Homework 1

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Problem 1

$$\begin{aligned} X &\sim Unif(0,1) \\ m &= \mathbb{E}(X) = 0.5 \\ \mathbb{E}(|X-m|) &= \mathbb{E}(|X-0.5|) \\ &= -E(X_{x\leq 0.5} - 0.5) + E(X_{x>0.5} - 0.5) \\ &= -(-0.25) + 0.25 \\ &= 0.5 \end{aligned}$$

Problem 2

By Bayes' Theorem, we can compute the posterior probability that coin 2 was chosen by:

$$\begin{aligned} P(c=2|h=1) &= \frac{P(h=1|c=2)P(c=2)}{P(h=1)} \\ &= \frac{P(h=1|c=2)P(c=2)}{P(h=1|c=2)P(c=2)+P(h=1|c=1)P(c=1)} \\ &= \frac{0.8\times 0.5}{0.8\times 0.5 + 0.5\times 0.5} \\ &= \frac{0.4}{0.65} \\ &\approx 0.615 \end{aligned}$$

Problem 3

Given Equation 2.3 and the fact that x and y are independent, we can show that $Pr(x|y=y^*) = Pr(x)$ by:

$$
\begin{aligned}
Pr(x|y=y^*) &= \frac{Pr(x, y=y^*)}{\int Pr(x, y=y^* * dx)} \\
&= \frac{Pr(x, y=y^*)}{Pr(y=y^*)} \\
&= \frac{Pr(x) \cdot Pr(y=y^*)}{Pr(y=y^*)} \\
&= Pr(x)
\end{aligned}
$$

Problem 4

The expected value of one roll of this biased die, x, is:

$$
\begin{aligned}
\mathbb{E}(x) &= 1 \cdot \frac{1}{12} + 2 \cdot \frac{1}{12} + 3 \cdot \frac{1}{12} + 4 \cdot \frac{1}{12} + 5 \cdot \frac{1}{6} + 6 \cdot \frac{1}{12} \\
&= \frac{10}{12} + \frac{5}{6} + \frac{6}{2} \\
&= \frac{56}{12} \\
&= \frac{14}{3} \\
&\approx 4.67
\end{aligned}
$$

The expected value of the sum of two rolls is:

$$
\begin{aligned}
\mathbb{E}(2x) &= 2\mathbb{E}(x) \\
&= 2(\frac{14}{3}) \\
&= \frac{28}{3} \\
&\approx 9.34
\end{aligned}
$$

Problem 5

Using the relations given in Exercise 2.9, we can show that $\mathbb{E}[(x-\mu)^2] = \mathbb{E}[x^2] - E[x]E[x]$ by:

$$
\begin{aligned}
\mathbb{E}[(x-\mu)^2] &= \mathbb{E}[x^2 - 2x\mu + \mu^2] \\
&= \mathbb{E}[x^2] - \mathbb{E}[2x\mu] + \mathbb{E}[\mu^2] \\
&= \mathbb{E}[x^2] - 2\mathbb{E}[x\mu] + \mathbb{E}[x]\mathbb{E}[x] \\
&= \mathbb{E}[x^2] - 2\mathbb{E}[x]\mathbb{E}[x] + \mathbb{E}[x]\mathbb{E}[x] \\
\mathbb{E}[(x-\mu)^2] &= \mathbb{E}[x^2] - E[x]E[x]
\end{aligned}
$$