.72	2.04	2.34	1.84
6.31	4.32	1.66	2.64	2.63	2.02	2.11	2.45
6.61	4.58	1.67	2.81	2.67	1.87	2.26	2.06
6.33	4.39	1.64	2.73	2.73	2.06	2.12	2.11
6.41	4.37	1.66	2.78	2.52	1.97	2.16	2.13
5.76	4.19	1.56	3.01	2.91	2.14	2.33	2.30
6.23	4.25	1.61	2.81	2.66	1.92	2.24	2.14
6.37	4.47	1.64	2.77	2.65	1.96	2.09	2.06
6.53	4.42	1.77	2.74	2.43	2.18	2.18	2.04
6.03	4.29	1.67	2.82	2.73	1.98	1.95	2.24
6.05	4.32	1.60	2.83	2.75	1.91	2.25	2.14
6.41	4.45	1.70	2.74	2.63	2.00	2.10	1.94
6.27	4.43	1.68	2.77	2.64	2.12	2.21	1.94
6.26	4.42	1.70	2.70	2.64	2.03	1.84	2.25
6.31	4.27	1.63	2.80	2.65	2.09	2.27	1.95
6.39	4.45	1.75	2.70	2.55	2.03	2.02	2.01
6.18	4.33	1.61	2.81	2.68	1.99	2.04	2.17
6.37	4.34	1.67	2.78	2.67	2.09	2.21	1.91
6.28	4.33	1.66	2.73	2.66	2.07	2.06	2.11
6.23	4.40	1.67	2.75	2.64	2.02	2.03	2.09
6.37	4.37	1.72	2.67	2.60	2.01	2.07	2.03
6.36	4.42	1.69	2.74	2.62	2.11	2.16	2.01
6.23	4.36	1.65	2.77	2.65	2.01	2.21	2.09
6.30	4.33	1.62	2.75	2.69	2.06	2.23	2.08
6.42	4.44	1.69	2.76	2.57	2.13	2.14	1.95
6.42	4.44	1.71	2.72	2.59	1.99	2.08	1.98
6.32	4.34	1.69	2.70	2.63	2.11	2.15	2.04
6.14	4.35	1.65	2.76	2.67	2.09	2.07	2.15
6.25	4.36	1.66	2.73	2.65	2.06	2.17	2.10
6.23	4.31	1.63	2.77	2.68	2.00	2.06	2.11
6.39	4.38	1.69	2.71	2.59	2.04	2.13	1.99
6.35	4.40	1.68	2.72	2.62	2.01	2.17	2.
6.27	4.31	1.69	2.69	2.64	2.08	2.13	2.08
6.39	4.44	1.71	2.69	2.61	2.10	2.07	2.05
6.31	4.37	1.69	2.71	2.62	2.04	2.10	2.04
6.31	4.37	1.68	2.72	2.61	2.12	2.20	2.02
6.26	4.32	1.64	2.74	2.63	2.00	2.22	2.12
6.30	4.34	1.64	2.75	2.64	1.99	2.14	2.06
6.34	4.37	1.65	2.72	2.60	2.03	2.17	2.09
6.42	4.39	1.69	2.71	2.59	2.08	2.16	1.99
6.33	4.38	1.64	2.77	2.61	1.98	2.19	2.00
6.34	4.39	1.68	2.71	2.62	2.02	2.14	2.02
6.34	4.37	1.68	2.70	2.63	2.06	2.19	2.02
6.33	4.37	1.65	2.73	2.61	2.04	2.20	2.08

```
  6.30    4.36    1.66    2.72    2.64    2.05    2.16    2.08
  6.31    4.38    1.69    2.72    2.63    2.06    2.11    2.03
  6.34    4.35    1.67    2.71    2.61    2.02    2.13    2.08

Columns 9 through 16

  4.31    8.17    5.52    5.54    8.42    6.43    5.41    8.90
  4.64    4.42    5.44    5.46    6.32    6.14    5.83    6.84
  2.33    2.20    2.42    2.71    3.11    3.63    3.16    3.25
  2.97    3.81    3.39    3.40    4.00    4.19    4.90    4.04
  3.09    4.63    3.78    3.69    3.73    4.28    4.27    4.94
  2.11    3.43    2.54    2.94    3.50    3.12    3.91    3.17
  1.86    2.39    2.61    2.99    2.35    2.38    2.85    3.44
  2.12    2.74    1.98    2.44    2.20    2.13    3.31    2.77
     0    1.49    0.80    1.62    1.87    1.34    1.09    1.25
  1.18       0    1.23    1.31    1.41    1.67    1.56    1.37
  0.72    1.41       0    1.08    1.25    1.27    1.47    1.25
  1.41    1.37    1.03       0    1.55    1.97    2.02    1.81
  1.13    1.26    0.99    1.39       0    1.17    1.81    1.84
  0.97    1.40    1.14    1.88    1.31       0    2.12    1.30
  0.74    1.40    1.14    1.76    2.24    1.78       0    1.47
  0.90    1.22    0.95    1.47    1.66    1.23    1.28       0
  0.95    1.10    1.15    1.52    1.44    1.41    1.27    1.31
  1.01    1.25    0.84    1.00    0.93    1.40    1.66    1.54
  0.72    1.03    0.92    1.06    1.22    1.47    1.33    1.18
  1.06    1.66    0.99    1.08    1.41    1.83    1.74    1.91
  0.81    1.13    0.76    1.07    1.16    1.52    1.24    1.47
  0.69    1.28    0.77    1.29    1.18    1.28    1.17    1.10
  0.84    0.94    0.93    1.22    1.57    1.17    1.43    1.11
  0.76    1.11    0.84    1.09    1.23    1.18    1.44    1.05
  0.63    1.15    0.57    1.13    1.03    1.15    1.24    1.16
  0.92    1.14    0.96    1.13    0.87    1.16    1.54    1.18
  0.63    1.04    0.94    1.32    1.29    1.04    1.32    1.12
  0.64    1.22    0.67    1.15    1.43    1.24    1.21    1.01
  0.70    1.24    0.62    1.21    1.05    1.20    1.23    1.24
  0.77    1.10    0.68    1.07    1.06    1.13    1.29    1.04
  0.70    1.08    0.93    1.21    1.10    1.30    1.35    1.36
  0.85    1.08    0.77    1.06    1.10    1.29    1.33    1.16
  0.69    1.16    0.75    1.17    0.96    1.07    1.31    1.11
  0.75    1.07    0.90    1.13    1.18    1.28    1.38    1.28
  0.89    1.09    0.81    1.01    1.00    1.23    1.51    1.28
  0.76    1.00    0.83    1.07    1.05    1.26    1.32    1.23
  0.66    1.10    0.75    1.15    1.10    1.15    1.24    1.13
  0.78    1.20    0.84    1.06    1.07    1.29    1.40    1.28
  0.73    0.99    0.66    1.08    0.92    1.04    1.30    1.
  0.67    1.05    0.66    1.08    1.14    1.10    1.24    0.91
```

```
  0.76    0.99    0.66    0.96    1.07    1.28    1.27    1.13
  0.78    1.06    0.68    1.08    0.88    1.12    1.30    1.13
  0.78    1.22    0.79    1.05    1.01    1.21    1.45    1.27
  0.79    1.03    0.78    1.01    1.02    1.26    1.32    1.17
  0.66    1.01    0.74    1.16    1.18    1.25    1.08    1.11
  0.71    1.01    0.71    1.01    1.09    1.19    1.27    1.03
  0.72    1.03    0.81    1.12    1.11    1.21    1.31    1.20
  0.78    1.16    0.76    1.03    1.02    1.25    1.36    1.22
  0.73    1.15    0.67    1.02    1.05    1.19    1.32    1.11
  0.71    1.04    0.75    1.08    1.09    1.24    1.20    1.08
  0.74    1.04    0.72    1.04    0.91    1.18    1.32    1.19
  0.72    1.10    0.75    1.07    1.04    1.18    1.30    1.12
  0.78    1.05    0.73    1.01    1.03    1.23    1.27    1.08
  0.71    1.09    0.71    1.02    1.17    1.24    1.25    1.
  0.72    1.11    0.72    1.07    1.11    1.26    1.27    1.17
  0.74    1.09    0.73    1.00    1.05    1.25    1.31    1.12
  0.80    1.10    0.72    0.99    0.95    1.24    1.34    1.19
  0.75    1.19    0.70    1.05    1.12    1.25    1.31    1.16
  0.73    1.14    0.70    1.04    1.05    1.22    1.29    1.15
  0.77    1.10    0.72    0.98    1.05    1.25    1.33    1.14
  0.73    1.07    0.67    0.99    1.06    1.24    1.26    1.07
  0.71    1.04    0.69    1.02    1.06    1.22    1.25    1.08
  0.74    1.06    0.75    1.06    1.01    1.15    1.34    1.12
  0.74    1.09    0.75    1.02    1.06    1.27    1.31    1.17

Columns 17 through 24

  7.60    8.54   11.42    8.57   11.23   10.40   10.33    9.40
  6.40    6.93    8.00    7.05    7.29    7.77    7.51    7.37
  3.29    3.07    3.20    3.54    3.40    3.77    3.46    3.77
  4.22    3.99    4.30    5.04    4.61    4.92    5.39    5.
  4.57    4.58    5.12    4.57    5.61    4.80    6.03    5.20
  3.64    2.87    3.18    3.16    3.53    3.26    4.17    3.39
  2.82    3.25    3.21    2.85    2.97    3.12    3.79    3.13
  2.75    2.36    3.62    2.92    2.90    3.06    3.60    3.15
  1.42    1.72    1.48    1.81    1.46    1.37    1.42    1.38
  1.41    1.48    1.26    2.11    1.57    1.77    1.23    1.50
  1.53    1.12    1.31    1.26    1.00    0.98    1.75    1.22
  2.01    1.27    1.80    1.39    1.47    1.90    2.21    1.60
  1.54    1.15    1.62    1.79    1.52    1.46    2.28    1.54
  1.67    1.77    1.84    2.07    1.51    1.36    1.59    1.48
  1.31    1.94    2.07    2.02    1.99    1.87    1.75    1.80
  1.30    1.61    1.41    1.91    1.67    1.27    1.42    1.27
     0    1.31    1.36    1.70    1.15    1.35    1.17    1.28
  1.29       0    1.11    1.09    0.85    0.99    1.76    0.90
  1.24    1.02       0    1.25    0.83    0.88    1.35    0.73
```

1.53	1.06	1.37	0	0.92	1.32	1.99	1.23
0.94	0.82	0.90	0.78	0	0.88	1.33	0.89
1.11	0.88	0.75	1.15	0.85	0	1.56	0.76
1.00	1.21	1.39	1.52	1.19	1.44	0	0.82
1.09	0.82	0.66	1.10	0.80	0.80	1.00	0
1.03	0.83	0.77	0.97	0.51	0.53	1.31	0.61
1.23	1.02	0.80	1.37	1.08	0.90	1.57	0.69
0.84	1.10	0.88	1.33	0.91	1.00	0.77	0.55
0.94	0.93	0.97	1.02	0.70	0.77	0.90	0.54
0.95	0.80	0.94	0.95	0.51	0.48	1.44	0.75
0.85	0.81	0.88	1.05	0.60	0.73	1.09	0.55
0.94	1.01	0.61	1.10	0.66	0.97	1.20	0.70
0.88	0.59	0.84	0.97	0.51	0.70	1.12	0.53
0.95	0.89	0.70	1.12	0.70	0.58	1.33	0.53
0.92	0.87	0.71	0.98	0.63	0.96	0.94	0.48
0.98	0.60	0.89	0.98	0.58	0.92	1.16	0.54
0.85	0.74	0.63	1.02	0.48	0.82	1.07	0.51
0.83	0.88	0.72	1.03	0.53	0.70	1.05	0.48
0.96	0.71	0.68	0.89	0.56	0.77	1.28	0.50
0.95	0.82	0.70	1.20	0.68	0.65	1.20	0.53
0.95	0.84	0.68	1.12	0.67	0.60	1.03	0.36
0.94	0.63	0.67	0.97	0.43	0.71	1.10	0.50
0.94	0.75	0.79	1.07	0.61	0.63	1.32	0.58
1.04	0.75	0.76	0.88	0.63	0.79	1.32	0.47
0.87	0.68	0.66	1.02	0.50	0.77	1.14	0.50
0.70	0.90	0.76	1.09	0.50	0.72	0.98	0.63
0.91	0.78	0.61	1.07	0.57	0.72	1.02	0.39
0.86	0.77	0.67	1.01	0.51	0.79	0.98	0.43
0.95	0.66	0.71	0.90	0.52	0.69	1.28	0.47
0.97	0.71	0.72	0.94	0.53	0.63	1.22	0.43
0.80	0.73	0.61	1.07	0.50	0.62	1.11	0.51
0.94	0.78	0.66	1.05	0.55	0.75	1.27	0.57
0.89	0.75	0.65	1.00	0.53	0.67	1.14	0.42
0.87	0.72	0.69	1.02	0.55	0.68	1.16	0.50
0.98	0.76	0.58	1.01	0.60	0.61	1.09	0.32
0.91	0.69	0.68	0.90	0.42	0.61	1.15	0.45
0.99	0.72	0.57	0.96	0.55	0.65	1.21	0.39
0.97	0.64	0.71	0.95	0.52	0.68	1.32	0.52
0.97	0.70	0.76	0.84	0.49	0.60	1.22	0.45
0.93	0.71	0.70	0.92	0.48	0.63	1.22	0.46
0.94	0.68	0.69	0.94	0.51	0.72	1.18	0.45
0.98	0.71	0.60	0.97	0.53	0.61	1.19	0.42
0.91	0.72	0.61	1.00	0.48	0.65	1.12	0.44
0.89	0.75	0.67	1.05	0.56	0.74	1.11	0.43
0.96	0.69	0.58	0.94	0.50	0.67	1.20	0.42

Columns 25 through 32

12.76	9.40	8.85	8.10	14.00	9.90	10.85	10.79
8.51	9.03	7.75	7.81	9.93	8.72	9.56	8.34
3.62	4.04	4.09	3.93	4.33	4.38	4.27	4.52
5.10	4.86	5.40	5.44	5.48	5.71	5.64	5.98
5.91	4.77	5.49	5.27	6.07	5.70	6.11	6.07
3.55	3.66	4.13	3.77	3.78	4.42	4.15	4.43
3.26	3.73	3.11	3.67	3.52	3.88	3.52	3.79
3.21	3.37	3.30	2.98	3.40	3.19	3.83	3.30
1.49	1.84	1.21	1.19	1.85	1.54	1.62	1.66
1.71	1.48	1.53	1.81	2.08	1.74	1.84	1.82
0.82	1.44	1.47	1.03	0.99	1.15	1.55	1.31
1.84	1.84	2.12	1.77	2.07	1.86	2.32	1.74
1.50	1.19	1.50	1.66	1.64	1.50	1.66	1.63
1.37	1.87	1.35	1.60	1.68	1.60	1.93	1.81
2.22	2.36	1.63	1.54	2.64	1.85	1.94	2.04
1.41	1.55	1.50	1.44	1.81	1.30	1.91	1.67
1.38	1.61	1.10	1.26	1.46	1.14	1.23	1.28
0.96	1.20	1.17	0.97	1.03	0.96	1.30	0.79
0.78	0.86	0.85	0.99	1.14	1.04	0.79	1.08
1.13	1.80	1.57	1.14	1.27	1.27	1.43	1.29
0.53	1.48	1.00	0.73	0.66	0.71	0.88	0.67
0.51	0.99	1.00	0.74	0.57	0.89	1.00	0.84
1.41	1.43	0.73	0.85	1.73	0.94	1.25	1.03
0.69	0.79	0.61	0.60	0.94	0.67	0.85	0.64
0	0.99	0.74	0.53	0.37	0.54	0.69	0.62
0.90	0	0.89	1.05	1.14	0.83	1.09	0.94
0.88	1.06	0	0.74	1.15	0.78	0.61	0.89
0.72	1.24	0.76	0	0.89	0.56	0.94	0.57
0.29	1.18	0.87	0.56	0	0.55	0.85	0.57
0.49	0.91	0.73	0.49	0.56	0	0.81	0.40
0.64	1.05	0.52	0.90	0.89	0.81	0	0.96
0.54	1.05	0.81	0.50	0.58	0.38	0.92	0
0.41	0.67	0.58	0.64	0.57	0.47	0.62	0.63
0.64	1.05	0.49	0.67	0.88	0.61	0.47	0.71
0.62	0.98	0.77	0.67	0.71	0.43	0.87	0.43
0.51	0.94	0.54	0.65	0.69	0.49	0.46	0.54
0.43	0.94	0.47	0.50	0.59	0.38	0.46	0.53
0.54	0.88	0.65	0.60	0.62	0.48	0.56	0.54
0.40	0.65	0.65	0.67	0.63	0.40	0.75	0.59
0.45	0.77	0.61	0.44	0.69	0.39	0.77	0.50
0.42	0.95	0.74	0.52	0.60	0.42	0.74	0.39
0.38	0.75	0.73	0.64	0.48	0.32	0.73	0.48
0.51	0.80	0.67	0.60	0.63	0.44	0.61	0.58
0.52	0.83	0.65	0.60	0.63	0.36	0.62	0.41

```
  0.54    1.11    0.58    0.53    0.69    0.46    0.57    0.54
  0.47    0.76    0.58    0.51    0.70    0.35    0.63    0.47
  0.50    0.99    0.46    0.56    0.70    0.49    0.48    0.52
  0.45    0.84    0.69    0.52    0.52    0.36    0.64    0.38
  0.37    0.81    0.69    0.44    0.50    0.29    0.69    0.39
  0.46    0.89    0.61    0.52    0.57    0.40    0.61    0.41
  0.42    0.74    0.64    0.69    0.61    0.43    0.53    0.60
  0.41    0.80    0.56    0.50    0.55    0.33    0.56    0.43
  0.48    0.79    0.71    0.54    0.58    0.26    0.71    0.36
  0.48    0.78    0.67    0.42    0.68    0.44    0.73    0.44
  0.34    0.98    0.66    0.40    0.48    0.40    0.62    0.34
  0.42    0.72    0.67    0.52    0.61    0.42    0.62    0.46
  0.44    0.78    0.74    0.58    0.53    0.36    0.69    0.39
  0.38    0.95    0.75    0.36    0.45    0.36    0.73    0.33
  0.36    0.86    0.67    0.45    0.47    0.31    0.62    0.38
  0.47    0.82    0.70    0.50    0.58    0.31    0.69    0.37
  0.38    0.76    0.71    0.48    0.57    0.38    0.69    0.42
  0.38    0.83    0.63    0.49    0.56    0.35    0.61    0.41
  0.45    0.77    0.54    0.56    0.61    0.32    0.57    0.47
  0.42    0.81    0.64    0.52    0.58    0.43    0.56    0.45

Columns 33 through 40

 12.33   14.19   15.31   12.97   12.60   12.99   13.54   11.72
 10.26   10.59   11.21    9.15   10.46   10.24   10.20   10.46
  4.55    4.56    4.86    4.90    5.00    5.25    5.22    5.28
  5.81    6.37    6.50    6.48    6.64    6.53    6.78    6.86
  6.01    6.76    7.04    7.20    6.64    7.26    7.28    7.03
  4.23    4.59    5.12    5.20    4.84    4.88    5.32    5.03
  4.04    4.22    4.78    3.73    4.16    4.22    4.40    4.47
  3.65    4.34    3.78    3.86    4.13    3.89    4.05    4.13
  1.75    1.83    2.36    1.67    1.69    2.07    1.77    1.67
  1.99    1.84    2.03    1.79    2.09    2.30    1.87    2.00
  1.28    1.62    1.51    1.57    1.38    1.73    1.31    1.38
  2.37    2.47    2.19    1.99    2.43    2.19    2.18    2.21
  1.52    2.14    1.91    1.76    1.87    2.06    1.58    1.82
  1.65    1.83    1.88    1.97    1.91    2.11    1.72    1.85
  2.44    2.69    3.08    2.25    2.35    2.38    2.45    2.01
  1.67    2.06    1.99    1.95    1.83    1.94    1.70    1.49
  1.40    1.49    1.67    1.39    1.37    1.42    1.54    1.52
  1.15    1.44    0.99    1.08    1.32    1.14    1.11    1.16
  0.91    0.90    1.17    0.83    1.02    1.01    0.98    0.95
  1.52    1.55    1.42    1.51    1.51    1.29    1.79    1.73
  0.97    0.81    0.80    0.65    0.73    0.85    0.95    1.08
  0.69    1.22    1.24    1.11    0.93    1.08    0.93    0.82
  1.50    1.22    1.55    1.09    1.30    1.28    1.33    1.07
```

```
  0.75    0.62    0.81    0.70    0.74    0.66    0.80    0.52
  0.47    0.77    0.80    0.67    0.48    0.76    0.53    0.50
  0.75    1.23    1.04    0.94    1.09    0.99    0.77    0.91
  0.84    0.66    1.21    0.67    0.63    0.84    0.94    0.77
  0.90    0.80    1.04    0.85    0.68    0.78    0.92    0.60
  0.60    0.98    0.84    0.79    0.64    0.80    0.70    0.70
  0.53    0.73    0.51    0.57    0.44    0.56    0.48    0.48
  0.68    0.49    0.92    0.47    0.57    0.64    0.94    0.92
  0.72    0.72    0.42    0.57    0.63    0.60    0.62    0.65
     0    0.80    0.76    0.56    0.44    0.54    0.42    0.44
  0.72       0    0.68    0.38    0.45    0.47    0.88    0.76
  0.73    0.66       0    0.46    0.67    0.56    0.70    0.77
  0.56    0.36    0.51       0    0.36    0.41    0.60    0.63
  0.42    0.43    0.67    0.34       0    0.42    0.54    0.45
  0.49    0.58    0.46    0.40    0.47       0    0.73    0.65
  0.37    0.80    0.66    0.57    0.52    0.73       0    0.37
  0.49    0.68    0.74    0.60    0.48    0.64    0.39       0
  0.64    0.61    0.50    0.43    0.51    0.61    0.47    0.50
  0.37    0.84    0.55    0.54    0.52    0.61    0.28    0.46
  0.44    0.66    0.46    0.49    0.50    0.24    0.68    0.60
  0.52    0.57    0.38    0.24    0.45    0.36    0.48    0.54
  0.65    0.60    0.86    0.45    0.34    0.64    0.61    0.57
  0.49    0.53    0.56    0.36    0.39    0.47    0.40    0.30
  0.57    0.26    0.61    0.21    0.29    0.43    0.63    0.57
  0.44    0.65    0.40    0.39    0.45    0.22    0.55    0.52
  0.41    0.67    0.47    0.47    0.41    0.37    0.45    0.35
  0.47    0.61    0.62    0.36    0.37    0.47    0.45    0.44
  0.37    0.68    0.54    0.34    0.44    0.49    0.39    0.53
  0.36    0.53    0.48    0.31    0.28    0.30    0.44    0.39
  0.48    0.68    0.45    0.42    0.45    0.44    0.41    0.43
  0.54    0.63    0.67    0.54    0.51    0.51    0.52    0.25
  0.51    0.53    0.55    0.40    0.35    0.41    0.55    0.46
  0.44    0.61    0.52    0.41    0.47    0.38    0.47    0.37
  0.45    0.75    0.42    0.41    0.53    0.40    0.43    0.49
  0.52    0.66    0.52    0.57    0.45    0.39    0.62    0.48
  0.41    0.62    0.47    0.40    0.35    0.32    0.47    0.41
  0.50    0.63    0.39    0.38    0.45    0.32    0.49    0.46
  0.47    0.67    0.55    0.48    0.48    0.48    0.41    0.31
  0.45    0.56    0.52    0.34    0.36    0.46    0.37    0.34
  0.39    0.51    0.44    0.28    0.32    0.36    0.41    0.42
  0.45    0.54    0.49    0.31    0.41    0.30    0.51    0.45

Columns 41 through 48

 16.28   17.20   14.92   14.95   12.14   13.92   12.57   15.83
 11.57   12.08   11.84   12.08   10.78   11.21   12.32   12.63
```

5.15	5.54	5.67	5.71	5.78	5.90	5.79	5.90
7.47	7.02	6.86	7.52	7.89	7.66	7.72	7.55
7.73	7.48	7.33	7.70	7.62	8.21	7.51	7.94
5.58	5.28	4.91	6.06	6.24	5.98	5.59	5.49
4.75	4.43	4.48	5.23	4.63	4.91	5.22	5.23
4.47	4.31	4.31	4.45	4.50	4.47	4.90	4.64
2.27	2.50	2.29	2.21	1.71	1.79	1.97	2.50
2.06	2.26	2.43	2.14	2.20	2.12	2.26	2.57
1.40	1.38	1.62	1.68	1.64	1.62	1.82	1.71
2.18	2.27	2.28	2.53	2.55	2.28	2.86	2.59
2.20	1.82	2.00	2.01	1.95	2.03	1.96	2.13
2.27	1.92	2.14	2.26	2.34	2.16	2.37	2.41
3.00	3.38	3.12	2.85	2.06	2.45	2.57	3.18
1.95	2.08	2.36	2.07	2.10	2.08	2.45	2.32
1.85	1.84	1.83	1.68	1.39	1.72	1.63	1.79
1.20	1.19	1.21	1.17	1.37	1.27	1.26	1.18
0.98	1.15	1.11	1.03	1.20	0.93	1.14	1.19
1.69	1.71	1.43	1.77	1.85	1.85	1.72	1.53
0.75	0.97	1.01	0.86	0.74	1.01	0.91	0.89
1.00	0.93	1.18	1.21	1.09	1.16	1.26	1.09
1.63	1.97	1.64	1.48	1.27	1.09	1.18	1.76
0.69	0.94	0.73	0.83	1.04	0.64	0.73	0.82
0.60	0.54	0.71	0.76	0.65	0.66	0.70	0.69
1.23	0.88	0.99	1.03	1.29	0.94	1.27	1.07
1.10	1.31	1.08	1.04	0.85	0.83	0.69	1.16
0.87	1.22	1.03	0.99	0.80	0.76	0.82	1.00
0.77	0.63	0.87	0.87	0.69	0.89	0.92	0.78
0.49	0.48	0.63	0.47	0.59	0.48	0.73	0.53
0.89	0.97	0.77	0.81	0.86	0.87	0.66	0.90
0.41	0.64	0.73	0.51	0.71	0.62	0.69	0.53
0.74	0.43	0.55	0.61	0.65	0.52	0.72	0.54
0.63	0.95	0.64	0.62	0.80	0.68	0.33	0.71
0.55	0.63	0.52	0.44	0.89	0.62	0.62	0.52
0.44	0.65	0.53	0.30	0.54	0.45	0.24	0.46
0.54	0.65	0.54	0.49	0.39	0.40	0.36	0.53
0.56	0.62	0.27	0.37	0.73	0.55	0.46	0.25
0.56	0.33	0.70	0.53	0.66	0.45	0.74	0.62
0.50	0.65	0.73	0.60	0.65	0.35	0.68	0.66
0	0.57	0.66	0.39	0.57	0.41	0.53	0.51
0.57	0	0.58	0.48	0.61	0.48	0.71	0.48
0.62	0.54	0	0.48	0.81	0.54	0.52	0.29
0.37	0.48	0.46	0	0.56	0.31	0.46	0.30
0.64	0.86	0.88	0.63	0	0.55	0.55	0.77
0.34	0.57	0.55	0.33	0.54	0	0.48	0.47
0.46	0.75	0.58	0.45	0.55	0.46	0	0.54
0.46	0.43	0.28	0.31	0.63	0.41	0.48	0

```
  0.38    0.40    0.38    0.39    0.61    0.31    0.55    0.25
  0.39    0.58    0.65    0.33    0.37    0.37    0.50    0.46
  0.55    0.35    0.46    0.36    0.58    0.37    0.55    0.42
  0.39    0.45    0.38    0.26    0.51    0.25    0.39    0.26
  0.35    0.39    0.53    0.23    0.53    0.30    0.61    0.34
  0.40    0.66    0.61    0.53    0.67    0.32    0.61    0.53
  0.31    0.58    0.52    0.44    0.51    0.42    0.39    0.35
  0.36    0.46    0.41    0.36    0.64    0.25    0.51    0.31
  0.45    0.31    0.40    0.30    0.61    0.35    0.57    0.22
  0.43    0.54    0.46    0.54    0.66    0.51    0.57    0.31
  0.36    0.42    0.38    0.33    0.52    0.32    0.48    0.20
  0.32    0.45    0.39    0.24    0.61    0.28    0.53    0.23
  0.30    0.44    0.52    0.40    0.60    0.27    0.59    0.38
  0.24    0.47    0.52    0.29    0.47    0.19    0.45    0.37
  0.42    0.45    0.39    0.23    0.56    0.24    0.39    0.32
  0.34    0.50    0.38    0.28    0.59    0.28    0.40    0.25

Columns 49 through 56

 15.51   16.07   17.14   16.70   17.17   16.38   17.87   16.60
 12.38   13.29   13.98   13.32   14.26   13.31   13.51   13.46
  6.25    5.94    6.31    6.28    6.33    6.64    6.39    6.95
  8.08    7.93    8.02    7.95    8.47    8.62    8.50    9.34
  8.41    8.50    8.39    8.52    8.82    8.96    9.43    9.18
  6.02    6.08    6.24    5.79    6.64    6.16    6.12    7.03
  5.15    5.67    5.51    5.26    6.35    5.77    5.72    5.71
  4.55    5.00    5.13    5.04    5.07    5.48    5.30    5.47
  2.07    2.25    2.42    2.38    2.47    2.12    2.39    2.44
  2.51    2.36    2.44    2.56    2.49    2.55    2.73    2.67
  1.56    1.81    1.66    1.74    1.78    1.77    1.85    1.91
  2.44    2.91    2.84    2.72    2.95    2.66    2.90    2.68
  2.21    2.29    1.97    2.29    2.34    2.50    2.54    2.45
  2.32    2.61    2.50    2.55    2.68    2.81    2.54    2.70
  2.81    2.88    3.41    3.21    3.26    3.04    3.21    3.08
  2.17    2.25    2.53    2.37    2.31    2.33    2.55    2.38
  1.97    1.61    2.00    1.82    1.92    2.14    1.91    2.23
  1.27    1.39    1.44    1.42    1.42    1.47    1.39    1.37
  1.15    1.13    1.12    1.18    1.27    1.14    1.23    1.05
  1.78    2.03    1.97    1.89    2.05    2.07    1.79    1.95
  1.03    0.99    1.07    1.05    1.08    1.24    0.82    1.09
  1.05    1.05    1.27    1.10    1.25    1.08    1.14    1.22
  1.51    1.63    1.94    1.72    1.76    1.59    1.71    1.66
  0.72    0.89    1.06    0.76    0.97    0.59    0.82    0.70
  0.58    0.71    0.67    0.65    0.80    0.78    0.59    0.71
  1.10    1.31    1.07    1.18    1.19    1.17    1.41    1.09
  1.08    1.05    1.24    1.00    1.32    1.14    1.13    1.18
```

```
  0.79    0.89    1.29    0.95    1.02    0.79    0.80    0.99
  0.73    0.81    0.92    0.83    0.93    1.03    0.73    0.96
  0.41    0.59    0.65    0.47    0.40    0.67    0.64    0.66
  1.00    0.91    0.80    0.83    1.14    1.21    0.97    1.02
  0.56    0.60    0.87    0.63    0.53    0.67    0.53    0.69
  0.53    0.65    0.52    0.49    0.68    0.73    0.70    0.61
  0.77    0.78    0.84    0.66    0.90    0.92    0.75    0.77
  0.59    0.80    0.72    0.64    0.64    0.85    0.74    0.76
  0.58    0.45    0.49    0.40    0.58    0.73    0.49    0.57
  0.47    0.46    0.57    0.35    0.61    0.65    0.44    0.55
  0.46    0.57    0.55    0.38    0.58    0.71    0.54    0.51
  0.56    0.61    0.48    0.54    0.54    0.67    0.73    0.62
  0.45    0.55    0.75    0.52    0.57    0.35    0.58    0.50
  0.46    0.45    0.61    0.48    0.43    0.51    0.42    0.47
  0.44    0.60    0.41    0.50    0.46    0.72    0.66    0.57
  0.41    0.72    0.51    0.43    0.63    0.73    0.62    0.48
  0.41    0.37    0.41    0.29    0.28    0.57    0.46    0.43
  0.70    0.47    0.82    0.65    0.73    0.80    0.60    0.78
  0.35    0.39    0.51    0.31    0.35    0.35    0.46    0.31
  0.56    0.50    0.66    0.42    0.68    0.68    0.45    0.58
  0.26    0.48    0.46    0.28    0.39    0.56    0.39    0.34
     0    0.46    0.49    0.25    0.34    0.40    0.35    0.29
  0.45       0    0.57    0.34    0.38    0.51    0.39    0.50
  0.49    0.54       0    0.39    0.48    0.73    0.60    0.51
  0.25    0.33    0.40       0    0.34    0.46    0.34    0.32
  0.31    0.36    0.47    0.31       0    0.47    0.49    0.39
  0.36    0.47    0.73    0.44    0.50       0    0.44    0.32
  0.33    0.38    0.62    0.34    0.49    0.46       0    0.40
  0.26    0.44    0.46    0.29    0.37    0.32    0.39       0
  0.28    0.49    0.38    0.36    0.32    0.56    0.47    0.33
  0.29    0.54    0.70    0.41    0.50    0.49    0.29    0.46
  0.14    0.38    0.45    0.20    0.32    0.45    0.27    0.30
  0.21    0.43    0.47    0.25    0.24    0.43    0.41    0.29
  0.25    0.41    0.51    0.34    0.32    0.28    0.39    0.19
  0.28    0.28    0.43    0.24    0.29    0.37    0.31    0.27
  0.34    0.40    0.35    0.16    0.36    0.53    0.45    0.40
  0.28    0.38    0.43    0.24    0.39    0.42    0.31    0.18

Columns 57 through 64

 18.90   21.27   18.82   17.21   18.32   20.48   19.55   16.90
 15.06   15.45   14.42   15.14   14.75   15.00   14.91   15.05
  6.69    6.78    7.12    7.14    7.05    7.09    7.47    7.51
  8.63    9.46    9.36    9.03    9.50    9.31    9.55    9.91
  9.06    9.99   10.00    9.65    9.67   10.86   10.13    9.62
  6.51    6.67    7.11    6.79    6.97    6.89    7.34    7.39
```

6.18	6.41	6.08	6.41	6.17	6.22	5.96	6.48
5.35	5.67	5.48	5.42	5.73	5.86	5.70	6.08
2.97	2.93	2.58	2.62	2.71	2.82	2.80	2.74
2.78	3.13	2.93	2.82	2.81	2.88	2.86	2.98
1.84	1.86	1.95	1.98	1.84	2.03	2.11	2.12
2.88	3.20	2.99	2.90	2.83	3.02	3.05	3.09
2.29	2.93	2.65	2.54	2.62	2.80	2.63	2.57
2.75	2.83	2.70	3.04	2.93	2.82	2.66	3.09
3.84	3.99	3.34	3.19	3.35	3.41	3.66	3.50
2.71	2.58	2.46	2.61	2.37	2.40	2.62	2.93
2.16	2.39	2.13	2.09	2.31	2.06	2.13	2.28
1.27	1.59	1.45	1.45	1.49	1.59	1.63	1.43
1.33	1.48	1.31	1.38	1.18	1.22	1.33	1.22
1.91	1.84	1.93	2.08	2.11	2.17	2.28	2.04
1.08	0.98	0.97	1.24	1.20	1.09	1.23	1.09
1.27	1.14	1.26	1.38	1.13	1.23	1.46	1.42
2.16	2.28	1.85	1.66	1.92	1.87	1.83	1.83
1.04	0.94	0.90	0.90	0.81	0.88	0.88	0.85
0.75	0.67	0.67	0.87	0.70	0.69	0.85	0.80
1.20	1.51	1.32	1.40	1.30	1.44	1.25	1.31
1.53	1.53	1.29	1.28	1.32	1.20	1.10	1.21
1.30	1.04	0.96	0.98	1.04	1.03	1.18	1.09
0.88	0.77	0.77	1.02	0.96	0.93	1.06	1.08
0.62	0.60	0.52	0.51	0.63	0.60	0.53	0.78
1.13	1.23	1.05	1.13	1.15	0.99	0.93	1.02
0.65	0.54	0.59	0.64	0.71	0.69	0.75	0.77
0.67	0.78	0.60	0.80	0.70	0.69	0.58	0.70
1.00	0.92	0.82	0.81	0.91	0.82	0.72	0.78
0.63	0.76	0.70	0.60	0.85	0.84	0.65	0.80
0.62	0.75	0.56	0.56	0.67	0.47	0.42	0.46
0.73	0.66	0.47	0.61	0.65	0.51	0.44	0.55
0.50	0.58	0.44	0.45	0.64	0.65	0.51	0.42
0.61	0.82	0.67	0.75	0.59	0.56	0.56	0.75
0.78	0.68	0.58	0.66	0.47	0.48	0.62	0.68
0.52	0.58	0.50	0.48	0.40	0.35	0.59	0.48
0.41	0.69	0.53	0.59	0.57	0.57	0.55	0.71
0.48	0.58	0.47	0.48	0.63	0.70	0.52	0.49
0.35	0.63	0.40	0.28	0.49	0.38	0.29	0.36
0.97	0.94	0.72	0.81	0.83	0.65	0.78	0.79
0.57	0.63	0.41	0.38	0.34	0.26	0.32	0.38
0.77	0.73	0.58	0.62	0.68	0.51	0.47	0.51
0.25	0.41	0.23	0.27	0.45	0.48	0.41	0.29
0.33	0.31	0.15	0.25	0.29	0.34	0.40	0.36
0.57	0.65	0.44	0.47	0.49	0.32	0.47	0.47
0.40	0.77	0.51	0.54	0.57	0.47	0.40	0.50
0.41	0.47	0.23	0.30	0.39	0.30	0.19	0.27

```
   0.35    0.53    0.34    0.24    0.36    0.35    0.37    0.47
   0.65    0.54    0.46    0.48    0.31    0.40    0.60    0.46
   0.52    0.32    0.29    0.45    0.43    0.34    0.53    0.36
   0.36    0.47    0.30    0.31    0.19    0.30    0.41    0.20
      0    0.54    0.31    0.27    0.39    0.43    0.45    0.36
   0.48       0    0.31    0.44    0.46    0.51    0.63    0.50
   0.32    0.30       0    0.25    0.33    0.31    0.38    0.29
   0.30    0.43    0.23       0    0.32    0.34    0.34    0.30
   0.38    0.44    0.31    0.33       0    0.23    0.48    0.35
   0.42    0.49    0.29    0.31    0.24       0    0.34    0.28
   0.43    0.59    0.36    0.35    0.47    0.35       0    0.37
   0.36    0.46    0.26    0.28    0.30    0.28    0.36       0
```

(g) classify.m

```
function label = classify(img, likelihood, prior)
  compressed = code(img);
  n = size(compressed, 2);
  d = size(likelihood.M, 2);
  post = zeros(d, n);
  for i=1:d
    [v, delta] = normalValue(compressed, likelihood.M(:, i), likelihood.S(:, i));
    post(i, :) = v * prior(i);
  end
  label = zeros(1, n);
  for j=1:n
    col = post(:, j);
    argmax = find( col == max(col));
    label(j) = argmax-1;
  end
end
```

(h) errorStats.m

```
function [E, errorRate, pCgT] = errorStats(computedLabel, trueLabel)
  labels = unique(computedLabel);
  n = size(computedLabel, 2);
  l = size(labels, 2);
  E = zeros(l, l);
  pCgT = zeros(l, l);
  for i=1:l
    for j=1:l
      E(i, j) = sum( (trueLabel == i-1) & (computedLabel == j-1) );
      pCgT(i, j) = sum((computedLabel==i-1) & (trueLabel==j-1))/sum(trueLabel==j-1);
```

```
    end
  end
  errorRate = 1 - (sum(diag(E))/n);
end
```

Output:

The error rate is 12.3%.

```
E =

  Columns 1 through 7

         928           0           9           3           1          21          12
           0        1074          13           9           1          11           6
          19           1         884          32          13           6          11
           4           1          19         877           2          50           3
           1           2          19           0         851          10          14
           4           1          11          67           8         743          12
          13           3          14           3          12          40         870
           3          22          40           2          17          15           5
           5           0          16          34           9          43           2
          13           2          28           9          69          13           1

  Columns 8 through 10

           2           3           1
           1          20           0
          16          46           4
          15          27          12
           3           7          75
           9          33           4
           0           2           1
         859          16          49
           8         845          12
          23          10         841

errorRate =

    0.1228

pCgT =

  Columns 1 through 8

    0.947         0    0.018    0.004    0.001    0.004    0.014    0.003
```

```
    0.946    0.001    0.001    0.002    0.001    0.003    0.021
    0.011    0.857    0.019    0.019    0.012    0.015    0.039
    0.008    0.031    0.868         0    0.075    0.003    0.002
    0.001    0.013    0.002    0.867    0.009    0.013    0.017
    0.010    0.006    0.050    0.010    0.833    0.042    0.015
    0.005    0.011    0.003    0.014    0.013    0.908    0.005
    0.001    0.016    0.015    0.003    0.010         0    0.836
    0.018    0.045    0.027    0.007    0.037    0.002    0.016
         0    0.004    0.012    0.076    0.004    0.001    0.048

Columns 9 through 10

    0.013
    0.002
    0.028
    0.009
    0.068
    0.013
    0.001
    0.023
    0.010
    0.833
```

(i) Given the instructions, column `j` in `pCgT` indicates values where the true digit was `j-1`. Thus, each column of `pCgT` should sum to one, as they do.

(j) If the classifier's error rate is p and errors on different digits are independent, the probability that the classifier gets a five-digit zip code wrong is $p_Z = 1 - (1 - p^5)$. For my error rate, $p = 0.123$, $p_Z = 1 - (1 - 0.123)^5 = 0.481$. For the best available rate today, $p = 0.002$, $p_Z = 1 - (1 - 0.002)^5 \approx 0.001$.

(k) If the state-of-the-art digit classifier were used, approximately 3,984,032 zip codes would be misclassified in the US each day.

(l) The posterior $p(\hat{w}|x)$ can take on values between 0.1 and 1. Because $\hat{w}$ is the argmax of a variable with 10 possible values, it must be at least 0.1. Because it is a probability, its upper bound is 1.

(m) An automatic zip code scanner could use the posterior value to determine whether there is sufficient confidence int he automated classification, or if the zip code needs further review (eg by a human). Some threshold, such as $\prod_{i=1}^{5} p(\hat{w}_i|x_i) < 0.9$, could be chosen so that misclassifications by the automated system are kept to a minimum while still saving the time of human reviewers.

(n) The assumption that errors on adjacent digits are mutually independent is not valid. First, all five digits in a handwritten zip code are typically generated by the same human hand. If a person has bad handwriting (1's that can easily be mistaken for 7's, for example), then errors will be correlated across digits they write. Secondly, it's possible that one digit will overlap or be smeared with a subsequent digit, making them both difficult to classify.