# 502 Controller

Equations of motion for the interesting parts:

$$
\begin{aligned}
\ddot{x} &= \frac{\left(\sin\left(\phi\right)\ \sin\left(\psi\right) + \cos\left(\phi\right)\ \cos\left(\psi\right)\ \sin\left(\theta\right)\right)\ \left(u_1 + u_2 + u_3 + u_4\right)}{m} \\
\ddot{y} &= -\frac{\left(\cos\left(\psi\right)\ \sin\left(\phi\right) - \cos\left(\phi\right)\ \sin\left(\psi\right)\ \sin\left(\theta\right)\right)\ \left(u_1 + u_2 + u_3 + u_4\right)}{m} \\
\ddot{z} &= \frac{\cos\left(\phi\right)\ \cos\left(\theta\right)\ \left(u_1 + u_2 + u_3 + u_4\right)}{m} - g \\
\dot{\omega}_1 &= \frac{l\,u_2 - l\,u_4 + I_{22}\,\omega_2\,\omega_3 - I_{33}\,\omega_2\,\omega_3}{I_{11}} \\
\dot{\omega}_2 &= -\frac{l\,u_1 - l\,u_3 + I_{11}\,\omega_1\,\omega_3 - I_{33}\,\omega_1\,\omega_3}{I_{22}} \\
\dot{\omega}_3 &= \frac{\sigma\,u_1 - \sigma\,u_2 + \sigma\,u_3 - \sigma\,u_4 + I_{11}\,\omega_1\,\omega_2 - I_{22}\,\omega_1\,\omega_2}{I_{33}}
\end{aligned}
$$

Common approach seems to be grouping the control inputs together in the following order:

$$
\begin{aligned}
U_1 &= \frac{u_1 + u_2 + u_3 + u_4}{m} \\
U_2 &= \frac{l(u_2 - u_4)}{I_{11}} \\
U_3 &= \frac{l(u_1 - u_3)}{I_{22}} \\
U_4 &= \frac{\sigma(u_1 - u_2 + u_3 - u_4)}{I_{33}}
\end{aligned}
$$

For these, we can rewrite these as a matrix multiplication

$$
\begin{bmatrix}
\frac{1}{m} & \frac{1}{m} & \frac{1}{m} & \frac{1}{m} \\
0 & \frac{l}{I_{11}} & 0 & -\frac{l}{I_{11}} \\
\frac{l}{I_{22}} & 0 & -\frac{l}{I_{22}} & 0 \\
\frac{\sigma}{I_{33}} & -\frac{\sigma}{I_{33}} & \frac{\sigma}{I_{33}} & -\frac{\sigma}{I_{33}}
\end{bmatrix}
\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix}
=
\begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix}
$$

This is full rank, so it is invertible, which is of the following form:

$$
\begin{pmatrix}
\frac{1}{4\,m} & 0 & \frac{1}{2\,l_{22}} & \frac{1}{4\,l_{33}} \\
\frac{1}{4\,m} & \frac{1}{2\,l_{11}} & 0 & -\frac{1}{4\,l_{33}} \\
\frac{1}{4\,m} & 0 & -\frac{1}{2\,l_{22}} & \frac{1}{4\,l_{33}} \\
\frac{1}{4\,m} & -\frac{1}{2\,l_{11}} & 0 & -\frac{1}{4\,l_{33}}
\end{pmatrix}
$$

Given this inverse, we can calculate the individual values of the control inputs g\frac{\partial }{\partial t}iven the signals.

Which I verified and obtained through the following matlab script:

```
syms m l11 l22 l33 'positive'


mat = [ m   m   m m;
        0 l11  0 -l11;
       l22  0  -l22 0;
       l33 -l33 l33 -l33;];

% Prints out 4
rank(mat)
% Prints out a valid
latex(inv(mat))
```

This (along with some rearranging) transforms the above equations:

$$\begin{aligned}
\ddot{z} &= U_1 \frac{\cos(\phi)\,\cos(\theta)}{m} - g \\
\dot{\omega}_1 &= U_2 + \omega_2\,\omega_3\,\frac{I_{22} - I_{33}}{I_{11}} \\
\dot{\omega}_2 &= -U_3 - \omega_1\,\omega_3\,\frac{I_{11} - I_{33}}{I_{22}} \\
\dot{\omega}_3 &= U_4 + \omega_1\,\omega_2\,\frac{I_{11} - I_{22}}{I_{33}}
\end{aligned}$$

Now, to begin devising the controller, the overall structure of the controller is as follows:

$$U_i(t) = U_{eq}(t) + U_D(t)$$

$U_D$ is the drawing phase of the SMC, and $U_{eq}$ is the sliding phase.

We can write the drawing phase part as:

$$U_D = k_D\,\mathrm{sign}(s(t))$$

However, this has a lot of problems with chattering near the sliding surface, so we can rewrite this as:

$$U_D = k_D\,\frac{s(t)}{|s(t)| + \delta}$$

This makes it behave like a proportional controller near the sliding surface, and saturated at $k_D$ elsewhere. Both $k_D$ and $\delta$ are tuned parameters, which we can start at 1 and then tune.

$s(t)$ is the sliding surface, which we can start out with as

$$s(t) = \dot{e} + \lambda e$$

Where $e$ is the error of the desired characteristic. Because we have only four actuators, we can only control 4 degrees of freedom. For this, we can control $z$, $\phi$, $\theta$, $\psi$.

We can start with deriving the control law for $z$:

$$\begin{aligned} e &= z_d - z \\ s &= (\dot{z}_d - \dot{z}) + \lambda(z_d - z) \end{aligned}$$

The system is in a sliding condition when $\dot{s} = 0$, we can find $\dot{s}$ :

$$\dot{s} = (\ddot{z}_d - \ddot{z}) + \lambda(\dot{z}_d - \dot{z})$$

We can substitute in our equation for $\ddot{z}$:

$$\dot{s} = (\ddot{z}_d + g - U_1 \frac{\cos(\phi)\,\cos(\theta)}{m} - g) + \lambda(\dot{z}_d - \dot{z})$$

When the system is in sliding condition ($\dot{s} = 0$) $U_d = 0$, and therefore $U_{eq} = 0$, so we can substitute it in:

$$\begin{aligned} 0 &= (\ddot{z}_d + g - U_{eq} \frac{\cos(\phi)\,\cos(\theta)}{m}) + \lambda(\dot{z}_d - \dot{z}) \\ U_{eq} &= (\ddot{z} + g + \lambda(\dot{z}_d - \dot{z})) \frac{m}{\cos(\phi)\,\cos(\theta)} \end{aligned}$$

So we can write the full control law for z as follows:

$$U_1(t) = (\ddot{z} + g + \lambda(\dot{z}_d - \dot{z})) \frac{m}{\cos(\phi)\,\cos(\theta)} + k_D \frac{s(t)}{|s(t)| + \delta}$$

The other terms can be similarly solved for. Note that because I am controlling $\omega$, the euler angles are in reference to the body fixed frame, which will be denoted with a $_b$ notation.

Goal for tonight:

- [ ] Calculate other three U's
- [ ] If I finish that, get the force conversion figured out

PROBLEM IS THAT NOTES DID NOT INCLUDE ACCELERATIONS FROM PREVIOUS FRAMES, MULTIPLY BY ADJOINT FROM PREVIOUS FRAME (FIRST WOULD BE MO1).

$$\dot{T}^{-1}\omega + T^{-1}\dot{\omega}$$