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HW4

All work herein is mine.

1. a. $\exists_y\ p(y) \vee [\exists_y (q(y) \Rightarrow (\exists_x (p(x) \vee q(x,y,c))))]$

$\exists_y\ p(y) \vee [\exists_y\ \neg q(y) \vee \exists_x\ p(x) \vee q(x,y,c))]$

$\exists_y\ p(y) \vee [\exists_z\ \neg q(z) \vee \exists_x\ p(x) \vee q(x,z,c))]$

$\exists_y\ p(y) \vee \exists_z \exists_x\ q(z) \vee p(x) \vee q(x,z,c))$

$\underset{e_1}{\exists_y}\ \underset{e_2}{\exists_z}\ \underset{e_3}{\exists_x}\ p(y) \vee q(z) \vee p(x) \vee q(x,z,c)$

$p(e_1(y)) \vee q(e_2(z)) \vee p(e_3(x)) \vee q(e_3(x), e_2(z), C)$

$\{\{p(e_1(y)), q(e_2(z))\}, \{q(e_3(x)), q(e_3(x), e_2(z), C)\}\}$

b. $\forall_x \forall_y \forall_z\ d(x,y) \wedge d(y,z) \rightarrow d(x,z)$

$\forall_x \forall_y \forall_z\ \neg(d(x,y) \wedge d(y,z)) \vee d(x,z)$

$\underset{f_1}{\forall_x}\ \underset{f_2}{\forall_y}\ \underset{f_3}{\forall_z}\ \neg d(x,y) \vee \neg d(y,z) \vee d(x,z)$

$\neg d(f_1(x), f_2(y)) \vee \neg d(f_2(y), f_3(z)) \vee d(f_1(x), f_3(z))$

$\{\{\neg d(f_1(x), f_2(y)), \neg d(f_2(y), f_3(z))\}, \{d(f_1(x), f_3(z))\}\}$

c. $(P \vee Q) \wedge (\neg P \Rightarrow (Q \vee R))$

$(P \vee Q) \wedge (\cancel{\neg\neg} P \vee (Q \vee R))$

$\cancel{(P \vee Q)} \wedge (P \vee Q) \vee (P \vee R)$

$(P \vee Q) \vee (P \vee R)$

$\{\{P, Q\}, \{P, R\}\}$

2. a. T b. F c. F d. F e. F f. F → logic expanded below

3. a. $\mathcal{M}(x)\ p(x)$

$p(1) = T$
$p(2) = F$
$p(3) = T$
$= 2$

b. $\forall_x\ \mathcal{M}_y\ m(y,x) \Rightarrow p(x)$

$m(1,1) \Rightarrow T = T$
$m(2,1) \Rightarrow T = T$
$m(3,1) \Rightarrow T = T$
$m(1,2) \Rightarrow F = T$
$m(2,2) \Rightarrow F = T$
$m(3,2) \Rightarrow F = F$
$m(1,3) \Rightarrow T = T$
$m(2,3) \Rightarrow T = T$
$m(3,3) \Rightarrow T = T$
$= 8$

4. $\exists_x \exists_y\ (gray(x) \wedge silver(y) \wedge loves(x,y))$

$gray(dog(x)) \wedge silver(dog(y)) \wedge loves(dog(x), dog(y))$

This sentence say there exists 2 dogs, one gray and the other silver, and the gray dog loves the silver dog.

C: Kaiser is either gray or silver and loves Ursula

$(gray(dog(x)) \vee silver(dog(x)) \wedge loves(dog(x), dog(y))$

$gray(dog(x)) \wedge silver(dog(y)) \wedge loves(dog(x), dog(y)) \wedge \neg(gray(dog(x)) \vee silver(dog(x)) \wedge loves(dog($

$gray(dog(x)) \wedge silver(dog(y)) \wedge loves(dog(x), dog(y)) \wedge \neg gray(dog(x)) \wedge \neg silver(dog(x)) \wedge \neg loves(dog($

$silver(dog(y)) \wedge \neg silver(dog(x))$

We know that this is satisfiable due to the negated terms.

2 (expanded).

a. $(1, \exists x)$, $(2, \exists x)$, $(3, \exists x)$

b. $(x, 1)$, $(x, 2)$, ~~$(x, 3)$~~

c. $(1,1)$, ~~$(2,2)$~~, ~~$(3,3)$~~

d. $(1, x)$, $(2, x)$, ~~$(3, x)$~~

e. $(x, 1)$, $(x, 2)$, $(x, 3)$

f. $(1,1)$, ~~$(2,2)$~~, ~~$(3,3)$~~

5. FOL

a. $(p(x) \Rightarrow g(x)) \Rightarrow (np(x) \Rightarrow r(x))$

b. $\forall_x \ g(x) \vee r(x)$

c. $\exists_x \ np(x) \Rightarrow (p(x) \Rightarrow g(x))$

d. $p(O_1) \Rightarrow T$

e. $p(O_2) \Rightarrow F$

Robot

a. [V [∧, p(x), g(x)], [V, [¬, np(x)], r(x)]

b. ~~[V, [¬, [np(x)]], [r(x)]]~~ [forall, x, [V, [g(x)], [r(x)]]]

c. [exists, x, [V, [¬, [np(x)]], [V [¬, [p(x)]], [g(x)]]]]

d. [V, [¬, [p(O₁)]], T]

e. [V, [¬, [p(O₂)]], F]

Negated conclusion: [¬, [exists, x, [r(x)]]]

$a \wedge b \wedge c \wedge \cancel{d}^{\,a\ T} \wedge \cancel{e}^{\,a\ F} \wedge nqc \Rightarrow F$

6. a. $\neg M(B)$

b. $(M(A) \vee M(B)) \wedge N(C)$

c. $I(B, A, C) \Rightarrow N(C)$

d. $\exists_x M(x) \wedge \forall_x N(x)$

e. $I(B, B, C)$

f. $\neg \exists_x \exists_y I(x, y, A)$

g. $\exists_x I(x, B, A) \Rightarrow \neg N(B)$

h. $\forall_x \neg \exists_y \ W(x, y)$