

Notes: I did not work with anyone else on this exam or refer to resources other than the supplied article, course notes, textbook, and course Piazza page.

Problem 1

A We can write out the expected log likelihood as:

$$\begin{aligned}
\mathbb{E}[\ell_c] &= \mathbb{E}[\log \prod_{i=1}^n (2\pi)^{p/2} |\Psi|^{-1/2} \exp\{-\frac{1}{2}[x_i - \mu - \Lambda z_i]' \Psi^{-1} [x_i - \mu - \Lambda z_i]\}] \\
&= c - \frac{n}{2} \log |\Psi| - \\
&\quad \sum_{i=1}^n \mathbb{E}[\frac{1}{2}(x_i \Psi^{-1} x_i' - 2x_i \Psi^{-1} \mu' - 2x_i \Psi^{-1} \Lambda z_i + \mu \Psi^{-1} \mu' + 2\mu \Psi^{-1} \Lambda z_i + z_i' \Lambda' \Psi^{-1} \Lambda z_i)] \\
&= c - \frac{n}{2} \log |\Psi| - \\
&\quad \sum_{i=1}^n \mathbb{E}[\frac{1}{2}x_i \Psi^{-1} x_i' - x_i \Psi^{-1} \mu' - x_i \Psi^{-1} \Lambda z_i + \frac{1}{2}\mu \Psi^{-1} \mu' + \mu \Psi^{-1} \Lambda z_i + \frac{1}{2}z_i' \Lambda' \Psi^{-1} \Lambda z_i] \\
&= c - \frac{n}{2} \log |\Psi| - \sum_{i=1}^n \left\{ \frac{1}{2}x_i \Psi^{-1} x_i' - x_i \Psi^{-1} \mu' - x_i \Psi^{-1} \Lambda \mathbb{E}[z_i | x_i] + \right. \\
&\quad \left. \frac{1}{2}\mu \Psi^{-1} \mu' + \mu \Psi^{-1} \Lambda \mathbb{E}[z_i | x_i] + \frac{1}{2}tr(\Lambda' \Psi^{-1} \Lambda E[z_i z_i' | x_i]) \right\}
\end{aligned}$$

From this we can determine that the expected sufficient statistics are $\mathbb{E}[z_i | x_i]$ and $E[z_i z_i' | x_i]$.