

3.1 (a) 0.

$q_1 0 \sqcup$   
 $\sqcup q_2 \sqcup$   
 $\sqcup \sqcup q_{accept}$

(b) 00.

$q_1 0 0 \sqcup$   
 $\sqcup q_2 0 \sqcup$   
 $\sqcup x q_3 \sqcup$   
 $\sqcup q_5 x \sqcup$   
 $q_5 \sqcup x \sqcup$   
 $\sqcup q_2 x \sqcup$   
 $\sqcup x q_2 \sqcup$   
 $\sqcup x \sqcup q_{accept}$

(c) 000.

$q_1 0 0 0 \sqcup$   
 $\sqcup q_2 0 0 \sqcup$   
 $\sqcup x q_3 0 \sqcup$   
 $\sqcup x 0 q_4 \sqcup$   
 $\sqcup x 0 \sqcup q_{reject}$

(d) 000000.

$q_1 000000 \sqcup$   
 $\sqcup q_2 000000 \sqcup$   
 $\sqcup x 0 q_4 000 \sqcup$   
 $\sqcup x 0 x q_3 00 \sqcup$   
 $\sqcup x 0 x 0 q_4 0 \sqcup$   
 $\sqcup x 0 x 0 x q_3 \sqcup$   
 $\sqcup x 0 x 0 q_5 x \sqcup$   
 $\sqcup x 0 x q_5 0 x \sqcup$   
 $\sqcup x 0 q_5 x 0 x \sqcup$   
 $\sqcup x q_5 0 x 0 x \sqcup$   
 $\sqcup q_5 x 0 x 0 x \sqcup$   
 $q_2 \sqcup x 0 x 0 x \sqcup$   
 $\sqcup q_2 x 0 x 0 x \sqcup$   
 $\sqcup x q_2 0 x 0 x \sqcup$   
 $\sqcup x x q_3 x 0 x \sqcup$   
 $\sqcup x x x 0 q_4 x \sqcup$   
 $\sqcup x x x 0 x q_4 \sqcup$   
 $\sqcup x x x 0 x \sqcup q_{reject}$

3.2 (a) 11.

$q_1 11 \sqcup$   
 $x q_3 1 \sqcup$   
 $x 1 q_3 \sqcup$   
 $x 1 q_{reject} \sqcup$

(b)  $1\#1$ .

$q_1 1 \# 1 \sqcup$   
 $x q_3 \# 1 \sqcup$   
 $x \# q_5 1 \sqcup$   
 $x q_6 \# x \sqcup$   
 $q_7 x \# x \sqcup$   
 $x q_1 \# x \sqcup$   
 $x \# q_8 x \sqcup$   
 $x \# x q_8 \sqcup$   
 $x \# x \sqcup q_{accept}$

(c)  $1\#\#1$ .

$q_1 1 \# \# 1 \sqcup$   
 $x \# q_5 \# 1 \sqcup$   
 $x \# q_{reject} \# 1 \sqcup$

(d)  $10\#11$ .

$q_1 1 0 \# 1 1 \sqcup$   
 $x q_3 0 \# 1 1 \sqcup$   
 $x 0 q_3 \# 1 1 \sqcup$   
 $x 0 \# q_5 1 1 \sqcup$   
 $x 0 q_5 \# x 1 \sqcup$   
 $x 0 q_6 \# x 1 \sqcup$   
 $x q_7 0 \# x 1 \sqcup$   
 $q_7 x 0 \# x 1 \sqcup$   
 $x q_1 0 \# x 1 \sqcup$   
 $x x q_2 \# x 1 \sqcup$   
 $x x \# q_4 x 1 \sqcup$   
 $x x \# x q_4 1 \sqcup$   
 $x x \# x q_{reject} 1 \sqcup$

(e)  $10\#10$ .

$q_1 1 0 \# 1 0 \sqcup$   
 $x q_3 0 \# 1 0 \sqcup$   
 $x 0 q_3 \# 1 0 \sqcup$   
 $x 0 \# q_5 1 0 \sqcup$   
 $x 0 q_6 \# x 0 \sqcup$   
 $x q_7 0 \# x 0 \sqcup$   
 $q_7 x 0 \# x 0 \sqcup$   
 $x q_1 0 \# x 0 \sqcup$   
 $x x q_2 \# x 0 \sqcup$   
 $x x \# q_4 x 0 \sqcup$   
 $x x \# x q_4 0 \sqcup$   
 $x x \# q_6 x x \sqcup$   
 $x x q_6 \# x x \sqcup$   
 $x q_7 x \# x x \sqcup$   
 $x x q_1 \# x x \sqcup$   
 $x x \# q_8 x x \sqcup$   
 $x x \# x q_8 x \sqcup$   
 $x x \# x x q_8 \sqcup$   
 $x x \# x x \sqcup q_{accept}$

3.3 Solution in book.

3.4 An enumerator  $E$  can be defined as a 2-tape Turing machine with the transition function:

$$\delta : Q \times \Gamma_{tape} \longrightarrow Q \times \Gamma_{tape} \times \Gamma_{print} \times \{L, R, S\} \times \{R, S\}$$

The language enumerated by the enumerator is  $L \subseteq \Gamma_p^*$ .

3.5 Solution in book.

3.6 For any given  $S_i$ , there is no guarantee  $M$  will not loop (i.e. not accept or reject the input), meaning there is no guarantee the algorithm will complete.

3.7 The machine will not *accept* or *reject* on any input, i.e. will loop on all inputs.

3.8 (a) Solution in book.

(b) On input string  $w$ :

1. Scan the tape and mark the first two 0s that have not been marked. If no unmarked 0 is found, go to stage 4. Otherwise, move the head back to the front of the tape.
  2. Scan the tape and mark the first 1 that has not been marked. If no unmarked 1 is found, *reject*.
  3. Move the head back to the front of the tape and go to stage 1.
  4. Move the head back to the front of the tape. Scan the tape to see if any unmarked 1s remain. If none are found, *accept*, otherwise, *reject*.
- (c) On input string  $w$ :
1. Run the previously specified TM on input string  $w$ .
  2. If the machine accepts, then *reject*, otherwise *accept*.