

- 7.1 (a) True.
(b) False.
(c) False.
(d) True.
(e) True.
(f) True.

- 7.2 (a) False.
(b) True.
(c) True.
(d) True.
(e) False.
(f) False.

7.3 Following Euclids algorithm:

- (a) Yes.

$$\begin{aligned} &GCD(10505, 1274) \\ &= GCD(1274, 313) \\ &= GCD(313, 22) \\ &= GCD(22, 5) \\ &= GCD(5, 2) \\ &= GCD(2, 1) \\ &= GCD(1, 1) \\ &= 1 \end{aligned}$$

- (b) No.

$$\begin{aligned} &GCD(8024, 7289) \\ &= GCD(7289, 740) \\ &= GCD(740, 629) \\ &= GCD(629, 111) \\ &= GCD(111, 74) \\ &= GCD(74, 37) \\ &= 37. \end{aligned}$$

7.4

7.5 Let $\phi = (x \vee y) \wedge (x \vee \bar{y}) \wedge (\bar{x} \vee y) \wedge (\bar{x} \vee \bar{y})$. The following truth-table shows that ϕ is not satisfiable.

x	y	$(x \vee y)$	$(x \vee \bar{y})$	$(\bar{x} \vee y)$	$(\bar{x} \vee \bar{y})$	ϕ
0	0	0	1	1	1	0
0	1	1	0	1	1	0
1	0	1	1	0	1	0
1	1	1	1	1	0	0

7.6 Let $\langle L_i, M_i \rangle$ be a polytime language and decider, such that $L_i = \{w \mid M_i\langle w \rangle \text{ accepts}\}$, and M_i always halts in polynomial time.

Let $L_{\cup} = L_i \cup L_j$, $L_{\circ} = L_i \circ L_j$, $\bar{L}_i = \{w \in \Sigma^* \mid w \notin L_i\}$ be languages. To show that $L_{\cup}, L_{\circ}, \bar{L}_i \in P$, we construct respective polynomial time deciders $M_{\cup}, M_{\circ}, \bar{M}_i$.

M_{\cup} = “On input w :

1. Run $M_i\langle w \rangle$. If M_i accepts, *accept*.
2. Run $M_j\langle w \rangle$. If M_j accepts, *accept*.
3. If neither $M_i\langle w \rangle$ nor $M_j\langle w \rangle$ accepted, *reject*.”

M_{\circ} = “On input w :

1. For each position $k = 0$ to $|w|$, divide w into substrings $w = w_1w_2$, where w_1 is the first k symbols in w .
2. Run $M_i\langle w_1 \rangle$ and $M_j\langle w_2 \rangle$. If both accept, *accept*.
3. If no k exists such that $M_i\langle w_1 \rangle$ and $M_j\langle w_2 \rangle$ both *accept*, *reject*.”

\bar{M}_i = “On input w :

1. Run M_i on w . If M_i accepts *reject*. If M_i rejects, *accept*.

7.7 Let $\langle L_i, N_i \rangle$ be a language and non-deterministic Turing machine, such that N_i decides L_i in polynomial time.

Let $L_{\cup} = L_i \cup L_j$, $L_{\circ} = L_i \circ L_j$ be languages. To show that $L_{\cup}, L_{\circ} \in P$, we construct respective polynomial time non-deterministic deciders.

N_{\cup} = “On input w :

1. Non-deterministically branch to simulate both $N_i\langle w \rangle$ and $N_j\langle w \rangle$.
2. If either branch accepts, *accept*, else *reject*.”

N_{\circ} = “On input w :

1. Non-deterministically branch for each position $k = 0$ to $|w|$, with each branch dividing w into substrings $w = w_1w_2$, where w_1 is the first k symbols in w .
2. For each branch, run $N_i\langle w_1 \rangle$ and $N_j\langle w_2 \rangle$. If both accept, *accept*.
3. If no branch accepts, *reject*."