Homework 1

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Problem 1

$$\begin{aligned}
X &\sim Unif(0,1) \\
m &= \mathbb{E}(X) = 0.5 \\
\mathbb{E}(|X-m|) &= \mathbb{E}(|X-0.5|) \\
&= -E(X_{x\leq 0.5} - 0.5) + E(X_{x>0.5} - 0.5) \\
&= -(-0.25) + 0.25 \\
&= 0.5
\end{aligned}$$

Problem 2

By Bayes' Theorem, we can compute the posterior probability that coin 2 was chosen by:

$$\begin{aligned}
P(c=2|h=1) &= \frac{P(h=1|c=2)P(c=2)}{P(h=1)} \\
&= \frac{P(h=1|c=2)P(c=2)}{P(h=1|c=2)P(c=2)+P(h=1|c=1)P(c=1)} \\
&= \frac{0.8\times 0.5}{0.8\times 0.5 + 0.5\times 0.5} \\
&= \frac{0.4}{0.65} \\
&\approx 0.615
\end{aligned}$$

Problem 3

Given Equation 2.3 and the fact that x and y are independent, we can show that $Pr(x|y=y^*) = Pr(x)$ by:

$$\begin{aligned}
Pr(x|y = y^*) &= \frac{Pr(x, y = y^*)}{\int Pr(x, y = y^* dx)} \\
&= \frac{Pr(x, y = y^*)}{Pr(y = y^*)} \\
&= \frac{Pr(x) \cdot Pr(y = y^*)}{Pr(y = y^*)} \\
&= Pr(x)
\end{aligned}$$

Problem 4

The expected value of one roll of this biased die, x, is:

$$\begin{aligned}
\mathbb{E}(x) &= 1 \cdot \frac{1}{12} + 2 \cdot \frac{1}{12} + 3 \cdot \frac{1}{12} + 4 \cdot \frac{1}{12} + 5 \cdot \frac{1}{6} + 6 \cdot \frac{1}{12} \\
&= \frac{10}{12} + \frac{5}{6} + \frac{6}{2} \\
&= \frac{56}{12} \\
&= \frac{14}{3} \\
&\approx 4.67
\end{aligned}$$

The expected value of the sum of two rolls is:

$$\begin{aligned}
\mathbb{E}(2x) &= 2\mathbb{E}(x) \\
&= 2(\frac{14}{3}) \\
&= \frac{28}{3} \\
&\approx 9.34
\end{aligned}$$

Problem 5

Using the relations given in Exercise 2.9, we can show that $\mathbb{E}[(x - \mu)^2] = \mathbb{E}[x^2] - E[x]E[x]$ by:

$$
\begin{aligned}
\mathbb{E}[(x-\mu)^2] &= \mathbb{E}[x^2 - 2x\mu + \mu^2] \\
&= \mathbb{E}[x^2] - \mathbb{E}[2x\mu] + \mathbb{E}[\mu^2] \\
&= \mathbb{E}[x^2] - 2\mathbb{E}[x\mu] + \mathbb{E}[x]\mathbb{E}[x] \\
&= \mathbb{E}[x^2] - 2\mathbb{E}[x]\mathbb{E}[x] + \mathbb{E}[x]\mathbb{E}[x] \\
\mathbb{E}[(x-\mu)^2] &= \mathbb{E}[x^2] - E[x]E[x]
\end{aligned}
$$

Problem 6

(a) isProbability.m:

```
% indicate whether a given matrix P is a valid probability distribution
function valid = isProbability(P)
  if ~ismatrix(P)
      error('Input must be a matrix')
  end
  nonnegative = all(all(P >= 0));
  total = sum(sum(P, 1));
  normalized = (abs(total - 1) <= 0.0001);
  valid = nonnegative & normalized;
end
```

(b)

```
% check validity of some matrices
>> isProbability([0 1; 1 0])

ans =

     0

>> isProbability([0 -0.2; 0.7 0.5])

ans =

     0

>> isProbability([1 2 1; 3 0 1] / 8)

ans =

     1
```

(c) marginals.m:

```
% compute marginal distributions from a joint probability distribution
function [Px, Py] = marginals(P)
  if ~isProbability(P)
      error('Input must be a valid probability matrix')
  end
  Px = sum(P, 2);
  Py = sum(P, 1);
end
```

(d)

```
>> P = [0 1 2 1; 0 9 0 3] / 16;
>> [Px, Py] = marginals(P)

Px =

    0.2500
    0.7500


Py =

         0    0.6250    0.1250    0.2500
```