

502 Controller

Equations of motion for the interesting parts:

$$\begin{aligned}\ddot{x} &= \frac{(\sin(\phi) \sin(\psi) + \cos(\phi) \cos(\psi) \sin(\theta)) (u_1 + u_2 + u_3 + u_4)}{m} \\ \ddot{y} &= -\frac{(\cos(\psi) \sin(\phi) - \cos(\phi) \sin(\psi) \sin(\theta)) (u_1 + u_2 + u_3 + u_4)}{m} \\ \ddot{z} &= \frac{\cos(\phi) \cos(\theta) (u_1 + u_2 + u_3 + u_4)}{m} - g \\ \dot{\omega}_1 &= \frac{l u_2 - l u_4 + I_{22} W_2 W_3 - I_{33} W_2 W_3}{I_{11}} \\ \dot{\omega}_2 &= -\frac{l u_1 - l u_3 + I_{11} W_1 W_3 - I_{33} W_1 W_3}{I_{22}} \\ \dot{\omega}_3 &= \frac{\sigma u_1 - \sigma u_2 + \sigma u_3 - \sigma u_4 + I_{11} W_1 W_2 - I_{22} W_1 W_2}{I_{33}}\end{aligned}$$

Common approach seems to be grouping the control inputs together in the following order:

$$\begin{aligned}U_1 &= \frac{u_1 + u_2 + u_3 + u_4}{m} \\ U_2 &= \frac{l(u_2 - u_4)}{I_{11}} \\ U_3 &= \frac{l(u_1 - u_3)}{I_{22}} \\ U_4 &= \frac{\sigma(u_1 - u_2 + u_3 - u_4)}{I_{33}}\end{aligned}$$

For these, we can rewrite these as a matrix multiplication

$$\begin{bmatrix} \frac{1}{m} & \frac{1}{m} & \frac{1}{m} & \frac{1}{m} \\ 0 & \frac{l}{I_{11}} & 0 & -\frac{l}{I_{11}} \\ \frac{l}{I_{22}} & 0 & -\frac{l}{I_{22}} & 0 \\ \frac{\sigma}{I_{33}} & -\frac{\sigma}{I_{33}} & \frac{\sigma}{I_{33}} & -\frac{\sigma}{I_{33}} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix}$$

This is full rank, so it is invertible, which is of the following form:

$$\begin{pmatrix} \frac{1}{4m} & 0 & \frac{1}{2I_{22}} & \frac{1}{4I_{33}} \\ \frac{1}{4m} & \frac{1}{2I_{11}} & 0 & -\frac{1}{4I_{33}} \\ \frac{1}{4m} & 0 & -\frac{1}{2I_{22}} & \frac{1}{4I_{33}} \\ \frac{1}{4m} & -\frac{1}{2I_{11}} & 0 & -\frac{1}{4I_{33}} \end{pmatrix}$$

Given this inverse, we can calculate the individual values of the control inputs given the signals. Which I verified and obtained through the following matlab script:

```
syms m l11 l22 l33 'positive'
```

```
mat = [ m   m   m   m;
        0  l11  0 -l11;
        l22  0 -l22  0;
        l33 -l33 l33 -l33];
```

```
% Prints out 4
```

```
rank(mat)
```

```
% Prints out a valid
```

```
latex(inv(mat))
```

This (along with some rearranging) transforms the above equations:

$$\begin{aligned}\ddot{z} &= U_1 \frac{\cos(\phi) \cos(\theta)}{m} - g \\ \dot{\omega}_1 &= U_2 + W_2 W_3 \frac{I_{22} - I_{33}}{I_{11}} \\ \dot{\omega}_2 &= -U_3 - W_1 W_3 \frac{I_{11} - I_{33}}{I_{22}} \\ \dot{\omega}_3 &= U_4 + W_1 W_2 \frac{I_{11} - I_{22}}{I_{33}}\end{aligned}$$

Now, to begin devising the controller, the overall structure of the controller is as follows:

$$U_i(t) = U_{eq}(t) + U_D(t)$$

U_D is the drawing phase of the SMC, and U_{eq} is the sliding phase.

We can write the drawing phase part as:

$$U_D = k_D \text{sign}(s(t))$$

However, this has a lot of problems with chattering near the sliding surface, so we can rewrite this as:

$$U_D = k_D \frac{s(t)}{|s(t)| + \delta}$$

This makes it behave like a proportional controller near the sliding surface, and saturated at k_D elsewhere. Both k_D and δ are tuned parameters, which we can start at 1 and then tune.

$s(t)$ is the sliding surface, which we can start out with as

$$s(t) = \dot{e} + \lambda e$$

Where e is the error of the desired characteristic. Because we have only four actuators, we can only control 4 degrees of freedom. For this, we can control z, ϕ, θ, ψ .

We can start with deriving the control law for z :

$$\begin{aligned} e &= z_d - z \\ s &= (\dot{z}_d - \dot{z}) + \lambda(z_d - z) \end{aligned}$$

The system is in a sliding condition when $\dot{s} = 0$, we can find \dot{s} :

$$\dot{s} = (\ddot{z}_d - \ddot{z}) + \lambda(\dot{z}_d - \dot{z})$$

We can substitute in our equation for \ddot{z} :

$$\dot{s} = (\ddot{z}_d + g - U_1 \frac{\cos(\phi) \cos(\theta)}{m} - g) + \lambda(\dot{z}_d - \dot{z})$$

When the system is in sliding condition ($\dot{s} = 0$) $U_d = 0$, and therefore $U_{eq} = 0$, so we can substitute it in:

$$\begin{aligned} 0 &= (\ddot{z}_d + g - U_{eq} \frac{\cos(\phi) \cos(\theta)}{m}) + \lambda(\dot{z}_d - \dot{z}) \\ U_{eq} &= (\ddot{z}_d + g + \lambda(\dot{z}_d - \dot{z})) \frac{m}{\cos(\phi) \cos(\theta)} \end{aligned}$$

So we can write the full control law for z as follows:

$$U_1(t) = (\ddot{z}_d + g + \lambda(\dot{z}_d - \dot{z})) \frac{m}{\cos(\phi) \cos(\theta)} + k_D \frac{s(t)}{|s(t)| + \delta}$$

The other terms can be similarly solved for. Note that because I am controlling ω , the euler angles are in reference to the body fixed frame, which will be denoted with a $_b$ notation.

Goal for tonight:

- ☐ Calculate other three U's
- ☐ If I finish that, get the force conversion figured out

PROBLEM IS THAT NOTES DID NOT INCLUDE ACCELERATIONS FROM PREVIOUS FRAMES, MULTIPLY BY ADJOINT FROM PREVIOUS FRAME (FIRST WOULD BE MO1).

$$\dot{T}^{-1}\omega + T^{-1}\dot{\omega}$$