STA561: Probabilistic machine learning

Variational Inference (11/4/13)

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1 Introduction

2 Ising Model

Before proceeding with variational inference, it is helpful to review the Ising model. The main idea behind the Ising model is a lattice of unobserved variables $(x_1, ...x_n)$, each with its own (noisy) observation $(y_1, ..., y_n)$). For example, we can think of the lattice as pixels in a black and white image ($x_i \in \{-1, 1\}$), with a noisy grayscale observation of the pixels ($y_i \in R$). Our goal in this case would be to obtain a de-noised version of the image. More generally, we wish to draw inferences about the unobserved lattice X from the observed values Y. Figure 1 illustrates an Ising model for $n = 9$, with the latent nodes colored white and the observed nodes shaded grey.

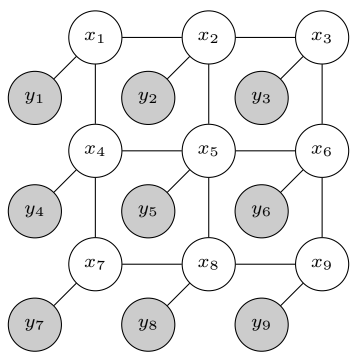

Figure 1: Schematic of an Ising Model

We now define the potential functions of the Ising model:

$$\begin{aligned} \psi_s(x_s) &= p(y_i|x_i) \equiv L_i(x_i) \\ \psi_{st}(x_s x_t) &= W_{st} x_s x_t \end{aligned}$$

Continuing with our image example above, we could set $W_{st} = 1$. In general, we set W to positive values if we want neighbors to agree, and negative values if we want them to differ.

Now we can specify our priors. Let $N(i)$ be a function that returns the first-degree neighbors of node i. For example, in Figure 1, calling $N(x_1)$ would return nodes x_2 and x_4.

$$p(x) = \frac{1}{z_0} \exp\{-\sum_{i=1}^{n} \sum_{j \in N(i)} x_i x_j\}$$

$$p(y|x) = \prod_{i=1}^{n} \exp\{-L_i(x_i)\}$$
$$p(x|y) = \frac{1}{z} \exp\{-\sum_{i=1}^{n} \sum_{j \in N(i)} x_i x_j - \sum_{i=1}^{n} L_i(x_i)\}$$

2.1 Mean Field Version of the Ising Model

Having seen an example of a basic Ising model, we now turn our attention to how we can analyze the mean field version of such a model. We do this by "breaking" the edges between the latent variables, and assigning a mean parameter μ to each of them. The new structure is illustrated in Figure 2, with the same color coding as above.

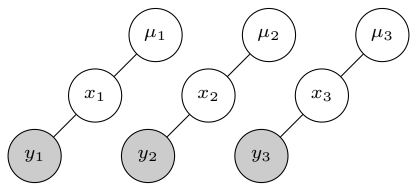

Figure 2: Mean Field Version of an Ising Model

3 Loopy Belief Propogation