Homework 1

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Problem 1

todo

$$
\begin{aligned}
X &\sim Unif(0,1) \\
m &= \mathbb{E}(X) = 0.5 \\
\mathbb{E}(|X-m|) &= \mathbb{E}(|X-0.5|) \\
&= -E(X_{x\leq 0.5} - 0.5) + E(X_{x>0.5} - 0.5) \\
&= -(-0.25) + 0.25 \\
&= 0.5
\end{aligned}
$$

Problem 2

By Bayes' Theorem, we can compute the posterior probability that coin 2 was chosen by:

$$
\begin{aligned}
P(c=2|h=1) &= \frac{P(h=1|c=2)P(c=2)}{P(h=1)} \\
&= \frac{P(h=1|c=2)P(c=2)}{P(h=1|c=2)P(c=2)+P(h=1|c=1)P(c=1)} \\
&= \frac{0.8\times 0.5}{0.8\times 0.5 + 0.5\times 0.5} \\
&= \frac{0.4}{0.65} \\
&\approx 0.615
\end{aligned}
$$

Problem 3

Given Equation 2.3 and the fact that x and y are independent, we can show that $Pr(x|y=y^*) = Pr(x)$ by:

$$
\begin{aligned}
Pr(x|y=y^*) &= \frac{Pr(x,y=y^*)}{\int Pr(x,y=y^* * dx)} \\
&= \frac{Pr(x,y=y^*)}{Pr(y=y^*)} \\
&= \frac{Pr(x)\cdot Pr(y=y^*)}{Pr(y=y^*)} \\
&= Pr(x)
\end{aligned}
$$

Problem 4

The expected value of one roll of this biased die, x, is:

$$
\begin{aligned}
\mathbb{E}(x) &= 1\cdot\frac{1}{12}+2\cdot\frac{1}{12}+3\cdot\frac{1}{12}+4\cdot\frac{1}{12}+5\cdot\frac{1}{6}+6\cdot\frac{1}{12} \\
&= \frac{10}{12}+\frac{5}{6}+\frac{6}{2} \\
&= \frac{56}{12} \\
&= \frac{14}{3} \\
&\approx 4.67
\end{aligned}
$$

The expected value of the sum of two rolls is:

$$
\begin{aligned}
\mathbb{E}(2x) &= 2\mathbb{E}(x) \\
&= 2(\frac{14}{3}) \\
&= \frac{28}{3} \\
&\approx 9.34
\end{aligned}
$$

Problem 5

todo: problem 2.10