

*Homework Notes:* I did not work with anyone else on this homework or refer to resources other than the course notes, textbook, and course Piazza page.

### Problem 1

**A** Using the normal equation  $\hat{\beta} = (X^T X)^{-1}(X^T Y)$ , we can calculate  $\hat{\beta}$  in  $\mathcal{R}$  with the following program (after loading the data as `lindata`):

Listing 1: R Code for 1A

```
1 X = as.matrix(lindata[1:1000, 1:2])
2 Y = as.matrix(lindata[1:1000, 3])
3 X.prime.X = t(X) %*% X
4 X.prime.X.inverse = solve(X.prime.X)
5 X.prime.Y = t(X) %*% Y
6 beta.hat = X.prime.X.inverse %*% X.prime.Y
7 beta.hat #=> 3.0117879, 0.7832938
```

Letting  $\beta_i$  correspond to  $X_i$ ,  $\hat{\beta}_1 = 3.01$ ,  $\hat{\beta}_2 = 0.78$ .

**B** We can also estimate  $\beta$  using online stochastic gradient descent using  $\mathcal{R}$ 's `optim` function for minimization:

Listing 2: R Code for 1B

```
8 linreg = function(X, Y){
9   RSS = function(b, x, y){
10    residuals = y - (x %*% b)
11    sq.resid = residuals^2
12    rss = sum(sq.resid)
13    return(rss)
14  }
15  results = optim(rep(0, ncol(X)), RSS,
16    hessian=TRUE, method="BFGS", x=X, y=Y)
17  list(beta=results$par,
18    vcov=solve(results$hessian),
19    converged=results$convergence==0)
20 }
21 model = linreg(X, Y)
22 model$beta #=> 3.0117879, 0.7832938
```

Using this method we reach the same results as above:  $\hat{\beta}_1 = 3.01$ ,  $\hat{\beta}_2 = 0.78$ .

### C

**D**

**Problem 2**