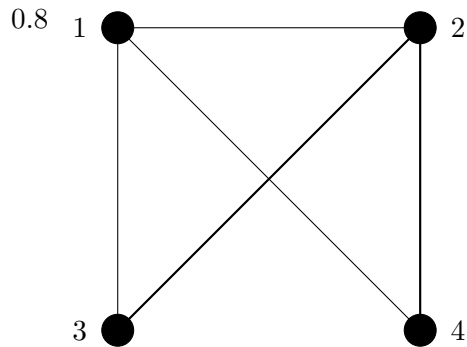


- 0.1 (a) Odd natural numbers.  
 (b) Even integers.  
 (c) Even natural numbers.  
 (d) Even natural numbers and positive multiples of three.  
 (e) Palendromes.  
 (f) Empty set.
- 0.2 (a)  $\{1, 10, 100\}$   
 (b)  $\{5, 6, 7, 8, \dots\}, \{n \in \mathbb{Z} \mid n > 5\}$   
 (c)  $\{0, 1, 2, 3, 4, 5\}, \{n \in \mathbb{N} \mid n < 5\}$   
 (d)  $\{\mathbf{aba}\}, \{w \mid w \text{ is the string } \mathbf{aba}\}$   
 (e)  $\{\epsilon\}, \{w \mid w \text{ is the empty string}\}$   
 (f)  $\emptyset$
- 0.3 (a) No.  
 (b) Yes.  
 (c)  $A$   
 (d)  $B$   
 (e)  $\{(x, x), (x, y), (y, x), (y, y), (z, x), (z, y)\}$   
 (f)  $\{\emptyset, \{x\}, \{y\}, \{x, y\}\}$
- 0.4 (a)  $a \times b$   
 (b)  $\sum_{k=0}^n \binom{n}{k} = 2^n$
- 0.5 (a) 7  
 (b)  $\{6, 7\}, \{1, 2, 3, 4, 5\}$   
 (c) 6  
 (d)  $\{6, 7, 8, 9, 10\}, \{1, 2, 3, 4, 5\} \times \{6, 7, 8, 9, 10\}$   
 (e)  $g(4, f(4)) = g(4, 7) = 8$
- 0.6 (a)  $\approx$   
 (b)  $\leq$   
 (c) *isAdjacent*
- 0.7  $\deg(1) = 3, \deg(2) = 3, \deg(3) = 2, \deg(4) = 2$



0.9  $G = (\{1, 2, 3, 4, 5, 6\}, \{(1, 4), (1, 5), (1, 6), (2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6)\})$

0.10 Take a graph  $G = (V, E)$ , where  $|V| \geq 2$ . To prove the statement true, we need to show it is not possible to construct  $G$  without two vertices having the same degree.

As each edge connects a pair of vertices, the set of all possible degrees for a given vertex  $v \in G$  is  $D = \{0, 1, \dots, (|V| - 1)\}$ . For the statement to be true, no two vertices may have the same degree, implying a bijection between  $D$  and  $V$ . Thus,  $V$  must include vertices  $v, u$  where  $D(v) = (|V| - 1)$ ,  $D(u) = 0$ . This is a contradiction, which completes the proof.

0.11 The proof only establishes that horses in  $H_1$  and  $H_2$  have the same colour when  $|H_1| = |H_2| = 1$ . It does not establish that horses in  $H_1 \cup H_2$  are the same colour.

0.12 (a) Prove  $S(n) = 1 + 2 + \dots + n = \frac{1}{2}n(n + 1)$ .

- Base case:

$$S(1) = \frac{1}{2}(1)(2) = 1$$

- Inductive case:

$$\begin{aligned} S(n) &= S(n - 1) + n \\ &= \frac{1}{2}(n - 1)(n - 1 + 1) + n \\ &= \frac{1}{2}(n^2 + n) \\ &= \frac{1}{2}n(n + 1) \end{aligned}$$

(b) Prove  $C(n) = 1^3 + 2^3 + \dots + n^3 = \frac{1}{4}n^2(n + 1)^2$ .

- Base case:

$$C(1) = \frac{1}{4}(1)^2(2)^2 = 1$$

- Inductive case:

$$\begin{aligned}
 C(n) &= C(n-1) + n^3 \\
 &= \frac{1}{4}(n-1)^2 n^2 + n^3 \\
 &= \frac{1}{4}(n^4 - 2n^3 + n^2) + n^3 \\
 &= \frac{1}{4}(n^4 + 2n^3 + n^2) \\
 &= \frac{1}{4}n^2(n+1)^2
 \end{aligned}$$

0.13 Division by  $a - b = 0$ .

0.14 Solution in book.

0.15 Solution in book.