

- 4.1 (a) Yes. M accepts 0100.
 (b) No. M rejects 011.
 (c) No. No input string is given.
 (d) No. R is not a DFA.
 (e) No. $L(M)$ is not empty.
 (f) Yes. By definition, $L(M) = L(M)$.

4.2 We define the language EQ_{DR} as

$$EQ_{DR} = \{\langle A, R \rangle \mid A \text{ is a DFA, } R \text{ is a regular expression, and } L(A) = L(R)\}$$

To show EQ_{DR} is decidable, we construct Turing machine M which decides EQ_{DR} .

$M =$ “On input $\langle A, R \rangle$

1. Construct DFA B which recognises R (see theorem 1.39).
2. Run $EQ_{DFA}\langle A, B \rangle$ (see theorem 4.5). If $EQ_{DFA}\langle A, B \rangle$ accepts, *accept*, else *reject*.”

4.3 To show ALL_{DFA} is decidable, we construct Turing machine M which decides ALL_{DFA} :

$M =$ “On input $\langle A \rangle$, where A is a DFA

1. Construct DFA A' by running algorithm *MINIMIZE* on A (see problem 7.25).
2. If A' consists of a single accepting state, *accept*, else *reject*.”