Logical Vectors

Week 5

Loosely follows Chapter 5

Logical Vectors

When comparing vectors, a logical vector results

Essentially an element-wise comparison operation

```
a = 1:5;
b = [0 2 3 5 6];
a == b

ans =
        [ 0 1 1 0 0 ]
```

Avoid Division By Zero

- With limits, can encounter zeros
- We want to “simulate” a zero
- MATLAB will give NaN

Avoid Division By Zero Example

x = -4*pi : pi / 20 : 4*pi;

y = sin(x) ./ x;

Where x=0 you'll receive NaN in y

logicalVector = x == 0;

The fix:

x = x + logicalVector*eps;

OR

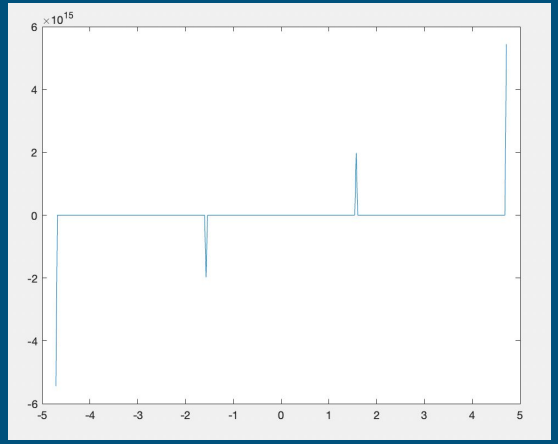
x(logicalVector)=eps;

Avoid Infinity

- With limits, encounter near infinity
- We want to simulate “infinity”
- This can throw plots off drastically

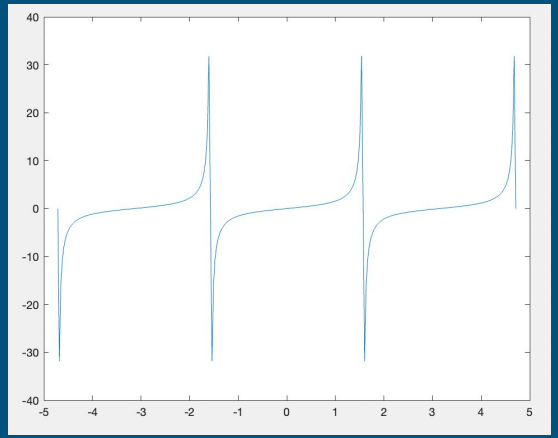
Avoid Infinity Example

```
x = -3/2*pi : pi/100 : 3/2*pi;

y = tan(x);

plot(x, y)
```



The fix:

```
y = y .* (abs(y) < 1e10);
```



Counting and Comparing Matches Example

```
r = rand(1,7)

r =

    0.8147    0.9058    0.1270    0.9134    0.6324    0.0975    0.2785

sum( r < 0.5 )

ans =

     3
```

Behind the scenes:

```
r < 0.5

ans =

  1×7 logical array

   0   0   1   0   0   1   1
```

Counting and Comparing Matches

all()

any()

sum()

Relational Operations

- Perform element-wise comparisons between two arrays
- They return a logical array of the same size
  - Element values will be 1 or 0 for true or false respectively

Relational Operators for Logical Operations

- `<` Less than
- `>` Greater than
- `==` Equal to (logical equality)
- `~` Not
- `<=` Less than or equal to (≤)
- `>=` Greater than or equal to (≥)
- `~=` Not equal to (≠)
- `xor()` Exclusive OR

Logical Data Type

- The logical data type is another MATLAB data type.
- It is either equal to 1 (true) or 0 (false)

Input	Output (ans =)	Data Type
4 < 5	1	Logical Array
4 <= 5	1	Logical Array
4 > 5	0	Logical Array
4 >= 5	0	Logical Array
4 ~= 5	1	Logical Array

Round-off Error

- The == and ~= commands can produce confusing results due to round-off error.
- Example

```
a = 0;
b = sin(pi);
a == b

ans =
        0
```

Round-off Error

What happened?

- MATLAB computes sin(pi) to within eps. (A number approximately zero)
- The == operator properly concludes that 0 and a number which is approximately zero are different.
- The best way to fix this is to show things to be approximately equal.

```
abs(a-b) < 1.0e-14
ans =
        1
```

Logical Operators

- Logical, or Boolean, operators produce a logical result

Operator	Operation
&	Logical AND
|	Logical OR
~	Logical NOT

Logical Operators

- Example
  - a = true;
  - b = false;
  - c = true;
- What are the results of the following
  - a & b
  - b & c
  - a & c
  - a | b
  - b | c
  - a | c
  - c | a
  - b | b

Hierarchy of Operations

- Logical operators are evaluated **after** arithmetic and relational operations.

Order of Operations

1. Arithmetic
2. Relational Operators
3. All ~ (NOT) operators
4. All & (AND) operators
5. All | (OR) operators

Example

You need the **full** condition for each condition you wish to meet.

60 <= finalExam < 70

(60 <= finalExam) & (finalExam < 70)

(a ~= 0) | (b ~= 0) | (c ~= 0)

~((a == 0) & (b == 0) & (c == 0))

Logical Functions

- Functions in MATLAB that implement logical operations
- For example
  - The find() function returns the index numbers of the matrix that meet the logical statement given.

```
v = [1, 3, 5, 6, 3, 4; 6, 36, 6, 7, 843, 5];
[row, col] = find(v == 3)
row =
        1
        1
col  =
        2
        5
```

find()

We can use functions to manipulate data more efficiently.

Get a subscript of all non-zero values in a

find(a)

Make a vector containing only non-zero values in a

a = a(find(a))

Logical Functions

- Other examples
  - ischar() returns 1 if ( ) contains character data
  - isinf() returns 1 if ( ) contains infinity (inf)
  - isnan() returns 1 if ( ) contains not a number (nan)
  - isnumeric() returns 1 if ( ) contains numeric data
  - isempty() returns 1 if ( ) is empty (x = [ ])
  - any() returns 1 if any element in ( ) is non-zero
  - all() returns 1 if all elements in ( ) are non-zero
  - exist() returns 1 if ( ) exists as a workspace variable

any() and all()

Can be useful in if statements

```
if all(a >= 1)
    % do something
end
```

Matrices

Week 5

Loosely follows Chapters 6

Subscripts

- Elements within a matrix are referenced with two subscripts.

- matrix(3) Returns third row, first column (Different for vectors)
- matrix(3,1) Returns third row, first column
- matrix(1:3, 4:6) First through third row of fourth through sixth column
- matrix(:, 4:6) All rows in fourth through sixth column

Subscripts

- Elements referenced in subscripts can also be written to using the assignment operator.
- matrix(1:2, 2:3) = ones(2) sets values to ones

- The keyword end can be used to reference the last element in a row or column.
- matrix(:, 2:end) Retrieves all elements but the first column.

Duplicating Rows and Columns

- Sometimes referred to as tiling
- repmat() allows you to repeat a matrix

```
vectorA = [ 1, 2, 3 ];
repmat(vectorA, 3, 1)          % Or repmat(vectorA, [3, 1])
ans =
     1     2     3
     1     2     3
     1     2     3
```

Extracting Rows and Columns

- Logical arrays can be used to extract rows or columns
- a(:, logical([1 0 1])) Extracts the first and third columns

- Positional values in an array can also be used
- a(:, [1 3]) Performs the same as above

Elementary Matrices

- There are a set of functions for generating elementary matrices
  - ones()
  - zeros()
  - rand()
  - eye()
- See more with the elmat command

Special Matrices

- There are a set of functions for generating special matrices
  - pascal()
  - magic()
  - gallery()
  - hankel()
- For more use the help elmat command

Matrices and Functions

- Most functions perform actions by column
  - all() returns 1 for each column where all elements are non-zero
  - any() returns 1 for each column where any elements are non-zero
  - sum() returns a vector with the sum of each column of elements
  - mean() returns a vector with the mean of each column of elements
- To perform a function on a matrix and obtain a scalar value, functions must be nested or performed in steps.

```
colSum = sum(matrix)
totalSum = sum(colSum)
```

```
totalSum = sum(sum(matrix))
```

Manipulating Matrices

- There are many functions for easily transforming matrices.
  - diag()
  - flipud()
  - fliplr()
  - rot90()
  - tril()
- For more see help
- Think of ways you can achieve different transformations by calling multiple functions.
- triu() == rot90(rot90(tril(rot90(rot90(matrix)))))

Matrix Multiplication

- Remember matrixA * matrixB is different from matrixA .* matrixB
- Matrix Multiplication steps
  - Multiply each column vector by each row vector and sum the results

$$\begin{bmatrix}1 & 2 & 3\end{bmatrix} \times \begin{bmatrix}4\\5\\6\end{bmatrix} = 1 \times 4 + 2 \times 5 + 3 \times 6 = 22$$

$$\begin{bmatrix}1 & 2\\3 & 4\\5 & 6\end{bmatrix} \times \begin{bmatrix}1 & 3 & 5 & 7\\2 & 4 & 6 & 8\end{bmatrix} = \begin{bmatrix}1*1+2*2 & 1*3+2*4 & 1*5+2*6 & 1*7+2*8\\3*1+4*2 & 3*3+4*4 & 3*5+4*6 & 3*7+4*8\\5*1+6*2 & 5*3+6*4 & 5*5+6*6 & 5*7+6*8\end{bmatrix}$$

$$= \begin{bmatrix}1+4 & 3+8 & 5+12 & 7+16\\3+8 & 9+16 & 15+24 & 21+32\\5+12 & 15+24 & 25+36 & 35+48\end{bmatrix} = \begin{bmatrix}5 & 11 & 17 & 23\\11 & 25 & 39 & 53\\17 & 39 & 61 & 83\end{bmatrix}$$

Matrix Multiplication Rules

- Matrix multiplication is not commutative (A*B ~= B*A)
- Columns of matrix A must match rows of matrix B
  - Resulting matrix is rowsXcols (the remaining values)
- To multiply a matrix by a vector in that respective order, the vector must be a column vector
  - Remember, cols of matrix A must match rows of matrix B

Matrix Exponentiation

- Matrix operation A^2 == A*A
- The matrix must be a square matrix
  - Think about why