Homework 1

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Problem 1

(a) integrate.m:

```
function i = integrate(p, dx, dy)
  if nargin < 2
    error('dx and/or dy must be specified')
  end

  if isvector(p)
    if (nargin == 3 && ~isempty(dy))
      error('dy is specified but p is one-dimensional')
    end
    total = p(1) + p(end) + sum(p(2:end-1))*2;
    i = dx/2 * total;
    return
  end

  if ~isvector(p) && (nargin == 2 || isempty(dy))
    total = p(: , 1) + p(:, end) + sum(p(:, 1:end), 2)*2;
    i = dx/2 * total;
    return % returns a column vector
  end

  if isempty(dx) && ~isempty(dy)
    i = transpose(integrate(transpose(p), dy));
    return % returns a row vector
  end

  if nargin==3
    % use Fubini's thm
    i = integrate( integrate(p, dx),  dy);
    return % returns a scalar
  end

end
```

The marginal distribution $p(x)$, computed both analytically and numerically, is shown in Figure 1 below.

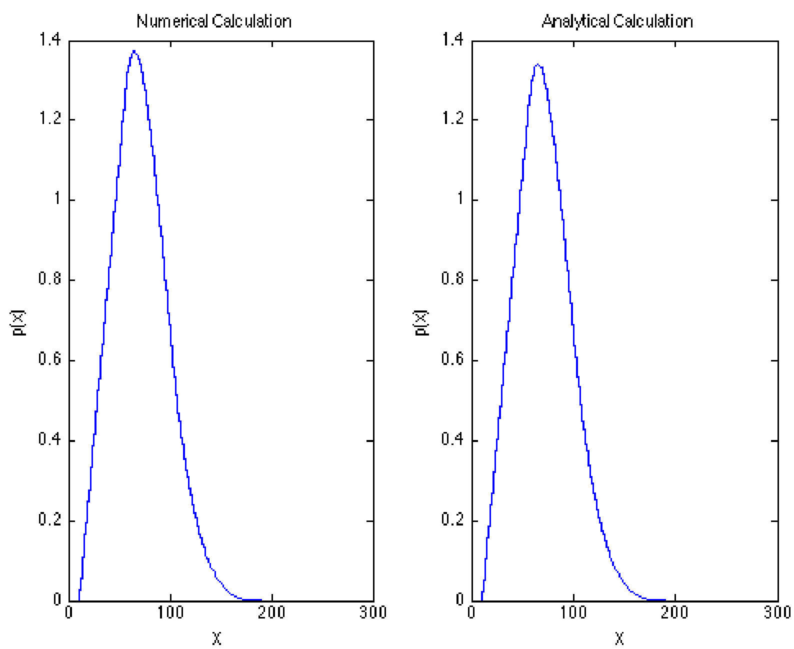

Figure 1: Marginal Distribution of x

(b) The root-mean-square discrepancy between the numerical and the analytical calcuations is 0.0155.

(c)

```
x = linspace(-0.1, 2.1, 201);
y = linspace(0, 1, 101);
P = pXYa(x, y)
dy = 0.01
pxn = integrate(P, [], dy)

[nrows, ncols] = size(P)
pygxn = P ./ (ones(nrows, 1) * pxn)
pygxn(isnan(pygxn)) = 0
contour(x, y, pygxn, 20, 'Color', 'k')
```

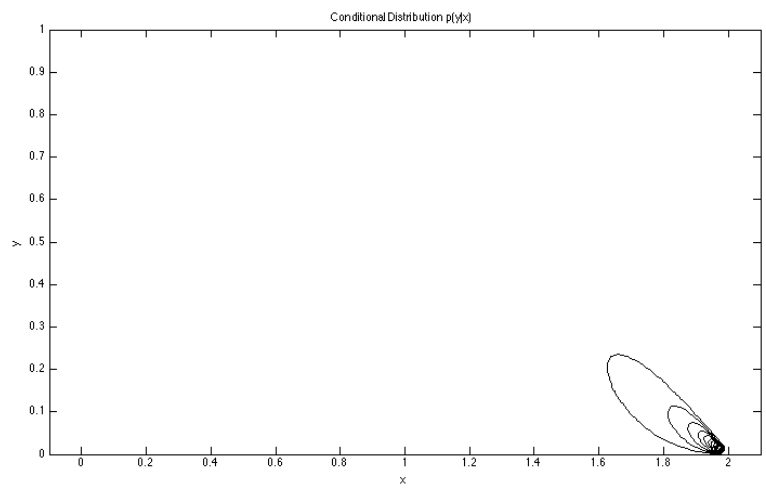

Figure 2: Conditional Distribution of $p(y|x)$

(d) Figure 2 above shows that X and Y are not independent because the probability of low values of Y is highly concentrated on the area where $1.6 < X < 2$. If the two variables were independent, the contours in the x direction would be horizontal lines, indicating that values of x do not give us information about the probability of y values.

(e) To compute samples of a joint probability distribution $p'(x, y)$ with the same marginals as above, I computed the marginals and multiplied them together.

This is consistent with the independence assumption. The code snippet and contour plot are shown below.

```
pxn = integrate(pxya, [], dy)
pyn = integrate(pxya, dx)
pxy_indep = pyn * pxn

ppygx = pxy_indep ./ (ones(nrows, 1) * pxn)
ppygx(isnan(ppygx)) = 0
contour(x, y, ppygx, 20, 'Color', 'k')
xlabel('x')
ylabel('y')
title('Conditional Distribution p(y|x) When Independent')
```

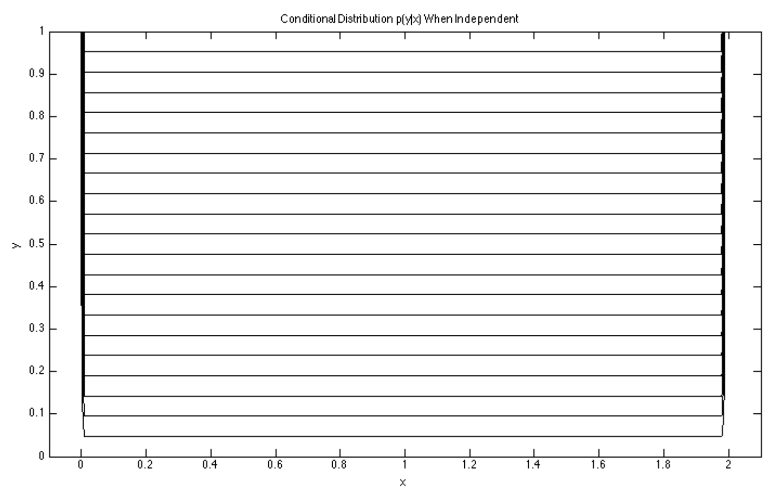

Figure 3: Conditional Distribution of $p(y|x)$ Under Independence$

(f) If there are $m+1$ samples in each of x and y, calculating the two-dimensional integral using Fubini's theorem and the trapezoidal rule takes m^2 sums.

(g) In general, computing a d-dimensional integral using Fubini's theorem and the trapezoidal rule takes m^d steps for $m+1$ samples.

(h) Since the complexity is exponential, using Fubini's theorem and the trapezoidal rule to compute high-dimensional integrals is very time consuming.

Problem 2

(a) The vectors in `cimg` are 12.25 ($\frac{28^2}{64}$) times shorter than the original images strung into vectors.

(b) A pair of images (original on the left, reconstructed on the right) is shown for each digit below.

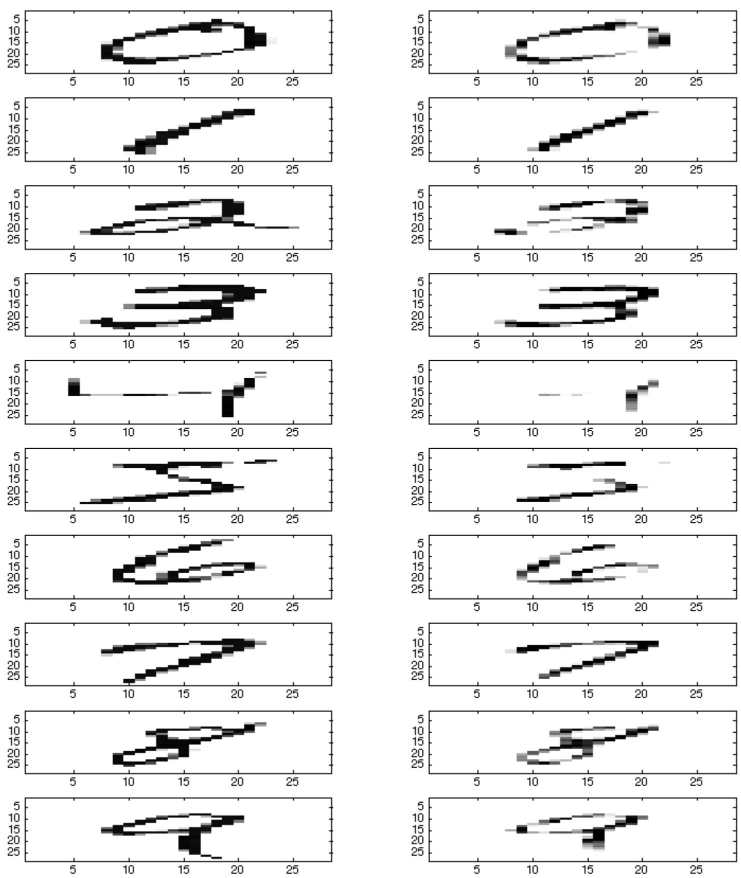

Figure 4: Original and Reconstructed Digits

(c) normalModel.m

```
function [likelihood, prior] = normalModel(X, L)
  X = double(X);
  [d, n] = size(X)
  labels = unique(L);
  l = size(labels, 2)
  m = zeros(d, l);
  s = zeros(d, l);
  prior = zeros(l, 1);
  size(prior)
  for i=1:l
    x = X(:, L==labels(i));
    k = size(x, 2);
    m(:, i) = mean(transpose(x));
    s(:, i) = std(transpose(x)).^2;
    prior(i, :) = k/n
  end
  likelihood = struct('M', m, 'S', s)
end
```

(d) The figure below shows samples drawn at random from the generative model, along with their labels.

(e) normalValue.m

```
function [v, delta]  = normalValue(X, m, s)
  [d, n] = size(X)
  mu = m * ones(1, n);
  diffsq = (X - mu).^2;
  quot = diffsq ./ (s * ones(1,n));
  delta = sqrt(sum(quot, 1));
  Sigma = diag(s);
  v = mvnpdf(transpose(X), transpose(m), Sigma);
end
```

(f) distances.m

```
function D = distances(L)
  [l, d] = size(L.M);
  D = zeros(l);
  for i = 1:l
    for j = 1:l
      s = diag( L.S(j, :) );
      D(i, j) = mahal( L.M(i, :), L.M(j, :),  s) ;
```

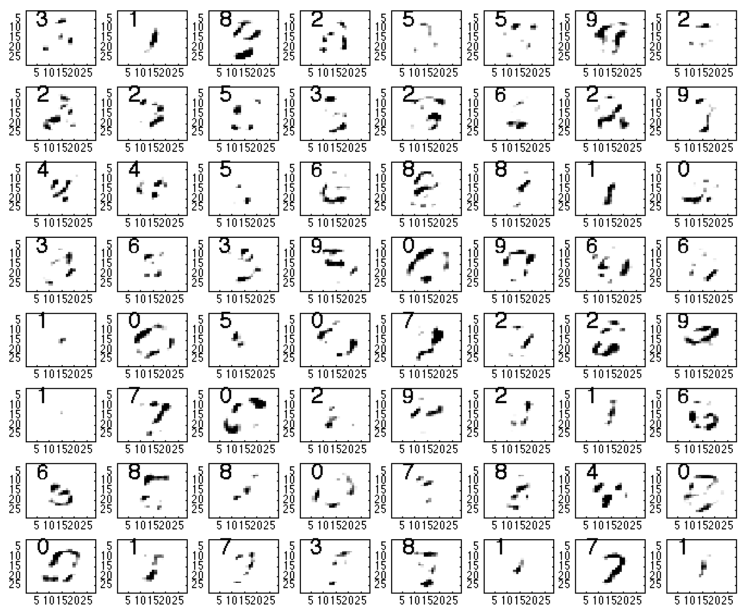

Figure 5: Sampled Images

```
    end
  end
end

function m = mahal(mu1, mu2, s2)
  m = sqrt( sum(( (mu1-mu2).^2 ) / s2 ));
end
```

The values for D are appended at the end of this submission.

(g) classify.m

```
function label = classify(img, likelihood, prior)
  compressed = code(img);
  n = size(compressed, 2);
  d = size(likelihood.M, 2);
  post = zeros(d, n);
  for i=1:d
    [v, delta] = normalValue(compressed, likelihood.M(:, i), likelihood.S(:, i));
    post(i, :) = v * prior(i);
  end
  label = zeros(1, n);
  for j=1:n
    col = post(:, j);
    argmax = find( col == max(col));
    label(j) = argmax-1;
  end
end
```

(h) errorStats.m

```
function [E, errorRate, pCgT] = errorStats(computedLabel, trueLabel)
  labels = unique(computedLabel);
  n = size(computedLabel, 2);
  l = size(labels, 2);
  E = zeros(l, l);
  pCgT = zeros(l, l);
  for i=1:l
    for j=1:l
      E(i, j) = sum( (trueLabel == i-1) & (computedLabel == j-1) );
      pCgT(i, j) = sum((computedLabel==i-1) & (trueLabel==j-1))/sum(trueLabel==j-1);
    end
  end
  errorRate = 1 - (sum(diag(E))/n);
end
```

Output:

The error rate is 12.3%.

```
E =

  Columns 1 through 7

        928           0           9           3           1          21          12
          0        1074          13           9           1          11           6
         19           1         884          32          13           6          11
          4           1          19         877           2          50           3
          1           2          19           0         851          10          14
          4           1          11          67           8         743          12
         13           3          14           3          12          40         870
          3          22          40           2          17          15           5
          5           0          16          34           9          43           2
         13           2          28           9          69          13           1

  Columns 8 through 10

          2           3           1
          1          20           0
         16          46           4
         15          27          12
          3           7          75
          9          33           4
          0           2           1
        859          16          49
          8         845          12
         23          10         841

errorRate =

    0.1228

pCgT =

  Columns 1 through 8

    0.947        0    0.018    0.004    0.001    0.004    0.014    0.003
    0        0.946    0.001    0.001    0.002    0.001    0.003    0.021
    0.009    0.011    0.857    0.019    0.019    0.012    0.015    0.039
    0.003    0.008    0.031    0.868        0    0.075    0.003    0.002
    0.001    0.001    0.013    0.002    0.867    0.009    0.013    0.017
    0.021    0.010    0.006    0.050    0.010    0.833    0.042    0.015
```

```
   0.012     0.005     0.011     0.003     0.014     0.013     0.908     0.005
   0.002     0.001     0.016     0.015     0.003     0.010         0     0.836
   0.003     0.018     0.045     0.027     0.007     0.037     0.002     0.016
   0.001         0     0.004     0.012     0.076     0.004     0.001     0.048

 Columns 9 through 10

   0.005     0.013
       0     0.002
   0.016     0.028
   0.035     0.009
   0.009     0.068
   0.044     0.013
   0.002     0.001
   0.008     0.023
   0.868     0.010
   0.012     0.833
```

(i) Given the instructions, column `j` in `pCgT` indicates values where the true digit was `j-1`. Thus, each column of `pCgT` should sum to one, as they do.

(j) If the classifier's error rate is p and errors on different digits are independent, the probability that the classifier gets a five-digit zip code wrong is $p_Z = 1-(1-p^5)$. For my error rate, $p = 0.123$, $p_Z = 1-(1-0.123)^5 = 0.481$. For the best available rate today, $p = 0.002$, $p_Z = 1-(1-0.002)^5 \approx 0.001$.

(k) If the state-of-the-art digit classifier were used, approximately 3,984,032 zip codes would be misclassified in the US each day.

(l) The posterior $p(\hat{w}|x)$ can take on values between 0.1 and 1. Because $\hat{w}$ is the argmax of a variable with 10 possible values, it must be at least 0.1. Because it is a probability, its upper bound is 1.

(m) An automatic zip code scanner could use the posterior value to determine whether there is sufficient confidence int he automated classification, or if the zip code needs further review (eg by a human). Some threshold, such as $\prod_{i=1}^{5} p(\hat{w}_i|x_i) < 0.9$, could be chosen so that misclassifications by the automated system are kept to a minimum while still saving the time of human reviewers.

(n) The assumption that errors on adjacent digits are mutually independent is not valid. First, all five digits in a handwritten zip code are typically generated by the same human hand. If a person has bad handwriting (1's that can easily be mistaken for 7's, for example), then errors will be correlated across digits they write. Secondly, it's possible that one digit will overlap or be smeared with a subsequent digit, making them both difficult to classify.

D =

Columns 1 through 8

0	7.18	4.86	7.19	5.26	4.35	4.93	7.52
5.72	0	3.49	5.65	5.46	4.48	5.46	6.06
6.05	4.53	0	4.53	3.30	2.88	3.57	3.83
7.24	5.26	3.81	0	4.39	3.59	3.75	3.80
8.01	6.00	2.86	4.35	0	3.56	3.32	3.62
6.86	4.80	2.13	3.62	3.35	0	3.26	3.35
6.95	5.05	2.48	3.06	2.82	2.32	0	3.15
7.26	5.04	2.64	3.08	3.08	2.88	3.03	0
5.77	4.52	1.78	2.80	2.99	1.81	1.96	2.69
6.02	4.17	1.49	3.08	2.84	2.58	2.53	2.66
6.50	4.72	1.73	2.70	2.88	1.94	2.39	2.06
6.72	4.52	1.83	2.58	2.99	2.30	2.98	2.23
6.98	4.63	1.88	2.85	2.58	2.39	2.05	1.89
6.32	4.62	1.85	3.05	2.89	2.12	2.13	1.90
5.38	4.31	1.61	3.16	2.93	2.35	2.29	2.98
5.93	4.42	1.76	2.88	2.77	2.35	2.69	2.24
5.92	4.26	1.86	2.77	2.74	2.43	2.08	2.13
6.72	4.23	1.76	2.67	2.72	2.04	2.34	1.84
6.31	4.32	1.66	2.64	2.63	2.02	2.11	2.45
6.61	4.58	1.67	2.81	2.67	1.87	2.26	2.06
6.33	4.39	1.64	2.73	2.73	2.06	2.12	2.11
6.41	4.37	1.66	2.78	2.52	1.97	2.16	2.13
5.76	4.19	1.56	3.01	2.91	2.14	2.33	2.30
6.23	4.25	1.61	2.81	2.66	1.92	2.24	2.14
6.37	4.47	1.64	2.77	2.65	1.96	2.09	2.06
6.53	4.42	1.77	2.74	2.43	2.18	2.18	2.04
6.03	4.29	1.67	2.82	2.73	1.98	1.95	2.24
6.05	4.32	1.60	2.83	2.75	1.91	2.25	2.14
6.41	4.45	1.70	2.74	2.63	2.00	2.10	1.94
6.27	4.43	1.68	2.77	2.64	2.12	2.21	1.94
6.26	4.42	1.70	2.70	2.64	2.03	1.84	2.25
6.31	4.27	1.63	2.80	2.65	2.09	2.27	1.95
6.39	4.45	1.75	2.70	2.55	2.03	2.02	2.01
6.18	4.33	1.61	2.81	2.68	1.99	2.04	2.17
6.37	4.34	1.67	2.78	2.67	2.09	2.21	1.91
6.28	4.33	1.66	2.73	2.66	2.07	2.06	2.11
6.23	4.40	1.67	2.75	2.64	2.02	2.03	2.09
6.37	4.37	1.72	2.67	2.60	2.01	2.07	2.03
6.36	4.42	1.69	2.74	2.62	2.11	2.16	2.01
6.23	4.36	1.65	2.77	2.65	2.01	2.21	2.09
6.30	4.33	1.62	2.75	2.69	2.06	2.23	2.08
6.42	4.44	1.69	2.76	2.57	2.13	2.14	1.95
6.42	4.44	1.71	2.72	2.59	1.99	2.08	1.98

6.32	4.34	1.69	2.70	2.63	2.11	2.15	2.04
6.14	4.35	1.65	2.76	2.67	2.09	2.07	2.15
6.25	4.36	1.66	2.73	2.65	2.06	2.17	2.10
6.23	4.31	1.63	2.77	2.68	2.00	2.06	2.11
6.39	4.38	1.69	2.71	2.59	2.04	2.13	1.99
6.35	4.40	1.68	2.72	2.62	2.01	2.17	2.
6.27	4.31	1.69	2.69	2.64	2.08	2.13	2.08
6.39	4.44	1.71	2.69	2.61	2.10	2.07	2.05
6.31	4.37	1.69	2.71	2.62	2.04	2.10	2.04
6.31	4.37	1.68	2.72	2.61	2.12	2.20	2.02
6.26	4.32	1.64	2.74	2.63	2.00	2.22	2.12
6.30	4.34	1.64	2.75	2.64	1.99	2.14	2.06
6.34	4.37	1.65	2.72	2.60	2.03	2.17	2.09
6.42	4.39	1.69	2.71	2.59	2.08	2.16	1.99
6.33	4.38	1.64	2.77	2.61	1.98	2.19	2.00
6.34	4.39	1.68	2.71	2.62	2.02	2.14	2.02
6.34	4.37	1.68	2.70	2.63	2.06	2.19	2.02
6.33	4.37	1.65	2.73	2.61	2.04	2.20	2.08
6.30	4.36	1.66	2.72	2.64	2.05	2.16	2.08
6.31	4.38	1.69	2.72	2.63	2.06	2.11	2.03
6.34	4.35	1.67	2.71	2.61	2.02	2.13	2.08

Columns 9 through 16

4.31	8.17	5.52	5.54	8.42	6.43	5.41	8.90
4.64	4.42	5.44	5.46	6.32	6.14	5.83	6.84
2.33	2.20	2.42	2.71	3.11	3.63	3.16	3.25
2.97	3.81	3.39	3.40	4.00	4.19	4.90	4.04
3.09	4.63	3.78	3.69	3.73	4.28	4.27	4.94
2.11	3.43	2.54	2.94	3.50	3.12	3.91	3.17
1.86	2.39	2.61	2.99	2.35	2.38	2.85	3.44
2.12	2.74	1.98	2.44	2.20	2.13	3.31	2.77
0	1.49	0.80	1.62	1.87	1.34	1.09	1.25
1.18	0	1.23	1.31	1.41	1.67	1.56	1.37
0.72	1.41	0	1.08	1.25	1.27	1.47	1.25
1.41	1.37	1.03	0	1.55	1.97	2.02	1.81
1.13	1.26	0.99	1.39	0	1.17	1.81	1.84
0.97	1.40	1.14	1.88	1.31	0	2.12	1.30
0.74	1.40	1.14	1.76	2.24	1.78	0	1.47
0.90	1.22	0.95	1.47	1.66	1.23	1.28	0
0.95	1.10	1.15	1.52	1.44	1.41	1.27	1.31
1.01	1.25	0.84	1.00	0.93	1.40	1.66	1.54
0.72	1.03	0.92	1.06	1.22	1.47	1.33	1.18
1.06	1.66	0.99	1.08	1.41	1.83	1.74	1.91
0.81	1.13	0.76	1.07	1.16	1.52	1.24	1.47
0.69	1.28	0.77	1.29	1.18	1.28	1.17	1.10

0.84	0.94	0.93	1.22	1.57	1.17	1.43	1.11
0.76	1.11	0.84	1.09	1.23	1.18	1.44	1.05
0.63	1.15	0.57	1.13	1.03	1.15	1.24	1.16
0.92	1.14	0.96	1.13	0.87	1.16	1.54	1.18
0.63	1.04	0.94	1.32	1.29	1.04	1.32	1.12
0.64	1.22	0.67	1.15	1.43	1.24	1.21	1.01
0.70	1.24	0.62	1.21	1.05	1.20	1.23	1.24
0.77	1.10	0.68	1.07	1.06	1.13	1.29	1.04
0.70	1.08	0.93	1.21	1.10	1.30	1.35	1.36
0.85	1.08	0.77	1.06	1.10	1.29	1.33	1.16
0.69	1.16	0.75	1.17	0.96	1.07	1.31	1.11
0.75	1.07	0.90	1.13	1.18	1.28	1.38	1.28
0.89	1.09	0.81	1.01	1.00	1.23	1.51	1.28
0.76	1.00	0.83	1.07	1.05	1.26	1.32	1.23
0.66	1.10	0.75	1.15	1.10	1.15	1.24	1.13
0.78	1.20	0.84	1.06	1.07	1.29	1.40	1.28
0.73	0.99	0.66	1.08	0.92	1.04	1.30	1.
0.67	1.05	0.66	1.08	1.14	1.10	1.24	0.91
0.76	0.99	0.66	0.96	1.07	1.28	1.27	1.13
0.78	1.06	0.68	1.08	0.88	1.12	1.30	1.13
0.78	1.22	0.79	1.05	1.01	1.21	1.45	1.27
0.79	1.03	0.78	1.01	1.02	1.26	1.32	1.17
0.66	1.01	0.74	1.16	1.18	1.25	1.08	1.11
0.71	1.01	0.71	1.01	1.09	1.19	1.27	1.03
0.72	1.03	0.81	1.12	1.11	1.21	1.31	1.20
0.78	1.16	0.76	1.03	1.02	1.25	1.36	1.22
0.73	1.15	0.67	1.02	1.05	1.19	1.32	1.11
0.71	1.04	0.75	1.08	1.09	1.24	1.20	1.08
0.74	1.04	0.72	1.04	0.91	1.18	1.32	1.19
0.72	1.10	0.75	1.07	1.04	1.18	1.30	1.12
0.78	1.05	0.73	1.01	1.03	1.23	1.27	1.08
0.71	1.09	0.71	1.02	1.17	1.24	1.25	1.
0.72	1.11	0.72	1.07	1.11	1.26	1.27	1.17
0.74	1.09	0.73	1.00	1.05	1.25	1.31	1.12
0.80	1.10	0.72	0.99	0.95	1.24	1.34	1.19
0.75	1.19	0.70	1.05	1.12	1.25	1.31	1.16
0.73	1.14	0.70	1.04	1.05	1.22	1.29	1.15
0.77	1.10	0.72	0.98	1.05	1.25	1.33	1.14
0.73	1.07	0.67	0.99	1.06	1.24	1.26	1.07
0.71	1.04	0.69	1.02	1.06	1.22	1.25	1.08
0.74	1.06	0.75	1.06	1.01	1.15	1.34	1.12
0.74	1.09	0.75	1.02	1.06	1.27	1.31	1.17

Columns 17 through 24

7.60	8.54	11.42	8.57	11.23	10.40	10.33	9.40

6.40	6.93	8.00	7.05	7.29	7.77	7.51	7.37
3.29	3.07	3.20	3.54	3.40	3.77	3.46	3.77
4.22	3.99	4.30	5.04	4.61	4.92	5.39	5.
4.57	4.58	5.12	4.57	5.61	4.80	6.03	5.20
3.64	2.87	3.18	3.16	3.53	3.26	4.17	3.39
2.82	3.25	3.21	2.85	2.97	3.12	3.79	3.13
2.75	2.36	3.62	2.92	2.90	3.06	3.60	3.15
1.42	1.72	1.48	1.81	1.46	1.37	1.42	1.38
1.41	1.48	1.26	2.11	1.57	1.77	1.23	1.50
1.53	1.12	1.31	1.26	1.00	0.98	1.75	1.22
2.01	1.27	1.80	1.39	1.47	1.90	2.21	1.60
1.54	1.15	1.62	1.79	1.52	1.46	2.28	1.54
1.67	1.77	1.84	2.07	1.51	1.36	1.59	1.48
1.31	1.94	2.07	2.02	1.99	1.87	1.75	1.80
1.30	1.61	1.41	1.91	1.67	1.27	1.42	1.27
0	1.31	1.36	1.70	1.15	1.35	1.17	1.28
1.29	0	1.11	1.09	0.85	0.99	1.76	0.90
1.24	1.02	0	1.25	0.83	0.88	1.35	0.73
1.53	1.06	1.37	0	0.92	1.32	1.99	1.23
0.94	0.82	0.90	0.78	0	0.88	1.33	0.89
1.11	0.88	0.75	1.15	0.85	0	1.56	0.76
1.00	1.21	1.39	1.52	1.19	1.44	0	0.82
1.09	0.82	0.66	1.10	0.80	0.80	1.00	0
1.03	0.83	0.77	0.97	0.51	0.53	1.31	0.61
1.23	1.02	0.80	1.37	1.08	0.90	1.57	0.69
0.84	1.10	0.88	1.33	0.91	1.00	0.77	0.55
0.94	0.93	0.97	1.02	0.70	0.77	0.90	0.54
0.95	0.80	0.94	0.95	0.51	0.48	1.44	0.75
0.85	0.81	0.88	1.05	0.60	0.73	1.09	0.55
0.94	1.01	0.61	1.10	0.66	0.97	1.20	0.70
0.88	0.59	0.84	0.97	0.51	0.70	1.12	0.53
0.95	0.89	0.70	1.12	0.70	0.58	1.33	0.53
0.92	0.87	0.71	0.98	0.63	0.96	0.94	0.48
0.98	0.60	0.89	0.98	0.58	0.92	1.16	0.54
0.85	0.74	0.63	1.02	0.48	0.82	1.07	0.51
0.83	0.88	0.72	1.03	0.53	0.70	1.05	0.48
0.96	0.71	0.68	0.89	0.56	0.77	1.28	0.50
0.95	0.82	0.70	1.20	0.68	0.65	1.20	0.53
0.95	0.84	0.68	1.12	0.67	0.60	1.03	0.36
0.94	0.63	0.67	0.97	0.43	0.71	1.10	0.50
0.94	0.75	0.79	1.07	0.61	0.63	1.32	0.58
1.04	0.75	0.76	0.88	0.63	0.79	1.32	0.47
0.87	0.68	0.66	1.02	0.50	0.77	1.14	0.50
0.70	0.90	0.76	1.09	0.50	0.72	0.98	0.63
0.91	0.78	0.61	1.07	0.57	0.72	1.02	0.39
0.86	0.77	0.67	1.01	0.51	0.79	0.98	0.43

0.95	0.66	0.71	0.90	0.52	0.69	1.28	0.47
0.97	0.71	0.72	0.94	0.53	0.63	1.22	0.43
0.80	0.73	0.61	1.07	0.50	0.62	1.11	0.51
0.94	0.78	0.66	1.05	0.55	0.75	1.27	0.57
0.89	0.75	0.65	1.00	0.53	0.67	1.14	0.42
0.87	0.72	0.69	1.02	0.55	0.68	1.16	0.50
0.98	0.76	0.58	1.01	0.60	0.61	1.09	0.32
0.91	0.69	0.68	0.90	0.42	0.61	1.15	0.45
0.99	0.72	0.57	0.96	0.55	0.65	1.21	0.39
0.97	0.64	0.71	0.95	0.52	0.68	1.32	0.52
0.97	0.70	0.76	0.84	0.49	0.60	1.22	0.45
0.93	0.71	0.70	0.92	0.48	0.63	1.22	0.46
0.94	0.68	0.69	0.94	0.51	0.72	1.18	0.45
0.98	0.71	0.60	0.97	0.53	0.61	1.19	0.42
0.91	0.72	0.61	1.00	0.48	0.65	1.12	0.44
0.89	0.75	0.67	1.05	0.56	0.74	1.11	0.43
0.96	0.69	0.58	0.94	0.50	0.67	1.20	0.42

Columns 25 through 32

12.76	9.40	8.85	8.10	14.00	9.90	10.85	10.79
8.51	9.03	7.75	7.81	9.93	8.72	9.56	8.34
3.62	4.04	4.09	3.93	4.33	4.38	4.27	4.52
5.10	4.86	5.40	5.44	5.48	5.71	5.64	5.98
5.91	4.77	5.49	5.27	6.07	5.70	6.11	6.07
3.55	3.66	4.13	3.77	3.78	4.42	4.15	4.43
3.26	3.73	3.11	3.67	3.52	3.88	3.52	3.79
3.21	3.37	3.30	2.98	3.40	3.19	3.83	3.30
1.49	1.84	1.21	1.19	1.85	1.54	1.62	1.66
1.71	1.48	1.53	1.81	2.08	1.74	1.84	1.82
0.82	1.44	1.47	1.03	0.99	1.15	1.55	1.31
1.84	1.84	2.12	1.77	2.07	1.86	2.32	1.74
1.50	1.19	1.50	1.66	1.64	1.50	1.66	1.63
1.37	1.87	1.35	1.60	1.68	1.60	1.93	1.81
2.22	2.36	1.63	1.54	2.64	1.85	1.94	2.04
1.41	1.55	1.50	1.44	1.81	1.30	1.91	1.67
1.38	1.61	1.10	1.26	1.46	1.14	1.23	1.28
0.96	1.20	1.17	0.97	1.03	0.96	1.30	0.79
0.78	0.86	0.85	0.99	1.14	1.04	0.79	1.08
1.13	1.80	1.57	1.14	1.27	1.27	1.43	1.29
0.53	1.48	1.00	0.73	0.66	0.71	0.88	0.67
0.51	0.99	1.00	0.74	0.57	0.89	1.00	0.84
1.41	1.43	0.73	0.85	1.73	0.94	1.25	1.03
0.69	0.79	0.61	0.60	0.94	0.67	0.85	0.64
0	0.99	0.74	0.53	0.37	0.54	0.69	0.62
0.90	0	0.89	1.05	1.14	0.83	1.09	0.94

0.88	1.06	0	0.74	1.15	0.78	0.61	0.89
0.72	1.24	0.76	0	0.89	0.56	0.94	0.57
0.29	1.18	0.87	0.56	0	0.55	0.85	0.57
0.49	0.91	0.73	0.49	0.56	0	0.81	0.40
0.64	1.05	0.52	0.90	0.89	0.81	0	0.96
0.54	1.05	0.81	0.50	0.58	0.38	0.92	0
0.41	0.67	0.58	0.64	0.57	0.47	0.62	0.63
0.64	1.05	0.49	0.67	0.88	0.61	0.47	0.71
0.62	0.98	0.77	0.67	0.71	0.43	0.87	0.43
0.51	0.94	0.54	0.65	0.69	0.49	0.46	0.54
0.43	0.94	0.47	0.50	0.59	0.38	0.46	0.53
0.54	0.88	0.65	0.60	0.62	0.48	0.56	0.54
0.40	0.65	0.65	0.67	0.63	0.40	0.75	0.59
0.45	0.77	0.61	0.44	0.69	0.39	0.77	0.50
0.42	0.95	0.74	0.52	0.60	0.42	0.74	0.39
0.38	0.75	0.73	0.64	0.48	0.32	0.73	0.48
0.51	0.80	0.67	0.60	0.63	0.44	0.61	0.58
0.52	0.83	0.65	0.60	0.63	0.36	0.62	0.41
0.54	1.11	0.58	0.53	0.69	0.46	0.57	0.54
0.47	0.76	0.58	0.51	0.70	0.35	0.63	0.47
0.50	0.99	0.46	0.56	0.70	0.49	0.48	0.52
0.45	0.84	0.69	0.52	0.52	0.36	0.64	0.38
0.37	0.81	0.69	0.44	0.50	0.29	0.69	0.39
0.46	0.89	0.61	0.52	0.57	0.40	0.61	0.41
0.42	0.74	0.64	0.69	0.61	0.43	0.53	0.60
0.41	0.80	0.56	0.50	0.55	0.33	0.56	0.43
0.48	0.79	0.71	0.54	0.58	0.26	0.71	0.36
0.48	0.78	0.67	0.42	0.68	0.44	0.73	0.44
0.34	0.98	0.66	0.40	0.48	0.40	0.62	0.34
0.42	0.72	0.67	0.52	0.61	0.42	0.62	0.46
0.44	0.78	0.74	0.58	0.53	0.36	0.69	0.39
0.38	0.95	0.75	0.36	0.45	0.36	0.73	0.33
0.36	0.86	0.67	0.45	0.47	0.31	0.62	0.38
0.47	0.82	0.70	0.50	0.58	0.31	0.69	0.37
0.38	0.76	0.71	0.48	0.57	0.38	0.69	0.42
0.38	0.83	0.63	0.49	0.56	0.35	0.61	0.41
0.45	0.77	0.54	0.56	0.61	0.32	0.57	0.47
0.42	0.81	0.64	0.52	0.58	0.43	0.56	0.45

Columns 33 through 40

12.33	14.19	15.31	12.97	12.60	12.99	13.54	11.72
10.26	10.59	11.21	9.15	10.46	10.24	10.20	10.46
4.55	4.56	4.86	4.90	5.00	5.25	5.22	5.28
5.81	6.37	6.50	6.48	6.64	6.53	6.78	6.86
6.01	6.76	7.04	7.20	6.64	7.26	7.28	7.03

4.23	4.59	5.12	5.20	4.84	4.88	5.32	5.03
4.04	4.22	4.78	3.73	4.16	4.22	4.40	4.47
3.65	4.34	3.78	3.86	4.13	3.89	4.05	4.13
1.75	1.83	2.36	1.67	1.69	2.07	1.77	1.67
1.99	1.84	2.03	1.79	2.09	2.30	1.87	2.00
1.28	1.62	1.51	1.57	1.38	1.73	1.31	1.38
2.37	2.47	2.19	1.99	2.43	2.19	2.18	2.21
1.52	2.14	1.91	1.76	1.87	2.06	1.58	1.82
1.65	1.83	1.88	1.97	1.91	2.11	1.72	1.85
2.44	2.69	3.08	2.25	2.35	2.38	2.45	2.01
1.67	2.06	1.99	1.95	1.83	1.94	1.70	1.49
1.40	1.49	1.67	1.39	1.37	1.42	1.54	1.52
1.15	1.44	0.99	1.08	1.32	1.14	1.11	1.16
0.91	0.90	1.17	0.83	1.02	1.01	0.98	0.95
1.52	1.55	1.42	1.51	1.51	1.29	1.79	1.73
0.97	0.81	0.80	0.65	0.73	0.85	0.95	1.08
0.69	1.22	1.24	1.11	0.93	1.08	0.93	0.82
1.50	1.22	1.55	1.09	1.30	1.28	1.33	1.07
0.75	0.62	0.81	0.70	0.74	0.66	0.80	0.52
0.47	0.77	0.80	0.67	0.48	0.76	0.53	0.50
0.75	1.23	1.04	0.94	1.09	0.99	0.77	0.91
0.84	0.66	1.21	0.67	0.63	0.84	0.94	0.77
0.90	0.80	1.04	0.85	0.68	0.78	0.92	0.60
0.60	0.98	0.84	0.79	0.64	0.80	0.70	0.70
0.53	0.73	0.51	0.57	0.44	0.56	0.48	0.48
0.68	0.49	0.92	0.47	0.57	0.64	0.94	0.92
0.72	0.72	0.42	0.57	0.63	0.60	0.62	0.65
0	0.80	0.76	0.56	0.44	0.54	0.42	0.44
0.72	0	0.68	0.38	0.45	0.47	0.88	0.76
0.73	0.66	0	0.46	0.67	0.56	0.70	0.77
0.56	0.36	0.51	0	0.36	0.41	0.60	0.63
0.42	0.43	0.67	0.34	0	0.42	0.54	0.45
0.49	0.58	0.46	0.40	0.47	0	0.73	0.65
0.37	0.80	0.66	0.57	0.52	0.73	0	0.37
0.49	0.68	0.74	0.60	0.48	0.64	0.39	0
0.64	0.61	0.50	0.43	0.51	0.61	0.47	0.50
0.37	0.84	0.55	0.54	0.52	0.61	0.28	0.46
0.44	0.66	0.46	0.49	0.50	0.24	0.68	0.60
0.52	0.57	0.38	0.24	0.45	0.36	0.48	0.54
0.65	0.60	0.86	0.45	0.34	0.64	0.61	0.57
0.49	0.53	0.56	0.36	0.39	0.47	0.40	0.30
0.57	0.26	0.61	0.21	0.29	0.43	0.63	0.57
0.44	0.65	0.40	0.39	0.45	0.22	0.55	0.52
0.41	0.67	0.47	0.47	0.41	0.37	0.45	0.35
0.47	0.61	0.62	0.36	0.37	0.47	0.45	0.44
0.37	0.68	0.54	0.34	0.44	0.49	0.39	0.53

0.36	0.53	0.48	0.31	0.28	0.30	0.44	0.39
0.48	0.68	0.45	0.42	0.45	0.44	0.41	0.43
0.54	0.63	0.67	0.54	0.51	0.51	0.52	0.25
0.51	0.53	0.55	0.40	0.35	0.41	0.55	0.46
0.44	0.61	0.52	0.41	0.47	0.38	0.47	0.37
0.45	0.75	0.42	0.41	0.53	0.40	0.43	0.49
0.52	0.66	0.52	0.57	0.45	0.39	0.62	0.48
0.41	0.62	0.47	0.40	0.35	0.32	0.47	0.41
0.50	0.63	0.39	0.38	0.45	0.32	0.49	0.46
0.47	0.67	0.55	0.48	0.48	0.48	0.41	0.31
0.45	0.56	0.52	0.34	0.36	0.46	0.37	0.34
0.39	0.51	0.44	0.28	0.32	0.36	0.41	0.42
0.45	0.54	0.49	0.31	0.41	0.30	0.51	0.45

Columns 41 through 48

16.28	17.20	14.92	14.95	12.14	13.92	12.57	15.83
11.57	12.08	11.84	12.08	10.78	11.21	12.32	12.63
5.15	5.54	5.67	5.71	5.78	5.90	5.79	5.90
7.47	7.02	6.86	7.52	7.89	7.66	7.72	7.55
7.73	7.48	7.33	7.70	7.62	8.21	7.51	7.94
5.58	5.28	4.91	6.06	6.24	5.98	5.59	5.49
4.75	4.43	4.48	5.23	4.63	4.91	5.22	5.23
4.47	4.31	4.31	4.45	4.50	4.47	4.90	4.64
2.27	2.50	2.29	2.21	1.71	1.79	1.97	2.50
2.06	2.26	2.43	2.14	2.20	2.12	2.26	2.57
1.40	1.38	1.62	1.68	1.64	1.62	1.82	1.71
2.18	2.27	2.28	2.53	2.55	2.28	2.86	2.59
2.20	1.82	2.00	2.01	1.95	2.03	1.96	2.13
2.27	1.92	2.14	2.26	2.34	2.16	2.37	2.41
3.00	3.38	3.12	2.85	2.06	2.45	2.57	3.18
1.95	2.08	2.36	2.07	2.10	2.08	2.45	2.32
1.85	1.84	1.83	1.68	1.39	1.72	1.63	1.79
1.20	1.19	1.21	1.17	1.37	1.27	1.26	1.18
0.98	1.15	1.11	1.03	1.20	0.93	1.14	1.19
1.69	1.71	1.43	1.77	1.85	1.85	1.72	1.53
0.75	0.97	1.01	0.86	0.74	1.01	0.91	0.89
1.00	0.93	1.18	1.21	1.09	1.16	1.26	1.09
1.63	1.97	1.64	1.48	1.27	1.09	1.18	1.76
0.69	0.94	0.73	0.83	1.04	0.64	0.73	0.82
0.60	0.54	0.71	0.76	0.65	0.66	0.70	0.69
1.23	0.88	0.99	1.03	1.29	0.94	1.27	1.07
1.10	1.31	1.08	1.04	0.85	0.83	0.69	1.16
0.87	1.22	1.03	0.99	0.80	0.76	0.82	1.00
0.77	0.63	0.87	0.87	0.69	0.89	0.92	0.78
0.49	0.48	0.63	0.47	0.59	0.48	0.73	0.53

0.89	0.97	0.77	0.81	0.86	0.87	0.66	0.90
0.41	0.64	0.73	0.51	0.71	0.62	0.69	0.53
0.74	0.43	0.55	0.61	0.65	0.52	0.72	0.54
0.63	0.95	0.64	0.62	0.80	0.68	0.33	0.71
0.55	0.63	0.52	0.44	0.89	0.62	0.62	0.52
0.44	0.65	0.53	0.30	0.54	0.45	0.24	0.46
0.54	0.65	0.54	0.49	0.39	0.40	0.36	0.53
0.56	0.62	0.27	0.37	0.73	0.55	0.46	0.25
0.56	0.33	0.70	0.53	0.66	0.45	0.74	0.62
0.50	0.65	0.73	0.60	0.65	0.35	0.68	0.66
0	0.57	0.66	0.39	0.57	0.41	0.53	0.51
0.57	0	0.58	0.48	0.61	0.48	0.71	0.48
0.62	0.54	0	0.48	0.81	0.54	0.52	0.29
0.37	0.48	0.46	0	0.56	0.31	0.46	0.30
0.64	0.86	0.88	0.63	0	0.55	0.55	0.77
0.34	0.57	0.55	0.33	0.54	0	0.48	0.47
0.46	0.75	0.58	0.45	0.55	0.46	0	0.54
0.46	0.43	0.28	0.31	0.63	0.41	0.48	0
0.38	0.40	0.38	0.39	0.61	0.31	0.55	0.25
0.39	0.58	0.65	0.33	0.37	0.37	0.50	0.46
0.55	0.35	0.46	0.36	0.58	0.37	0.55	0.42
0.39	0.45	0.38	0.26	0.51	0.25	0.39	0.26
0.35	0.39	0.53	0.23	0.53	0.30	0.61	0.34
0.40	0.66	0.61	0.53	0.67	0.32	0.61	0.53
0.31	0.58	0.52	0.44	0.51	0.42	0.39	0.35
0.36	0.46	0.41	0.36	0.64	0.25	0.51	0.31
0.45	0.31	0.40	0.30	0.61	0.35	0.57	0.22
0.43	0.54	0.46	0.54	0.66	0.51	0.57	0.31
0.36	0.42	0.38	0.33	0.52	0.32	0.48	0.20
0.32	0.45	0.39	0.24	0.61	0.28	0.53	0.23
0.30	0.44	0.52	0.40	0.60	0.27	0.59	0.38
0.24	0.47	0.52	0.29	0.47	0.19	0.45	0.37
0.42	0.45	0.39	0.23	0.56	0.24	0.39	0.32
0.34	0.50	0.38	0.28	0.59	0.28	0.40	0.25

Columns 49 through 56

15.51	16.07	17.14	16.70	17.17	16.38	17.87	16.60
12.38	13.29	13.98	13.32	14.26	13.31	13.51	13.46
6.25	5.94	6.31	6.28	6.33	6.64	6.39	6.95
8.08	7.93	8.02	7.95	8.47	8.62	8.50	9.34
8.41	8.50	8.39	8.52	8.82	8.96	9.43	9.18
6.02	6.08	6.24	5.79	6.64	6.16	6.12	7.03
5.15	5.67	5.51	5.26	6.35	5.77	5.72	5.71
4.55	5.00	5.13	5.04	5.07	5.48	5.30	5.47
2.07	2.25	2.42	2.38	2.47	2.12	2.39	2.44

2.51	2.36	2.44	2.56	2.49	2.55	2.73	2.67
1.56	1.81	1.66	1.74	1.78	1.77	1.85	1.91
2.44	2.91	2.84	2.72	2.95	2.66	2.90	2.68
2.21	2.29	1.97	2.29	2.34	2.50	2.54	2.45
2.32	2.61	2.50	2.55	2.68	2.81	2.54	2.70
2.81	2.88	3.41	3.21	3.26	3.04	3.21	3.08
2.17	2.25	2.53	2.37	2.31	2.33	2.55	2.38
1.97	1.61	2.00	1.82	1.92	2.14	1.91	2.23
1.27	1.39	1.44	1.42	1.42	1.47	1.39	1.37
1.15	1.13	1.12	1.18	1.27	1.14	1.23	1.05
1.78	2.03	1.97	1.89	2.05	2.07	1.79	1.95
1.03	0.99	1.07	1.05	1.08	1.24	0.82	1.09
1.05	1.05	1.27	1.10	1.25	1.08	1.14	1.22
1.51	1.63	1.94	1.72	1.76	1.59	1.71	1.66
0.72	0.89	1.06	0.76	0.97	0.59	0.82	0.70
0.58	0.71	0.67	0.65	0.80	0.78	0.59	0.71
1.10	1.31	1.07	1.18	1.19	1.17	1.41	1.09
1.08	1.05	1.24	1.00	1.32	1.14	1.13	1.18
0.79	0.89	1.29	0.95	1.02	0.79	0.80	0.99
0.73	0.81	0.92	0.83	0.93	1.03	0.73	0.96
0.41	0.59	0.65	0.47	0.40	0.67	0.64	0.66
1.00	0.91	0.80	0.83	1.14	1.21	0.97	1.02
0.56	0.60	0.87	0.63	0.53	0.67	0.53	0.69
0.53	0.65	0.52	0.49	0.68	0.73	0.70	0.61
0.77	0.78	0.84	0.66	0.90	0.92	0.75	0.77
0.59	0.80	0.72	0.64	0.64	0.85	0.74	0.76
0.58	0.45	0.49	0.40	0.58	0.73	0.49	0.57
0.47	0.46	0.57	0.35	0.61	0.65	0.44	0.55
0.46	0.57	0.55	0.38	0.58	0.71	0.54	0.51
0.56	0.61	0.48	0.54	0.54	0.67	0.73	0.62
0.45	0.55	0.75	0.52	0.57	0.35	0.58	0.50
0.46	0.45	0.61	0.48	0.43	0.51	0.42	0.47
0.44	0.60	0.41	0.50	0.46	0.72	0.66	0.57
0.41	0.72	0.51	0.43	0.63	0.73	0.62	0.48
0.41	0.37	0.41	0.29	0.28	0.57	0.46	0.43
0.70	0.47	0.82	0.65	0.73	0.80	0.60	0.78
0.35	0.39	0.51	0.31	0.35	0.35	0.46	0.31
0.56	0.50	0.66	0.42	0.68	0.68	0.45	0.58
0.26	0.48	0.46	0.28	0.39	0.56	0.39	0.34
0	0.46	0.49	0.25	0.34	0.40	0.35	0.29
0.45	0	0.57	0.34	0.38	0.51	0.39	0.50
0.49	0.54	0	0.39	0.48	0.73	0.60	0.51
0.25	0.33	0.40	0	0.34	0.46	0.34	0.32
0.31	0.36	0.47	0.31	0	0.47	0.49	0.39
0.36	0.47	0.73	0.44	0.50	0	0.44	0.32
0.33	0.38	0.62	0.34	0.49	0.46	0	0.40

0.26	0.44	0.46	0.29	0.37	0.32	0.39	0
0.28	0.49	0.38	0.36	0.32	0.56	0.47	0.33
0.29	0.54	0.70	0.41	0.50	0.49	0.29	0.46
0.14	0.38	0.45	0.20	0.32	0.45	0.27	0.30
0.21	0.43	0.47	0.25	0.24	0.43	0.41	0.29
0.25	0.41	0.51	0.34	0.32	0.28	0.39	0.19
0.28	0.28	0.43	0.24	0.29	0.37	0.31	0.27
0.34	0.40	0.35	0.16	0.36	0.53	0.45	0.40
0.28	0.38	0.43	0.24	0.39	0.42	0.31	0.18

Columns 57 through 64

18.90	21.27	18.82	17.21	18.32	20.48	19.55	16.90
15.06	15.45	14.42	15.14	14.75	15.00	14.91	15.05
6.69	6.78	7.12	7.14	7.05	7.09	7.47	7.51
8.63	9.46	9.36	9.03	9.50	9.31	9.55	9.91
9.06	9.99	10.00	9.65	9.67	10.86	10.13	9.62
6.51	6.67	7.11	6.79	6.97	6.89	7.34	7.39
6.18	6.41	6.08	6.41	6.17	6.22	5.96	6.48
5.35	5.67	5.48	5.42	5.73	5.86	5.70	6.08
2.97	2.93	2.58	2.62	2.71	2.82	2.80	2.74
2.78	3.13	2.93	2.82	2.81	2.88	2.86	2.98
1.84	1.86	1.95	1.98	1.84	2.03	2.11	2.12
2.88	3.20	2.99	2.90	2.83	3.02	3.05	3.09
2.29	2.93	2.65	2.54	2.62	2.80	2.63	2.57
2.75	2.83	2.70	3.04	2.93	2.82	2.66	3.09
3.84	3.99	3.34	3.19	3.35	3.41	3.66	3.50
2.71	2.58	2.46	2.61	2.37	2.40	2.62	2.93
2.16	2.39	2.13	2.09	2.31	2.06	2.13	2.28
1.27	1.59	1.45	1.45	1.49	1.59	1.63	1.43
1.33	1.48	1.31	1.38	1.18	1.22	1.33	1.22
1.91	1.84	1.93	2.08	2.11	2.17	2.28	2.04
1.08	0.98	0.97	1.24	1.20	1.09	1.23	1.09
1.27	1.14	1.26	1.38	1.13	1.23	1.46	1.42
2.16	2.28	1.85	1.66	1.92	1.87	1.83	1.83
1.04	0.94	0.90	0.90	0.81	0.88	0.88	0.85
0.75	0.67	0.67	0.87	0.70	0.69	0.85	0.80
1.20	1.51	1.32	1.40	1.30	1.44	1.25	1.31
1.53	1.53	1.29	1.28	1.32	1.20	1.10	1.21
1.30	1.04	0.96	0.98	1.04	1.03	1.18	1.09
0.88	0.77	0.77	1.02	0.96	0.93	1.06	1.08
0.62	0.60	0.52	0.51	0.63	0.60	0.53	0.78
1.13	1.23	1.05	1.13	1.15	0.99	0.93	1.02
0.65	0.54	0.59	0.64	0.71	0.69	0.75	0.77
0.67	0.78	0.60	0.80	0.70	0.69	0.58	0.70
1.00	0.92	0.82	0.81	0.91	0.82	0.72	0.78

0.63	0.76	0.70	0.60	0.85	0.84	0.65	0.80
0.62	0.75	0.56	0.56	0.67	0.47	0.42	0.46
0.73	0.66	0.47	0.61	0.65	0.51	0.44	0.55
0.50	0.58	0.44	0.45	0.64	0.65	0.51	0.42
0.61	0.82	0.67	0.75	0.59	0.56	0.56	0.75
0.78	0.68	0.58	0.66	0.47	0.48	0.62	0.68
0.52	0.58	0.50	0.48	0.40	0.35	0.59	0.48
0.41	0.69	0.53	0.59	0.57	0.57	0.55	0.71
0.48	0.58	0.47	0.48	0.63	0.70	0.52	0.49
0.35	0.63	0.40	0.28	0.49	0.38	0.29	0.36
0.97	0.94	0.72	0.81	0.83	0.65	0.78	0.79
0.57	0.63	0.41	0.38	0.34	0.26	0.32	0.38
0.77	0.73	0.58	0.62	0.68	0.51	0.47	0.51
0.25	0.41	0.23	0.27	0.45	0.48	0.41	0.29
0.33	0.31	0.15	0.25	0.29	0.34	0.40	0.36
0.57	0.65	0.44	0.47	0.49	0.32	0.47	0.47
0.40	0.77	0.51	0.54	0.57	0.47	0.40	0.50
0.41	0.47	0.23	0.30	0.39	0.30	0.19	0.27
0.35	0.53	0.34	0.24	0.36	0.35	0.37	0.47
0.65	0.54	0.46	0.48	0.31	0.40	0.60	0.46
0.52	0.32	0.29	0.45	0.43	0.34	0.53	0.36
0.36	0.47	0.30	0.31	0.19	0.30	0.41	0.20
0	0.54	0.31	0.27	0.39	0.43	0.45	0.36
0.48	0	0.31	0.44	0.46	0.51	0.63	0.50
0.32	0.30	0	0.25	0.33	0.31	0.38	0.29
0.30	0.43	0.23	0	0.32	0.34	0.34	0.30
0.38	0.44	0.31	0.33	0	0.23	0.48	0.35
0.42	0.49	0.29	0.31	0.24	0	0.34	0.28
0.43	0.59	0.36	0.35	0.47	0.35	0	0.37
0.36	0.46	0.26	0.28	0.30	0.28	0.36	0