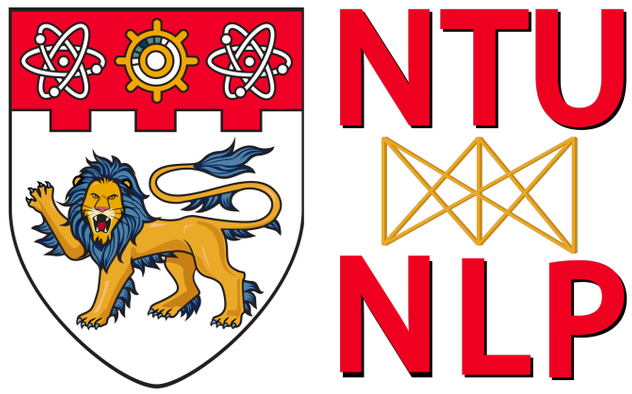
Deep Learning for Natural Language Processing

Shafiq Joty



Lecture 2: Machine Learning Basics

What is NLP (Recap)

NTU NLP

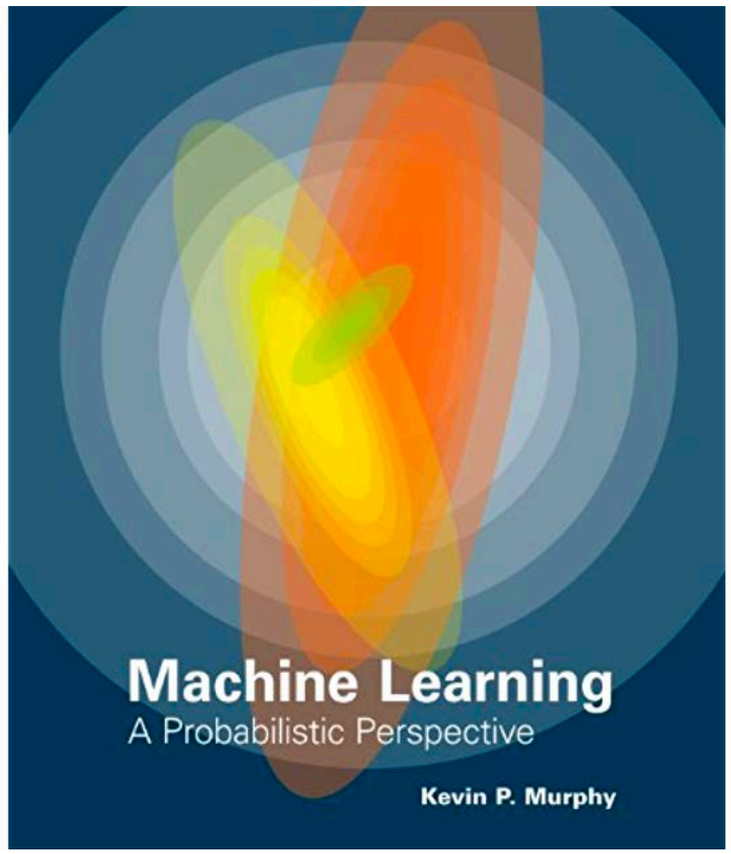
We study formalisms, models and algorithms to allow computers to perform useful tasks involving knowledge about human languages.

Different Neural Models (This course)
Traditionally:
Finite State Automata, Markov Models, Graphical Models

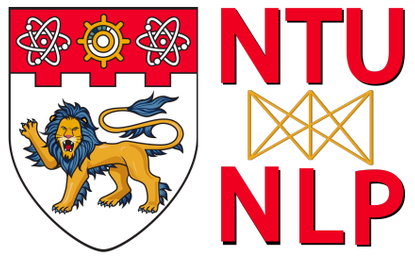
Lecture Plan

NTU NLP

- What is Machine Learning?
  - Supervised vs. unsupervised learning
  - Parametric vs. non-parametric models
- Linear Regression
- Logistic Regression
  - Parameter estimation
  - Gradient-based optimization & SGD
  - Multi-class classification

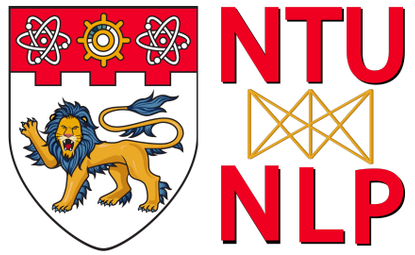


What is Machine Learning



A set of methods that can automatically detect patterns in data, and then use the uncovered patterns to predict future data, or to perform other kinds of decision making **under uncertainty**.

Types of ML

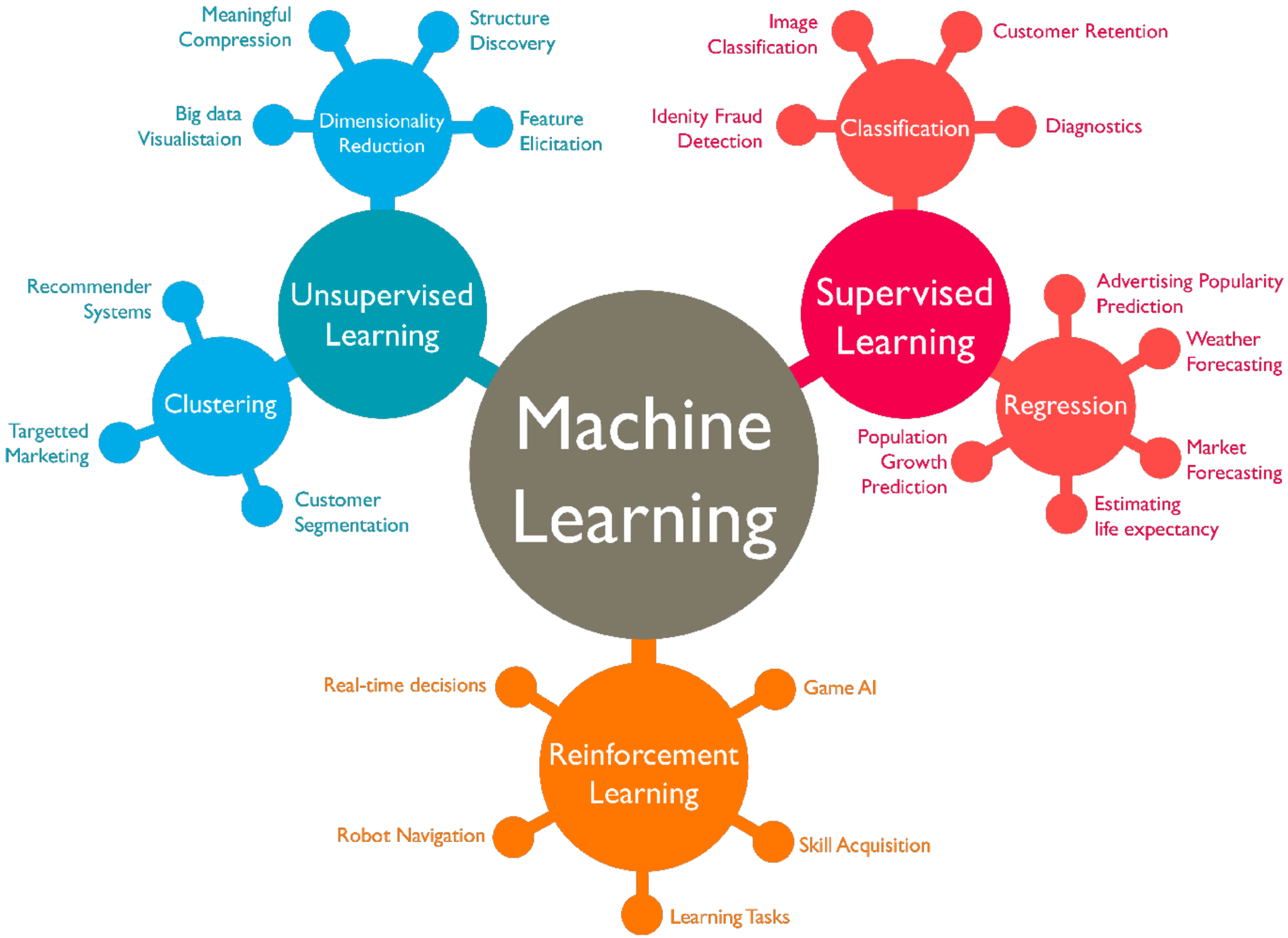


- **Supervised/Predictive learning:**
  To learn a mapping from inputs X to outputs Y, given a labelled set of input-output pairs.

- **Unsupervised/Descriptive learning:**
  To find interesting patterns in the data from given input. Sometimes called Knowledge Discovery.

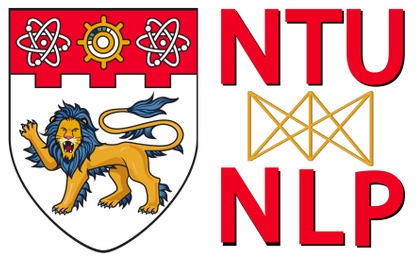
- **Reinforcement learning:**
  To learning how to act or behave when given occasional reward or punishment signals.

Types of ML



Source: Internet

Supervised Learning



Classification: To learn a mapping from inputs x to outputs y, where $y \in \{1, \ldots, C\}$, with C being the number of classes, which means y is categorical.

- If C = 2, this is called binary classification (in which case we often assume $y \in \{0, 1\}$
- If C > 2, this is called multiclass classification.

Examples

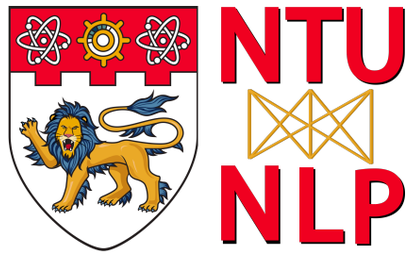
(1) Document classification and email spam filtering

(2) Sentiment classification

(3) Image classification and handwriting recognition

(4) Face detection and recognition

Supervised Learning

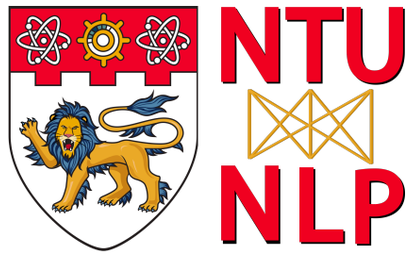


Regression: To learn a mapping from inputs x to outputs y, where y is real-valued.

Examples

1. Predict tomorrows stock market price given current market conditions and other possible side information.
2. Predict the age of a viewer watching a given video on YouTube.
3. Predict the location in 3d space of a robot arm end effector, given control signals (torques) sent to its various motors.
4. Predict the temperature at any location inside a building using weather data, time, door sensors.

Supervised Learning: Probabilistic Output



To handle ambiguous inputs, it is desirable to return a probability of being a member of some class C_i.

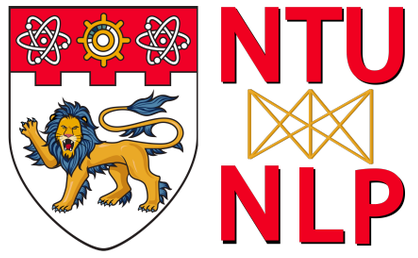
The probability distribution over possible labels $p(y|x, D)$, given the input vector x and the training set D.

Given the distribution, the inference can be computed as follows:

Prediction

$$\hat{y} = \hat{f}(\mathbf{x}) = \underset{c=1}{\overset{C}{\operatorname{argmax}}}\, p(y = c|\mathbf{x}, \mathcal{D})$$

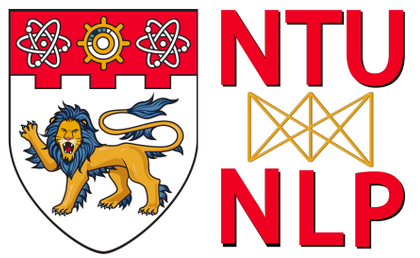
Unsupervised Learning



We are just given input data, without any outputs. The goal is to discover interesting structure in the data; this is sometimes called knowledge discovery.

1. Discovering clusters: To cluster data into groups and estimate which cluster each data sample belongs to.
2. Discovering latent factors: To project the data to a lower dimensional subspace which captures the essence of the data. It is also called "Dimension Reduction". (eg, PCA)
3. Discovering graph structure.
4. Matrix completion: To "fill in" missing data. Such as image impainting, collaborative filtering in the movie market.

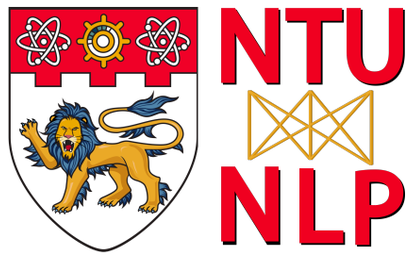
Parametric vs. non-parametric models



- Parametric model: the model has a fixed number of parameters
- Non-parametric model: the number of parameters grow with the size of training data set.

In general, parametric modeling methods has its advantage in computation, but they make strong assumptions about the data distributions.

Lecture Plan



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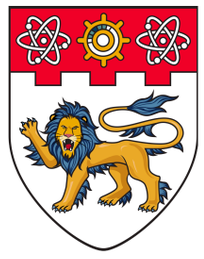
Machine Learning Steps



NTU NLP

3 Steps of parametric model:

1. Model Assumption. Also known as **inductive bias**.
2. Fit/Learn/Train the model parameters.
3. Apply/Test the model.

Linear Regression

NTU NLP

Problem Setup

Training Data is represented by:

$$\mathcal{D} = \{(\mathbf{x}_i, y_i)\}_{i=1}^{N}$$ Supervised

Here,
$\mathcal{D}$ is called the training set.
N is the number of training example.
$\mathbf{x}_i$ is a d-dimensional vector of features (also called attributes or covariates) i.e. #features is d.
y_i is called output or response variable.

For regression: y_i is real-valued i.e. $y_i \in \mathbb{R}$

Linear Regression

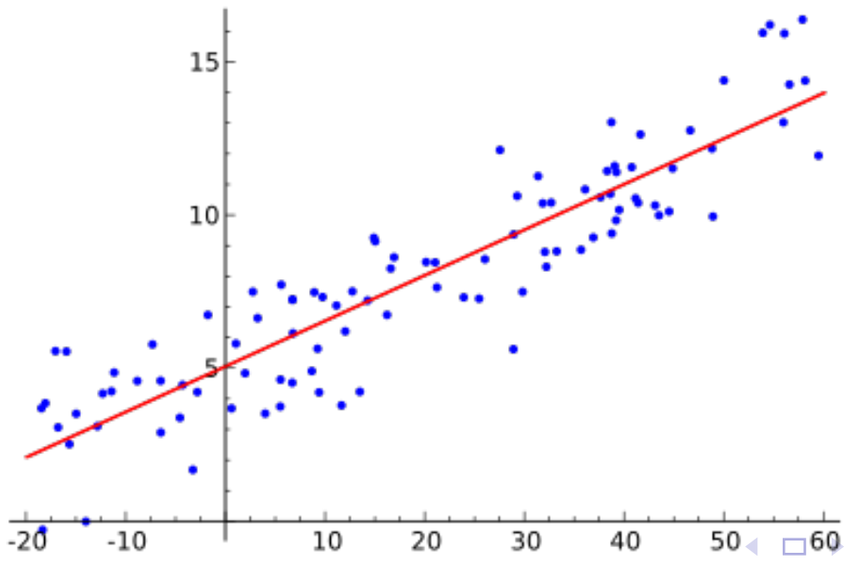
NTU NLP

What is it?

The basic idea behind a regression is that, you want to model the relationship between a real valued outcome variable y, and a vector of explanatory variables $\mathbf{x} = (x_1, x_2, \ldots, x_n)$.

A linear regression relates y to a linear predictor function of $\mathbf{x}$.
For a given data point i, the linear function is of the form:

$$\hat{y}_i = w_0 + w_1 x_{i1} + \ldots + w_d x_{id}$$



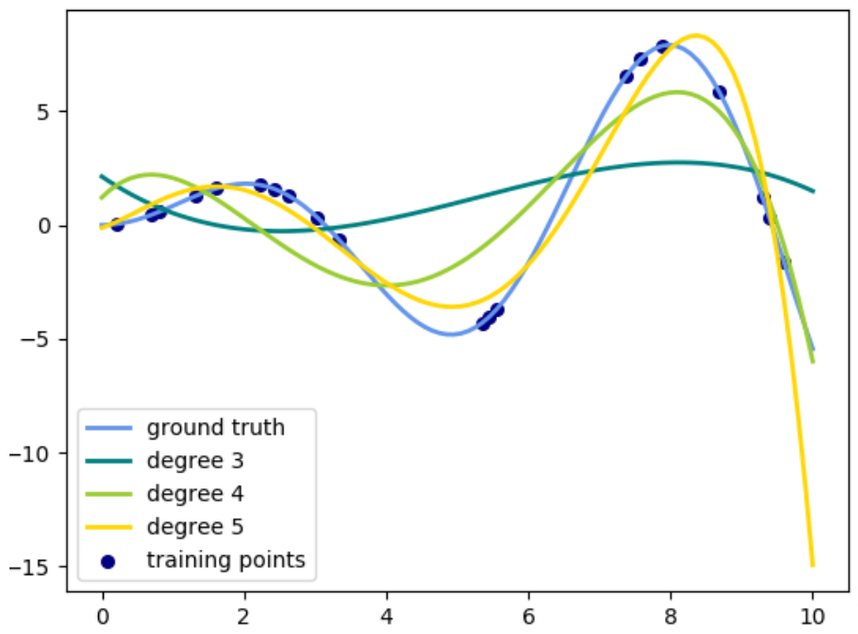
Linear Regression

NTU NLP

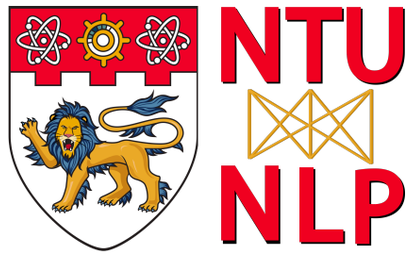
Remember

The function is linear in the parameters $\mathbf{w} = (w_0, w_1, ..., w_n)$, not necessarily in terms of the explanatory variables.

It's possible to use a non-linear function of explanatory variable e.g. $\mathbf{x}_d^2, \mathbf{x}_d * \mathbf{x}_{d+1}$ *etc*. This is known as Basis Function Expansion.



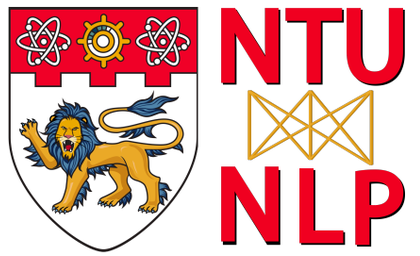
Linear Regression



Applications

- Predict tomorrows stock market price given current market conditions and other possible side information.
- Predict the age of a viewer watching a given video on YouTube.
- Predict the location in 3D space of a robot arm end effector, given control signals (torques) sent to its various motors.
- Predict the amount of Prostate Specific Antigen (PSA) in the body as a function of a number of different clinical measurements.
- Predict the temperature at any location inside a building using weather data, time, door sensors, etc.

Linear Regression



Least Square Formulation of the Loss

In this formulation, the least squares fit is the line that **minimizes** the sum of the squared distances between observed data and predicted values, i.e. it minimizes the **Residual Sum of Squares** (*RSS*):

$$\arg\min_{\mathbf{w}} \sum_{i=1}^{N} (y_i - \hat{y}_i)^2$$

where $\hat{y}_i = w_0 + w_1 x_i$ is the predicted outcome for the i^{th} observation.

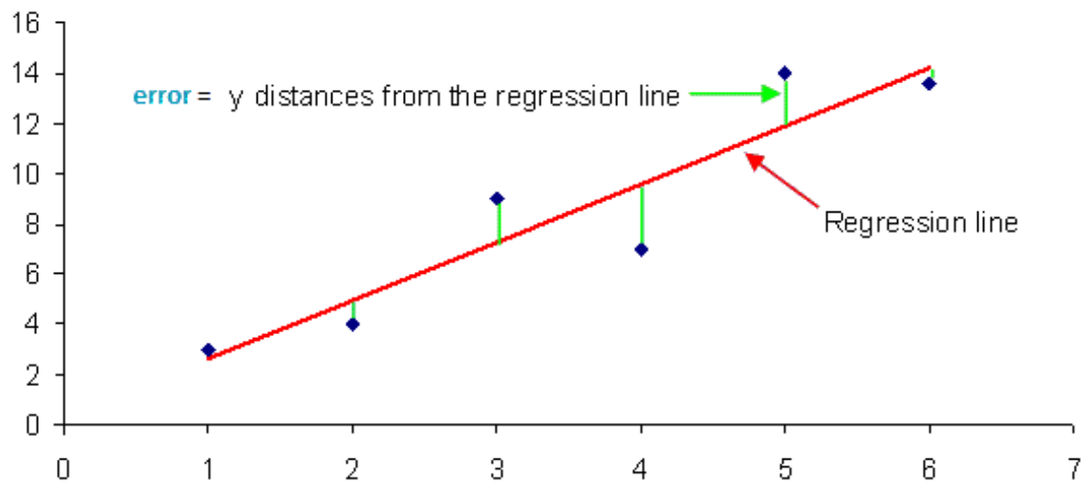
Linear Regression

Least Square Formulation

We can write the actual value of the i^{th} observation as:

$$y_i = w_0 + w_1 x_i + \epsilon_i$$

where ϵ_i is the residual, or error, we get for the i^{th} observation, i.e. the difference between our linear predictions and the true response.



Linear Regression

NTU NLP

Least Square Formulation

So our **objective function** is:

$$J(\theta) = \arg\min_{\theta} \sum_{i=1}^{N} (y_i - \hat{y}_i)^2$$

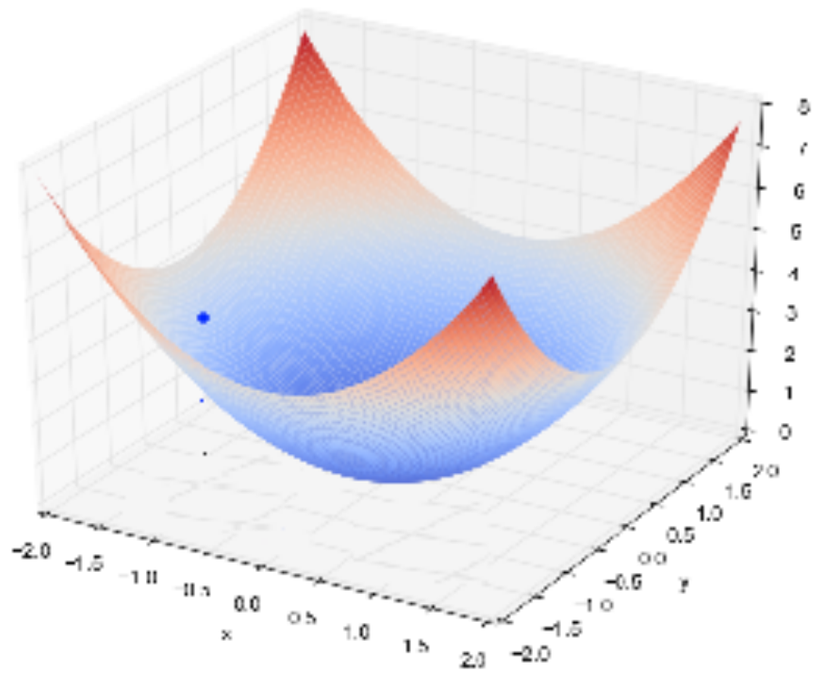
$$= \sum_{i=0}^{N} (y_i - \mathbf{x}_i^\mathsf{T}\mathbf{w})^2 = (y - \mathbf{X}\mathbf{w})^\mathsf{T}(y - \mathbf{X}\mathbf{w})$$

Vector notation

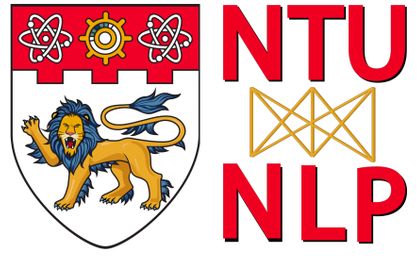
Linear Regression

Least Square Formulation

Notice that, we want to minimize $J(\theta)$.
We see that $J(\theta)$ is **Quadratic**. Hence it will be Bowl Shaped.



Linear Regression



Least Square Formulation

$$J(\theta) = (y - \mathbf{X}\mathbf{w})^\intercal (y - \mathbf{X}\mathbf{w}) = (y^\intercal - \mathbf{w}^\intercal \mathbf{X}^\intercal)(y - \mathbf{X}\mathbf{w})$$
$$= y^\intercal y - y^\intercal \mathbf{X}\mathbf{w} - \mathbf{w}^\intercal \mathbf{X}^\intercal y + \mathbf{w}^\intercal \mathbf{X}^\intercal \mathbf{X}\mathbf{w}$$

Taking partial derivative with respect to $\mathbf{w}$:

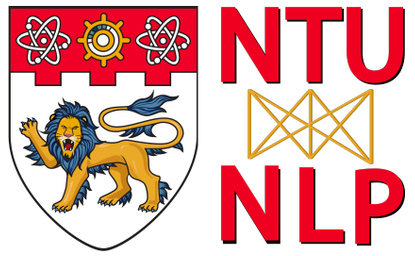
$$\frac{\partial}{\partial \mathbf{w}} J(\theta) = \frac{\partial}{\partial \mathbf{w}} (y^\intercal y - y^\intercal \mathbf{X}\mathbf{w} - \mathbf{w}^\intercal \mathbf{X}^\intercal y + \mathbf{w}^\intercal \mathbf{X}^\intercal \mathbf{X}\mathbf{w})$$
$$= 0 - 2\mathbf{X}^\intercal y + \mathbf{X}^\intercal \mathbf{X}\mathbf{w}$$

Equqating $\frac{\partial}{\partial \mathbf{w}} J(\theta)$ to 0 we get,

$$\frac{\partial}{\partial \mathbf{w}} J(\theta) = 0 = -2\mathbf{X}^\intercal y + \mathbf{X}^\intercal \mathbf{X}\mathbf{w}$$
$$\mathbf{w} = (\mathbf{X}^\intercal \mathbf{X})^{-1} \mathbf{X}^\intercal y$$

Closed form Solution

Linear Regression



Probabilistic Formulation

If we ever want to understand linear regression from a **Bayesian perspective** we need to start thinking probabilistically.

We need to *flip things over* and instead of thinking about the line minimizing the cost, think about it as maximizing the likelihood of the observed data.

As we will see, this amounts to the **exact same thing** - *mathematically speaking* - it's just a different way of looking at it.

Linear Regression

NTU NLP

Probabilistic Formulation

If we ever want to understand linear regression from a **Bayesian perspective** we need to start thinking probabilistically.

We have seen previously:

$$y_i = \mathbf{w}^\mathsf{T}\mathbf{x}_i + \epsilon_i$$

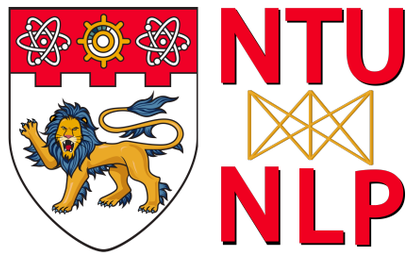
We often assume that ϵ_i normally distributed with a mean of 0 i.e.

$$\epsilon_i \sim \mathcal{N}(0, \sigma^2)$$

So, From the above equation we can say that:

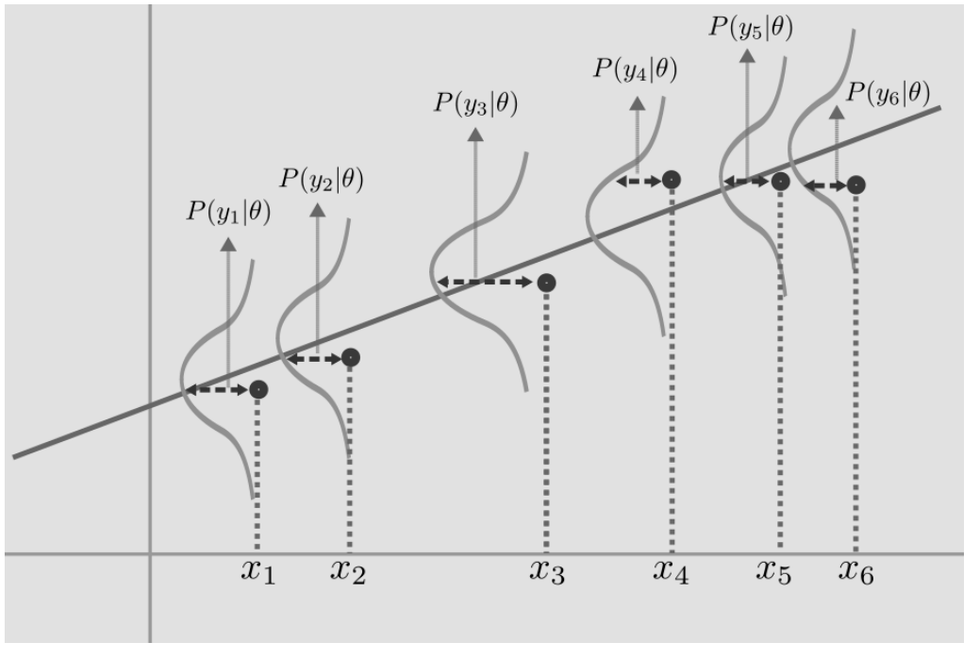
$$y_i \sim \mathcal{N}(\mathbf{w}^\mathsf{T}\mathbf{x}_i, \sigma^2)$$

Linear Regression

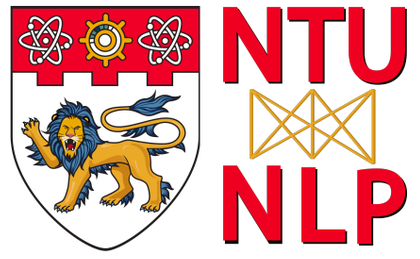


Probabilistic Formulation

It is common to assume the training examples are Independent and Identically Distributed, commonly abbreviated to **IID**.
So, we assume all our training examples are Normally distributed with mean $\mathbf{w}^\intercal \mathbf{x}_i$ and variance σ^2.



Linear Regression



Maximum Likelihood Estimation

We can write Linear regression as a model of the form:

$$p(y|\mathbf{x}, \boldsymbol{\theta}) = \mathcal{N}(y|\mathbf{w}^\intercal \mathbf{x}, \sigma^2)$$

Maximum likelihood Estimation

A common way to estimate the parameters of a statistical model is to compute the Maximum likelihood Estimation(MLE), which is defined as:

$$\hat{\boldsymbol{\theta}} \triangleq \underset{\boldsymbol{\theta}}{\operatorname{argmax}} \log p(\mathcal{D}|\boldsymbol{\theta})$$

Linear Regression

Maximum Likelihood Estimation

Log Likelihood

log function is a monotonically increasing function.

We know that maximizing the log of a value with respect to some parameter, **coincides with** maximizing the value itself.

So, the log-likelihood can be written as:

$$\ell(\boldsymbol{\theta}) \triangleq \log p(\mathcal{D}|\boldsymbol{\theta}) = \sum_{i=1}^{N} \log p(y_i|\mathbf{x}_i, \boldsymbol{\theta})$$

$$= \sum_{i=1}^{N} \log \left[\left(\frac{1}{2\pi\sigma^2} \right)^{\frac{1}{2}} \exp \left(-\frac{1}{2\sigma^2}(y_i - \mathbf{w}^\mathsf{T}\mathbf{x}_i)^2 \right) \right]$$

$$= \frac{-1}{2\sigma^2} RSS(\mathbf{w}) - \frac{N}{2} \log(2\pi\sigma^2)$$

RSS stands for Residual Sum of Squares $\quad RSS(\mathbf{w}) \triangleq \sum_{i=1}^{N} (y_i - \mathbf{w}^\mathsf{T}\mathbf{x}_i)^2$

Linear Regression

NTU NLP

Maximum Likelihood Estimation

RSS is defined by:

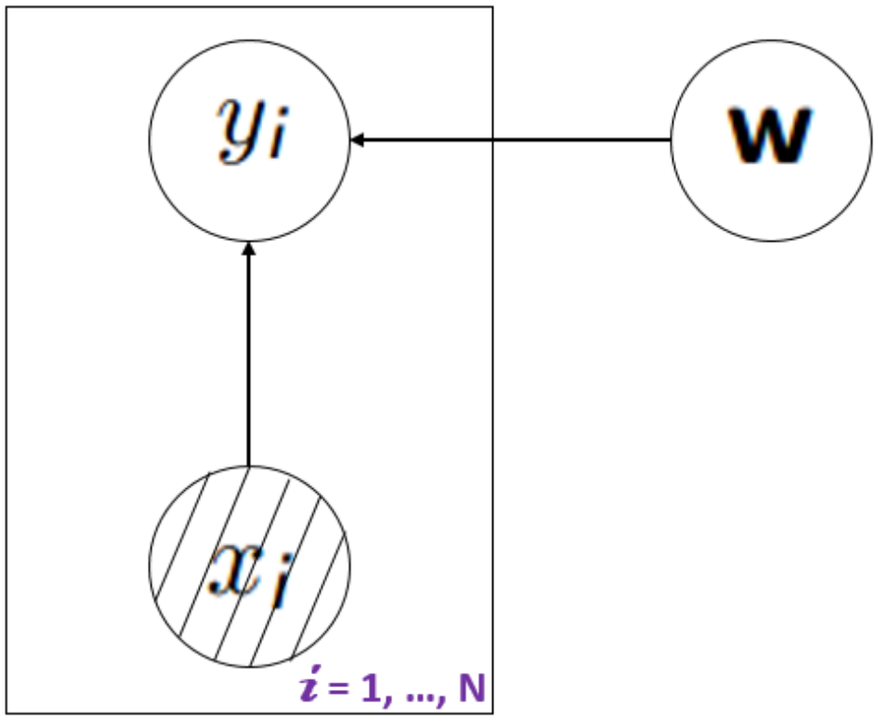
$$RSS(\mathbf{w}) \triangleq \sum_{i=1}^{N} (y_i - \mathbf{w}^\intercal \mathbf{x}_i)^2$$

The *RSS* is also called the Sum of Squared Errors.

$$\mathbf{w} = (\mathbf{X}^\intercal \mathbf{X})^{-1} \mathbf{X}^\intercal y$$

Linear Regression

Graphical Model Representation



Linear Regression

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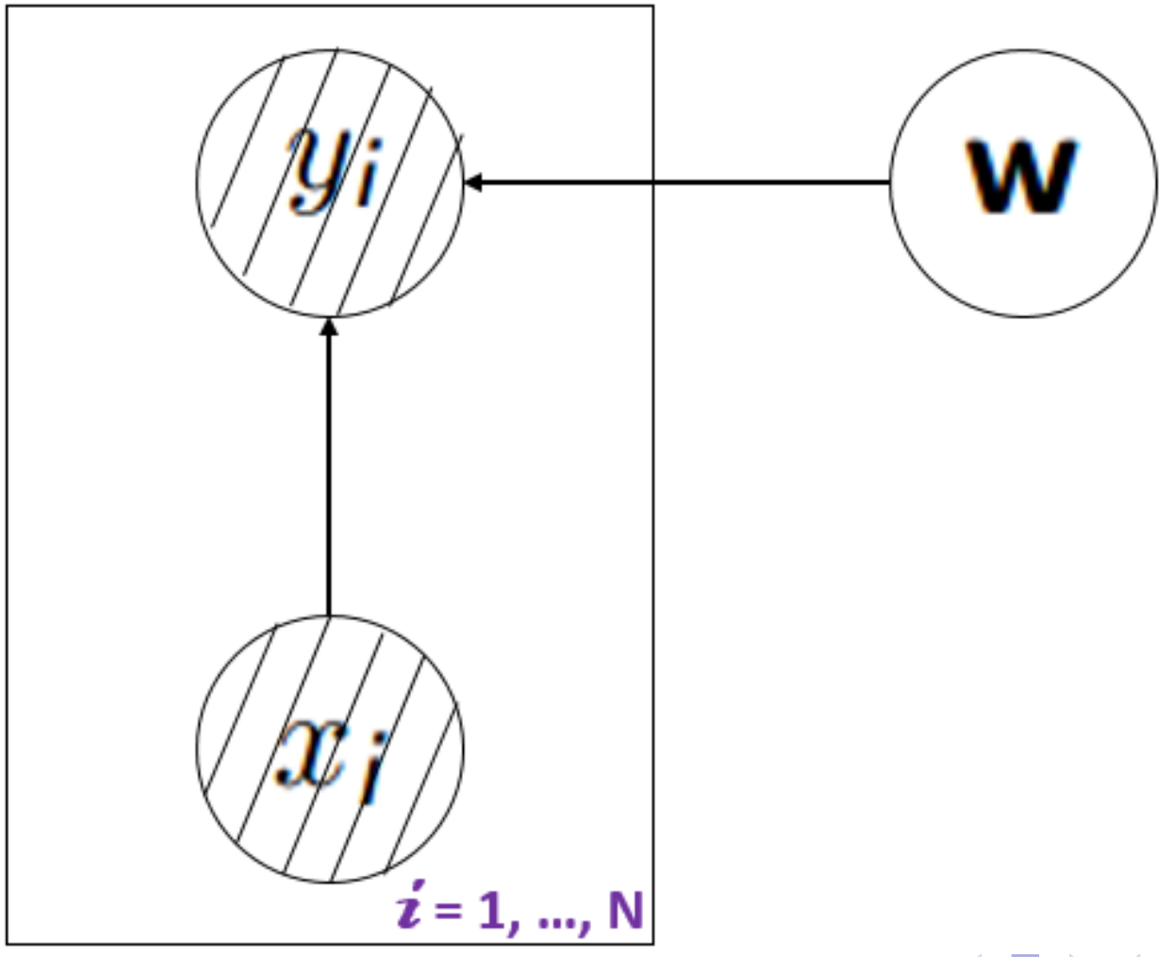
Graphical Model Representation (Training)

In training phase, both x_i and y_i is observed.
We want to estimate **w**.



Linear Regression

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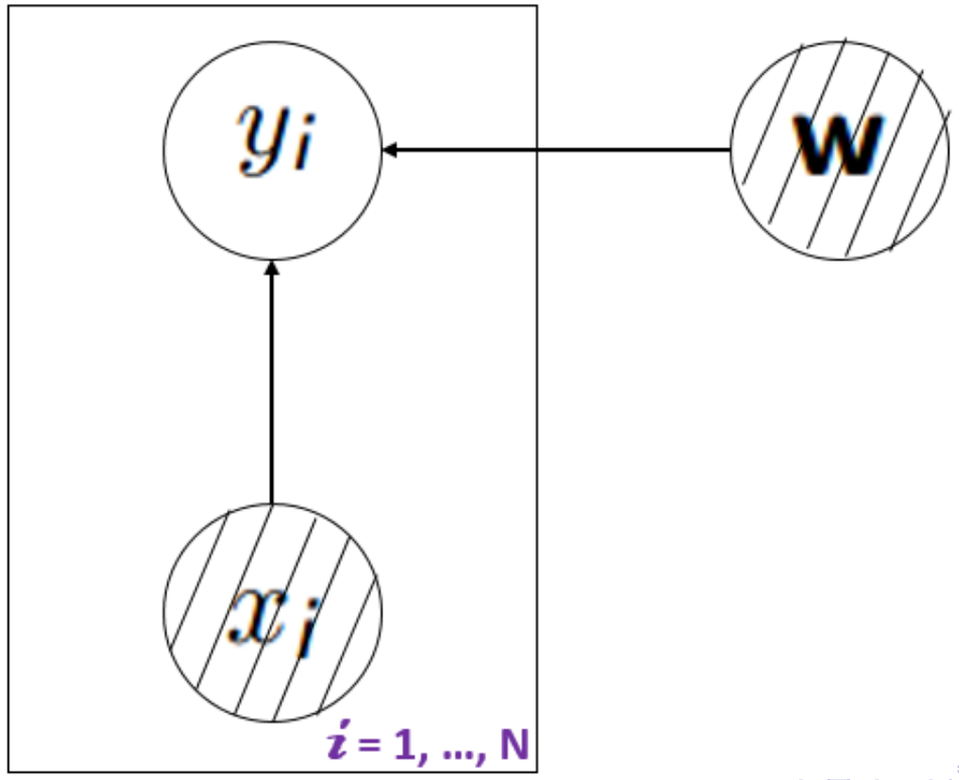
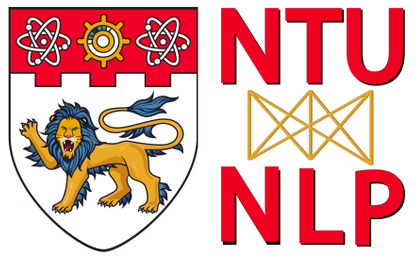
Graphical Model Representation (Testing)

Now we know x_i and **w**.
We will reveal y_i using these information.



Lecture Plan



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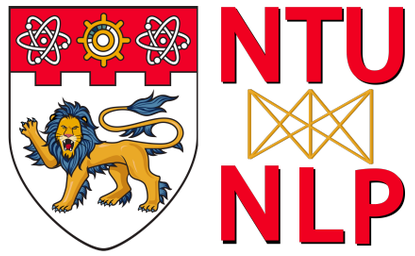
Logistic Regression



Problem Setup

Training Data is represented by

$$\mathcal{D} = \{(x_i, y_i)\}_{i=1}^{N}$$ Supervised

Here

$\mathcal{D}$ is called the training set.

N is the number of training examples.

x_i is a d-dimensional vector of features (also called attributes or covariates) i.e. #features is d.

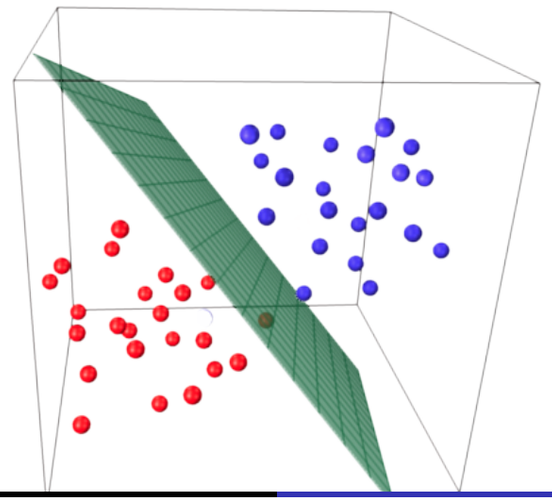
y_i is called output or response variable.

For classification: y_i is categorial i.e. $y_i \in \{1, .. C\}$

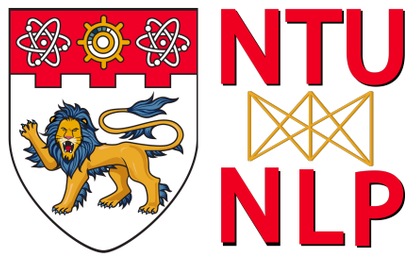
Logistic Regression

y_i is categorial i.e. $y_i \in \{1, .., C\}$

First, we will consider the case of **binary response variable** that is, where it can take only two values, "0" and "1".
Cases where the response variable has more than two outcome categories may be analyzed in **Multinomial Logistic Regression**.



Logistic Regression



Two Approaches to Probabilistic Classification

#1 Generative Approach

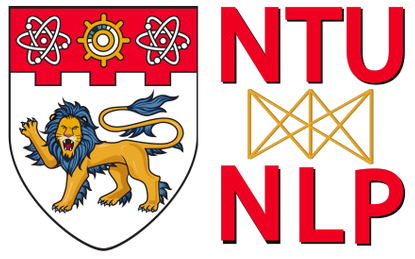
One way to build a probabilistic classifier is to create a joint model of the form $\mathcal{P}(y, \mathbf{x})$ and then to condition on $\mathbf{x}$, thereby deriving $\mathcal{P}(y|\mathbf{x})$.

#2 Discriminative Approach

In this approach we fit a model of the form $\mathcal{P}(y|\mathbf{x})$ directly.

Discriminative Approach is the approach we adopt for logistic regression. In particular, we will assume discriminative models which are linear in the parameters.

Logistic Regression



We can **generalize** linear regression to the (binary) classification setting by making two changes.

First, we replace the Gaussian distribution for y with a *Bernoulli distribution*. That is, we use

$$\mathcal{P}(y|\mathbf{x}, \mathbf{w}) = \mathit{Ber}(y|\mu(\mathbf{x}))$$

where $\mu(\mathbf{x}) = \mathbb{E}[y|\mathbf{x}] = \mathcal{P}(y = 1|\mathbf{x})$

Second, we compute a linear combination of the inputs as before, but then we *pass this through a sigmoid function* that ensures $0 \leq \mu(\mathbf{x}) \leq 1$. So we define $\mu(\mathbf{x}) = \mathit{sigm}(\mathbf{w}^\intercal \mathbf{x})$

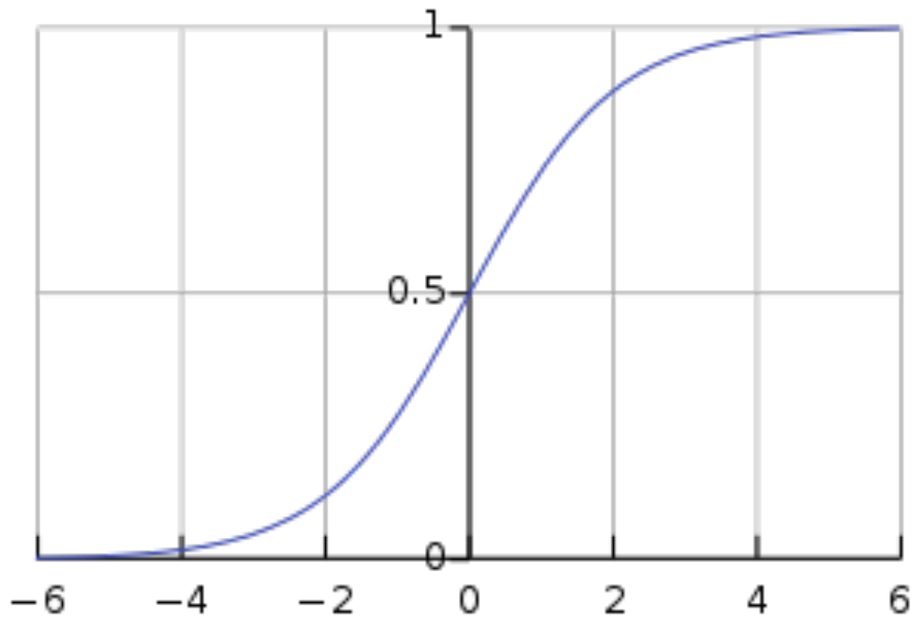
Logistic Regression

NTU NLP

Sigmoid Function

Sigmoid is a squashing function.
It maps the whole real line to [0,1].
It is also known as *logistic* or *logit* function
The term "sigmoid" means S-shaped. This is defined as:

$$sigm(\eta) \triangleq \frac{1}{1+\exp(-\eta)} = \frac{e^{\eta}}{e^{\eta}+1}$$



Logistic Regression

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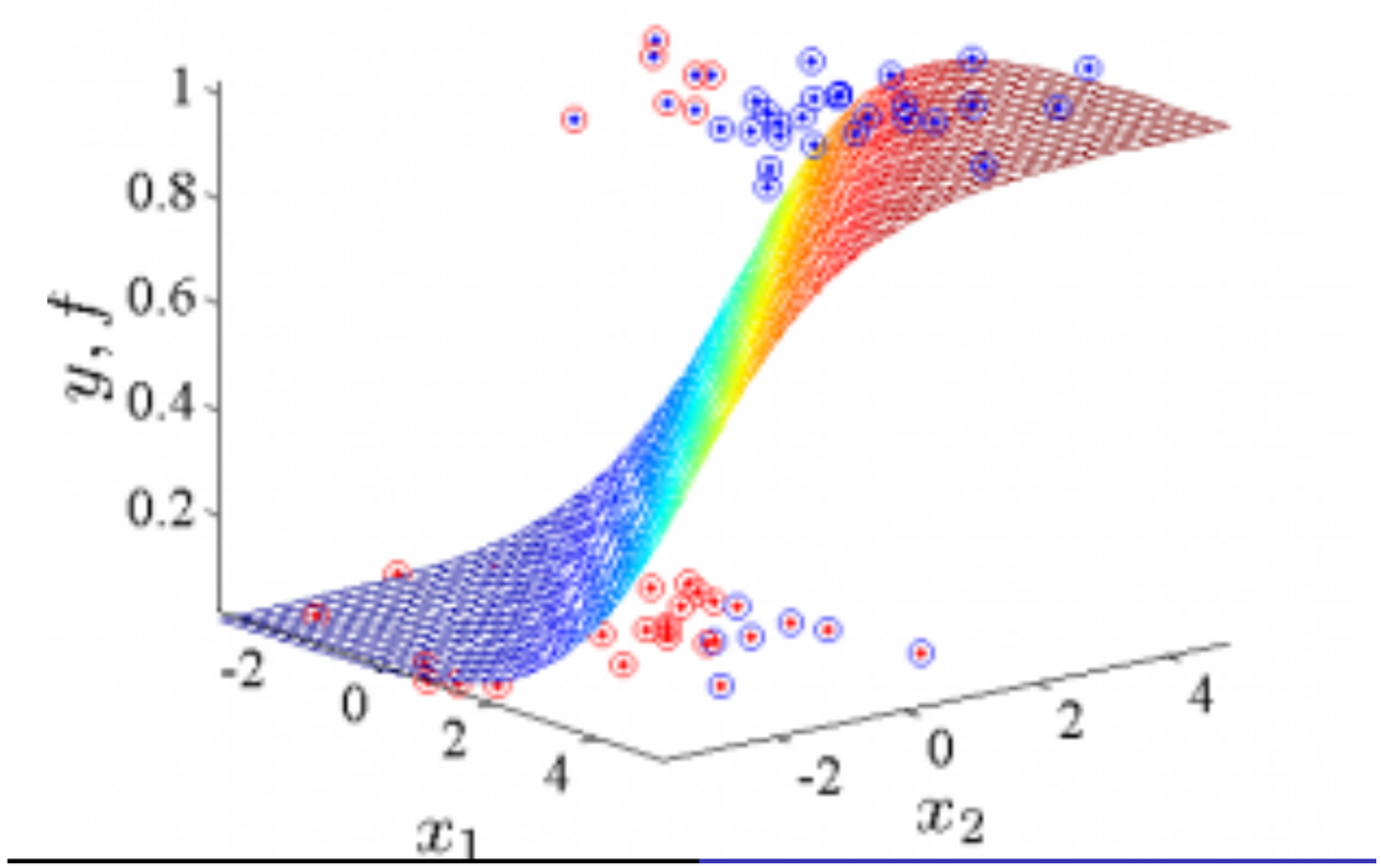
Model Definition

$$\mathcal{P}(y|\mathbf{x}, \mathbf{w}) = Ber(y|sigm(\mathbf{w}^\mathsf{T}\mathbf{x}))$$



Logistic Regression

— At the decision boundary, both classes are **equiprobable**. Thus:

$$\mathcal{P}(y = 1|\mathbf{x}, \mathbf{w}) = \mathcal{P}(y = 0|\mathbf{x}, \mathbf{w})$$

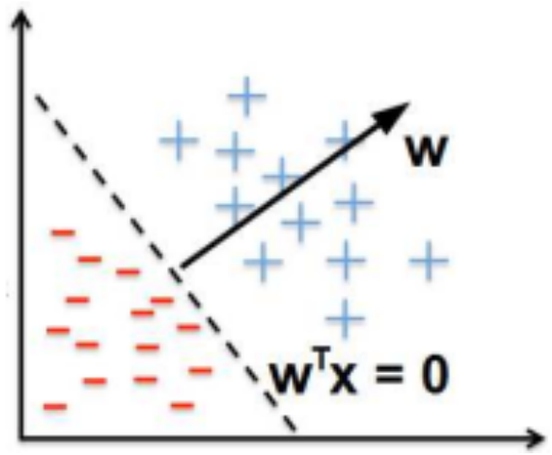
$$\frac{\exp(\mathbf{w}^\intercal \mathbf{x})}{1 + \exp(\mathbf{w}^\intercal \mathbf{x})} = \frac{1}{1 + \exp(\mathbf{w}^\intercal \mathbf{x})}$$

$$\exp(\mathbf{w}^\intercal \mathbf{x}) = 1$$

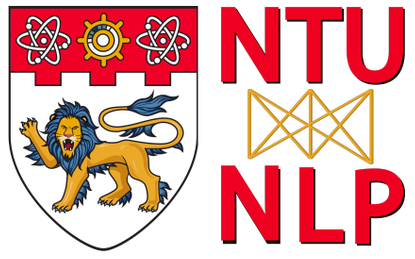
$$\mathbf{w}^\intercal \mathbf{x} = 0$$

Thus the decision boundary of Logistic Regression is nothing but a linear hyperplane.

- High positive score $\mathbf{w}^\intercal \mathbf{x}$: High probability of label 1
- High negative score $\mathbf{w}^\intercal \mathbf{x}$: Low probability of label 1



Logistic Regression



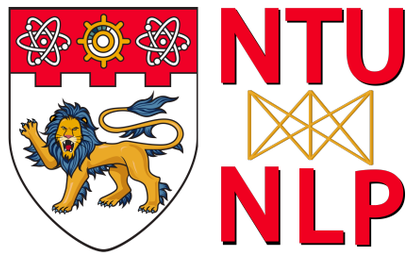
Instead of **maximizing** the log-likelihood, we can equivalently **minimize** the negative log likelihood or NLL.

The negative log-likelihood for logistic regression is given by

$$NLL(\mathbf{w}) = -\sum_{i=1}^{N} \log[\mu_i^{\mathbb{1}(y_i=1)} \times (1-\mu_i^{\mathbb{1}(y_i=0)})]$$
$$= -\sum_{i=1}^{N} [y_i \log \mu_i + (1-y_i) \log(1-\mu_i)]$$

This is also called the Cross Entropy Error Function.

Logistic Regression



Entropy

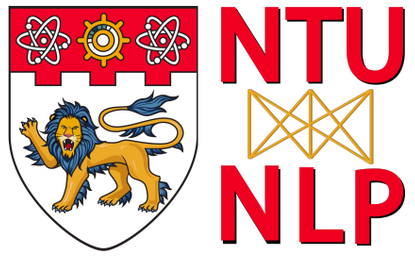
The entropy of a random variable X with diestribution p, denoted by $\mathbb{H}(\mathrm{X})$, is a measure of its **uncertainty**. It is defined by

$$\mathbb{H}(\mathrm{X}) \triangleq -\sum_{k=1}^{K} \mathcal{P}(\mathrm{X}=k) \log \mathcal{P}(\mathrm{X}=k)$$

Cross Entropy

Cross entropy of two probability distribution p and q is denoted by $\mathbb{H}(p,q)$ and is defined by $\mathbb{H}(p,q) \triangleq -\sum_{k} p_k \log q_k$

Logistic Regression



Unlike linear regression, we can no longer write down the MLE in closed form. Instead, we need to use an **optimization algorithm** to compute it.

For this, we need to derive the gradient and Hessian.

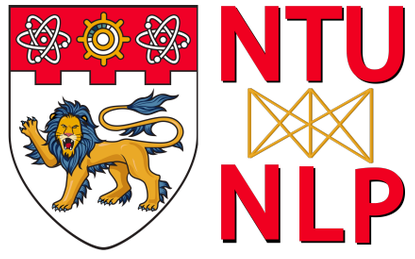
$$\mathbf{g} = \frac{d}{d\mathbf{w}} f(\mathbf{w}) = -\sum_i (\mu_i - y_i)\mathbf{x}_i = \mathbf{X}^\mathsf{T}(\mu - \mathrm{y})$$

$$\mathbf{H} = \frac{d}{d\mathbf{w}} \mathbf{g}(\mathbf{w})^\mathsf{T} = -\sum_i (\nabla_\mathbf{w} \mu_i \mathbf{x}_i^\mathsf{T} = -\sum_i \mu_i(1-\mu_i)\mathbf{x}_i\mathbf{x}_i^\mathsf{T} = \mathbf{X}^\mathsf{T}\mathbf{S}\mathbf{X}$$

where $\mathrm{S} \triangleq diag(\mu_i(1-\mu_i))$

One can also show that **H** is positive definite.
Hence the *NLL* is convex and has a unique global minimum.

Multiclass Logistic Regression



Logistic regression can be extended to handle more than two classes. This is called **Multi-class logistic regression**.

Also called Multinomial logistic regression. Sometimes called a Maximum Entropy Classifier.

Multiclass Logistic Regression

NTU NLP

In this case, response variable $y_i \in \{0, 1, 2, ..., (k-1)\}$

Model Specification:

$$\mathcal{P}(y = c|\mathbf{x}, \mathbf{W}) = \frac{\exp(\mathbf{w}_c^\mathsf{T}\mathbf{x})}{\sum_{c'=1}^{C} \exp(\mathbf{w}_{c'}^\mathsf{T}\mathbf{x})}$$

Softmax

Multiclass Logistic Regression

Let, $\mu_{ic} = \mathcal{P}(y_i = c|\mathbf{x}_i, \mathbf{W}) = \mathcal{S}(\eta_i)_c$
Where $\eta_i = \mathbf{W}^\intercal \mathbf{x}_i$
Also Let, $y_i c = \mathbb{I}(y_i = c)$

With this the log likelihood can be written as:

$$\ell(\mathbf{W}) = \log \prod_{i=1}^{N} \prod_{c=1}^{C} \mu_{ic}^{y_{ic}} = \sum_{i=1}^{N} \sum_{c=1}^{C} y_{ic} \log \mu_{ic}$$

$$= \sum_{i=1}^{N} \left[\left(\sum_{c=1}^{C} y_{ic} \mathbf{w}_c^\intercal \mathbf{x}_i \right) - \log \left(\sum_{c'=1}^{C} \exp(\mathbf{w}_{c'}^\intercal \mathbf{x}_i) \right) \right]$$

Can do MLE for $\mathbf{W}$ similar to the binary logistic regression case.

Gradient Descent

Parameter estimation

Perhaps the simplest algorithm for unconstrained optimization is gradient descent, also known as steepest descent. This can be written as follows:

$$\theta_{k+1} = \theta_k - \eta_k \mathbf{g}_k$$

where η_k is the step size or learning rate.

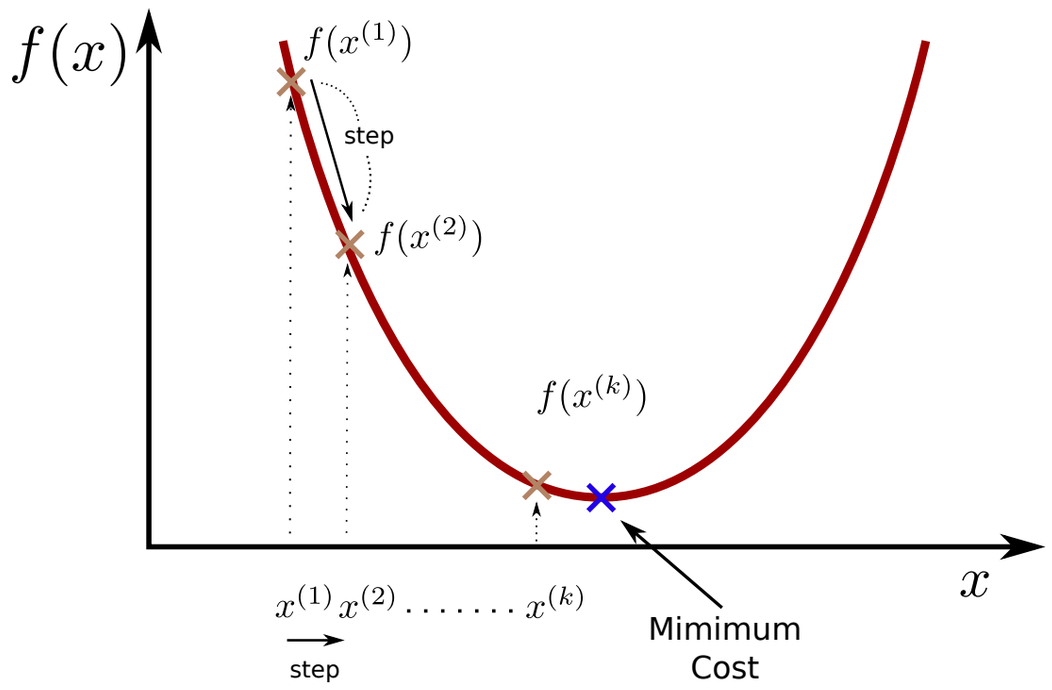


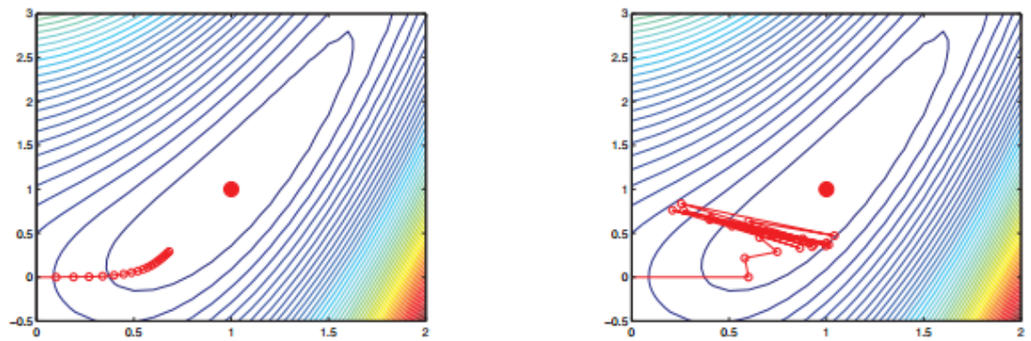
Figure: Illustration of gradient descent

Gradient Descent

The main issue in gradient descent is:

> How should we set the step size?

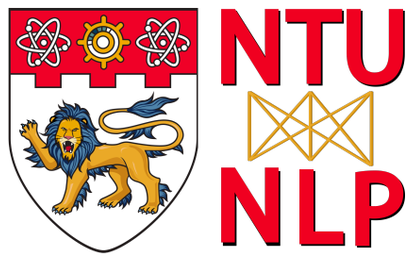
This turns out to be quite tricky. If we use a constant learning rate, but make it **too small**, convergence will be very slow, but if we make it **too large**, the method can fail to converge at all.



(a)

(b)

Gradient Descent

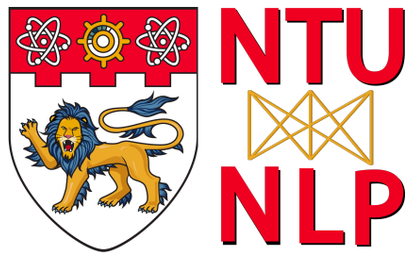


Several ways to remedy this.

- Choose the optimal step size η by line-search.
- Add a momentum term to the updates.
- Use methods such as Conjugate Gradient
- Use second-order methods (e.g., Newtons method) to exploit the **curvature of the objective function**. This requires the Hessian matrix.

Use Adaptive Methods with Stochastic Gradient Descent

Online Learning



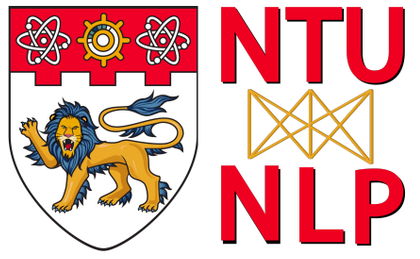
Traditionally machine learning is performed **offline**, which means we have a batch of data, and we optimize an equation of the following form

$$f(\theta) = \frac{1}{N} \sum_{i=1}^{N} f(\theta, \mathbf{z}_i)$$

where $\mathbf{z}_i = (\mathbf{x}_i, y_i)$ in the supervised case, or just $\mathbf{x}_i$ in the unsupervised case, and $f(\theta, \mathbf{z}_i)$ is some kind of loss function.

In **frequentist decision theory**, the average loss is called the Risk, so this overall approach is called Empirical Risk Minimization or ERM.

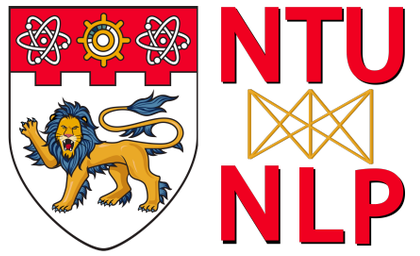
Online Learning



If we have streaming data, we need to perform online learning, so we can update our estimates as each new data point arrives rather than waiting until **the end** (which may never occur).

Even if we have a batch of data, we might want to treat it like a stream if it is too large to hold in main memory.

Online Learning

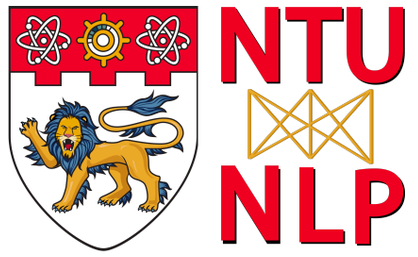


Suppose that at each step, "nature" presents a sample $\mathbf{z}_k$ and the "learner" must respond with a parameter estimate θ_k.

One simple algorithm for online learning is online gradient descent, which is as follows: at each step k, update the parameters using

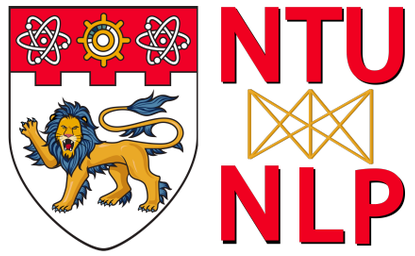
$$\theta_{k+1} = \mathit{proj}_{\Theta}(\theta_k - \eta_k \mathbf{g}_k)$$

Online Learning



Suppose we receive an infinite stream of samples from the distribution. **One way** to optimize stochastic objectives is to perform the update at each step. This is called stochastic gradient descent or SGD.

Online Learning



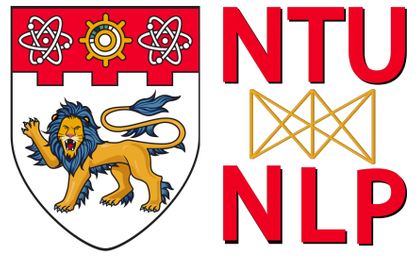
The need to adjust these tuning parameters is one of the **main drawback** of stochastic optimization.
One simple heuristic is as follows:

Store an initial subset of the data, and try a range of η values on this subset; then choose the one that results in the fastest decrease in the objective and apply it to all the rest of the data.

Early Stopping

Note that this may not result in convergence, but the algorithm can be terminated when the performance improvement on a hold-out set plateaus.

Online Learning



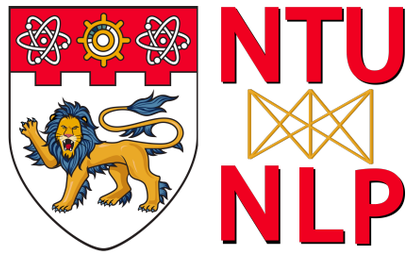
One drawback of SGD is that it uses the **same step size** for all parameters.
We now briefly present a method known as **adagrad** (short for adaptive gradient), which is similar in spirit to a diagonal Hessian approximation.

In particular, if $\theta_i(k)$ is parameter i at time k, and $\mathbf{g}_i(k)$ is its gradient, then we make an update as follows:

$$\theta_i(k+1) = \theta_i(k) - \eta \frac{\mathbf{g}_i(k)}{\tau_0 + \sqrt{s_i(k)}}$$

where the diagonal step size vector is the gradient vector squared, summed over all time steps.

Online Learning



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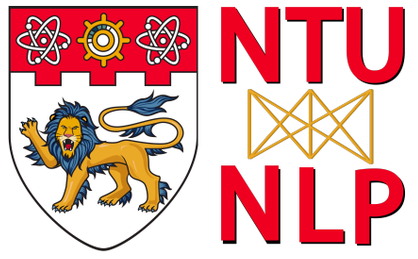
This can be recursively updated as follows:

$$s_i(k) = s_i(k-1) + \mathbf{g}_i(k)^2$$

The result is a per-parameter step size that adapts to the curvature of the loss function.

Online Learning

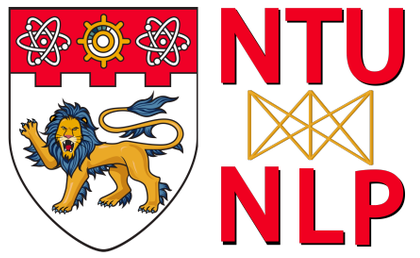


If we don't have an infinite data stream, we can simulate one by sampling data points at random from our training set.

In theory, we should **sample with replacement**, although in practice it is usually better to randomly permute the data and sample without replacement, and then to repeat.

A single such pass over the entire data set is called an **epoch**.

Online Learning



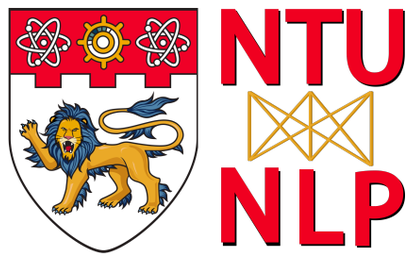
In this ofine case, it is often better to compute the gradient of a mini-batch of B data cases.

If $\mathbf{B = 1}$, this is ststandard SGD,

and if $\mathbf{B = N}$, this is standard steepest descent.

Typically B $\approx$ 100 is used.

Online Learning



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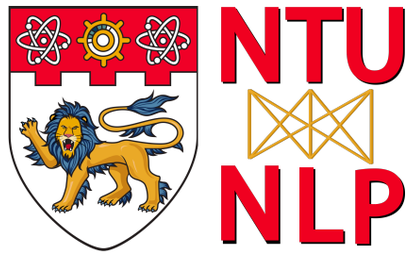
Although a simple first-order method, SGD performs surprisingly well on some problems, especially ones with **large data sets**.

Online Learning

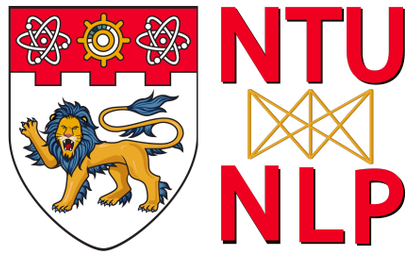


Although a simple first-order method, SGD performs surprisingly well on some problems, especially ones with **large data sets**.

The intuitive reason for this is that one can get a fairly good estimate of the gradient by looking at just a few examples.
Carefully evaluating precise gradients using large datasets is often a waste of time, since the algorithm will have to recompute the gradient again anyway at the next step.

It is often a better use of computer time to have a noisy estimate and to move rapidly through parameter space.

Online Learning

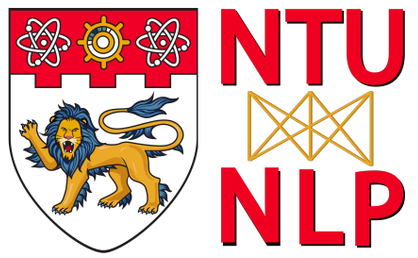


It is often a better use of computer time to have a noisy estimate and to move rapidly through parameter space.

As an extreme example, suppose we double the training set by duplicating every example.

Batch methods will take **twice** as long,
but *online methods* will be **unaffected**, since the direction of the gradient has not changed (*doubling the size of the data changes the magnitude of the gradient, but that is irrelevant, since the gradient is being scaled by the step size anyway*).

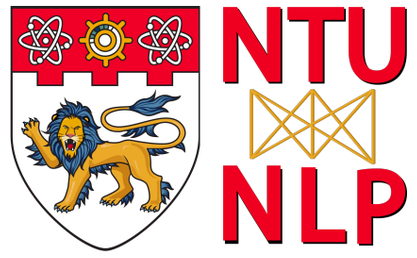
Online Learning



In addition to enhanced speed, SGD is often **less prone** to getting stuck in shallow local minima, **because** it adds a certain amount of "noise".

Consequently it is quite popular in the machine learning community for fitting models with non-convex objectives, such as neural networks and deep belief networks.

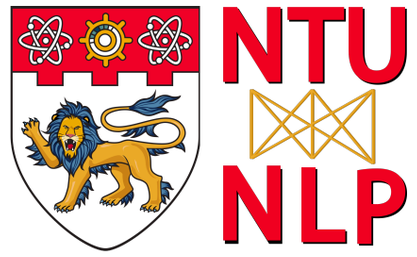
Online Learning



Algorithm 2 Stochastic Gradient Descent Algorithm

initialize θ, η
repeat
  Randomly permute data
  for $i = 1 : N$ **do**
    $\mathbf{g} = \nabla f(\theta, \mathbf{z}_i)$
    $\theta \leftarrow proj_{\Theta}(\theta - \eta \mathbf{g})$
    Update η
  end for
until *converged*

Online Learning



Let us consider how to compute the **MLE for linear regression** in an online fashion.
The online gradient at iteration k is given by

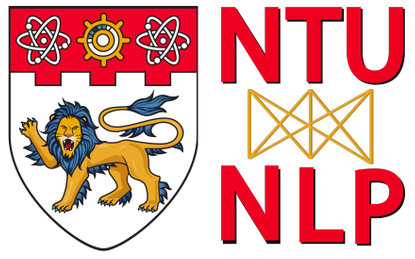
$$\mathbf{g}_k = \mathbf{x}_i(\theta_k^\mathsf{T}\mathbf{x}_i - y_i)$$

where $i = i(k)$ is the training example to use at iteration k.

After computing the gradient, we take a step along it as follows:

$$\theta_{k+1} = \theta_k - \eta_k(\hat{y_k} - y_k)\mathbf{x}_k$$

Online Learning



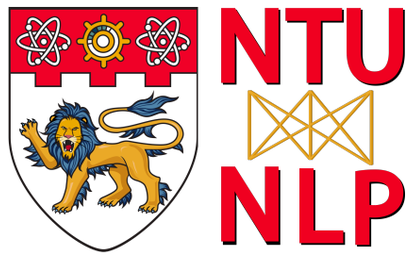
After computing the gradient, we take a step along it as follows:

$$\theta_{k+1} = \theta_k - \eta_k(\hat{y_k} - y_k)\mathbf{x}_k$$

Interpretation: it is the feature vector $\mathbf{x}_k$ weighted by the difference between what we predicted, $\hat{y_k} = \theta_k^\mathsf{T}\mathbf{x}_k$, and the true response, y_k; hence the gradient acts like an **error signal**.

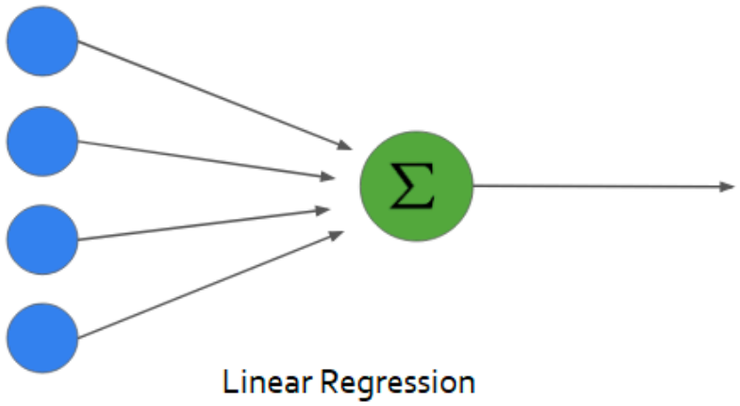
This algorithm is called the Least Mean Squares or LMS algorithm

Outline of Models

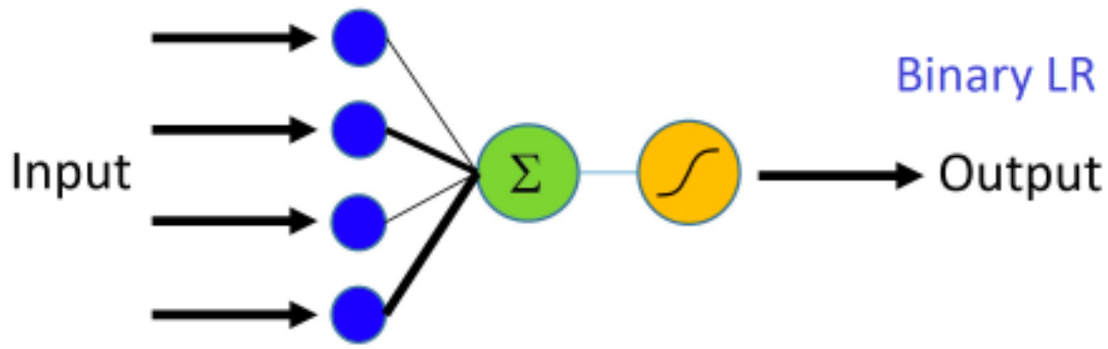


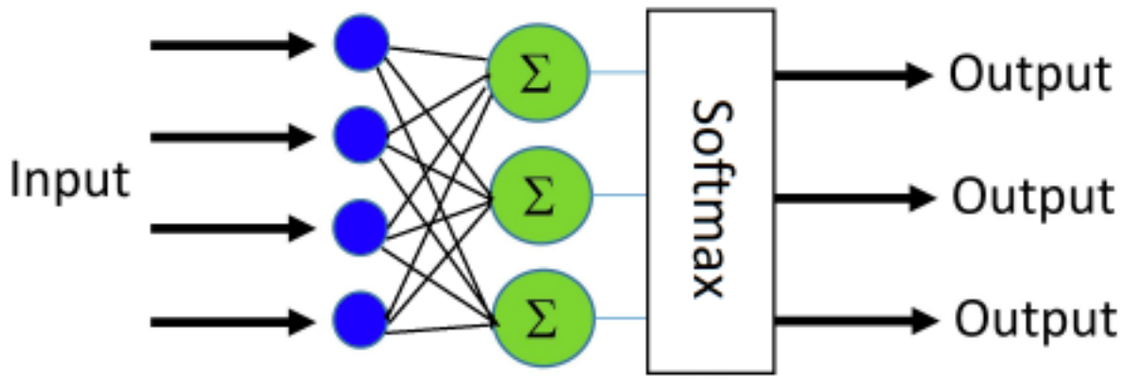
Linear Models	GD, SGD, L-BFGS
Feedforward NN/MLP	SGD + Backprop
Window-based Methods with MLP	SGD + Backprop
Convolutional Nets	SGD + Backprop
Recurrent Nets	SGD+BPTT
Recursive Nets	SGD+BPTS
Transformer Nets	SGD+Backprop

Summary



Linear Regression





SGD

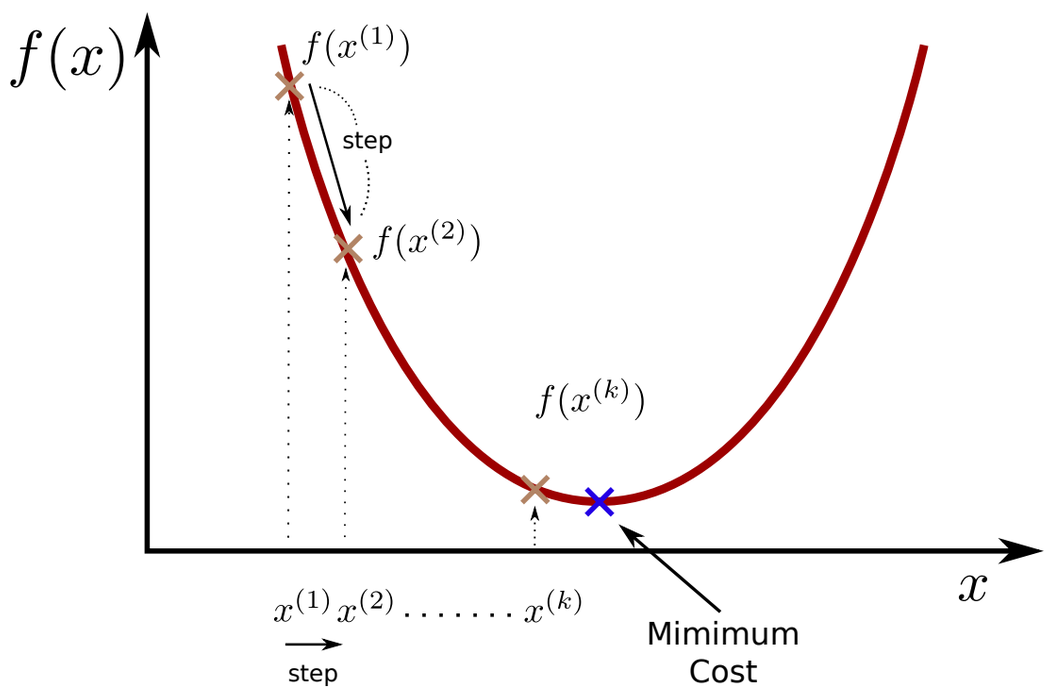


Figure: Illustration of gradient descent

$$\theta_{k+1} = \theta_k - \eta_k \mathbf{g}_k$$