7.1 (a) True.
(b) False.
(c) False.
(d) True.
(e) True.
(f) True.

7.2 (a) False.
(b) True.
(c) True.
(d) True.
(e) False.
(f) False.

7.3 Following Euclids algorithm:

(a) Yes.

$$\begin{aligned}
&GCD(10505, 1274) \\
&= GCD(1274, 313) \\
&= GCD(313, 22) \\
&= GCD(22, 5) \\
&= GCD(5, 2) \\
&= GCD(2, 1) \\
&= GCD(1, 1) \\
&= 1
\end{aligned}$$

(b) No.

$$\begin{aligned}
&GCD(8024, 7289) \\
&= GCD(7289, 740) \\
&= GCD(740, 629) \\
&= GCD(629, 111) \\
&= GCD(111, 74) \\
&= GCD(74, 37) \\
&= 37.
\end{aligned}$$

7.4

7.5 Let $\phi = (x \vee y) \wedge (x \vee \bar{y}) \wedge (\bar{x} \vee y) \wedge (\bar{x} \vee \bar{y})$. The following truth-table shows that ϕ is not satisfiable.

x	y	$(x \vee y)$	$(x \vee \bar{y})$	$(\bar{x} \vee y)$	$(\bar{x} \vee \bar{y})$	ϕ
0	0	0	1	1	1	0
0	1	1	0	1	1	0
1	0	1	1	0	1	0
1	1	1	1	1	0	0

7.6 Let $\langle L_i, M_i\rangle$ be a polytime language and decider, such that $L_i = \{w \mid M_i\langle w\rangle \text{ accepts}\}$, and M_i always halts in polynomial time.

Let $L_\cup = L_i \cup L_j$, $L_\circ = L_i \circ L_j$, $\bar{L_i} = \{w \in \Sigma^* \mid w \notin L_i\}$ be languages. To show that $L_\cup, L_\circ, \bar{L_i} \in$ P, we construct respective polynomial time deciders $M_\cup, M_\circ, \bar{M_i}$.

$M_\cup$ = "On input w:

1. Run $M_i\langle w\rangle$. If M_i accepts, *accept*.
2. Run $M_j\langle w\rangle$. If M_j accepts, *accept*.
3. If neither $M_i\langle w\rangle$ nor $M_j\langle w\rangle$ accepted, *reject*."

$M_\circ$ = "On input w:

1. For each position $k = 0$ to $|w|$, divide w into substrings $w = w_1 w_2$, where w_1 is the first k symbols in w.
2. Run $M_i\langle w_1\rangle$ and $M_j\langle w_2\rangle$. If both accept, *accept*.
3. If no k exists such that $M_i\langle w_1\rangle$ and $M_j\langle w_2\rangle$ both *accept*, *reject*."

$\bar{M_i}$ = "On input w:

1. Run M_i on w. If M_i accepts *reject*. If M_i rejects, *accept*.